Hydraulics

(Open Channel flow part)

Lecture notes

TU, IOE
B.E. (Civil), Second year/second part

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Chapter 5: Basics of open channel flow

5.1 Introduction

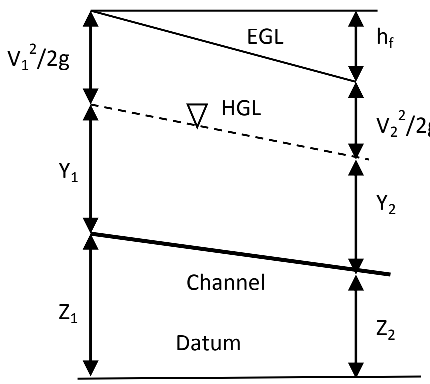
Open Channel flow/Free surface flow

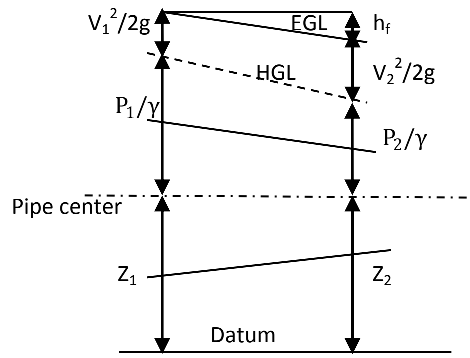
An open channel is a conduit for flow which has a free surface, which is subjected to atmospheric pressure. The free surface is actually an interface between two fluids of different density and will have constant pressure (atmospheric pressure). In case of moving fluid, the motion is caused by gravity, and the pressure distribution within the fluid is generally hydrostatic. Open channel flows are almost always turbulent and unaffected by surface tension.

Examples: flow in streams, rivers, canals, partially filled sewers

Difference between open channel and pipe flow

Aspect	Open channel	Pipe flow
Condition	Uncovered, have free surface, atmospheric pressure at free surface	Covered, no free surface
Cross-section	Any shape, e.g. rectangular, Parabolic, triangular, trapezoidal, circular, irregular	Generally circular cross section
Cause of flow	Flow due to gravity	Flow due to pressure
Surface roughness	Varies between wide limits, varies place to place	depends upon the material of the pipe
Velocity distribution	The maximum velocity at little distance below the water surface. The shape of the velocity profile dependent on the channel roughness.	The maximum velocity at the center of the flow and reducing to zero at the pipe wall velocity distribution symmetrical about the pipe axis
Piezometric head	Z+y where y = depth of flow; HGL coincides with the water surface	$Z+P/\gamma$, where p= pressure in pipe. HGL does not coincide with water surface
Surface tension	Negligible	Dominant for small diameter





Open channel flow Pipe flow

5.2 Types of flows

Basic variables: Velocity (V) or discharge (Q), flow depth (y)

a. Based on time criterion

I. Steady flow: flow propertiesat any point do not change with time (dv/dt= 0, dy/dt =0, dQ/dt =0), e.g. flow of water through a channel at constant rate
II. Unsteady flow:flow propertiesat any point do not change with time (dv/dt≠ 0, dy/dt≠ 0, dQ/dt≠ 0), e.g. flood flow

b. Based on space criterion

I. Uniform flow: flow properties do not vary with distance (dv/dx= 0, dy/dx =0) e.g. flow through a channel of constant cross-section
II. Non-uniform or varied flow: flow properties vary with distance (dv/dx ≠ 0, dy/dx ≠ 0) e.g. flow through natural channel

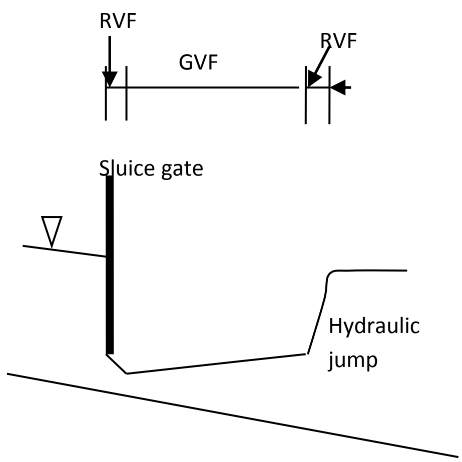
c. Based on both time and space criteria

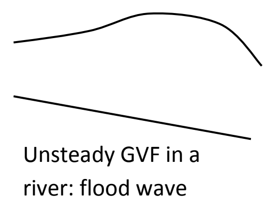
- Steady uniform flow, e.g. flow through a channel of constant cross-section at a constant rate
- Unsteady uniform flow,e.g. flow through a channel of constant cross-section at varying rate
- Steady non-uniform flow, e.g. flow behind a dam
- Unsteady non-uniform flow, e.g. flood flow

Steady uniform flow is the fundamental type of flow treated in open channel hydraulics. Unsteady uniform flow is rare. For such flow, depth of flow varies with time, but remains constant with distance, which is conceptually possible, but not possible theoretically.

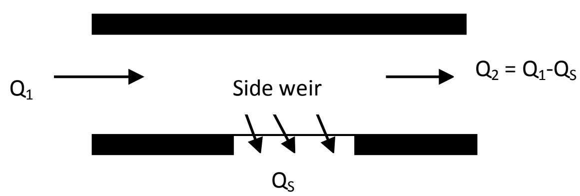
Classification of non-uniform flow

The non-uniform flow is classified as either gradually varied flow or rapidly varied flow. If the depth of flow changes rather slowly with distance, the flow is said to be gradually varied flow GVF, Examples of steady GVF:flow profile behind the dam, flow profile d/s of sluice gate from the vena-contract. Example of unsteady GVF: the passage of flood wave in a river. If the depth of flow changes rapidly over a relatively short distance, the flow is said to be rapidly varied flow, e.g. hydraulic jump, flow over a weir, flow below the sluice gate up to vena-contracta, hydraulic drop, surge (sudden change in flow that increases or decreases the depth), bore (surge of tidal origin).





Spatially varied flow (SVF): If there is addition to or withdrawal of flow from the system, the resulting varied flow is known as spatially varied flow.
Example of steady SVF: flow over a side weir
Example of unsteady SVF: flow due to rainfall, flow through gutters



Side weir (Plan)

c. Depending on the effect of viscosity relative to inertia
Reynolds number (Re) = Ratio of inertia force to viscous force
$Re = \frac{\rho VL}{\mu}$ $or \frac{VL}{\upsilon}$where ρ =density of fluid, V = mean velocity of flow, L = Characteristic length, μ = dynamic viscosity, υ = kinematic viscosity
For open channel flow, hydraulic radius (R) is taken as characteristic length.
R = cross-sectional area/wetted perimeter
I. Laminar flow: strong viscous force, very small velocity, motion of fluid in layers, occurs for Reynold no.<500 in open channel flow, e.g. flow through smooth pipe having low velocity, groundwater flow, a thin film of liquid flowing down an inclined/vertical plane

II. Turbulent flow: weak viscous force, irregular motion of fluid, occurs for Reynold no.≥ 2000in open channel flow, e.g. flow through river, high velocity flow in a conduit of large size

[For pipe flow: Re<2000: laminar and Re>4000: turbulent

For open channel, R= D/4 (D= dia. of pipe)=L. So the value is ¼ of pipe flow. But for turbulent flow, the limit is usually taken as 2000.)

d. Depending on the effect of gravity relative to inertia

Froude number (F_r) = Ratio of inertia force to gravity force

$F_r = \frac{V}{\sqrt{gL}}$ where V = mean velocity of flow, g = acceleration due to gravity, L = Characteristic length.

For open channel, L =hydraulic depth (D) where D = Cross-sectional area/width of free surface = A/T

Based on F_r, the flow is classified into three types.

I. Subcritical (tranquil): $F_r<1$, low velocity, large depth, flow controlled by downstream conditions, occurs on flat streams

II. Critical: $F_r = 1$

III. Supercritical (shooting or rapid): $F_r>1$, high velocity, small depth, flow controlled by upstream conditions, occurs on steep streams

5.3 Classification of open channel

1. On the basis of cross- sectional form of the channel
    a. Natural channel- irregular shape, developed in a natural way and not significantly improved by humans, e.g. River, streams, estuaries etc.
    b. Artificial channel- developed by human efforts, regular shape, e.g. irrigation and power canals, navigation channels, gutters and drainage ditches etc.
       Rectangular channel, trapezoidal channel, triangular channels etc
2. According to shape of channel
    a. Prismatic channel –constant cross-section and bottom slope along the length
    b. Non- prismatic channel –Varying cross-section or bottom slope along the length
3. According to type of boundary
    a. Rigid boundary channel –Not deformable boundary, e.g. lined canal, sewers and non-erodible canals
    b. Mobile boundary channel – Boundaries undergo deformation due to the continuous erosion and deposition due to the flow.

Types of artificial open channel

- Canal: long channel of mild slope
- Flume: channel of wood, metal, concrete or masonry, built above the ground surface to convey flow across depression
- Chute: steep slope channel
- Drop: Steep slope channel, in which the change in elevation is effected in a short distance

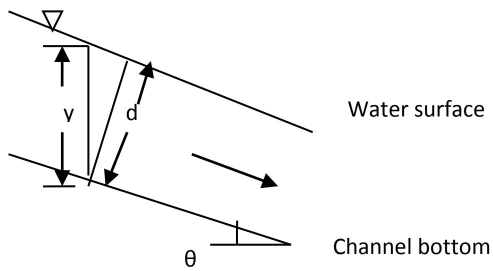
- Culvert flowing partially full: covered channel used to convey a flow under highways, railroad embankments or runways

5.4 Geometric properties of channel section

Channel section: cross-section of the channel taken normal to the direction of flow

The following are the geometric elements of basic importance.

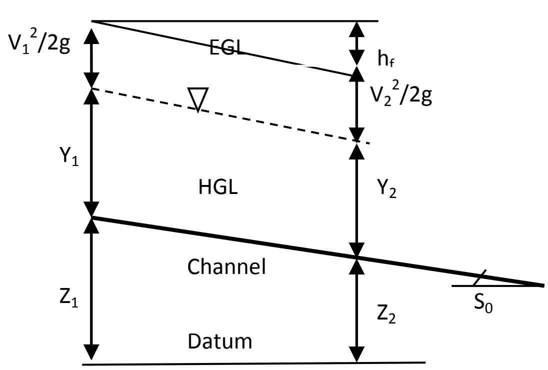
a) Depth of flow (y): Vertical distance from the lowest point of the channel section to the water surface
b) Depth of flow section (d): Depth of flow normal to the direction of flow.



c) Stage: elevation of water surface relative to datum
d) Top width (T): Width of channel section at the water surface.
e) Flow area or water area (A) : cross sectional area of the flow normal to the direction of flow.
f) Wetted perimeter (p): the length of the line which is the interface between the fluid and the channel boundary.
g) Hydraulic radius (R): the ratio of the flow area (A) to the wetted perimeter (P).
   R=A/P
h) Hydraulic depth (D): Ratio of the flow area (A) to the top width (T)
   D=A/T
i) Section factor for critical flow computation (Z): product of the flow area and the square root of the hydraulic depth.

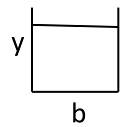
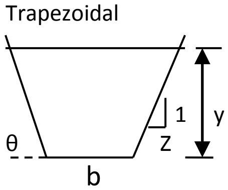
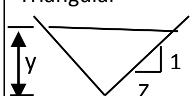
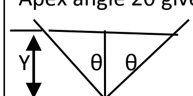
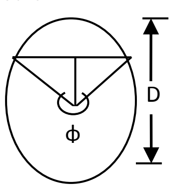
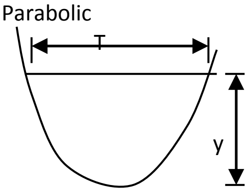
$$Z = A\sqrt{D}$$

j) Channel slope or bed slope or bottom slope or longitudinal slope (S_0) : Inclination of the channel bed
k) Water surface slope (S_w): slope made by water surface
l) Energy slope (S_f): The energy gradient line represents the sum of pressure head, velocity head and elevation head at any point along the channel. The angle of inclination of energy gradient line with the horizontal is called energy slope.

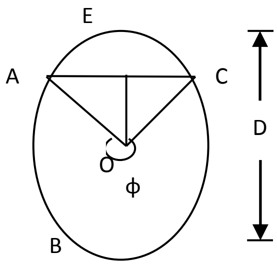


$S_f = h_f/L$, where h_f = Head loss between 1 and 2, L = Length of reach

Channel section geometric elements of channels of common shape

Channel	A	P	R	T	D
Rectangular 	*by*	*b+2y*	$\frac{by}{b+2y}$	*b*	*y*
Trapezoidal 	*(b+Zy)y*	$b+2y\sqrt{1+Z^2}$	$\frac{(b+Zy)y}{b+2y\sqrt{1+Z^2}}$	*b+2Zy*	$\frac{(b+Zy)y}{b+2Zy}$
$tan\theta = 1/z$ In terms of θ	$\left(b+\frac{y}{tan\theta}\right)y$	$\left(b+\frac{2y}{sin\theta}\right)$	$\frac{\left(b+\frac{y}{tan\theta}\right)y}{\left(b+\frac{2y}{sin\theta}\right)}$	$\left(b+\frac{2y}{tan\theta}\right)$	$\frac{\left(b+\frac{y}{tan\theta}\right)y}{\left(b+\frac{2y}{tan\theta}\right)}$
Triangular 	Zy^2	$2y\sqrt{1+Z^2}$	$\frac{Zy}{2\sqrt{1+Z^2}}$	*2Zy*	*y/2*
Apex angle 2θ given 	$y^2 tan\theta$	$\frac{2y}{cos\theta}$	$\frac{ysin\theta}{2}$	$2ytan\theta$	*y/2*
Circular 	$\frac{D^2}{8}(\varphi - Sin\varphi)$	ϕD/2	$\frac{D}{4}(1 - Sin\varphi/\varphi)$	$DSin\frac{\varphi}{2}$	$\frac{D}{8}\frac{(\varphi - Sin\varphi)}{Sin\frac{\varphi}{2}}$
Parabolic 	$\frac{2}{3}Ty$	$T+\frac{8}{3}\frac{y^2}{T}$	$\frac{2T^2y}{3T^2+8y^2}$	$\frac{3}{2}\frac{A}{y}$	$\frac{2}{3}y$

Circular section



Flow depth above center

Flow area (A) = Area of circle- (Area of sector AOCE-Area of triangle AOC)

$$= \frac{\pi}{4}D^2 - \left[\frac{2\pi-\varphi}{2\pi}\frac{\pi}{4}D^2 - \frac{1}{2}D\,sin\left(\frac{2\pi-\varphi}{2}\right)\frac{D}{2}cos\left(\frac{2\pi-\varphi}{2}\right)\right]$$

$$= \frac{D^2}{8}(\varphi + \sin(2\pi-\varphi))$$

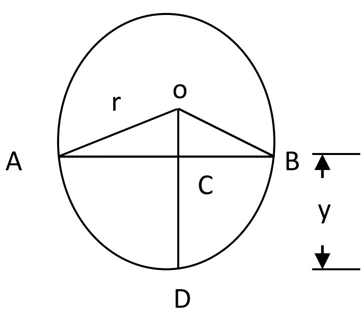
$$= \frac{D^2}{8}(\varphi - Sin\varphi)$$

Wetted perimeter (P) = $\frac{\pi D}{2\pi}\varphi = \frac{\varphi D}{2}$

(P is πD for angle 2π)

Top width (T) = AC = $D\,sin\left(\frac{2\pi-\varphi}{2}\right)$ = $Dsin\frac{\varphi}{2}$

Flow depth below center



θ = angle AOC = angle COB, radius of circle = r, y = depth of flow

Wetted perimeter (p) = $\frac{2\pi r}{2\pi}2\theta$ = 2rθ

Wetted area (A) = Area ADBA =Area of sector OADBO-Area of ΔOAB

$$= \frac{\pi r^2}{2\pi}2\theta - \frac{1}{2}.2rsin\theta.rcos\theta = r^2\left(\theta - \frac{1}{2}sin2\theta\right)$$

In terms of dia. (D)

$$= \frac{D^2}{8}(2\theta - sin2\theta)$$

Chapter 6: Uniform flow in open channel

Conditions of uniform flow in a prismatic channel

Uniform flow in channel occurs under following conditions.

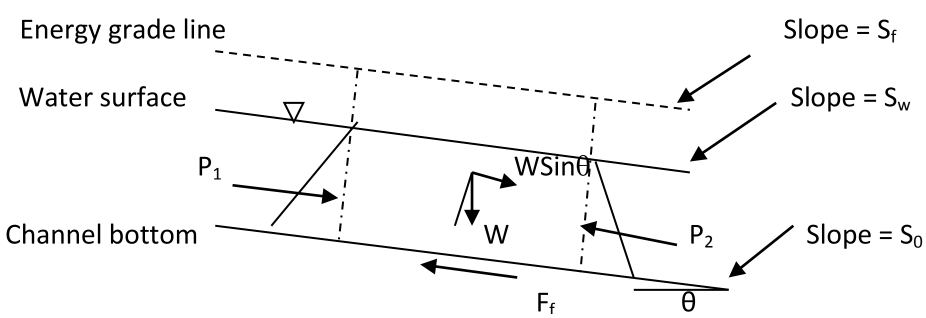
I. The depth, flow area, velocity and discharge at every section of the channel reach are constant.
II. The total energy line, water surface and the channel bottom are all parallel, or $S_f = S_w = S_o = S$
where S_f = energy line slope, S_w = water surface slope and S_o = channel bottom slope.

Although truly uniform flow seldom occurs in nature, the concept of uniform flow is central to understanding the solution of most problems in open channel hydraulics.

In general, uniform flow can occur only in very long, straight prismatic channels where a terminal velocity of flow can be achieved, i.e. the head loss due to turbulent flow is exactly balanced by the reduction in potential energy due to the uniform decrease in the elevation of the bottom of the channel.

6.1 Shear stress on the boundary

When water flows in an open channels resistance is encountered by the water as it flows downstream.



Let W = weight of water contained, L = Length of channel reach, A =cross-sectional area, θ = angle of inclination of channel bottom with the horizontal, P_1 and P_2 = pressure force at 1 and 2, V_1 and V_2 = Velocity at 1 and 2, τ_o = Boundary shear stress acting over the area of contact, γ = Specific weight of water

Applying momentum equation

$$P_1 - P_2 + W sin\theta - F_f = \rho Q(V_2 - V_1)$$

For uniform flow, $P_1 = P_2$ and $V_1 = V_2$,and $S_o = S_w = S_f = S = \tan\theta = \sin\theta$
With these simplifications,

$$W sin\theta = F_f$$

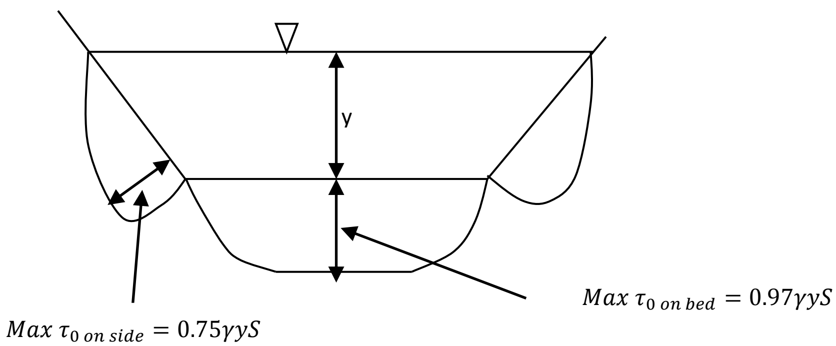
$\gamma ALS = \tau_0 PL$
This shows that the force of resistance is equal to the component of gravity force.

$$\tau_0 = \frac{\gamma AS}{P}$$

$\tau_0 = \gamma RS$
Where R = hydraulic radius

Shear stress distribution

- Average shear stress on the boundary, $\tau_0 = \gamma RS$
- Average shear stress on the boundary is not uniformly distributed due to the turbulence and secondary circulation.
- $\tau_0 = 0$ at the intersection of water surface with the boundary and at the corners of the boundary.
- Local maxima of τ_0 occurs on the bed and sides.



Example: Shear stress distribution for b/y = 4 and z:1 = 1.5:1

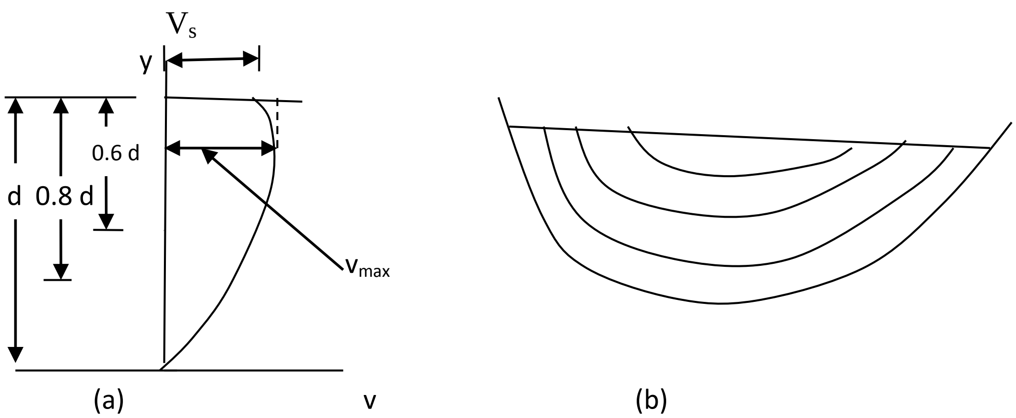
6.2 Velocity distribution in open channels

The velocity of flow at any channel section is not uniformly distributed due to the presence of a free surface and the frictional resistance along the channel boundary. The velocity is zero at the solid boundary and gradually increases with the increase in distance from the boundary. The maximum velocity occurs at some distance below the free surface. At the free surface, the velocity is less than the maximum value due to the secondary circulation and air resistance. Secondary circulation is due to the turbulence in the flow from the channel boundary to the channel center.

