Hw10_p2

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$$S = \frac{\partial S}{\partial x} = e^x$$

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0$$

$S = e^x$

$V(x,t) = C(S,t)$

$$\Rightarrow \frac{\partial C}{\partial S}\cdot\frac{\partial S}{\partial x} \Rightarrow \frac{\partial C}{\partial S}\cdot S$$

$$\frac{\partial V}{\partial x} = \frac{\partial C}{\partial S}\cdot S$$

$$\frac{\partial^2 V}{\partial x^2} = \frac{\partial^2 C}{\partial S^2}\cdot S^2$$

or, $$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2\left(\frac{\partial^2 V}{S^2\partial x^2} - \frac{\partial V}{S^2\partial x}\right) + rS\frac{\partial C}{\partial S} - rV = 0$$

or, $$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2\frac{\partial^2 V}{\partial x^2} - \frac{1}{2}\sigma^2\frac{\partial V}{\partial x} + r\frac{\partial V}{\partial x} - rV = 0$$

or, $$\boxed{\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2\frac{\partial^2 V}{\partial x^2} - \left(r - \frac{\sigma^2}{2}\right)\frac{\partial V}{\partial x} - rV = 0}$$

Ans