

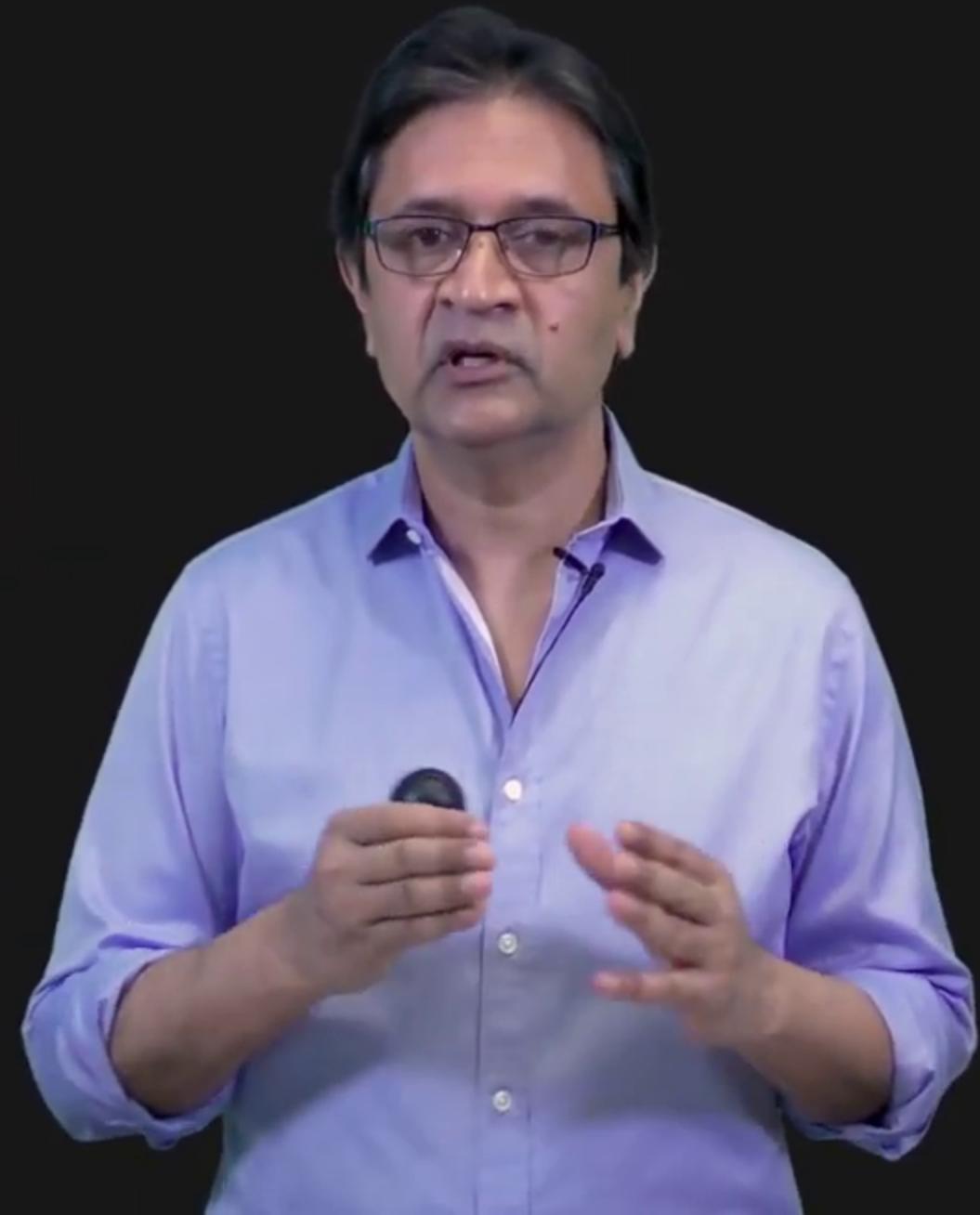
# Motion Field and Optical Flow

Shree K. Nayar

Columbia University

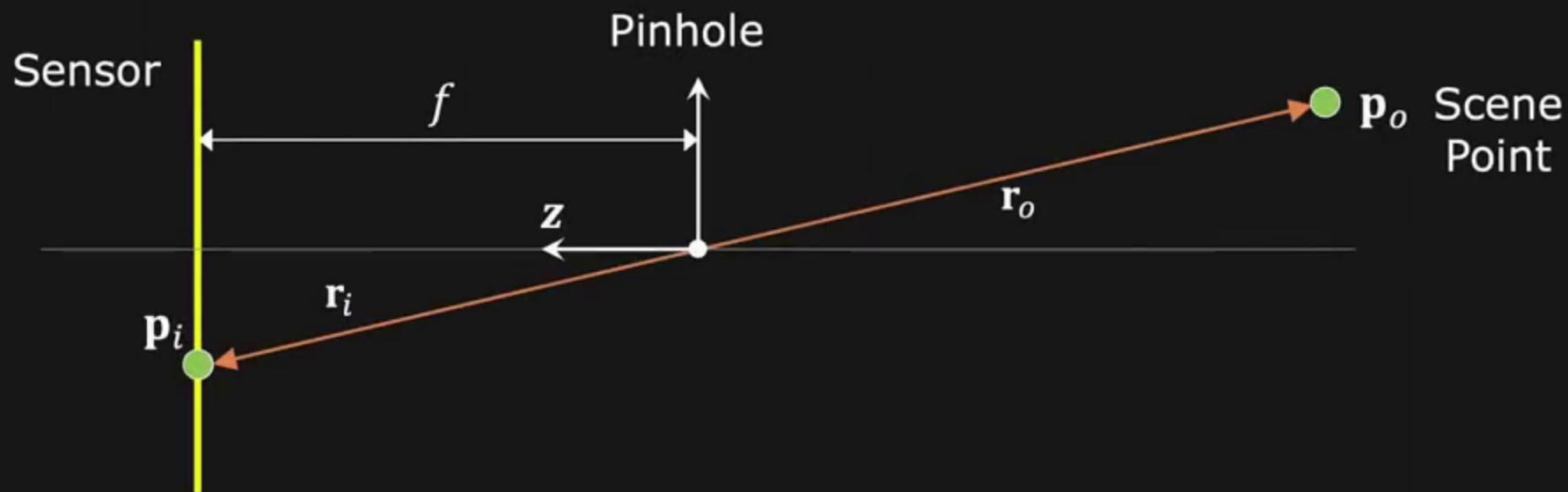
Topic: Motion and Optical Flow, Module: Reconstruction II

First Principles of Computer Vision



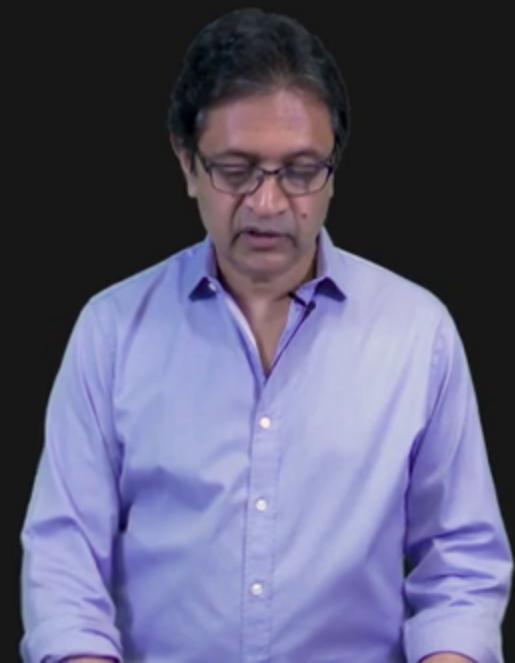
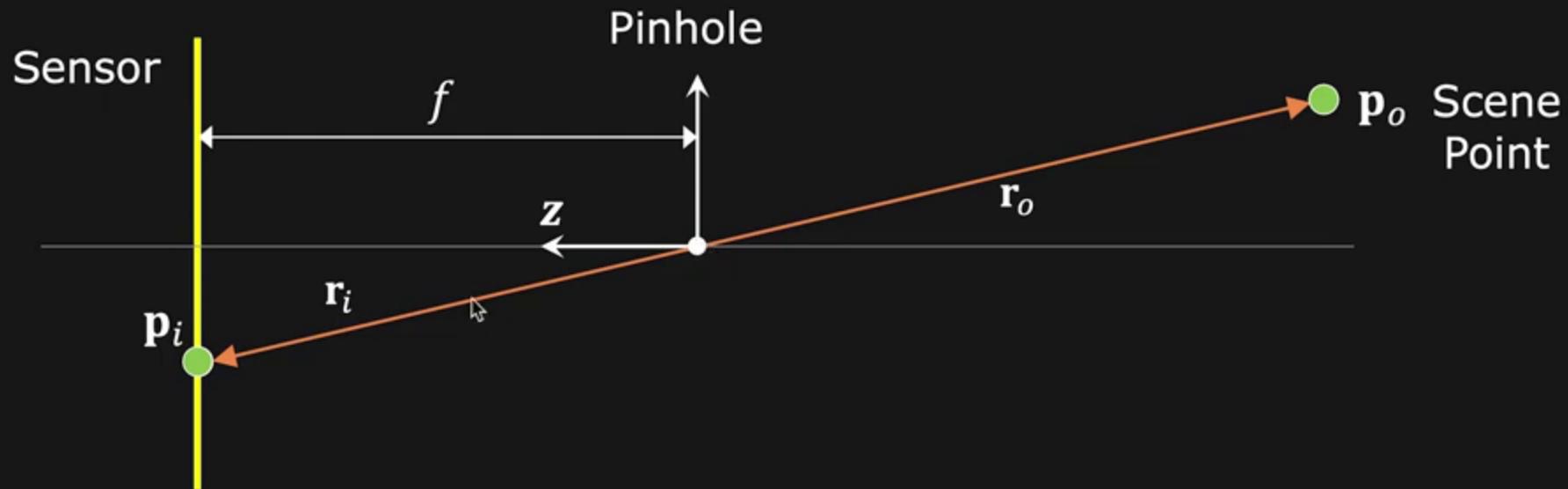
# Motion Field