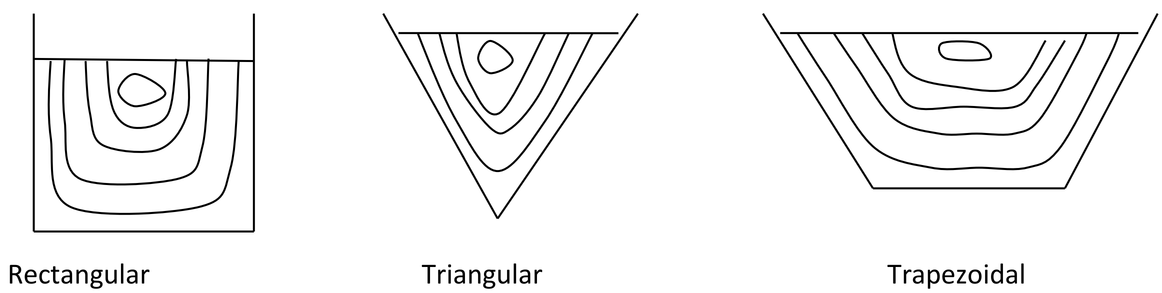
Some of the features of velocity distribution in open channel are as follows.

- As we go away from the bank to the center section, the point of maximum velocity shifts upward and at the central section, the maximum velocity occurs almost at the free surface.

- The velocity profile in vertical is logarithmic up to the maximum velocity point.
- Maximum velocity usually occurs below the free surface at a distance of 0.05 to 0.25 of the depth of flow. The higher value is applicable to section closer to the bank.
- For deep narrow channel, the location of maximum velocity point will be much lower than for a wider channel of same depth. In a broad, rapid and shallow stream or in a very smooth channel, the maximum velocity may often be found at the free surface.
- The mean velocity (V_{av}) = velocity at 0.6d where d= depth from the free surface
  or, V_{av} = (V at 0.2d + V at 0.8d)/2
- The mean velocity in the vertical = 0.85 to 0.95 times the surface velocity. The smaller value applies to the shallower streams.



Velocity distribution in open channel in (a) vertical direction, (b) across section



Velocity profiles for different cross-sections

Velocity profile for laminar flow in open channel

The velocity distribution in laminar flow in open channel is parabolic.

$v = \frac{\gamma}{\mu} S \left(Dy - \frac{y^2}{2} \right)$ where D = Depth of channel, y = flow depth, S = slope of channel

Velocity profile for turbulent flow in open channel

For turbulent flow, velocity distribution can be obtained from Prandtl's the mixing length hypothesis (as in pipe flow)

$\frac{v}{V_*} = 5.75 log_{10}\,(y/y')$ where v = velocity at any point in vertical, V_* = shear velocity, y = flow depth, y' = small depth from wall at which v is zero.
The velocity distribution in turbulent flow is logarithmic.

Simplifications (as in pipe flow)
For smooth surfaces: $\frac{v}{V_*} = 5.75 log_{10}\left(\frac{V_* y}{\upsilon}\right) + 5.5$
For rough surfaces: $\frac{v}{V_*} = 5.75 log_{10}(y/k) + 8.5$

6.3 Velocity distribution coefficients

The computation of kinetic energy at a flow section is usually based on the average velocity thereby ignoring the effect of non-uniformity of velocity distribution across the section. As a result of non-uniform distribution of velocities over a channel section, the velocity head of flow is generally greater than the value computed by using the expression $V^2/2g$, where V is the mean velocity. The true velocity is expressed as $\alpha V^2/2g$, where α is known as energy coefficient (energy correction factor) or Coriolis coefficient (first proposed by G. Coriolis).

The non-uniform distribution of velocities also affects the computation of momentum in open channel flow. The momentum of fluid passing through a channel section per unit time is expressed as $\beta\rho QV$, where β is known as momentum coefficients (momentum correction factor) or Boussinesq coefficient (first proposed by J. Boussinesq); ρ is density; Q is discharge, V is mean velocity.

Both coefficients are equal to or greater than unity. In general for straight prismatic channel α varies from 1.03 to 1.36 and β varies from 1.01 to 1.12. For straight and prismatic channel, the effect of non-uniform velocity distribution on the computed velocity head and momentum is small. Therefore, these coefficients are often assumed to be unity.

Expression for mean velocity (V)

Let v is the velocity of flowing fluid at any point through any elementary area dA.
$V = \frac{\int vdA}{A}$
where A = cross-sectional area of channel

Expression for velocity distribution coefficients

Energy coefficient (α)

Mass of fluid flowing per unit time = ρvdA

Kinetic energy of fluid =$\frac{(\rho v dA)v^2}{2} = \frac{\rho}{2} v^3 dA$

Total kinetic energy possessed by the fluid across the entire cross section=$\int \frac{\rho}{2} v^3 dA$

The kinetic energy of the flowing fluid in terms of the mean velocity 'V' of flow= $\alpha \frac{(\rho VA)V^2}{2} = \alpha \frac{\rho}{2} V^3 A$

Equating above two expressions,

$$\alpha = \frac{\int v^3 dA}{V^3 A}$$

Momentum coefficient (β)

Mass of fluid flowing per unit time = $\rho v dA$

Momentum of the fluid =$(\rho v dA)v$ = $\rho v^2 dA$

Total momentum by the fluid across the entire cross section = $\int \rho v^2 \, dA$

Momentum of fluid in terms of mean velocity V = $\beta \rho V^2 A$

Equating above two expressions,

$$\beta = \frac{\int v^2 dA}{V^2 A}$$

Expressions for rectangular channel of width b and water depth (y_0)

$$V = \frac{\int v dA}{A} = \frac{\int_0^{y_0} v b dy}{b y_0} = \frac{1}{y_0} \int_0^{y_0} v dy$$

$$\alpha = \frac{\int v^3 dA}{V^3 A} = \frac{\int_0^{y_0} v^3 b dy}{V^3 b y_0} = \frac{\int_0^{y_0} v^3 dy}{V^3 y_0}$$

$$\beta = \frac{\int v^2 dA}{V^2 A} = \frac{\int_0^{y_0} v^2 b dy}{V^2 b y_0} = \frac{\int_0^{y_0} v^2 dy}{V^2 y_0}$$

If v is known or expressed as an algebraic function of y, then α and β can be obtained by integration. If the relationship for v-y does not exist, the result of actual measurement or graphical method is used to determine the coefficients.

Graphical approach for given velocity distribution curve

$V = \frac{1}{y_0} \int_0^{y_0} v dy = \frac{Area\ under\ v{-}y\ curve}{y_0}$

$\alpha = \frac{\int_0^{y_0} v^3 dy}{V^3 y_0} = \frac{Area\ under\ v^3{-}y\ curve}{V^3 y_0}$

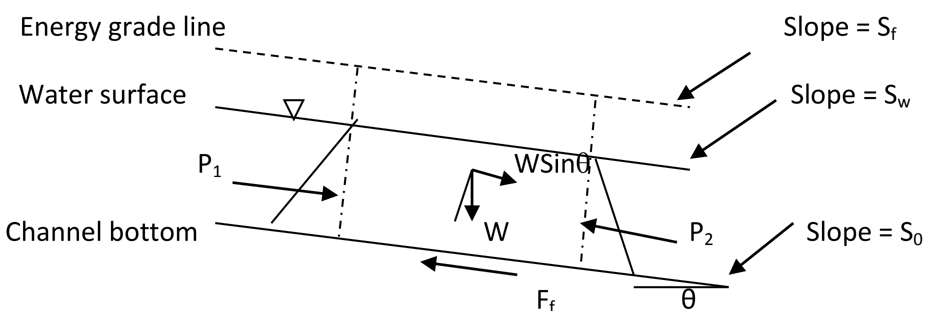
$\beta = \frac{\int_0^{y_0} v^2 dy}{V^2 y_0} = \frac{Area\ under\ v^2{-}y\ curve}{V^2 y_0}$

6.4 Computation of velocity in uniform flow: Resistance equations

a. Chezy equation

The Chezy's formula is based on the following two assumptions:

- The force resisting the flow per unit area is proportional to the square of the velocity.
- The component of gravity force causing the flow is equal to the force of resistance



Let W= weight of water contained, L = Length of channel reach, A =cross-sectional area, θ = angle of inclination of channel bottom with the horizontal, P_1 and P_2 = pressure force at 1 and 2, V_1 and V_2 = Velocity at 1 and 2, τ_o = Boundary shear stress acting over the area of contact, γ = Specific weight of water

Applying momentum equation

$$P_1 - P_2 + Wsin\theta - F_f = \rho Q(V_2 - V_1)$$

For uniform flow, $P_1 = P_2$ and $V_1 = V_2$, and $S_o = S_w = S_f = S = \tan\theta = \sin\theta$

With these simplifications

$Wsin\theta = F_f$

Resistive force $(F_f) \propto Surface\ area\ x\ V^2$

Resistive force $(F_f) = KPLV^2$ where K = coefficient, P = Wetted perimeter

$$\gamma ALS = KPLV^2$$

$$V = \sqrt{\frac{\gamma}{K}}\sqrt{RS}$$

$$V = C\sqrt{RS}$$

Where R = A/P = Hydraulic radius and C= $\sqrt{\frac{\gamma}{K}}$ = flow resistance factor known as Chezy's coefficient

This is Chezy's equation for uniform flow.

(b) Manning's formula

The Manning's formula for uniform flow in open channel flow is

$$V = \frac{1}{n} R^{2/3} {S_f}^{1/2}$$

where n = Manning's roughness coefficient or rugosity Coefficient, R= Hydraulic radius, S_f = slope of the energy line, which is equal to bed slope (S) for uniform flow. For uniform flow,

$$V = \frac{1}{n} R^{2/3} S^{1/2}$$

Manning's formula is empirical in nature. Due to its simplicity and satisfactory results it gives to practical applications, the Manning's formula has become the most widely used formula for all uniform flow computation for open channels.

(c) Darcy-Weisbach equation

For pipe flow, Darcy-Weisbach equation for head loss is

$$h_f = \frac{fLV^2}{2gD}$$

where h_f = head loss, f = friction factor, L = Length of pipe, V = Mean velocity, D = Diameter of pipe.

An open channel can be assumed to be a pipe cut into half.

Hydraulic radius (R) = $\frac{A}{P} = \frac{\pi D^2/4}{\pi D} = \frac{D}{4}$ or D = 4R

$$h_f = \frac{fLV^2}{2g.4R}$$

$$V = \sqrt{\frac{8gRh_f}{fL}}$$

$$V = \sqrt{\frac{8g}{f}} \sqrt{R\frac{h_f}{L}} = \sqrt{\frac{8g}{f}} \sqrt{RS}$$

$$V = C\sqrt{RS}$$

where $= \sqrt{\frac{8g}{f}}$, h_f/L = S = Slope of energy line = Bed slope for uniform flow

Relationship between Chezy C, Manning n and Darcy-Weisbach f

Chezy formula: $V = C\sqrt{RS}$ (a)

Manning's formula: $V = \frac{1}{n} R^{2/3} S^{1/2}$ (b)

Darcy-Weisbach formula: $V = \sqrt{\frac{8g}{f}} \sqrt{RS}$ (c)

From equations a and b

$$C\sqrt{RS} = \frac{1}{n} R^{2/3} S^{1/2}$$

$$C = \frac{1}{n} R^{1/6} \qquad \text{(d)}$$

From equations a and c, $C = \sqrt{\frac{8g}{f}}$ (e)

From equations d and e

$$\frac{1}{n} R^{1/6} = \sqrt{\frac{8g}{f}}$$

$$f = \frac{8gn^2}{R^{1/3}}$$

Equations for f

In terms of roughness, Reynold number (Rn) = $\frac{V_* K}{\upsilon}$ where V* = shear velocity, k = roughness height and υ = kinematic viscosity

$V_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{\rho g R S}{\rho}} = \sqrt{gRS}$

Rn<4: smooth

Rn>60: rough

4<Rn<60: transition

f values

Re = Reynold number = $\frac{VD}{\upsilon} = \frac{4VR}{\upsilon}$ where D= 4R and R = Hydraulic radius

a. Laminar: $f = \frac{64}{Re}$

b. Turbulent

I. Smooth

Re 4000 to 10^5, Blasius equation: $f = \frac{0.316}{Re^{1/4}}$

For Re>10^5 : $\frac{1}{\sqrt{f}} = 2.0 log_{10}\left(Re\sqrt{f}\right) - 0.8$

II. Rough

$\frac{1}{\sqrt{f}} = -2.0 log_{10}(K/4R) + 1.14$

III. Transition

$\frac{1}{\sqrt{f}} = -2.0 log_{10}\left(\frac{k}{14.84R} + \frac{2.51}{Re\sqrt{f}}\right)$

Approximate solution of above equation by Jain

$$\frac{1}{\sqrt{f}} = 1.14 - 2.0log_{10}\left(\frac{k}{4R} + \frac{21.25}{Re^{0.9}}\right)$$

Factors affecting Manning's n

The value of n is highly variable and depends on a number of factors. These factors are

i) Surfaces roughness:
- Roughness of wetted perimeter producing retarding effect on flow
- Low n for fine grains, high n for coarse grain

ii) Vegetation:
- Vegetation is a type of roughness,
- Increases n.

iii) Channel irregularity:
- Irregularities in wetted perimeter and cross section, size and shape along the channel length
- Gradual or uniform variations will not appreciably affect the value of n, while abrupt changes increase n.

iv) Channel alignment:
- Smooth curve with large radius will give a relatively low value of n whereas sharp curve with meandering will increase n.

v) Silting and scouring:
- Silting may decrease n.
- Scouring may increase n and depends on the nature of the material deposited.

vi) Obstruction:
- Presence of logs jams, bridge piers and the like tends to increase n.

vii) Shape and size of channel
- An increase in hydraulic radius may either increase or decrease n depending on the condition of channels.

viii) Stage and discharge:
- n decreases with increase in stage and discharge
- n increases at high stages if the banks are rough and grassy.
- For flood plain, n depends on surface condition or vegetation. (usually larger than channel)

ix) Seasonal change:
- Owing to the seasonal growth of aquatic plants, grass, weeds and trees in the channel or on the banks, the value of n may increase in the growing season and diminish in the dormant season

x) Suspended materials and bed load:
- The suspended materials and the bed load whether moving or not moving would consume energy and cause head loss or increase roughness.

Determination of Manning's n

In applying the Manning's formula, the greatest difficulty lies in the determination of the roughness coefficient n. Manning’s n cannot be measured directly and there is no exact method of selecting the n value.

Following approaches can be adopted for selecting n.
a. Using references, e.g. Table of Manning’s by Chow
b. Determination of n by velocity or discharge measurement
c. To understand the factors that affect n and to acquire a basic knowledge of the problem and narrow the wide range of guesswork
Recognizing several primary factors affecting the roughness coefficient **Cowan** developed a procedure for estimating the value of n
$n = (n_0+n_1+n_2+n_3+n_4)m_5$
Where n_0= n value for a straight, uniform and smooth channel
n_1= value for the effect of surface irregularities
n_2= Value for the variation in shape and size of the channel
n_3= Value for obstruction
n_4= Value for vegetation and flow conditions
m_5= correction factor for meandering of channels
d. Grain size analysis
Strickler formula

$$n = \frac{d_{50}^{1/6}}{21.1}$$

where d_{50} is in meters and represents the particle size in which 50% of the bed material is finer. For mixtures of bed material with considerable coarse-grained size, the modified equation is

$$n = \frac{d_{90}^{1/6}}{26}$$

where d_{90} is in meters and represents the particle size in which 90% of the bed material is finer.

Equivalent roughness or composite roughness (n)

Equivalent roughness is the weighted average value of roughness coefficients for different parts of perimeter.
Divide the water area into N parts having wetted perimeter P_1, P_2, ……., P_N and roughness coefficients n_1, n_2, ………..,n_N.
Approaches for estimating equivalent roughness
a. Considering constant mean velocities in all sub-areas
Horton and Einstein equation

$$n = \left[\frac{\left(\sum_{i=1}^{N} P_i n_i^{3/2}\right)}{P}\right]^{2/3}$$

b. Considering total resistance force = sum of sub-area resistance forces

Pavlovskij equation

$$n = \left[\frac{\left(\sum_{i=1}^{N} P_i n_i^2\right)}{P}\right]^{1/2}$$

c. Considering total discharge = sum of sub-area discharges

Lotter equation

$$n = \frac{PR^{5/3}}{\sum_{i=1}^{N}\left(\frac{P_i R_i^{5/3}}{n_i}\right)}$$

Values of n for some cases

SN	Surface characteristics	Range of n
1) Lined channel with straight alignment		
a	Metal	0.011-0.030 (Normal: 0.012)
b	Cement	0.010-0.015
c	Wood	0.010-0.018
d	Concrete	0.011-0.027 (Normal: 0.013)
e	Masonry	0.017-0.035
f	Asphalt	0.013-0.016
g	Vegetal lining	0.03-0.5
2) Natural channels		
a	Minor streams Streams in plain Mountain streams	 0.025-0.015 0.03-0.07
b	Flood plains	0.025-0.2
c	Major streams	0.025-0.1

6.5 Terms used in the computation of uniform flow

Conveyance and section factor

Using Manning's equation

$$V = \frac{1}{n} R^{2/3} S^{1/2}$$

$$Q = AV = \frac{1}{n} A R^{2/3} S^{1/2} = K\sqrt{S}$$

where $K = \frac{1}{n} A R^{2/3}$ conveyance of the channel section, which is a measure of carrying capacity of the channel section.

$$K = \frac{Q}{\sqrt{S}}$$

The expression $AR^{2/3}$ is called the section factor for uniform flow computation.

$$AR^{2/3} = nk$$

$$AR^{2/3} = \frac{nQ}{\sqrt{S}}$$

Normal depth

For given condition of n, Q and S, there is only one possible depth for maintaining a uniform flow. This depth is called normal depth. Thus the normal depth is defined as the depth of flow at which a given discharge flows as uniform flow in a given channel.

Normal discharge

When n and s are known for at a channel section, there is only one discharge for maintaining a uniform flow through the section. This discharge is normal discharge.

Hydraulic exponent

The relationship between conveyance (K) and depth (y) is given by

$$K^2 = Cy^N$$

where C is constant and N is hydraulic exponent for computation of uniform flow.

Taking logarithm on both sides and differentiating w.r.t. y,

2lnK = lnC+Nlny

$\frac{d(lnK)}{dy} = \frac{N}{2y}$ (a)

Using Manning's equation

$K = \frac{1}{n} AR^{2/3}$

Taking logarithm on both sides and differentiating w.r.t. y,

$lnK = \ln\left(\frac{1}{n}\right) + lnA + \frac{2}{3} lnR$

$\frac{d(lnK)}{dy} = \frac{1}{A}\frac{dA}{dy} + \frac{2}{3R}\frac{d(A/P)}{dy}$

dA/dy = T

$\frac{d(lnK)}{dy} = \frac{T}{A} + \frac{2}{3R}\left[\frac{1}{P}\frac{dA}{dy} - \frac{A}{P^2}\frac{dP}{dy}\right]$

$\frac{d(lnK)}{dy} = \frac{T}{A} + \frac{2}{3A/P}\left[\frac{1}{P}T - \frac{A}{P^2}\frac{dP}{dy}\right]$

$\frac{d(lnK)}{dy} = \frac{5T}{3A} - \frac{2}{3P}\frac{dP}{dy}$ (b)

Equating a and b

$\frac{N}{2y} = \frac{5T}{3A} - \frac{2}{3P}\frac{dP}{dy}$

$N = \frac{2y}{3A}\left[5T - 2R\frac{dP}{dy}\right]$

6.6 Most (Hydraulically) efficient channel section

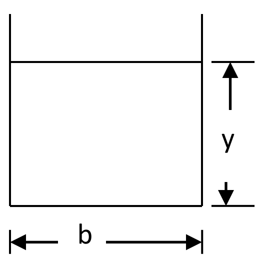
Using Manning's equation

$$Q = \frac{1}{n} A R^{2/3} S^{1/2} = \frac{A^{5/3} S^{1/2}}{n P^{2/3}}$$

A channel section is said to be most efficient when it can pass a maximum discharge for a given flow area (A), roughness coefficient (n) and slope (S). For given values of n and S, the discharge is maximum for a given area of cross-section when wetted perimeter (P) is minimum. This channel is also called the best section. Of all the various possible open channel cross sections for a given area of flow, the semi circular channel is the most efficient due to the smallest perimeter. However, there are practical limitations to using such cross-section, such as difficulty in construction, steepest stable slope etc

The most economical channel is that channel which requires minimum cost for construction. For minimum wetted perimeter, the cross-sectional area is also minimum. So, the cost of excavation and the lining is minimum for hydraulically efficient channel. However, in actual practice, the banks are higher than the full supply level and the most efficient channel does not necessarily result in minimum excavation or minimum length of lining.

a. Most efficient rectangular section



Bottom width = b and depth of flow = y
Flow area (A) = by = const.
Wetted perimeter (P) = b + 2y = $\frac{A}{y} + 2y$
For P to be minimum with A as constant, dP/dy = 0
Differentiating w.r.t. y

$$\frac{dP}{dy} = -\frac{A}{y^2} + 2 = 0$$

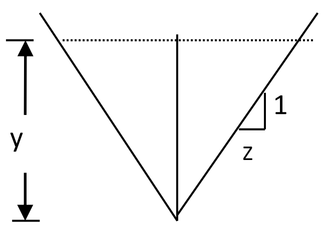
$$A = 2y^2$$

$$by = 2y^2$$

b = 2y
Hydraulic radius $(R) = \frac{A}{P} = \frac{by}{b+2y} = \frac{2y.y}{2y+2y} = \frac{y}{2}$

Thus for most efficient rectangular channel, the width should two times the depth of flow and hydraulic radius should be half the depth of flow. In other words, the depth of flow is equal to half bottom width i.e. when the channel section is half-square, best section is obtained.

b) Most efficient triangular section



Side slopes of triangular channel = z:1 and depth of flow = y

Flow area (A) = zy^2 (a)

Wetted perimeter (P) = $2y\sqrt{z^2+1}$ (b)

From (a), $y = \sqrt{\frac{A}{Z}}$

Substituting y in (b)

$$P = 2\sqrt{\frac{A}{Z}}\sqrt{z^2+1}$$

$$P^2 = \frac{4(z^2+1)}{z}A = \left(4z + \frac{4}{z}\right)A$$

For the section to be most efficient with given area, dP/dz = 0

Differentiating w.r.t. z

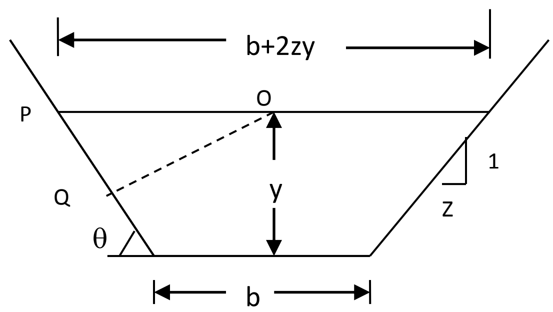
$$2P\frac{dP}{dz} = \left(4 - \frac{4}{z^2}\right)A = 0$$

Z = 1

Hence a triangular channel section will be most economical when each of its side slopes is 1:1 and sloping sides makes an angles of 45^0 with vertical. In other words, a triangular section with a central angle of 90^0 is the best.

