# Engineering Economics

Lecture 3

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# Chapter 4
# Time Is Money

- Interest: The Cost of Money
- Economic Equivalence
- Development of Interest Formulas
- Unconventional Equivalence Calculations

# Mrs. Rosalind Setchfield's Decision Problem

- She gave up annual lottery earnings of $32,639 for 9 years with a lump sum payment of $140,000.
- Did she make a right decision?

# Take or Not to Take the Offer

| Year | Installment | Year | Installment | Reduced Payment |
|---|---|---|---|---|
| 1988 | $65,277 | 1995 | $65,277 | $32,639 |
| 1989 | 65,277 | 1996 | 65,277 | 32,639 |
| 1990 | 65,277 | 1997 | 65,277 | 32,639 |
| 1991 | 65,277 | 1998 | 65,277 | 32,639 |
| 1992 | 65,277 | 1999 | 65,277 | 32,639 |
| 1993 | 65,277 | 2000 | 65,277 | 32,639 |
| 1994 | 65,277 | 2001 | 65,277 | 32,639 |
| | | 2002 | 65,277 | 32,639 |
| | | 2003 | 65,277 | 32,639 |
| | | 2004 | 65,277 | |
| | | 2005 | 65,277 | |
| | | 2006 | 65,277 | |
| | | 2007 | 65,277 | |

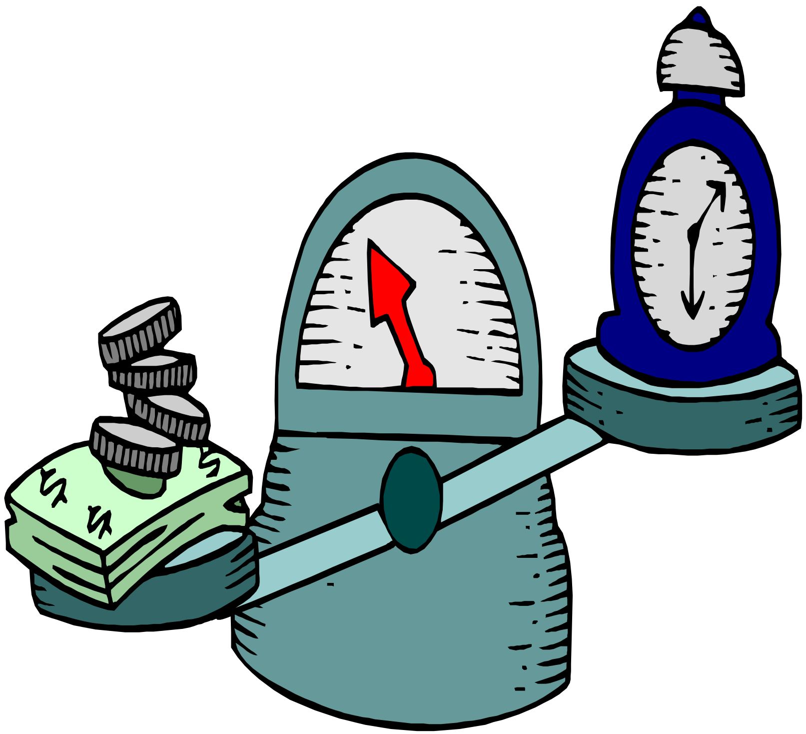
$140,000 lump sum payment now

# What Do We Need to Know?

- To make such comparisons (Ms. Rosalind Setchfield's decision problem), we must be able to compare the value of money at different point in time.
- To do this, we need to develop a method for reducing a sequence of benefits and costs to a single point in time. Then, we will make our comparisons on that basis.

# Time Value of Money

- Money has a time value because it can earn more money over time (earning power).
- Time value of money is measured in terms of interest rate.
- Interest is the cost of money—a cost to the borrower and an earning to the lender



# Repayment Plans

| End of Year | Receipts | Payments | |
|---|---|---|---|
| | | Plan 1 | Plan 2 |
| Year 0 | $20,000.00 | $200.00 | $200.00 |
| Year 1 | | 5,141.85 | 0 |
| Year 2 | | 5,141.85 | 0 |
| Year 3 | | 5,141.85 | 0 |
| Year 4 | | 5,141.85 | 0 |
| Year 5 | | 5,141.85 | 30,772.48 |
| $P$ = $20,000, $A$ = $5,141.85, $F$ = $30,772.48 | | | |

# Cash Flow Diagram

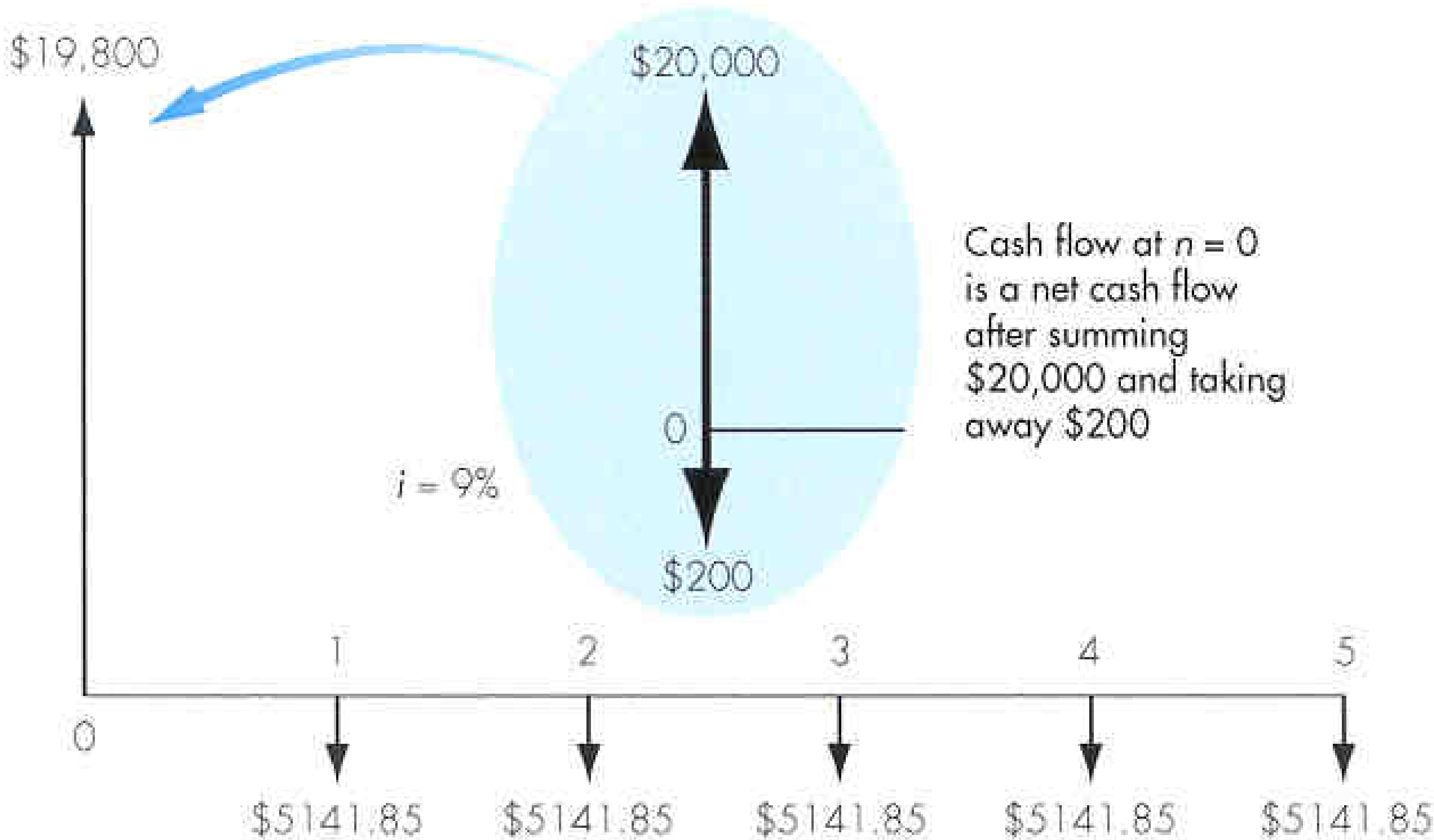



**Figure 4.1** A cash flow diagram for plan 1 of the loan repayment example summarized in Table 4.1

# Methods of Calculating Interest

- Simple interest: the practice of charging an interest rate only to an initial sum (principal amount).
- Compound interest: the practice of charging an interest rate to an initial sum and to any previously accumulated interest that has not been withdrawn.

# Simple Interest

- *P* = Principal amount
- *i* = Interest rate
- *N* = Number of interest periods
- Example:
  - *P* = $1,000
  - *i* = 8%
  - *N* = 3 years

| End of Year | Beginning Balance | Interest earned | Ending Balance |
|---|---|---|---|
| 0 | | | $1,000 |
| 1 | $1,000 | $80 | $1,080 |
| 2 | $1,080 | $80 | $1,160 |
| 3 | $1,160 | $80 | $1,240 |

# Compound Interest

- *P* = Principal amount
- *i* = Interest rate
- *N* = Number of interest periods
- Example:
  - *P* = $1,000
  - *i* = 8%
  - *N* = 3 years

| End of Year | Beginning Balance | Interest earned | Ending Balance |
|---|---|---|---|
| 0 | | | $1,000 |
| 1 | $1,000 | $80 | $1,080 |
| 2 | $1,080 | $86.40 | $1,166.40 |
| 3 | $1,166.40 | $93.31 | $1,259.71 |

