Intrinsic and Extrinsic Matrices

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Columbia University

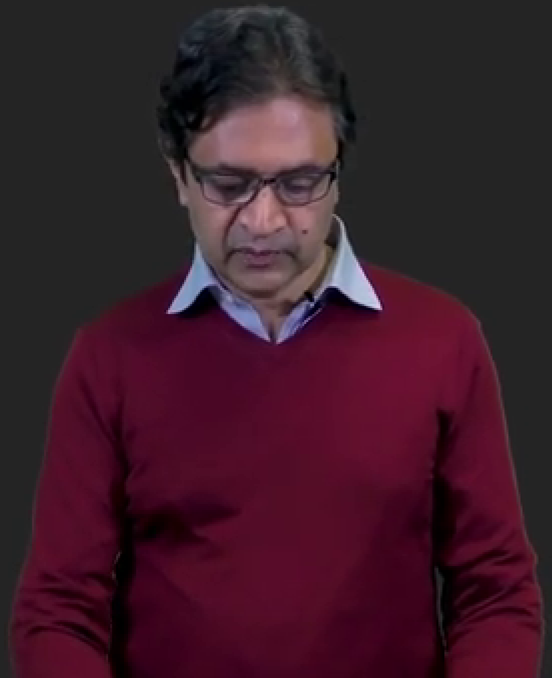
Topic: Camera Calibration, Module: Reconstruction II

First Principles of Computer Vision

Extracting Intrinsic/Extrinsic Parameters

We know that:

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & 0 & o_x & 0 \\ 0 & f_y & o_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{M_{int}} \underbrace{\begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{M_{ext}}$$



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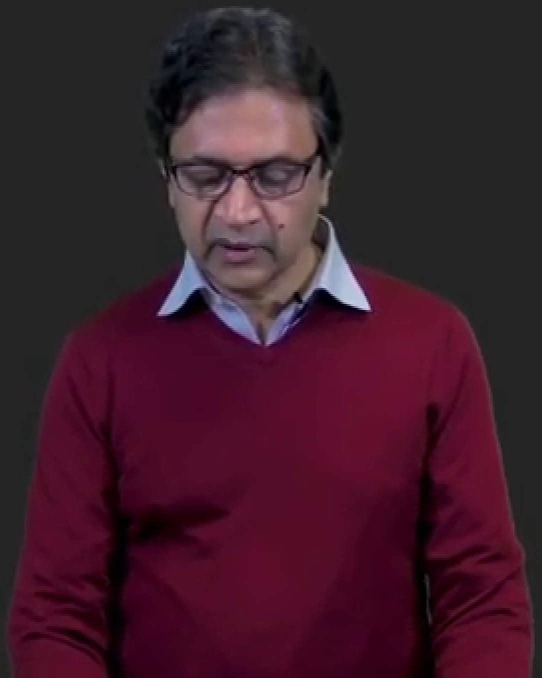

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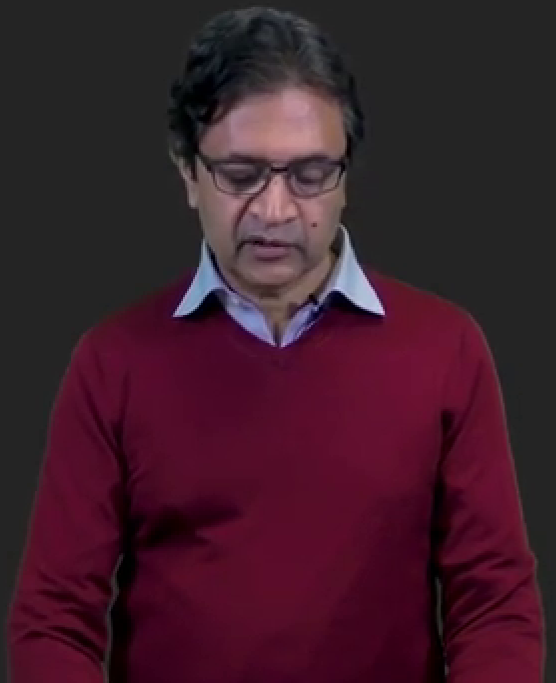

