Engineering Economics

Lecture 6

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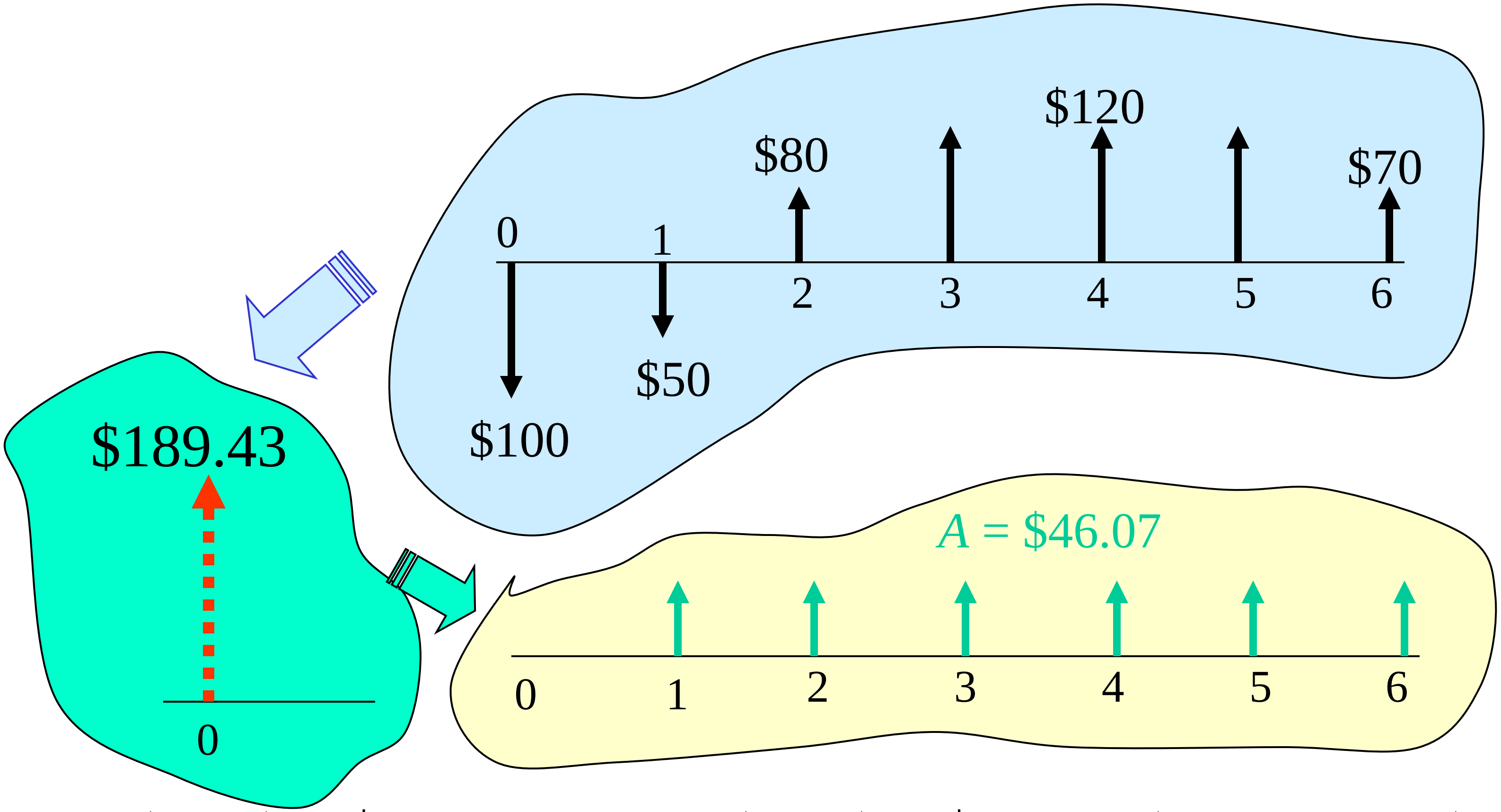
Chapter 8
Annual Equivalent Worth Analysis

- Annual equivalent criterion
- Applying annual worth analysis
- Mutually exclusive projects
- Design economics

Annual Worth Analysis

- Principle: Measure investment worth on annual basis
- Benefit: By knowing annual equivalent worth, we can:
  - Seek consistency of report format
  - Determine unit cost (or unit profit)
  - Facilitate unequal project life comparison

