Exercise for MA-INF 2201 Computer Vision WS22/23
Submission on 03.12.2023

1. A function is *submodular* when it satisfies the equation:

$$P(\alpha, \beta) + P(\lambda, \tau) \geq P(\lambda, \beta) + P(\alpha, \tau)$$

for all $\alpha, \beta, \tau, \lambda$ such that $\tau > \beta$ and $\alpha > \lambda$.

Show that the following functions are *submodular* and if not present a counter-example.

(a) $P(\omega_m, \omega_n) = sin(|\omega_m - \omega_n|)$, where $|.|$ is the absolute function

(b) $P(\omega_m, \omega_n) = \begin{cases} (\omega_m - \omega_n)^2 & \text{if } |\omega_m - \omega_n| \leq 1 \\ 1 & \text{else} \end{cases}$

(c) $P(\omega_m, \omega_n) = c(\omega_m - \omega_n)^2$ for $c > 0$

(4 points)

2. Provide a graph structure using the *alpha expansion* method that encodes the initial state of 6 nodes (a,b,c,d,e,f) with initial states $\beta\beta\gamma\alpha\alpha\gamma$ for the case where the label α is expanded. ***(4 points)***

Programming Exercises

3. Denoise the binary image using a Markov random field. Read the noisy binary image in *images/noisy_binary_img.png*

3.1. Create a graph for the image using all the pixels as nodes. Each pixel (node) is connected to the "source node" and the "sink node" with directed edges as well as the directed edges between its left, top, right, and bottom neighboring pixels. ***(2 points)***

3.2. The Unaries are defined by

$$\begin{aligned} P(x_n|w_n = 0) &= Bern_{x_n}[\rho] \\ P(x_n|w_n = 1) &= Bern_{x_n}[1 - \rho] \end{aligned}$$

where *Bern* is the Bernoulli distribution.
The pairwise costs are defined as follows:

$$P(w_m = 0, w_n = 0) = P(w_m = 1, w_n = 1) = \theta_s$$
$$P(w_m = 0, w_n = 1) = P(w_m = 1, w_n = 0) = \theta_d$$

where θ_s and θ_d represent the cost of assigning identical and distinct labels to neighboring pixels, respectively. Use different combinations of θ_s and θ_d and display the denoised output of each.

i. What is the impact of changing θ_s and θ_d on the output?
ii. Which combination gives you the best denoised output?

Note: For min-cut/max-flow algorithm install "PyMaxflow" ***(4 points)***

4. Extend the algorithm in question 3 for a grayscale image *images/noisy_grayscale_img.png* using Alpha Expansion Algorithm. There are only three labels $[l_1, l_2, l_3]$ where l_1=1, l_2=2 and l_3=3 corresponding to gray values of (0,128,255) respectively. Unary costs are defined as:

$$\begin{aligned} P(x_n = l_i | w_n = l_i) &= \rho_{l_i} \\ P(x_n = l_j | w_n = l_i) &= \frac{(1 - \rho_{l_i})}{2} \ \forall i \neq j \end{aligned}$$

where, $\rho_{l_i} = 0.05$. Define the pairwise cost using Potts Model:

$$P(\omega_m, \omega_n) = (1 - \boldsymbol{\delta}(\omega_m - \omega_n))$$

where $\boldsymbol{\delta}(x) = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{else} \end{cases}$ is the Kronecker delta function.

(6 points)

* For programming exercises, you can also use Jupyter Notebook to submit your solution.