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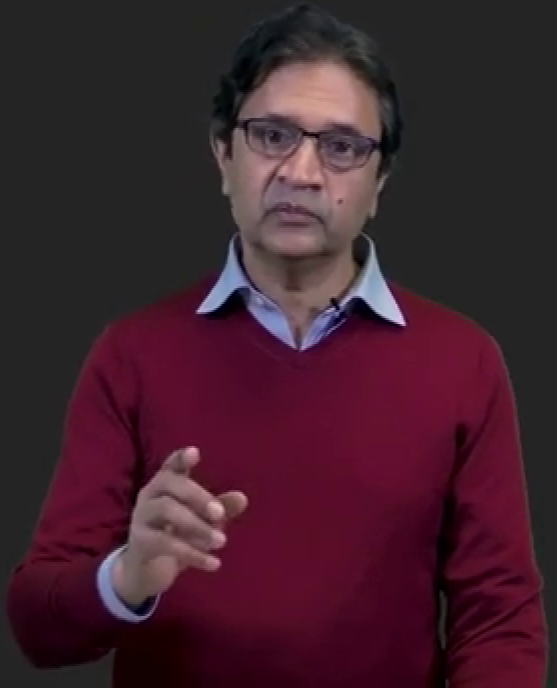

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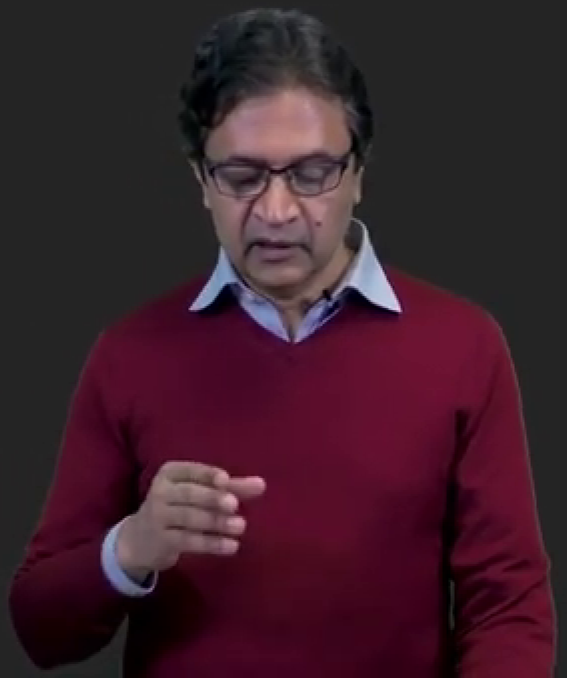

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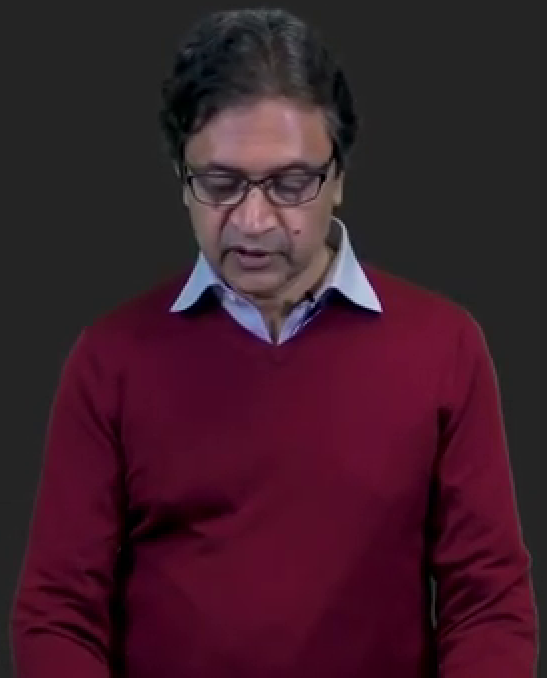
Given that K is an Upper Right Triangular matrix and R is an Orthonormal matrix, it is possible to uniquely "decouple" K and R from their product using "QR factorization".