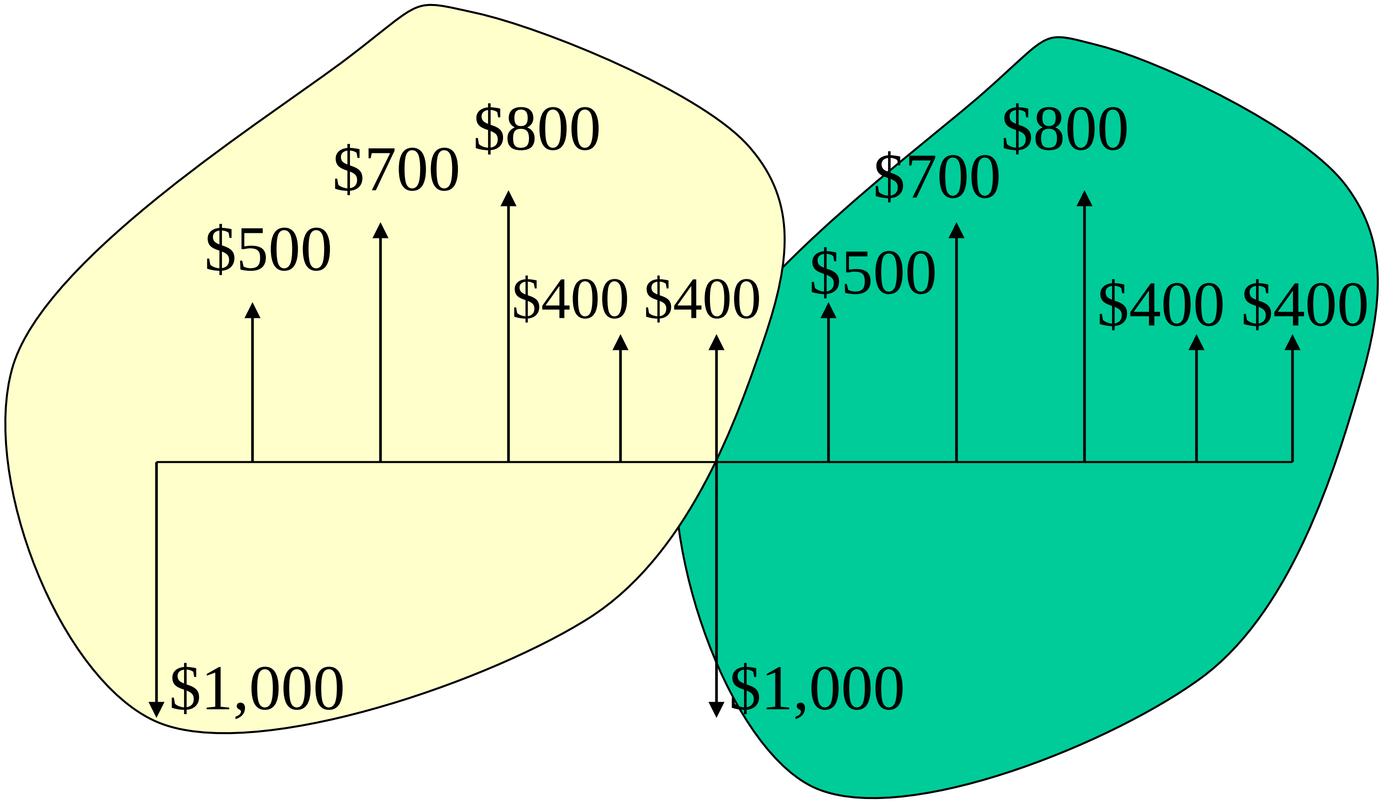
Computing Equivalent Annual Worth



PW(12%) = $189.43

AE(12%) = $189.43($A/P$, 12%, 6)
= $46.07

Annual Equivalent Worth - Repeating Cash Flow Cycles



Repeating cycle

- **First Cycle**:

$$PW(10\%) = -\$1{,}000 + \$500\,(P/F, 10\%, 1) + \ldots + \$400\,(P/F, 10\%, 5) = \$1{,}155.68$$

$$AE(10\%) = \$1{,}155.68\,(A/P, 10\%, 5) = \boxed{\$304.87}$$

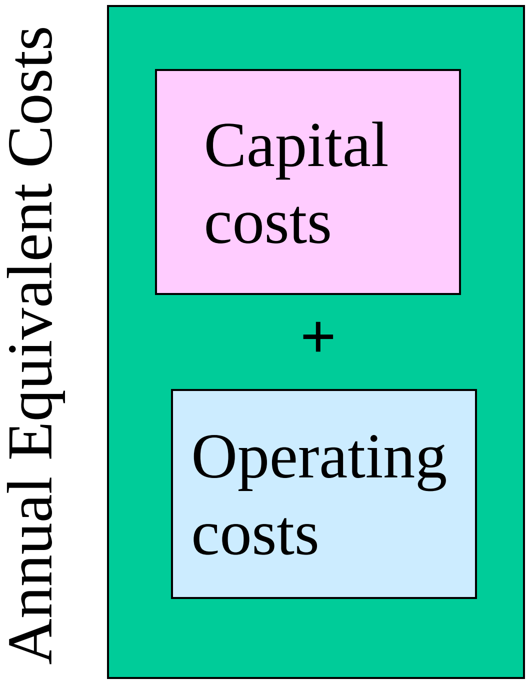
- **Both Cycles**:

$$PW(10\%) = \$1{,}155.68 + \$1{,}155.68\,(P/F, 10\%, 5) + \ldots + \$400\,(P/F, 10\%, 5) = \$1{,}873.27$$

$$AE(10\%) = \$1{,}873.27\,(A/P, 10\%, 10) = \boxed{\$304.87}$$

Annual Equivalent Cost

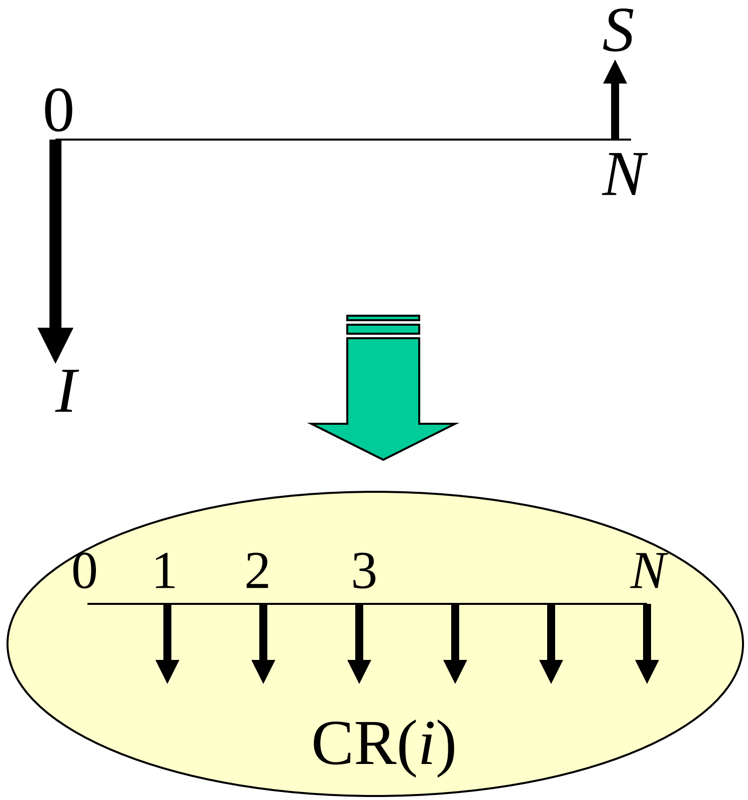
- When only costs are involved, the AE method is called the annual equivalent cost.
- Revenues must cover two kinds of costs: Operating costs and capital costs.



