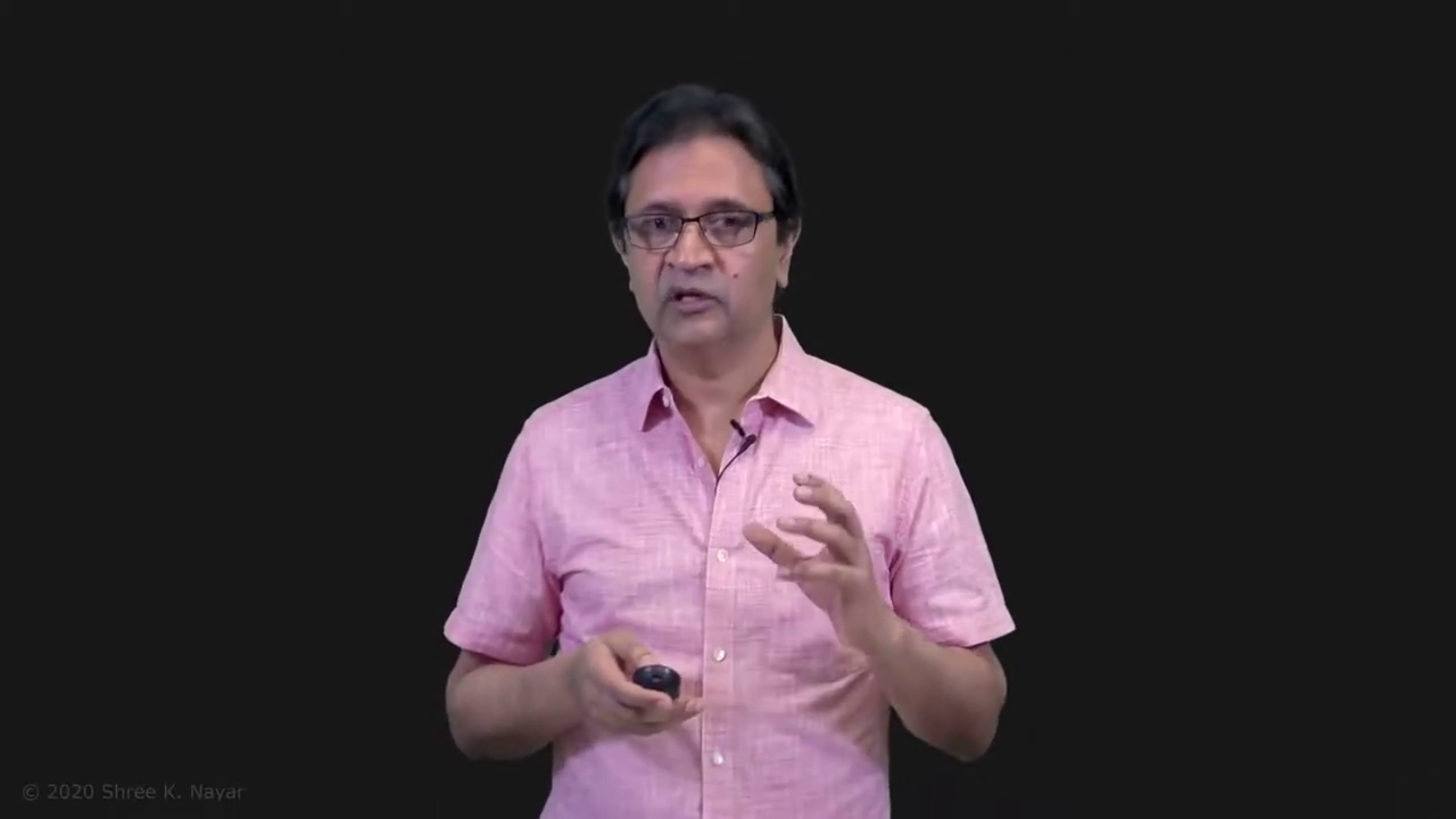
Tomasi-Kanade Factorization

Shree K. Nayar

Columbia University

Topic: Structure from Motion, Module: Reconstruction II

First Principles of Computer Vision



Rank of Observation Matrix

$$\underset{2F\times N}{W} = \underset{2F\times 3}{M} \times \underset{3\times N}{S}$$

We know:

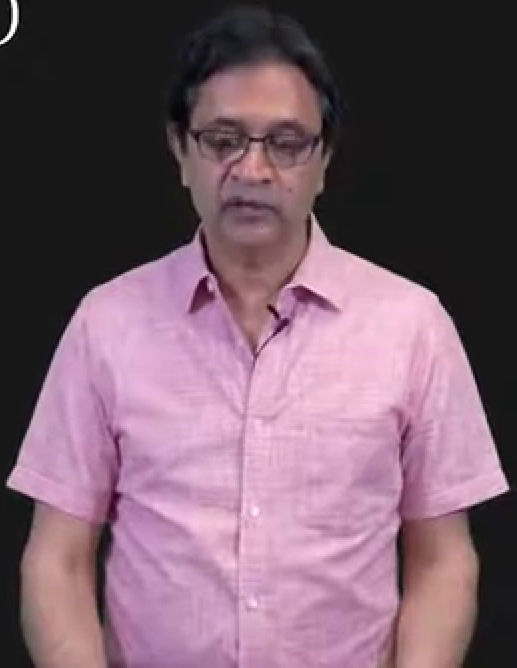
$$Rank(MS) \leq Rank(M) \qquad Rank(MS) \leq Rank(S)$$

$$\Rightarrow \quad Rank(MS) \leq \min(3,2F) \qquad Rank(MS) \leq \min(3,N)$$

$$\Rightarrow \quad Rank(W) = Rank(MS) \leq \min(3,N,2F)$$

$$\boxed{\text{Rank Theorem: } Rank(W) \leq 3}$$

[Tomasi 1992]



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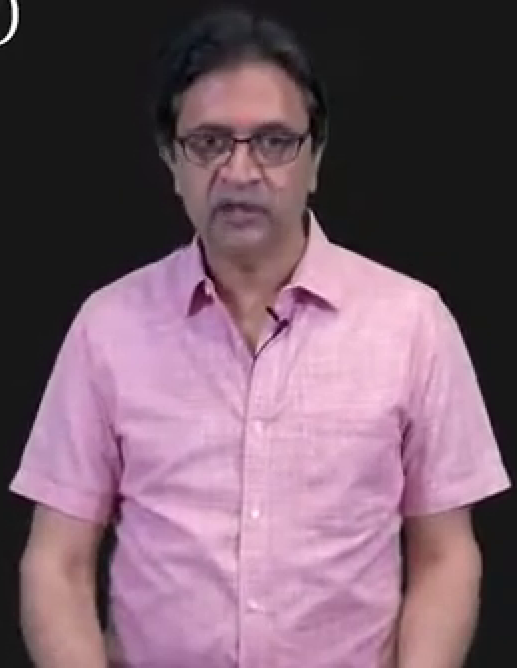

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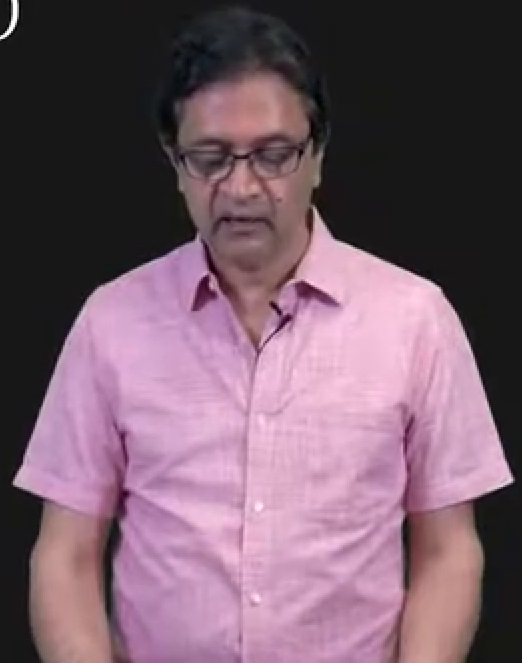

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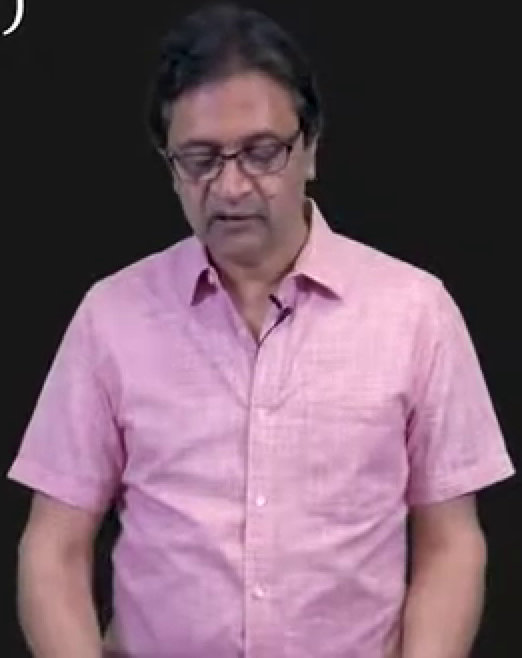

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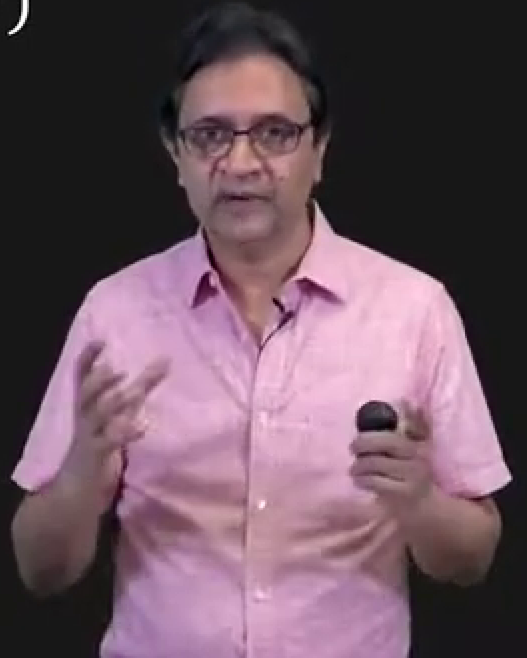

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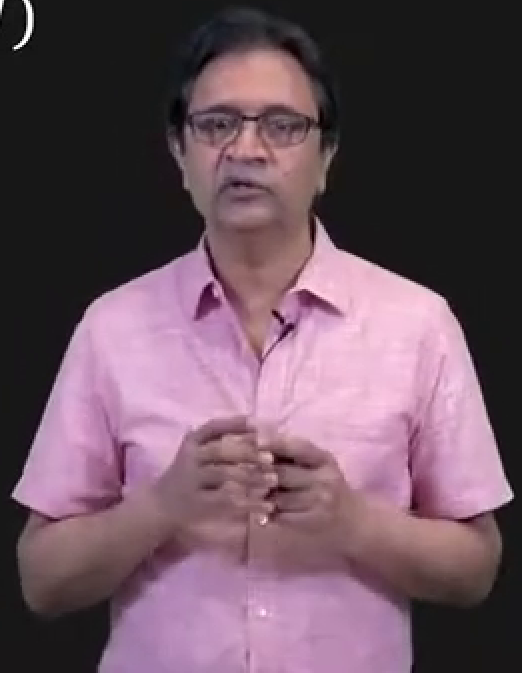
We can "factorize" W into M and S!

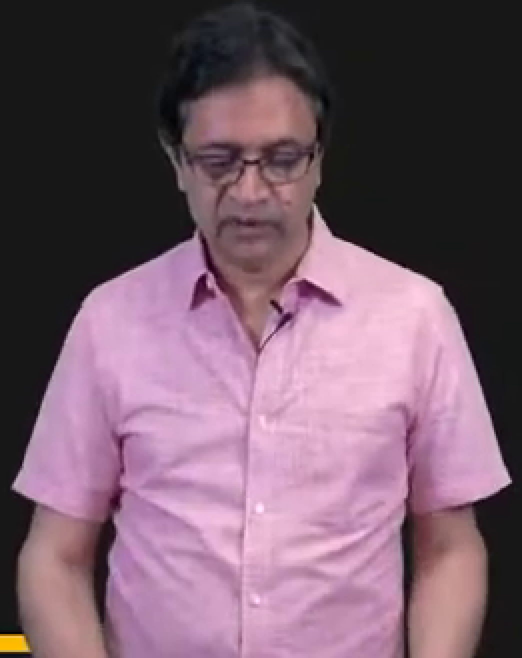
[Tomasi 1992]



Singular Value Decomposition (SVD)

For any matrix A there exists a factorization:

$$A_{M \times N} = U_{M \times M} \cdot \Sigma_{M \times N} \cdot {V^T}_{N \times N}$$



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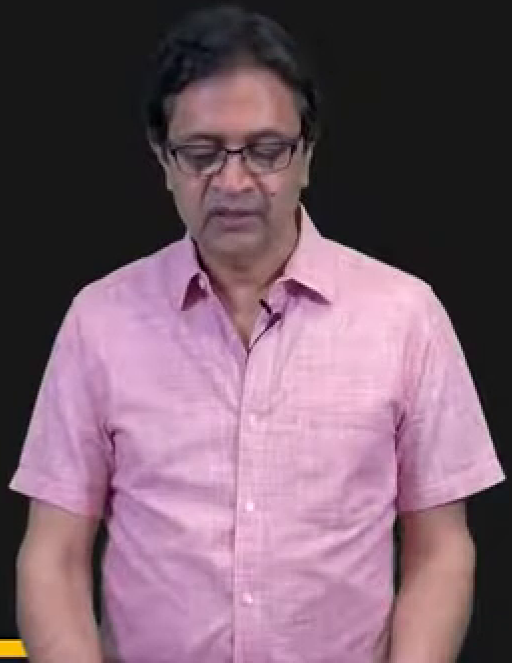