Image velocity of a point that is moving in the scene



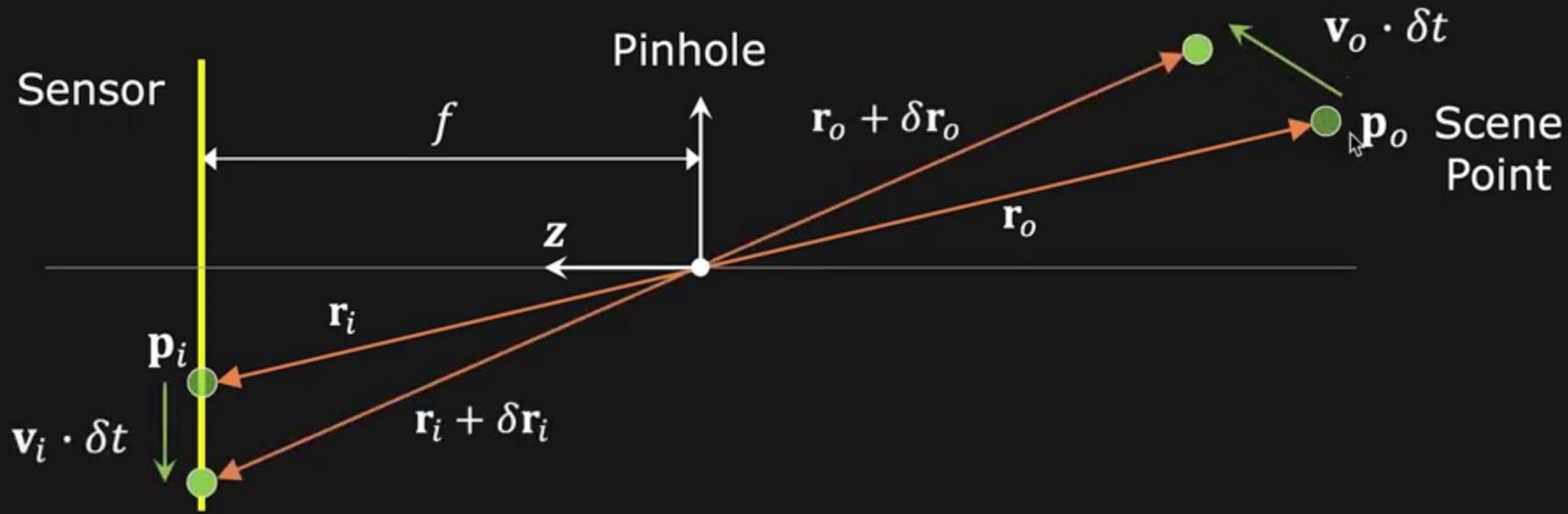
# Motion Field

Image velocity of a point that is moving in the scene



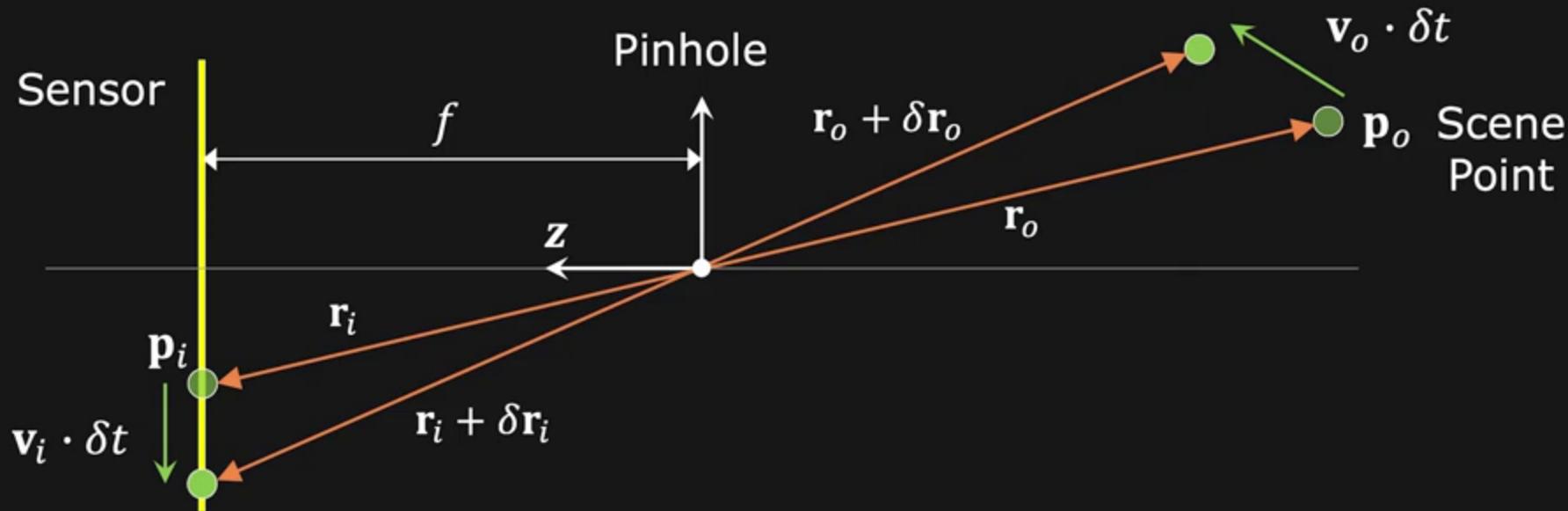
# Motion Field

Image velocity of a point that is moving in the scene



# Motion Field

Image velocity of a point that is moving in the scene



$$\text{Scene Point Velocity: } \mathbf{v}_o = \frac{d\mathbf{r}_o}{dt}$$



# Motion Field

Image velocity of a point that is moving in the scene

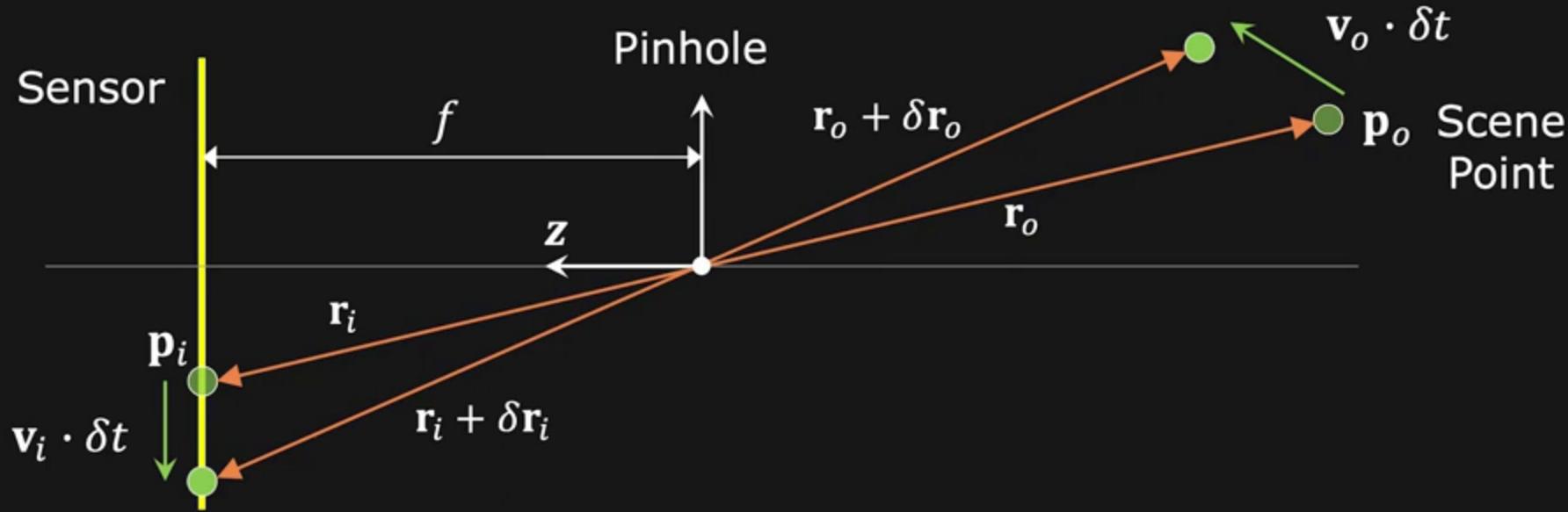


Image Point Velocity:  $\mathbf{v}_i = \frac{d\mathbf{r}_i}{dt}$   
(Motion Field)

Scene Point Velocity:  $\mathbf{v}_o = \frac{d\mathbf{r}_o}{dt}$



# Motion Field

Image velocity of a point that is moving in the scene

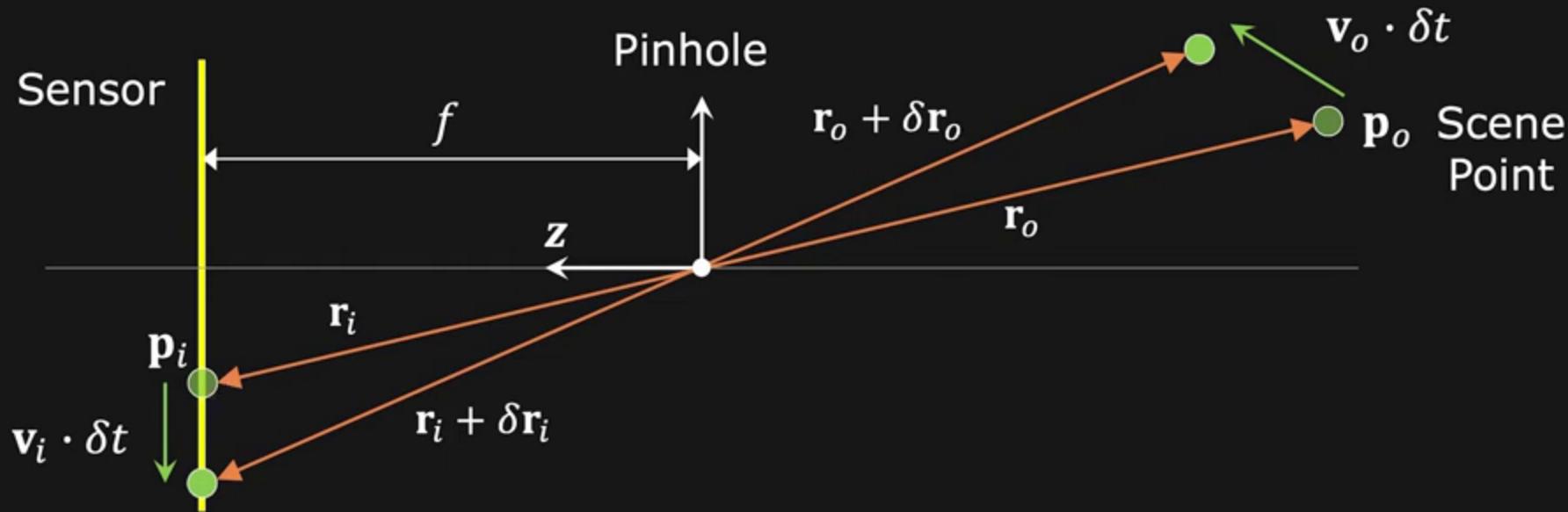


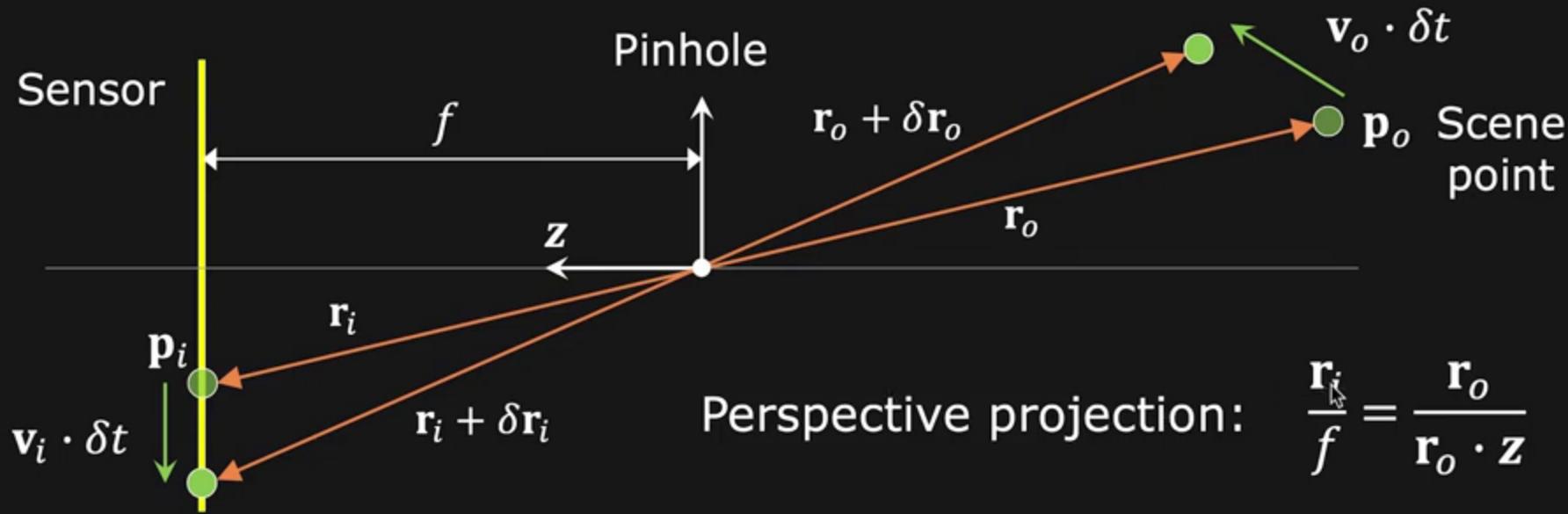
Image Point Velocity:  $\mathbf{v}_i = \frac{d\mathbf{r}_i}{dt}$   
(Motion Field)

