Jacobs University
Fall 2022

November 9, 2022

Stochastic Methods Lab

Assignment Sheet 8

Due on November 16, 2022

Note: The work is to be submitted via `git`, as discussed in class. The coding language is Python. Please make sure that your code actually runs and produces the requested output. Please make your code readable for the instructor and TA, and include comments wherever necessary. Please submit .py source code, not jupyter notebooks. Theoretical questions may be submitted as a scan of handwritten notes or typed up (e.g., using LaTeX). The submission deadline is midnight of the stated due date.

Problem 2 [6 points]

Let $X = X(t)$ be an Itô process, i.e., a solution to the stochastic differential equation

$$dX = f(X,t)\,dt + g(X,t)\,dW,$$

interpreted in the sense of the Itô stochastic integral. Let $F(X,t)$ be twice continuously differentiable. With this exercise we would like to verify the Itô formula from class numerically for the example when $X(t)$ is geometric Brownian motion with $\mu = 0.5$ and $\sigma = 3$, and where

$$F(X,t) = (1+t)^2\,\cos(X).$$

(a) Apply the Itô formula to F to derive the corresponding SDE for F (theoretical exercise).

(b) Compare the numerical (Euler Maruyama) solution to the SDE from (a) with the given solution F in a plot.

Problem 2 [8 points]

Look up stock option quotes for European or American call options on the stock of a major corporation (make sure you choose a non-dividend paying stock). Plot the implied volatility (i.e., the parameter σ given the market value of the option) vs. the strike price, while the time to maturity is fixed. (The applicable interest rate is the spot rate for zero coupon bonds of the same maturity.) Here it would be easiest to use the Black-Scholes formula for the option pricing. Make sure to mark the current stock price and some historical volatility (which you have to look up) in the plot, and to label the plot nicely.

Problem 3 [6 points]

Let $W(t)$ be the standard Brownian motion (starting at $W(0) = 0$). Now consider the first time a Brownian motion crosses the value -1 or 1 (i.e., the first time it leaves the interval $(-1, 1)$). Find numerically the expectation value and variance of this "hitting time" distribution.