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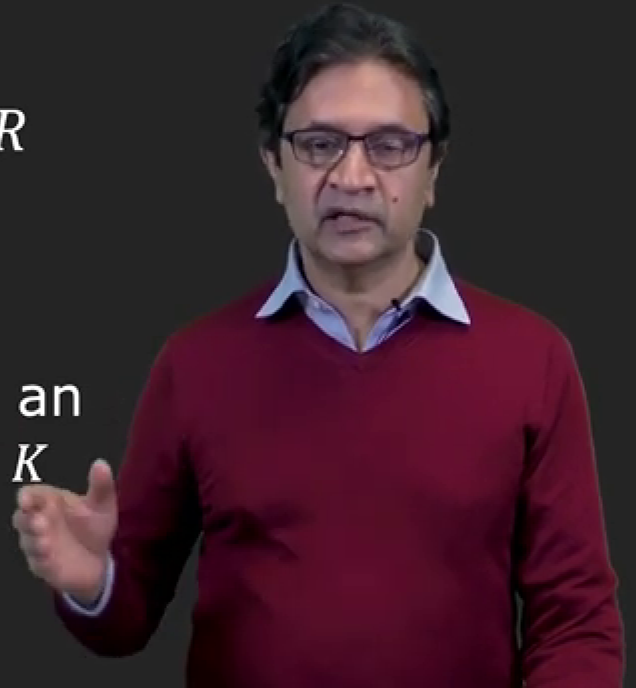

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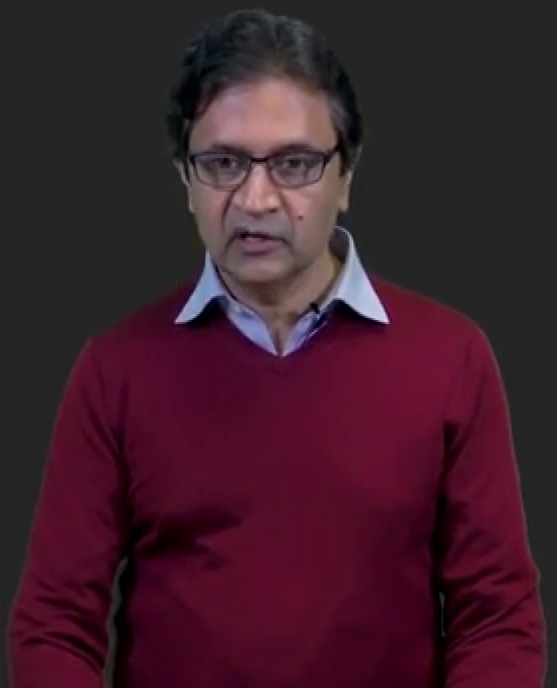

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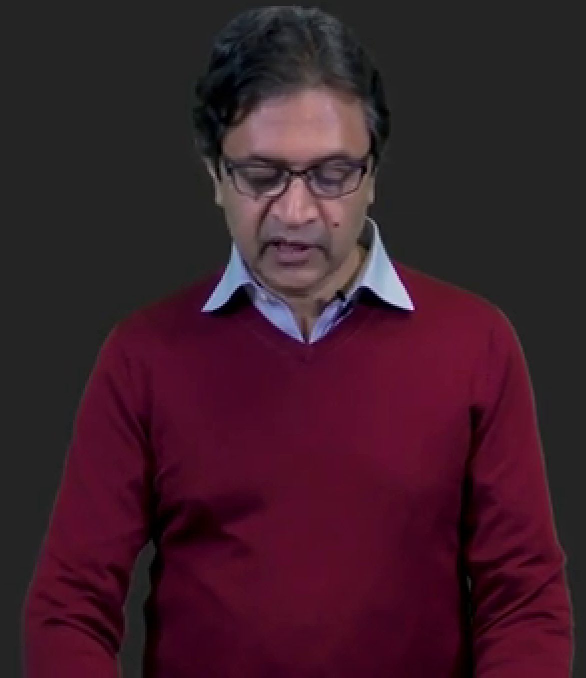