Scene Point Velocity:  $\mathbf{v}_o = \frac{d\mathbf{r}_o}{dt}$



# Motion Field

Image velocity of a point that is moving in the scene



# Motion Field

Image velocity of a point that is moving in the scene

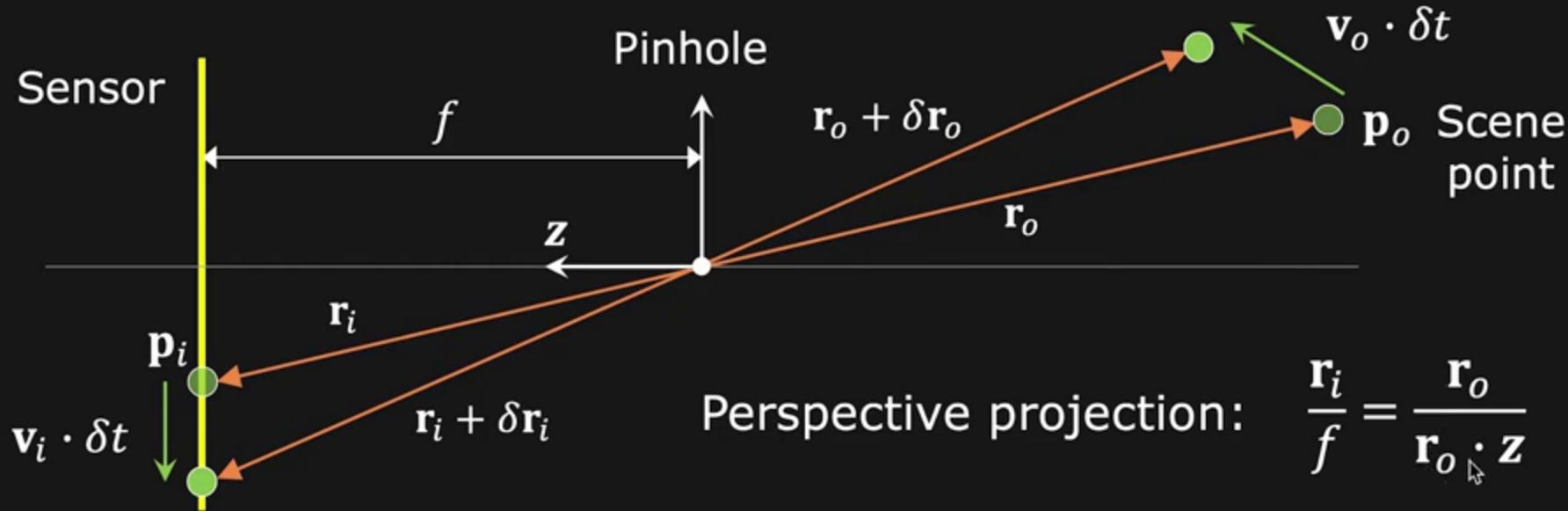


Image Point Velocity:  $\mathbf{v}_i = \frac{d\mathbf{r}_i}{dt}$   
(Motion Field)



# Motion Field

Image velocity of a point that is moving in the scene

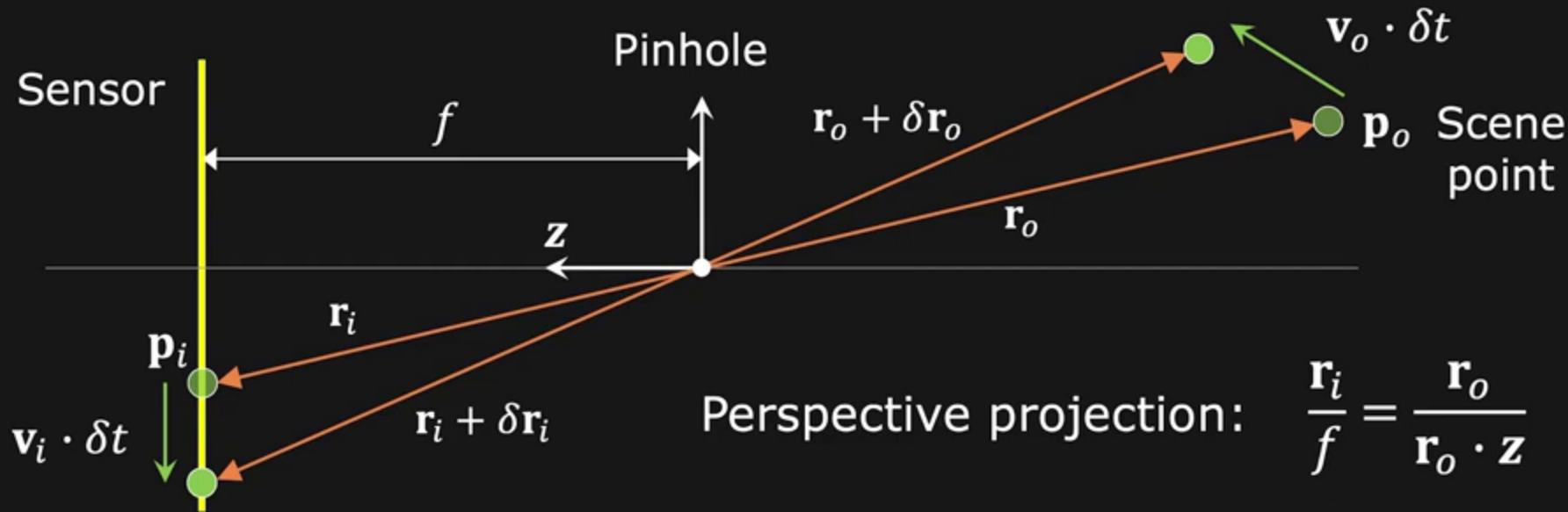
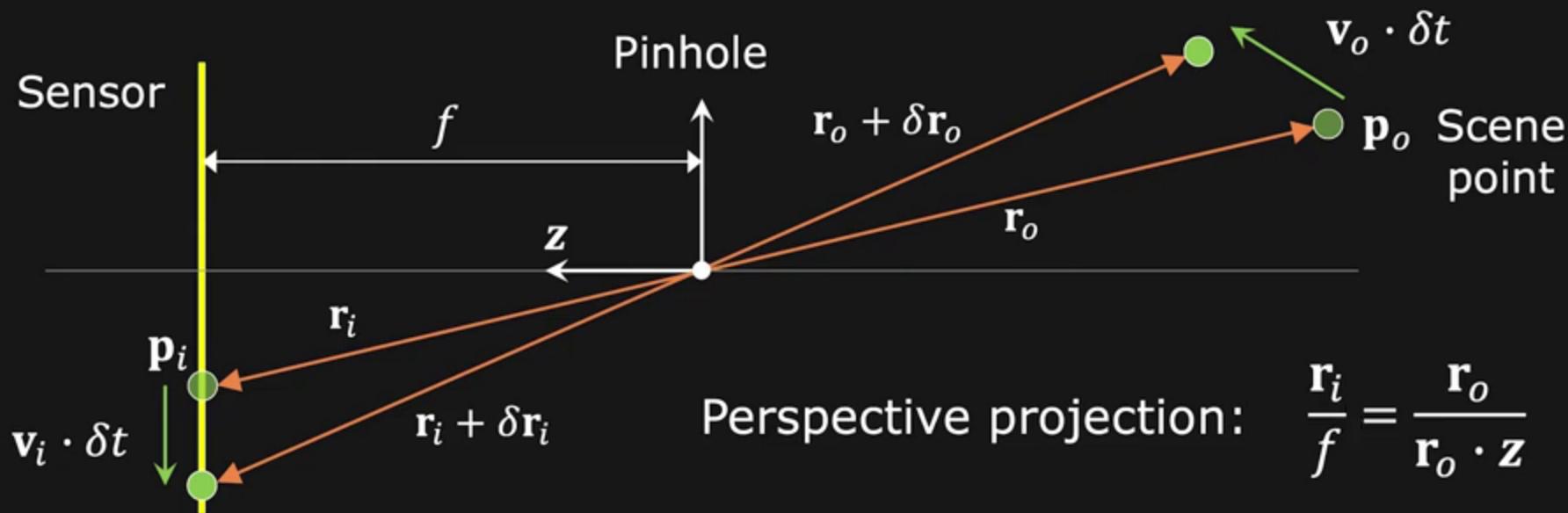


Image Point Velocity:  $\mathbf{v}_i = \frac{d\mathbf{r}_i}{dt}$   
(Motion Field)



# Motion Field

Image velocity of a point that is moving in the scene



$$\text{Perspective projection: } \frac{\mathbf{r}_i}{f} = \frac{\mathbf{r}_o}{\mathbf{r}_o \cdot \mathbf{z}}$$

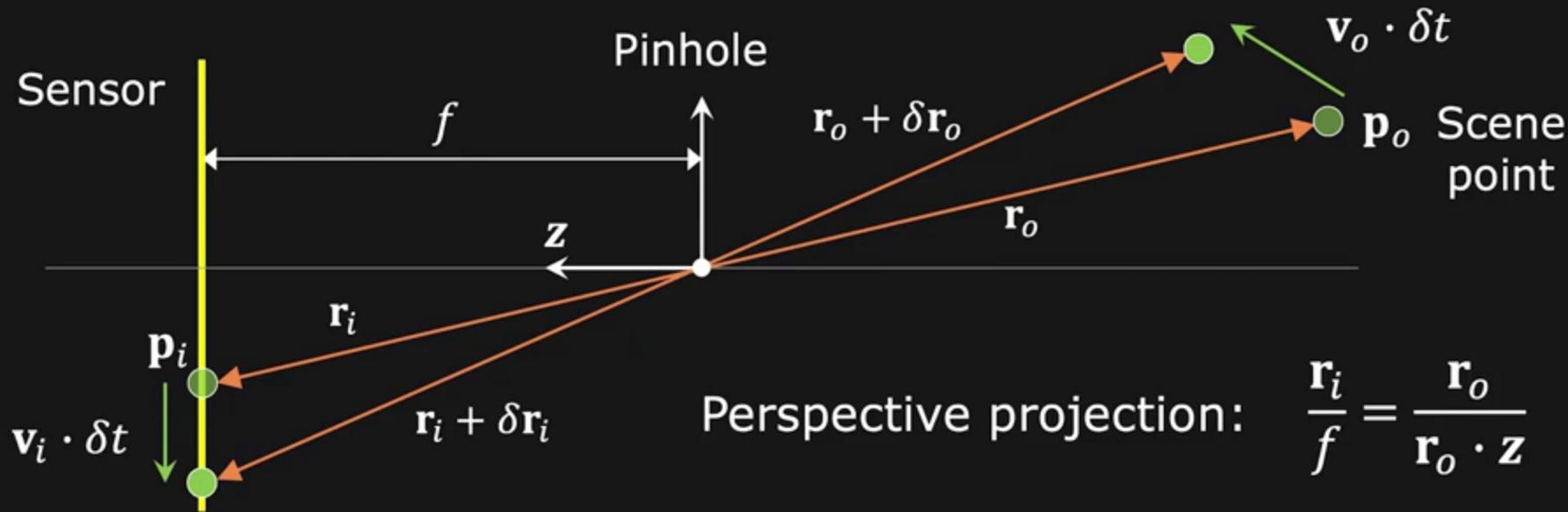
$$\text{Image Point Velocity: } \mathbf{v}_i = \frac{d\mathbf{r}_i}{dt} = f \frac{(\mathbf{r}_o \cdot \mathbf{z})\mathbf{v}_o - (\mathbf{v}_o \cdot \mathbf{z})\mathbf{r}_o}{(\mathbf{r}_o \cdot \mathbf{z})^2}$$

(Motion Field)



# Motion Field

Image velocity of a point that is moving in the scene



$$\text{Perspective projection: } \frac{\mathbf{r}_i}{f} = \frac{\mathbf{r}_o}{\mathbf{r}_o \cdot \mathbf{z}}$$

