Jacobs University
Fall 2022

October 4, 2022

Stochastic Methods Lab

Assignment Sheet 4

Due on October 18, 2022

Note: The work is to be submitted via `git`, as discussed in class. The coding language is Python. Please make sure that your code actually runs and produces the requested output. Please make your code readable for the instructor and TA, and include comments wherever necessary. Please submit .py source code, not jupyter notebooks. Theoretical questions may be submitted as a scan of handwritten notes or typed up (e.g., using LaTeX). The submission deadline is midnight of the stated due date.

Problem 1 [2 points]

In class, we analyzed how to price an option for a binary model of stock prices with one time step. Would a similar analysis work if we allowed the stock to have three possible values in the future instead of just two? Explain your answer.

Problem 2 [12 points]

Implement the binomial tree in python via backwards induction. Use only one `for` loop to go from one step to the previous one, and implement all other operations with vectors. In more detail, implement a function

```
binomial_tree(payoff, n, rp, sigma, S, K, T)
```

that returns the price of the option at time $T = 0$. The arguments of the function are:

- `payoff`, a function that takes the stock price `S` (possibly a vector) and strike price `K` as arguments and returns the payoff,
- `n`, the number of steps,
- `rp`, the risk-free period interest rate,
- `sigma`, the volatility,
- `S`, the initial stock price,
- `K`, the strike price,
- `T`, the maturity.

Use the calibration of the model that we discussed in class, i.e., use the parameters

$$u = \frac{1}{d} = \exp\left(\sigma\sqrt{\frac{T}{n}}\right).$$

Test your code by pricing a European call option with strike price $K = 0.7$, risk-free period interest rate $r_p = 0.02$, volatility $\sigma = 0.5$, maturity $T = 1$, and initial stock price $S = 1$, using $n = 1000$ steps. (The result should be 0.3669.)

Problem 3 [**4 points**]

Modify your binomial tree algorithm from Problem 2 to price also American call and put options (i.e., the holder may exercise the option at any time before expiration). Plot the option price for different strike prices for American and European puts (i.e., two graphs in the same coordinate system) with some reasonable parameters. Is the price of an American put higher or lower than that of a European put with otherwise identical parameters?

Problem 4 [**2 points**]

Consider two portfolios: A) You buy one call and sell one put, for the same stock with price S at time 0 and $S(T)$ at expiration T, and with same strike price K. B) You buy one stock and borrow bonds worth K at time T. Then use a "no-arbitrage argument" to derive a relationship between the prices of European calls and puts. The resulting formula is called the "put-call parity".