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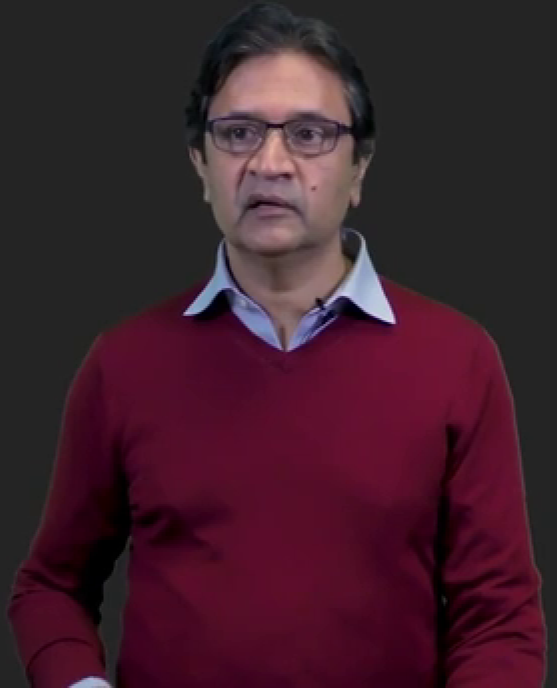

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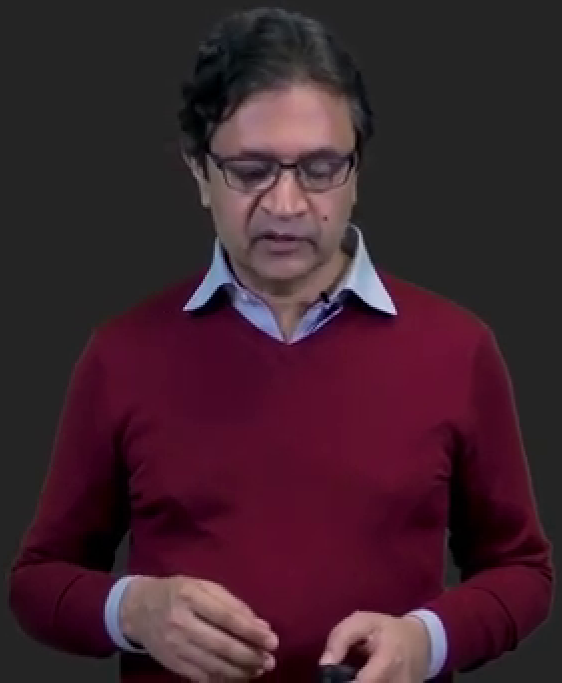
Other Intrinsic Parameters

Pinholes do not exhibit image distortions. But, lenses do!



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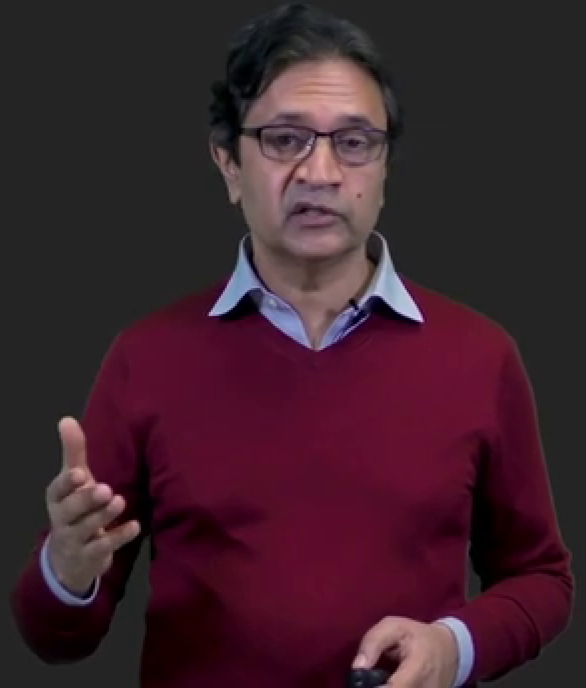