Capital (Ownership) Costs

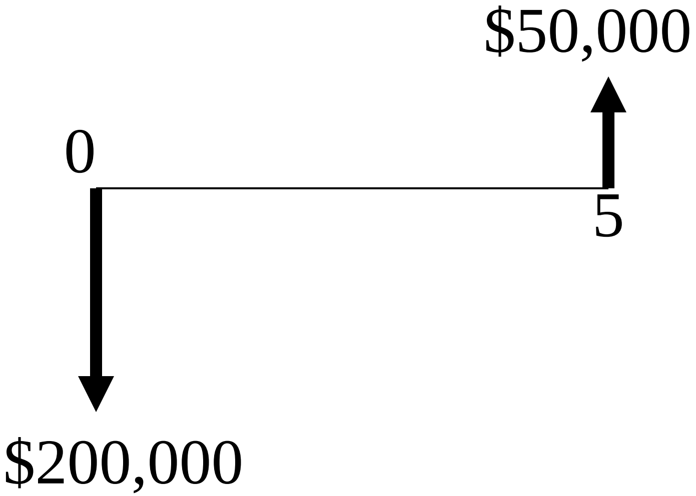
- Def: The cost of owning an equipment is associated with two transactions—(1) its initial cost (*I*) and (2) its salvage value (*S*).
- Capital costs: Taking into these sums, we calculate the capital costs as:

$$CR(i) = I(A/P,i,N) - S(A/F,i,N)$$
$$= (I-S)(A/P,i,N) + iS$$



Example - Capital Cost Calculation

- **Given**:
  - I = $200,000
  - N = 5 years
  - S = $50,000
  - i = 20%
- **Find**: CR(20%)



$$CR(i) = (I - S)(A/P,\ i,\ N) + iS$$

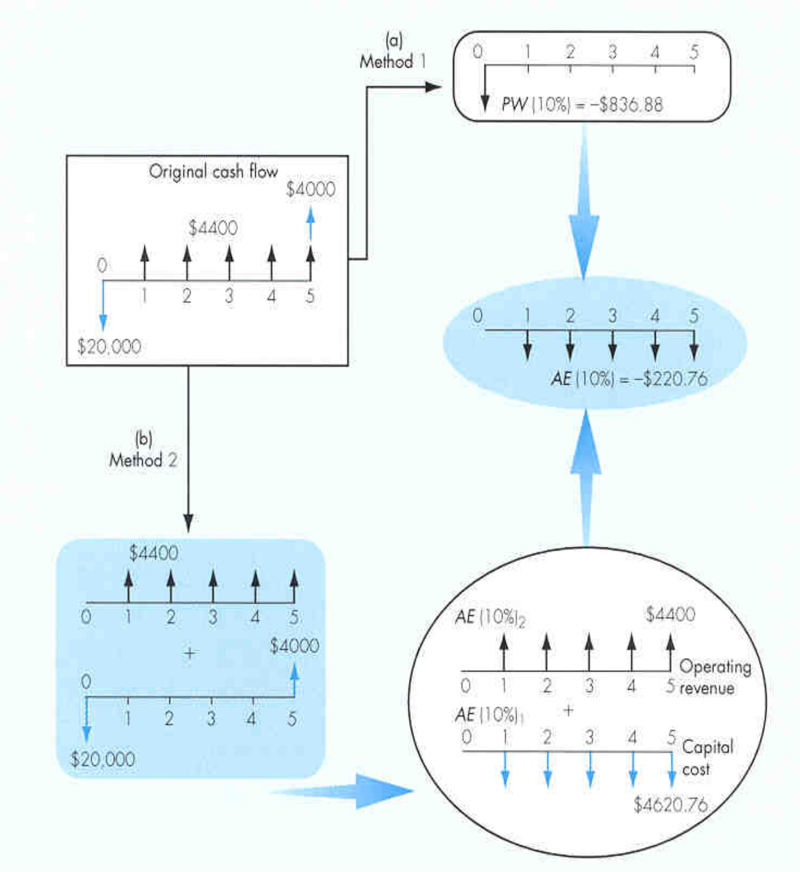
$$CR(20\%) = (\$200{,}000 - \$50{,}000)(A/P,\ 20\%,\ 5) + (0.20)\$50{,}000 = \$60{,}157$$

Justifying an investment based on AE Method

- Given: I = \$20,000, S = \$4,000, N = 5 years, i = 10%
- Find: see if an annual revenue of \$4,400 is enough to cover the capital costs.
- Solution:

  CR(10%) = \$4,620.76

- Conclusion: Need an additional annual revenue in the amount of \$220.76.

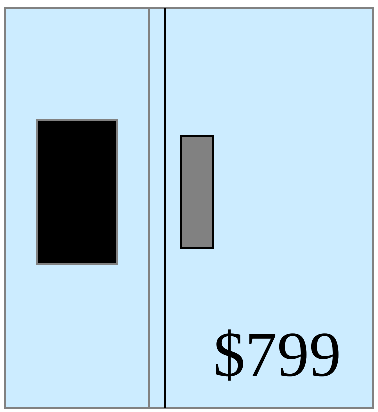


Applying Annual Worth Analysis

- Unit Cost (Profit) Calculation
- Unequal Service Life Comparison
- Minimum Cost Analysis

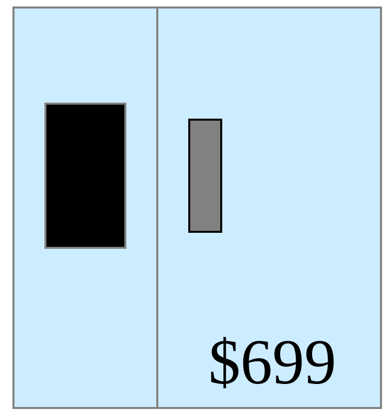
Which Brand Would you Pick?

Brand A



$55/Year

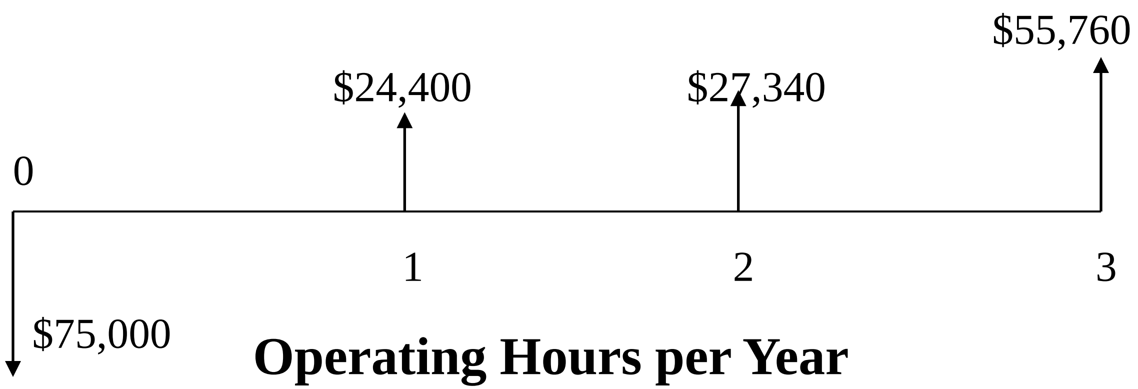
Brand B



$85/Year

How Would you calculate the hourly operating cost?

