

HW4 (Stochastic Methods Lab)Problem 4Given:

Portfolio A: Buy one call and sell one put for the same stock with price S at time 0 and $S(T)$ at expiration T ,

Strike price (same) K .

Portfolio B:

Buy one stock and borrow bonds worth K at time T . Then use a "non-arbitrage argument" to derive a relationship between the prices of European calls and puts.

put-call parity.

↳ if you want to get the upside of owning the stock still mitigating the downside in case it goes down

The resulting formula is called the "put-call parity."

Put Call parity formula: $C + K = P + S$
Let

$C \rightarrow$ price of call option

$P \rightarrow$ price of put option

$K \rightarrow$ strike price.

$S \rightarrow$ stock price at $t=0$

$S(T) \rightarrow$ stock price at expiration ($t=T$)

Let there is loan X .

The initial cash flow = $C - P - S + K$. (from portfolio A) — (1)

At expiration,

Case I:

Call option worthless and put option will be worth $X - S(T)$

Case II:

Put Option worthless and call option will be worth $S(T) - X$

After exercising everything and repaying X ,
net future cash flow = 0.

\Rightarrow No arbitrage principle applied which sets up initial cash flow to zero in either case.

$\therefore (1) = 0$.

$\Rightarrow C - P - S + K = 0$

$\Rightarrow \boxed{C + K = P + S} \leftarrow$ put-call parity.