Introduction to Heat Transfer

Chapter XII : Introduction to Heat Transfer

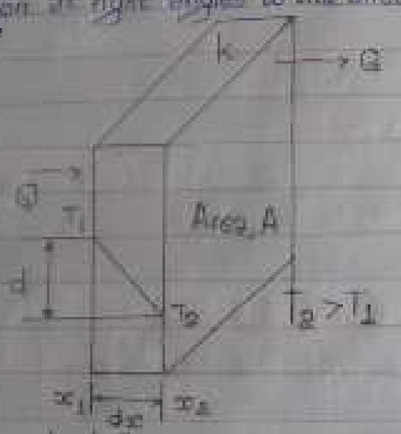
Basic Concepts and Modes of Heat Transfer

Heat transfer is defined as the transmission of energy from one region to [illegible] as a result of temperature difference between them.

1. Conduction:- The mode of heat transfer between two bodies and two parts of the same body through molecules which are more or less sta-tionary is known as conduction.

According to Fourier's Law of Conduction.

"The rate of flow of heat through a simple homogeneous solid is directly proportional to the area of the section at right angles to the direction of heat flow, and the temperature gradient"



$$Q \propto A \cdot \frac{dT}{dx}$$

$$Q = -k \cdot A \cdot \frac{dT}{dx}$$

or, $$\boxed{q = \frac{Q}{A} = -k \cdot \frac{dT}{dx}}$$

where, Q = rate of heat flow

A = area $\perp^{ar}$ to the direction of heat flow

$\frac{dT}{dx}$ = temperature gradient

k = constant of proportionality known as thermal conductivity of the body

q = heat flux

The -ve sign indicates that the heat flow is towards negative temperature gradient i.e. towards the lower temperature region.

2. Convection: Convection is defined as a process of heat transfer by the combined action of heat conduction and actual movement of the fluids.

- Convection requires a medium for heat transfer.

Radiation :- It is the phenomenon of heat transfer from the source to the receiver without heating the intervening medium i.e. medium is not required for heat transfer.

Electrical Analogy for Thermal Resistance.

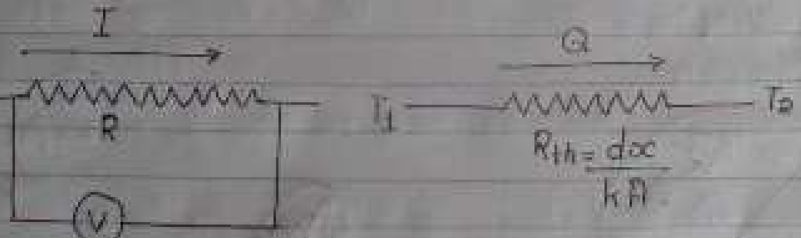
From Ohm's Law,

$$\text{Current } (I) = \frac{\text{Voltage Potential } (dV)}{\text{Electric Resistance } (R_e)}$$

And, from Fourier's Law,

$$\text{Heat flow rate } (Q) = \frac{\text{Temperature Potential } (dt)}{\frac{dx}{kA}}$$

- Electric current (amperes) is analogous to thermal heat flow rate (kW).
- Electric voltage (volts) is analogous to thermal temperature difference (deg. Celsius).
- Electric resistance (Ohms) is analogous to quantity $\frac{dx}{kA}$. This quantity is called thermal resistance.



Electrical System   Equivalent thermal circuit

$$\therefore \text{Thermal Resistance } (R_{th}) = \frac{dx}{kA}$$

One-dimensional steady state heat conduction through a plane wall/flat plate.

Let us consider a plane wall having thickness "L", inside temperature T_1 and outside temperature T_2.