Hydraulic radius $(R) = \frac{A}{P} = \frac{zy^2}{2y\sqrt{z^2+1}} = \frac{1.y^2}{2y\sqrt{1^2+1}} = \frac{y}{2\sqrt{2}}$

c) Most efficient trapezoidal section



b = Base width of the channel, y= Depth of flow, θ = Angle made by sides with horizontal and Side slope = z:1

Flow area (A) = (b+zy)y (a)

Wetted perimeter (P) = $b + 2y\sqrt{z^2 + 1}$ (b)

From a), $b = \frac{A}{y} - zy$

Substituting b in (b)

$$P = \frac{A}{y} - zy + 2y\sqrt{z^2 + 1}$$

I. Varying y with Keeping A and z fixed

For economical section dP/dy = 0

Differentiating w.r.t. y

$$\frac{dP}{dy} = -\frac{A}{y^2} - z + 2\sqrt{z^2 + 1} = 0$$

$$A = \left(2\sqrt{z^2 + 1} - z\right)y^2$$

$$(b + zy)y = \left(2\sqrt{z^2 + 1} - z\right)y^2$$

$$b + 2zy = 2\left(\sqrt{z^2 + 1}\right)y$$

i.e. top width (T) = twice the length of sloping sides

Hydraulic radius $(R) = \frac{A}{P} = \frac{(b+zy)y}{b+2y\sqrt{z^2+1}} = \frac{(b+zy)y}{b+b+2zy} = \frac{y}{2}$

Consider O as center and OQ as perpendicular to the side. Referring to triangle OPQ

$$OQ = OPsin\theta = \frac{T}{2}\frac{1}{\sqrt{z^2+1}} = \frac{2\left(\sqrt{z^2+1}\right)y}{2}\frac{1}{\sqrt{z^2+1}} = y$$

Thus the proportions of the hydraulically efficient trapezoidal channel section will be such that a semi-circle can be inscribed in it.

II. Varying z with Keeping A and y fixed

For economical section dP/dz = 0

Differentiating w.r.t. z

$$\frac{dP}{dz} = -y + \frac{2y}{2\left(\sqrt{z^2+1}\right)}2z = 0$$

$$2z = \left(\sqrt{z^2+1}\right)$$

$$z = \frac{1}{\sqrt{3}}$$

0r, $\theta = 60^0$

The most efficient section in this case is one half of hexagon.

III. Varying y and z keeping A fixed

For efficient section, $b + 2zy = 2\left(\sqrt{z^2+1}\right)y$

Substituting the value of side slope, $z = \frac{1}{\sqrt{3}}$ in above equation

$$b + \frac{2}{\sqrt{3}}y = 2\left(\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}\right)y = \frac{2y}{\sqrt{3}}$$

$$A = (b + zy)y = \left(\frac{2y}{\sqrt{3}} + \frac{y}{\sqrt{3}}\right)y = \sqrt{3}y^2$$

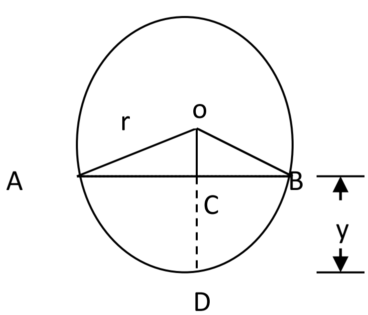
Wetted perimeter (P) = $b + 2y\sqrt{z^2+1} = \frac{2y}{\sqrt{3}} + 2y\left(\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}\right) = \frac{6y}{\sqrt{3}} = 3b$

R = A/P = y/2

Thus for a side slope of 60^0, the length of sloping side is equal to the base width of the trapezoidal section. Hence the most economical section of a trapezoidal channel will be a half hexagon.

d) Most efficient circular section

In case of circular channels running partially full, both the wetted perimeter and wetted area vary with depth of flow. The most efficient section is designed both for condition of maximum mean velocity and maximum discharge.



θ = angle AOC = angle COB, radius of circle = r, y = depth of flow

Wetted perimeter (p) = $\frac{2\pi r}{2\pi}2\theta$ = 2rθ

Wetted area (A) = Area ADBA =Area of sector OADBO-Area of ΔOAB

$$= \frac{\pi r^2}{2\pi}2\theta - \frac{1}{2}.2rsin\theta.rcos\theta = r^2\left(\theta - \frac{1}{2}sin2\theta\right)$$

I. Condition for maximum velocity

Using Chezy's formula

$V = C\sqrt{RS} = C\sqrt{(A/P)S}$

The flow velocity will have a maximum value when A/P is maximum.

$\frac{d(A/P)}{d\theta}$=0   (Where A and P both are function of θ)

$$\frac{P\frac{dA}{d\theta} - A\frac{dP}{d\theta}}{P^2} = 0$$

$$P\frac{dA}{d\theta} - A\frac{dP}{d\theta} = 0$$

$$\frac{dA}{d\theta} = \frac{d\left[r^2\left(\theta - \frac{1}{2}sin2\theta\right)\right]}{d\theta} = r^2(1 - cos2\theta)$$

$$\frac{dP}{d\theta} = 2r$$

Substituting the values of P, A, dA/dθ and dP/dθ

$$2r\theta.r^2(1 - cos2\theta) - r^2\left(\theta - \frac{1}{2}sin2\theta\right).2r = 0$$

$$-\theta cos2\theta + \frac{sin2\theta}{2}$$

$$tan2\theta = 2\theta$$

Solving by trial and error

2θ = 257.5^0

θ = 128.75^0

Depth of flow (y) = OD-OC=r-rcosθ= r(1-cosθ)=r(1-cos128.75) = 1.62r =0.81D where D = diameter of circular channel

Thus maximum velocity occurs when the depth of flow is 0.81 times diameter of the circular channel.

Hydraulic radius $(R) = \frac{A}{P} = \frac{r^2\left(\theta - \frac{1}{2}sin2\theta\right)}{2r\theta} = \frac{r\left(\theta - \frac{1}{2}sin2\theta\right)}{2\theta}$

θ = 128.75^0 =128.75$\frac{\pi}{180}$=2.247 rad

$$R = \frac{r\left(2.247 - \frac{1}{2}sin257.5^0\right)}{2x2.247} = 0.611r = 0.3D$$

Thus for maximum mean velocity in a channel of circular section hydraulic radius equals 0.3 times the channel diameter.

Instead of Chezy, if Manning’s equation is used,

$$V = \frac{1}{n}R^{2/3}S^{1/2} = \frac{1}{n}\left(\frac{A}{P}\right)^{2/3} S^{1/2}$$

The flow velocity will have a maximum value when A/P is maximum. i.e. $\frac{d(A/P)}{d\theta}$ =0. So the derivation is same as above.

II. Condition for maximum discharge
Using chezy’s equation

$$Q = AC\sqrt{RS} = AC\sqrt{\frac{A}{P}S} = C\sqrt{\frac{A^3}{P}S}$$

Maximum discharge is obtained when $\frac{A^3}{P}$ is maximum.

$$\frac{d(A^3/P)}{d\theta} = 0$$

$$\frac{Px3A^2\frac{dA}{d\theta} - A^3\frac{dP}{d\theta}}{P^2} = 0$$

$$3P\frac{dA}{d\theta} - A\frac{dP}{d\theta} = 0$$

Substituting the values of P, A, dA/dθ and dp/dθ

$$3.2r\theta.r^2(1 - cos2\theta) - r^2\left(\theta - \frac{1}{2}sin2\theta\right).2r = 0$$

Dividing by $2r^3$

$$4\theta - 6\theta cos2\theta + sin2\theta = 0$$

By trial and error, $2\theta = 308^0$

$\theta = 154^0$

Depth of flow (y)= OD-OC=r-rcosθ= r(1-cosθ)=r(1-cos154) = 1.898r =0.95D where D = diameter of circular channel

Thus maximum discharge occurs when the depth of flow is 0.95 times diameter of the circular channel.

Hydraulic radius $(R) = \frac{A}{P} = \frac{r^2\left(\theta - \frac{1}{2}sin2\theta\right)}{2r\theta} = \frac{r\left(\theta - \frac{1}{2}sin2\theta\right)}{2\theta}$

$\theta = 154^0 = 154\frac{\pi}{180}$=2.687 rad

$$R = \frac{r\left(2.687 - \frac{1}{2}sin308^0\right)}{2x2.687} = 0.573r = 0.29D$$

Thus for maximum discharge in a channel of circular section hydraulic radius equals 0.29 times the channel diameter.

Instead of Chezy, if Manning's formula is used,

$$Q = \frac{1}{n}AR^{2/3}S^{1/2} = \frac{1}{n}\frac{A^{5/3}}{P^{2/3}}S^{1/2} = \frac{1}{n}\left(\frac{A^5}{P^2}\right)^{1/3} S^{1/2}$$

For maximum discharge $\frac{A^5}{P^2}$ should be maximum.

$$\frac{d\left(\frac{A^5}{P^2}\right)}{d\theta} = 0$$

$$\frac{P^2 x 5A^4 \frac{dA}{d\theta} - A^5 x 2P \frac{dP}{d\theta}}{P^4} = 0$$

$$5P\frac{dA}{d\theta} - 2A\frac{dP}{d\theta} = 0$$

Substituting the values of P, A, dA/dθ and dp/dθ

$$5.2r\theta . r^2(1 - cos2\theta) - 2r^2\left(\theta - \frac{1}{2}sin2\theta\right).2r = 0$$

Dividing by $2r^3$

$$3\theta - 5\theta cos2\theta + sin2\theta = 0$$

By trial and error, $2\theta = 302.36^0$

$\theta = 151.18^0$

Depth of flow (y) = OD-OC=r-rcosθ= r(1-cosθ)=r(1-cos151.18) = 1.876r =0.938D where D = diameter of circular channel

Hydraulic radius $(R) = \frac{A}{P} = \frac{r^2\left(\theta - \frac{1}{2}sin2\theta\right)}{2r\theta} = \frac{r\left(\theta - \frac{1}{2}sin2\theta\right)}{2\theta} = 0.58r$

For maximum discharge in a circular channel, y = 0.938D and R = 0.29D.

Summary of properties for the most efficient channel

I. Rectangular

Condition: B = 2y

A = By =$2y^2$

P = B+2y = 4y

R= A/P =y/2

II. Triangular

Condition: Z:1 = 1:1

A= $Zy^2 = y^2$

$P = 2y\sqrt{z^2+1} = 2\sqrt{2}y$

R= A/P = $y/2\sqrt{2}$ = 0.2536y

III. Trapezoidal

a) If Z:1 is given

Condition: $b + 2zy = 2\left(\sqrt{z^2+1}\right)y$

b) If Z:1 is not given

Condition: Z:1 = $\frac{1}{\sqrt{3}}: 1$, $b = 2y/\sqrt{3}$, P = 3b

A = (b+zy)y = $\left(\frac{2y}{\sqrt{3}} + \frac{y}{\sqrt{3}}\right) y = \sqrt{3}y^2 = 1.732y^2$
R = A/P = y/2

IV. Circular
Condition (using Chezy):
Hydraulic radius (R) = 0.3D, y = 0.81D (for maximum velocity)
Hydraulic radius (R) = 0.29D, y = 0.95D (for maximum discharge)
Condition (using Manning):
Hydraulic radius (R) = 0.3D, y = 0.81D (for maximum velocity)
Hydraulic radius (R) = 0.29D, y = 0.938D (for maximum discharge)

Semi-circular section
$A = \frac{\pi D^2}{8}$, $P = \frac{\pi D}{2}$, $R = \frac{D}{4}$, y = D

6.8 Types of flow problems in uniform flow and their solutions

The basic variables in uniform flow are: discharge (Q), mean velocity of flow (V), normal depth (y_n), roughness coefficient (n), channel slope (S), and the geometric elements. When any four of the variables are given, the remaining two unknowns can be determined by the two equations.

The following are the six types of problems and the method of solutions.
a. To find Q given y, n, S and geometric elements
Solution: use $Q = \frac{1}{n}AR^{2/3}S^{1/2}$
b. To find V given y, n, S and geometric elements
Solution: use $V = \frac{1}{n}R^{2/3}S^{1/2}$
c. To find n given Q, y, S and geometric elements
Solution: use $Q = \frac{1}{n}AR^{2/3}S^{1/2}$ to compute n.
d. To find S given Q, y, n and geometric elements
Solution: use $Q = \frac{1}{n}AR^{2/3}S^{1/2}$ to compute S.
e. To find geometric elements given Q, y, n and S
Solution: Apply Manning's equation and solve for unknown. Solve by trial and error if no direct solution exists.
f. To find normal depth (y_n) given Q, n, S and geometric elements
Solution: Solve by trial and error

Trial and error method to find normal depth (y_n)

- Compute A, P and R in terms of y_n.
- From Manning's equation

$$Q = \frac{1}{n} A R^{2/3} S^{1/2}$$

or, $AR^{2/3} = \frac{nQ}{\sqrt{S}}$

Substitute the values of variables.

- Find y by graphical or trial and error or by programming.

a. Trial and error approach (analytical approach)

Assume different values of y, compute LHS of equation. Perform computations until LHS becomes almost equal to the numerical value in the RHS of the given. The value of y for which LHS is almost equal to RHS is the normal depth.

b. Graphical approach

For different values of y, compute corresponding section factor $AR^{2/3}$. Plot a graph of $AR^{2/3}$ versus y. For the computed value of section factor obtained from the expression $\frac{nQ}{\sqrt{S}}$, find the corresponding y value from the graph. This gives the normal depth.

Trial and error approach to find geometric elements

- Similar to the normal depth approach

Wide rectangular channel

b is very large compared to y.

A = by, P = b+2y

$R = \frac{A}{P} = \frac{by}{b+2y} = \frac{by}{b\left(1+\frac{2y}{b}\right)}$

For b very large, y/b is very small, which can be neglected.

So R = y for wide rectangular channel.

Deep gorge

y is very large compared to b.

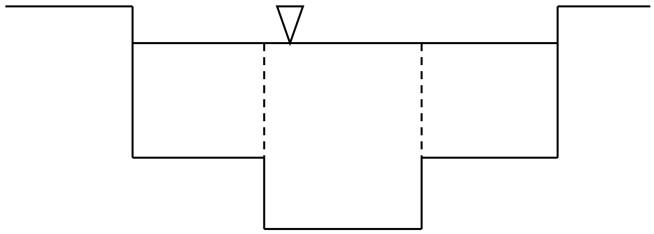
A = by, P = b+2y

$R = \frac{A}{P} = \frac{by}{b+2y} = \frac{by}{y\left(2+\frac{b}{y}\right)}$

For y very large, b/y is very small, which can be neglected.

So R = y/2 for deep gorge.

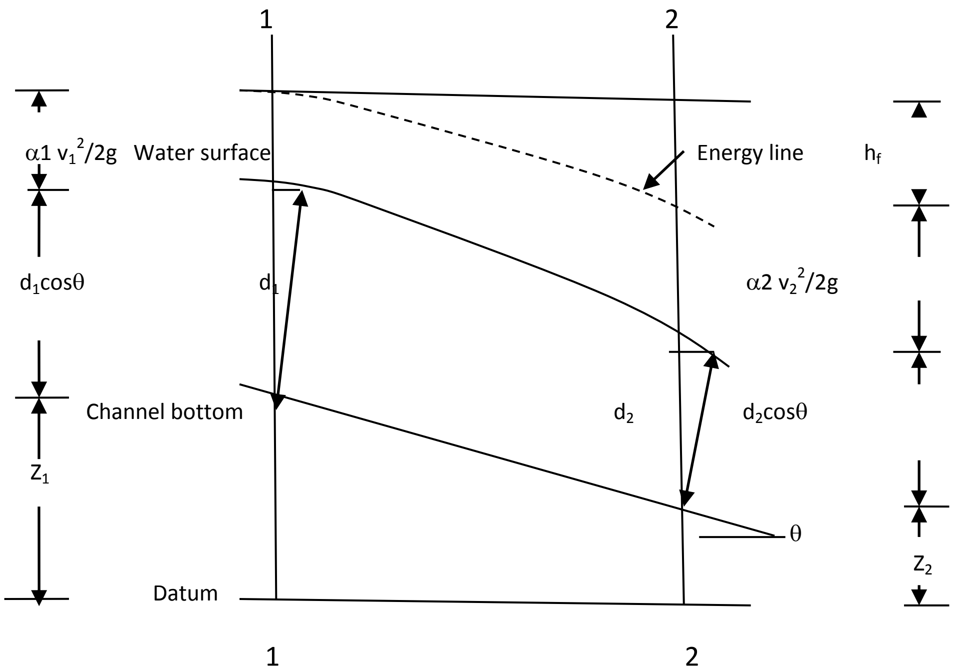
Compound channel



- Divide the channel into sub-sections and compute discharge for each sub-section. Then sum up.

Chapter 7: Energy and momentum principle in open channel

7.1 Energy principle (Bernoulli's equation) in open channel flow



The total energy of water at any point through a channel section is equal to the sum of elevation head, pressure head and velocity head.

$$H = Z + d\cos\theta + \alpha\frac{V^2}{2g}$$

H = total head, Z = Datum head, d = depth of water, θ = slope angle of channel bottom, $V^2/2g$ = velocity head, α = velocity distribution coefficient

According to the principle of conservation of energy for two sections 1 and 2,

Total energy head at section 1 = total energy head at section 2 + loss of energy between two sections

$$Z_1 + d_1\cos\theta + \alpha_1\frac{{V_1}^2}{2g} = Z_2 + d_2\cos\theta + \alpha_2\frac{{V_2}^2}{2g} + h_f$$

For a channel of small slope, $\theta \approx 0$ and $\cos\theta = 1$ and $d \approx y$

$$Z_1 + y_1 + \alpha_1\frac{{V_1}^2}{2g} = Z_2 + y_2 + \alpha_2\frac{{V_2}^2}{2g} + h_f$$

where y_1 and y_2 = water depth at 1 and 2

This equation is known as Bernoulli's equation.

When $\alpha_1 = \alpha_2 = 1$ and $h_f = 0$, the Bernoulli's equation is

$$Z_1 + y_1 + \frac{{V_1}^2}{2g} = Z_2 + y_2 + \frac{{V_2}^2}{2g}$$

7.2 Specific energy

The Specific energy in a channel section is defined as the energy per unit weight of water measured with respect to the channel bottom. Thus Specific energy E at any section is the sum of the depth of flow at that section and the velocity head. With Z = 0, the total energy is the specific energy (E), which is given by

$$E = d\cos\theta + \alpha\frac{V^2}{2g}$$

For a channel of small slope, $\theta \approx 0$, and taking $\alpha = 1$,

$$E = y + \frac{V^2}{2g}$$

This shows that the specific energy is equal to the sum of water depth and velocity head. i.e. specific energy = static energy + kinetic energy

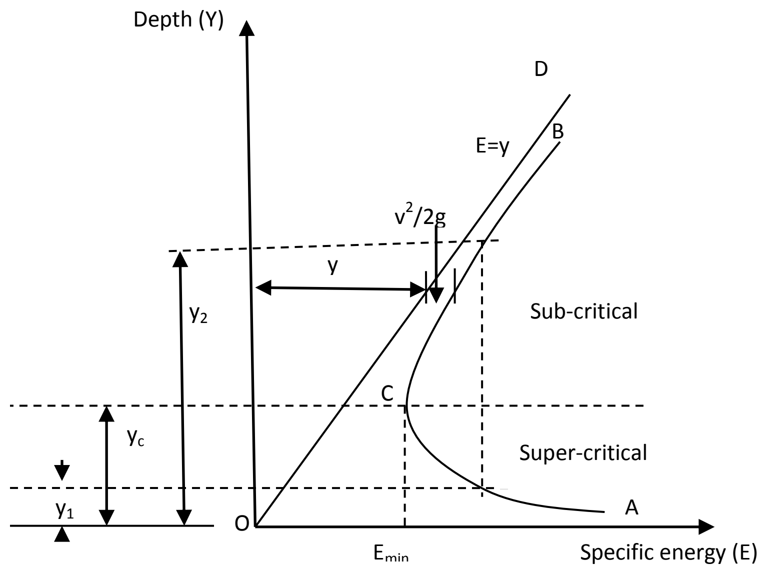
With V = Q/A, above expression becomes

$$E = y + \frac{Q^2}{2gA^2}$$

This shows that for given channel section and discharge Q, the specific energy is a function of the depth of the flow only.

Specific energy curve/diagram

Specific energy curve is the graphical plot of depth of flow versus specific energy for a given channel section and discharge.



Specific energy diagram for discharge (Q)

Features of specific energy diagram

a. At any point on the specific energy curve, the ordinate represents the depth and the abscissa represents the specific energy. , which is equal to the sum of pressure head and velocity head.
b. The specific energy curve has two limbs AC (upper) and BC (lower).
c. The plot of pressure head (E=y) is straight line, which passes through the origin O and has an angle of inclination equal to 45^0. The plot of velocity head is parabolic. Specific energy curve is obtained by summing pressure head and velocity head.
d. The specific energy curve is asymptotic to the horizontal axis for small value of y and asymptotic to the 45^0 line for high value of y.