# Comparing Simple to Compound Interest

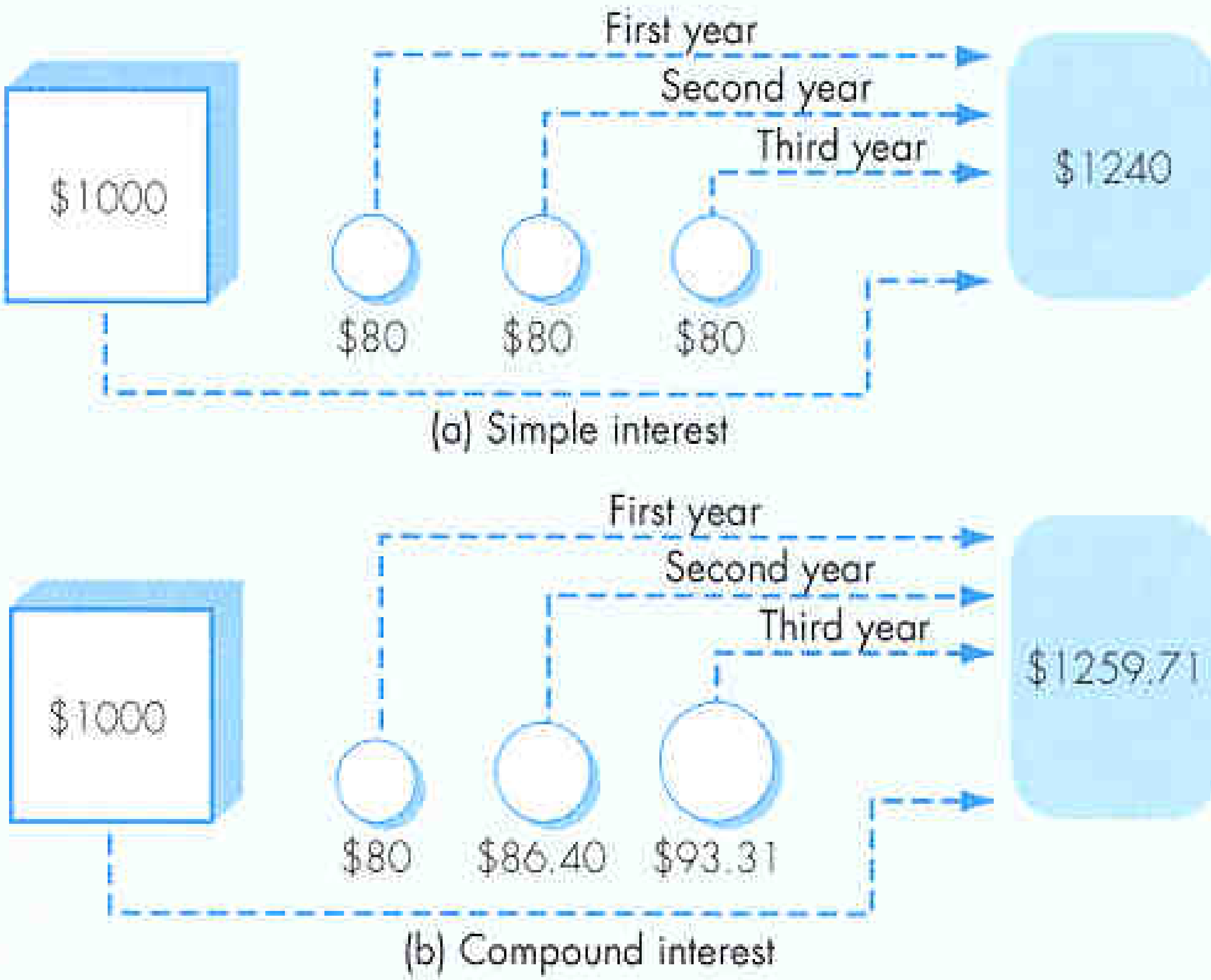



**Figure 4.2** Two methods of calculating the balance when $1000 at 8% interest is deposited for 3 years (Example 4.1)

# Economic Equivalence

- What do we mean by “economic equivalence?”
- Why do we need to establish an economic equivalence?
- How do we establish an economic equivalence?

# Economic Equivalence

- Economic equivalence exists between cash flows that have the same economic effect and could therefore be traded for one another.
- Even though the amounts and timing of the cash flows may differ, the appropriate interest rate makes them equal.

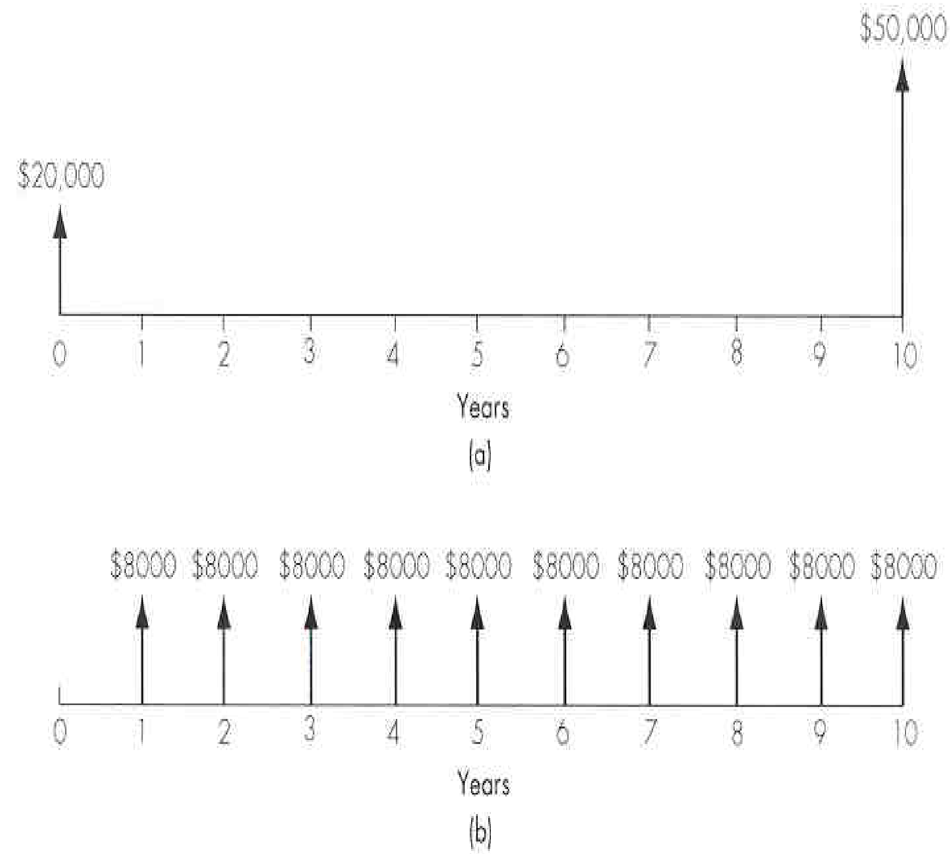


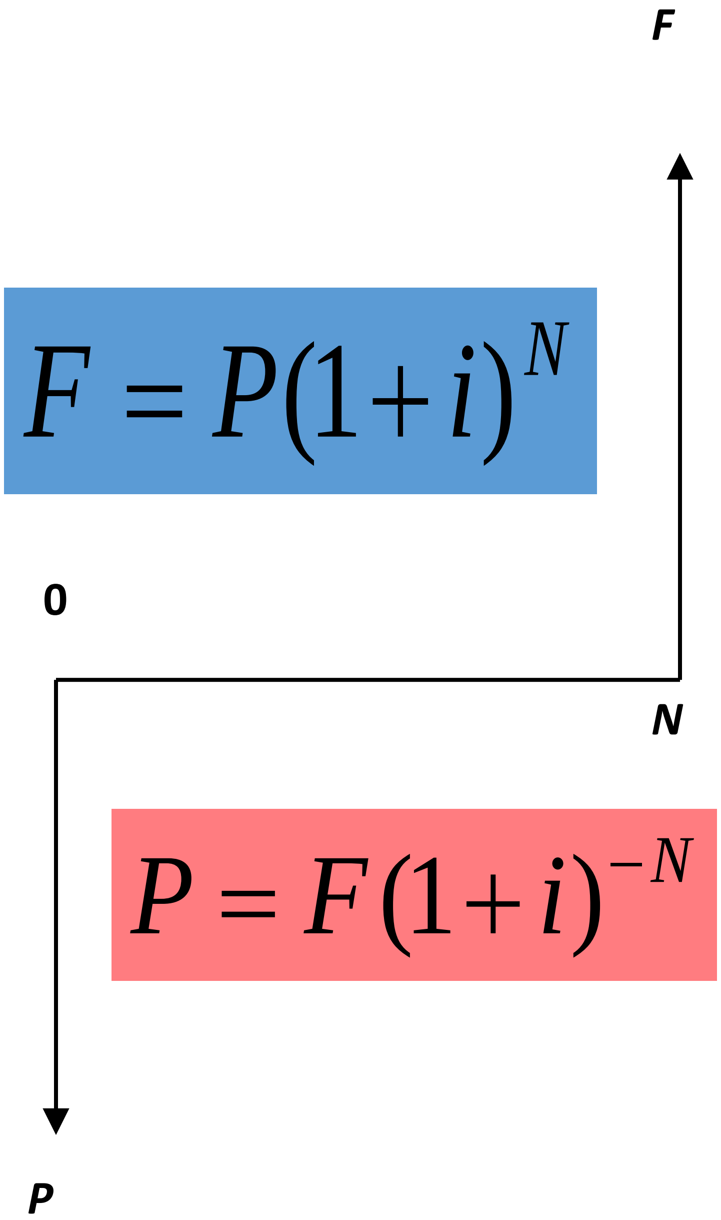

**Figure 4.3** Which option would you prefer? (a) Two payments ($20,000 now and $50,000 at the end of 10 years) or (b) ten equal annual receipts in the amount of $8000

# Typical Repayment Plans for a Bank Loan of $20,000

| | Repayments | | |
|---|---|---|---|
| | **Plan 1** | **Plan 2** | **Plan 3** |
| **Year 1** | $5,141.85 | 0 | $1,800.00 |
| **Year 2** | 5,141.85 | 0 | 1,800.00 |
| **Year 3** | 5,141.85 | 0 | 1,800.00 |
| **Year 4** | 5,141.85 | 0 | 1,800.00 |
| **Year 5** | 5,141.85 | $30,772.48 | 21,800.00 |
| **Total of payments** | $25,709.25 | $30,772.48 | $29,000.00 |
| **Total interest paid** | $5,709.25 | $10,772.48 | $9,000.00 |

- If you deposit $P$ dollars today for $N$ periods at $i$, you will <u>have</u> $F$ dollars at the end of period $N$.
- $F$ dollars at the end of period $N$ is <u>equal</u> to a single sum $P$ dollars now, if your earning power is measured in terms of interest rate $i$.

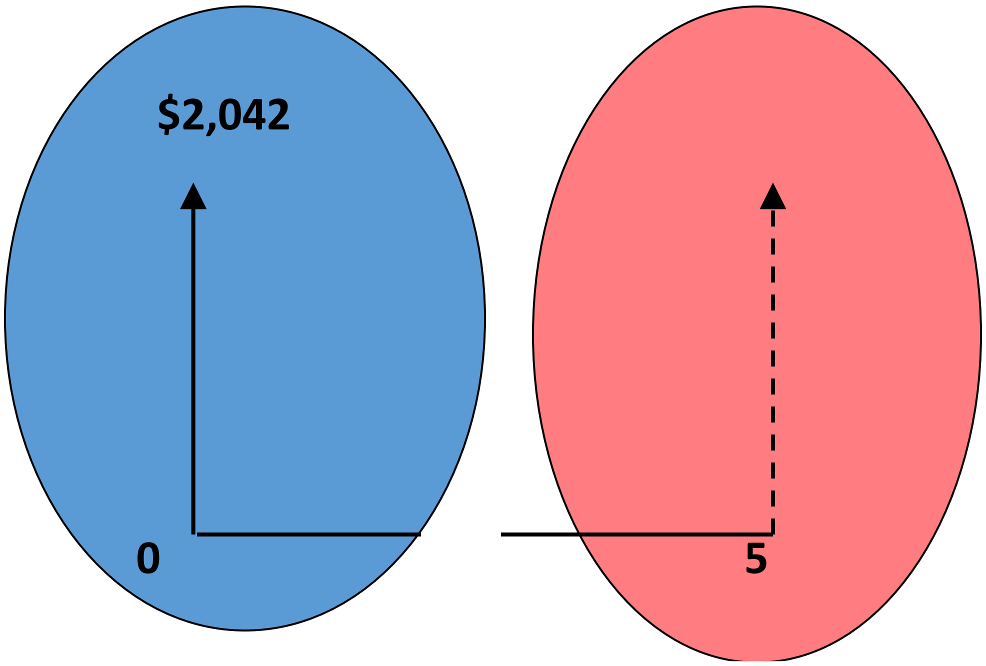
$$F = P(1+i)^N$$




$$P = F(1+i)^{-N}$$

# Equivalence Between Two Cash Flows

- Step 1: Determine the <u>base period</u>, say, year 5.
- Step 2: Identify the <u>interest rate</u> to use.
- Step 3: Calculate <u>equivalence value</u>.