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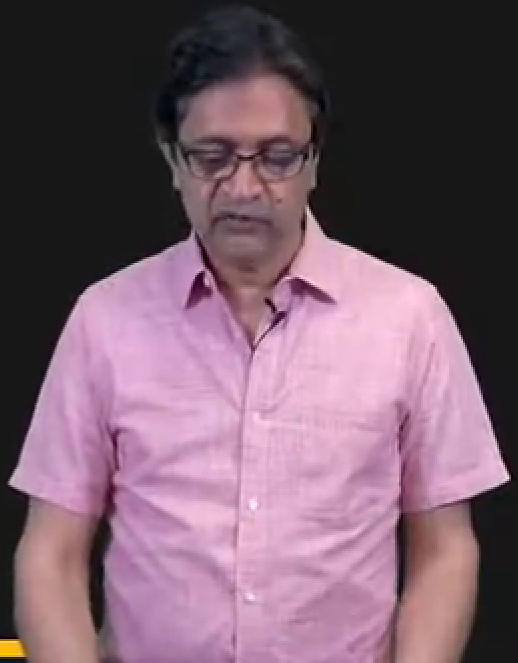

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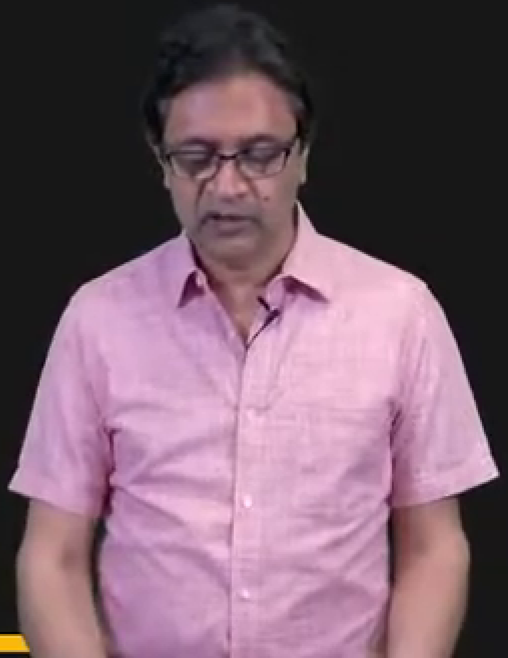
MATLAB: `[U,S,V] = svd(A)`



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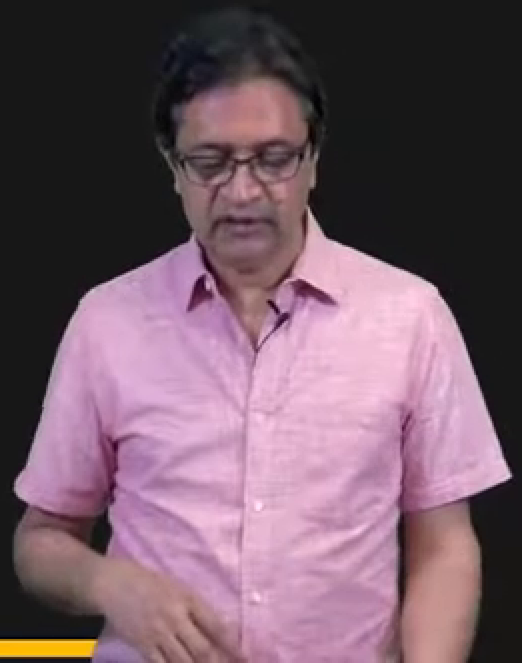

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$$\Sigma_{M\times N} = \begin{pmatrix} \sigma_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & 0 & \dots & 0 \\ 0 & 0 & 0 & \sigma_4 & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & \sigma_N \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \end{pmatrix}$$

$\sigma_1, \dots, \sigma_N$: Singular Values

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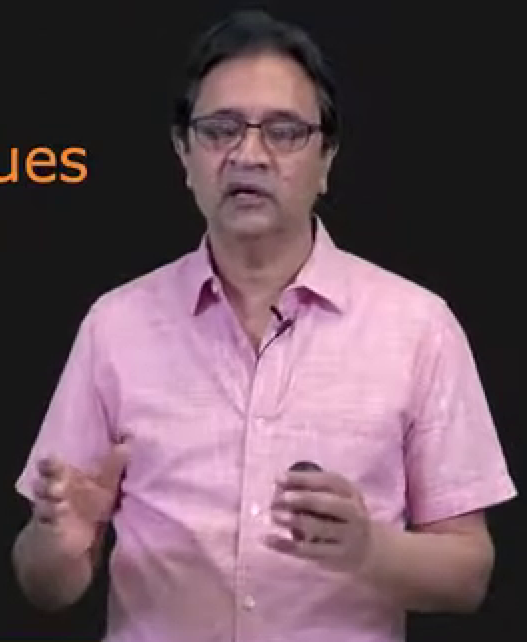

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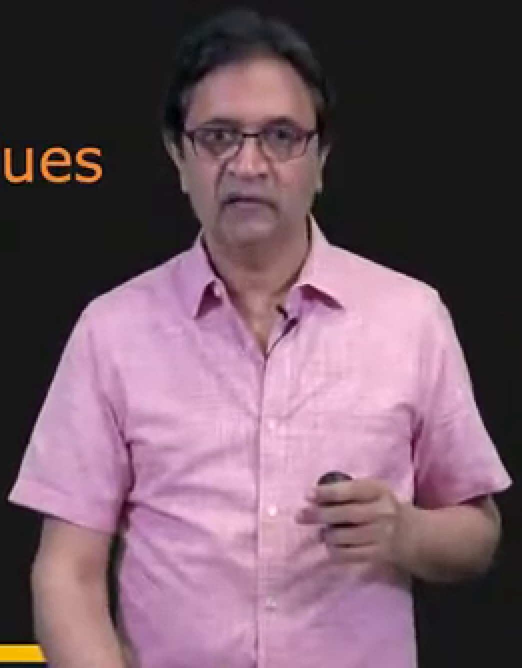

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$\sigma_1, \dots, \sigma_N$: Singular Values

If $Rank(A) = r$ then A has r non-zero singular values.

Enforcing Rank Constraint

Using SVD: $W = U \Sigma V^T$

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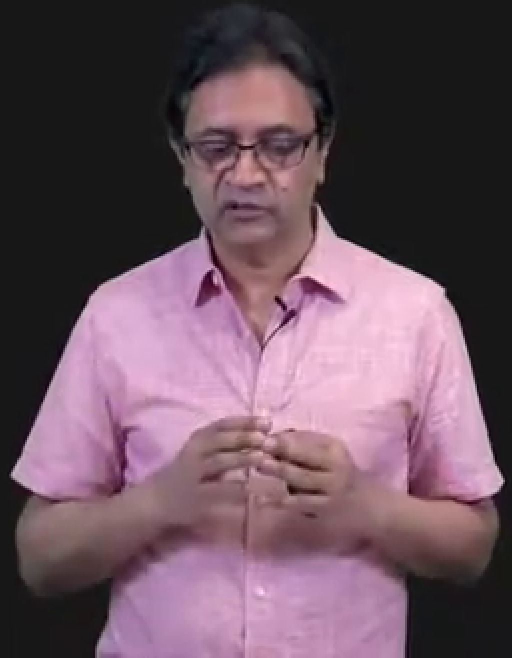
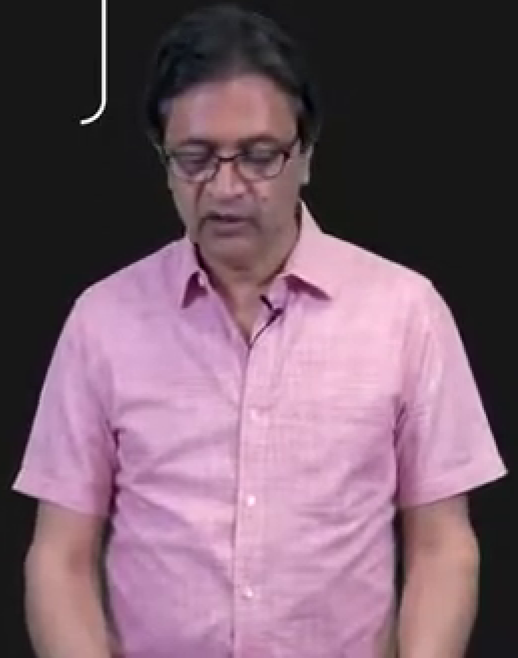


Enforcing Rank Constraint

Using SVD: $W = U \Sigma V^T$

$$= \underbrace{\begin{bmatrix} & & \\ & U & \\ & & \end{bmatrix}}_{2F\times 2F} \underbrace{\begin{bmatrix} \sigma_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & 0 & \dots & 0 \\ 0 & 0 & 0 & \sigma_4 & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & \sigma_N \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \end{bmatrix}}_{2F\times N} \underbrace{\begin{bmatrix} & & \\ & V^T & \\ & & \end{bmatrix}}_{N\times N}$$



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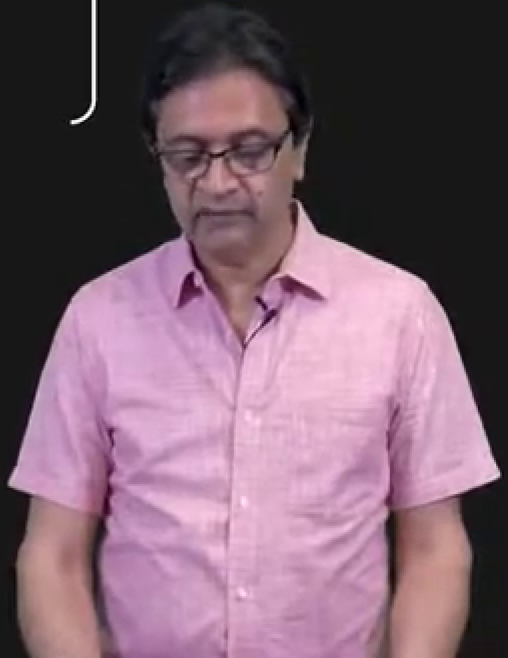
$2F \times 2F$ $\qquad$ $2F \times N$ $\qquad$ $N \times N$



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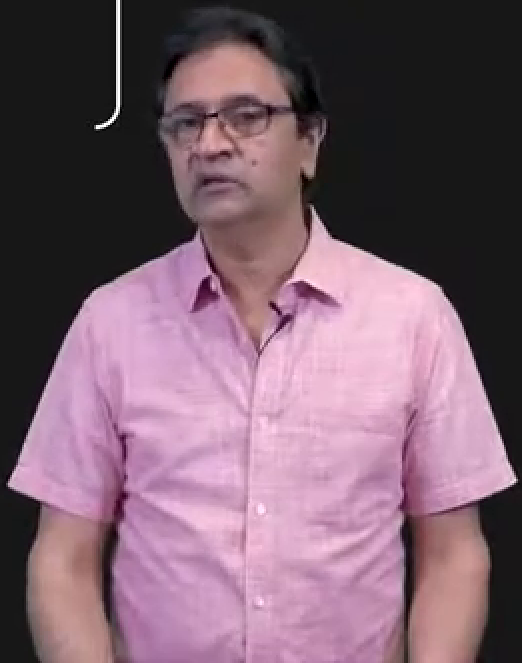
$$
= \begin{bmatrix} & & \\ & U & \\ & & \end{bmatrix}
\begin{bmatrix}
\sigma_1 & 0 & 0 & 0 & \ldots & 0 \\
0 & \sigma_2 & 0 & 0 & \ldots & 0 \\
0 & 0 & \sigma_3 & 0 & \ldots & 0 \\
0 & 0 & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ldots & \vdots
\end{bmatrix}
\begin{bmatrix} & & \\ & V^T & \\ & & \end{bmatrix}
$$

$2F \times 2F$ $2F \times N$ $N \times N$

Since $Rank(W) \leq 3$, $Rank(\Sigma) \leq 3$.



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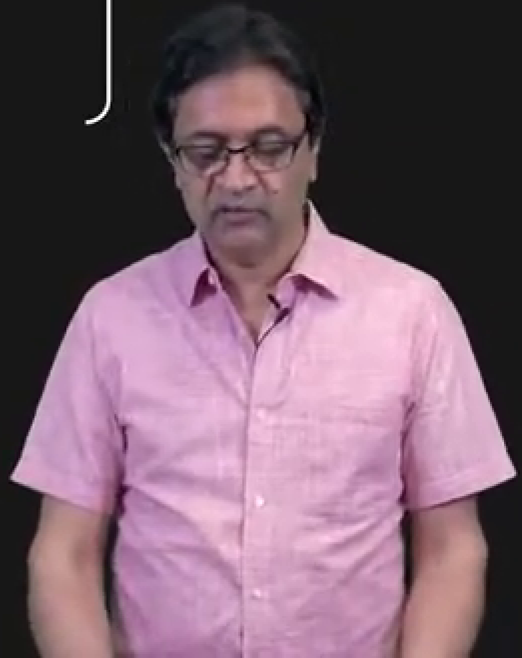

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$$
= \begin{bmatrix} \\ \\ & U & \\ \\ \\ \end{bmatrix}
\begin{bmatrix}
\sigma_1 & 0 & 0 & 0 & \dots & 0 \\
0 & \sigma_2 & 0 & 0 & \dots & 0 \\
0 & 0 & \sigma_3 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \dots & \vdots
\end{bmatrix}
\begin{bmatrix} \\ \\ & V^T & \\ \\ \\ \end{bmatrix}
$$

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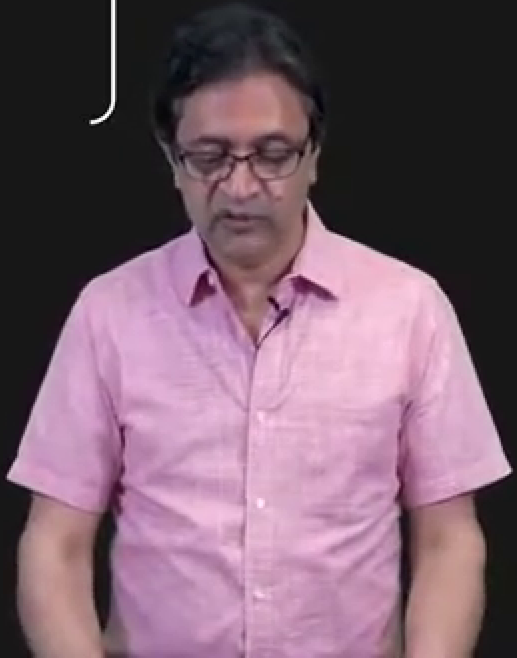