$E = y + \frac{V^2}{2g}$, Taking V=q/y where q = discharge per unit width
$y \to 0, V = \frac{q}{y} \to \infty$. So $E \to \infty$ (asymptotic to X-axis)
$y \to \infty, V = \frac{q}{y} \to 0$. So $E \to \infty$ (asymptotic to E=y line)

e. Specific energy curve shows that at a certain point (point C in the figure), the specific energy is minimum. The condition of minimum specific energy is called critical flow condition and the corresponding depth is called critical depth (y_c). The velocity at critical depth is called critical velocity (V_c).
f. The curve shows that, for a given specific energy other than minimum specific energy, there are two possible depth of flow, e.g. low stage $y_1 < y_c$ and high stage $y_2 > y_c$. These two depths for the same specific energy are called alternative depths.
g. The zone above the critical state is the sub-critical zone and below it is supercritical zone.
$y>y_c$: subcritical ($V<V_C$, $F_r<1$)
$y=y_c$: critical ($V=V_C$, $F_r=1$)
$y<y_c$: supercritical ($V>V_C$, $F_r>1$)
h. Specific energy increases with the increase of depth for subcritical flow, whereas it increases with the decrease of depth for supercritical flow.
h. The specific energy curve shifts towards right for increase in discharge.

7.3 Criterion for critical state for given discharge

Specific energy (E) is given by

$$E = y + \frac{Q^2}{2gA^2}$$

Differentiating w.r.t. y (keeping Q constant)

$$\frac{dE}{dy} = 1 - \frac{Q^2}{gA^3}\frac{dA}{dy}$$

dE/dy = 0

dA/dy = top width (T)

(For an element of thickness dy and top width T, elementary area (dA) = Tdy)

$$1 - \frac{Q^2T}{gA^3} = 0$$

$$\frac{Q^2T}{gA^3} = 1$$

or, $\frac{Q^2}{g} = \frac{A^3}{T}$

This is the condition for minimum specific energy for constant discharge.

Other form,

$$\frac{Q^2}{A^2g} = \frac{A}{T}$$

Substituting V= Q/A and Hydraulic depth (D) = A/T

$$\frac{V^2}{g} = D$$

$$\frac{V^2}{2g} = \frac{D}{2}$$

This is the criterion for critical flow, which shows that the velocity head is equal to half the hydraulic depth. The above equation can also be written as

$$\frac{V}{\sqrt{gD}} = 1$$

Fr = 1

where $\frac{V}{\sqrt{gD}} = Fr$

This shows that Froude number (Fr) is unity for critical flow. In other words, specific energy is minimum at critical state.

7.4 Computation of critical depth

Given data: Q and geometry

- Express cross-section area and top width in terms of given geometric data and critical depth (y_c).
- Substitute values of Q, A and T in equation $\frac{Q^2}{g} = \frac{A^3}{T}$ and solve for y_c. (In case of no direct solution, solve by trial and error method).

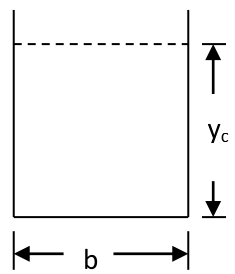
Computation of critical velocity (V_c)

$$V_c = \sqrt{gD} = \sqrt{gA/T}$$

Computation of minimum specific energy $(E_c) = y_c + \frac{V_c^2}{2g}$

Critical depth computation of some channel sections

a. Rectangular



Flow area (A) = b y_c and top width (T) = b
y_c = critical depth of flow
At critical state,

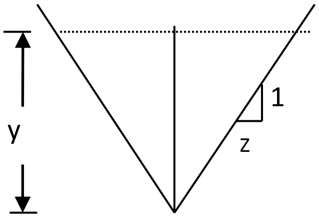
$$\frac{Q^2}{g} = \frac{A^3}{T}$$
$$\frac{Q^2}{g} = \frac{(by_c)^3}{b}$$
$$y_c = \left(\frac{Q^2}{b^2 g}\right)^{1/3}$$

or, $y_c = \left(\frac{q^2}{g}\right)^{1/3}$ where q = discharge per unit width
Critical velocity $(V_c) = \sqrt{gD} = \sqrt{gA/T}$
$= \sqrt{g\frac{by_c}{b}} = \sqrt{gy_c}$

The minimum specific energy E_c corresponding to critical depth and critical velocity is given by

$$E_c = y_c + \frac{V_c^2}{2g} = y_c + \frac{gy_c}{2g}$$
$$E_c = \frac{3}{2}y_c$$

b) Triangular channel



Side slopes of triangular channel = z:1 and depth of flow = y
Flow area (A) = zy_c^2

Top width (T) = $2zy_c$

At critical state,

$$\frac{Q^2}{g} = \frac{A^3}{T}$$

$$\frac{Q^2}{g} = \frac{(zy_c^2)^3}{2zy_c}$$

$$y_c = \left(\frac{2Q^2}{gz^2}\right)^{1/5}$$

Critical velocity $(V_c) = \sqrt{gD} = \sqrt{gA/T}$

$= \sqrt{g\,\frac{zy_c^2}{2zy_c}} = \sqrt{g\,\frac{y_c}{2}}$

The minimum specific energy E_c corresponding to critical depth and critical velocity is given by

$$E_c = y_c + \frac{V_c^2}{2g} = y_c + \frac{gy_c}{4g}$$

$$E_c = \frac{5}{4}y_c$$

c. Parabolic

$T = k\sqrt{y_c}$

$A = \frac{2}{3}Ty_c = \frac{2}{3}k\sqrt{y_c}y_c = \frac{2}{3}ky_c^{3/2}$

At critical state,

$$\frac{Q^2}{g} = \frac{A^3}{T}$$

$$\frac{Q^2}{g} = \frac{(\frac{2}{3}ky_c^{3/2})^3}{k\sqrt{y_c}}$$

$$y_c = \left(\frac{27Q^2}{8gk^2}\right)^{1/4}$$

Critical velocity $(V_c) = \sqrt{gD} = \sqrt{gA/T}$

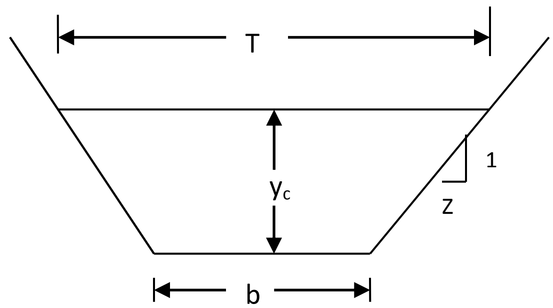
$= \sqrt{g\,\frac{\frac{2}{3}ky_c^{3/2}}{k\sqrt{y_c}}} = \sqrt{g\,\frac{2y_c}{3}}$

The minimum specific energy E_c corresponding to critical depth and critical velocity is given by

$$E_c = y_c + \frac{V_c^2}{2g} = y_c + \frac{2gy_c}{6g}$$

$$E_c = \frac{4}{3}y_c$$

d) Trapezoidal



A= $(b+zy_c)y_c$
T = $b+2zy_c$

$$\frac{Q^2}{g} = \frac{A^3}{T}$$
$$\frac{Q^2}{g} = \frac{[(b+zy_c)y_c]^3}{(b+2zy_c)}$$
$$\frac{Q^2}{g} = \frac{\left[b^2\left(1+z\frac{y_c}{b}\right)\frac{y_c}{b}\right]^3}{\left(1+2z\frac{y_c}{b}\right)\mathrm{b}}$$
$$\frac{Q^2}{gb^5} = \frac{\left[\left(1+z\frac{y_c}{b}\right)\frac{y_c}{b}\right]^3}{\left(1+2z\frac{y_c}{b}\right)}$$

No direct solution exists. Solve for y_c by trial and error.

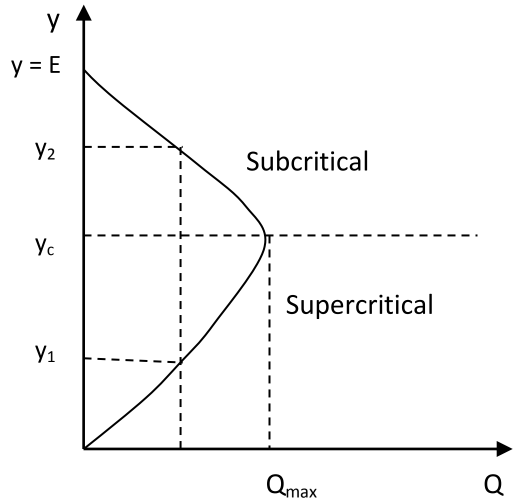
7.5 Depth-discharge diagram

Condition for maximum discharge for a given value of specific energy
Specific energy (E) is given by

$$E = y + \frac{Q^2}{2gA^2}$$
$$E - y = \frac{Q^2}{2gA^2}$$
$$Q = A\sqrt{2g}\sqrt{E-y}$$

A =f(y) for a given channel and Q=f(y) for a given specific energy.

- At y=0, A=0, then Q=0
- At y=E, the expression (E-y) =0, then Q=0
- Q is maximum at an intermediate depth within y=0 and y=E



Depth-discharge diagram

For maximum discharge, dQ/dy =0.
Differentiating w.r.t. y

$$\frac{dQ}{dy} = \sqrt{2g}\left[-\frac{A}{2\sqrt{E-y}} + \sqrt{E-y}\,\frac{dA}{dy}\right] = 0$$

dA/dy = Top width (T)

$$T\sqrt{E-y} = \frac{A}{2\sqrt{E-y}}$$

$$2(E-y) = \frac{A}{T}E - y = \frac{Q^2}{2gA^2}$$

From above two expressions

$$\frac{Q^2}{gA^2} = \frac{A}{T}$$

$$\frac{Q^2T}{gA^3} = 1$$

Or, $\frac{Q^2}{g} = \frac{A^3}{T}$
In terms of velocity

$$\frac{Q^2}{A^2g} = \frac{A}{T}$$

$\frac{V^2}{g} = D$ where Hydraulic depth (D) = A/T

$$\frac{V}{\sqrt{gD}} = 1$$

i.e. $F_r = 1$

For the discharge to be maximum for a given specific energy, the flow should be critical.

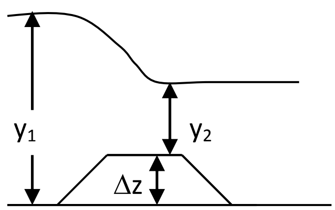
7.6 Application of specific energy concept

The concept of specific energy and critical depth are extremely useful in the analysis of flow problems, such as flow through transitions (reduction in channel width), flow over raised channel floors, flow through sluice gate opening.

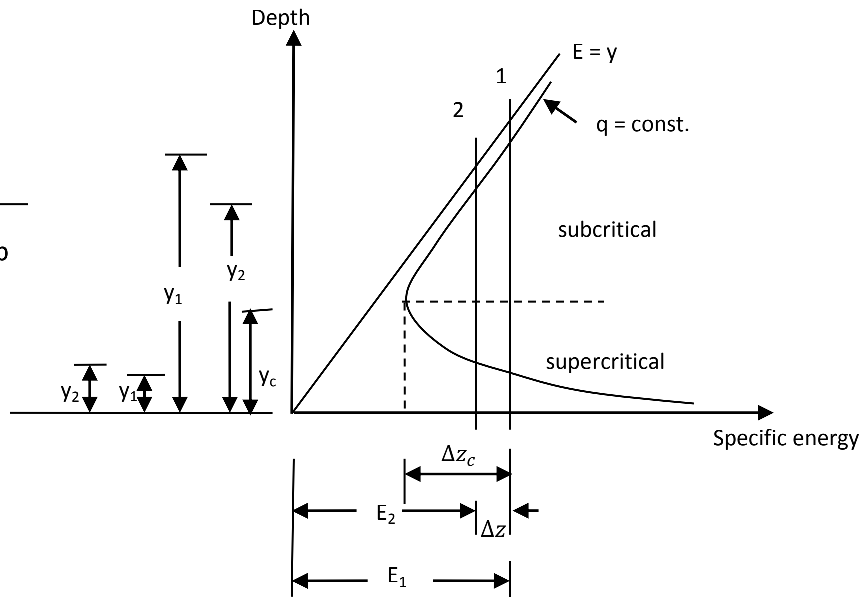
I. Flow through transitions

A channel transition refers to a certain stretch of channel in which the cross-section of the channel varies.

b. Rise in bed level of channel section (Channel with hump)



Channel transition with hump



Specific energy diagram for channel bed rise

Consider a channel of constant width (b) carrying a discharge Q with a rise in bed level (ΔZ) in a certain reach. Let ΔZ_c is the minimum height of hump under critical condition.

q_1=q_2 =q = discharge per unit width = constant

Specific Energy equation for hump: $E_1=E_2+\Delta Z$

Analysis of flow for hump

y_1 = flow depth at section 1, y_2 = flow depth at section 2, y_c = flow depth at section for critical condition.

First compute ΔZ_c by computing E_1 and E_2 which is equal to E_c at section 2. So, $\Delta Z_c = E_1 - E_C$

Case I: $\Delta Z < \Delta Z_c$

Flow profile:

- y_1 = constant for both subcritical and supercritical flow
- y_2 reduces for subcritical flow and increases for supercritical flow until y_2 is equal to y_c.

Given: Q, y_1, ΔZ

To find: y_2

Compute $E_1 = y_1 + \frac{Q^2}{2gA_1{}^2}$

$E_1=E_2+\Delta Z$

$E_2=E_1-\Delta Z$

$E_2>E_c$ where E_c is minimum specific energy.

$$E_2 = y_2 + \frac{Q^2}{2gA_2{}^2}$$

Find the value of y_2 by trial and error method.

Case II: $\Delta Z= \Delta Z_c$

Flow profile:

- y_1 = constant for both subcritical and supercritical flow
- $y_2 = y_c$ for both subcritical and supercritical flow

Given: Q, y_1

To find: ΔZ_c (minimum rise or minimum size of hump under critical condition)

In this case, energy is minimum and the flow is critical at 2.

$E_2 = E_c$

$E_1=E_2+\Delta Z_c$

or, $\Delta Z_c = E_1 - E_2 = E_1 - E_C$

For rectangular section

$$E_c = 1.5y_c = 1.5\left(\frac{q^2}{g}\right)^{1/3}$$

$$\Delta Z_c = E_1 - E_c = y_1 + \frac{v_1{}^2}{2g} - \frac{3}{2}\left(\frac{y_1{}^2 v_1{}^2}{g}\right)^{1/3}$$

$$= y_1\left[1 + \frac{v_1{}^2}{2gy_1} - 1.5\left(\frac{v_1{}^2}{gy_1}\right)^{1/3}\right]$$

$$= y_1\left[1 + \frac{F_{r1}{}^2}{2} - 1.5F_{r1}{}^{2/3}\right]$$

where F_{r1} = Froude no. at section 1 = $\frac{v_1}{\sqrt{gy_1}}$

Case III: $\Delta Z > \Delta Z_c$

In this case, $(E_1-\Delta Z=E_2)<E_c$ which is physically not possible. The flow cannot take place with the available specific energy (choking condition). The flow will be critical at section 2 ($E_2= E_c$, $y_2=y_c$) and the upstream depth will be changed.

Flow profile:

- y_2 remains constant at y_c for both subcritical and supercritical flow.
- y_1 increases for subcritical flow and decreases for supercritical flow.

Given: Q, ΔZ

To find: u/s depth (y_{1a})

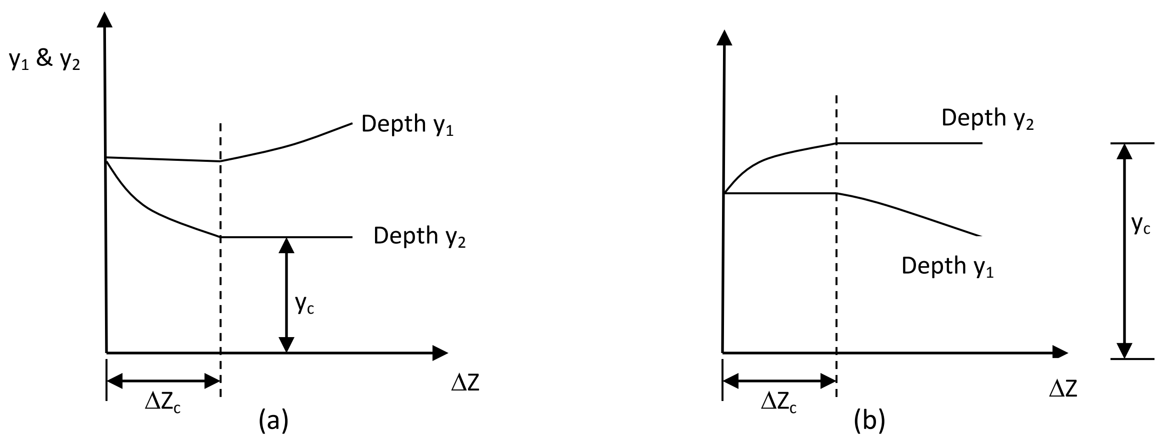
$$E_{1a} = \Delta Z + E_2$$

$$E_{1a} = \Delta Z + E_C = \Delta Z + 1.5E_C = \Delta Z + 1.5\left(\frac{q^2}{g}\right)^{1/3}$$

New u/s depth y1a is computed by

$$y_{1a} + \frac{q^2}{2gy_{1a}^2} = E_{1a}$$

Solve this equation by trial and error to get y_{1a}.



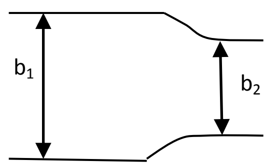
Variation of depth for (a) subcritical flow and (b) supercritical flow in case of hump

If loss if energy due to hump (h_L) is considered, then

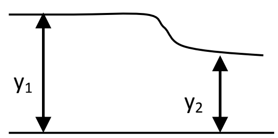
$E_1=E_2+\Delta Z+h_L$

b. Reduction in channel width

Let us consider that the width of channel is reduced from b_1 to b_2, the floor level remaining the same. Also neglect energy loss between two sections. For a constant discharge Q, the discharge per unit width in the approach channel is $q_1=Q/b_1$ and in the throat portion is $q_2= Q/b_2$. For subcritical approach flow, depth of flow at section 2 is less than that at section 1. For supercritical approach flow, depth of flow at section 2 is greater than that at section 1. If the width b_2 is reduced, q_2 increases and approaches q_c, i.e. the flow becomes critical at a particular width $b_2 = b_c$. The width b_c is therefore the limiting width at which maximum unit discharge q_{max} passes through the throat under critical condition. If the throat width is further reduced to b_3 less than b_c resulting in $q_3>q_{max}$ which is obviously not possible at the given value of specific energy E_1.The energy upstream must increase under this condition to make the flow at contraction at critical state.

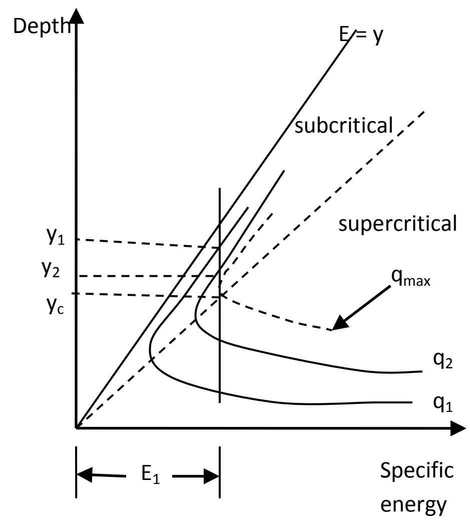


Width constriction



Subcritical flow approach

Supercritical approach flow



Specific energy diagram for width constriction

Analysis of flow for width constriction

b_1 = width at section 1, b_2 = width at section 2, b_c = width at section 2 under critical condition
y_1 = flow depth at section 1, y_2 = flow depth at section 2, y_c = flow depth at section for $b_2 = b_c$
First compute b_c by computing E_c at section 2. ($E_2 = E_c$. As $E_1 = E_2$, $E_1 = E_c$)
Case I: $b_2>b_c$
Flow profile:

- y_1 = constant for both subcritical and supercritical flow
- y_2 reduces for subcritical flow and increases for supercritical flow until b_2 is equal to b_c

Given: Q, y_1, b_1, b_2
To find: y_2
Compute $E_1 = y_1 + \frac{Q^2}{2gA_1{}^2}$
$E_2 = E_1$
$y_2 + \frac{Q^2}{2gA_2{}^2} = E_2$
Solve by trial and error to get y_2.

Case II: $b_2 = b_c$
Flow profile:

- y_1 = constant for both subcritical and supercritical flow

- $y_2 = y_c$ for both subcritical and supercritical flow

Given: Q, y_1, b_1

To find: minimum width at contraction (b_c) at critical condition

Compute $E_1 = y_1 + \frac{Q^2}{2gA_1{}^2}$

$E_1 = E_2$

$E_2 = E_C$

Hence, $E_1 = E_C$

For rectangular channel

$$E_1 = E_c = 1.5y_c$$

$$= 1.5\left(\frac{q^2}{g}\right)^{1/3} = 1.5\left(\frac{\left(\frac{Q}{b_c}\right)^2}{g}\right)^{1/3}$$

$$E_1^3 = 3.375\frac{Q^2}{gb_c^2}$$

$$b_c = 1.84\frac{Q}{E_1^{3/2}\sqrt{g}}$$

Case 3: $b_2 < b_c$

Flow profile:

- y_2 is critical with new critical depth, y_{c1} for both subcritical and supercritical flow. y_{c1} is computed by taking width = b_2. So $y_{c1} > y_c$. As specific energy at 2 (E_{c1}) rises, y_{c1} also rises.
- y_1 increases for subcritical flow and decreases for supercritical flow.

Given: Q, b_1, b_2

To compute: Depth at 2 and 1

Given discharge cannot passes through the section 2 with the available energy E_1 (choking condition). The specific energy at section 2 should be minimum and the upstream depth is changed to y_{1a}.