$$i = 6\%,$$

$$i = 8\%,$$

$$i = 10\%,$$

# Example 4.5 Equivalence

Various dollar amounts that will be <u>economically equivalent</u> to $3,000 in 5 years, given an interest rate of 8%

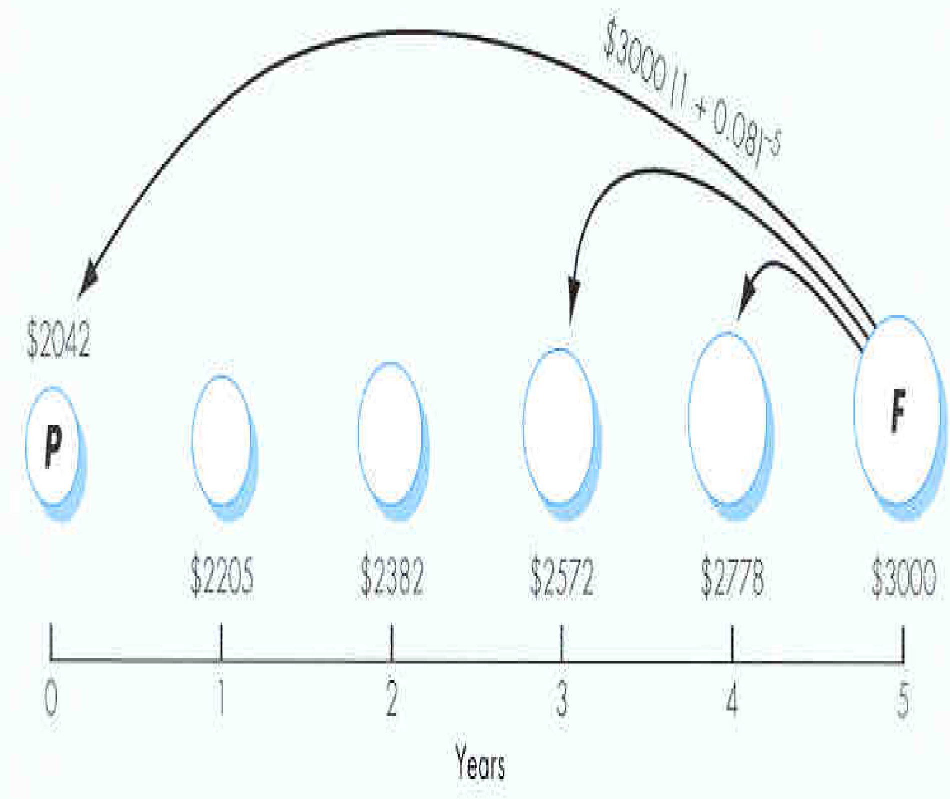



**Figure 4.4** Various dollar amounts that will be economically equivalent to $3000 in 5 years, given an interest rate of 8% (Example 4.3)

# Equivalent Cash Flows are Equivalent at Any Common Point In Time

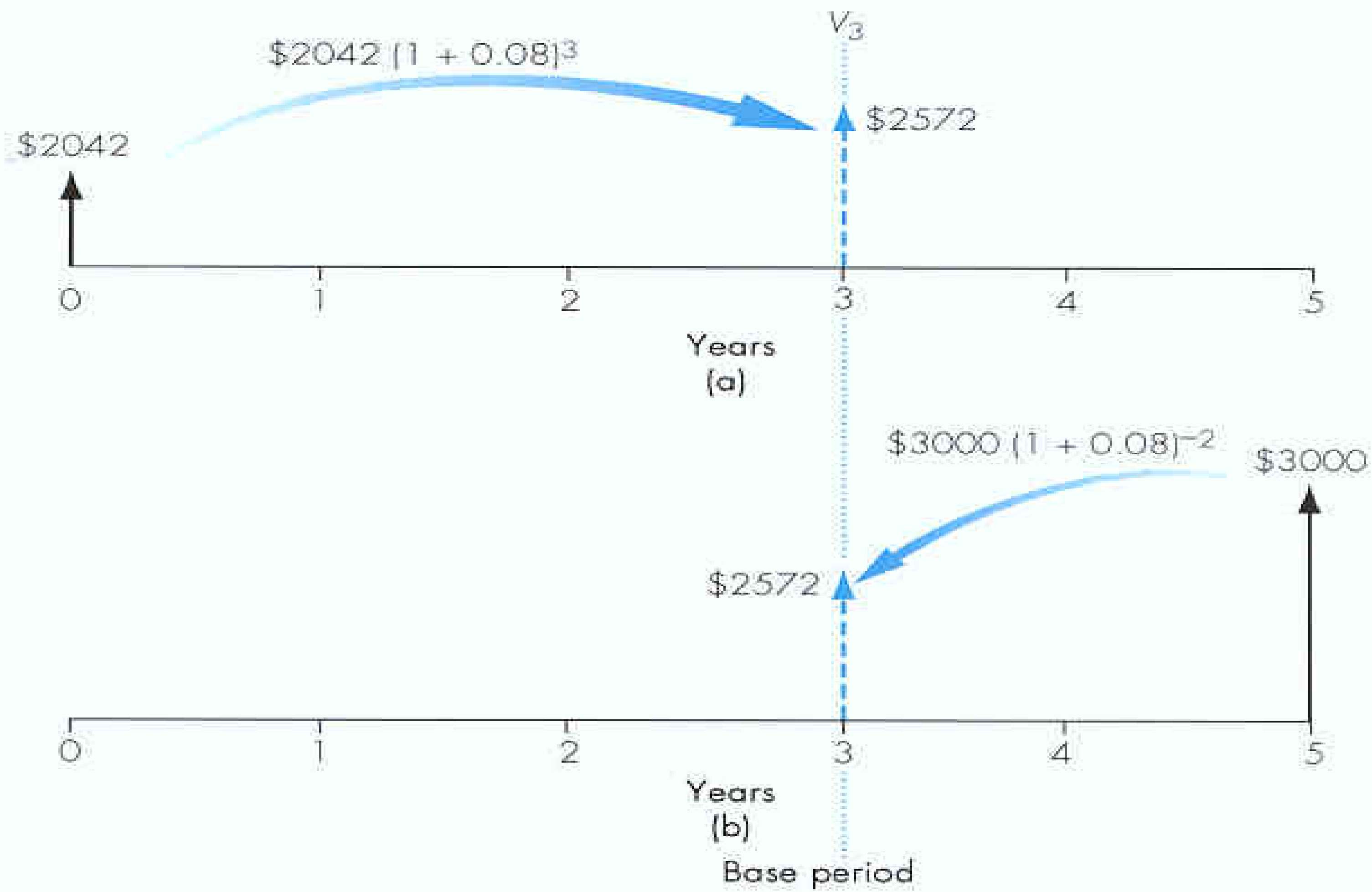



**Figure 4.5** Selection of a base period for an equivalence calculation (Example 4.4)

# Example 4.5 Equivalence Calculations with Multiple Payments

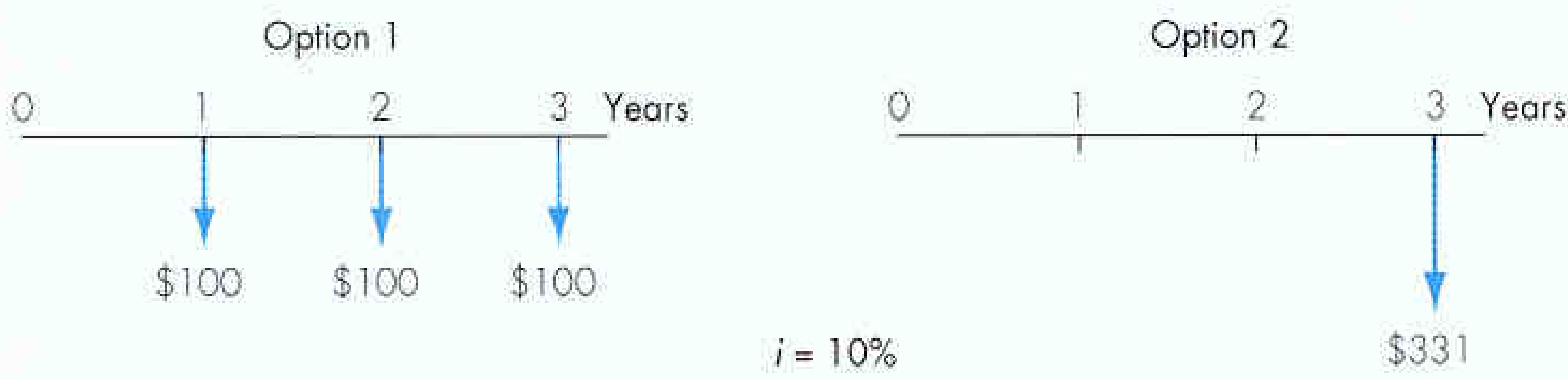



**Figure 4.6** Equivalent cash flow diagram for Option 1 and Option 2 (excluding the common principal payment $1,000 at the end of year 3) (Example 4.5)