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Using SVD: $$W = U\,\Sigma\,V^T$$

$$= \left[\begin{array}{c|c} U_1 & U_2 \end{array}\right]_{\underset{2F\times 2F}{3 \quad\quad 2F-3}} \begin{bmatrix} \sigma_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \end{bmatrix}_{2F\times N} \begin{array}{c} \begin{bmatrix} V_1^T \\ \hline V_2^T \end{bmatrix}_{N\times N} \end{array} \begin{array}{l} 3 \\ N-3 \end{array}$$

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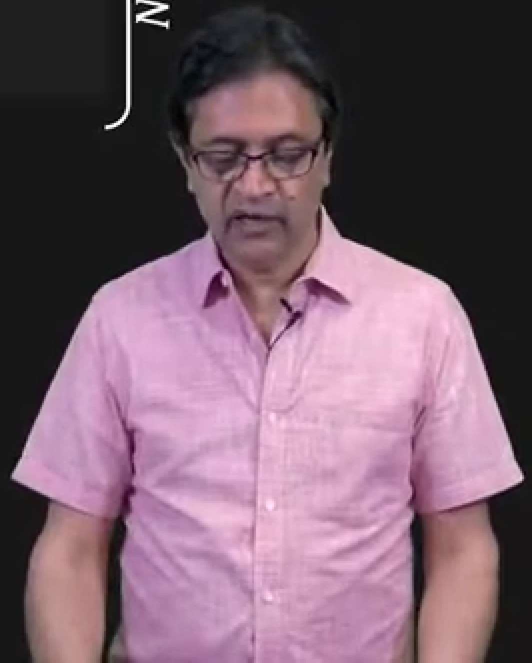

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$$
= \underbrace{\begin{pmatrix} \underbrace{U_1}_{3} & \underbrace{U_2}_{2F-3} \end{pmatrix}}_{2F\times 2F}
\underbrace{\begin{pmatrix}
\sigma_1 & 0 & 0 & 0 & \dots & 0 \\
0 & \sigma_2 & 0 & 0 & \dots & 0 \\
0 & 0 & \sigma_3 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \dots & \vdots
\end{pmatrix}}_{2F\times N}
\underbrace{\begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix}
\begin{matrix} 3 \\ N-3 \end{matrix}}_{N\times N}
$$

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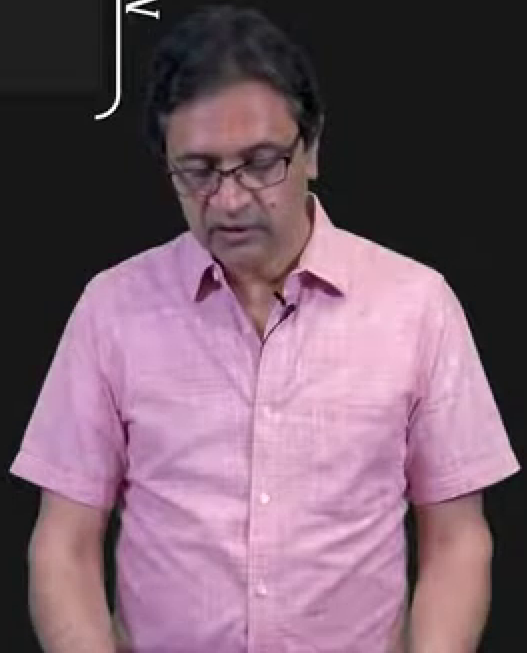

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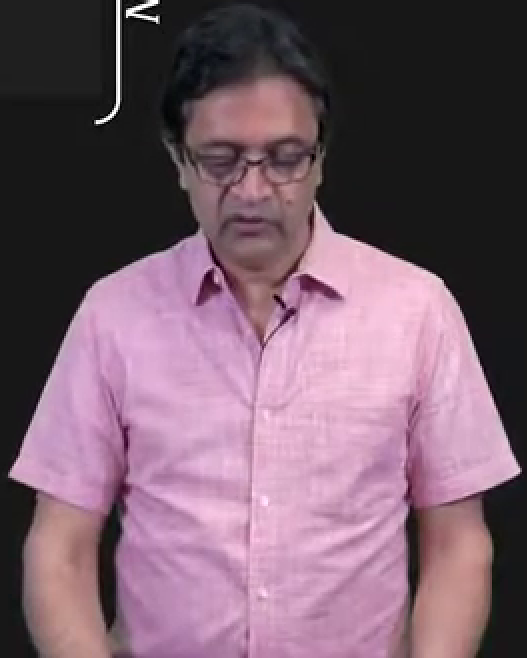

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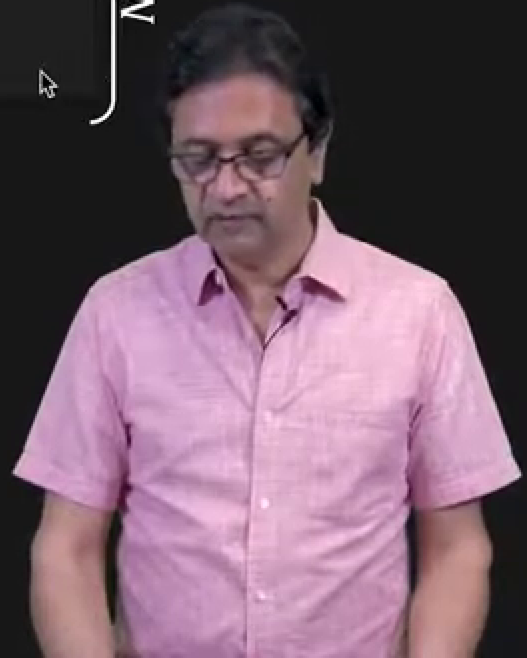
$$= \underset{2F\times 2F}{\left(\begin{array}{c|c} U_1 & U_2 \\ \underset{3}{} & \underset{2F-3}{} \end{array}\right)} \underset{2F\times N}{\begin{pmatrix} \sigma_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \end{pmatrix}} \underset{N\times N}{\begin{pmatrix} V_1^T \\ \hline V_2^T \end{pmatrix}} \begin{array}{l} 3 \\ N-3 \end{array}$$

Since $Rank(W) \leq 3$, $Rank(\Sigma) \leq 3$.



Enforcing Rank Constraint

Using SVD: $W = U\,\Sigma\,V^T$

$$= \underbrace{\begin{pmatrix} \underbrace{U_1}_{3} & \underbrace{U_2}_{2F-3} \end{pmatrix}}_{2F\times 2F} \underbrace{\begin{pmatrix} \sigma_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \end{pmatrix}}_{2F\times N} \underbrace{\begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix}\begin{matrix} 3 \\ N-3 \end{matrix}}_{N\times N}$$

Since $Rank(W) \le 3$, $Rank(\Sigma) \le 3$.

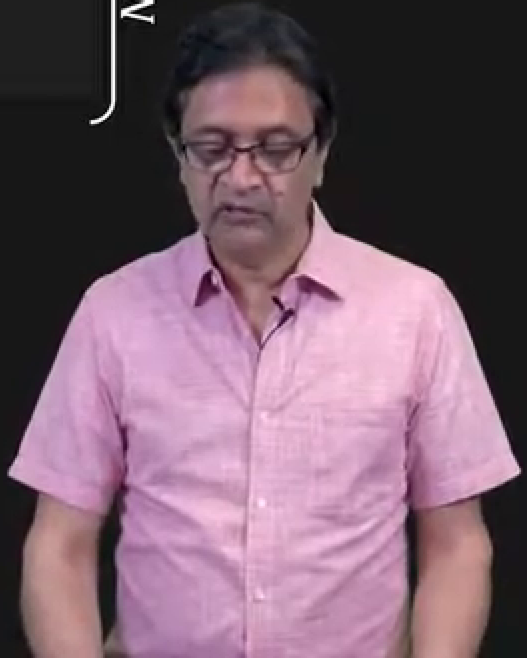
Enforcing Rank Constraint

Using SVD: $W = U \Sigma V^T$

$$
= \underbrace{\left[\begin{array}{c|c} U_1 & U_2 \end{array}\right]}_{2F\times 2F}
\underbrace{\begin{bmatrix}
\sigma_1 & 0 & 0 & 0 & \dots & 0 \\
0 & \sigma_2 & 0 & 0 & \dots & 0 \\
0 & 0 & \sigma_3 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \dots & \vdots
\end{bmatrix}}_{2F\times N}
\underbrace{\begin{bmatrix} V_1^T \\ \hline V_2^T \end{bmatrix}}_{N\times N}
\begin{array}{l} 3 \\ N-3 \end{array}
$$

$$
\begin{array}{cc} 3 & 2F-3 \end{array}
$$

Since $Rank(W) \leq 3$, $Rank(\Sigma) \leq 3$.



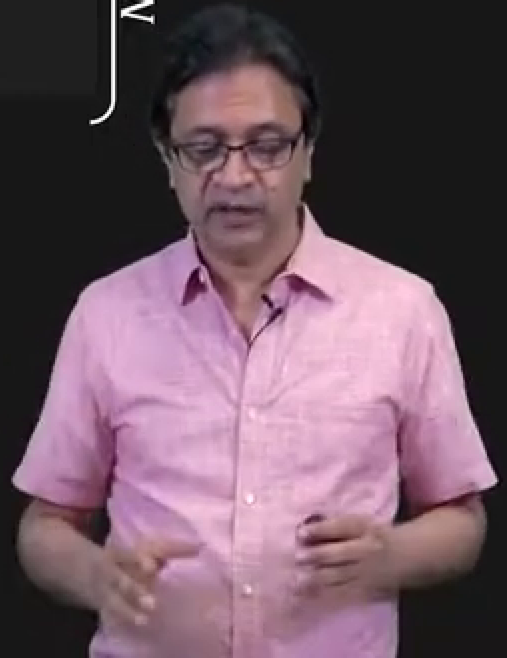
Enforcing Rank Constraint

Using SVD: $\quad W = U\,\Sigma\,V^T$

$$
= \underset{2F\times 2F}{\begin{array}{cc} \left[\begin{matrix} U_1 & U_2 \end{matrix}\right] \\ \;\;3 \quad\; 2F-3 \end{array}}
\underset{2F\times N}{\begin{bmatrix}
\sigma_1 & 0 & 0 & 0 & \dots & 0 \\
0 & \sigma_2 & 0 & 0 & \dots & 0 \\
0 & 0 & \sigma_3 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \dots & \vdots
\end{bmatrix}}
\underset{N\times N}{\begin{array}{cc} \left[\begin{matrix} V_1^T \\ V_2^T \end{matrix}\right] & \begin{matrix} 3 \\ N-3 \end{matrix} \end{array}}
$$

Since $Rank(W) \leq 3$, $Rank(\Sigma) \leq 3$.

Submatrices U_2 and V_2^T do not contribute to W.



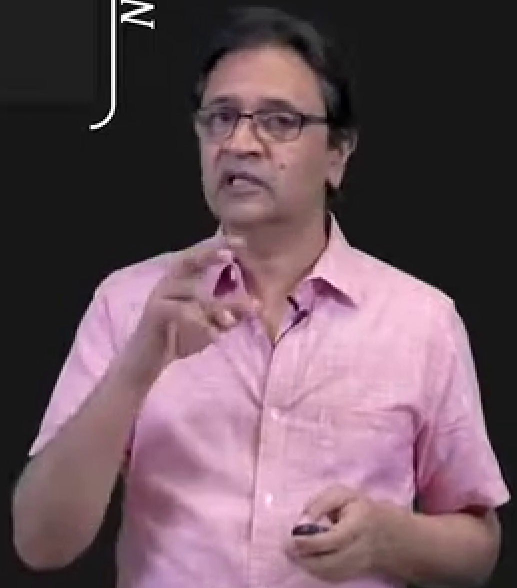
Enforcing Rank Constraint

Using SVD: $W = U\,\Sigma\,V^T$

$$= \underset{2F\times 2F}{\begin{array}{cc} \left(\begin{array}{cc} U_1 & U_2 \end{array}\right) \\ \begin{array}{cc} 3 & 2F-3 \end{array} \end{array}} \; \underset{2F\times N}{\begin{pmatrix} \sigma_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \end{pmatrix}} \; \underset{N\times N}{\begin{array}{cc} \begin{pmatrix} V_1^T \\ V_2^T \end{pmatrix} & \begin{array}{c} 3 \\ N-3 \end{array} \end{array}}$$

Since $Rank(W) \leq 3$, $Rank(\Sigma) \leq 3$.

Submatrices U_2 and V_2^T do not contribute to W.



