Exercise 03 for MA-INF 2201 Computer Vision WS23/24
05.11.2023
Submission on 12.11.2023

1. **Hough Transform for Line Detection**

   Given the image `shapes.png` of different shapes, read the image and convert it into a grayscale image.

   (a) Detect the lines by a Hough transform using `cv2.HoughLines`. Visualize the detections by drawing lines on the image.

   (b) Detect the lines with your implementation of `myHoughLines`. Visualize the detections and the accumulator.

   (5 Points)

2. **Convergence of K-Means**

   Given a set of n point $p_i \in \mathbb{R}^d, i \in \{1, 2, \ldots, n\}$ and the number of clusters k, the K-means clustering algorithm aim to find the centers of k clusters $c_j, j \in \{1, 2, \ldots, k\}$ by minimizing the average distance from n points to their assigned closest cluster centers. The loss function to be minimized can be formulated as:

   $$L(c) = \sum_{i=1}^{n} \min_{j \in \{1,\ldots,k\}} ||p_i - c_j||_2^2 \tag{1}$$

   To approximate the solution, the new assignment variables $z_i \in argmin_{j \in \{1,\ldots,k\}} ||p_i - c_j||_2^2$ for each data point p_i is introduced. The K-means clustering algorithm iterates between updating the variables z_i (*assignment step*) and updating the centers $c_j = \frac{1}{|\{i:z_i=j\}|} \sum_{i:z_i=j} p_i$ (*refitting step*). The algorithm stops when no change occurs during the *assignment step*.

   Please prove that K-means is guaranteed to converge (to a local optimum). Note: You need to prove that the loss function is guaranteed to decrease monotonically in each iteration until convergence. Prove this separately for the *assignment step* and the *refitting step*, provide your solution in a pdf file.

   (3 Points)

3. **K-Means for Segmentation as Clustering**

   Implement the function `myKmeans` and then use it to segment the image `flower.png` based on:

   (a) Intensity,

   (b) Color,

   (c) Intensity and (properly scaled) image position,

   (d) Other property that you choose as the feature space.

   Visualize the results for all the cases with $k = 2, 4, 6$. Analyze your results.

   (5 Points)

4. **Mean Shift**

   Implement the function **meanShift** and use it to:

   (a) Find the peaks in the accumulator. You should read the image `line.png` and use your implementation of `myHoughLines` in Question 1 (b) to get the accumulator of the detected lines in the image. Visualize the accumulator and the lines corresponding to the peaks.

   (b) Segment the image `flower.png` in Question 3. Compare the results with K-Means.

   (For more details: *D. Comaniciu and P. Meer. Mean Shift: A Robust Approach Toward Feature Space Analysis. IEEE Transactions on Pattern Analysis and Machine Intelligence 2002.*)

   (7 Points)