$E_2 = E_c$ and y_2 = critical depth (y_{c1})

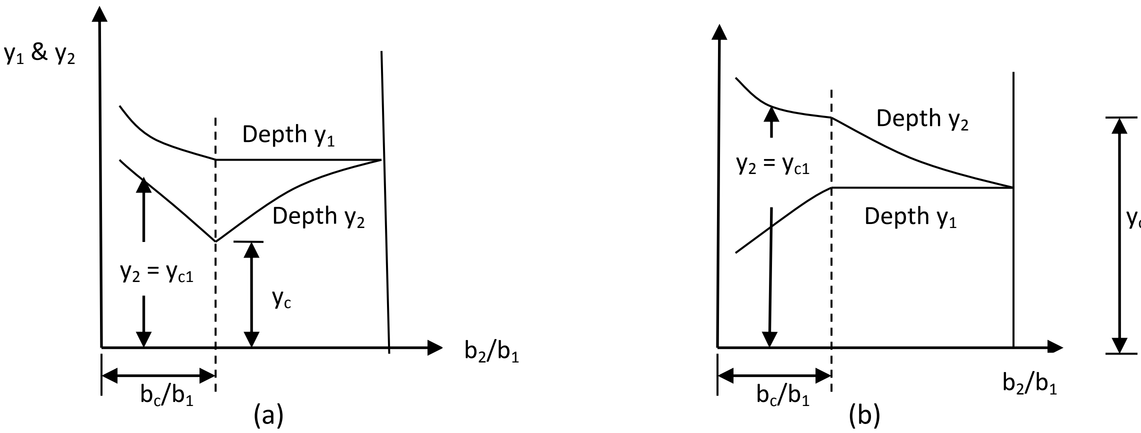
New specific energy upstream (E_{1a}) = E_2

$E_{1a} = E_c$

$$E_{1a} = E_c = 1.5y_{c1} = 1.5\left(\frac{\left(\frac{Q}{b_2}\right)^2}{g}\right)^{1/3}$$

$$E_{1a} = y_{1a} + \frac{q_1^2}{2gy_{1a}^2}$$

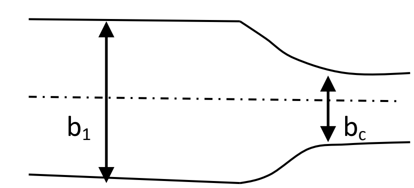
Find y_{1a} by trial and error.



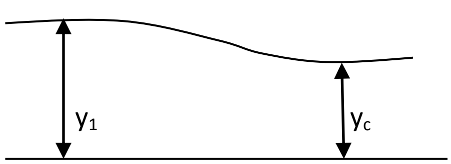
Variation of depth for (a) subcritical flow and (b) supercritical flow in case of width constriction

c) Venturiflume

It is a simple device for measuring discharge in open channel flow in which the width of contraction is equal to or less than critical width (b_c).



Venturiflume



Assumptions

- Head loss is negligible
- Flow level is horizontal
- Specific energy is constant
- Contraction is adequate to result critical flow

$$E_1 = y_1 + \frac{v_1^2}{2g}$$

$$E_c = \frac{3}{2} y_c$$

$$E_1 = E_C$$

$$y_1 + \frac{v_1^2}{2g} = \frac{3}{2} y_c$$

$$y_1 + \frac{v_1^2}{2g} = \frac{3}{2}\left(\frac{\left(\frac{Q}{b_2}\right)^2}{g}\right)^{1/3}$$

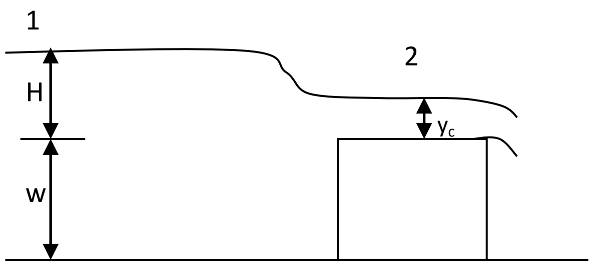
Neglecting velocity of approach $\left(\frac{v_1^2}{2g}\right)$

$$Q = 1.7 b_2 y_1^{3/2}$$

Thus discharge can be calculated by merely measuring the depth upstream of the throat.

d) Broad crested weir

Board crested weir is a simple device for measuring the discharge in open channel flow.



$E_1 = E_2 + w$

$H + w + \frac{v_1^2}{2g} = E_c + w$

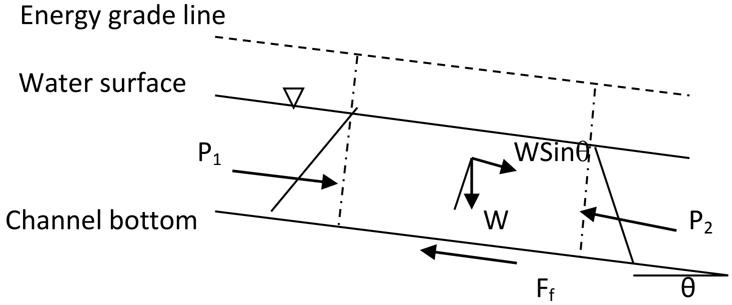
Neglecting velocity of approach

$$H = E_c$$

$$H = \frac{3}{2} y_c = \frac{3}{2}\left(\frac{(Q/B)^2}{g}\right)^{1/3}$$

$$Q = 1.704 B H^{3/2}$$

7.7 Momentum equation and specific force



Consider two sections 1 and 2 along the length of channel. Let P1 and P2 are the hydrostatic pressure force at section (1) and 2 respectively; W is the weight of water enclosed between two sections; and F_f is the resistance force due to friction. Applying the momentum principle,

Net force = Change in momentum

$$P_1 - P_2 + Wsin\theta - F_f = \rho Q(V_2 - V_1)$$

For a short reach of prismatic channel, θ = 0 and friction force, F_f can be neglected. Hence the momentum equation becomes

$$P_1 - P_2 = \frac{\gamma Q}{g}(V_2 - V_1)$$

$$\gamma A_1\overline{Z_1} - \gamma A_2\overline{Z_2} = \frac{\gamma Q}{g}\left(\frac{Q}{A_2} - \frac{Q}{A_1}\right)$$

where A_1 and A_2 are cross-sectional area, and $\overline{Z_1}$ and $\overline{Z_2}$ are the distances of centroids of the areas below the surface of flow. The above equation can be written as

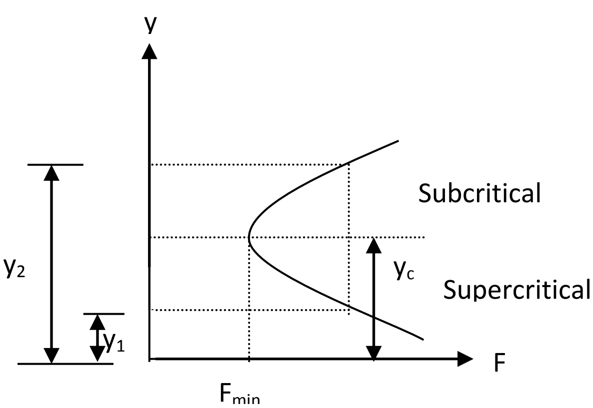
$$\frac{Q^2}{gA_1} + \overline{Z_1}A_1 = \frac{Q^2}{gA_2} + \overline{Z_2}A_2$$

The two sides of above equation are identical and may be represented by force F.

$$F = \frac{Q^2}{gA} + \bar{Z}A$$

The first term is the momentum flux per unit weight and the second term is the force per unit weight of the liquid. The sum of these two terms is known as specific force. The equation shows that the specific force is a function of depth.

Specific force curve



Specific force curve

The plot of specific force versus depth for a given channel section and discharge is called specific forces curve. The curve has two limbs. The lower limb approaches the horizontal axis asymptotically towards the right. The upper limb rises upward and extends indefinitely to the right.

For a given value of specific force, the curve has two possible depths y_1 and y_2. These depths are known as conjugate depths or sequent depths. The depth y_1 lies in the super critical region and is known as initial depth while the second depth y_2 corresponds to sub critical flow and is known as the sequent depth. These depths are also known as conjugate depths. For a particular depth (y_c), the specific force becomes minimum and the corresponding depth of flow is called critical depth.

a. Criterion for minimum specific force for a given discharge

$F = \frac{Q^2}{gA} + \bar{Z}A$ (a)

For a minimum value of specific force dF/dy = 0

Differentiating F w.r.t. y

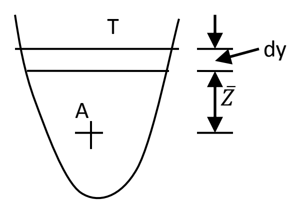
$\frac{dF}{dy} = -\frac{Q^2}{gA^2}\frac{dA}{dy} + \frac{d(\bar{Z}A)}{dy} = 0$ (b)

$d(\bar{Z}A)$ represents the change in static moment of water area about the free surface.
Moment of area about water surface = $\bar{Z}A$
For a change in depth dy,
Moment of area about water surface = Moment of A with arm $(\bar{Z} + dy)$ plus moment of elementary area = $\left[A(\bar{Z} + dy) + T\,dy\frac{dy}{2}\right]$

$$d(\bar{Z}A) = [A(\bar{Z} + dy) + T\,(dy)^2/2] - \bar{Z}A$$

Neglecting differential of higher order

$$d(\bar{Z}A) = Ady$$



With this, eq. b becomes

$$-\frac{Q^2}{gA^2}\frac{dA}{dy} + A = 0$$

Substituting dA/dy = top width (T)

$$\frac{Q^2T}{gA^3} = 1$$

Or, $\frac{Q^2}{g} = \frac{A^3}{T}$

In terms of velocity

$$\frac{Q^2}{A^2g} = \frac{A}{T}$$

$\frac{V^2}{g} = D$ where Hydraulic depth (D) = A/T

$$\frac{V}{\sqrt{gD}} = 1$$

i.e. $F_r = 1$

At critical state of flow, specific force is minimum for given discharge.

b. Criterion for maximum discharge for a given specific force

$F = \frac{Q^2}{gA} + \bar{Z}A$ (a)

Rearranging

$Q = \sqrt{g}\sqrt{A(F - \bar{Z}A)}$ (b)

For discharge to be maximum dQ/dy = 0

Differentiating Q w.r.t. y (keeping F fixed)

$\frac{dQ}{dy} = \sqrt{g}\left[\sqrt{(F - \bar{Z}A)}\frac{d(\sqrt{A})}{dy} + \sqrt{A}\frac{d\left(\sqrt{(F-\bar{Z}A)}\right)}{dy}\right] = 0$

$\sqrt{g}\left[\sqrt{(F - \bar{Z}A)}\frac{1}{2\sqrt{A}}\frac{dA}{dy} + \sqrt{A}\frac{1}{2\left(\sqrt{(F-\bar{Z}A)}\right)}x - \frac{d(\bar{Z}A)}{dy}\right] = 0$ ©

$d(\bar{Z}A)$ represents the change in static moment of water area about the free surface.

Moment of area about water surface = $\bar{Z}A$

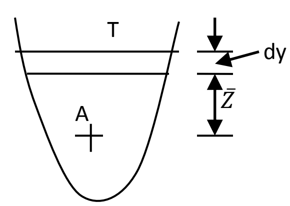
For a change in depth dy,

Moment of area about water surface = Moment of A with arm $(\bar{Z} + dy)$ plus moment of elementary area = $\left[A(\bar{Z} + dy) + T\,dy\frac{dy}{2}\right]$

$$d(\bar{Z}A) = [A(\bar{Z} + dy) + T\,(dy)^2/2] - \bar{Z}A$$

Neglecting differential of higher order

$$d(\bar{Z}A) = Ady$$



dA/dy = top width (T)

substituting dA/dy and $d(\bar{Z}A)$

$\sqrt{g}\left[\sqrt{(F - \bar{Z}A)}\frac{1}{2\sqrt{A}}T + \sqrt{A}\frac{1}{2\left(\sqrt{(F-\bar{Z}A)}\right)}x - A\right] = 0$

$\frac{\sqrt{g}}{2\left(\sqrt{A(F-\bar{Z}A)}\right)}[T(F - \bar{Z}A) - A^2] = 0$

$T(F - \bar{Z}A) - A^2 = 0$ (d)

From b,

$(F - \bar{Z}A) = \frac{Q^2}{gA}$ (e)

From d and e

$\frac{Q^2T}{gA^3} = 1$

Or, $\frac{Q^2}{g} = \frac{A^3}{T}$

In terms of velocity

$\frac{Q^2}{A^2g} = \frac{A}{T}$

$\frac{V^2}{g} = D$ where Hydraulic depth (D) = A/T

$\frac{V}{\sqrt{gD}} = 1$

i.e. $F_r = 1$

At critical state of flow, discharge is maximum for given specific force.

Summary of critical flow condition

Conditions to be fulfilled for the flow to be critical

- Specific energy is minimum for a given discharge.
- Discharge is maximum for a given specific energy.
- Specific force is minimum for a given discharge.
- Discharge is maximum for a given force.
- Froude number is unity.

Occurrence of critical flow conditions

- At a point where subcritical flow changes to supercritical flow (e.g. at a break from mild to steep slope)
- Entrance to a channel of steep slope from a reservoir
- Free outfall from a channel with a mild slope
- Change of bed level or channel width

Chapter 8: Gradually varied flow

Gradually varied flow (GVF) is a steady non-uniform flow in which the depth of flow varies gradually. In the GVF, the velocity varies along the channel and consequently the bed slope, water surface slope and energy slope will all differ from each other. The boundary friction is important in GVF.

The GVF may be caused due to one or more of the following factors:

- Change in shape and size of the cross-section
- change in slope of channel
- presence of obstruction, e.g. weir
- change in frictional forces at the boundary

Examples: flow upstream of a weir or dam, flow downstream of a sluice gate, flow in channels with break in bottom slope.

8.1 Differential Equation of GVF

Assumptions for deriving dynamic equation of GVF

- The uniform flow formula (i.e. Manning's or Chezy) is used to evaluate the energy slope of a gradually varied flow and the corresponding coefficient of roughness developed primarily for uniform flow are applicable to GVF.
- The bottom slope of a channel is very small so that the depth of flow is same whether the vertical or normal direction is used.
- The flow is steady.
- The pressure distribution over the channel section is hydrostatic.
- The channel is prismatic.
- The velocity distribution in the channel is fixed.
- The roughness coefficient is independent of the depth of flow and constant throughout the channel reach under consideration.
- The energy correction factor is unity.

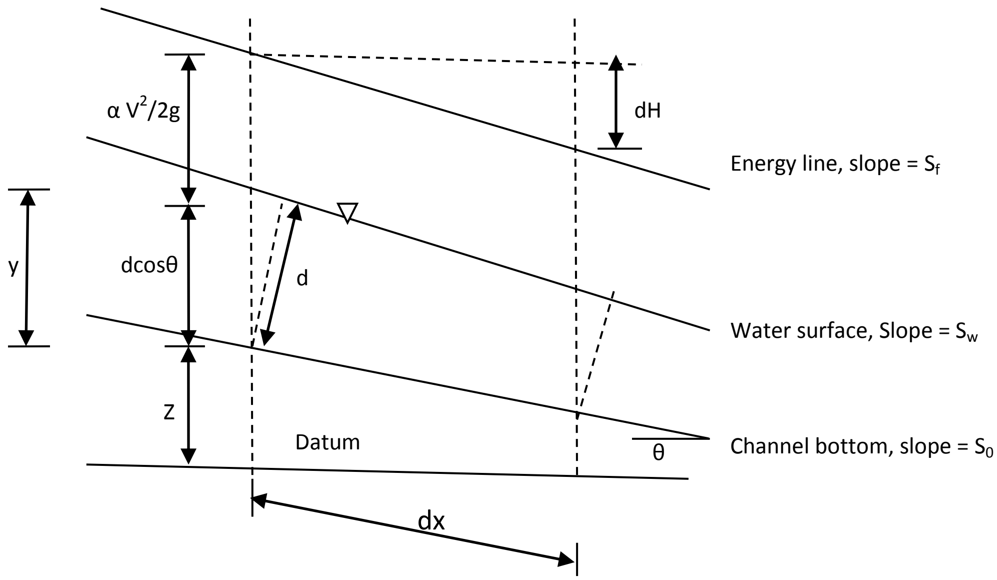
Consider the profile of GVF in the elementary length dx of an open channel. The total head above the datum at the upstream section is

$$H = z + dcos\theta + \alpha \frac{V^2}{2g}$$

where H = total head, z = vertical distance of channel bed above the datum, d = depth of flow normal to the direction of flow and V = mean velocity. For small slope, $dcos\theta \approx y$, where y = depth of flow. α =1

$$H = z + y + \frac{V^2}{2g}$$

Let S_f, S_w and S_0 are slope of energy line, slope of water surface and channel bottom slope respectively.



Taking the bottom of the channel as the x axis and the vertically upwards direction measured from the channel bottom as the y axis and differentiation w.r.t. x

$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right)$$

$$= \frac{dz}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{Q^2}{2gA^2}\right)$$

$$= \frac{dz}{dx} + \frac{dy}{dx} + \frac{d}{dy}\left(\frac{Q^2}{2gA^2}\right)\frac{dy}{dx}$$

$$= \frac{dz}{dx} + \frac{dy}{dx}\left[1 - \frac{Q^2}{gA^3}\frac{dA}{dy}\right]$$

$dH/dx = -S_f$, $dz/dx = -S_0$ and dA/dy =top width (T)

$$-S_f = -S_0 + \frac{dy}{dx}\left[1 - \frac{Q^2 T}{gA^3}\right]$$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2 T}{gA^3}}$$

This equation is known as the dynamic equation of gradually varied flow. It can also be expressed in terms of Froude number (F_r).

$$\frac{Q^2T}{gA^3} = \frac{V^2}{gA/T} = \frac{V^2}{gD} = F_r^2$$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2}$$

If dy/dx =0, S_o=S_f, the water surface is parallel to channel bottom, thus representing a uniform flow. When dy/dx is +ve the water surface is rising and when dy/dx is –ve , the water surface is falling.

Modified form of GVF

Form 1: GVF equation in terms of conveyance and section factor

K = conveyance at any depth y and K_n = conveyance corresponding to normal depth y_n.

$K = \frac{Q}{\sqrt{S_f}}$ and $K_n = \frac{Q}{\sqrt{S_0}}$

$S_f = \frac{Q^2}{K^2}$ and $S_0 = \frac{Q^2}{{K_n}^2}$

$$\frac{S_f}{S_0} = \frac{{K_n}^2}{K^2}$$

Let Z= section factor at depth y and Z_c = section factor at the critical depth y_c.

$Z = \sqrt{\frac{A^3}{T}}$ and $Z_c = \sqrt{\frac{{A_c}^3}{T}}$

$Z^2 = \frac{A^3}{T}$ and $Z_c^2 = \frac{{A_c}^3}{T} = \frac{Q^2}{g}$

$$\frac{Z_c^2}{Z^2} = \frac{Q^2T}{gA^3}$$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2T}{gA^3}} = \frac{S_0\left(1 - \frac{S_f}{S_0}\right)}{1 - \frac{Q^2T}{gA^3}}$$

Substituting the values of $\frac{S_f}{S_0}$ and $\frac{Q^2T}{gA^3}$

$$\frac{dy}{dx} = \frac{S_0(1 - K_n^2/K^2)}{1 - Z_c^2/Z^2}$$

This is another form of equation for GVF.

Form 2: Modified form of GVF equation considering wide rectangular channel

For wide rectangular channel, hydraulic radius (R) can be approximated as flow depth (y).

a. Using Manning's equation

Discharge = $\frac{1}{n}AR^{2/3}S_f^{1/2} = \frac{1}{n}byy^{2/3}S_f^{1/2} = \frac{1}{n}by^{5/3}S_f^{1/2}$

Discharge in terms of normal depth (y_n) is given by

Discharge = $\frac{1}{n}AR^{2/3}S_f^{1/2} = \frac{1}{n}by_n{y_n}^{2/3}S_f^{1/2} = \frac{1}{n}b{y_n}^{5/3}S_0^{1/2}$

$$\frac{1}{n}by^{5/3}S_f^{1/2} = \frac{1}{n}b{y_n}^{5/3}S_0^{1/2}$$

$$\frac{S_f}{S_0} = \left(\frac{y_n}{y}\right)^{10/3}$$

$$\frac{Q^2 T}{gA^3} = \frac{Q^2 b}{g(by)^3} = \frac{Q^2}{b^2 g} \cdot \frac{1}{y^3} = \frac{y_c^3}{y^3}$$

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2 T}{gA^3}} = \frac{S_0\left(1 - \frac{S_f}{S_0}\right)}{1 - \frac{Q^2 T}{gA^3}} \qquad \text{(a)}$$

Substituting the values of $\frac{S_f}{S_0}$ and $\frac{Q^2 T}{gA^3}$

$$\frac{dy}{dx} = \frac{S_0\left[1 - (y_n/y)^{10/3}\right]}{1 - (y_c/y)^3}$$

b. Using Chezy's formula

$$Q = CA\sqrt{RS_f} = Cby\sqrt{yS_f} = Cby^{3/2} S_f^{1/2}$$

Similarly in terms of normal depth,

$$Q = Cby_n{}^{3/2} S_0^{1/2} \frac{S_f}{S_0} = \left(\frac{y_n}{y}\right)^3$$

Substituting the values of $\frac{S_f}{S_0}$ and $\frac{Q^2 T}{gA^3}$ in (a)

$$\frac{dy}{dx} = \frac{S_0(1 - (y_n/y)^3)}{1 - (y_c/y)^3}$$

Slope of water surface

$\frac{dy}{dx}$ = slope of water surface with respect to channel bottom

Slope of water surface with respect to horizontal (S_w)

$S_w = S_0 - \frac{dy}{dx}$ for rising water level

$S_w = S_0 + \frac{dy}{dx}$ for falling water level

8.2 Classification of slopes

- Mild slope (M): channel bottom slope<critical slope, normal depth>critical depth
- Steep slope (S): channel bottom slope>critical slope, normal depth<critical depth
- Critical slope (C): Mild slope: channel bottom slope=critical slope, normal depth=critical depth
- Horizontal slope (H): Channel bottom slope = 0, Normal depth = ∞
- Adverse slope (A): Channel bottom slope <0 (negative), normal depth: imaginary or non-existent

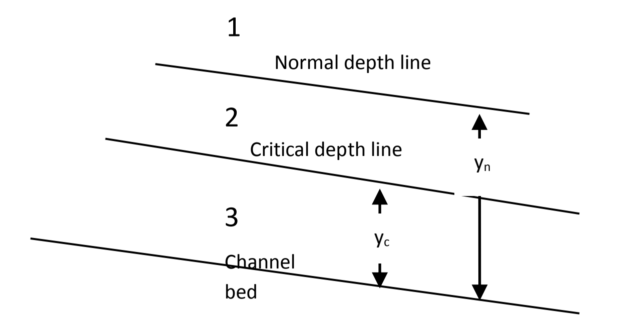
Sustaining and non-sustaining slope

A sustaining slope is a channel slope that falls in the direction of flow. Hence it is always positive. It may be critical, mild (subcritical) or steep (supercritical).