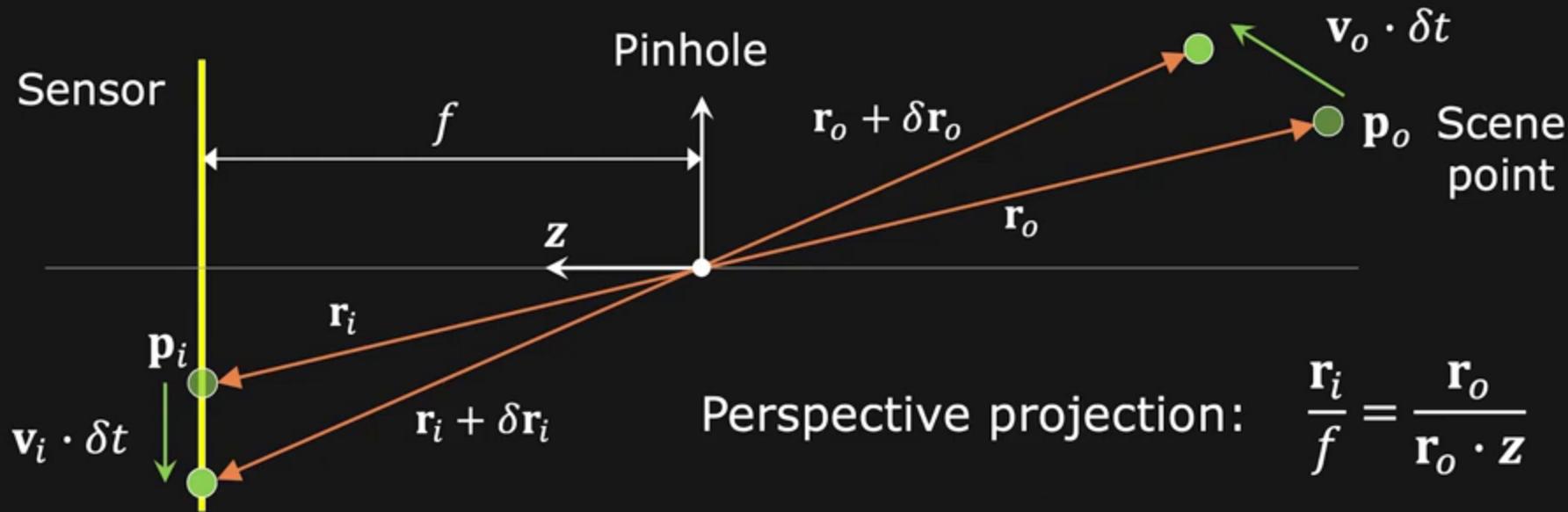
$$\text{Image Point Velocity: } \mathbf{v}_i = \frac{d\mathbf{r}_i}{dt} = f \frac{(\mathbf{r}_o \cdot \mathbf{z})\mathbf{v}_0 - (\mathbf{v}_o \cdot \mathbf{z})\mathbf{r}_0}{(\mathbf{r}_o \cdot \mathbf{z})^2}$$

(Motion Field)



# Motion Field

Image velocity of a point that is moving in the scene



$$\text{Perspective projection: } \frac{\mathbf{r}_i}{f} = \frac{\mathbf{r}_o}{\mathbf{r}_o \cdot \mathbf{z}}$$

$$\text{Image Point Velocity: } \mathbf{v}_i = \frac{d\mathbf{r}_i}{dt} = f \frac{(\mathbf{r}_o \cdot \mathbf{z})\mathbf{v}_0 - (\mathbf{v}_o \cdot \mathbf{z})\mathbf{r}_0}{(\mathbf{r}_o \cdot \mathbf{z})^2}$$

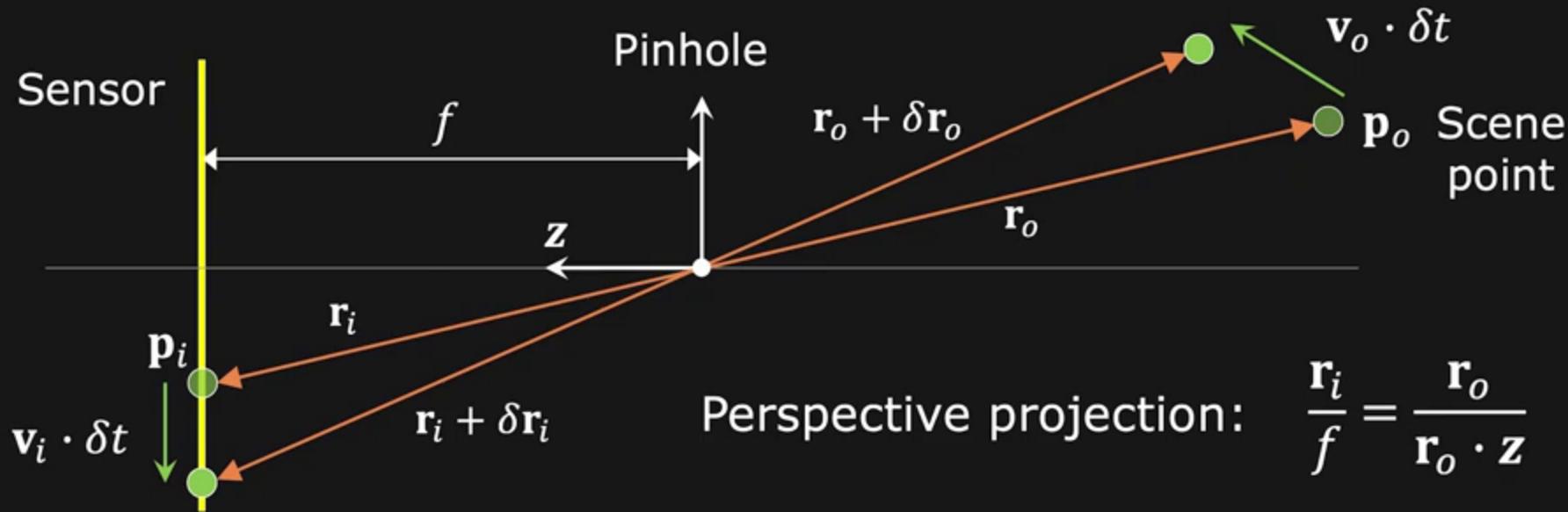
$$\mathbf{v}_i = f \frac{(\mathbf{r}_o \times \mathbf{v}_0) \times \mathbf{z}}{(\mathbf{r}_o \cdot \mathbf{z})^2}$$

[Horn 1981]



# Motion Field

Image velocity of a point that is moving in the scene



$$\text{Perspective projection: } \frac{\mathbf{r}_i}{f} = \frac{\mathbf{r}_o}{\mathbf{r}_o \cdot \mathbf{z}}$$

Image Point Velocity:  $\mathbf{v}_i = \frac{d\mathbf{r}_i}{dt} = f \frac{(\mathbf{r}_o \cdot \mathbf{z})\mathbf{v}_0 - (\mathbf{v}_o \cdot \mathbf{z})\mathbf{r}_0}{(\mathbf{r}_o \cdot \mathbf{z})^2}$   
(Motion Field)

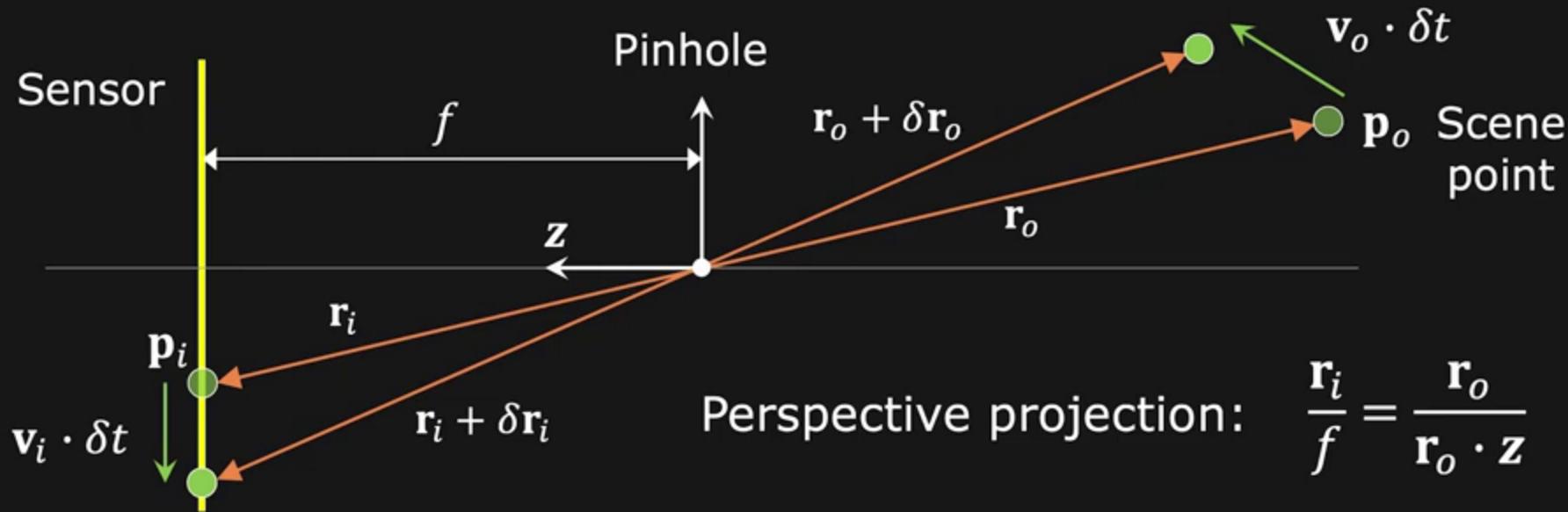
$$\mathbf{\hat{v}}_i = f \frac{(\mathbf{r}_o \times \mathbf{v}_0) \times \mathbf{z}}{(\mathbf{r}_o \cdot \mathbf{z})^2}$$

[Horn 1981]



# Motion Field

Image velocity of a point that is moving in the scene

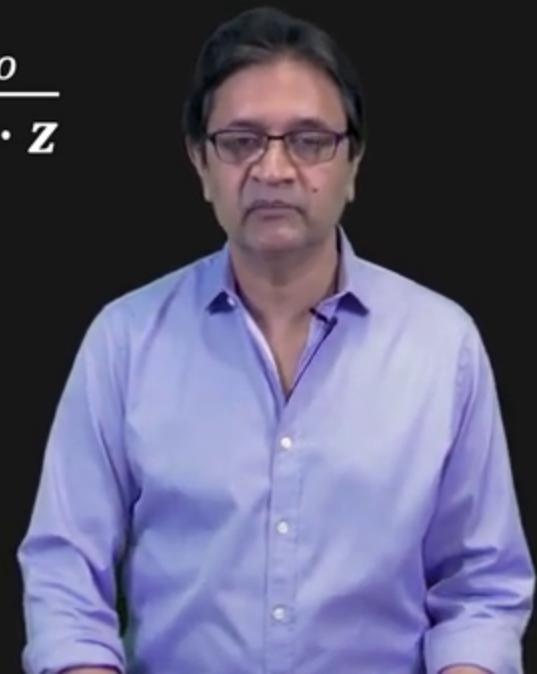


$$\text{Perspective projection: } \frac{\mathbf{r}_i}{f} = \frac{\mathbf{r}_o}{\mathbf{r}_o \cdot \mathbf{z}}$$

Image Point Velocity:  $\mathbf{v}_i = \frac{d\mathbf{r}_i}{dt} = f \frac{(\mathbf{r}_o \cdot \mathbf{z})\mathbf{v}_0 - (\mathbf{v}_o \cdot \mathbf{z})\mathbf{r}_0}{(\mathbf{r}_o \cdot \mathbf{z})^2}$   
(Motion Field)

$$\mathbf{v}_i = f \frac{(\mathbf{r}_o \times \mathbf{v}_0) \times \mathbf{z}}{(\mathbf{r}_o \cdot \mathbf{z})^2}$$

[Horn 1981]



# Optical Flow

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Motion of brightness patterns in the image



Image Sequence  
(2 frames)



# Optical Flow

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Motion of brightness patterns in the image



Image Sequence  
(2 frames)