Example 8.4
Equivalent Worth per Unit of Time



Operating Hours per Year

2,000 hrs.	2,000 hrs.	2,000 hrs.

- **PW (15%) = $3553**
- **AE (15%) = $3,553 (*A*/*P*, 15%, 3)**

  = $1,556
- **Savings per Machine Hour**

  = $1,556/2,000

  = $\boxed{\$0.78/\text{hr.}}$

Example 8.7
Breakeven Analysis

Problem:

At *i* = 6%, what should be the reimbursement rate per mile so that Sam can break even?

Year (n)	Miles Driven	O&M costs
1	14,500	$4,680
2	13,000	$3,624
3	11,500	$3,421
Total	39,000	$11,725

	First Year	Second Year	Third Year
Depreciation	$2,879	$1,776	$1,545
Scheduled maintenance	100	153	220
Insurance	635	635	635
Registration and taxes	78	57	50
Total ownership cost	$3,693	$2,621	$2,450
Nonscheduled repairs	35	85	200
Replacement tires	35	30	27
Accessories	15	13	12
Gasoline and taxes	688	650	522
Oil	80	100	100
Parking and tolls	135	125	110
Total operating cost	$988	$1,003	$971
Total of all costs	$4,680	$3,624	$3,421
Expected miles driven	14,500 miles	13,000 miles	11,500 miles

- **Equivalent annual cost of owning and operating the car**

$$[\$4{,}680\,(P/F, 6\%, 1) + \$3{,}624\,(P/F, 6\%, 2) + \$3{,}421\,(P/F, 6\%, 3)]\,(A/P, 6\%, 3)$$

$$= \boxed{\$3{,}933 \text{ per year}}$$

- **Equivalent annual Reimbursement**

Let X = reimbursement rate per mile

$$[14{,}500X(P/F, 6\%, 1) + 13{,}000X(P/F, 6\%, 2) + 11{,}500\,X\,(P/F, 6\%, 3)]\,(A/P, 6\%, 3)$$

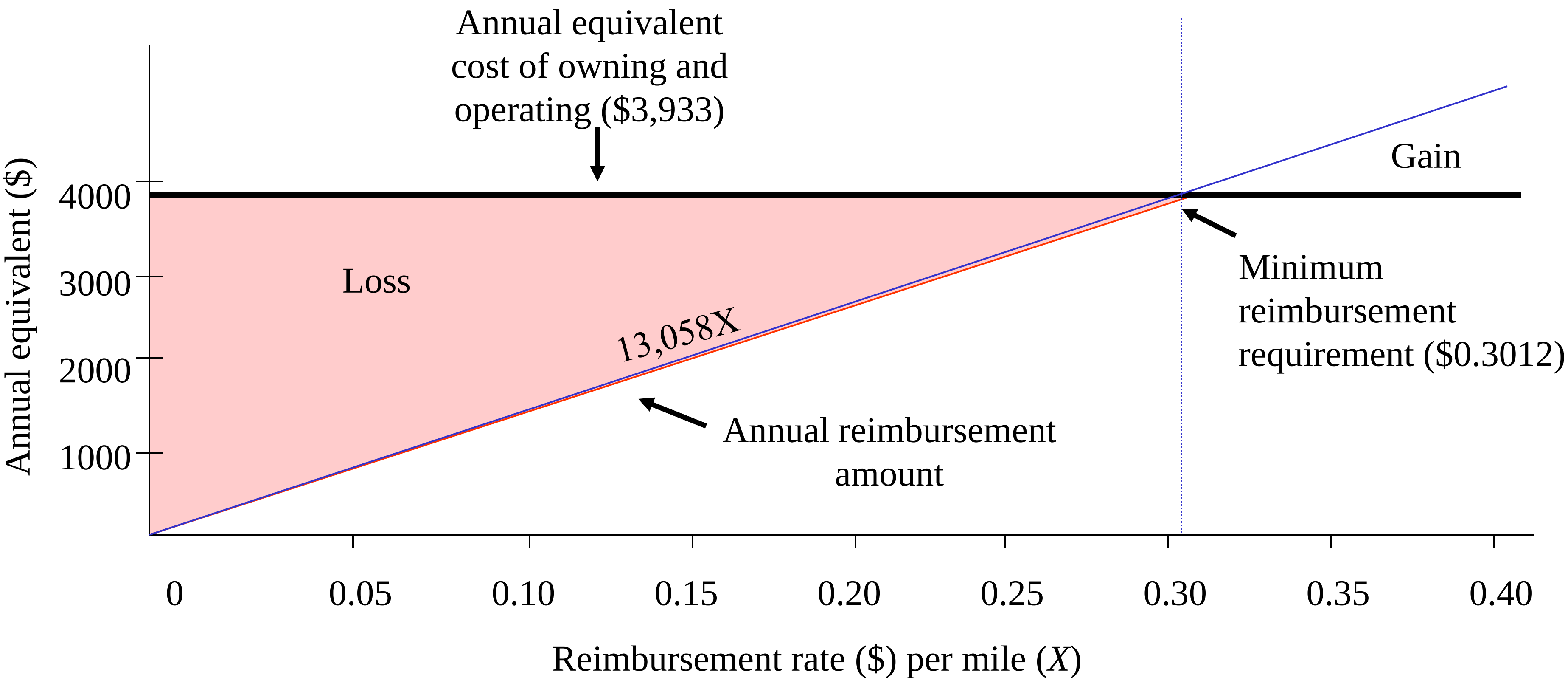
$$= \boxed{13{,}058X}$$

- **Break-even value**

$$13.058X = 3{,}933$$

$$\boxed{X = 30.12 \text{ cents per mile}}$$

Annual equivalent reimbursement as a function of cost per mile



Mutually Exclusive Alternatives with Equal Project Lives

	Standard Motor	Premium Efficient Motor
Size	25 HP	25 HP
Cost	$13,000	$15,600
Life	20 Years	20 Years
Salvage	$0	$0
Efficiency	89.5%	93%
Energy Cost	$0.07/kWh	$0.07/kWh
Operating Hours	3,120 hrs/yr.	3,120 hrs/yr.

(a) At i= 13%, determine the operating cost per kWh for each motor.
(b) At what operating hours are they equivalent?