Enforcing Rank Constraint

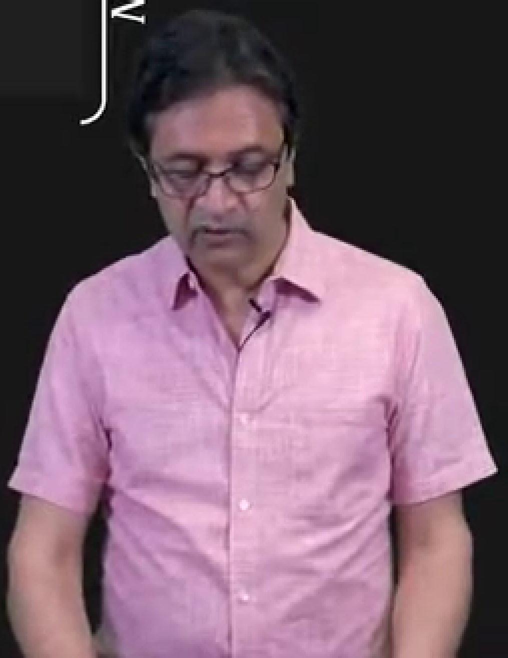
Using SVD: $W = U \Sigma V^T$

$$
= \underbrace{\left[\begin{array}{c|c} U_1 & U_2 \end{array}\right]}_{2F\times 2F}
\underbrace{\begin{bmatrix}
\sigma_1 & 0 & 0 & 0 & \dots & 0 \\
0 & \sigma_2 & 0 & 0 & \dots & 0 \\
0 & 0 & \sigma_3 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
0 & 0 & 0 & 0 & \dots & 0 \\
\vdots & \vdots & \vdots & \vdots & \dots & \vdots
\end{bmatrix}}_{2F\times N}
\underbrace{\begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}}_{N\times N}
\begin{array}{l} 3 \\ N-3 \end{array}
$$

$$\begin{array}{cc} 3 & 2F-3 \end{array}$$

$$\boxed{W = U_1 \; \Sigma_1 \; V_1^T}$$

$(2F\times 3)(3\times 3)(3\times P)$



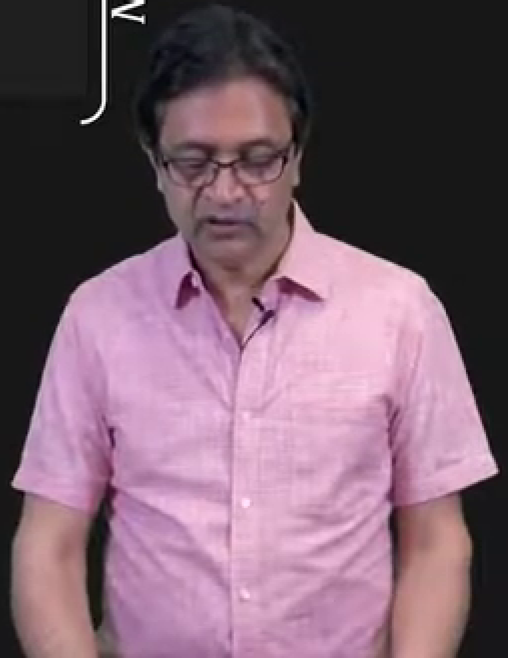
Enforcing Rank Constraint

Using SVD: $$W = U \Sigma V^T$$

$$= \underset{2F\times 2F}{\begin{array}{cc} \left[\begin{array}{c|c} U_1 & U_2 \end{array}\right] \\ \begin{array}{cc} 3 & \quad 2F-3 \end{array} \end{array}} \; \underset{2F\times N}{\begin{bmatrix} \sigma_1 & 0 & 0 & 0 & \dots & 0 \\ 0 & \sigma_2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \sigma_3 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots \end{bmatrix}} \; \underset{N\times N}{\begin{array}{cc} \begin{bmatrix} V_1^T \\ \hline V_2^T \end{bmatrix} & \begin{array}{c} 3 \\ N-3 \end{array} \end{array}}$$

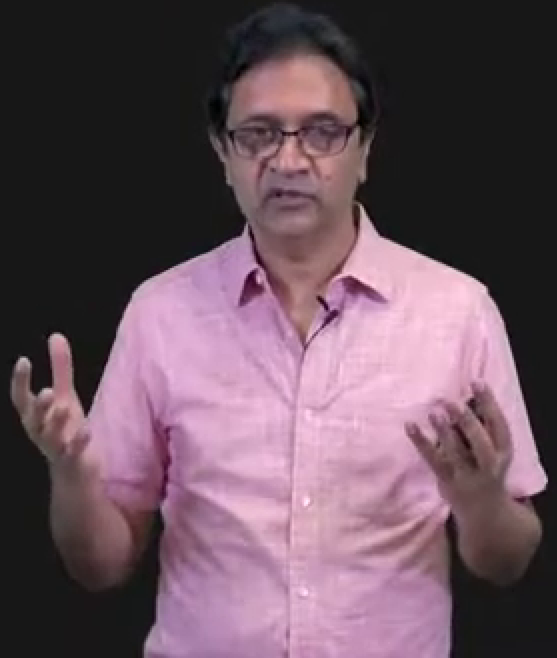
$$\boxed{W = U_1 \; \Sigma_1 \; V_1^T}$$

$$(2F\times 3)(3\times 3)(3\times P)$$



Factorization (Finding M, S)

$$W = U_1 \, (\Sigma_1)^{1/2} \;\; (\Sigma_1)^{1/2} V_1^T$$



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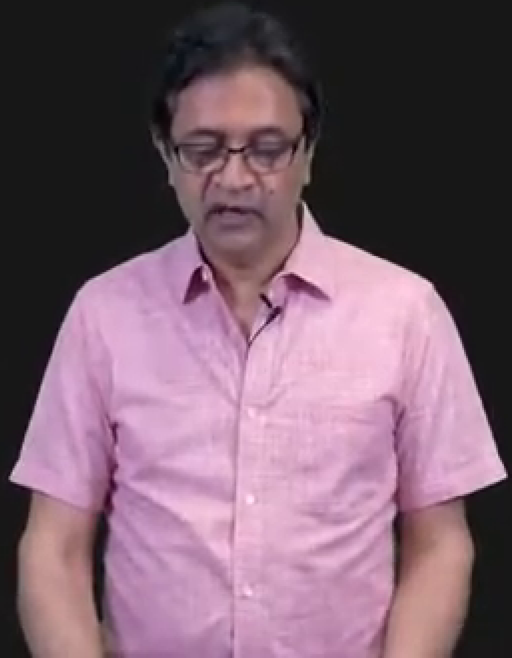
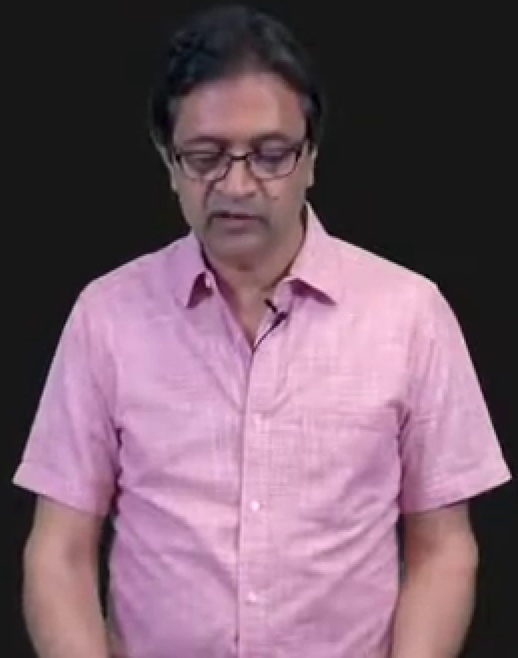


Factorization (Finding M, S)

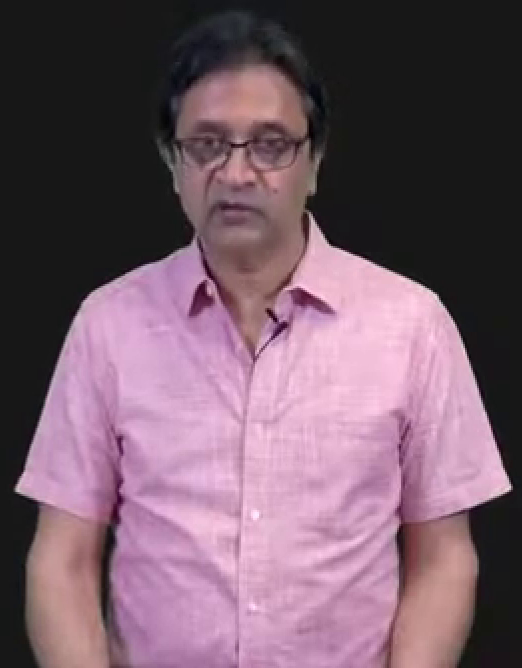
$$W = \underbrace{U_1 (\Sigma_1)^{1/2}}_{(2F\times 3)} \underbrace{(\Sigma_1)^{1/2} V_1^T}_{(3\times N)}$$



Factorization (Finding M, S)

$$W = \underbrace{U_1 (\Sigma_1)^{1/2}}_{(2F\times 3)} \underbrace{(\Sigma_1)^{1/2} V_1^T}_{(3\times N)}$$

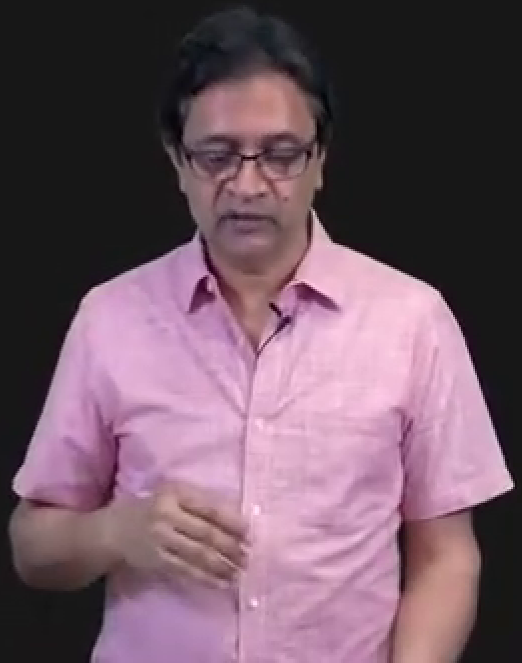
$= M?$ $= S?$



Factorization (Finding M, S)

$$W = \underbrace{U_1 (\Sigma_1)^{1/2}}_{(2F\times 3)} \; \underbrace{(\Sigma_1)^{1/2} V_1^T}_{(3\times N)}$$

$$= M? \qquad = S?$$



Factorization (Finding M, S)

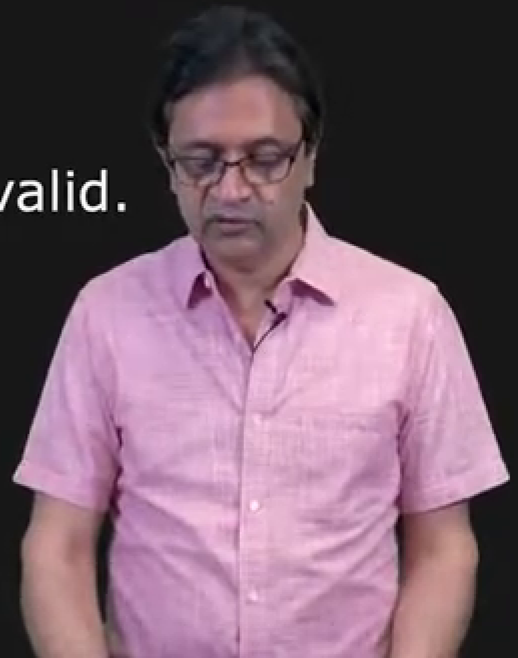
$$W = \underbrace{U_1 (\Sigma_1)^{1/2}}_{(2F\times3)} \underbrace{(\Sigma_1)^{1/2} V_1^T}_{(3\times N)}$$

$$= M? \qquad = S?$$

Not so fast. Decomposition not unique!

For any 3x3 non-singular matrix Q:

$$W = \underbrace{U_1 (\Sigma_1)^{1/2} Q}_{(2F\times3)} \underbrace{Q^{-1} (\Sigma_1)^{1/2} V_1^T}_{(3\times N)}$$ is also valid.



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$$W = \underbrace{U_1 (\Sigma_1)^{1/2}}_{(2F\times 3)} \; \underbrace{(\Sigma_1)^{1/2} V_1^T}_{(3\times N)}$$

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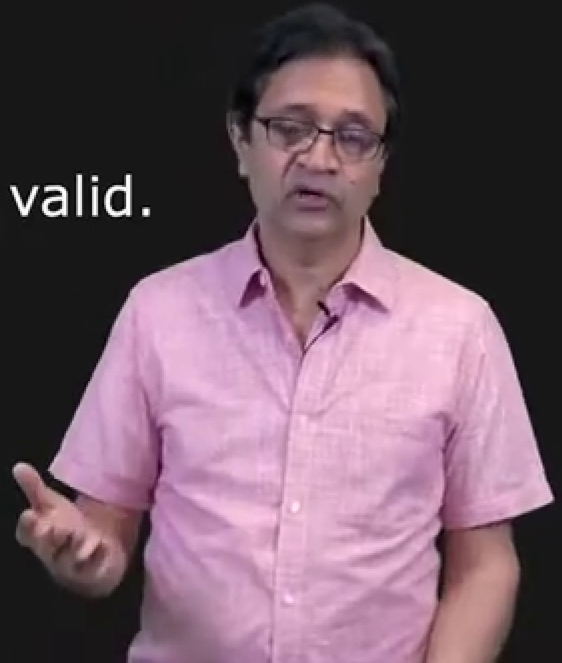

Factorization (Finding M, S)

$$W = \underbrace{U_1 (\Sigma_1)^{1/2}}_{(2F \times 3)} \underbrace{(\Sigma_1)^{1/2} V_1^T}_{(3 \times N)}$$

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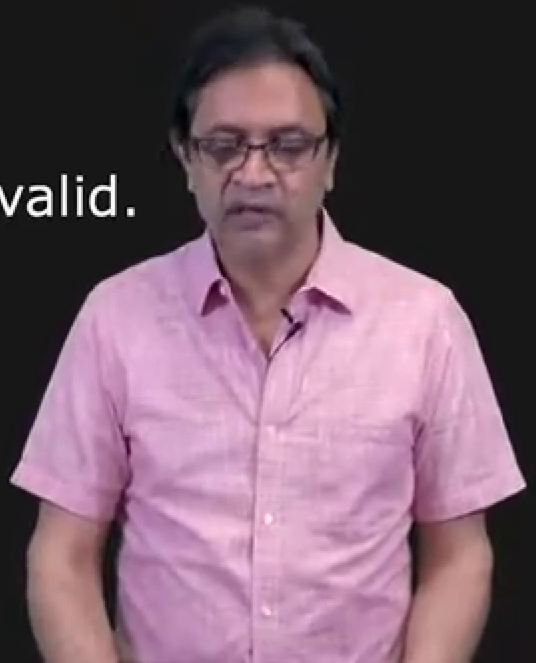

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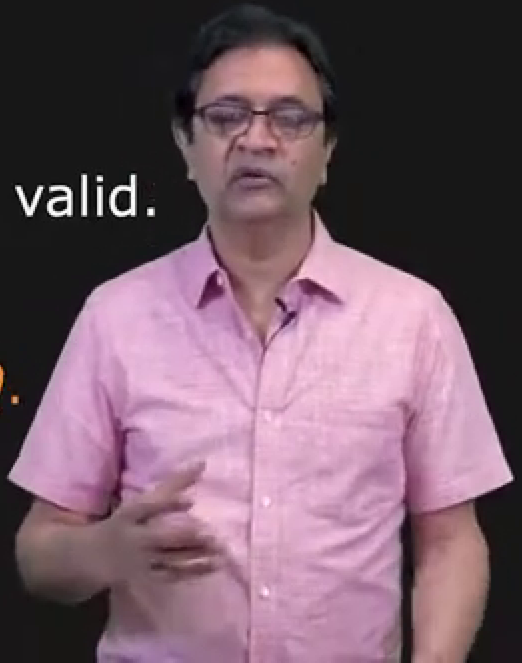
$= M$ $\quad\quad = S$ ... for some Q.