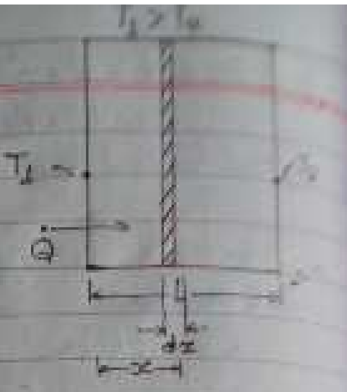
We have, $\dot{Q} = -KA \cdot \dfrac{dT}{dx}$

(Fourier law)

or, $\dot{Q} \cdot dx = -KA \cdot dT$

Integrating both sides,

$$\dot{Q} \cdot \int_0^L dx = -KA \int_{T_1}^{T_2} dT$$



or, $\dot{Q}(L-0) = -KA(T_2 - T_1)$

$\therefore \dot{Q} = \dfrac{-KA(T_2 - T_1)}{L}$

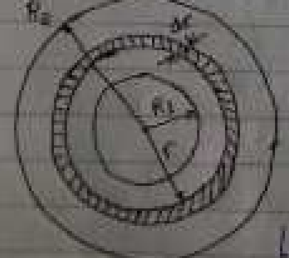
$\Rightarrow \dot{Q} = \dfrac{KA(T_1 - T_2)}{L}$

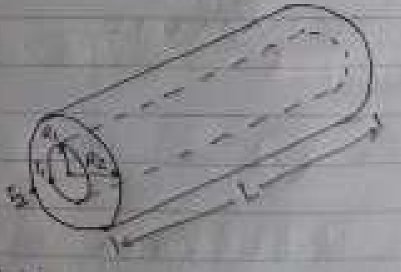
$$\dot{Q} = \frac{T_1 - T_2}{\left(L/KA\right)}$$

$\therefore$ Thermall resistance $(R_{th}) = \dfrac{L}{KA}$

Radial Steady State Heat Conduction through a Hollow Cylinder

Let us consider a hollow cylinder having inner radius R_1 and outer radius R_2 and length "L".





Let at $R = R_1$, $T = T_1$ and

$R = R_2$, $T = T_2$.

Using Fourier's law, $\dfrac{\dot{Q}}{A} \propto \dfrac{dT}{dr}$

$\Rightarrow \dot{Q} = -KA \dfrac{dT}{dr}$ $\Rightarrow \dot{Q} = -K \cdot 2\pi r L \cdot \dfrac{dT}{dr}$

$$\Rightarrow \dot{Q} \int_{R_1}^{R_2} \frac{dr}{r} = -K \cdot 2\pi L \int_{T_1}^{T_2} dT$$

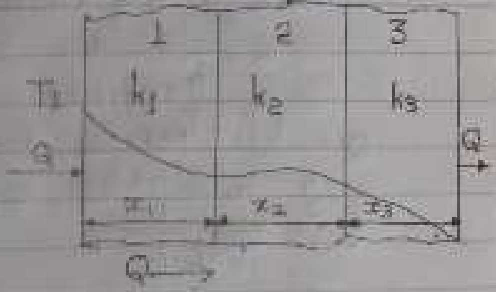
or, $\dot{Q} \cdot \log_e \dfrac{R_2}{R_1} = -2\pi kL(T_2 - T_1)$

$$\therefore \dot{Q} = \frac{(T_1 - T_2)}{\dfrac{\log_e R_2/R_1}{2\pi kL}} = \frac{T_1 - T_2}{R_{th}}$$

where, R_{th} = Thermal Resistance $= \dfrac{\log_e R_2/R_1}{2\pi kL}$.

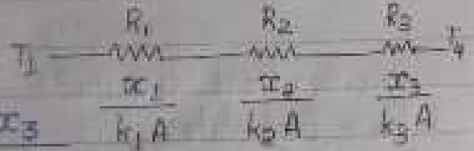
Heat Flow Through Composite Structures

1. One Dimensional Steady State Heat Conduction through a Composite Wall



- The three plane walls are of identical cross-section.
- The walls have different thicknesses – x_1, x_2, x_3 and different thermal conductivities – k_1, k_2, k_3

- $T_1 > T_4$. T_2 and T_3 are interface temperatures.



$$R_{t1} = \frac{x_1}{k_1 A} ;\; R_{t2} = \frac{x_2}{k_2 A} ;\; R_{t3} = \frac{x_3}{k_3 A}$$

$$\dot{Q} = \frac{k_1 A(T_1 - T_2)}{x_1} = \frac{k_2 A(T_2 - T_3)}{x_2} = \frac{k_3 A(T_3 - T_4)}{x_3}$$

or, $T_1 - T_2 = \dfrac{\dot{Q} x_1}{k_1 A}$; $T_2 - T_3 = \dfrac{\dot{Q} x_2}{k_2 A}$; $T_3 - T_4 = \dfrac{\dot{Q} x_3}{k_3 A}$