Solution:

(a):

- **Operating cost per kWh per unit**

$$\text{Input power} = \frac{\text{output power}}{\%\ \text{efficiency}}$$

Determine total input power

Conventional motor**:**

input power = 18.650 kW/ 0.895 = $\underline{20.838\text{kW}}$

PE motor**:**

input power = 18.650 kW/ 0.930 = $\underline{20.054\text{kW}}$

- **Determine total kWh per year with 3120 hours of operation**

  Conventional motor**:**

  3120 hrs/yr (20.838 kW) = <u>65,018 kWh/yr</u>

  PE motor**:**

  3120 hrs/yr (20.054 kW) = <u>62,568 kWh/yr</u>

- **Determine annual energy costs at $0.07/kwh:**

  Conventional motor**:**

  $0.07/kwh × 65,018 kwh/yr = <u>$4,551/yr</u>

  PE motor**:**

  $0.07/kwh × 62,568 kwh/yr = <u>$4,380/yr</u>

- **Capital cost**:

  Conventional motor:

  $\$13{,}000(A/P, 13\%, 20) = \underline{\$1{,}851}$

  PE motor:

  $\$15{,}600(A/P, 13\%, 20) = \underline{\$2{,}221}$

- **Total annual equivalent cost**:

  Conventional motor:

  $\text{AE}(13\%) = \$4{,}551 + \$1{,}851 = \underline{\$6{,}402}$

  Cost per kwh = \$6,402/58,188 kwh = $\boxed{\$0.1100/\text{kwh}}$

  PE motor:

  $\text{AE}(13\%) = \$4{,}380 + \$2{,}221 = \underline{\$6{,}601}$

  Cost per kwh = \$6,601/58,188 kwh = $\boxed{\$0.1134/\text{kwh}}$

(b) break-even Operating Hours = 6,742

	Conventional Motor	Premium Efficiency Motor
Output power (hp)	25	25
Operating hours per year	6,742	6,742
Efficiency (%)	89.5	93
Initial cost ($)	$ 13,000.00	$ 15,600.00
Salvage value ($)	0	0
Service life (year)	20	20
Utility rate ($/kWh)	$ 0.07	$ 0.07
Interest rate (%)	13	13
Capital cost ($/year)	$ 1,851	$ 2,221
Energy cost ($/year)	$ 9,834	$ 9,464
Total Equ. annual cost	$ 11,685	$ 11,685
Cost per kWh	$ 0.0929	$ 0.0929

	Annual Equivalent Cost	
Operating Hours	Conventional Motor	PE Motor
0	$ 1,851	$ 2,221
500	$ 2,580	$ 2,923
1,000	$ 3,309	$ 3,624
1,500	$ 4,039	$ 4,326
2,000	$ 4,768	$ 5,028
2,500	$ 5,497	$ 5,730
3,000	$ 6,227	$ 6,432
3,500	$ 6,956	$ 7,134
4,000	$ 7,685	$ 7,836
4,500	$ 8,415	$ 8,538
5,000	$ 9,144	$ 9,240
5,500	$ 9,873	$ 9,941
6,000	$ 10,603	$ 10,643
6,500	$ 11,332	$ 11,345
7,000	$ 12,061	$ 12,047
7,500	$ 12,791	$ 12,749
8,000	$ 13,520	$ 13,451
8,500	$ 14,249	$ 14,153
8,750	$ 14,614	$ 14,504

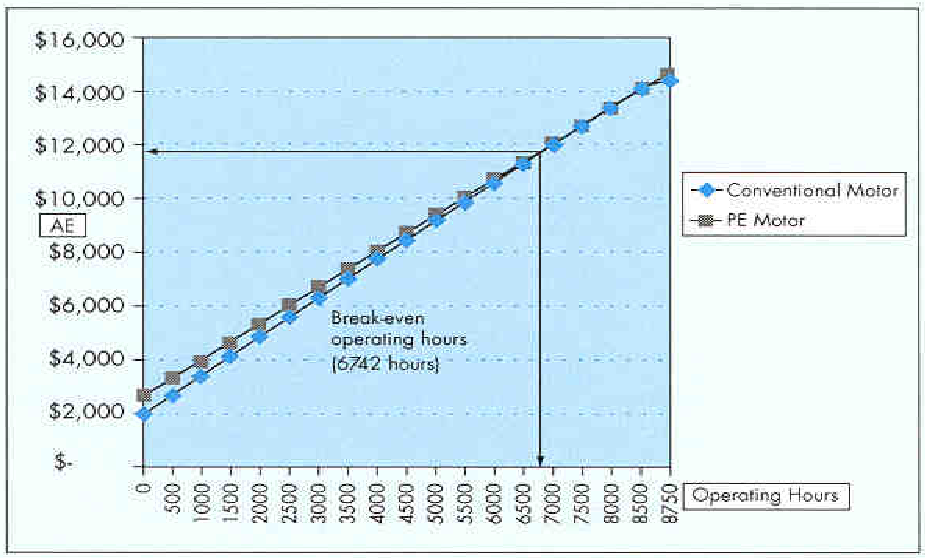
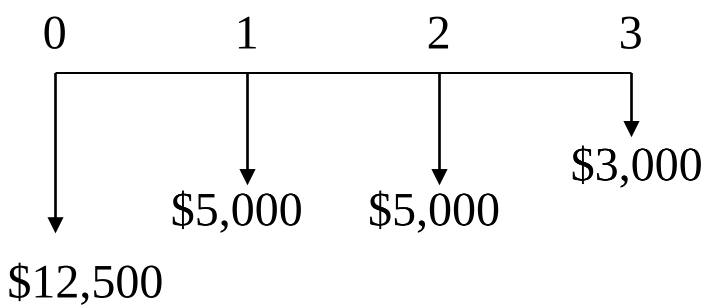


Figure 8.7 Excel's output: Break-even operating hours—a sensitivity graph

Mutually Exclusive Alternatives with Unequal Project Lives

Model A:

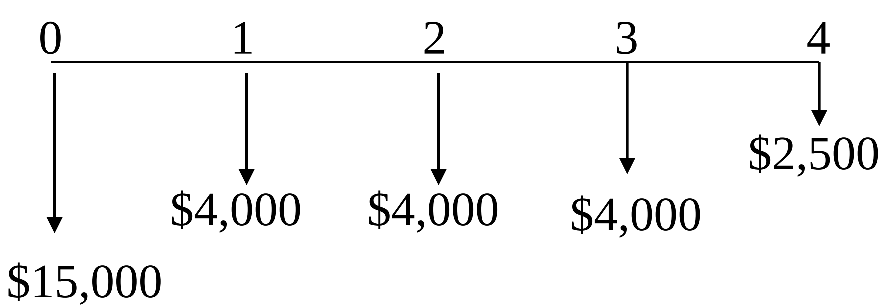


Required service Period = Indefinite

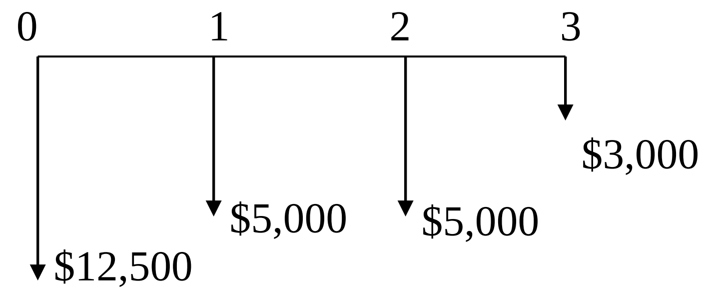
Analysis period = LCM (3,4) = 12 years

Least common multiple)