$$F_3 \text{ for } \$100 \text{ at } n = 1{:}\$100(1+0.10)^{3-1} = \$121$$

$$F_3 \text{ for } \$100 \text{ at } n = 2{:}\$100(1+0.10)^{3-2} = \$110$$

$$F_3 \text{ for } \$100 \text{ at } n = 3{:}\$100(1+0.10)^{3-3} = \$100$$

# Types of Cash Flows

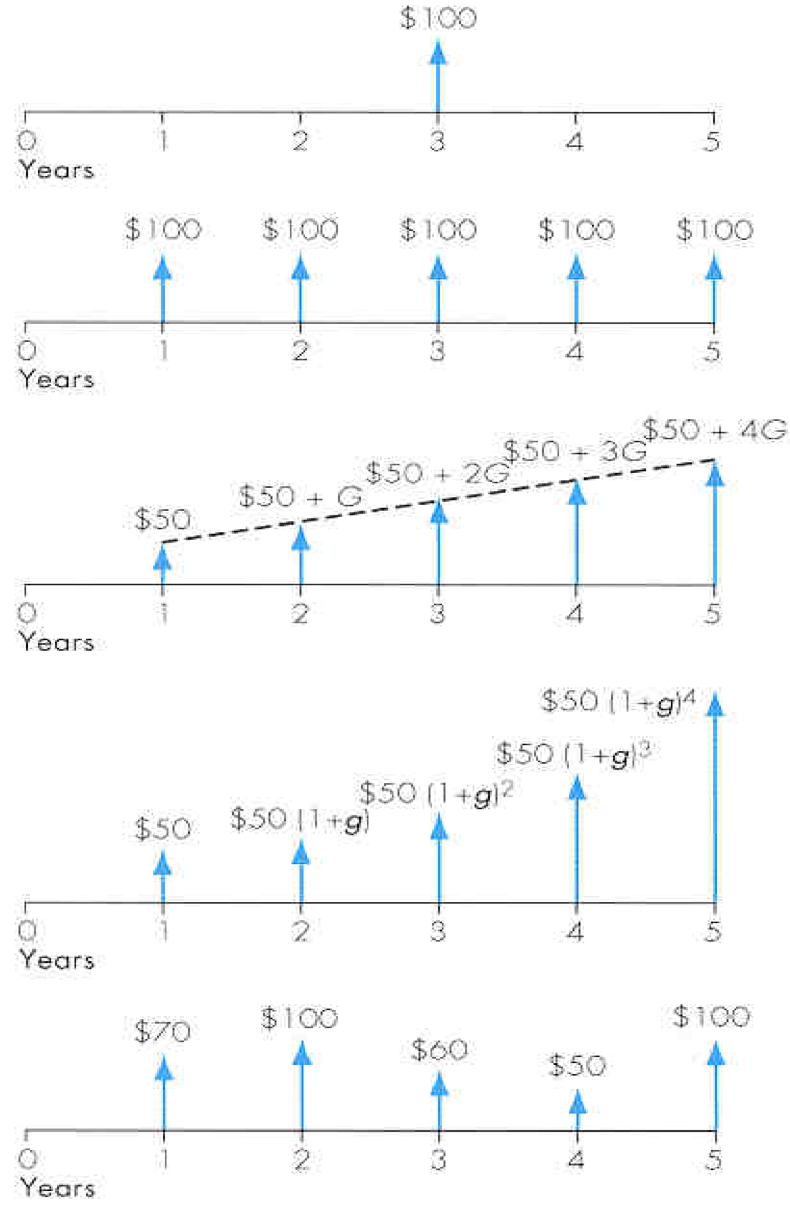
(a) Single cash flow

(b) Equal (uniform) payment series

(c) Linear gradient series

(d) Geometric gradient series

(e) Irregular payment series

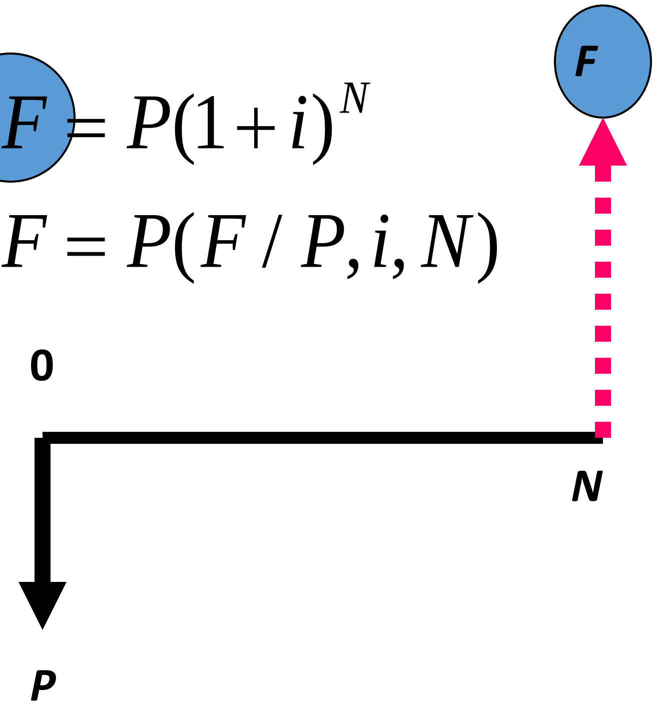


# Single Cash Flow Formula

- <u>Single payment compound amount factor</u> (growth factor)

$$F = P(1+i)^N$$

$$F = P(F/P, i, N)$$




- Given:

$$i = 10\%$$

$$N = 8 \text{ years}$$

$$P = \$2{,}000$$

- Soln:

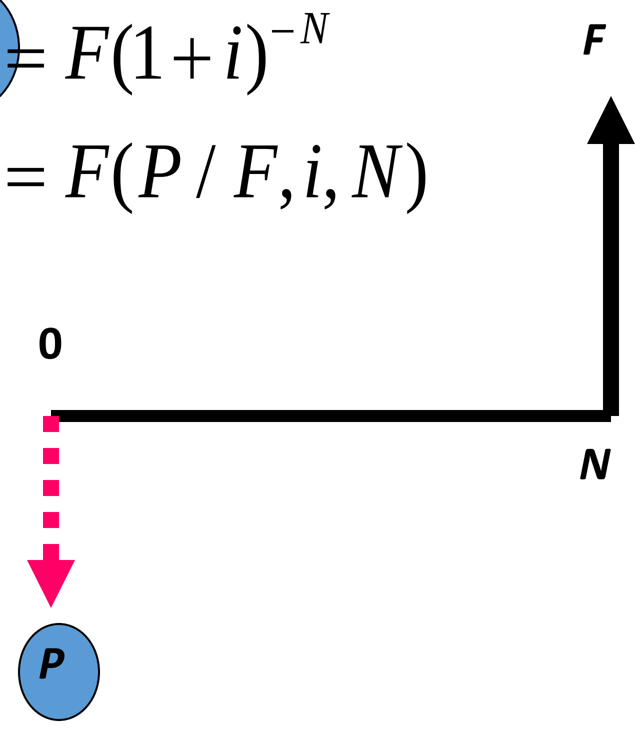
$$\begin{aligned} F &= \$2{,}000(1+0.10)^8 \\ &= \$2{,}000(F/P, 10\%, 8) \\ &= \$4{,}287.18 \end{aligned}$$

# Single Cash Flow Formula

- Single payment present worth factor (discount factor)

$$P = F(1+i)^{-N}$$

$$P = F(P/F, i, N)$$




- Given:

$$i = 12\%$$

$$N = 5 \text{ years}$$

$$F = \$1{,}000$$

- Soln:

$$\begin{aligned} P &= \$1{,}000(1+0.12)^{-5} \\ &= \$1{,}000(P/F, 12\%, 5) \\ &= \$567.40 \end{aligned}$$