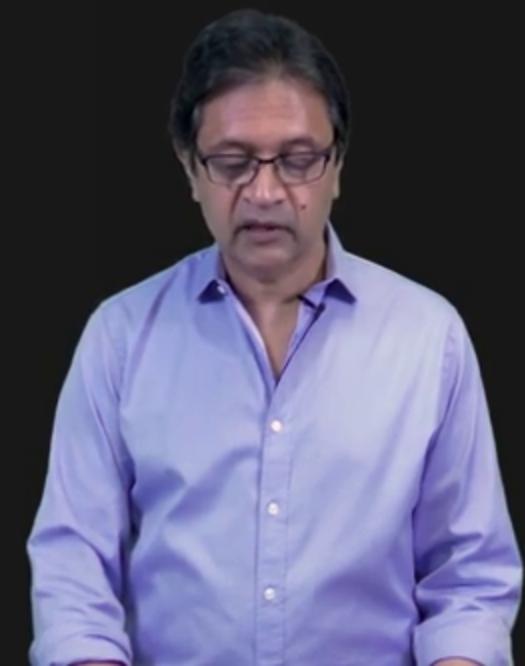
# Optical Flow

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Motion of brightness patterns in the image



Image Sequence  
(2 frames)



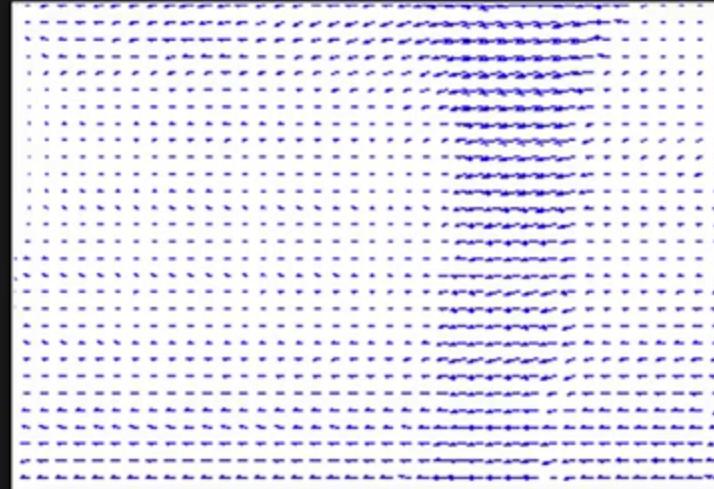
# Optical Flow

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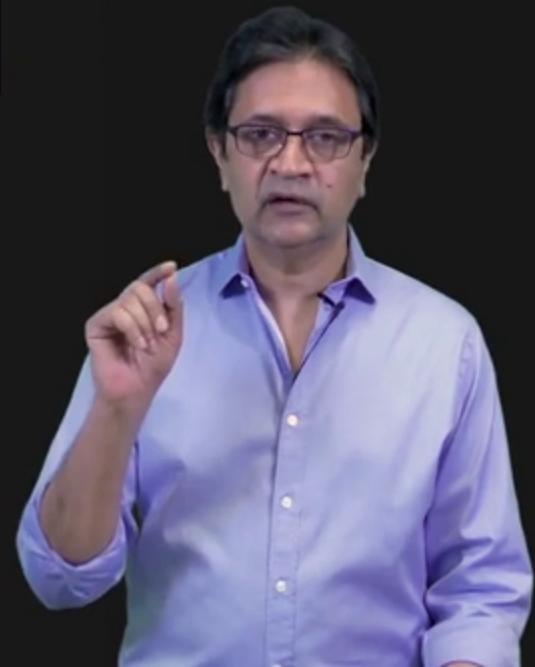
Motion of brightness patterns in the image



Image Sequence  
(2 frames)



Optical Flow

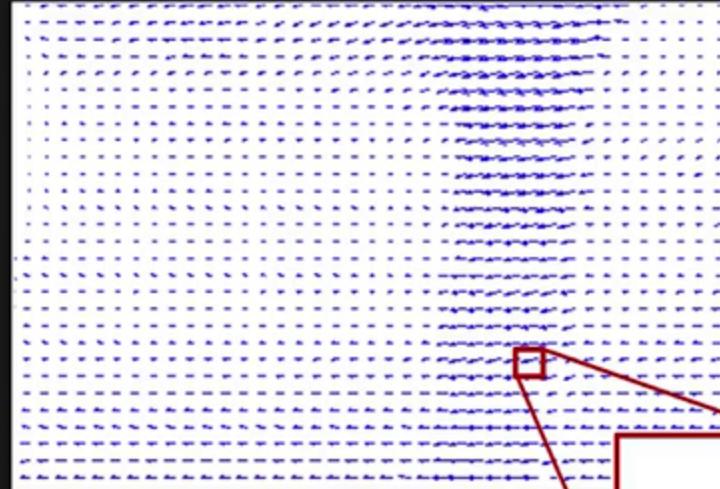


# Optical Flow

Motion of brightness patterns in the image



Image Sequence  
(2 frames)



Optical Flow

Velocity of  
brightness pattern

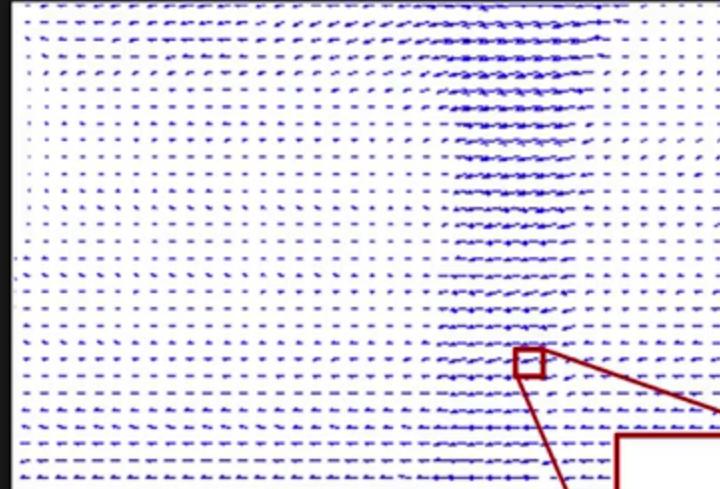


# Optical Flow

Motion of brightness patterns in the image



Image Sequence  
(2 frames)



Optical Flow

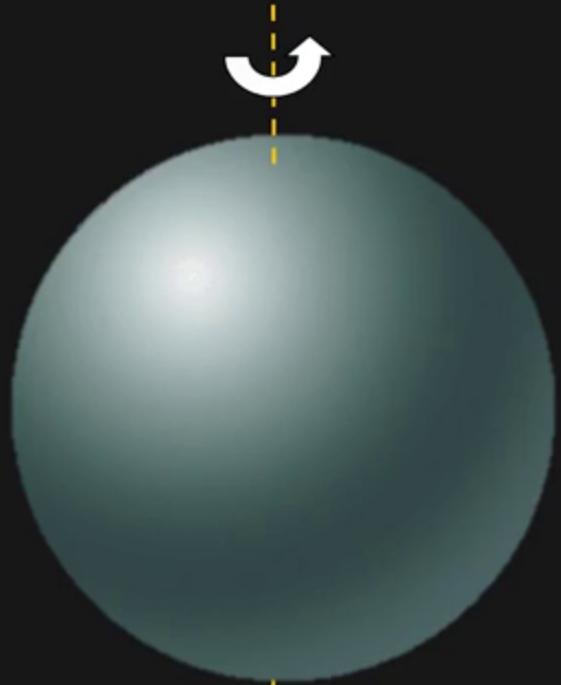
Velocity of  
brightness pattern

Ideally, Optical Flow = Motion Field

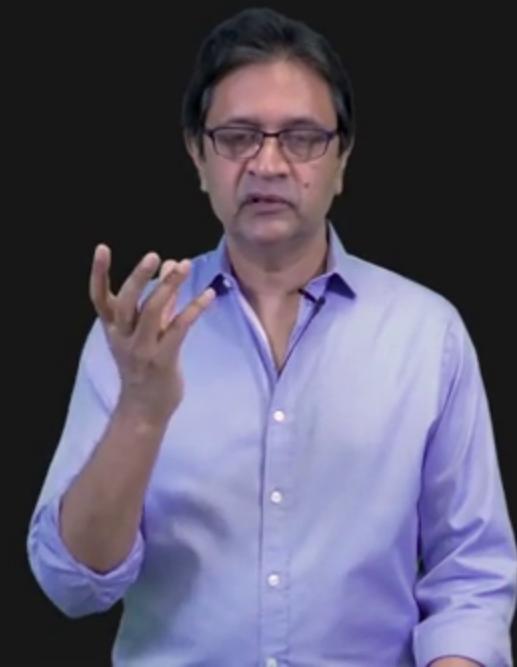


# When is Optical Flow $\neq$ Motion Field?

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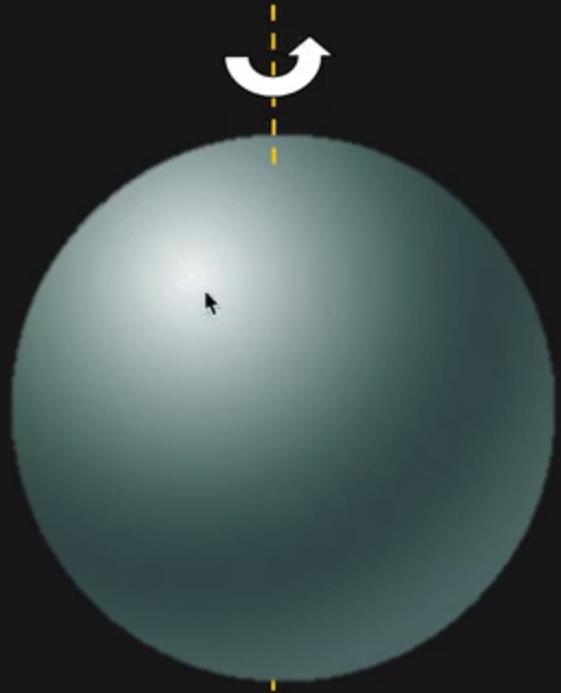


Spinning Sphere  
Stationary Light Source



# When is Optical Flow $\neq$ Motion Field?

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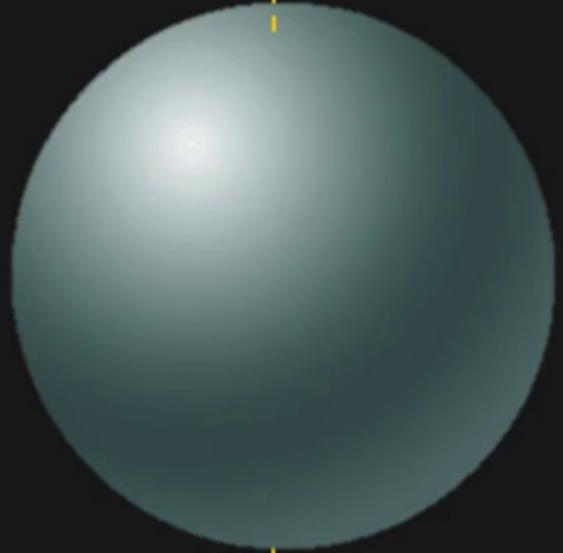