Model B:



Model A:



- First Cycle:

$$PW(15\%) = -\$12{,}500 - \$5{,}000\,(P/A, 15\%, 2) - \$3{,}000\,(P/F, 15\%, 3) = -\$22{,}601$$

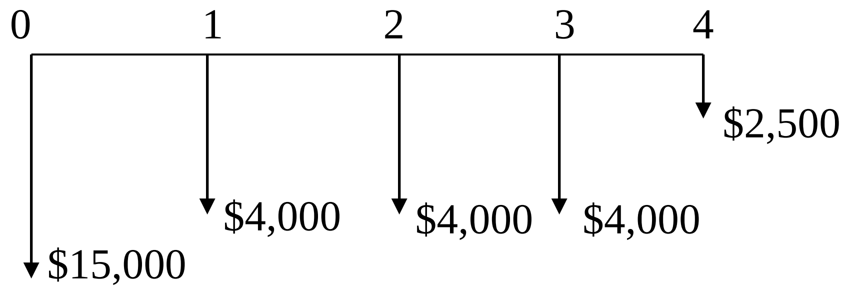
$$AE(15\%) = -\$22{,}601(A/P, 15\%, 3) = \boxed{-\$9{,}899}$$

- With 4 replacement cycles:

$$PW(15\%) = -\$22{,}601\,[1 + (P/F, 15\%, 3) + (P/F, 15\%, 6) + (P/F, 15\%, 9)] = -\$53{,}657$$

$$AE(15\%) = -\$53{,}657(A/P, 15\%, 12) = \boxed{-\$9{,}899}$$

Model B:



- First Cycle:

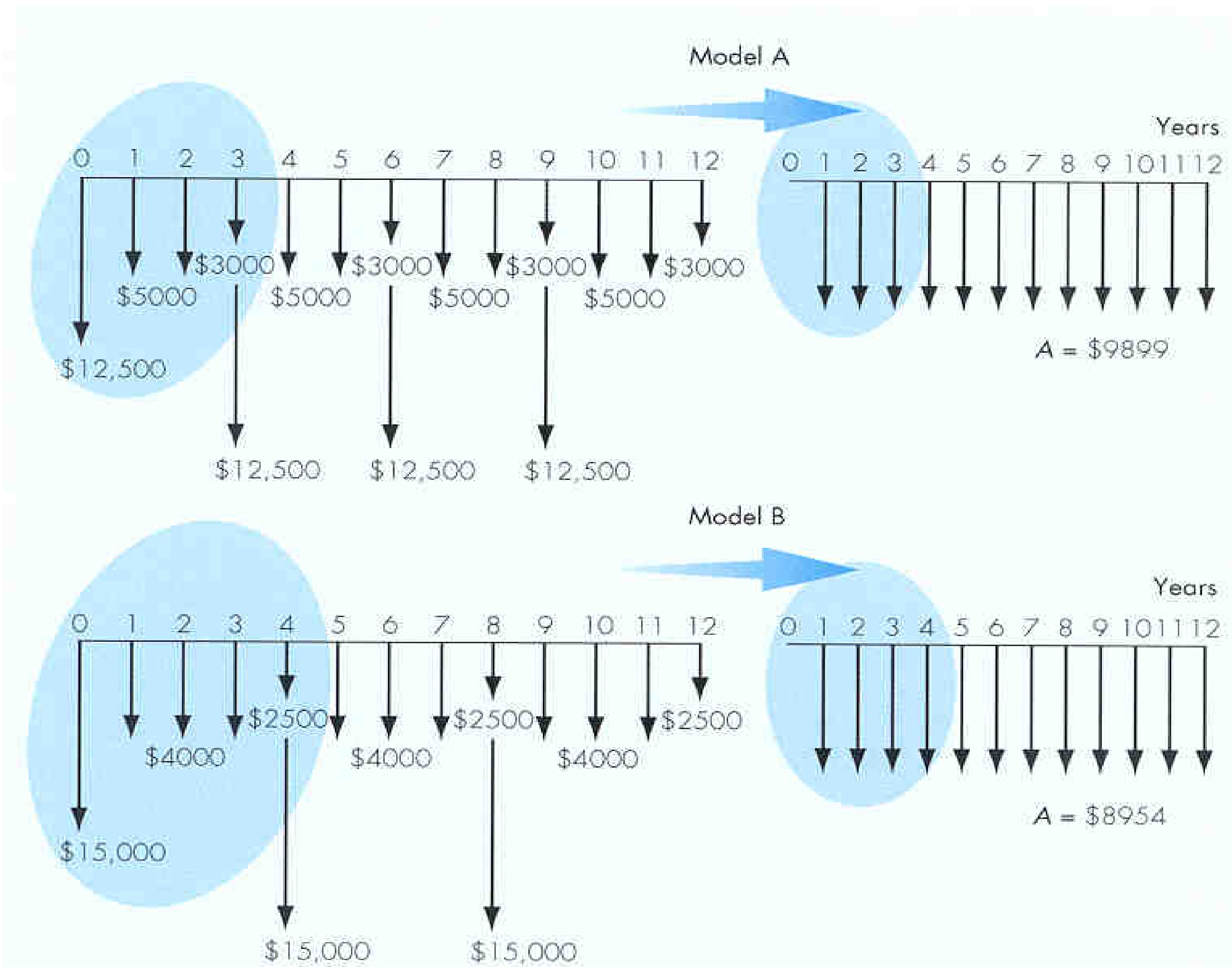
$$PW(15\%) = -\$15{,}000 - \$4{,}000\,(P/A, 15\%, 3) - \$2{,}500\,(P/F, 15\%, 4) = -\$25{,}562$$

$$AE(15\%) = -\$25{,}562(A/P, 15\%, 4) = \boxed{-\$8{,}954}$$

- With 3 replacement cycles:

$$PW(15\%) = -\$25{,}562\,[1 + (P/F, 15\%, 4) + (P/F, 15\%, 8)] = -\$48{,}534$$

$$AE(15\%) = -\$48{,}534(A/P, 15\%, 12) = \boxed{-\$8{,}954}$$



Model A
Years
0 1 2 3 4 5 6 7 8 9 10 11 12
$3000
$3000
$3000
$3000
$5000
$5000
$5000
$5000
$12,500
$12,500
$12,500
$12,500
A = $9899
Model B
Years
0 1 2 3 4 5 6 7 8 9 10 11 12
$2500
$2500
$2500
$4000
$4000
$4000
$15,000
$15,000
$15,000
A = $8954

