Hw7p4

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Problem 4.

Show that it is never optimal to exercise an American Call Option on a non-dividend-paying stock before expiration.

Proof

Explanation:

It is never optimal to exercise an American Call option on a non-dividend-paying stock before expiration, because such options consist of an intrinsic value along with time value.
As intrinsic value of such options is always greater than zero, and the option also has some cash time-value, it is always optimal when such options are exercised at expiration.
Otherwise, it may result into automatic loss of that time-value.

Proving Mathematically.

Let there be two portfolios:

Portfolio A:

One American call option with additional cash $Ke^{-r(T-t)}$ at time t.

Portfolio B:

One share.

At time $t < T$ (T being the maturity time), the cost of share is S_t.

Then,

Value of Portfolio A = $S_t - K + Ke^{-r(T-t)} < S_t$.

Value of Portfolio B = S_t

At time T,

Value of Portfolio A = $\max\{S_T, K\} \geq$ Value of Portfolio B.

Here we can see that,

Before T, value of portfolio A < Value of portfolio B.
whereas, At T, Value of portfolio A $\geq$ Value of portfolio B.

Thus, we prove that portfolio A is to be exercised at time T, ie: at maturity.

proved