Spinning Sphere  
Stationary Light Source

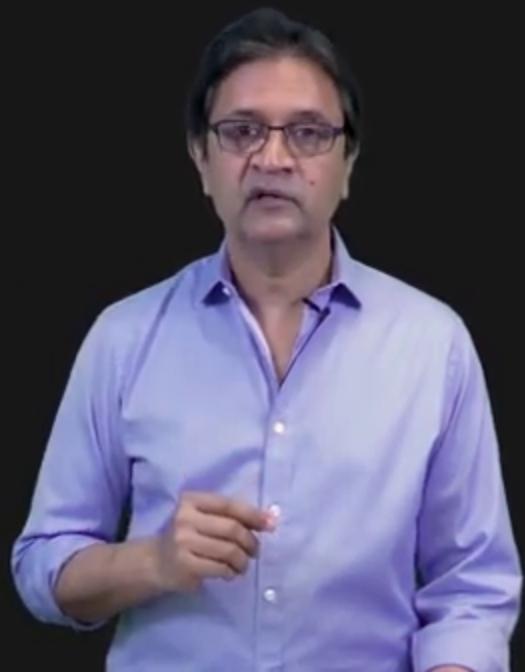


# When is Optical Flow $\neq$ Motion Field?

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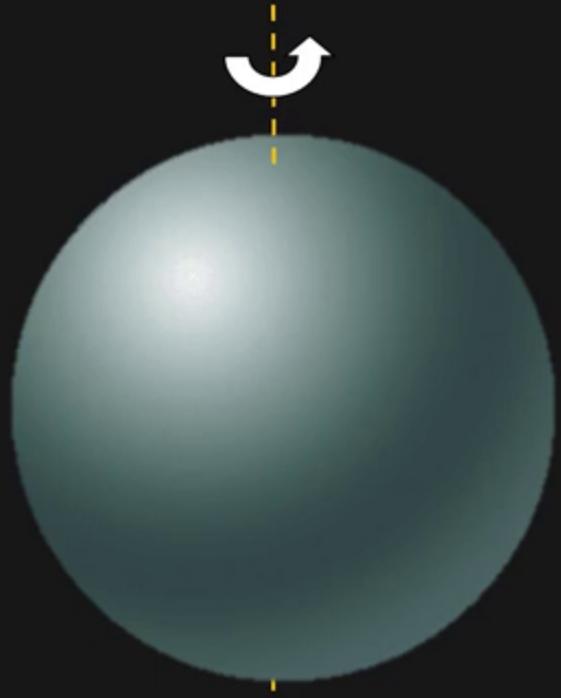


Spinning Sphere  
Stationary Light Source



# When is Optical Flow $\neq$ Motion Field?

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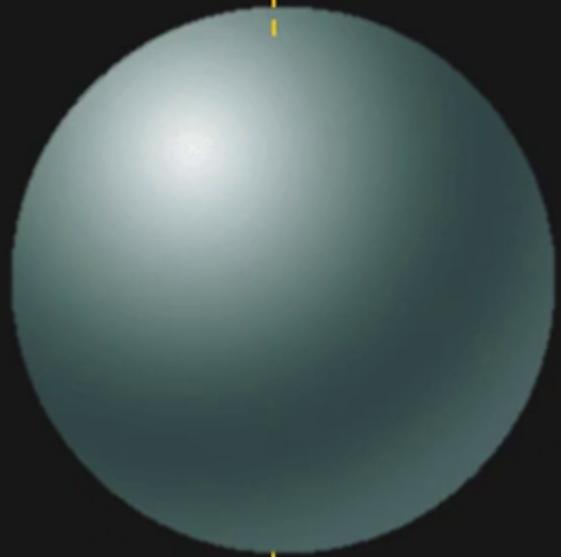


Spinning Sphere  
Stationary Light Source

Motion Field exists  
But no Optical Flow



# When is Optical Flow $\neq$ Motion Field?

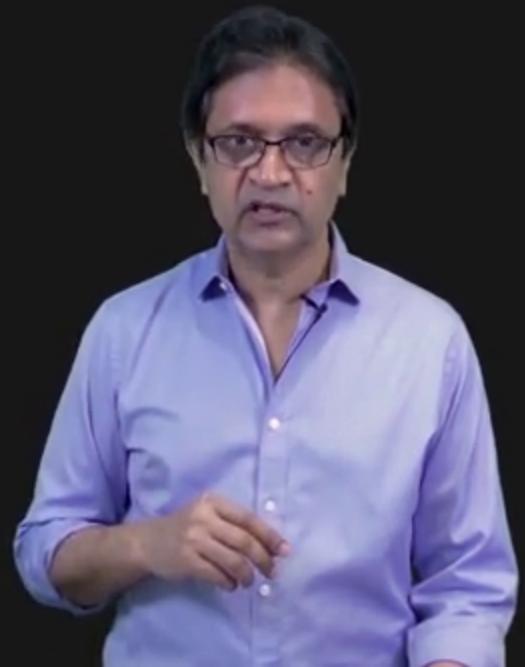


Spinning Sphere  
Stationary Light Source

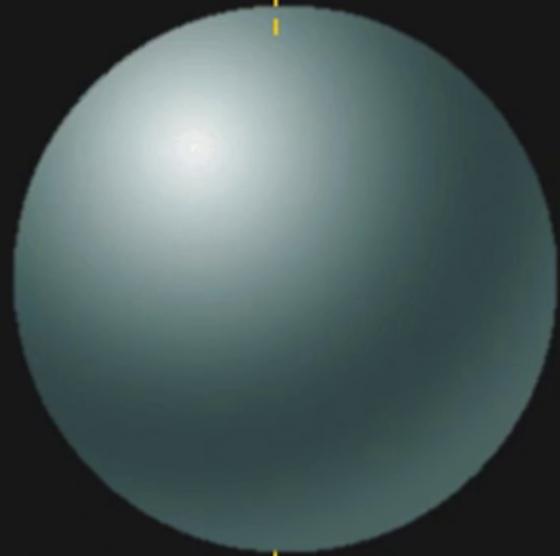
Motion Field exists  
But no Optical Flow



Stationary Sphere  
Moving Light Source

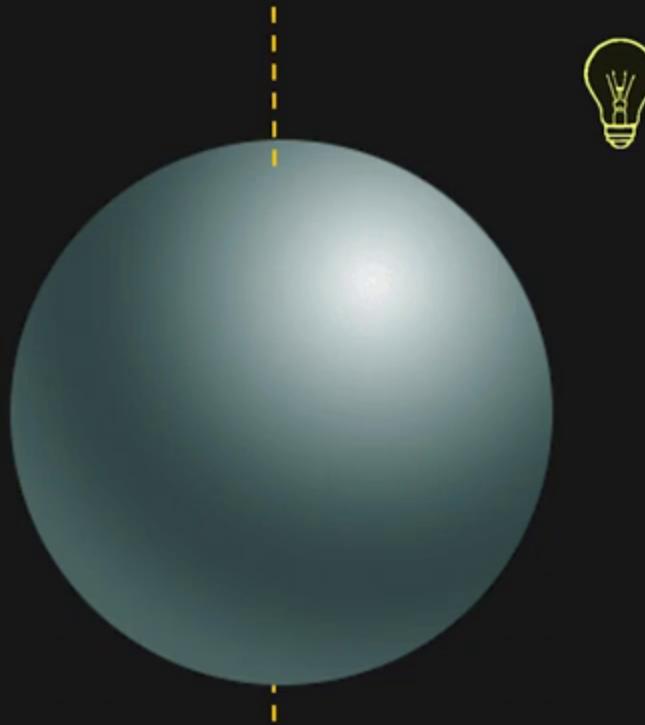


# When is Optical Flow $\neq$ Motion Field?

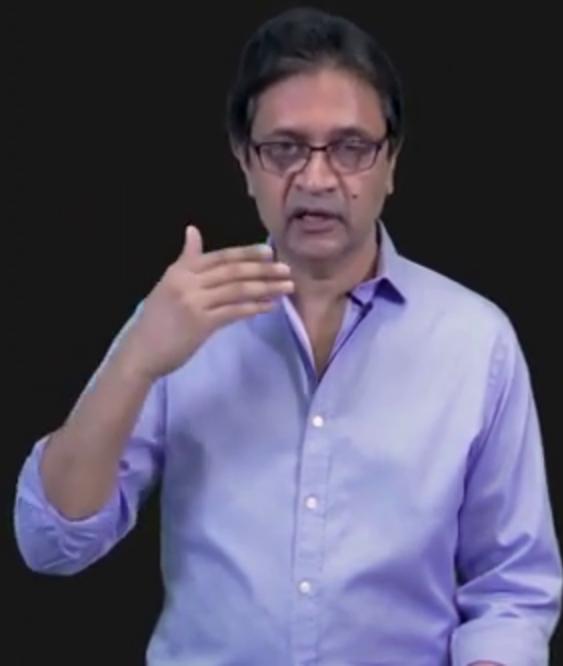


Spinning Sphere  
Stationary Light Source

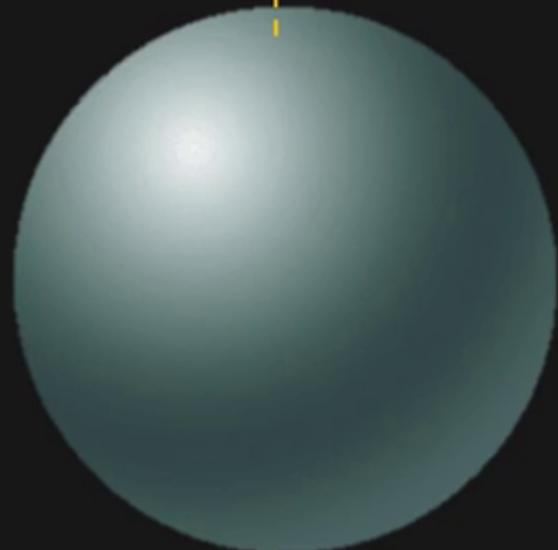
Motion Field exists  
But no Optical Flow



Stationary Sphere  
Moving Light Source

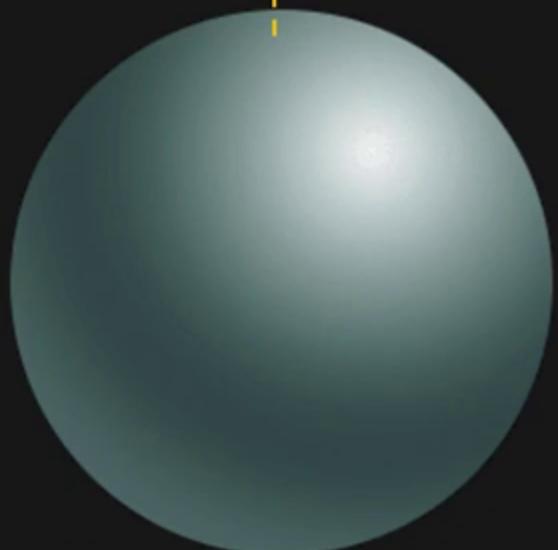


# When is Optical Flow $\neq$ Motion Field?



Spinning Sphere  
Stationary Light Source

Motion Field exists  
But no Optical Flow

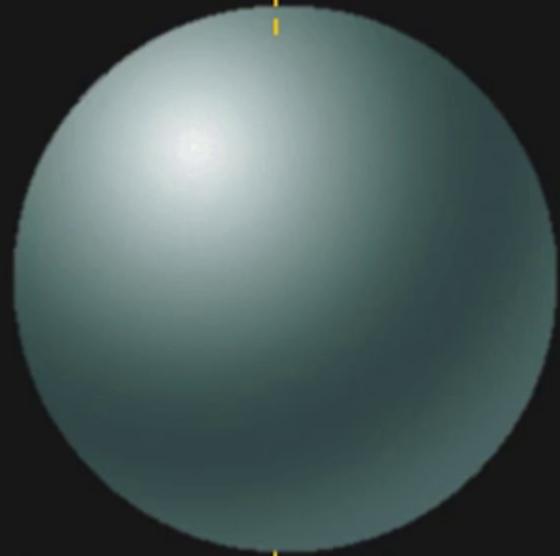


Stationary Sphere  
Moving Light Source

No Motion Field exists  
But there is Optical Flow



# When is Optical Flow $\neq$ Motion Field?



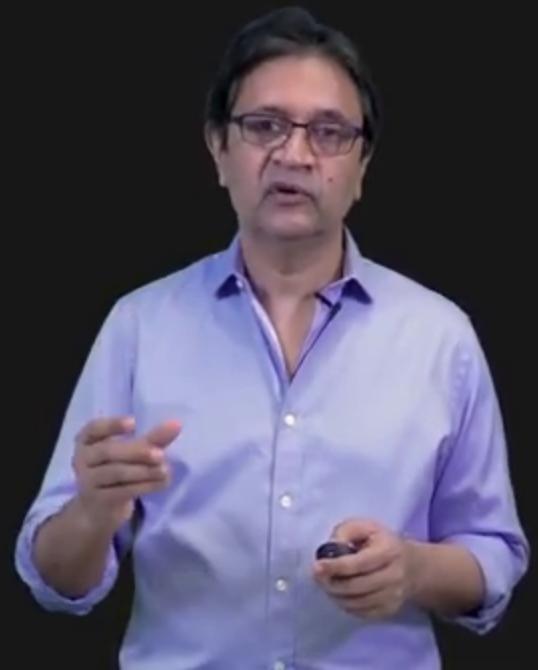
Spinning Sphere  
Stationary Light Source