Minimum Cost Analysis

- Concept: Total cost is given in terms of a specific design parameter
- Goal: Find the optimal design parameter that will minimize the total cost
- Typical Mathematical Equation:

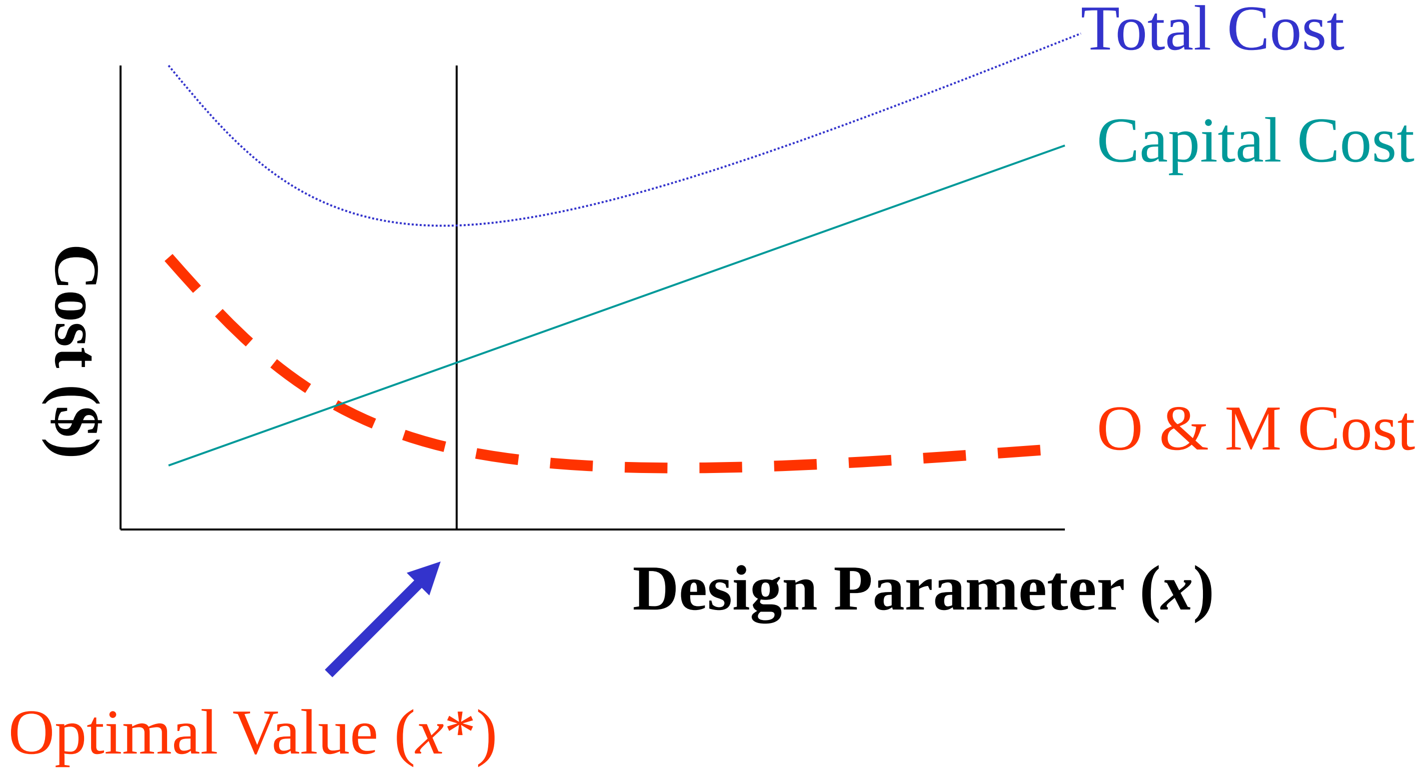
$$AE(i) = a + bx + \frac{c}{x}$$

where x is common design parameter

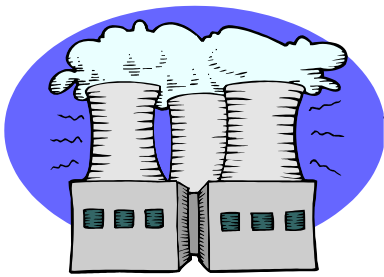
- Analytical Solution:

$$x^* = \sqrt{\frac{c}{b}}$$

Typical Graphical Relationship



Example 8.10 Optimal Cross-Sectional Area



Power Plant

Substation

A copper conductor

- Copper price: \$8.25/lb
- Resistivity: $0.8145421 \times 10^{-5} \Omega in^2/ft$
- Cost of energy: \$0.05/kwh
- density of copper: 555 lb/ft
- useful life: 25 years
- salvage value: \$0.75/lb
- interest rate: 9%

1,000 ft.
5,000 amps
24 hours
365 days

Operating Cost (Energy Loss)

- Energy loss in kilowatt-hour (L)

$$L = \frac{I^2 R}{1000A} T$$

I = current flow in amperes
R = resistance in ohms
T = number of operating hours
A = cross-sectional area

$$L = \frac{5000^2 (0.008145)}{1000A} (24 \times 365)$$
$$= \frac{1{,}783{,}755}{A} \text{kwh}$$

$$\text{Energy loss cost} = \frac{1{,}783{,}755}{A} \text{kwh}(\$0.05)$$
$$= \boxed{\frac{\$89{,}188}{A}}$$