How to find the matrix Q ?



Factorization (Finding M, S)

$$W = \underbrace{U_1 \, (\Sigma_1)^{1/2}}_{(2F\times 3)} \; \underbrace{(\Sigma_1)^{1/2} V_1^T}_{(3\times N)}$$

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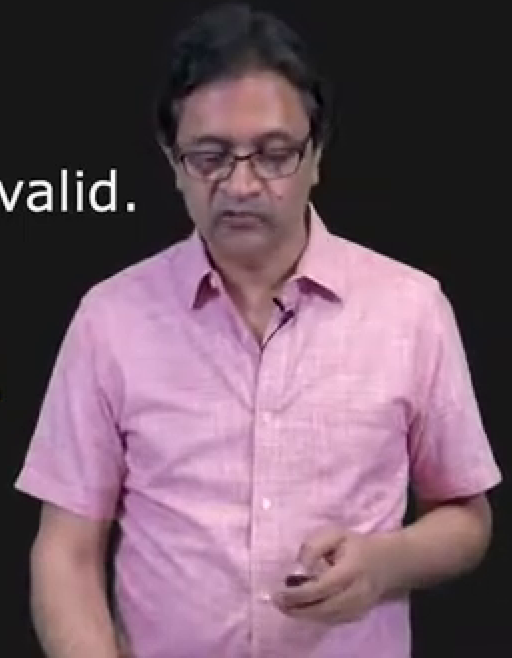
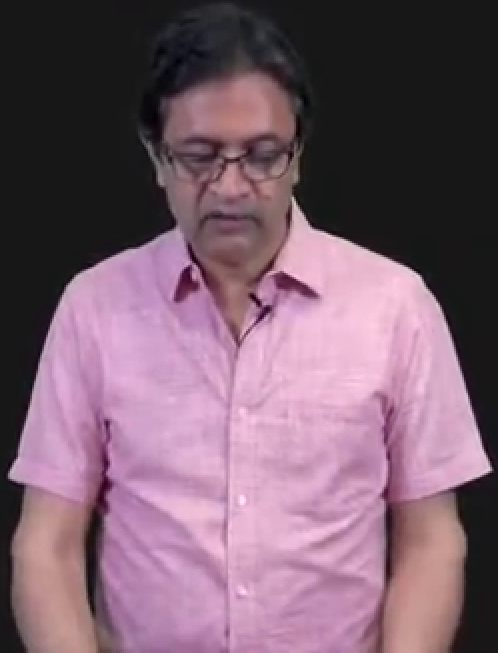


Orthonormality of M

The Motion Matrix M:

$$M = \begin{bmatrix} \mathbf{i}_1^T \\ \vdots \\ \mathbf{i}_F^T \\ \mathbf{j}_1^T \\ \vdots \\ \mathbf{j}_F^T \end{bmatrix} = \underbrace{U_1 (\Sigma_1)^{1/2}}_{\text{Computed}} Q$$



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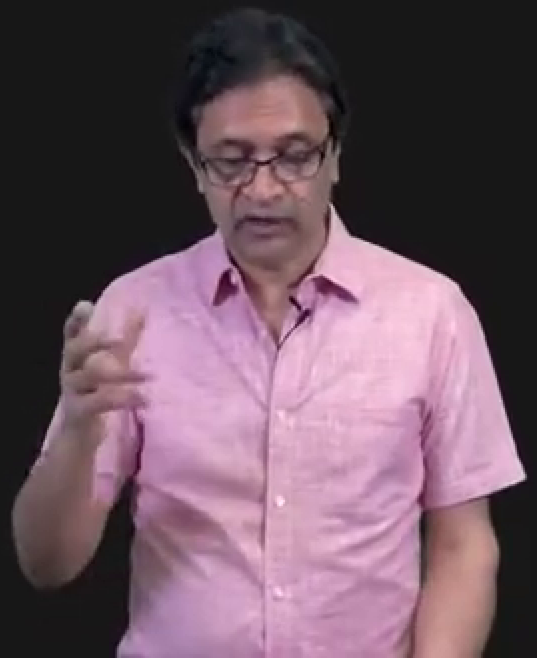

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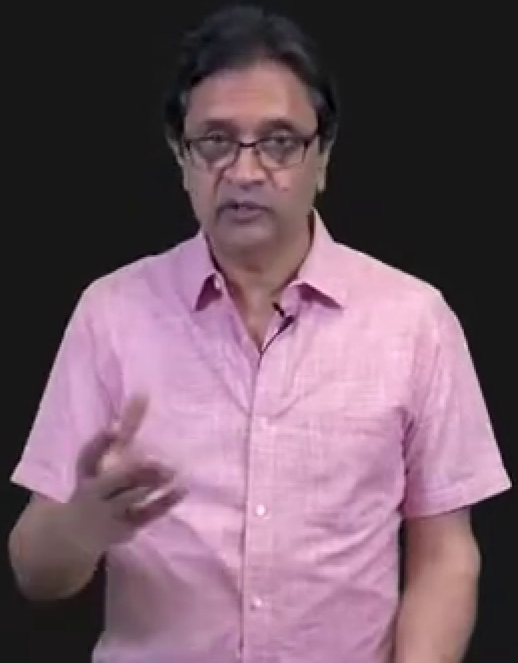

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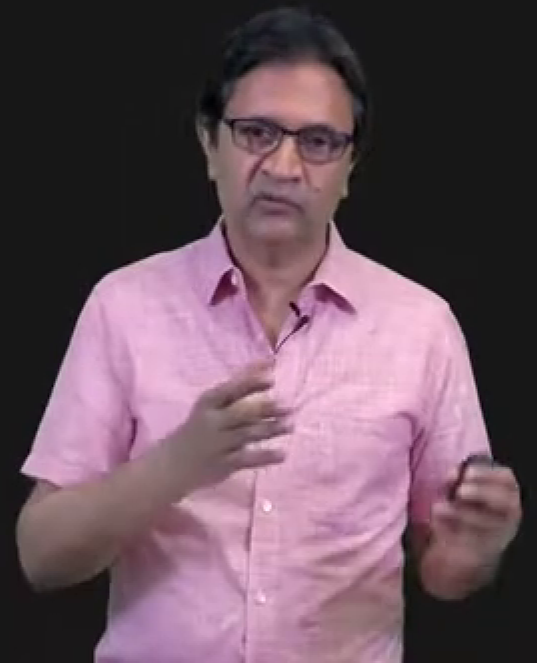

Orthonormality of M

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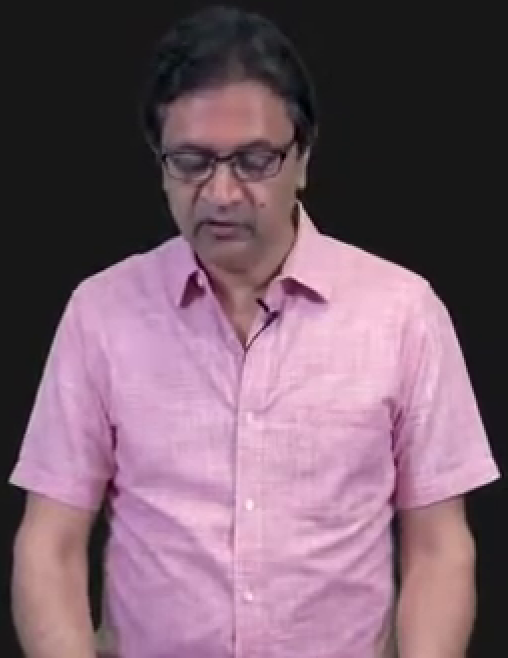
$$M = \begin{bmatrix} \mathbf{i}_1^T \\ \vdots \\ \mathbf{i}_F^T \\ \mathbf{j}_1^T \\ \vdots \\ \mathbf{j}_F^T \end{bmatrix} = \underbrace{\boxed{U_1 (\Sigma_1)^{1/2}}}_{\text{Computed}} Q = \underbrace{\boxed{\begin{bmatrix} \hat{\mathbf{i}}_1^T \\ \vdots \\ \hat{\mathbf{i}}_F^T \\ \hat{\mathbf{j}}_1^T \\ \vdots \\ \hat{\mathbf{j}}_F^T \end{bmatrix}}}_{\text{Computed}} Q$$



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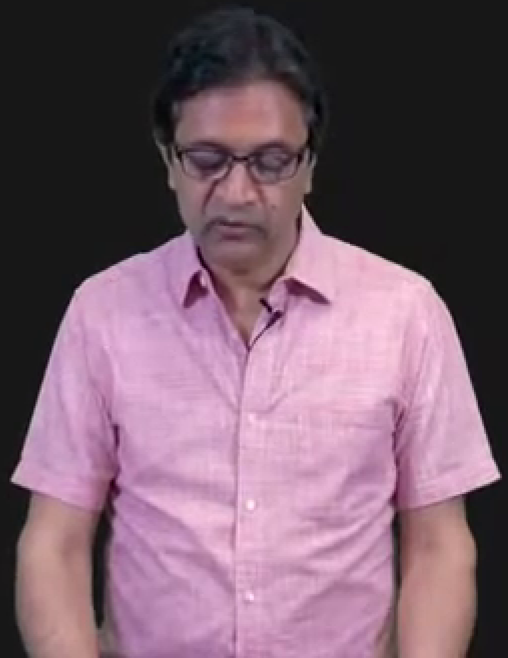
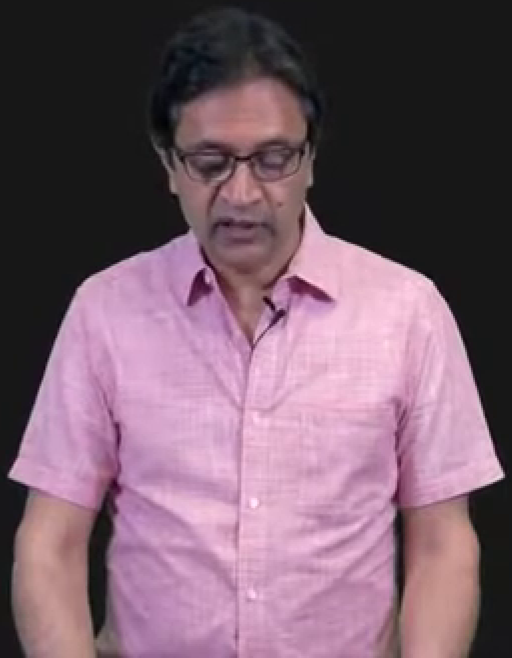


Orthonormality of M

The Motion Matrix M:

$$M = \begin{bmatrix} \mathbf{i}_1^T \\ \vdots \\ \mathbf{i}_F^T \\ \mathbf{j}_1^T \\ \vdots \\ \mathbf{j}_F^T \end{bmatrix} = \underbrace{\boxed{U_1 (\Sigma_1)^{1/2}}}_{\text{Computed}} Q = \underbrace{\boxed{\begin{bmatrix} \hat{\mathbf{i}}_1^T \\ \vdots \\ \hat{\mathbf{i}}_F^T \\ \hat{\mathbf{j}}_1^T \\ \vdots \\ \hat{\mathbf{j}}_F^T \end{bmatrix}}}_{\text{Computed}} Q = \begin{bmatrix} \hat{\mathbf{i}}_1^T Q \\ \vdots \\ \hat{\mathbf{i}}_F^T Q \\ \hat{\mathbf{j}}_1^T Q \\ \vdots \\ \hat{\mathbf{j}}_F^T Q \end{bmatrix}$$



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Orthonormality Constraints:

$$\boxed{\begin{aligned} \mathbf{i}_f \cdot \mathbf{i}_f &= \mathbf{i}_f^T \mathbf{i}_f = 1 \\ \mathbf{j}_f \cdot \mathbf{j}_f &= \mathbf{j}_f^T \mathbf{j}_f = 1 \\ \mathbf{i}_f \cdot \mathbf{j}_f &= \mathbf{i}_f^T \mathbf{j}_f = 0 \end{aligned}}$$

Orthonormality of M

The Motion Matrix M:

$$M = \begin{bmatrix} \mathbf{i}_1^T \\ \vdots \\ \mathbf{i}_F^T \\ \mathbf{j}_1^T \\ \vdots \\ \mathbf{j}_F^T \end{bmatrix} = \underbrace{U_1 (\Sigma_1)^{1/2}}_{\text{Computed}} Q = \underbrace{\begin{bmatrix} \hat{\mathbf{i}}_1^T \\ \vdots \\ \hat{\mathbf{i}}_F^T \\ \hat{\mathbf{j}}_1^T \\ \vdots \\ \hat{\mathbf{j}}_F^T \end{bmatrix}}_{\text{Computed}} Q = \begin{bmatrix} \hat{\mathbf{i}}_1^T Q \\ \vdots \\ \hat{\mathbf{i}}_F^T Q \\ \hat{\mathbf{j}}_1^T Q \\ \vdots \\ \hat{\mathbf{j}}_F^T Q \end{bmatrix}$$

Orthonormality Constraints:

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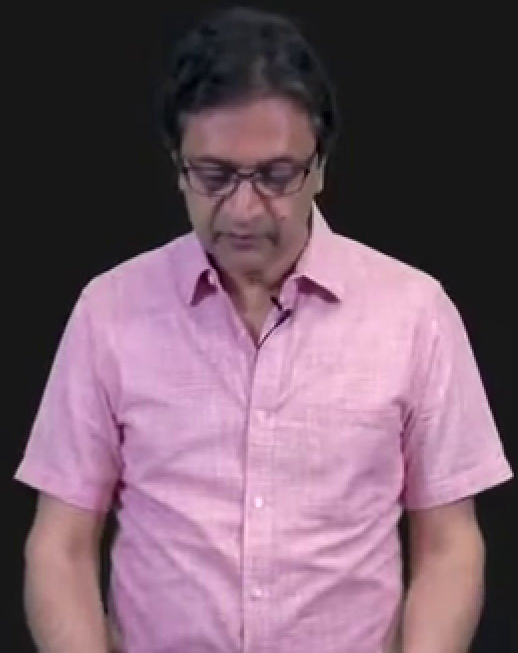