Motion Field exists  
But no Optical Flow



Stationary Sphere  
Moving Light Source

No Motion Field exists  
But there is Optical Flow

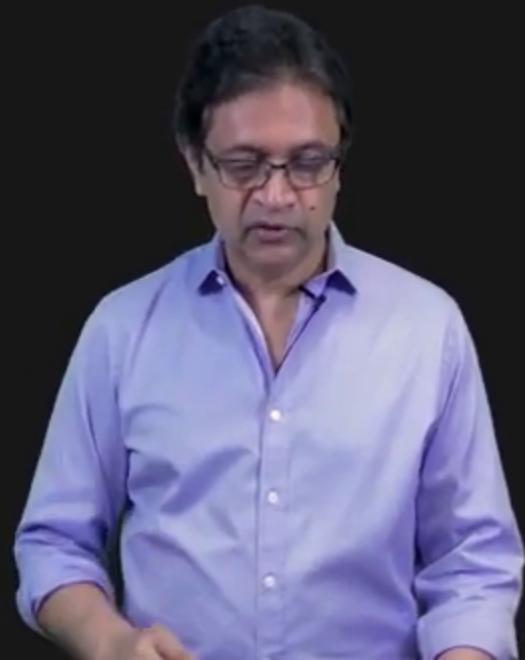


# When is Optical Flow $\neq$ Motion Field?

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Barber Pole  
Illusion

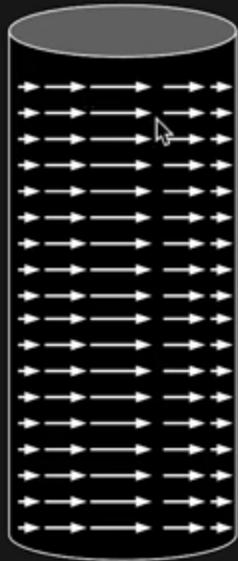


# When is Optical Flow $\neq$ Motion Field?

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Barber Pole  
Illusion



Motion Field

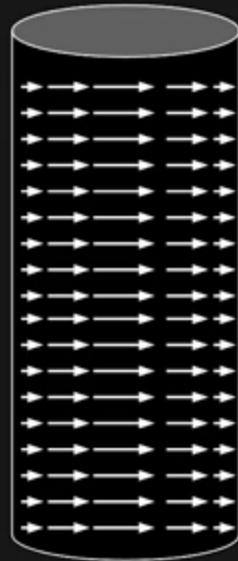


# When is Optical Flow $\neq$ Motion Field?

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Barber Pole  
Illusion



Motion Field

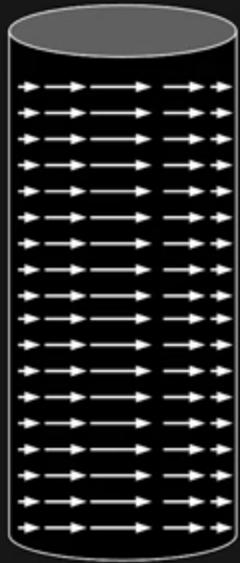


# When is Optical Flow $\neq$ Motion Field?

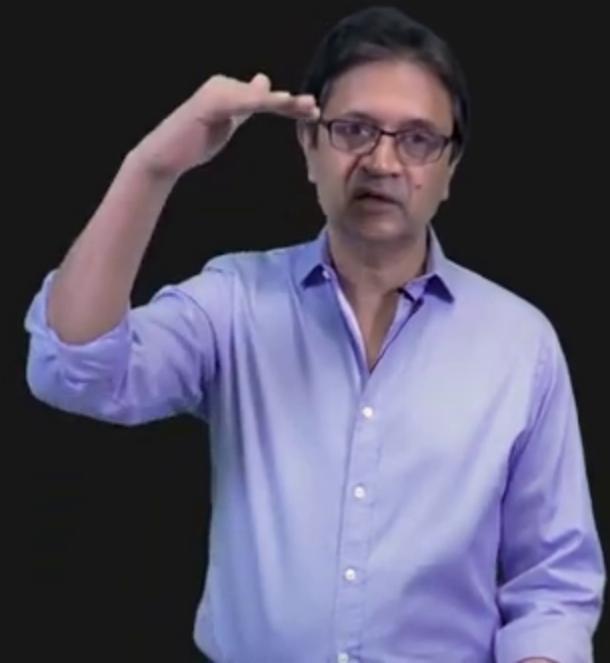
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Barber Pole  
Illusion



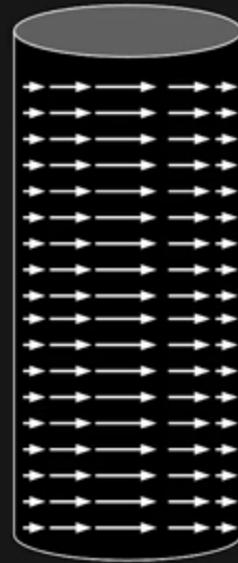
Motion Field



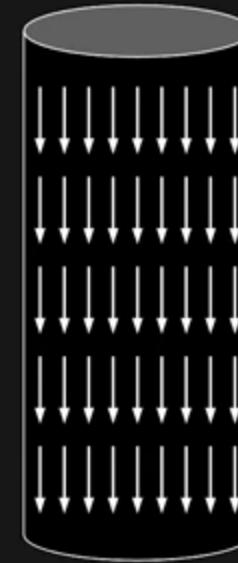
# When is Optical Flow $\neq$ Motion Field?



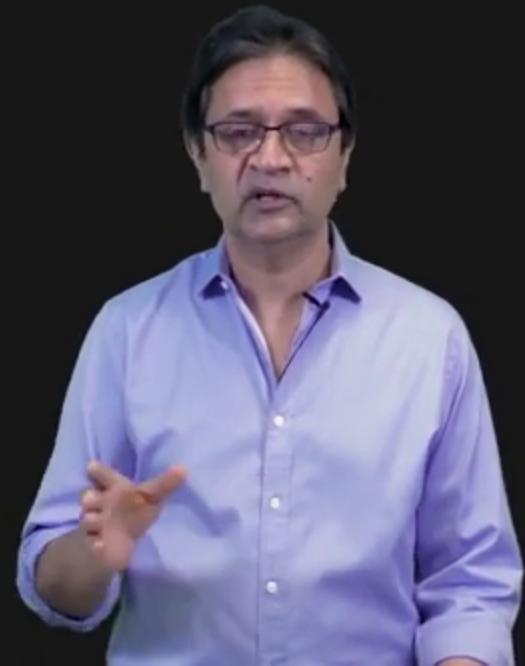
Barber Pole  
Illusion



Motion Field



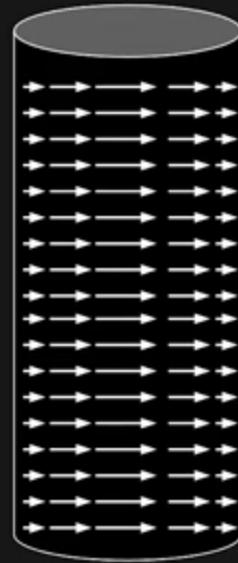
Optical Flow



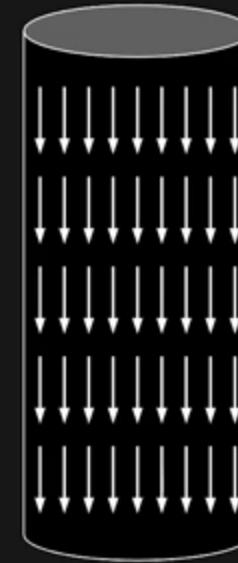
# When is Optical Flow $\neq$ Motion Field?