Material Costs

- Material weight in pounds

$$\frac{1000(12)(555)A}{12^3} = 3{,}854A$$

- Material cost (required investment)

$$\text{Total material cost} = 3{,}854A(\$8.25)$$

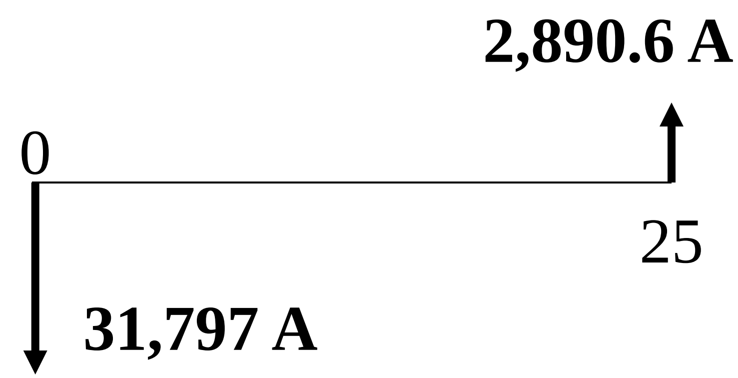
$$= 31{,}797A$$

- Salvage value after 25 years: ($0.75)(31,797$A$)

Capital Recovery Cost

Given:
Initial cost = $31,797*A*
Salvage value = $2,890.6A
Project life = 25 years
Interest rate = 9%

Find: CR(9%)



$$CR(9\%) = (31{,}797A - 2{,}890.6A)(A/P,\ 9\%,\ 25) + 2{,}890.6A(0.09)$$
$$= \boxed{3{,}203A}$$

Total Equivalent Annual Cost

- Total equivalent annual cost

  AE = Capital cost + Operating cost

  = Material cost + Energy loss

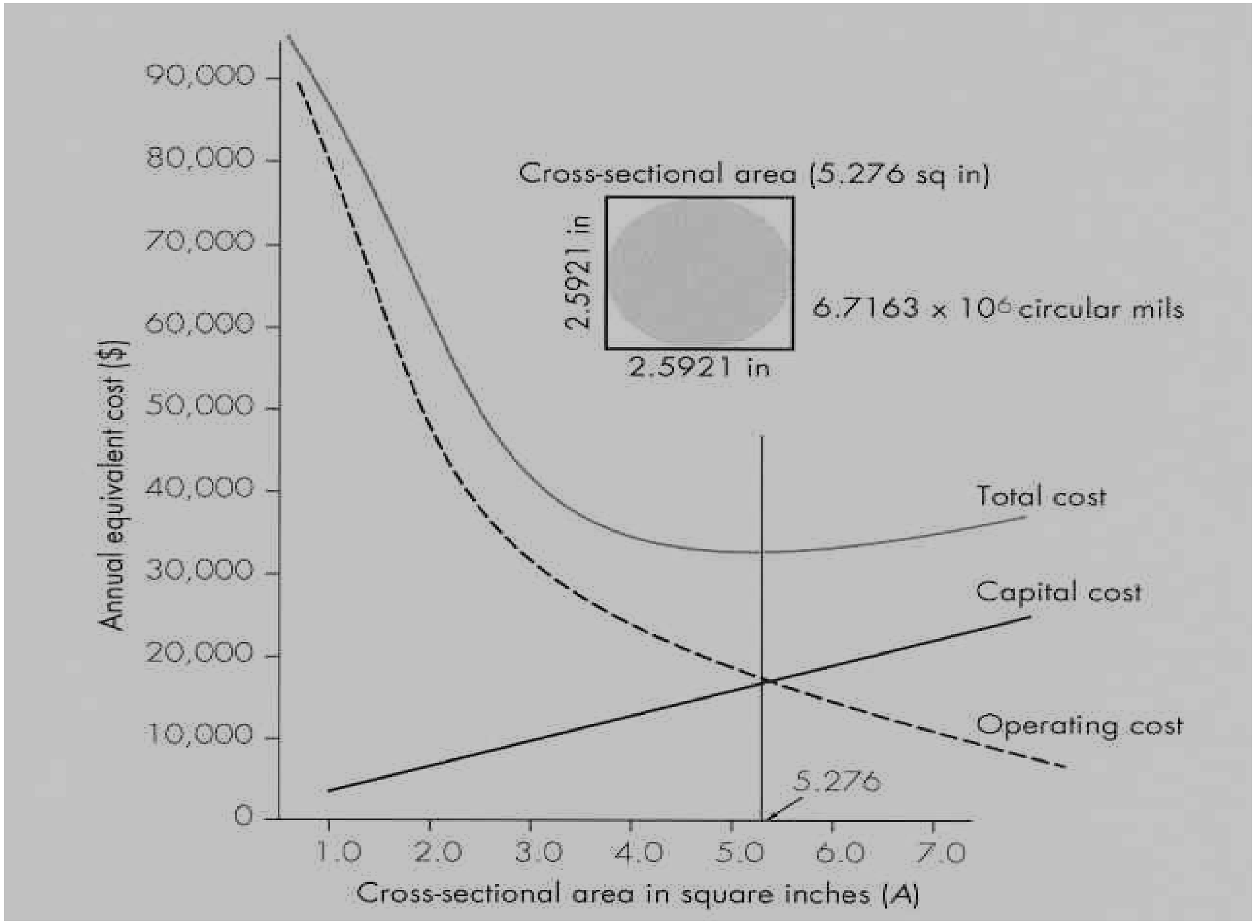
- Find the minimum annual equivalent cost

$$AE(9\%) = 3{,}203A + \frac{89{,}188}{A}$$

$$\frac{dAE(9\%)}{dA} = 3{,}203 - \frac{89{,}188}{A^2} = 0$$

$$\boxed{\begin{aligned} A^* &= \sqrt{\frac{89{,}188}{3{,}203}} \\ &= 5.276 \text{ in}^2 \end{aligned}}$$

Optimal Cross-sectional Area



Summary

- Annual equivalent worth analysis, or AE, is—along with present worth analysis—one of two main analysis techniques based on the concept of equivalence. The equation for AE is

$$AE(i) = PW(i)(A/P, i, N).$$

  AE analysis yields the same decision result as PW analysis.
- The capital recovery cost factor, or CR(i), is one of the most important applications of AE analysis in that it allows managers to calculate an annual equivalent cost of capital for ease of itemization with annual operating costs.

- The equation for CR(i) is

$$CR(i) = (I - S)(A/P, i, N) + iS,$$

where I = initial cost and S = salvage value.

- AE analysis is recommended over NPW analysis in many key real-world situations for the following reasons:

1. In many financial reports, an annual equivalent value is preferred to a present worth value.
2. Calculation of unit costs is often required to determine reasonable pricing for sale items.
3. Calculation of cost per unit of use is required to reimburse employees for business use of personal cars.
4. Make-or-buy decisions usually require the development of unit costs for the various alternatives.
5. Minimum cost analysis is easy to do when based on annual equivalent worth.

End of Lecture 6