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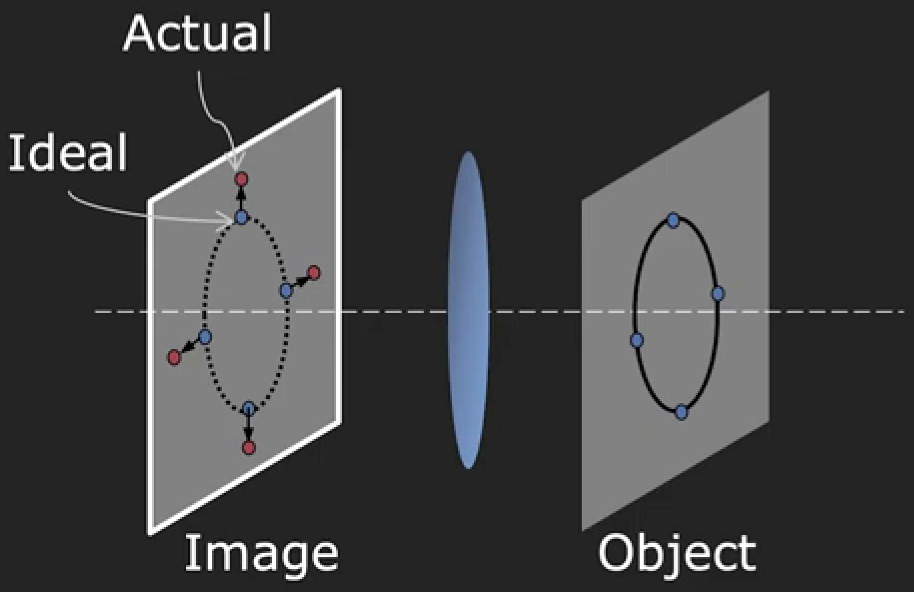
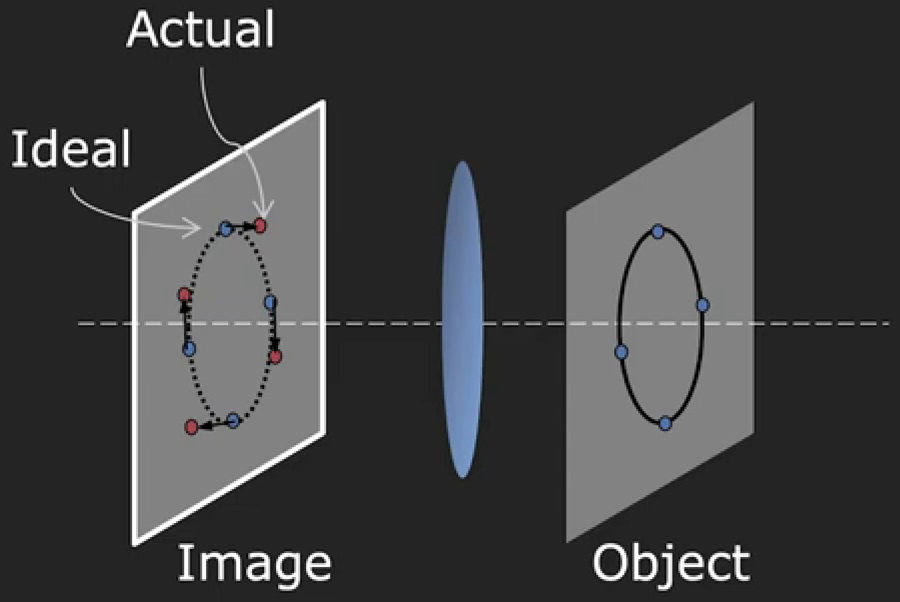
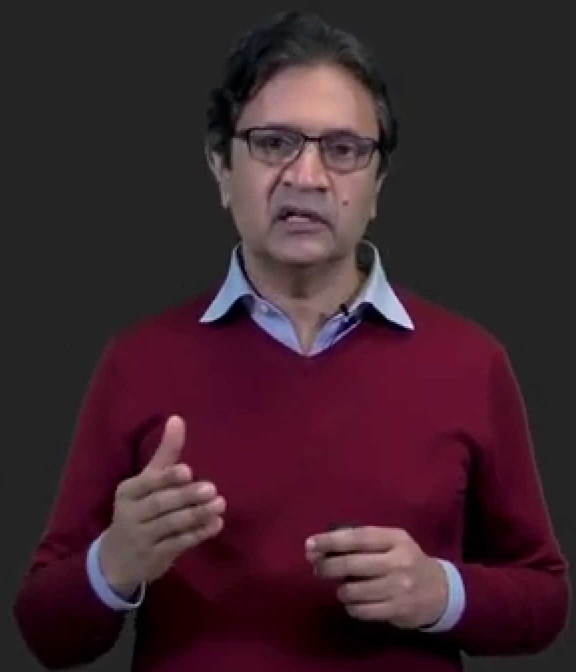