Barber Pole  
Illusion



Motion Field

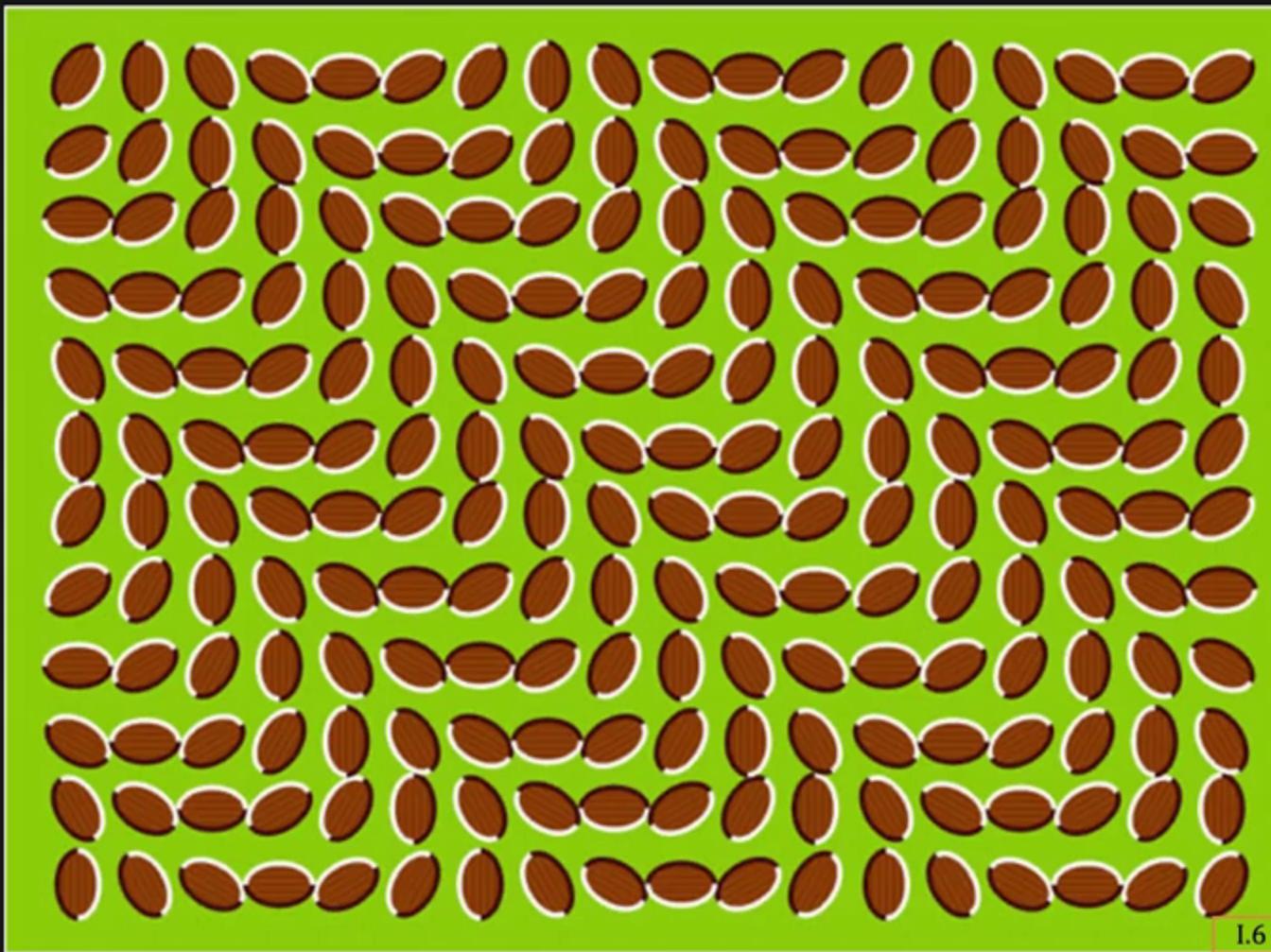


Optical Flow

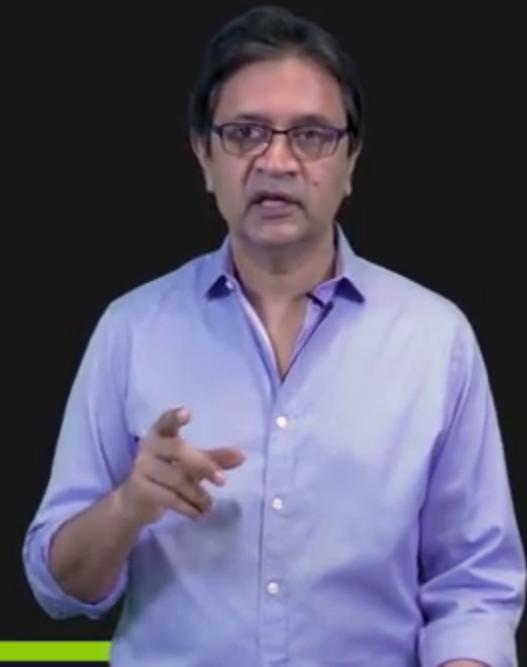


# Motion Illusions

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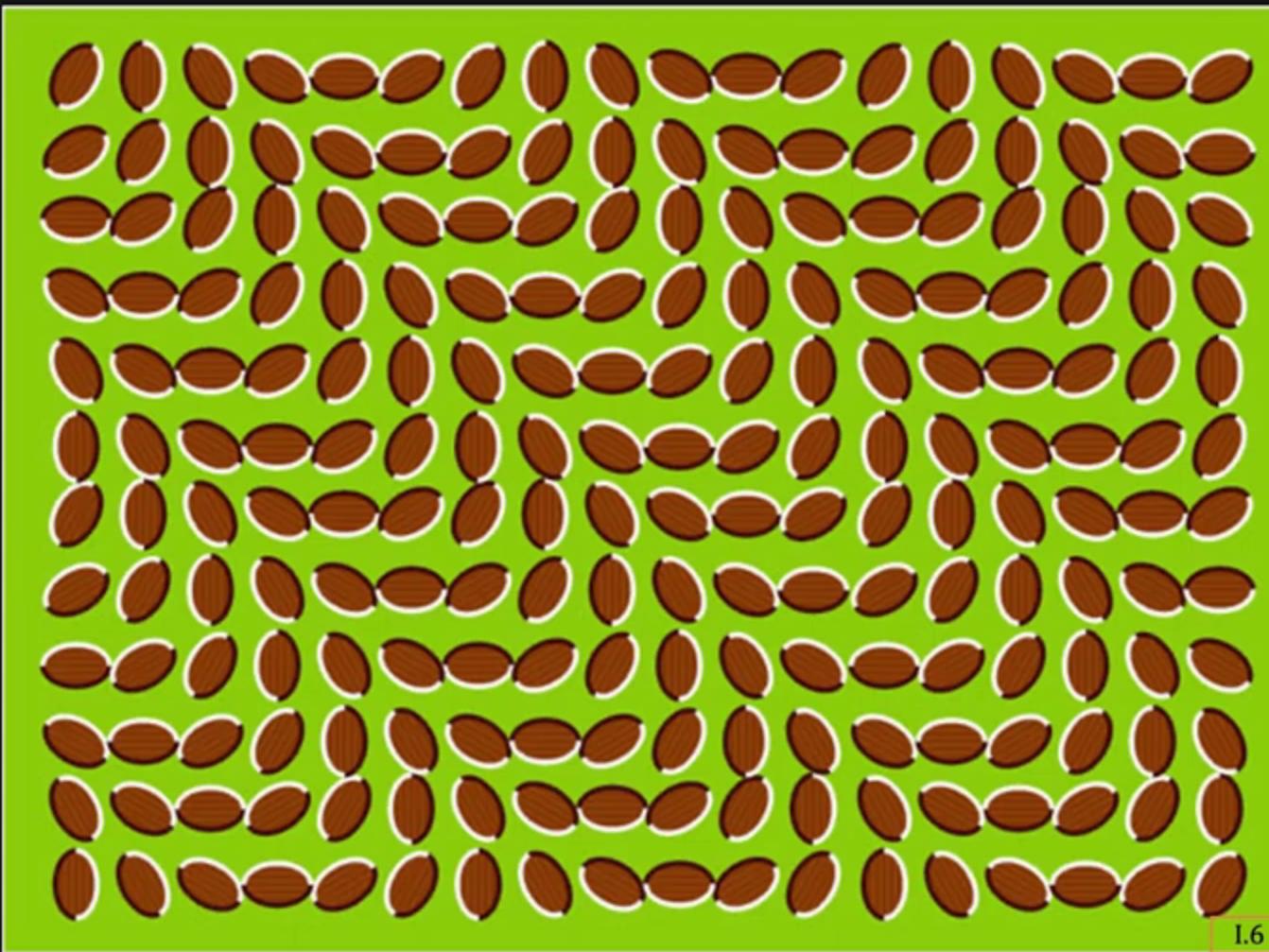


Donguri Wave Illusion

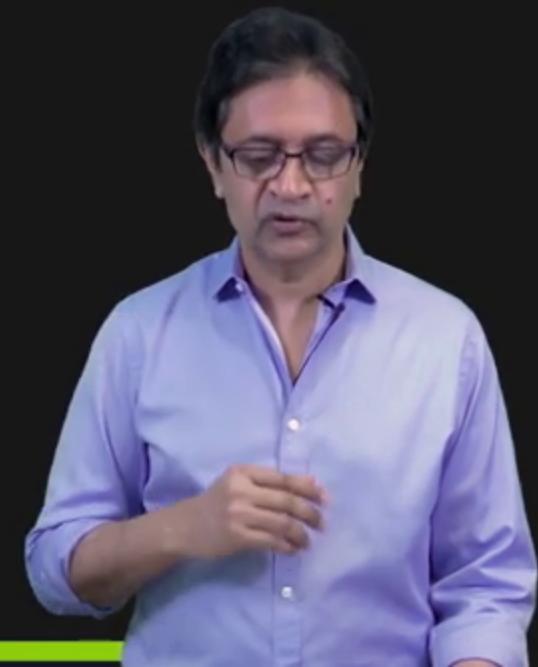


# Motion Illusions

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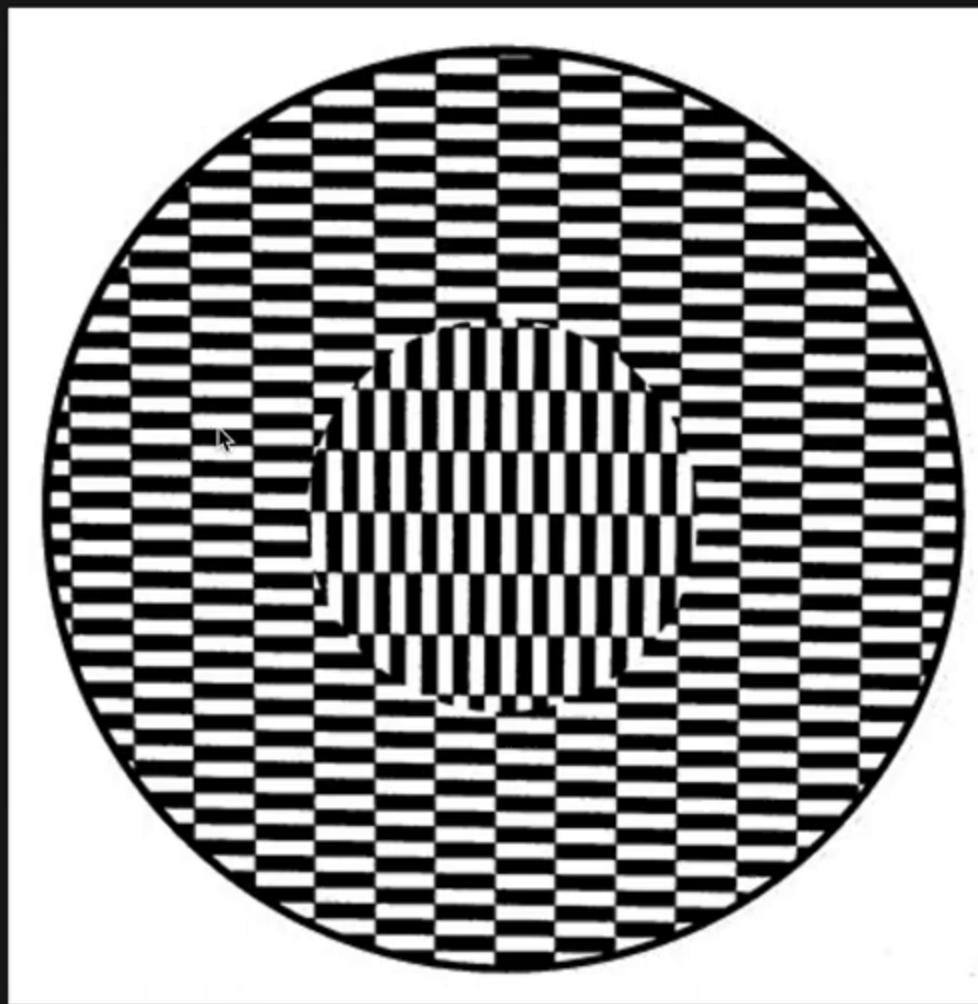


Donguri Wave Illusion



# Motion Illusions

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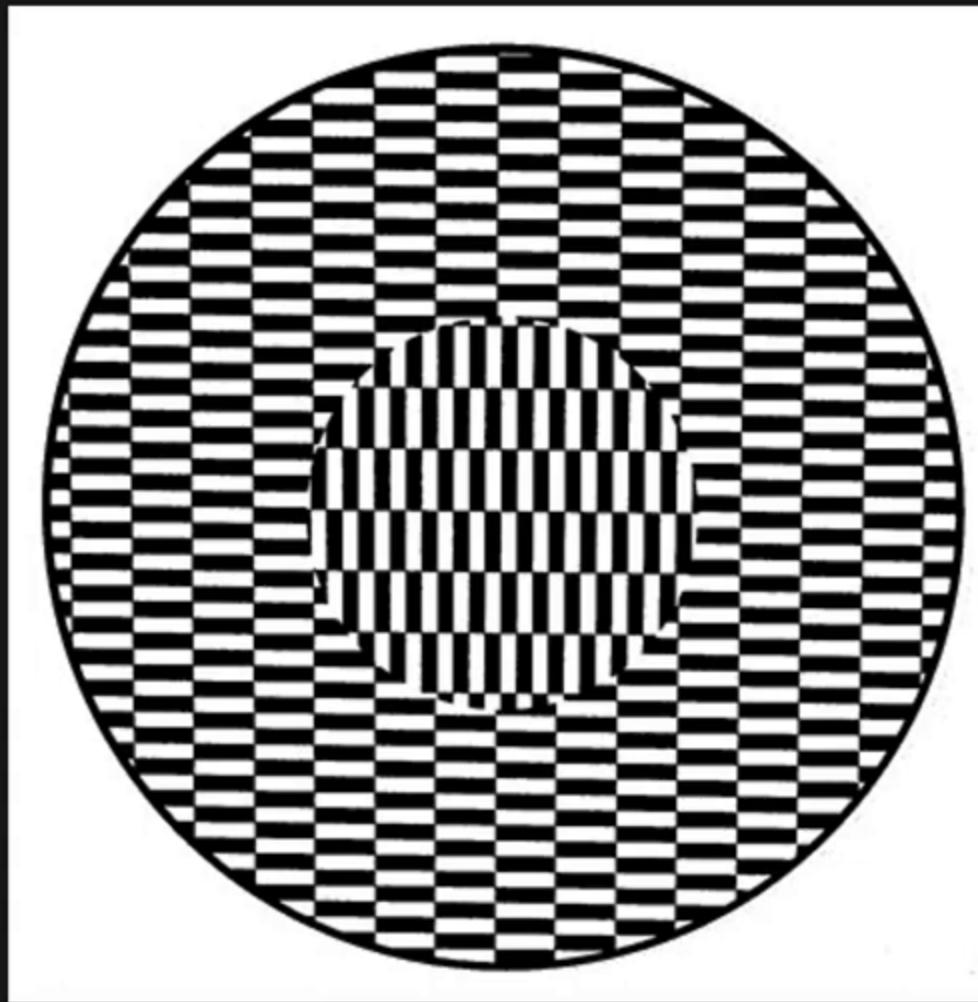


Ouchi Pattern



# Motion Illusions

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Ouchi Pattern

