In order to prove:

$$\mathrm{tr}[\boldsymbol{X^T}\boldsymbol{X}\omega z^T] = z^T\boldsymbol{X^T}\boldsymbol{X}\omega \qquad (1)$$

$$\mathrm{tr}[z\omega^T\boldsymbol{X^T}\boldsymbol{X}\omega z^T] = z^T z \cdot \omega^T\boldsymbol{X^T}\boldsymbol{X}\omega \qquad (2)$$

Let's first consider the dimensions of the given vectors and matrices.

$$\boldsymbol{X} \in \mathbb{R}^{m\times n}$$

$$\boldsymbol{X^T} \in \mathbb{R}^{n\times m}$$

$$z \in \mathbb{R}^{n\times 1}$$

$$z^T \in \mathbb{R}^{1\times n}$$

$$\omega \in \mathbb{R}^{n\times 1}$$

$$\omega^T \in \mathbb{R}^{1\times n}$$

We can see that:

1)

$$\boldsymbol{X^T}\boldsymbol{X}\omega \in \mathbb{R}^{n\times 1}$$

$$\boldsymbol{X^T}\boldsymbol{X}\omega z^T \in \mathbb{R}^{n\times n}$$

$$\mathrm{tr}[\boldsymbol{X^T}\boldsymbol{X}\omega z^T] \in \mathbb{R}^{1\times 1}$$

$$z^T\boldsymbol{X^T}\boldsymbol{X}\omega \in \mathbb{R}^{1\times 1}$$

2)

$$z\omega^T \in \mathbb{R}^{n\times n}$$

$$z\omega^T\boldsymbol{X^T}\boldsymbol{X}\omega \in \mathbb{R}^{n\times 1}$$

$$z\omega^T\boldsymbol{X^T}\boldsymbol{X}\omega z^T \in \mathbb{R}^{n\times n}$$

$$\mathrm{tr}[z\omega^T\boldsymbol{X^T}\boldsymbol{X}\omega z^T] \in \mathbb{R}^{1\times 1}$$

$$z^T z \cdot \omega^T\boldsymbol{X^T}\boldsymbol{X}\omega \in \mathbb{R}^{1\times 1}$$

In both cases we multiply two vectors of size $n \times 1$ and $1 \times n$ together. If we can proof that $\mathrm{Tr}(\mathbf{v}_2\mathbf{v}_1)$ equals $\mathbf{v}_1\mathbf{v}_2$, than we have a proof for (1) and (2) where:

$$\mathbf{v}_1 = \mathbf{X^T}\mathbf{X}\omega \text{ (1)}$$

$$\mathbf{v}_1 = \mathbf{z}\omega^{\mathbf{T}}\mathbf{X^T}\mathbf{X}\omega \text{ (2)}$$

$$\mathbf{v}_2 = \mathbf{z}^T$$

Let $\mathbf{v}_1$ be a column vector of size $n \times 1$ and $\mathbf{v}_2$ be a row vector of size $1 \times n$. The trace of the product $\mathbf{v}_2\mathbf{v}_1$ is given by:

$$\mathrm{Tr}(\mathbf{v}_2\mathbf{v}_1)$$

The product $\mathbf{v}_2\mathbf{v}_1$ results in an $n \times n$ matrix. Let's compute the product:

$$\mathbf{v}_2\mathbf{v}_1 = \begin{bmatrix} v_{21} \\ v_{22} \\ \vdots \\ v_{2n} \end{bmatrix} \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \end{bmatrix}$$

The resulting matrix $\mathbf{v}_2\mathbf{v}_1$ is:

$$\begin{bmatrix} v_{21}v_{11} & v_{21}v_{12} & \cdots & v_{21}v_{1n} \\ v_{22}v_{11} & v_{22}v_{12} & \cdots & v_{22}v_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{2n}v_{11} & v_{2n}v_{12} & \cdots & v_{2n}v_{1n} \end{bmatrix}$$

Now, the trace is the sum of the diagonal elements:

$$\mathrm{Tr}(\mathbf{v}_2\mathbf{v}_1) = v_{21}v_{11} + v_{22}v_{12} + \cdots + v_{2n}v_{1n}$$

Now, let's consider the product $\mathbf{v}_1\mathbf{v}_2$, which is a 1×1 matrix (a scalar):

$$\mathbf{v}_1\mathbf{v}_2 = \begin{bmatrix} v_{11} \\ v_{12} \\ \vdots \\ v_{1n} \end{bmatrix} \begin{bmatrix} v_{21} & v_{22} & \cdots & v_{2n} \end{bmatrix}$$

The result is:

$$\mathbf{v}_1\mathbf{v}_2 = v_{11}v_{21} + v_{12}v_{22} + \cdots + v_{1n}v_{2n}$$

Now, observe that these two expressions are the same:

$$\mathrm{Tr}(\mathbf{v}_2\mathbf{v}_1) = \mathbf{v}_1\mathbf{v}_2$$

Therefore, we have shown that the trace of the product $\mathbf{v}_2\mathbf{v}_1$ is equal to the trace of the product $\mathbf{v}_1\mathbf{v}_2$.