A non-sustaining slope may be horizontal or adverse. A horizontal slope is a zero slope. An adverse slope is a negative slope that rises in the direction of flow.

8.3 Water surface profile

For a given discharge and channel conditions, the normal depth and critical depth can be computed. A line drawn parallel to the channel bed and at height y_n is called normal depth line (NDL) and a line parallel to the channel bed and at height y_c is called the critical depth line (CDL). On the basis of these lines, the vertical space in a longitudinal section of the channel can be divided into three regions
Zone 1- space above both the critical and normal depths
Zone 2 – region lies between normal depth and critical depth
Zone 3 -region below both the normal and critical depth lines and above channel bed



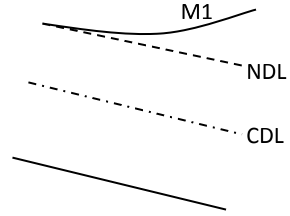
The following is the classification of flow profiles for different kinds of slopes.

I. M profile (Surface profile for Mild sloped channels)

All the three zones occur with channels of mild slopes and corresponding flow profiles are designed as M1, M2 and M3 profiles.

a. M1 profile

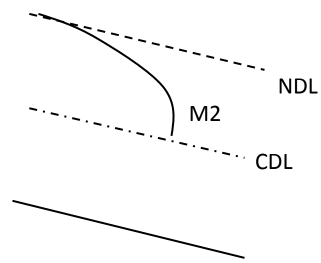
Condition: $y > y_n > y_c$
$\frac{dy}{dx} = \frac{S_0[1-(y_n/y)^{10/3}]}{1-(y_c/y)^3} = \frac{+ve}{+ve} = +ve$
$y \to y_n, \frac{dy}{dx} \to 0$: u/s end of the curve touches NDL.
$y \to \infty, \frac{dy}{dx} \to S_0$: d/s end of the curve is horizontal.
Type of profile: Backwater



Type of flow: Subcritical

Examples: flow behind dam, flow in a canal joining two reservoirs

b. M2 profile



Condition: $y_n > y > y_c$

$\frac{dy}{dx} = \frac{S_0[1-(y_n/y)^{10/3}]}{1-(y_c/y)^3} = \frac{-ve}{+ve} = -ve$

$y \to y_n, \frac{dy}{dx} \to 0$: u/s end of the curve is tangent to the NDL.

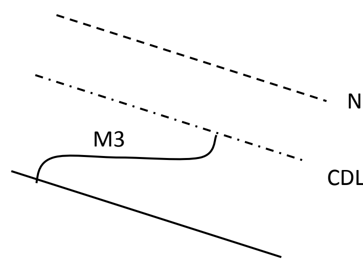
$y \to y_{c,}\ \frac{dy}{dx} \to \infty$: d/s end of the curve is normal to the CDL.

Type of profile: Drawdown

Type of flow: Subcritical

Examples: Flow over a free overfall, flow at the U/S end of a sudden enlargement in a mild channel

c. M3 profile



Condition: $y_n > y_c > y$

$\frac{dy}{dx} = \frac{S_0[1-(y_n/y)^{10/3}]}{1-(y_c/y)^3} = \frac{-ve}{-ve} = +ve$

$y \to y_c, \frac{dy}{dx} \to \infty$: d/s end of the curve is normal to the CDL.

$y \to 0, \frac{dy}{dx} \to \infty$: u/s end of the curve is normal to be bed.

Type of profile: Backwater

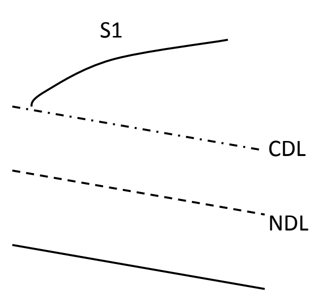
Type of flow: Supercritical

Examples: Flow downstream of a sluice gate, flow in a channel where bottom slope changes from steep to mild

II. S profile (Surface profile for Steep sloped channel)

All the three zones occur with channels of steep slopes and corresponding flow profiles are designed as S1, S2 and S3 profiles.

a) S1 profile



Condition: $y > y_c > y_n$

$\frac{dy}{dx} = \frac{S_0[1-(y_n/y)^{10/3}]}{1-(y_c/y)^3} = \frac{+ve}{+ve} = +ve$

$y \to y_c, \frac{dy}{dx} \to \infty$: u/s end of the curve is normal to the CDL.

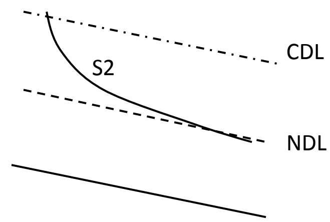
$y \to \infty, \frac{dy}{dx} \to S_0$: d/s end of the curve is horizontal.

Type of profile: Backwater

Type of flow: Subcritical

Examples: flow behind dam on a steep channel, flow behind an overflow weir

b) S2 profile



Condition: $y_c > y > y_n$

$\frac{dy}{dx} = \frac{S_0\left[1-(y_n/y)^{10/3}\right]}{1-(y_c/y)^3} = \frac{+ve}{-ve} = -ve$

$y \to y_c, \frac{dy}{dx} \to \infty$: u/s end of the curve is normal to the CDL.

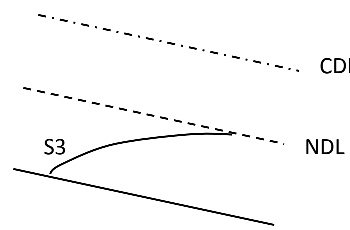
$y \to y_n, \frac{dy}{dx} \to 0$: d/s end of the curve is tangent to the NDL.

Type of profile: Drawdown

Type of flow: Supercritical

Examples: Flow from steep to steeper channel, Profile formed when there is sudden enlargement in a steep slope

c) S3 profile



Condition: $y_c > y_n > y$

$\frac{dy}{dx} = \frac{S_0\left[1-(y_n/y)^{10/3}\right]}{1-(y_c/y)^3} = \frac{-ve}{-ve} = +ve$

$y \to y_n, \frac{dy}{dx} \to 0$: d/s end of the curve is tangential to the NDL.

$y \to y_c, \frac{dy}{dx} \to \infty$: u/s end of the curve is normal to the bed.

Type of profile: Backwater

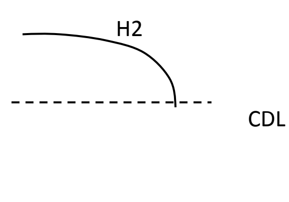
Type of flow: Supercritical

Examples: Flow down of the sluice gate, flow through channel where bottom slope change from steeper to steep

III. H profile (Surface profile in horizontal channels)

Since y_n is infinite, H1 profile does not exist. So there are two H profiles: H2 and H3 (Assume S_0 to be very small.)

a. H2 profile



Condition: $y > y_c$

$\frac{dy}{dx} = \frac{S_0 - S_f}{1-(y_c/y)^3} = \frac{-ve}{+ve} = -ve$

$y \to \infty, \frac{dy}{dx} \to S_0$: u/s end of the curve is horizontal.

$y \to y_c, \frac{dy}{dx} \to -\infty$: d/s end of the curve is normal to the CDL.

Type of profile: Drawdown

Type of flow: Subcritical

Example: Flow at free fall overran edge

b. H3 profile

Condition: $y < y_c$

$\frac{dy}{dx} = \frac{S_0 - S_f}{1-(y_c/y)^3} = \frac{-ve}{-ve} = +ve$

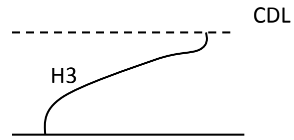
$y \to y_c, \frac{dy}{dx} \to -\infty$: d/s end of the curve is normal to CDL.

$y \to 0, \frac{dy}{dx} \to \infty$: u/s end of the curve is normal to bed.

Type of profile: Backwater

Type of flow: Supercritical

Example: Flow below a sluice gate provided in a horizontal channel



IV. C profile (Surface profile in critical-sloped channels)

As $y_n = y_c$, C2 profile does not exist. So there are two C profiles: C1 and C3.

a. C1 profile

Condition: $y > y_c = y_n$

$\frac{dy}{dx} = \frac{S_0[1-(y_n/y)^3]}{1-(y_c/y)^3} = \frac{+ve}{+ve} = +ve$

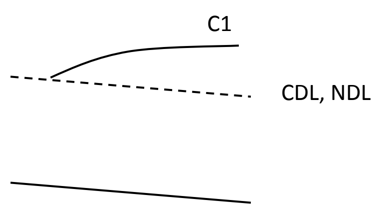
$y \to y_c, \frac{dy}{dx} \to S_0$: u/s end of the curve is horizontal.

$y \to \infty, \frac{dy}{dx} \to S_0$: d/s end of the curve is horizontal.

Type of profile: Backwater

Type of flow: Subcritical

Example: Flow behind an overflow weir, flow behind a sluice gate



b. C3 profile

Condition: $y < y_c = y_n$

$\frac{dy}{dx} = \frac{S_0[1-(y_n/y)^3]}{1-(y_c/y)^3} = \frac{-ve}{-ve} = +ve$

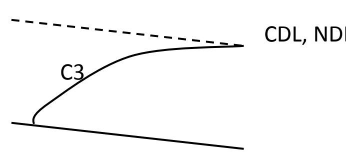
$y \to y_c, \frac{dy}{dx} \to S_0$: d/s end of the curve is horizontal.

$y \to 0, \frac{dy}{dx} \to \infty$: : u/s end of the curve is normal to the bed.

Type of profile: Backwater

Type of flow: Supercritical

Example: Flow below a sluice gate provided in a channel with critical slope



V. A profile (Surface profile is adverse-sloped channels)

Since S_0<0 (-ve) and y_n is imaginary, A1 profile does not exist. So there are two A profiles: A2 and A3.

a. A2 profile

Condition: $y < y_c$

$\frac{dy}{dx} = \frac{S_0 - S_f}{1-(y_c/y)^3} = \frac{S_0(1-S_f/S_0)}{1-(y_c/y)^3} = \frac{-ve}{+ve} = -ve$

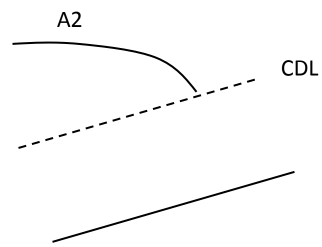
$y \to y_c, \frac{dy}{dx} \to -\infty$: d/s end of the curve is normal to CDL.

$y \to \infty, \frac{dy}{dx} \to S_0$: u/s end of the curve is horizontal.

Type of profile: Drawdown

Type of flow: Subcritical

Example: Flow profile at the D/S end of a adverse slope



b. A3 profile

Condition: $y<y_c$

$\frac{dy}{dx} = \frac{S_0 - S_f}{1-(y_c/y)^3} = \frac{S_0(1-S_f/S_0)}{1-(y_c/y)^3} = \frac{-ve}{-ve} = +ve$

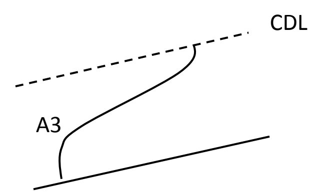
$y \to y_c, \frac{dy}{dx} \to -\infty$: d/s end of the curve is normal to CDL.

$y \to 0, \frac{dy}{dx} \to -\infty$: u/s end of the curve is normal to bed.

Type of profile: Backwater

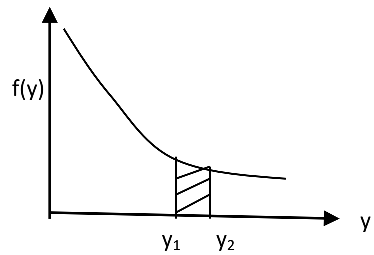
Type of flow: Supercritical

Example: Flow profile below a sluice gate provided in a adverse channel



8.4 Computation of GVF

a. Graphical integration method



This method is used to compute the distance from the given depth.

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - F_r^2}$$

x =Distance between x_1 and x_2

$$x = x_2 - x_1 = \int_{x_1}^{x_2} dx = \int_{y_1}^{y_2} \frac{dx}{dy} dy$$

$$= \int_{y_1}^{y_2} \left(\frac{1 - F_r^2}{S_0 - S_f}\right) dy = \int_{y_1}^{y_2} f(y) dy$$

Take different value of y and compute corresponding f(y), and plot f(y) vs y curve. The area enclosed between y_1 and y_2 gives the distance.

If the interval is very small, the area can be computed by multiplying the average value of f(y) with the interval dy.

b. Numerical approach

I. Direct integration method

Bresse's method

Bresse's method of integrating the varied flow equation is applicable in the case of very wide rectangular channels. In this method Chezy's formula is used for the evaluation of the effect of frictional resistance to the flow.

Using Chezy's formula

$$\frac{dy}{dx} = \frac{S_0(1 - (y_n/y)^3)}{1 - (y_c/y)^3}$$

Let $y/y_n = u$

$$\frac{dy}{dx} = \frac{S_0(1 - 1/u^3)}{1 - (y_c/y_n)^3(1/u^3)}$$

$$S_0 dx\,(u^3 - 1) = (u^3 - (y_c/y_n)^3) dy$$

$$S_0 dx = ((y_c/y_n)^3 - u^3)\frac{1}{(1-u^3)}dy$$

$y = uy_n$
$dy = y_n du$

$$dx = \frac{y_n}{S_0}((y_c/y_n)^3 - u^3)\frac{1}{(1-u^3)}du$$

$$dx = \frac{y_n}{S_0}((y_c/y_n)^3 - 1 + 1 - u^3)\frac{1}{(1-u^3)}du$$

$$dx = \frac{y_n}{S_0}\left[du + ((y_c/y_n)^3 - 1)\frac{du}{1-u^3}\right]$$

Integrating

$$x = \frac{y_n}{S_0}\left[u - (1 - (y_c/y_n)^3)\int \frac{du}{1-u^3}\right] + Constant$$

The distance x can be calculated from this equation if the value of the integral $\int \frac{du}{1-u^3}$ is known. The function $\int \frac{du}{1-u^3}$ is known as Bresse's varied flow function, which is given by

$$\int \frac{du}{1-u^3} = \varphi(u) = \frac{1}{6}ln\frac{u^2+u+1}{(u-1)^2} - \frac{1}{\sqrt{3}}tan^{-1}\frac{\sqrt{3}}{2u+1}$$

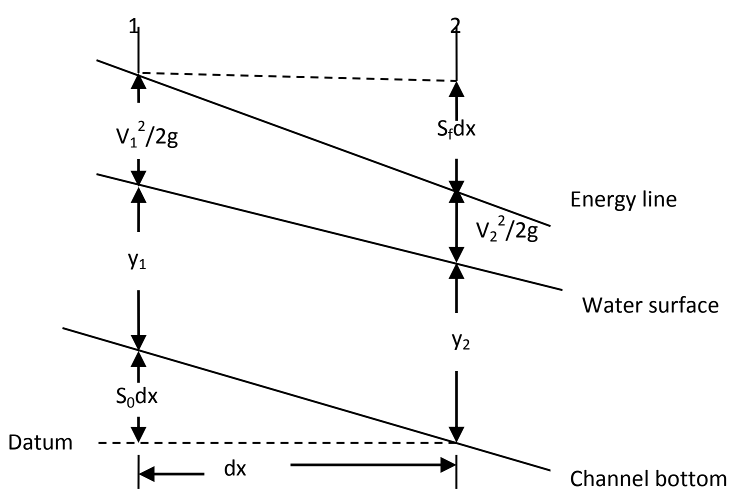
$\left(\frac{y_c}{y_n}\right)^3 = \frac{C^2 S_0}{g}$ (Using chezy)

For two sections

$$x2 - x1 = \frac{y_n}{S_0}\left[(u2 - u1) - \left(1 - \left(\frac{y_c}{y_n}\right)^3\right)\left(\varphi(u2) - \varphi(u1)\right)\right]$$

II. Direct step method

In this method, the entire length of the channel is divided into short reaches and the computation is carried out step by step from one end of the reach to the other.



Consider a channel reach of length dz. Equating the total energies at section 1 and 2

$S_0 dx + y_1 + V_1^2/2g = y_2 + V_2^2/2g + S_f dx$

where V_1 and V_2 are mean velocity at section 1 and 2, S_0 is bed slope and S_f is energy line slope.

$E_1 = y_1 + V_1^2/2g$ and $E_2 = y_2 + V_2^2/2g$

where E_1 and E_2 are the specific energy at section 1 and 2.

From these two equations,

$S_0 dx + E_1 = E_2 + S_f dx$

$$dx = \frac{E_2 - E_1}{S_0 - S_f} = \frac{\Delta E}{S_0 - S_f}$$

where ΔE represent the change in specific energy between the section 1 and 2. For S_f, average friction slope is used in the computation.

$$dx = \frac{\Delta E}{S_0 - \bar{S_f}}$$

For the computation of the length x of the surface profile the various quantities that should be known, are: discharge Q, the channel shape, bottom slope S_0, Manning's n or Chezy's C and the depth of flow at one of the section (at one of the control point). The procedure for computation is as follows

Tabular form

y	A	P	R	V	E	S_f	$\bar{S_f}$	Δx	x

- For the first value of y, compute A, P, R, V, E and S_f.
- Take another value of y (increasing/decreasing from previous), and compute all variables.
- Perform computation by taking different values of y to obtain the required flow profile. (Assumed depth will be either more or less than the known depth at control section, depending on the surface profile (rising or falling)).

Formulae:

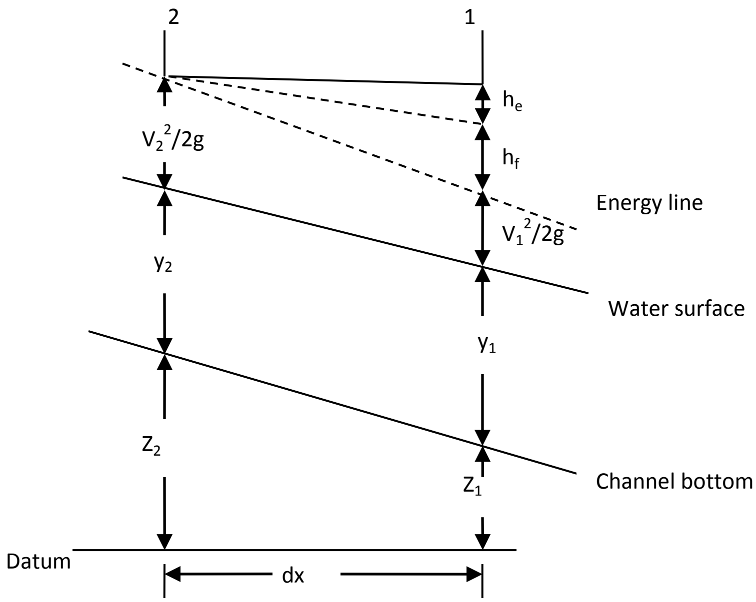
R=A/P, V= Q/A, E = y +V^2/2g, $S_f = \frac{n^2V^2}{R^{4/3}}$ using n or $S_f = \frac{V^2}{C^2R}$ using C

$\bar{S_f} = \frac{1}{2}\left[\left(S_f\right)_{i-1} + \left(S_f\right)_i\right]$

$\Delta x = \frac{E_i - E_{i-1}}{S_0 - \bar{S_f}}$, x = cumulative sum of Δx

III. Standard step method

Direct step method is suitable for prismatic channels, but there are some basic difficulties in applying it to natural channels. In natural channels, the cross-sectional shapes are likely to vary from section to section and also cross-sectional information is known only at a few locations along the channel. In such case, standard step method is applied.



Consider section 1 downstream of section 2. Proceed the calculation upstream.

Applying Bernoulli's equation at 1 and 2

$Z_1 + y_1 + V_1^2/2g + h_f + h_e = Z_2 + y_2 + V_2^2/2g$

h_f = friction loss, h_e = eddy loss

$h_f = S_f\, dx$

$H_1 = Z_1 + y_1 + V_1^2/2g$ = elevation of energy line at 1

$H_2 = Z_2 + y_2 + V_2^2/2g$ =elevation of energy line at 2

$H_2 = H_1 + h_f + h_e$

The frictional loss, h_f is estimated by

$$h_f = \bar{S_f} dx = \frac{1}{2}(S_{f1} + S_{f2})dx$$

where $S_f = \frac{n^2V^2}{R^{4/3}}$ using n or $S_f = \frac{V^2}{C^2R}$ using C

$$h_e = C_e \left|\frac{V_1^2 - V_2^2}{2g}\right|$$

C_e = coefficient

For prismatic channel, C_e = 0.

<u>Trial and error approach</u>

Solution in tabular form (trial and error)

Stn.	Z	y	A	P	R	V	H(1)	S_f	$\bar{S_f}$	Δx	h_f	h_e	H(2)

Z= water surface elevation, y = depth of flow

- For station 1, Z is known. Compute y from the given information. (e.g. y= Z-El of dam site-S_0 dx). Then compute A, P, R, V, H(1) and S_f. H(2) = H(1) in the first step.
- For station 2, assume a value of Z or y and compute all variables. Check if H(1) is close to H(2). If not, take another value of Z and repeat the procedure until H(1) is almost equal to H(2).
- Perform similar computations for all remaining stations to obtain a flow profile.