Now, for the overall temp. difference, we have,

$$T_1 - T_4 = (T_1 - T_2) + (T_2 - T_3) + (T_3 - T_4)$$

or, $T_1 - T_4 = \dot{Q}\left(\dfrac{x_1}{k_1 A} + \dfrac{x_2}{k_2 A} + \dfrac{x_3}{k_3 A}\right)$

or, $\dot{Q} = \dfrac{(T_1 - T_4)}{\dfrac{x_1}{k_1 A} + \dfrac{x_2}{k_2 A} + \dfrac{x_3}{k_3 A}}$

$$\therefore \boxed{\dot{Q} = \frac{(T_1 - T_4)}{R_{t1} + R_{t2} + R_{t3}} = \frac{T_1 - T_4}{R_{equivalent}}}$$

where $R_{equivalent} = R_{t1} + R_{t2} + R_{t3}$.

For a composite wall made up of a number of plane walls.

$$\dot{Q} = \frac{(T_1 - T_{n+1})}{\sum_{i=1}^{n} \frac{x_i}{k_i A}}$$

2. Radial Steady State Heat Conduction through a Multilayer Tube.

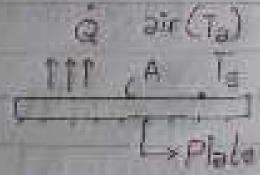
Proceeding in a similar way as described for a composite wall,

$$\dot{Q} = \frac{T_1 - T_4}{R_{equivalent}}$$

$$\dot{Q} = \frac{(T_1 - T_{n+1})}{\sum_{i=1}^{n} \frac{\ln\left(\frac{r_{i+1}}{r_i}\right)}{2\pi k_i L}}$$ for a composite cylinder made up of n 'number' of cylinders.

Convection Heat Transfer

1. For a flat plate



$$\frac{Q}{A} \propto dT \text{ (Newton's Law of Cooling)}$$

$$\Rightarrow \dot{Q} = hA(T_s - T_A)$$

$$\therefore \dot{Q} = \frac{T_s - T_A}{1/hA} = \frac{T_s - T_A}{R_{th}}$$

h = convective heat transfer coefficient

2. For Cylinder

$$\dot{Q} = hA(T_s - T_A)$$

$$\therefore \dot{Q} = \frac{T_s - T_A}{1/hA} \quad \therefore \dot{Q} = \frac{T_s - T_A}{R_{th}}$$

Overall Heat Transfer Coefficient for Plane Wall and Tube

The heat transfer rate could be expressed as

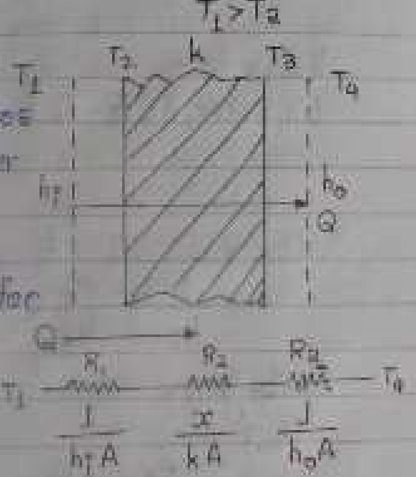
$$\dot{Q} = UA\Delta T$$

where, U is the overall heat transfer coefficient for the surface.

Also, we have, $\dot{Q} = \dfrac{\Delta T}{R_{th,\,total}}$

Thus the overall heat transfer coefficient (U) is defined as

$$U = \frac{1}{A(R_{th,\,total})}$$



Here we consider a wall with surface temperatures T_2 and T_3 and two different fluids at the inner and outer surface of the wall at temperatures T_1 and T_4. The wall has a thermal conductivity k and the fluids have the convective heat transfer coefficient of h_i and h_o.

