Proof

$$\int g(x-u)g(u)du = \frac{1}{2\pi\sigma^2} \int \exp\left(-\frac{u^2 + (x-u)^2}{2\sigma^2}\right) du$$

$$= \frac{1}{2\pi\sigma^2} \int \exp\left(-\frac{2u^2 + x^2 - 2xu}{2\sigma^2}\right) du$$

$$= \frac{1}{2\pi\sigma^2} \int \exp\left(-\frac{1}{\sigma^2}u^2 + \frac{x}{\sigma^2}u\right) du \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$= \frac{1}{2\pi\sigma^2} \sqrt{\sigma^2\pi} \exp\left(\sigma^2\frac{x^2}{4\sigma^4}\right) \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$= \frac{1}{\sqrt{2\pi(\sqrt{2}\sigma)^2}} \exp\left(-\frac{x^2}{2(\sqrt{2}\sigma)^2}\right)$$