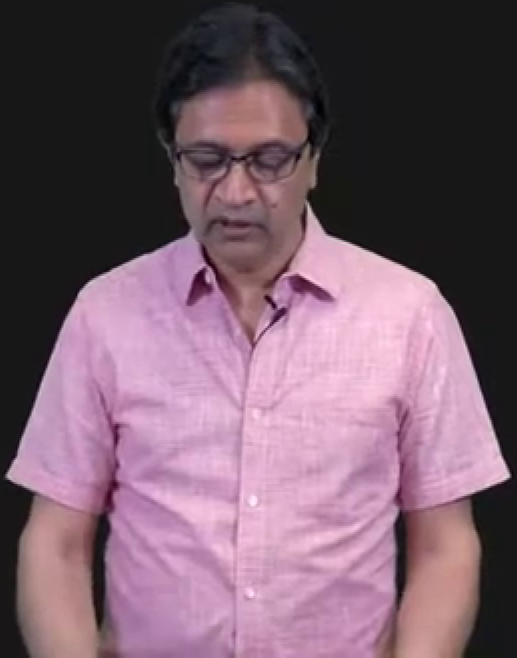
Orthonormality of M

The Motion Matrix M:

$$M = \begin{bmatrix} \mathbf{i}_1^T \\ \vdots \\ \mathbf{i}_F^T \\ \mathbf{j}_1^T \\ \vdots \\ \mathbf{j}_F^T \end{bmatrix} = \underbrace{U_1(\Sigma_1)^{1/2}}_{\text{Computed}} Q = \underbrace{\begin{bmatrix} \hat{\mathbf{i}}_1^T \\ \vdots \\ \hat{\mathbf{i}}_F^T \\ \hat{\mathbf{j}}_1^T \\ \vdots \\ \hat{\mathbf{j}}_F^T \end{bmatrix}}_{\text{Computed}} Q = \begin{bmatrix} \hat{\mathbf{i}}_1^T Q \\ \vdots \\ \hat{\mathbf{i}}_F^T Q \\ \hat{\mathbf{j}}_1^T Q \\ \vdots \\ \hat{\mathbf{j}}_F^T Q \end{bmatrix}$$

Orthonormality Constraints:

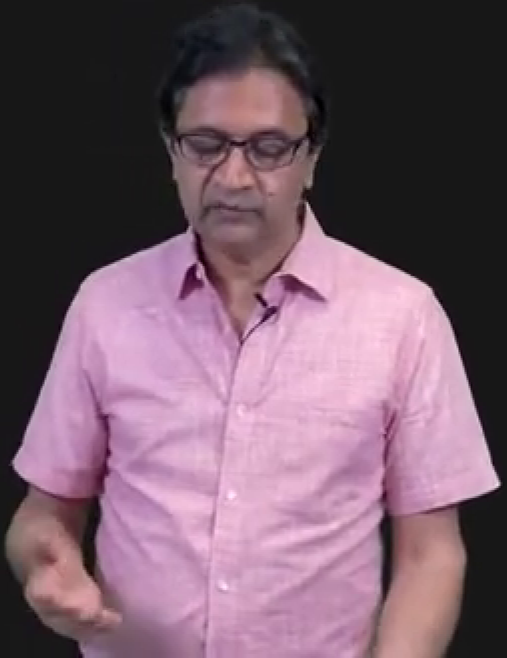
$$\boxed{\begin{aligned} \mathbf{i}_f \cdot \mathbf{i}_f &= \mathbf{i}_f^T \mathbf{i}_f = 1 \\ \mathbf{j}_f \cdot \mathbf{j}_f &= \mathbf{j}_f^T \mathbf{j}_f = 1 \\ \mathbf{i}_f \cdot \mathbf{j}_f &= \mathbf{i}_f^T \mathbf{j}_f = 0 \end{aligned}} \quad \Rightarrow \quad \boxed{\begin{aligned} \hat{\mathbf{i}}_f^T Q Q^T \hat{\mathbf{i}}_f &= 1 \\ \hat{\mathbf{j}}_f^T Q Q^T \hat{\mathbf{j}}_f &= 1 \\ \hat{\mathbf{i}}_f^T Q Q^T \hat{\mathbf{j}}_f &= 0 \end{aligned}}$$



Orthonormality of M

- We have computed $(\hat{\mathbf{i}}_f^T, \hat{\mathbf{j}}_f^T)$ for $f = 1, \dots, F$.

$$\left.\begin{aligned} \hat{\mathbf{i}}_f^T Q Q^T \hat{\mathbf{i}}_f &= 1 \\ \hat{\mathbf{j}}_f^T Q Q^T \hat{\mathbf{j}}_f &= 1 \\ \hat{\mathbf{i}}_f^T Q Q^T \hat{\mathbf{j}}_f &= 0 \end{aligned}\right\} \; Q \text{ is unknown.}$$



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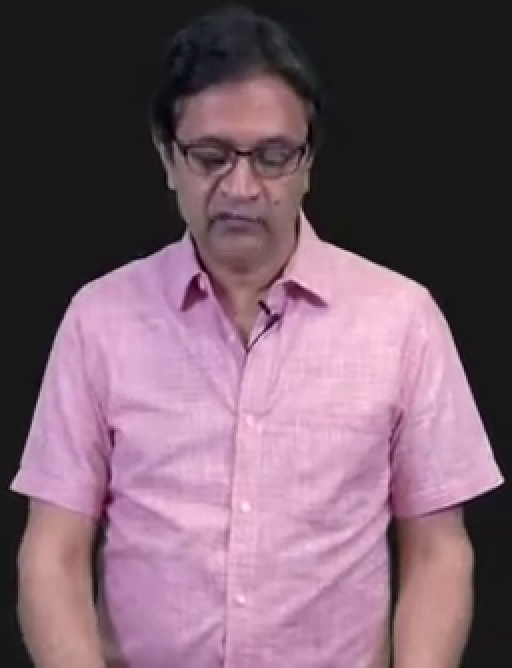

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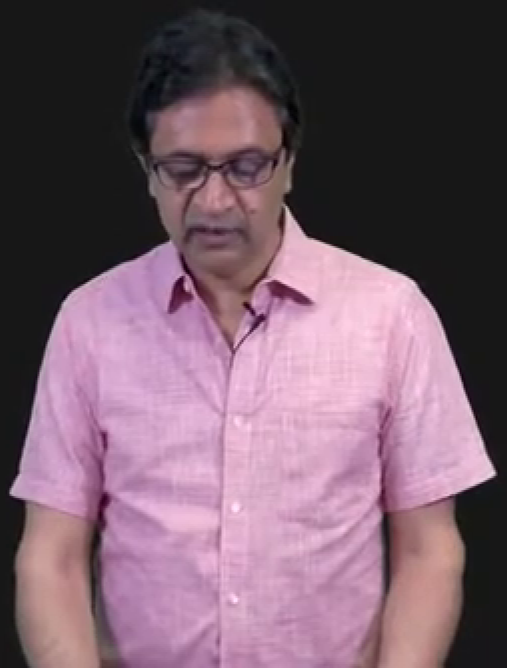

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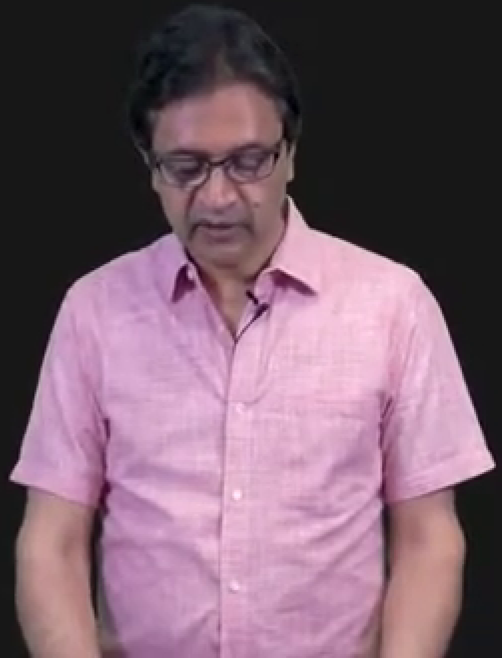
Camera Motion — Scene Structure



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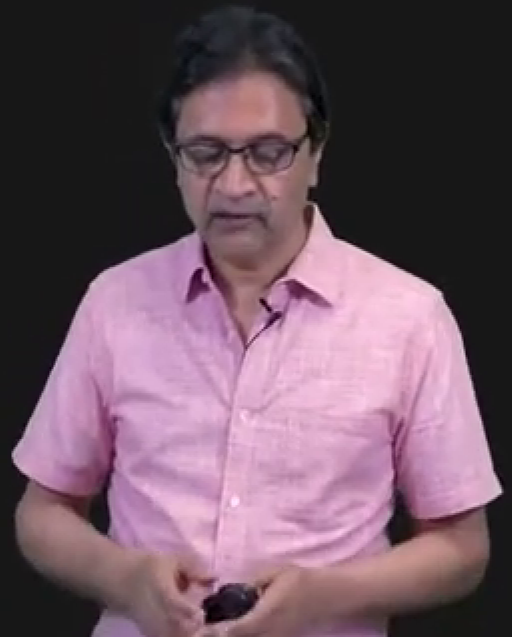
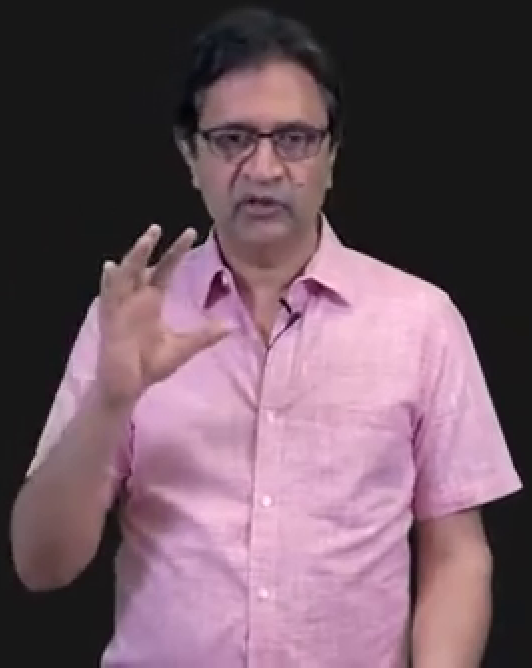


Summary: Orthographic SFM

1. Detect and track feature points.

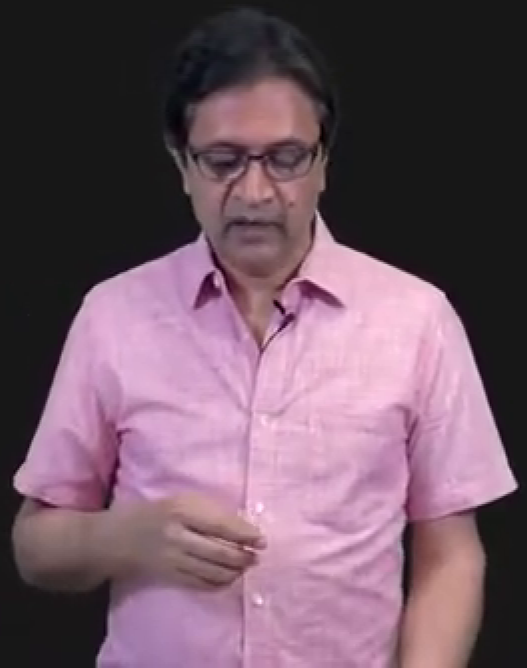


[Tomasi 1992]

Summary: Orthographic SFM

1. Detect and track feature points.

2. Create the centroid subtracted matrix W of corresponding feature points.



[Tomasi 1992]

Summary: Orthographic SFM

1. Detect and track feature points.

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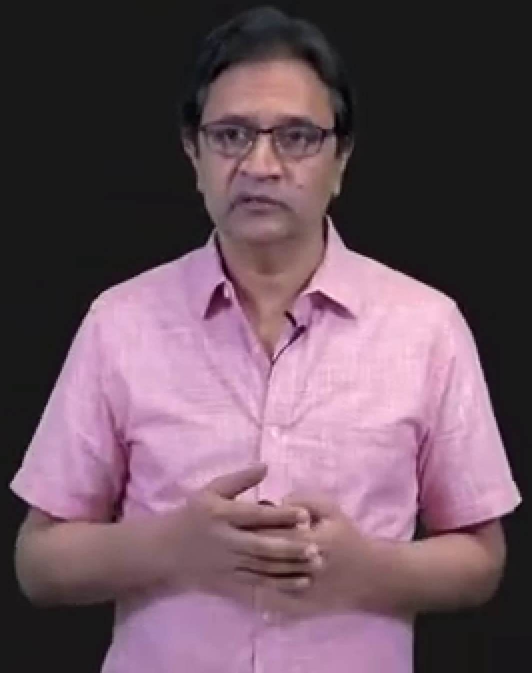
3. Compute SVD of W and enforce rank constraint.

$$W = U\,\Sigma\,V^T = \underset{(2F\times 3)}{U_1}\,\underset{(3\times 3)}{\Sigma_1}\,\underset{(3\times P)}{V_1^T}$$



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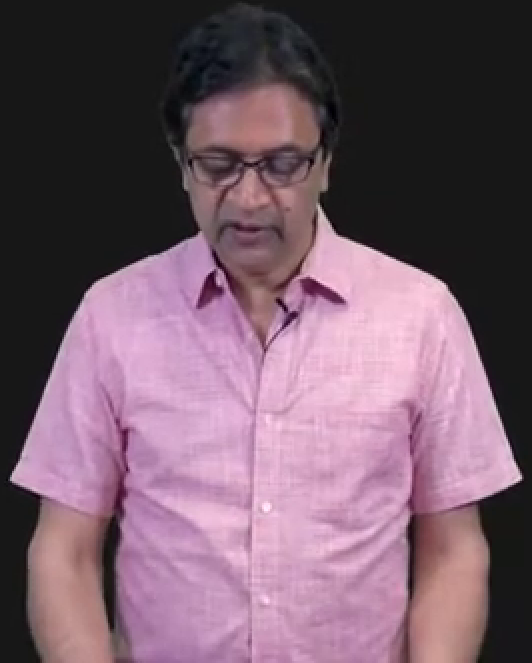