# Uneven Payment Series

$$P_1 = \$25{,}000(P/F, 10\%, 1)$$

$$P_2 = \$3{,}000(P/F, 10\%, 2)$$

$$P_4 = \$5{,}000(P/F, 10\%, 4)$$

$$P = P_1 + P_2 + P_4$$

$$= \$28{,}622$$

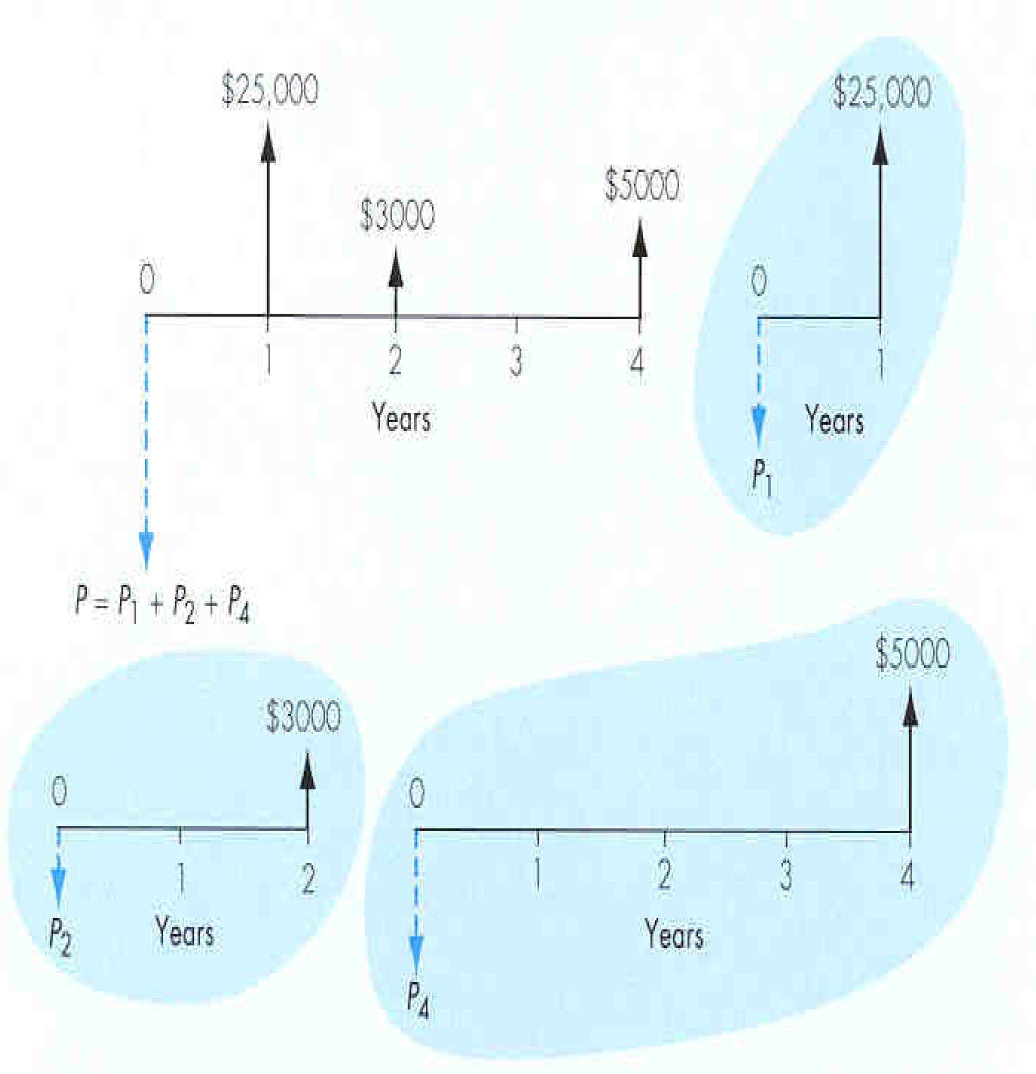


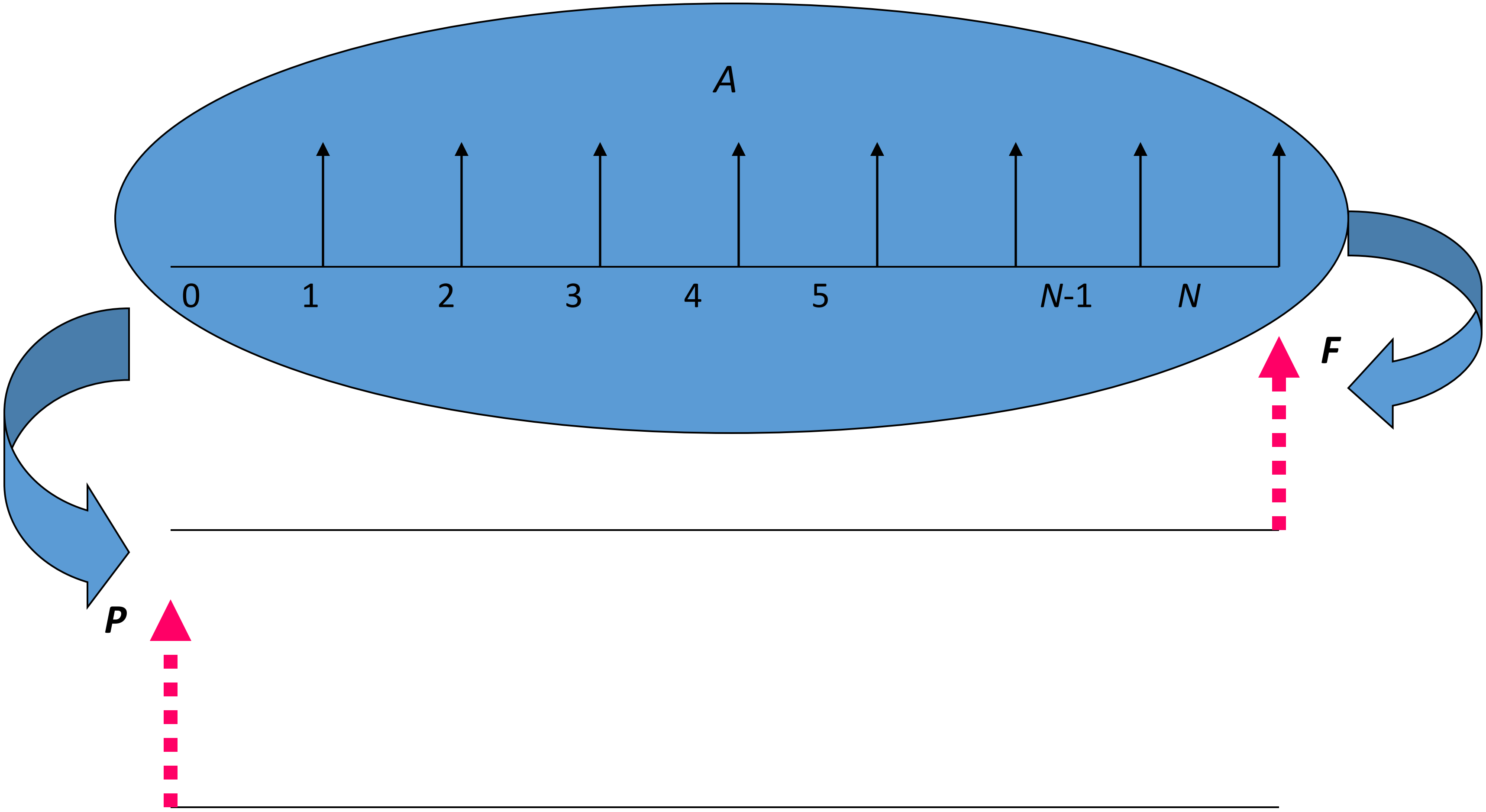

**Figure 4.14** Decomposition of uneven cash flow series (Example 4.11)

## Example 4.12 Calculating the Actual Worth of a Long-Term Contract

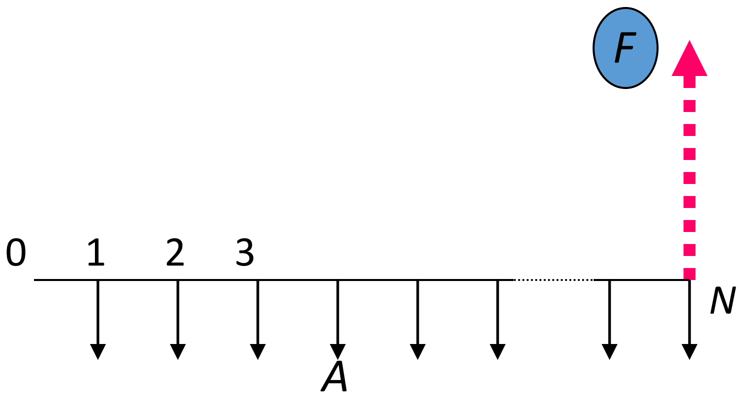
| | Beginning of Season | Contract Salary | Prorated Signing Bonus | Total Annual Payment |
|---|---|---|---|---|
| 0 | 2001 | $ 21,000,000 | $ 2,000,000 | $ 23,000,000 |
| 1 | 2002 | 21,000,000 | 2,000,000 | 23,000,000 |
| 2 | 2003 | 21,000,000 | 2,000,000 | 23,000,000 |
| 3 | 2004 | 21,000,000 | 2,000,000 | 23,000,000 |
| 4 | 2005 | 25,000,000 | 2,000,000 | 27,000,000 |
| 5 | 2006 | 25,000,000 | | 25,000,000 |
| 6 | 2007 | 27,000,000 | | 27,000,000 |
| 7 | 2008 | 27,000,000 | | 27,000,000 |
| 8 | 2009 | 27,000,000 | | 27,000,000 |
| 9 | 2010 | 27,000,000 | | 27,000,000 |

$$
\begin{aligned}
P &= \$23M(P/F,6\%,1) + \$23M(P/F,6\%,2) + \ \ldots \ + \$27M(P/F,6\%,9) \\
&= \$215.75M
\end{aligned}
$$

# Equal Payment Series



# Equal Payment Series Compound Amount Factor

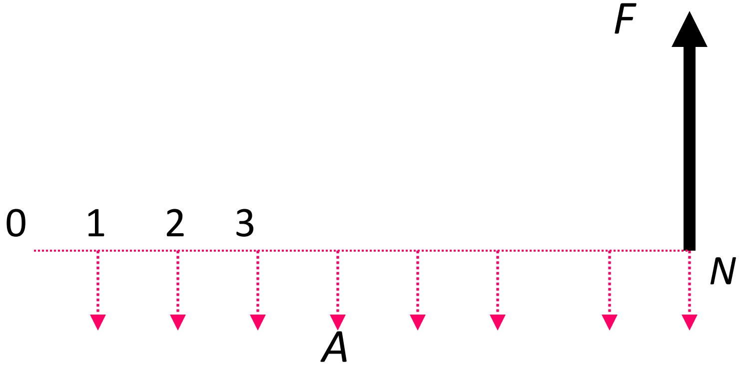



$$F = A\boxed{\frac{(1+i)^N - 1}{i}}$$
$$= A(F/A, i, N)$$

Example 4.13:

- Given: $A$ = \$3,000, $N$ = 10 years, and $i$ = 7%
- Find: $F$
- Solution: $F$ = \$3,000($F$/$A$,7%,10) = \$41,449.20

# Sinking Fund Factor

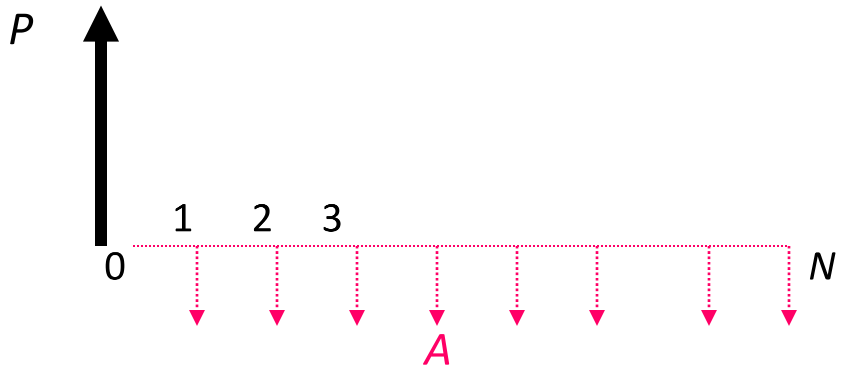



$$A = F\boxed{\frac{i}{(1+i)^{N}-1}}$$
$$= F(A/F, i, N)$$

<u>Example 4.15</u>:

- Given: $F$ = \$5,000, $N$ = 5 years, and $i$ = 7%
- Find: $A$
- Solution: $A$ = \$5,000($A/F$,7%,5) = \$869.50

# Capital Recovery Factor



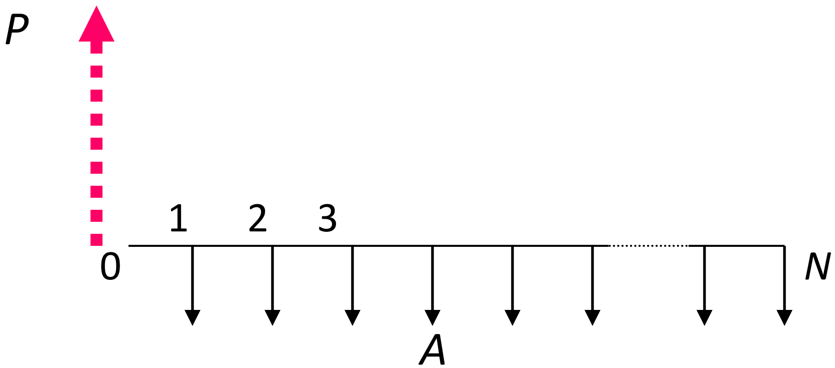

$$A = P\boxed{\frac{i(1+i)^N}{(1+i)^N - 1}}$$

$$= P(A/P, i, N)$$

Example 4.16:

- Given: $P$ = \$250,000, $N$ = 6 years, and $i$ = 8%
- Find: $A$
- Solution: $A$ = \$250,000($A/P$,8%,6) = \$54,075

# Equal Payment Series Present Worth Factor




$$P = A\boxed{\frac{(1+i)^N - 1}{i(1+i)^N}}$$
$$= A(P/A, i, N)$$

Example 4.18:

- Given: $A$ = \$32,639, $N$ = 9 years, and $i$ = 8%
- Find: $P$
- Solution: $P$ = \$32,639($P/A$,8%,9) = \$203,893

