Definition of convolution for functions h and f:

$$(h * f)(t) \stackrel{\text{Def.}}{=} \int h(\tau) \cdot f(t-\tau) d\tau$$

Associativity:

$$((f * g) * h)(t) = (f * (g * h))(t)$$

Proof:

$$\begin{aligned}
((f * g) * h)(t) &\stackrel{\text{Def.}}{=} \int (f * g)(\tau_1) \cdot h(t-\tau_1)\, d\tau_1 \\
&\stackrel{\text{Def.}}{=} \int \left[\int f(\tau_2) \cdot g(\tau_1 - \tau_2) d\tau_2 \right] \cdot h(t-\tau_1)\, d\tau_1 \\
\text{rearrange:} \quad &= \int \int f(\tau_2) \cdot g(\tau_1 - \tau_2) \cdot h(t-\tau_1)\, d\tau_2\, d\tau_1 \\
\text{rearrange:} \quad &= \int f(\tau_2) \cdot \left[\int g(\tau_1 - \tau_2) \cdot h(t-\tau_1)\, d\tau_1 \right] d\tau_2 \\
\text{shift by } \tau_2 \text{ (transl. invar.):} \quad &= \int f(\tau_2) \cdot \left[\int g((\tau_1 + \tau_2) - \tau_2) \cdot h(t-(\tau_1+\tau_2))\, d\tau_1 \right] d\tau_2 \\
&= \int f(\tau_2) \cdot \left[\int g(\tau_1) \cdot h((t-\tau_2)-\tau_1)\, d\tau_1 \right] d\tau_2 \\
&\stackrel{\text{Def.}}{=} \int f(\tau_2) \cdot (g * h)(t-\tau_2)\, d\tau_2 \\
&\stackrel{\text{Def.}}{=} (f * (g * h))(t)
\end{aligned}$$

$\square$