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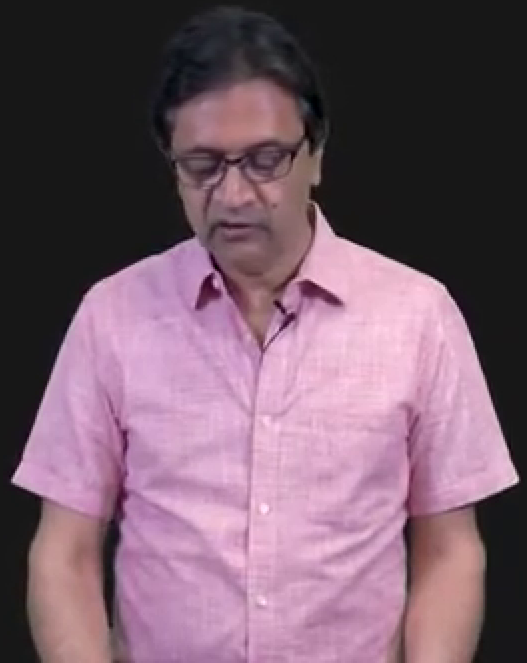

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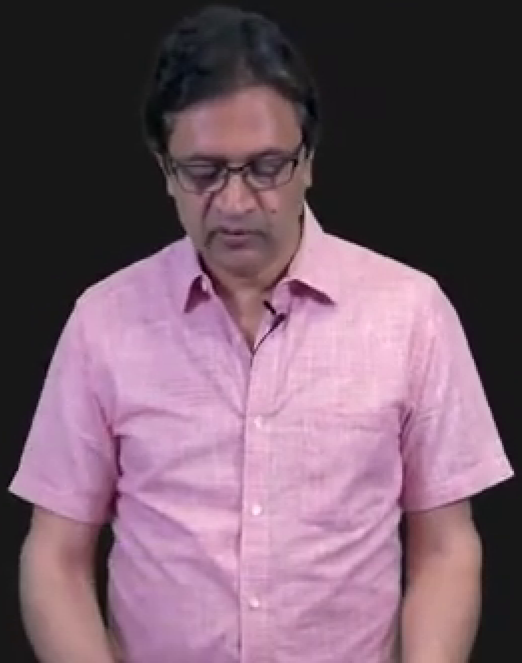
5. Find Q by enforcing the orthonormality constraint.

[Tomasi 1992]



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[Tomasi 1992]

Results

Input Image Sequence

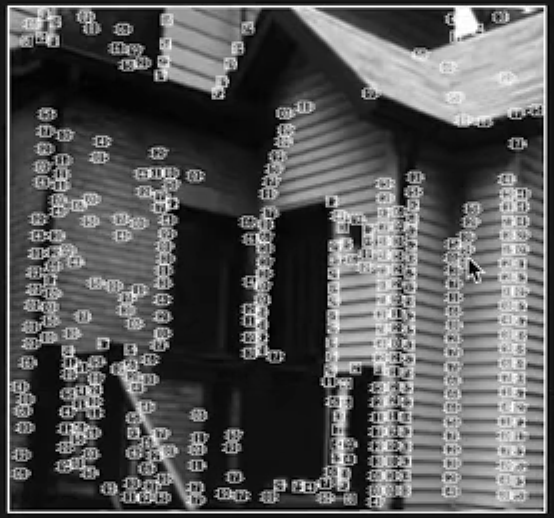
Results

Input Image Sequence

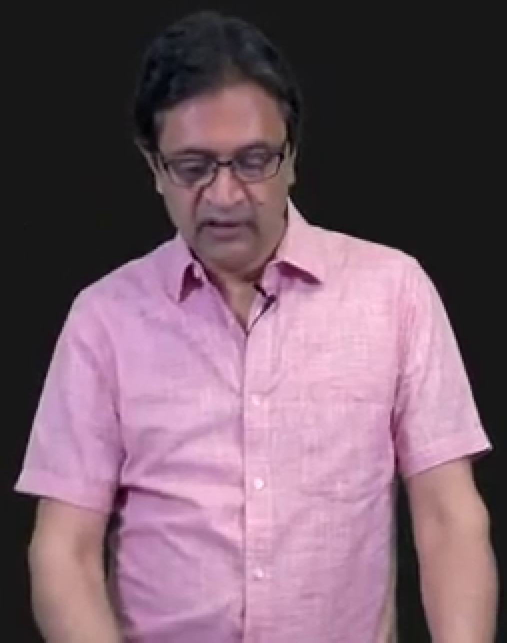
Estimated 3D Points

Results

Input Image Sequence



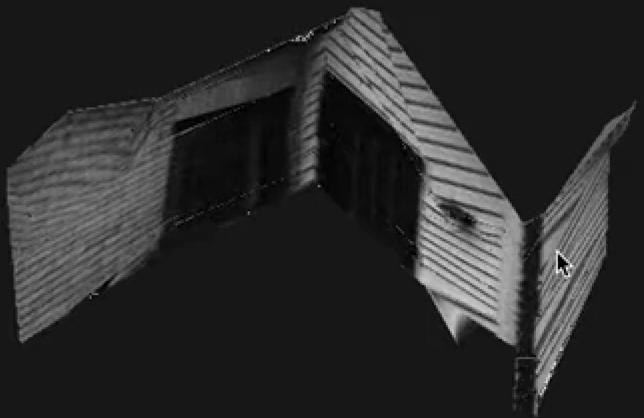
Tracked Features



Results

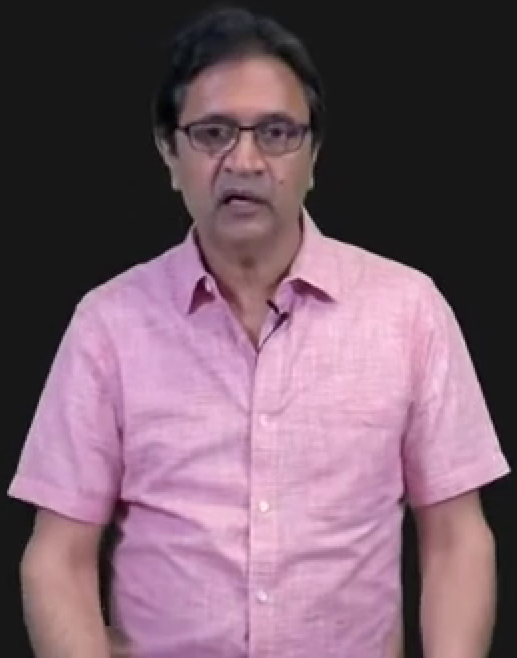
Input Image Sequence

Tracked Features



3D Reconstruction

3D Reconstruction



Results

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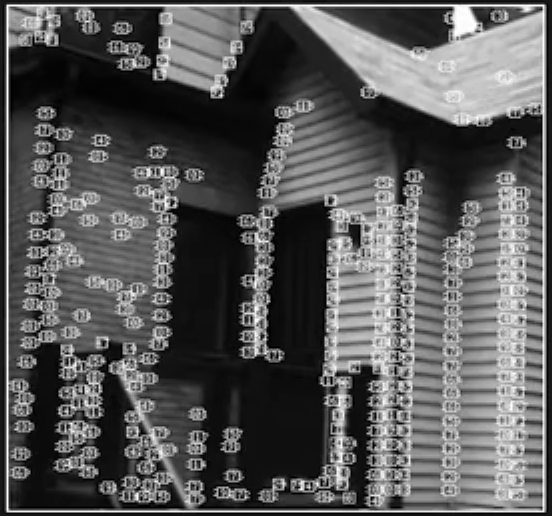
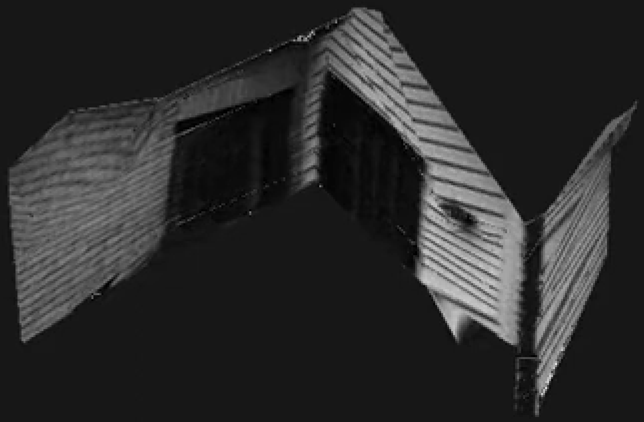


Tracked Features



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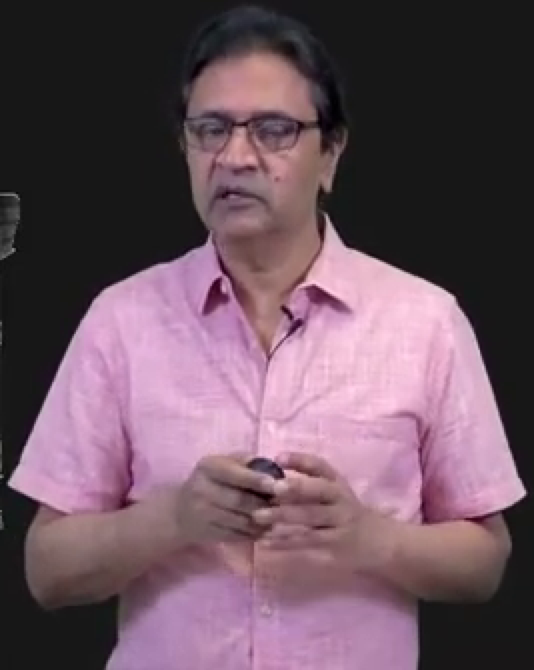

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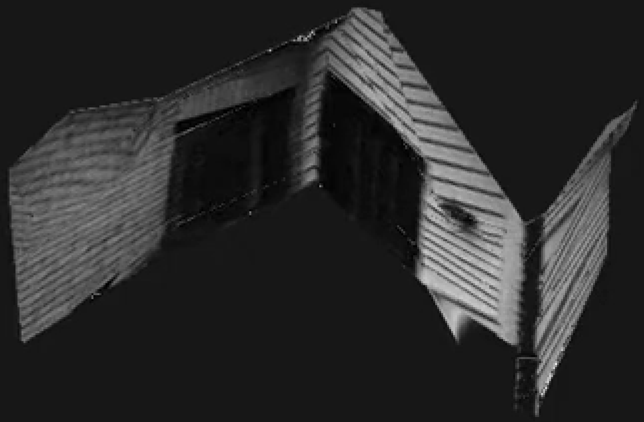

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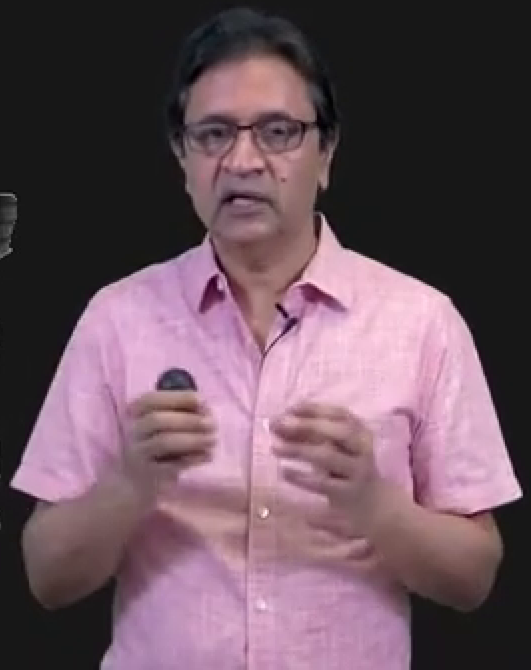

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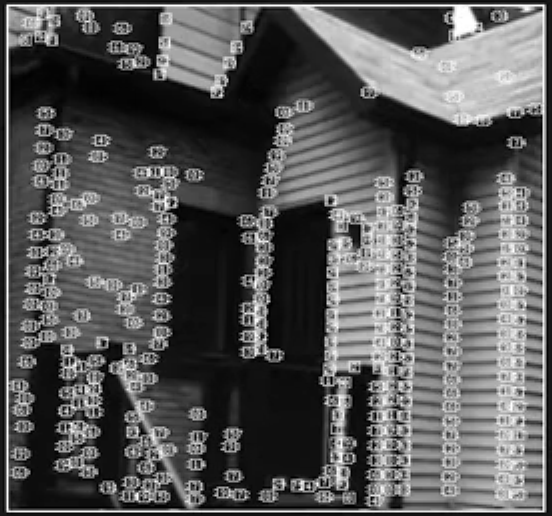

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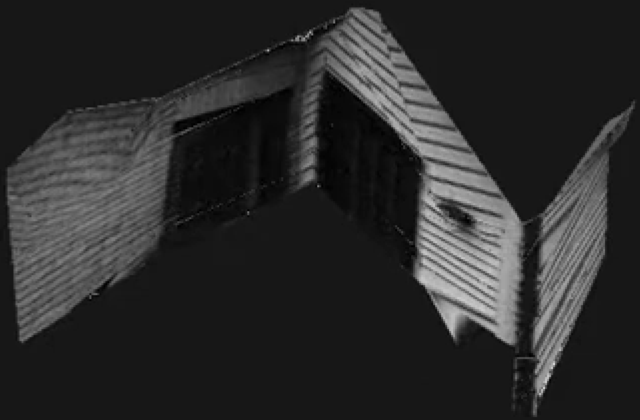