# Linear Gradient Series

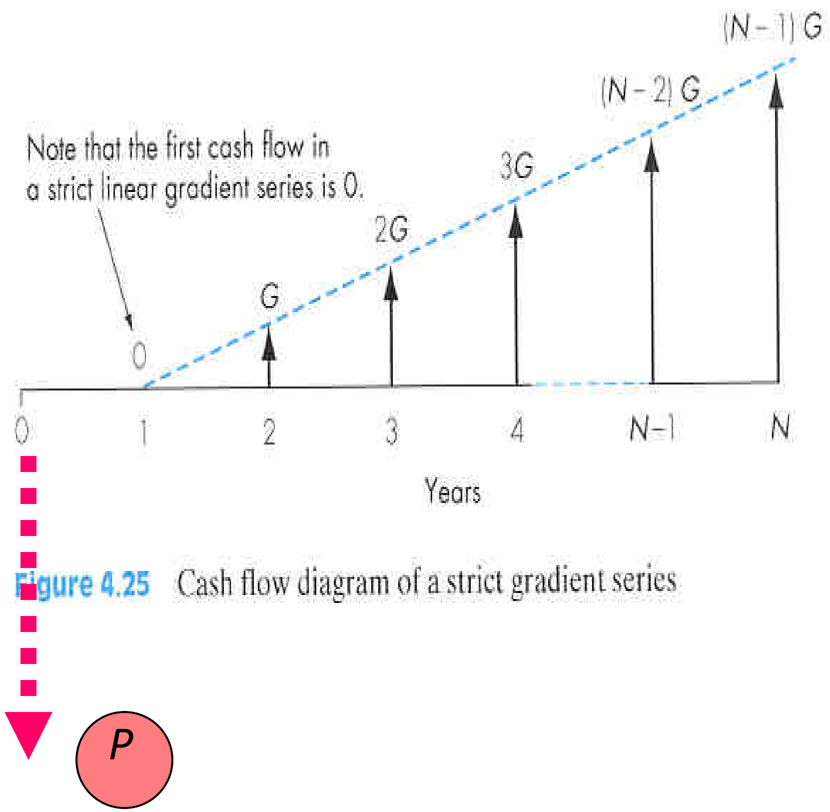



**Figure 4.25** Cash flow diagram of a strict gradient series

$$P = G\boxed{\frac{i(1+i)^N - iN - 1}{i^2(1+i)^N}}$$

$$= G(P/G, i, N)$$

# Gradient Series as a Composite Series

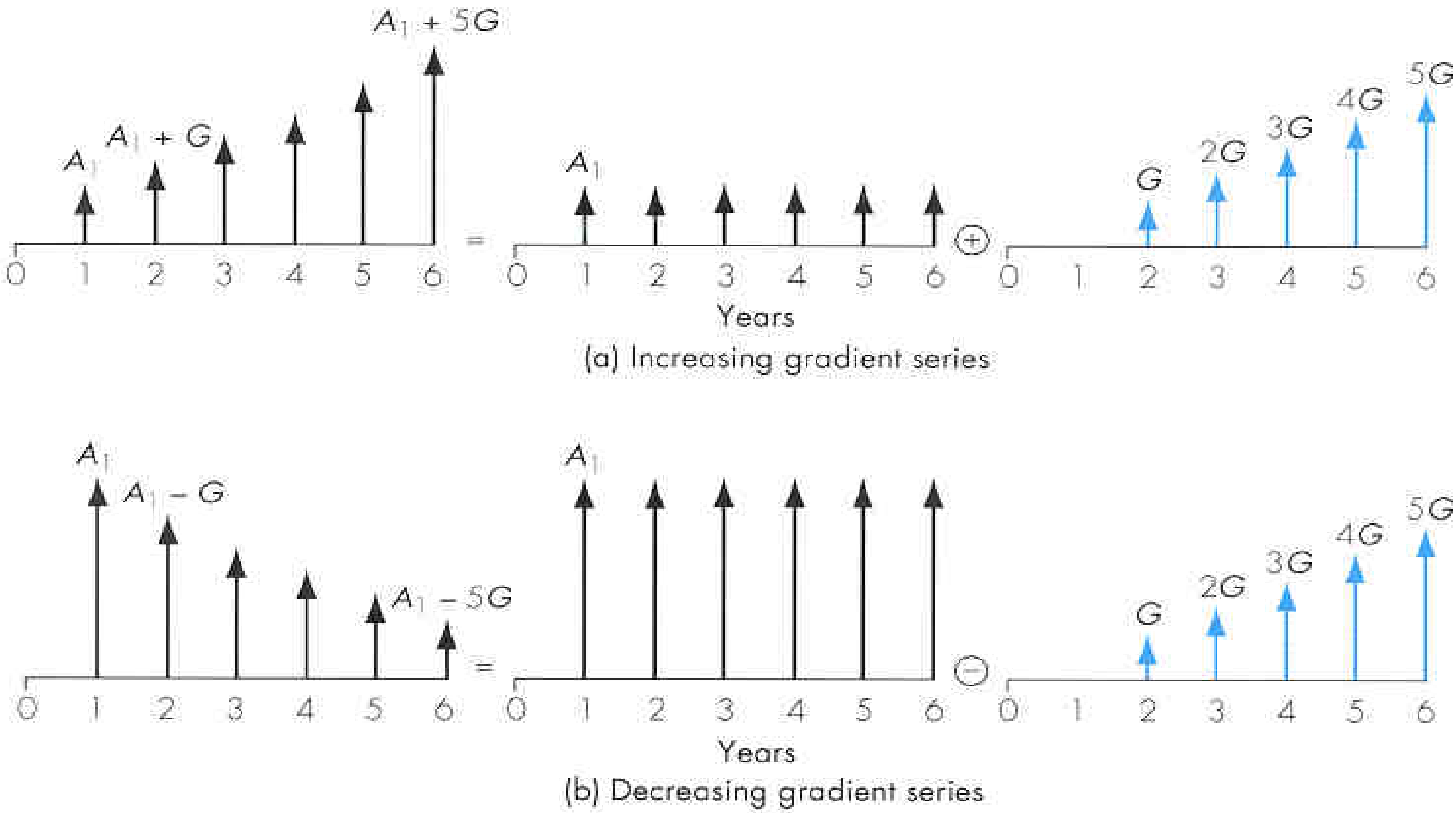
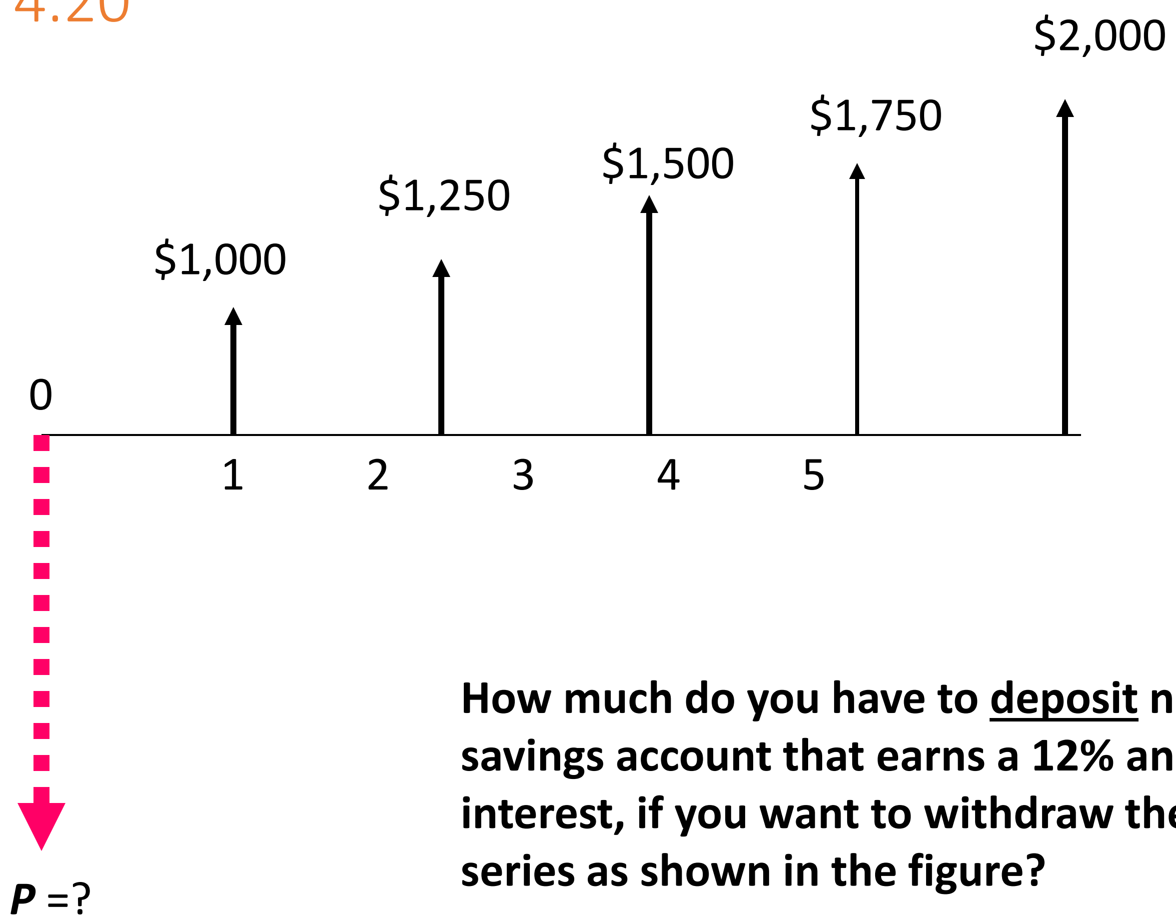


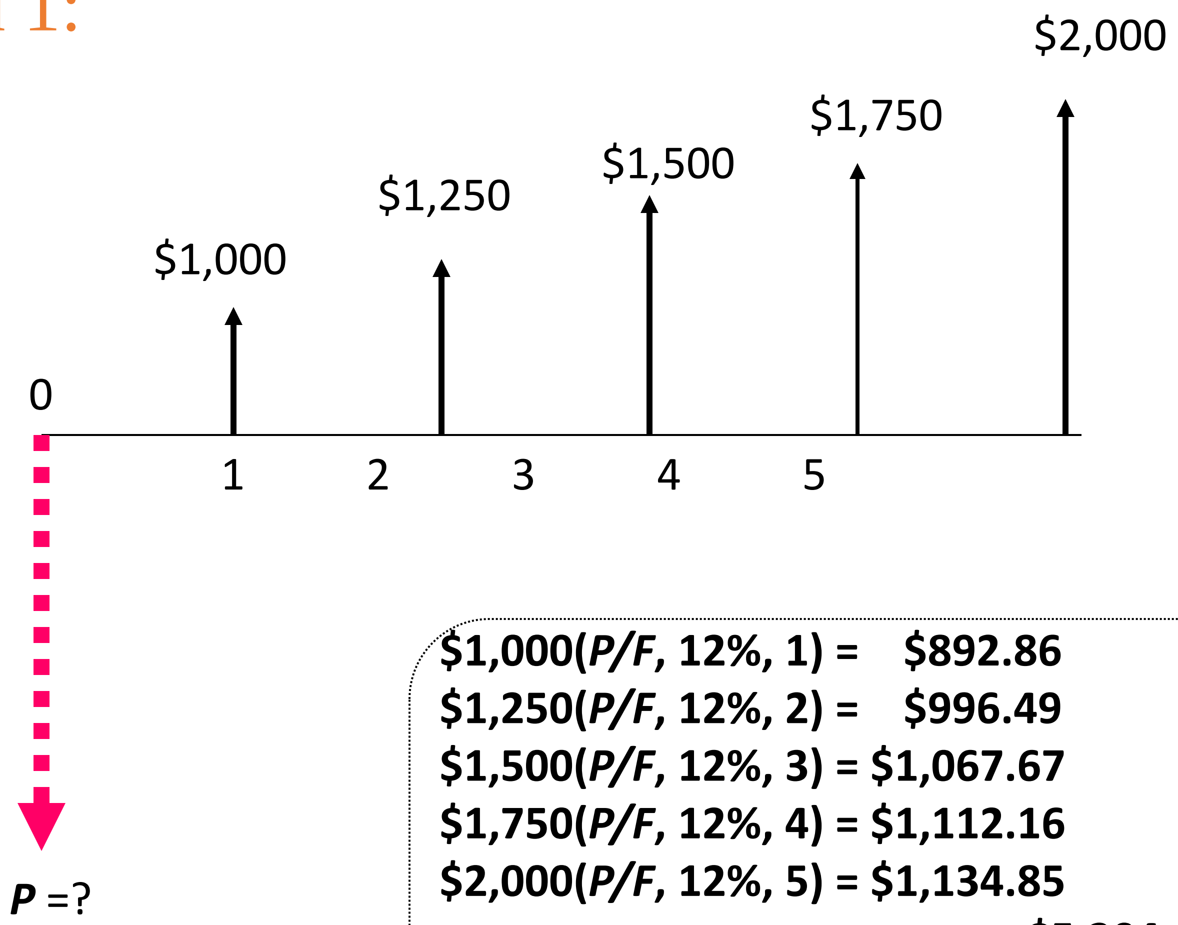

**Figure 4.27** Two types of linear gradient series as composites of a uniform series of $N$ payments of $A_1$ and the gradient series of increments of constant amount $G$