Then,

$$\dot{Q} = \frac{\Delta T}{R_{th,\,total}} = \frac{T_1 - T_4}{\frac{1}{h_i A} + \frac{x}{kA} + \frac{1}{h_o A}}$$

Thus the overall heat transfer coefficient is given as:

$$U = \frac{1}{A(R_{th,\,total})} = \frac{1}{A\left(\frac{1}{h_i A} + \frac{x}{kA} + \frac{1}{h_o A}\right)}$$

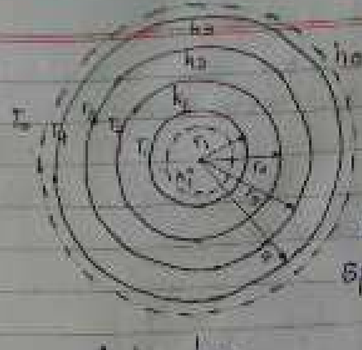
$$\therefore U = \frac{1}{\frac{1}{h_i} + \frac{x}{k} + \frac{1}{h_o}} \Rightarrow \boxed{\frac{1}{U} = \frac{1}{h_i} + \frac{x}{k} + \frac{1}{h_o}}$$

In case of a composite cylindrical tube, heat flow rate can be written as:

$$Q = UA(T_i - T_o)$$

Since the flow area varies for a cylindrical tube, it becomes necessary to specify the area on which U is based.

$T_i > T_o$



$$Q \longrightarrow$$

$$h_o \quad T_i \longrightarrow \underset{\frac{1}{h_i A}}{R_1} \longrightarrow \underset{\frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_1 L}}{R_2} \longrightarrow \underset{\frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_2 L}}{R_3} \longrightarrow \underset{\frac{\ln\left(\frac{r_4}{r_3}\right)}{2\pi k_3 L}}{R_4} \longrightarrow \underset{\frac{1}{h_o A}}{R_5}$$

Depending upon whether the inner or outer area is specified, two different values are defined for U_{in}

$$Q = U_{in} A_{in} (T_i - T_o) = U_{out} A_{out} (T_i - T_o)$$

And, we have,

$$U_{in} \cdot 2\pi r_1 L (T_i - T_o) = \frac{(T_i - T_o)}{\frac{1}{2\pi r_1 L h_i} + \frac{\ln(r_2/r_1)}{2\pi k_1 L} + \frac{\ln(r_3/r_2)}{2\pi k_2 L} + \frac{\ln\left(\frac{r_4}{r_3}\right)}{2\pi k_3 L} + \frac{1}{2\pi r_4 L h_o}}$$

$$\therefore U_{in} = \frac{1}{\frac{1}{h_i} + \frac{r_1}{k_1}\ln\left(\frac{r_2}{r_1}\right) + \frac{r_1}{k_2}\ln\left(\frac{r_3}{r_2}\right) + \frac{r_1}{k_3}\ln\left(\frac{r_4}{r_3}\right) + \frac{r_1}{r_4 h_o}}$$

Similarly,

$$U_{out} = \frac{1}{\frac{r_4}{r_1 h_i} + \frac{r_4}{k_1}\ln\left(\frac{r_2}{r_1}\right) + \frac{r_4}{k_2}\ln\left(\frac{r_3}{r_2}\right) + \frac{r_4}{k_3}\ln\left(\frac{r_4}{r_3}\right) + \frac{1}{h_o}}$$

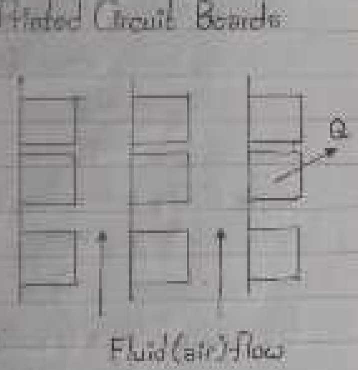
Nature of Convection: Free and Forced Convection

1. Forced Convection:- It is forced convection when the flow is caused by external means, such as by a fan, a pump, or atmospheric winds. The mechanical devices provide a definite circuit for the circulating curr… and that speeds up the heat transfer rate.

Examples of forced convection are cooling of internal combustion engines, air conditioning installations and nuclear reactors, condenser tubes and other heat exchange equipment. E.g use of fan to provide forced convection air cooling of hot electrical components on a stack of printed circuit boards.