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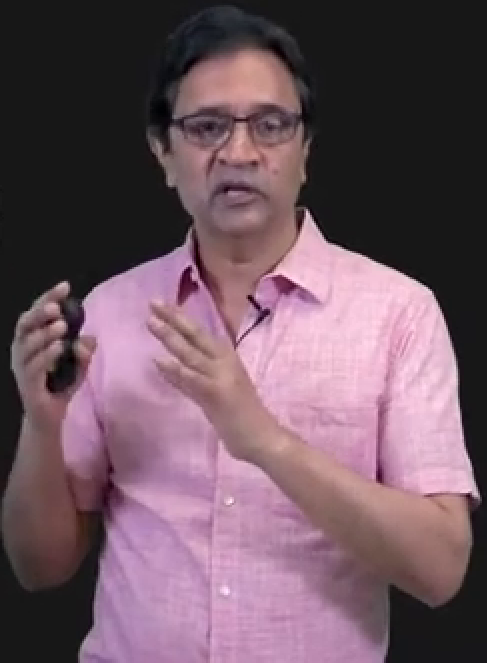
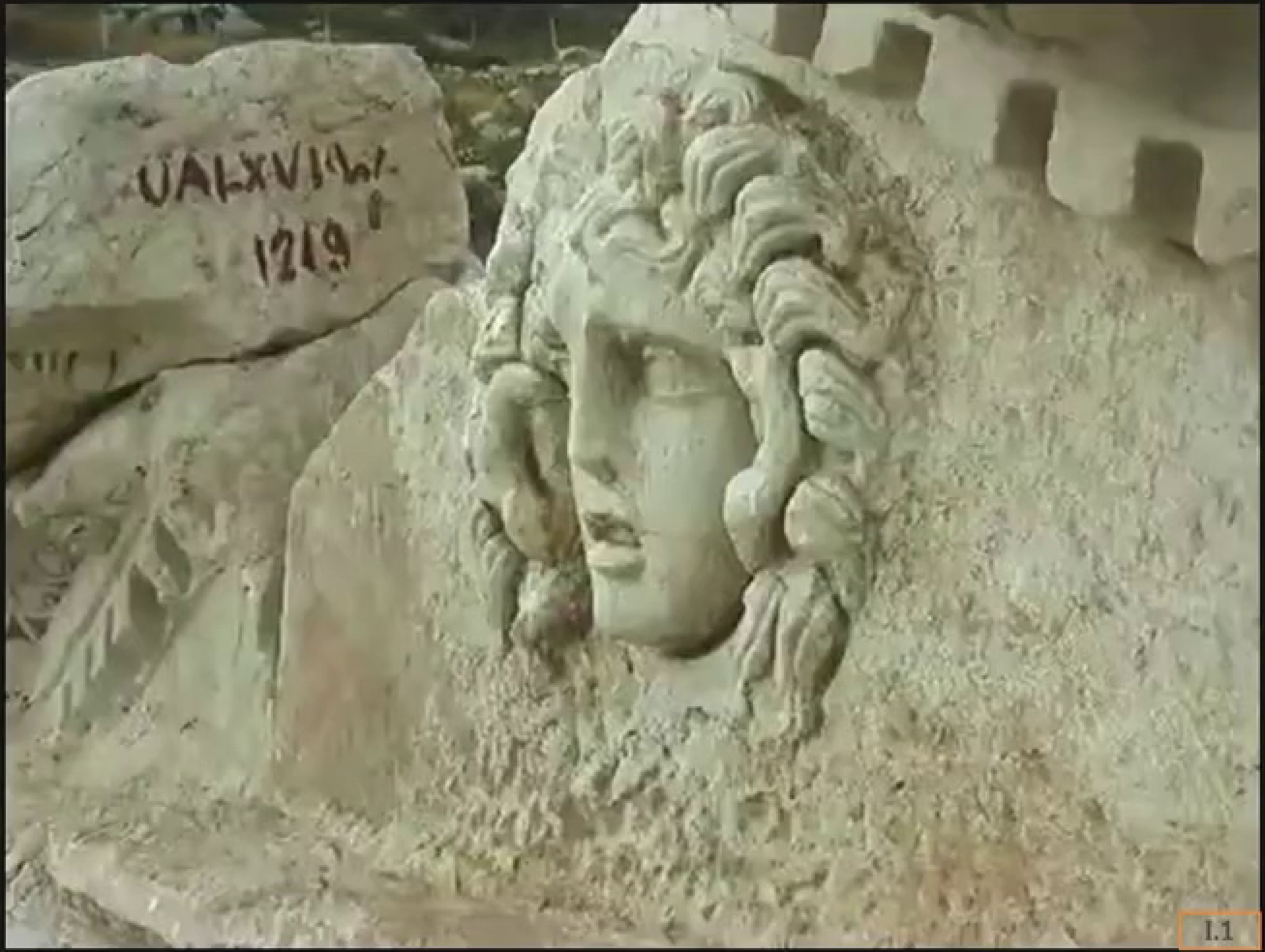


Structure From Motion: Result



[Pollefeys 2002]

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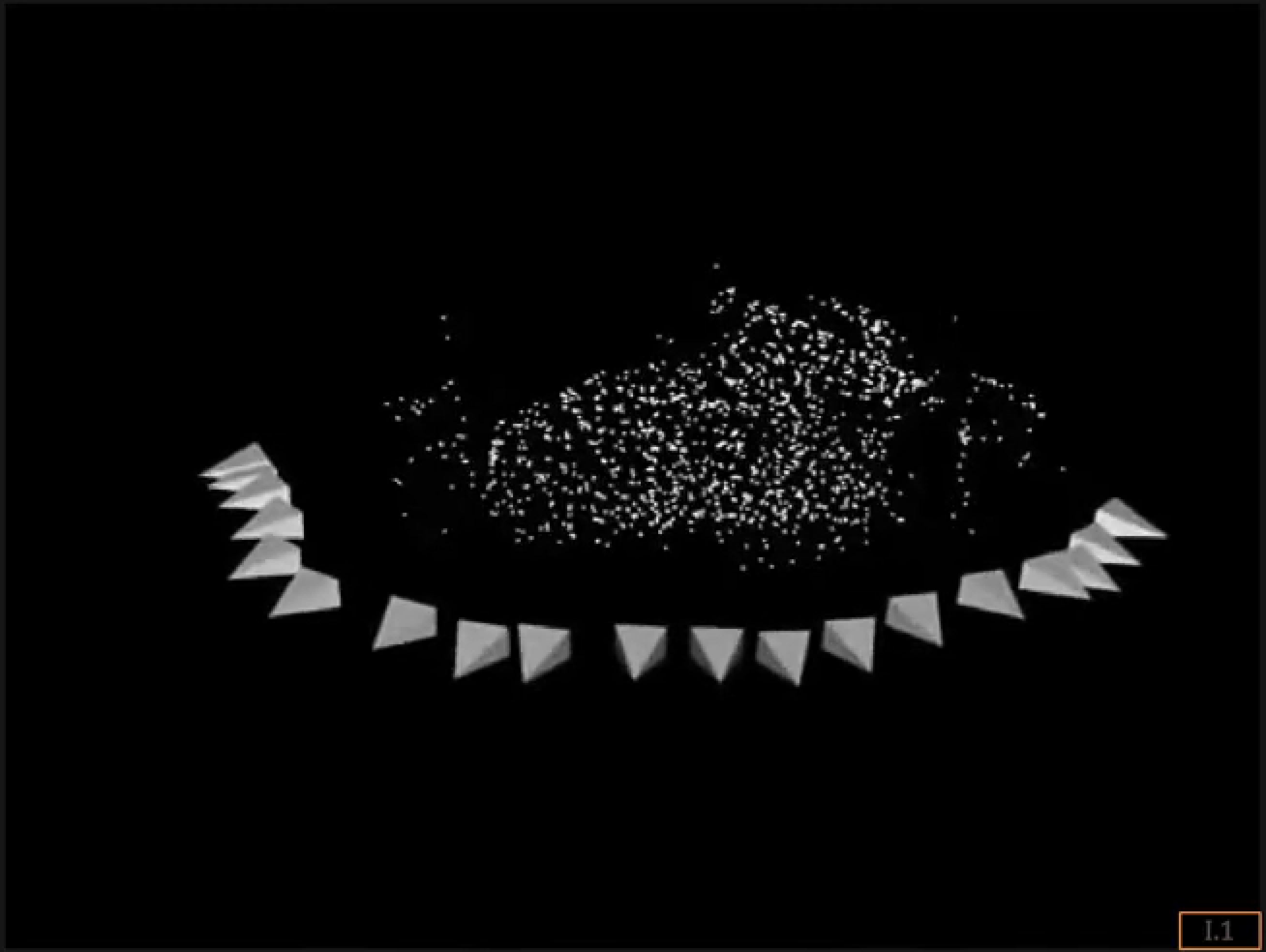

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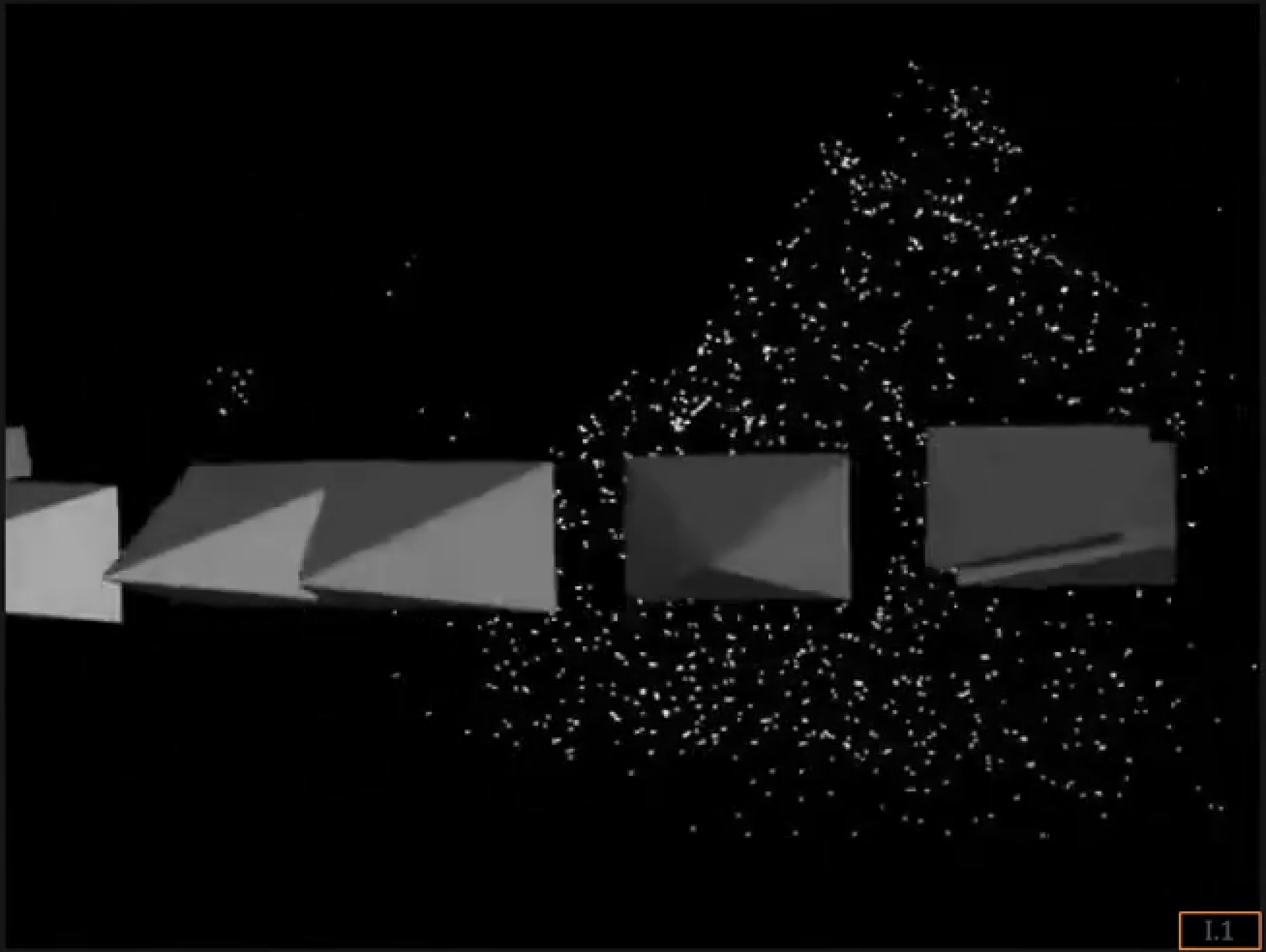

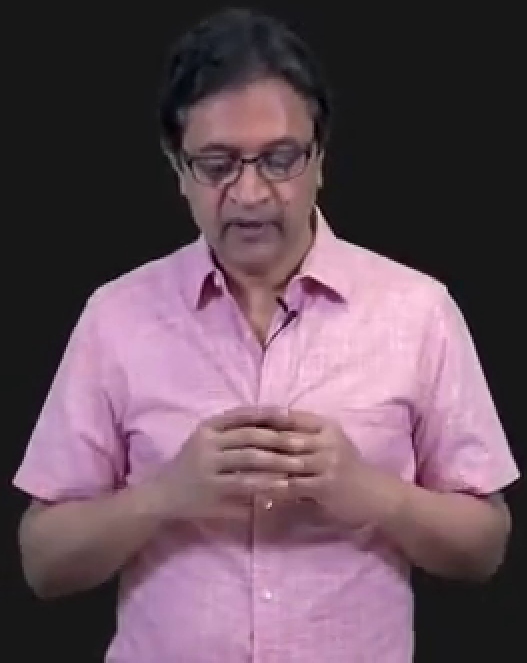

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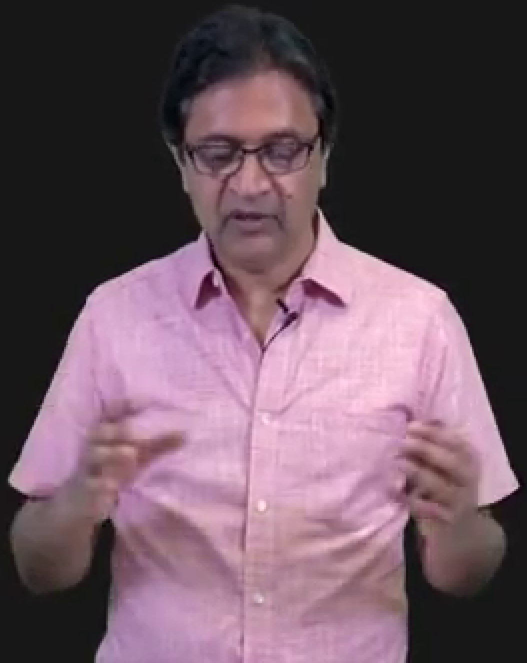

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References: Papers

[Tomasi 1992] Tomasi, C. and Kanade, T. “Shape and Motion from Image Streams under Orthography: A Factorization Method” IJCV, 1992.

[Pollefeys 2002] M. Pollefeys and L. Van Gool. Visual modeling: from images to images, The Journal of Visualization and Computer Animation, 13: 199-209, 2002.