# Example 4.20



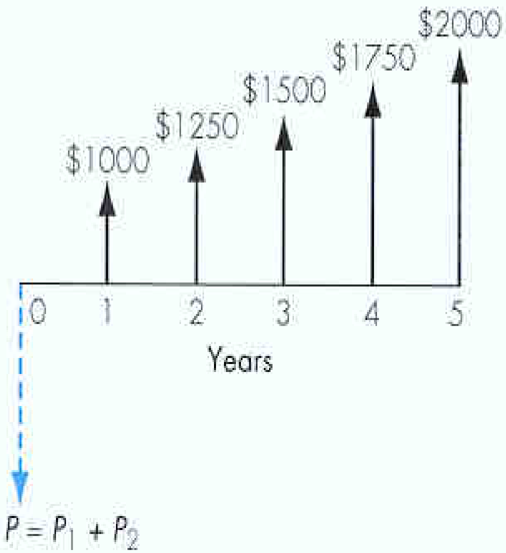

**How much do you have to <u>deposit</u> now in a savings account that earns a 12% annual interest, if you want to withdraw the annual series as shown in the figure?**

# Method 1:




**$1,000(*P/F*, 12%, 1) = $892.86**
**$1,250(*P/F*, 12%, 2) = $996.49**
**$1,500(*P/F*, 12%, 3) = $1,067.67**
**$1,750(*P/F*, 12%, 4) = $1,112.16**
**$2,000(*P/F*, 12%, 5) = $1,134.85**
**$5,204.03**

# Method 2:

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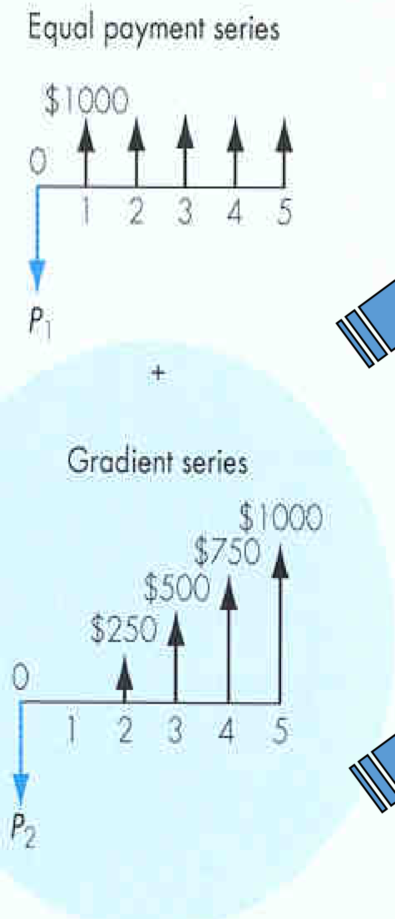



$$P_1 = \$1{,}000(P/A, 12\%, 5)$$
$$= \$3{,}604.80$$

$$P_2 = \$250(P/G, 12\%, 5)$$
$$= \$1{,}599.20$$

$$P = \$3{,}604.08 + \$1{,}599.20$$
$$= \$5{,}204$$

# Geometric Gradient Series

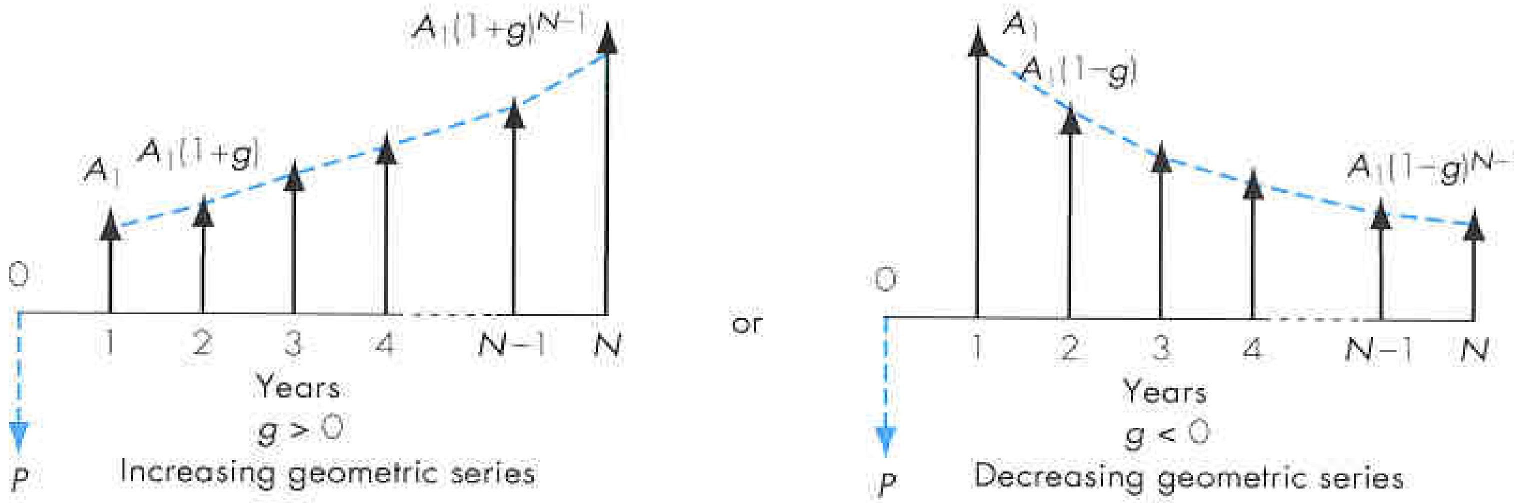
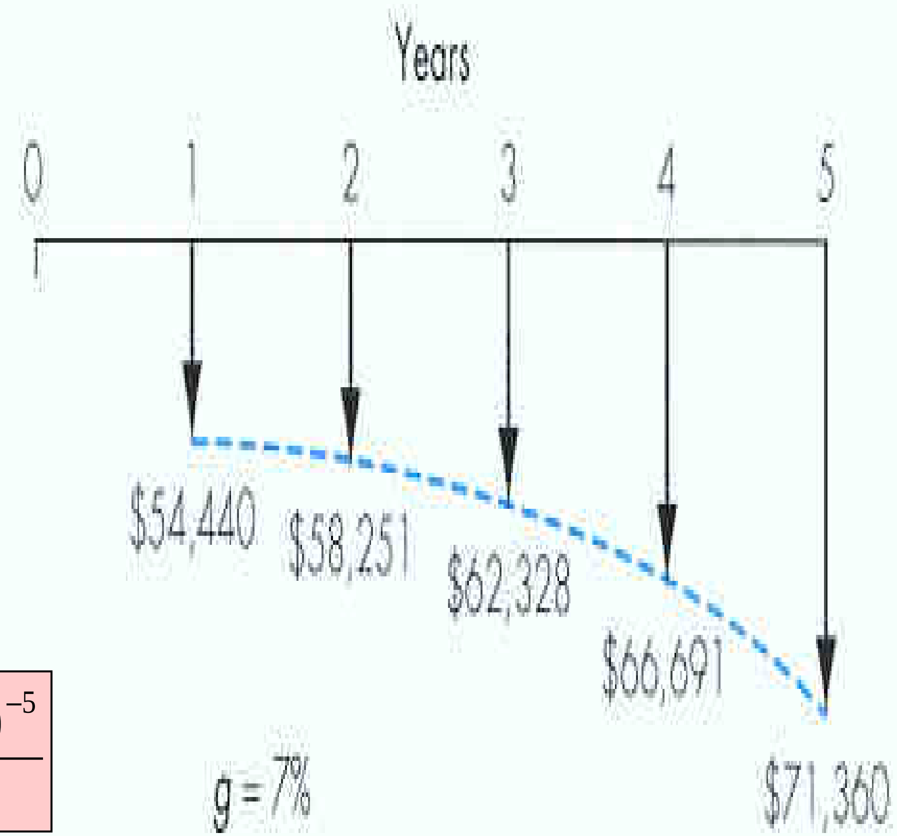



**Figure 4.32** A geometrically increasing or decreasing gradient series at a constant rate $g$

$$P = \begin{cases} A_1 \boxed{\dfrac{1-(1+g)^N(1+i)^{-N}}{i-g}}, & \text{if } i \neq g \\ NA_1/(1+i), & \text{if } i = g \end{cases}$$

# Example 4.24 Geometric Gradient: Find $P$, Given $A_1, g, i, N$

- Given:
  - $g$ = 7%
  - $i$ = 12%
  - $N$ = 5 years
  - $A_1$ = \$54,440
- Find: $P$