Forced convection depends on the following properties namely viscosity, density, thermal conductivity, specific heat, temperature difference between fluid and surface, fluid velocity, and characteristic linear dimension.

Free Convection :- Free (or natural) convection is the mode of heat transfer in which the flow is induced by buoyancy forces that arise from density differences caused by temperature variations in the fluid. Examples: the cooling of transmission lines, electric transformers and rectifiers; heat of rooms by use of radiators, heat transfer from hot pipes and ovens surrounded by cooler air.



Consider the free convection heat transfer that occurs from hot components on a vertical array of circuit boards in still air. Here, the air that makes contact with the components experiences an increase in temperature and hence a reduction in density. Since it is now lighter than the surrounding air, buoyancy forces induce a vertical motion in which warm air ascending from the boards is replaced by air inflow of cooler ambient air.

Heat Radiation

Heat radiation is due to the property of matter to emit and to absorb different kinds of radiation and the fact that an empty space is perfectly permeable to radiation and that the matter allows them to pass more or less.

Stefan's Law

The law states that the emissive power of a black body is directly proportional to the fourth power of its absolute temperature

i.e. $E_b = \sigma_b T^4$

where, E_b = emissive power of a black body

σ_b = Stefan-Boltzmann constant

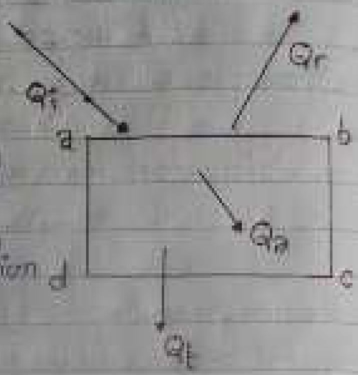
$= 5.67 \times 10^{-8}\ W/m^2K^4$

Consider that surface 1 at temperature T_1 is completely enclosed by another black surface at temperature T_2. The net radiant flux is then given by

$$E_b = \sigma_b (T_1^4 - T_2^4)$$

Absorptivity, Reflectivity and Transmissivity-

If G_i denotes the total incident radiation per unit time per unit area of a surface, G_α, G_ρ, G_τ represent respectively the amount of radiation absorbed, reflected and transmitted, then



absorptivity = fraction of incident radiation absorbed

$$\alpha = \frac{G_\alpha}{G_i}$$

reflectivity = fraction of incident radiation reflected

$$\rho = \frac{G_\rho}{G_i}$$

transmissivity = fraction of incident radiation transmitted

$$\tau = \frac{G_\tau}{G_i}$$

$\therefore \alpha + \rho + \tau = 1$ Since $G_i = G_\alpha + G_\rho + G_\tau$

(a) Black body: This body absorbs all the radiations, $\alpha = 1$, $\rho = \tau = 0$.
(b) White body: This body reflects all the radiations, $\rho = 1$, $\alpha = \tau = 0$.
(c) Transparent body: This body transmits all the radiations, $\tau = 1$, $\alpha = \rho = 0$
(d) Opaque body: This body does not transmit any radiation, $\alpha + \rho = 1$, $\tau = 0$

Black and Gray Bodies

The salient features of the black body are as follows:

a. A black body absorbs all the incident radiation regardless of wavelength and direction.
b. A black body neither reflects nor transmits any amount of incident radiation.

For a prescribed wavelength, a black body radiates the maximum energy possible at the temperature of the body.

The black body is a diffused emitter. This implies that the radiation emitted by a black surface is a function of wavelength and temperature but is independent of temperature.

Examples of black body (approximate): Hohlraum, isothermal furnaces with small apertures.

A gray body is a non-black body with a constant emissivity at all temperatures and throughout the entire range of wavelength.

The emissivity of the gray surface may be expressed as

$$\epsilon = \frac{E}{E_b} = \frac{\sigma T^4}{\sigma_b T^4} = \frac{\sigma}{\sigma_b}$$

$$\therefore \quad 0 < \epsilon < 1.$$

$$\therefore \boxed{E_b = \epsilon \sigma_b T^4} \quad W/m^2.$$