Formulae:

R= A/P, V= Q/A, H (1) = Z + V^2/2g, $S_f = \frac{n^2V^2}{R^{4/3}}$ using n or $S_f = \frac{V^2}{C^2R}$ using C

$\bar{S_f} = \frac{1}{2}\left[\left(S_f\right)_{i-1} + \left(S_f\right)_i\right]$

$h_f = \Delta x \bar{S_f}$

H(2) = $(H(2))_{i-1} + h_f + h_e$

Alternative solution for standard step

Neglecting h_e

$$H_2 = H_1 + h_f$$

$$H_2 = H_1 + \frac{1}{2}\left(S_{f1} + S_{f2}\right)(X_2 - X_1)$$

$$Z_2 + y_2 + \frac{Q^2}{2gA_2^2} - \frac{1}{2}S_{f2}(X_2 - X_1) - H_1 - \frac{1}{2}S_{f1}(X_2 - X_1) = 0$$

$$f(y_2) = Z_2 + y_2 + \frac{Q^2}{2gA_2^2} - H_1 - \frac{1}{2}\left(S_{f1} + S_{f2}\right)(X_2 - X_1)$$

In this equation, the values of variables at 1 are known. A_2 and S_{f2} are function of y_2. Hence y_2 may be determined by solving above equation. This equation may be solved for y_2 by numerical method such as Newton-Raphson method.

Chapter 9: Rapidly varied flow

9.1 Hydraulic jump

When the depth of flow changes rapidly from a low stage to a high stage, there is abrupt rise of water surface. This local phenomenon is known as hydraulic jump. The hydraulic jump occurs when a supercritical flow changes into subcritical flow. It is classified as rapidly varied flow. The flow in the hydraulic jump is accompanied by the formation of extremely turbulent rollers and hence there is a considerable dissipation of energy. The turbulent eddies break up into smaller ones as they move downstream. The energy is dissipated into heat through these small eddies. Further, air is entrained due to the breaking of number of wavelets on the surface.

The hydraulic jump occurs frequently in a canal below a regulating sluice, at the foot of a spillway, or at the place where a steep channel bottom slope suddenly turns flat.

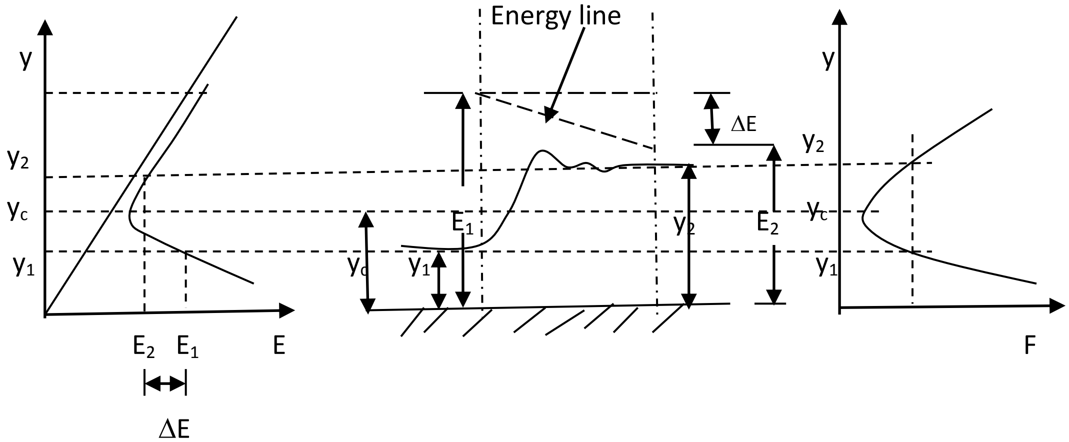
The energy equation is not applicable for the analysis of hydraulic jump because the hydraulic jump is associated with an appreciable loss of energy which is initially unknown. Therefore, momentum equation is used by considering the portion of the hydraulic jump as the control volume.

Applications of hydraulic jump

- As an energy dissipater, to dissipate the excess energy of flowing water downstream of hydraulic structure such as spillway and sluice gates
- Mixing of chemical
- Aeration of stream polluted by biodegradable waste
- Raising the water level in the channel for irrigation
- Desalination of seawater
- efficient operation of flow measurement flumes

Specific energy and specific force curves for hydraulic jump

Consider a hydraulic jump formed in a prismatic channel with horizontal floor carrying a discharge Q. The depth of flow before the jump (y_1) is called initial depth and that after the jump (y_2) is called sequent depth. The initial depth and the sequent depth are commonly known as conjugate depths. The specific force is same in case of conjugate depths, whereas the specific energy is same for alternate depths. Both the specific energy curve and the specific curve attain minimum value at the critical depth.



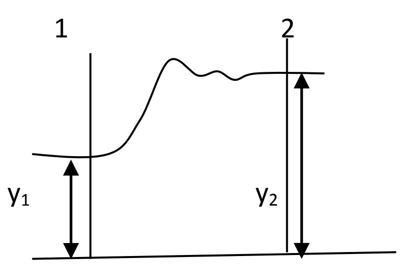
Specific energy curve Hydraulic jump Specific force curve

9.2 Hydraulic jump in a rectangular channel

Momentum equation for hydraulic jump

Assumptions

- Loss of head due to friction is negligible
- The flow is uniform and the pressure distribution is hydrostatic before and after the jump
- Channel is horizontal hence the weight component in the direction is neglected
- The momentum correction factor is unity.
- The flow is steady



Consider two sections 1 and 2 before and after the jump. Based on above assumptions, the only external forces acting on the mass of water between sections 1 and 2 are hydrostatic pressures P_1 and P_2 at sections 1 and 2 respectively. Let Q be the discharge flowing through the channel and V_1 and V_2 be the velocities at section 1 and 2 respectively. W is the weight of water enclosed between two sections; and F_f is the resistance force due to friction.

The momentum equation for the jump is given by

$$P_1 - P_2 + W sin\theta - F_f = \rho Q(V_2 - V_1)$$

For a short reach of prismatic channel, θ = 0 and friction force, F_f can be neglected.

$$P_1 - P_2 = \rho Q(V_2 - V_1)$$

a. Expression for sequent depth

Consider unit width of the channel. The discharge per unit width $q = V_1y_1=V_2y_2$

$P_1 = \gamma A_1 \overline{x_1} = \gamma(1.y_1).\frac{y_1}{2} = \frac{1}{2}\gamma y_1^2$ and $P_2 = \gamma A_2 \overline{x_2} = \gamma(1.y_2).\frac{y_2}{2} = \frac{1}{2}\gamma y_2^2$

Substituting the values of P_1 and P_2

$$\frac{1}{2}\gamma y_1^2 - \frac{1}{2}\gamma y_2^2 = \rho q(V_2 - V_1)$$

From continuity equation: $q = y_1 V_1 = y_2 V_2$

$V_1 = q/y_1$, $V_2 = q/y_2$

Substituting the values of V_1 and V_2

$$\frac{1}{2}\rho g(y_1^2 - y_2^2) = \rho q\left(\frac{q}{y_2} - \frac{q}{y_1}\right)$$

$$\frac{g}{2}(y_1 + y_2)(y_1 - y_2) = q^2\left(\frac{y_1 - y_2}{y_1 y_2}\right)$$

$\frac{2q^2}{g} = y_1 y_2(y_1 + y_2)$ (I)

Dividing both sides by y_1 and simplifying

$$y_2^2 + y_1 y_2 - \frac{2q^2}{g y_1} = 0$$

Solving for y_2

$$y_2 = \frac{-y_1 \pm \sqrt{y_1^2 + 4\frac{2q^2}{g y_1}}}{2}$$

As negative root is not possible

$$y_2 = -\frac{y_1}{2} + \sqrt{\left(\frac{y_1}{2}\right)^2 + \frac{2q^2}{g y_1}}$$

This is the relationship between conjugate depths

Conjugate depths in terms of Froude number

Substituting $q = y_1 v_1$

$$y_2 = -\frac{y_1}{2} + \sqrt{\left(\frac{y_1}{2}\right)^2 + \frac{2y_1^2 {V_1}^2}{g y_1}}$$

$$y_2 = -\frac{y_1}{2} + \sqrt{\left(\frac{y_1}{2}\right)^2\left(1 + \frac{8{V_1}^2}{g y_1}\right)}$$

$$\frac{{V_1}^2}{gy_1} = F_{r1}^2$$

$$y_2 = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)$$

Similarly

For y_1

$y_1 = -\frac{y_2}{2} + \sqrt{\left(\frac{y_2}{2}\right)^2 + \frac{2q^2}{gy_2}}$ and $y_1 = \frac{y_2}{2}\left(-1 + \sqrt{1 + 8F_{r2}^2}\right)$

The above equation is known as Belanger momentum equation.

$F_{r1}>1$

$F_{r2}<1$

The higher F_{r1}, the lower F_{r2}

Alternate derivation: Expression for sequent depth using specific force analysis

Specific force at 1 = specific force at 2

$$\frac{Q^2}{gA_1} + A_1\overline{x_1} = \frac{Q^2}{gA_2} + A_2\overline{x_2}$$

For rectangular channel of bottom width b,

$$\frac{Q^2}{gby_1} + by_1\frac{y_1}{2} = \frac{Q^2}{gby_2} + by_2\frac{y_2}{2}$$

$$\frac{b}{2}\left(\frac{2Q^2}{gb^2y_1} + y_1^2\right) = \frac{b}{2}\left(\frac{2Q^2}{gb^2y_2} + y_2^2\right)$$

$$y_2^2 - y_1^2 = \frac{2Q^2}{gb^2}\left(\frac{1}{y_1} - \frac{1}{y_2}\right)$$

$$(y_2 + y_1)(y_2 - y_1) = \frac{2q^2}{g}\left(\frac{y_2 - y_1}{y_1y_2}\right)$$

$\frac{2q^2}{g} = y_1y_2(y_1 + y_2)$ (I)

Dividing both sides by y_1 and simplifying

$$y_2^2 + y_1y_2 - \frac{2q^2}{gy_1} = 0$$

Solving for y_2

$$y_2 = \frac{-y_1 \pm \sqrt{y_1^2 + 4\frac{2q^2}{gy_1}}}{2}$$

As negative root is not possible

$$y_2 = -\frac{y_1}{2} + \sqrt{\left(\frac{y_1}{2}\right)^2 + \frac{2q^2}{gy_1}}$$

This is the relationship between conjugate depths

Conjugate depths in terms of Froude number

Substituting q = y_1 v_1

$$y_2 = -\frac{y_1}{2} + \sqrt{\left(\frac{y_1}{2}\right)^2 + \frac{2y_1^2 {V_1}^2}{gy_1}}$$

$$y_2 = -\frac{y_1}{2} + \sqrt{\left(\frac{y_1}{2}\right)^2 \left(1 + \frac{8{V_1}^2}{gy_1}\right)}$$

$$\frac{{V_1}^2}{gy_1} = F_{r1}^2$$

$$y_2 = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)$$

Similarly

For y_1

$y_1 = -\frac{y_2}{2} + \sqrt{\left(\frac{y_2}{2}\right)^2 + \frac{2q^2}{gy_2}}$ and $y_1 = \frac{y_2}{2}\left(-1 + \sqrt{1 + 8F_{r2}^2}\right)$

b. Relationship between downstream and upstream Froude numbers

$F_{r1} = \frac{V_1}{\sqrt{gy_1}}$ and $F_{r2} = \frac{V_2}{\sqrt{gy_2}}$

or, $V_1 = F_{r1}\sqrt{gy_1}$ and $V_2 = F_{r2}\sqrt{gy_2}$ (a)

From continuity

$V_1 y_1 = V_2 y_2$ (b)

From a and b

$$y_1\, F_{r1}\sqrt{gy_1} = y_2\, F_{r2}\sqrt{gy_2}$$

$\frac{y_1}{y_2} = \left(\frac{F_{r2}}{F_{r1}}\right)^{2/3}$ (c)

Also,

$$y_2 = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)$$

$\frac{y_1}{y_2} = \frac{2}{-1+\sqrt{1+8F_{r1}^2}}$ (d)

Equating c and d

$$\left(\frac{F_{r2}}{F_{r1}}\right)^{2/3} = \frac{2}{-1 + \sqrt{1 + 8F_{r1}^2}}$$

$$F_{r2}^2 = \frac{8F_{r1}^2}{\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)^3}$$

c. Expression for energy loss in terms of conjugate depths (Analysis using specific energy)

$$\Delta E = E_1 - E_2$$
$$= \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$$

From continuity $V_1 = q/y_1$, $V_2 = q/y_2$

$$= \left(y_1 + \frac{q^2}{2gy_1^2}\right) - \left(y_2 + \frac{q^2}{2gy_2^2}\right)$$
$$= \frac{q^2}{2g}\left(\frac{y_2^2 - y_1^2}{y_1^2 y_2^2}\right) - (y_2 - y_1)$$

Substituting $\frac{q^2}{g} = \frac{1}{2} y_1 y_2 (y_1 + y_2)$ from eq. (i)

$$\Delta E = \frac{1}{4} y_1 y_2 (y_1 + y_2)\left(\frac{y_2^2 - y_1^2}{y_1^2 y_2^2}\right) - (y_2 - y_1)$$

$$\Delta E = \frac{(y_2 - y_1)^3}{4 y_1 y_2}$$

d. Energy loss in terms of velocity

$$\Delta E = E_1 - E_2$$
$$= \left(y_1 + \frac{V_1^2}{2g}\right) - \left(y_2 + \frac{V_2^2}{2g}\right)$$
$$= \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) - (y_2 - y_1)$$

From continuity $y_1 = q/V_1$, $y_2 = q/V_2$

$$= \left(\frac{V_1^2 - V_2^2}{2g}\right) - \left(\frac{q}{V_2} - \frac{q}{V_1}\right)$$
$$= \left(\frac{V_1^2 - V_2^2}{2g}\right) - q\left(\frac{V_1 - V_2}{V_1 V_2}\right)$$

We have, $\frac{2q^2}{g} = y_1 y_2 (y_1 + y_2)$

$$\frac{2q^2}{g} = \frac{q^2}{V_1 V_2}\left(\frac{q}{V_1} + \frac{q}{V_2}\right)$$

$$q = \frac{2(V_1 V_2)^2}{g(V_1 + V_2)}$$

Substituting the value of q

$$\Delta E = \left(\frac{V_1^2 - V_2^2}{2g}\right) - \frac{2(V_1 V_2)^2}{g(V_1 + V_2)}\left(\frac{V_1 - V_2}{V_1 V_2}\right)$$

$$\Delta E = \frac{(V_1 - V_2)^3}{2g(V_1 + V_2)}$$

e. Other characteristics of jump

I. Relative loss= $\Delta E/E_1$ where E_1 = specific energy before jump

Relative loss in terms of Froude numbers

$$\frac{\Delta E}{E_1} = \frac{\frac{(y_2 - y_1)^3}{4y_1y_2}}{y_1 + \frac{V_1^2}{2g}}$$

$$= \frac{\frac{y_1^3\left(\frac{y_2}{y_1} - 1\right)^3}{4y_1y_2}}{y_1\left(1 + \frac{V_1^2}{2gy_1}\right)} = \frac{\left(\frac{y_2}{y_1} - 1\right)^3}{4\frac{y_2}{y_1}\left(1 + \frac{V_1^2}{2gy_1}\right)}$$

We have,

$y_2 = \frac{y_1}{2}\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)$ or, $\frac{y_2}{y_1} = \left(-\frac{1}{2} + \frac{1}{2}\sqrt{1 + 8F_{r1}^2}\right)$

$$= \frac{\left(\left(-\frac{1}{2} + \frac{1}{2}\sqrt{1 + 8F_{r1}^2}\right) - 1\right)^3}{4\left(-\frac{1}{2} + \frac{1}{2}\sqrt{1 + 8F_{r1}^2}\right)\left(1 + \frac{F_{r1}^2}{2}\right)} = \frac{\left(-3\sqrt{1 + 8F_{r1}^2}\right)^3}{8(2 + F_{r1}^2)\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)}$$

II. Height of jump (h_j) = $y_2 - y_1$

Relative height = $\frac{h_j}{E_1}$

Relative height in terms of Froude numbers

$\frac{h_j}{E_1} = \frac{y_2 - y_1}{y_1 + \frac{V_1^2}{2g}} = \frac{y_1\left(\frac{y_2}{y_1} - 1\right)}{y_1\left(1 + \frac{V_1^2}{2gy_1}\right)}$

Substituting for y_2/y_1

$$\frac{h_j}{E_1} = \frac{\left(-\frac{1}{2} + \frac{1}{2}\sqrt{1 + 8F_{r1}^2}\right) - 1}{\left(1 + \frac{F_{r1}^2}{2}\right)} = \frac{-3 + \sqrt{1 + 8F_{r1}^2}}{(2 + F_{r1}^2)}$$

III. Length of jump = $6(y_2 - y_1)$

IV. Efficiency of jump = E_2/E_1 where E_2 = specific energy after jump and E_1 = specific energy before jump

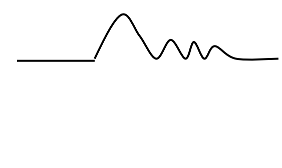
V. Power dissipated by the jump = $\gamma Q(\Delta E)$

9.3 Classification of jump

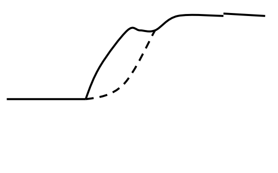
a. Based on the Froude number

Based on the Froude's number (F_{r1}) of the supercritical flow, the jump can be classified into following five categories.

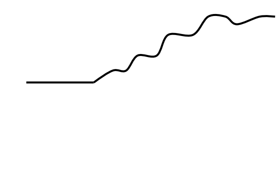
- Undular jump: $1.0 < F_{r1} < 1.7$, Undulating water surface with very small ripples on the surface, insignificant energy loss
- Weak jump: $1.7 < F_{r1} < 2.5$, A series of small roller on the surfaces of water, low energy loss.
- Oscillating jump: $2.5 < F_{r1} < 4.5$, Oscillation between the bed and the surface due to high velocity, moderate energy loss
- Steady jump: $4.5 < F_{r1} < 9.0$, Well established, significant energy loss
- Strong jump: $F_{r1} > 9.0$, Rough (wavy) water surface, very efficient energy dissipation



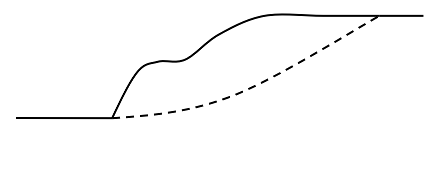
Undular jump



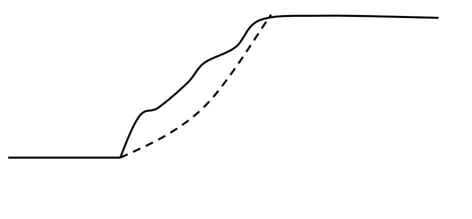
Weak jump



Oscillating jump



Steady jump



Strong jump

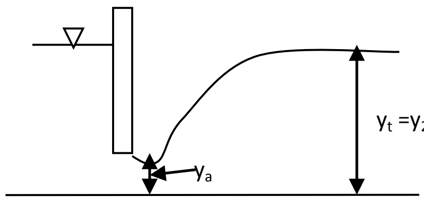
b. Based on tail water depth

The depth downstream of a hydraulic structure is called tailwater depth.
y_t = tailwater depth, y_a = Depth at the vena-contracta, y_2 = sequent depth to y_a

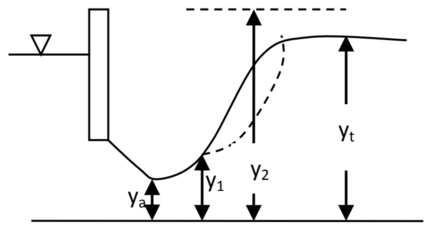
Free jump: The jump with sequent depth equal to or less than y_2 is called free jump. When $y_t = y_2$, a free jump will form at the vena-contracta.

Repelled jump: If $y_t < y_2$, the jump is repelled downstream of the vena-contracta through an M_3 curve (or may be H_3). The depth at the toe of the jump is larger than y_a. Such a jump is called a repelled jump.

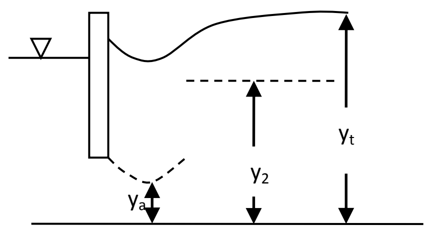
Submerged jump: If $y_t > y_2$, the jump is no longer free but gets drowned out. Such a jump is called drowned jump or submerged jump. The loss of energy in a submerged jump is smaller than that in a free jump.



Jump at vena-contracta



Repelled jump



Submerged jump

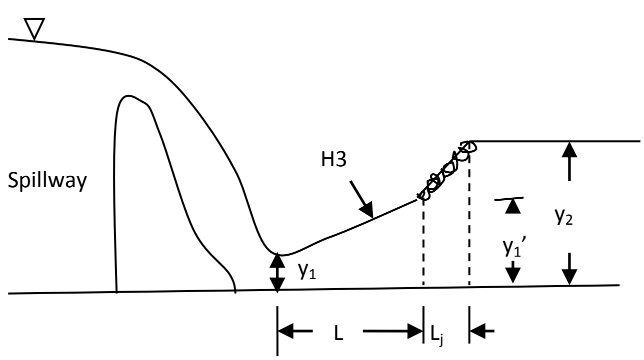
9.4 Location of jump

The formation of jump depends upon the initial depth, sequent depth and Froude number of flow. The position of jump can be fixed by introducing an obstruction or can be determined by using Belanger equation in case of naturally occurring jump.

$$y_1 = \frac{y_2}{2}\left(-1 + \sqrt{1 + 8F_{r2}^2}\right)$$

For given value of y_2, y_1 can be calculated using this equation. The length of jump can be computed by jump equation and the length of flow profile formed before or after the jump can be calculated by using any gradually varied profile computation method.

a. Jump below overfall spillway



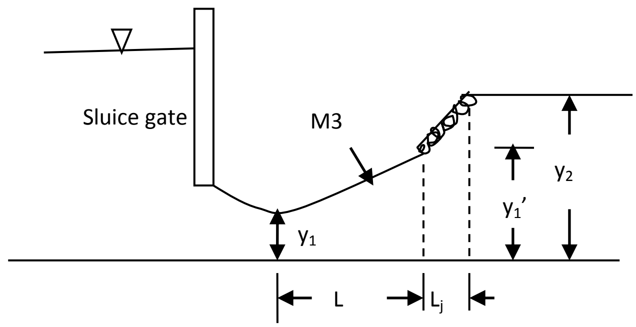
y_1 = Depth at vena-contracta (given)
y_2 = known or equal to y_n if not given
y_1' = depth sequent to y_2
Compute y_1' from Belanger equation.