Radial Distortion



Tangential Distortion



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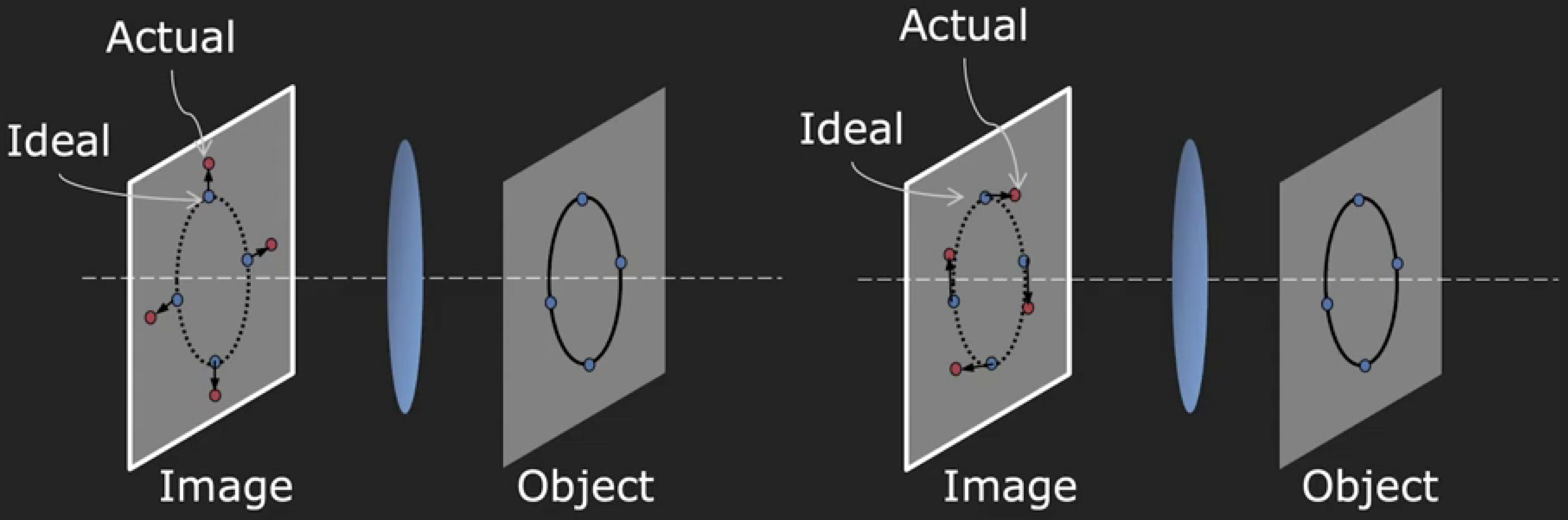