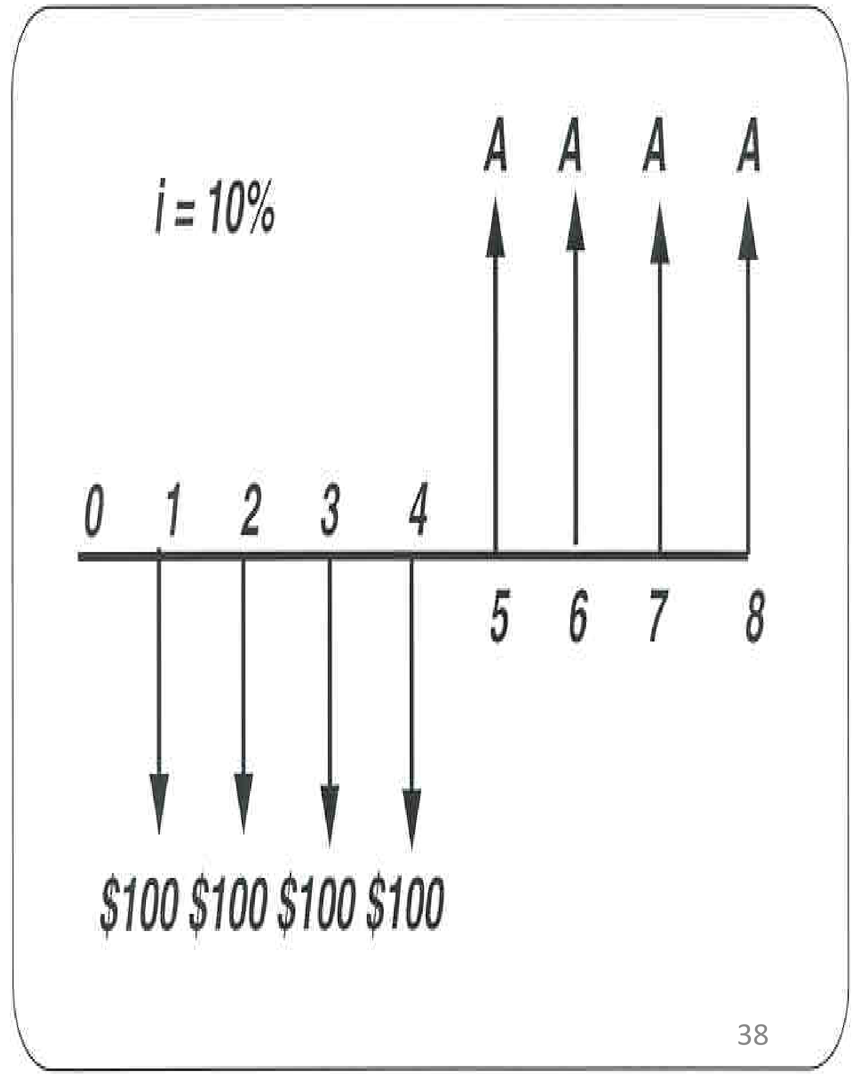



$$P = \$54{,}440 \boxed{\frac{1-(1+0.07)^5(1+0.12)^{-5}}{0.12-0.07}}$$

$$= \$151{,}109$$

# Unconventional Equivalence Calculations

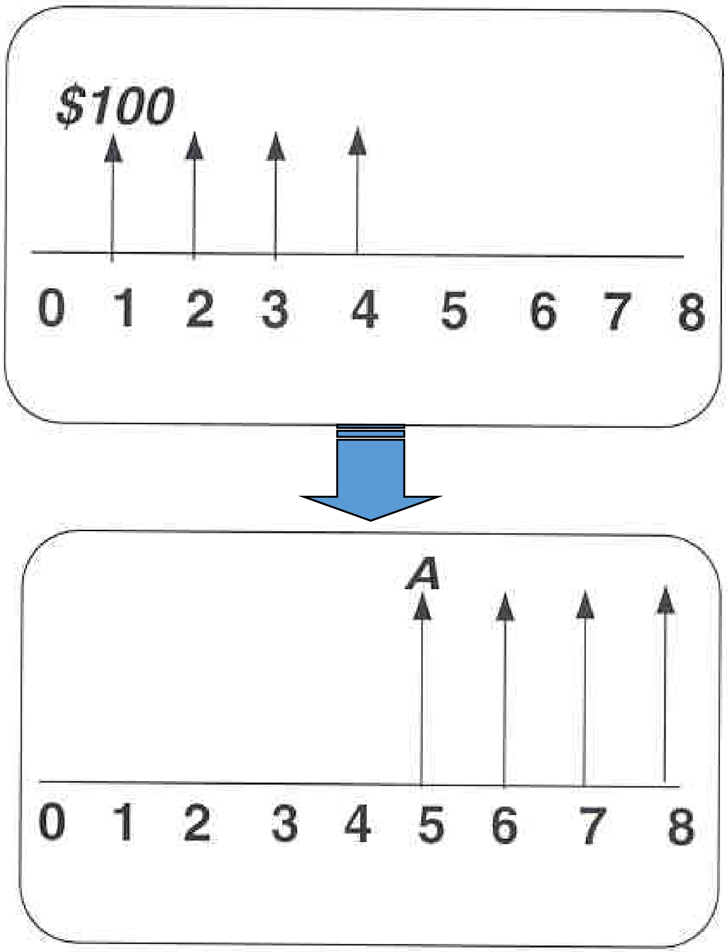
**Situation 1**: If you make 4 annual deposits of \$100 in your savings account which earns a 10% annual interest, what equal annual amount can be withdrawn over 4 subsequent years?



# Unconventional Equivalence Calculations

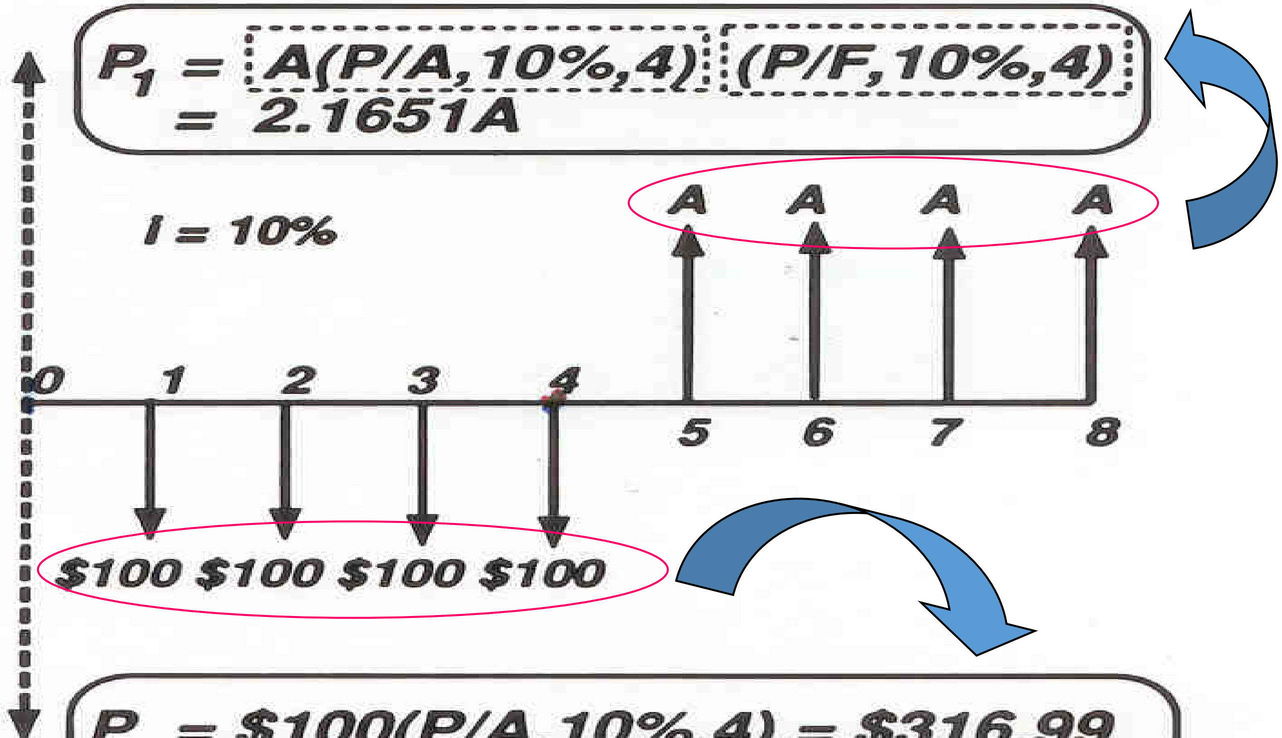
- Situation 2:

  What value of $A$ would make the two cash flow transactions equivalent if $i$ = 10%?



# Method 1: Establish economic equivalence at period 0

$$P_1 = A(P/A, 10\%, 4)(P/F, 10\%, 4) = 2.1651A$$




$$P_2 = \$100(P/A, 10\%, 4) = \$316.99$$

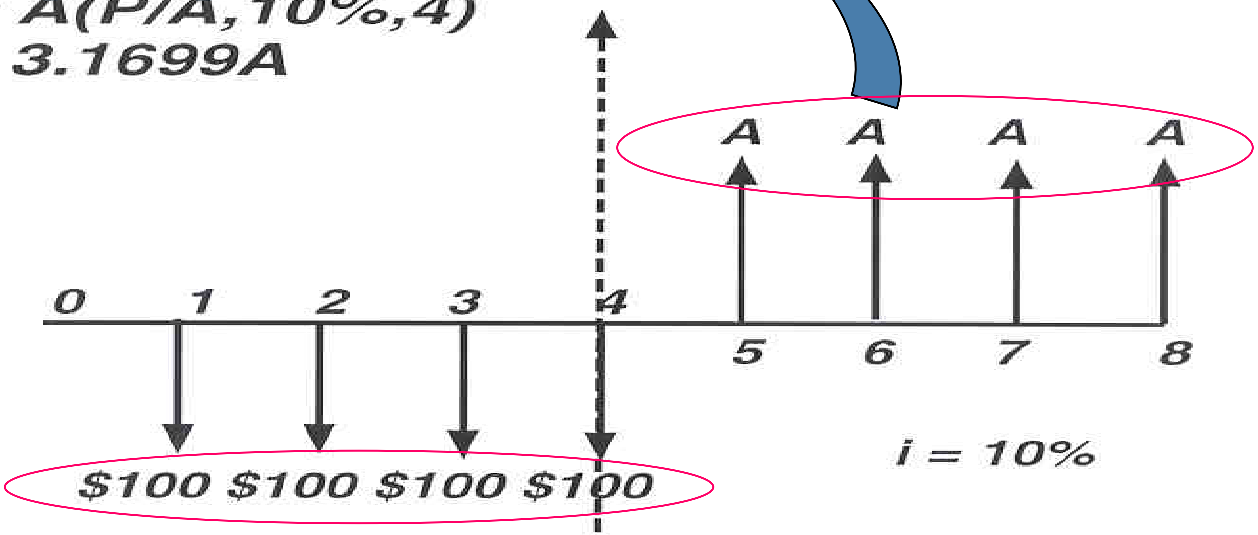
$$P_1 = P_2$$

$$2.1651A = \$316.99$$

$$\boxed{A = \$146.41}$$

## Method 2: Establish economic equivalence at period 4

$$V_{4,1} = A(P/A, 10\%, 4) = 3.1699A$$




$$V_{4,2} = \$100(F/A, 10\%, 4) = \$464.10$$

$$3.1699A = \$464.10$$

$$\boxed{A = \$146.41}$$

# Summary

- Money has a time value because it can earn more money over time.
- Economic equivalence exists between individual cash flows and/or patterns of cash flows that have the same value. Even though the amounts and timing of the cash flows may differ, the appropriate interest rate makes them equal.
- The purpose of developing various interest formulas was to facilitate the economic equivalence computation.

# Assignment 1

- Chapter 2: 2.1, 2.3
- Chapter 3: 3.3, 3.5
- Chapter 4: 4.1, 4.5, 4.7, 4.9, 4.13, 4.19, 4.22, 4.26, 4.35, 4.40, 4.45, 4.49, 4.52

# End of Lecture 3