$L_j = 6(y_2-y_1)$

Compute length (L) of H3 profile by direct step or other method taking depths at two ends as y_1 and y_1'.

Location of jump = distance L from vena-contracta.

b. Jump below sluice gate in mild slope



y_1 = Depth at vena-contracta (given)

y_2 = known or equal to y_n if not given

y_1' = depth sequent to y_2

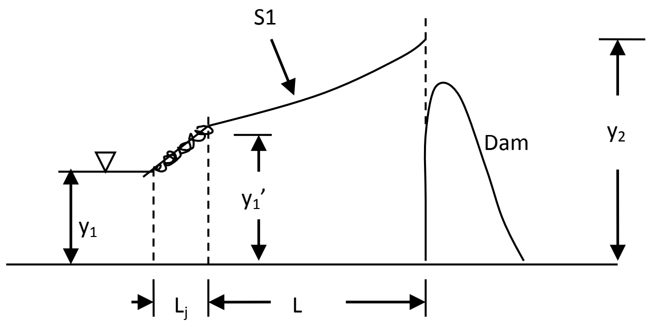
Compute y_1' from Belanger equation.

$L_j = 6(y_2-y_1)$

Compute length (L) of M3 profile by direct step or other method taking depths at two ends as y_1 and y_1'.

Location of jump = distance L from vena-contracta.

c. Jump in steep slope with barrier, e.g. dam



y_2 = Known depth at dam section

y_1 = Normal depth

y_1' = Depth sequent to y_1

y_2 should be greater than y_1' for forming jump.

Compute y_1' from Belanger equation.

$L_j = 6(y_1'-y_1)$

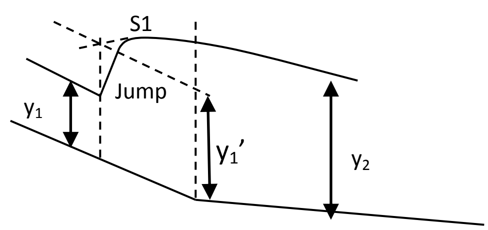
Compute length (L) of S1 profile by direct step or other method taking depths at two ends as y_1' and y_2.

Location of jump = distance $L+L_j$ from dam.

d. Jump formed due to change in slope from steep to mild/flat

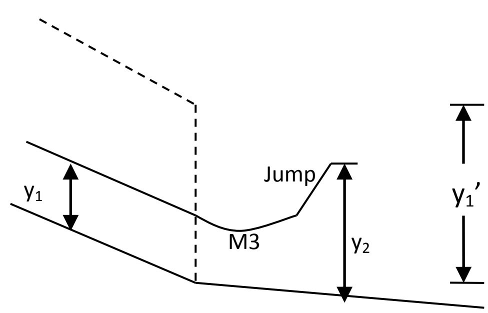
y_1 = normal depth at u/s channel (known or compute from given data)
y_2 = normal depth at d/s channel (known or compute from given data)
y_1' = Depth sequent to y_1 (Compute from Belanger equation)

(I) If $y_1' < y_2$, the jump will be formed on the u/s channel.



Compute the length (L) of S1 profile by direct step or other method taking depths at two ends as y_2 and y_1'.
location of jump = Distance L upstream from the point of change in slope.

(II) If $y_1' > y_2$, the jump will be formed on the d/s channel.



y_2' = Depth sequent to y_2 (Compute from Belanger equation)
Compute the length (L) of M3 profile by direct step or other method taking depths at two ends as y_1 and y_2'.
Location of jump = Distance L downstream from the point of change in slope.

Chapter 10: Flow in mobile boundary channel

10.1 Introduction

Sediment (Alluvium)

- loose non-cohesive material transported or deposited by the action of flowing water

Types of sediments

Bed load: sediment which moves on or near bed, movement by sliding or rolling
Suspended load: sediment which is in suspension

Types of Open channel

a. Rigid boundary channel (Non erodible boundary)

Rigid boundary channels are those channels whose bed and banks are made up of non-erodible material. This type of channel can resist erosion satisfactorily. The resistance to the water flowing in a rigid boundary channel depends only on the nature of the boundary surface and it can be determined up to fair degree of accuracy. The dimensions of the channel can be calculated by uniform flow equations, such as Manning or Chezy.

b. Mobile boundary channel (Erodible boundary channel)

Mobile boundary channels are those channels whose boundary is made up of loose soil which can be easily eroded and transported be the flowing water. The mobile boundary channel includes rivers and unlined alluvial canals. In case of mobile boundary channels, the resistance to the flowing water depends not only on the boundary surface but also on the condition of bed and the banks of the channel. Uniform flow formula is insufficient condition for designing movable boundary. This is because stability of erodible channel depends mainly on the properties of the material for the channel body.

10.2 Minimum permissible velocity approach for the design of rigid boundary channel

It represents the lowest velocity which will prevent both sedimentation and vegetation growth. In general 0.6 to 0.9 m/s will prevent both sedimentation and vegetation growth when the silt load in the flow is low. This is an important criterion for designing rigid boundary channel.

Maximum permissible velocity

It is the greatest mean velocity that will not cause erosion of the channel bed and bank. This is an important criterion for designing movable boundary channel.

Permissible velocity method for design of channels

In this method, the channel size is selected such that the mean velocity of flow for the designing discharge is less than permissible flow velocity. The permissible velocity basically depends on type of soil, the size of particles and the depth of flow and alignment of channel.

Recommended permissible velocity for 1m depths

Material	V m/s
Fine sand	0.6
Coarse sand	1.2
Earthen channel	
Sandy -silt	0.6
Silt - clay	1.1
Clay	1.8
Grass lined earthen channel	
Sandy-silt	1.8
Silt-clay	2.1
Self stone	2.4
Hard rock	6.1

Suggested side slope

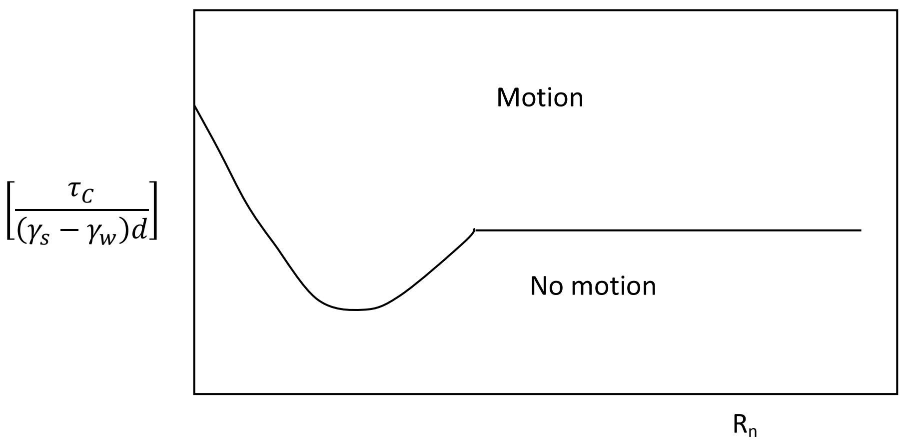
Material	Side slope H:V
Rock	Nearly vertical
Stiff clay	0.5:1 to 1:1
Loose sandy clay	2:1
Sandy loam	3:1

10.3 Incipient motion condition

At low discharge, the sediment remains stationary on the bed and clear water flows over the sediment. At low flow, depth is low and shear stress is also small. If the discharge is gradually increased, a stage will come when the shear stress begins to exceed the force opposing the movement of particles. In such stage, particles on the bed begin to move intermittently. This condition in which sediment just begins to move is called incipient motion condition or the critical condition. The bed shear stress corresponding to incipient motion is called critical shear stress or critical tractive force. When average shear stress on the bed (τ_0) is equal to or greater than the critical value (τ_c), the particles on the bed starts to move in the direction of flow. The knowledge of incipient motion condition is useful in fixing the bed slope or depth of clear water flow to reservoir to minimize silting problem in reservoir.

10.4 Shield's diagram for the studying incipient motion

According to Shield, the tractive force exerted by the flowing water on the sediment to cause motion is equal to boundary shear, τ_0. The relative magnitude of tractive force and resistance to motion of uniform sediment grain is expressed by the dimensionless ratio $\left[\frac{\tau_c}{(\gamma_s-\gamma_w)d}\right]$, which is a significant parameter in the bed load movement. In this expression, γ_s and γ_w are specific weight of sediment and water respectively and d is the grain diameter. The dimensional less ratio is termed as entrainment function or non-dimensional shear stress. Another non-dimensionless number is the Shear Reynold's number (R_n), which is given by $R_n = \frac{V*d}{\upsilon}$, where V_* = shear velocity, υ = kinematic viscosity



Shield's curve

By plotting $\left[\frac{\tau_C}{(\gamma_s-\gamma_w)d}\right]$ against $\frac{V*d}{\upsilon}$, a curve is obtained which is known as Shield's curve. This curve is used to establish the criteria for incipient sediment motion. The salient features of the curve are as follows:

- For $R_n \leq 2$, the grain is completely enclosed in laminar sub-layer and τ_C is not affected by the particle size.
- For $2 < R_n < 400$, the flow is in transition stage where both the particle size and fluid viscosity affect τ_C.
- For R_n >400, the curve becomes horizontal. At that time the value of the entrainment function becomes constant at about 0.06 (independent of Reynolds no.), thereby indicating that in turbulent flow τ_C is proportional to $(\gamma_s - \gamma_w)d$. That means τ_C is a function of particle size only. For $R_n > 400$ and the entrainment function greater than or equal to 0.06, the sediment attains incipient motion condition.
- The minimum value of the entrainment function = 0.03 at R_n = 10. That means for R_n > 10, the entrainment function should be greater than or equal to 0.03 for the sediment motion to occur.

If the median size of the particle (d) is greater than 6mm, then critical shear stress is given by

$$\tau_c = 0.06(\gamma_s - \gamma_w)d$$

Taking sp gr of sediment = 2.65,

$\tau_c = 0.06(2.65x9810 - 9810)d/1000$ = 0.905d

If d<6mm, trial and error approach has to be adopted to find critical shear stress from Shield's curve. Swamee and Mittal have expressed the relationship for Shield's curve for d<6mm as

$\tau_c = 0.155 + \frac{0.409d^2}{(1+0.177d^2)^{1/2}}$

where d is in mm and τ_c is in Pa.

Determination of size of sediments (d_c) that will not be removed from the bed (valid for d≥6mm)

$$\frac{\tau_c}{(\gamma_s - \gamma_w)d} = 0.06$$

$$\tau_c = 0.06(\gamma_s - \gamma_w)d_c$$

$$\gamma_w RS = 0.06(\gamma_s - \gamma_w)d_c$$

where R = Hydraulic radius and S = longitudinal slope

Taking sp gr of sediment = 2.65,

$$9810xRS = 0.06(2.65x9810 - 9810)d_c$$

d_c = 10RS

10.5 Alluvial channel

An alluvial channel is defined as channel which transports water as well as sediment and the sediment transported by the channel has same properties as the material of the channel boundary. Such a channel is said to be stable if the sediment inflow into a channel reach is equal to sediment outflow. Under such an equilibrium condition, the bed of the channel neither rises nor falls. Obviously, the shape, longitudinal slope and cross-sectional dimensions of such a stable channel depend on the discharge, the size of the sediment and the sediment load to be carried.

10.6 Design of mobile boundary channel

a. Minimum permissible velocity method

Design procedure

- Select Manning's n for the given material of channel boundary.
- Select side slope for given material.
- Find permissible velocity.
- From permissible velocity and S, calculate R using $V = \frac{1}{n}R^{2/3}S^{1/2}$
- For given discharge Q, find area A = Q/V. Solve the equation for A and R simultaneously and find b and y.
- Final criteria : V< Vpermissible

b . Tractive force method

Tractive force is the drag force exerted by the flowing water on the sediment particles, thereby causing their motion. This force is due to boundary shear stress, which is equal to the tractive force per unit area. The tractive force required to initiate general movement of grains is called critical tractive force. It is the function of material size and the sediment concentration.

Critical tractive force approach for the design of stable channel

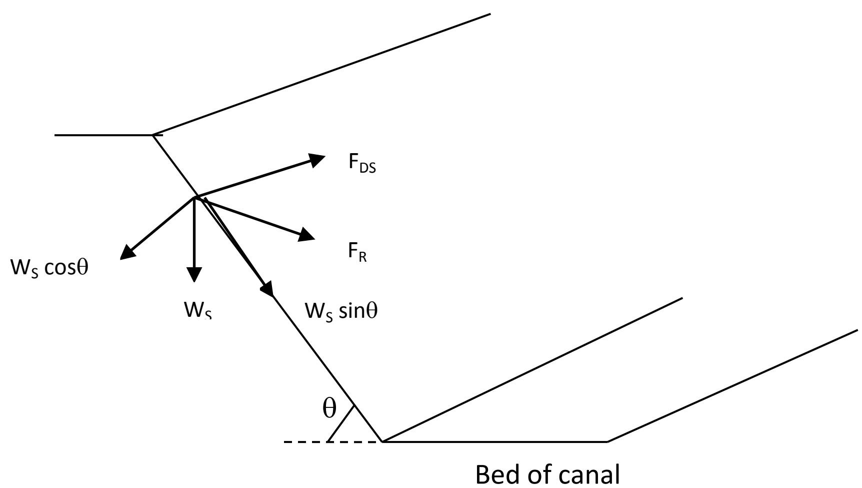
This approach attempts to restrict shear stress anywhere in the channel to a value less than the critical shear stress of bed material. If the channel carries clear water, there will be no deposition problem and hence the channel will be of stable cross-section.

Stability of particle on side slope

Consider a particle on the side of channel of inclination θ to the horizontal.

d= size of particle so that its effective area, a = C_1 d^2 where C_1 is coefficient

W_s = submerged weight of particle = $C_2(\gamma_s - \gamma_w)d^3$ where γ_s and γ_w are specific weight of sediment and water respectively , and C_2 is coefficient



Components of W_s are $W_s$sinθ and $W_s$cosθ. Due to the flow, a shear stress, τ_w exists on the particle situated on the slope. The drag force on the particle due to shear is

$$F_{Ds} = \tau_w a$$

Resultant force tending to move the particle is

$$F_R = \sqrt{F_{Ds}^2 + W_s^2 sin^2\theta}$$

Stabilizing force is

$$F_s = W_s cos\theta$$

At the condition of incipient motion, $\frac{F_R}{F_S} = tan\varphi$ where φ = angle of repose of sediment particles under water (maximum angle at which the pile of sediments will be accumulated without sliding). Substituting the values of F_R and F_S

$$\frac{F_{Ds}^2 + W_s^2 sin^2\theta}{W_s^2 cos^2\theta} = tan^2\varphi$$

$$F_{Ds} = W_s cos\theta\ tan\varphi \left[1 - \frac{tan^2\theta}{tan^2\varphi}\right]^{1/2}$$

For θ = 0, the drag force on a particle situated on a horizontal bed at the time of incipient motion is obtained. Thus if τ_b = shear stress on the bed of a channel,

$$F_{Db} = \tau_b\ a = W_s\ tan\varphi$$

$$\frac{F_{Ds}}{F_{Db}} = \frac{\tau_w}{\tau_b} = cos\theta\ \left[1 - \frac{tan^2\theta}{tan^2\varphi}\right]^{1/2} = K_1$$

$K_1 = cos\theta\ \left[1 - \frac{tan^2\theta}{tan^2\varphi}\right]^{1/2}$ Or, $K_1 = \left[1 - \frac{sin^2\theta}{sin^2\varphi}\right]^{1/2}$

$$\tau_w = K_1\tau_b$$

$\tau_b = K_2\tau_c$ where τ_c = critical shear stress and K_2 = coefficient

$$\tau_w = K_1K_2\tau_c$$

Value of K_2
Straight channel = 0.9
Slightly curved = 0.81
Moderately curved = 0.67
Very curved = 0.54

Design of chanel using tractive force method

I. Determine angle of repose φ .
II. Establish longitudinal slope from topographical consideration. Fix side slope from practical and constructional aspects.
III. Estimate Manning's n. (e.g. by using Strickler's formula, n= $d^{1/6}$/21.1 or from references)
IV. Compute critical shear stress by using Shield's curve or empirical equations.
V. Design the non-erodible channel to withstand the maximum shear stress that may occur anywhere in the perimeter of the channel.

For trapezoidal channel of normal depth y_0 and longitudinal slope S_0, maximum shear stress on the sides $(\tau_w)_{max}$ and bed $(\tau_b)_{max}$ can be given by

$$(\tau_w)_{max} = 0.75\gamma S_0 y_0$$

$$(\tau_b)_{max} = \gamma S_0 y_0$$

For non-erodibility condition,

$$\tau_w \le (\tau_w)_{max}$$

$K_1K_2\tau_c \le 0.75\gamma S_0 y_0$ (a)

and

$$\tau_b \le (\tau_b)_{max}$$

$K_2\tau_c \le \gamma S_0 y_0$ (b)

The lesser of the two values of y_0 obtained from a and b is adopted. With y_0 known, the width of channel is determined by using Manning's formula.

c. Regime theory approach

A channel in which neither silting nor scouring takes place is called regime channel or stable channel. This stable channel is said to be in a state of regime if the flow is such that silting and scouring need no special attention. The basis of designing such an ideal channel is that whatever silt has entered the channel at its canal head, it is always kept in suspension and not allowed to settle anywhere along its course. Simultaneously, velocity of water is such that it does not produce local silt by erosion of channel bed or sides.

Lacey's regime theory

According to Lacey, dimensions of bed width, depth and slope of channel attain a state of equilibrium with time which is called regime state. Lacey defined a regime channel as a channel which carries constant discharge under uniform flow in an unlimited incoherent alluvium having the same characteristics as that transported without changing bottom slope, shape or size of cross-section over a period of time. Thus in regime channel, there will be suspended load, bed load and formation of bed forms. In the initial state, depth, width and longitudinal slope may change. The continuous action of water overcomes the resistance of banks and sets up a condition such that the channel adjusts its complete section, then final regime condition is reached. After attaining regime state, the dimensions of the channel will remain constant over time.

Design procedure by Lacey's theory

Values of discharge (Q), sand size (d_{mm} in mm), side slope Z:1 (if not given assume 0.5:1) are given.

- Compute silt factor (f_s) by using eq. $f_s = 1.76\sqrt{d_{mm}}$.
- Compute longitudinal slope by using eq. $S = \frac{0.0003 f_s^{5/3}}{Q^{1/6}}$.
- Compute hydraulic radius by using eq. $R = 0.48\left(\frac{Q}{f_s}\right)^{1/3}$.
- Compute wetted perimeter by using eq. $P = 4.75\sqrt{Q}$.
- Compute cross sectional area (A) from P and R (A=PR).
- For trapezoidal section

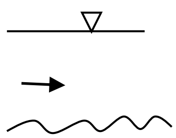
$$A = (b + Zy)y$$
$$P = b + 2y\sqrt{1 + Z^2}$$

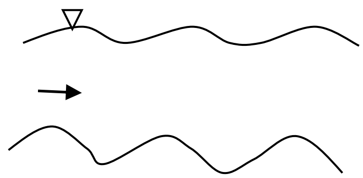
- Substitute the values of A and P, and solve for y and b.

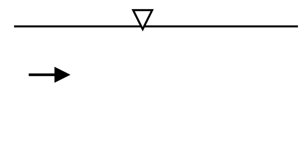
10.7 Bed forms in Alluvial Stream

When average shear stress on the bed of alluvial channel (τ_0) is greater than the critical value (τ_c), the particles on the bed starts to move in the direction of flow. Depending on the sediment size, fluid and flow condition, the bed and water surface attain different forms. The features that form on the bed under different conditions are called bed forms. The characteristics of different bed forms are known as regime of flow.

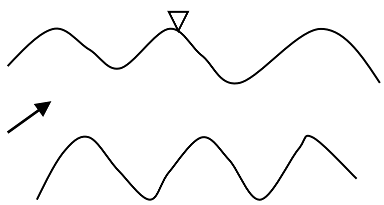
Types of bed forms (deformations)

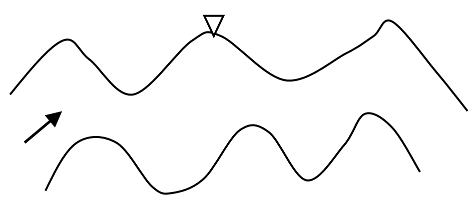






Ripples Dunes Washed out dunes





Standing wave Antidunes

a. Plane bed with no sediment motion: When $\tau_0 < \tau_c$, the sediments on the bed do not move. The bed remains plane and the channel behaves as a rigid boundary channel.

b. Ripples and Dunes

Ripples: When τ_0 is moderately greater than τ_c, the sediment particles start moving and small unsymmetrical triangular undulations appear on the bed. These are known as ripples. The length of these undulations is less than 0.4m and the height is less than 40mm. The sediment size is normally below 0.6mm. The sediments move by sliding or rolling. Water surface is fairly smooth.

Dunes: With the increase in discharge, the ripples grow in size with flat upstream face and steep downstream face. They are larger in size than ripples and are called dunes. Water surface is not smooth and some of the particles may remain in suspension. Their length varies from 0.3m to several m and height from 30mm to several cm. The sediment is eroded on u/s side. This process results in the apparent d/s movement of the dunes at a speed much less than mean stream velocity. The ripples or dunes or both offer a relatively large resistance to flow as compared with plane bed. Both are formed when Fr<1 (subcritical condition).

c. Transition

If the discharge is increased in duned bed, dunes are washed away leaving only a small undulation. In some cases, the dunes may be completely washed out creating plane bed. A very small increase in discharge on such a bed may lead to the formation of sinusoidal waves on the bed and water surface, which are known as standing waves. These two types of bed form are designated as transition. Fr is relatively high and bed form is unstable. The transition regime of flow offers relatively low resistance to flow.

d. Antidunes: With further increase in discharge, the intensity of sediment transport increases and symmetrical bed and water surface waves appear. Although the sediment moves d/s, the crest of the bed wave move u/s. These undulations are called antidunes. The waves gradually grow steeper and then break. Antidunes occur in supercritical flow and the sediment transport rate will be very high. However, the resistance to flow is small compared to ripples and dunes.