Radial Distortion　　Tangential Distortion

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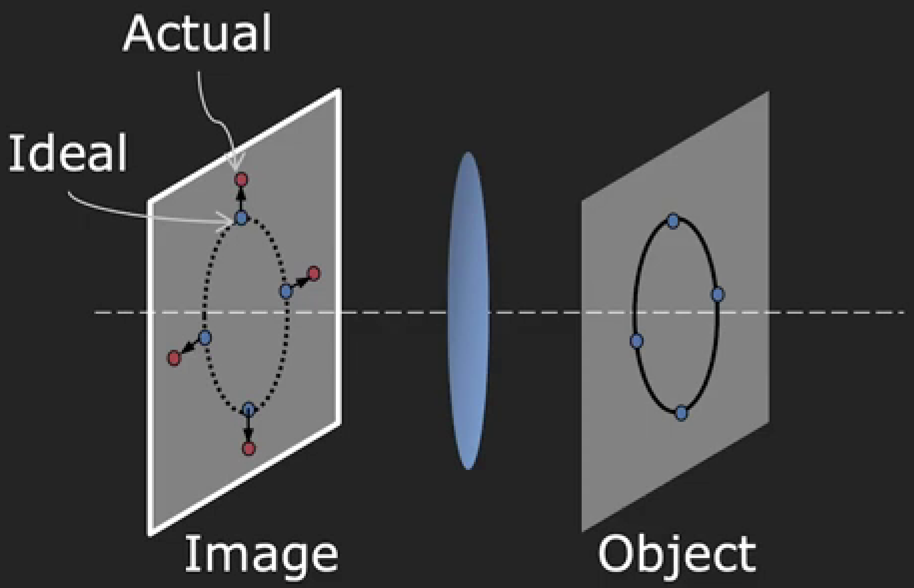

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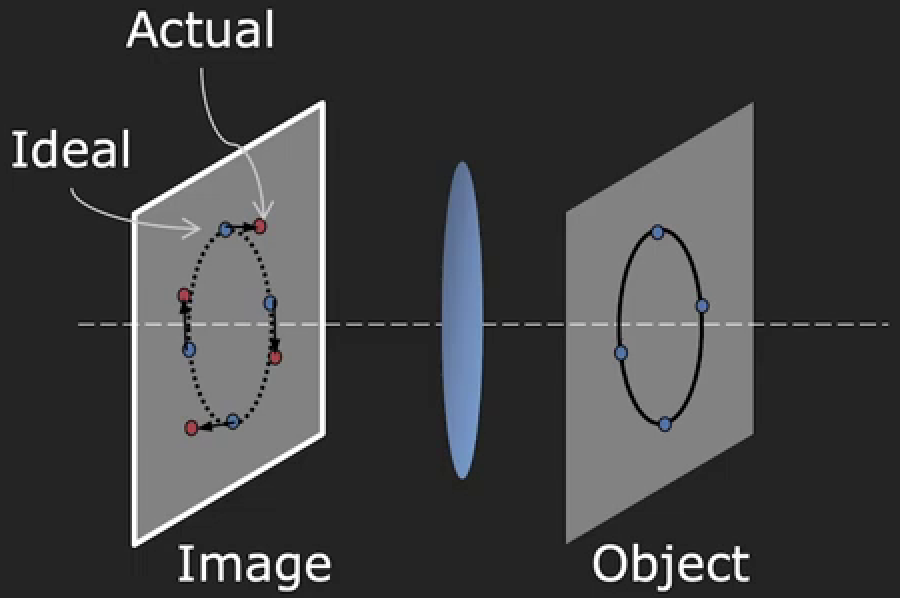

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