

Observation Matrix

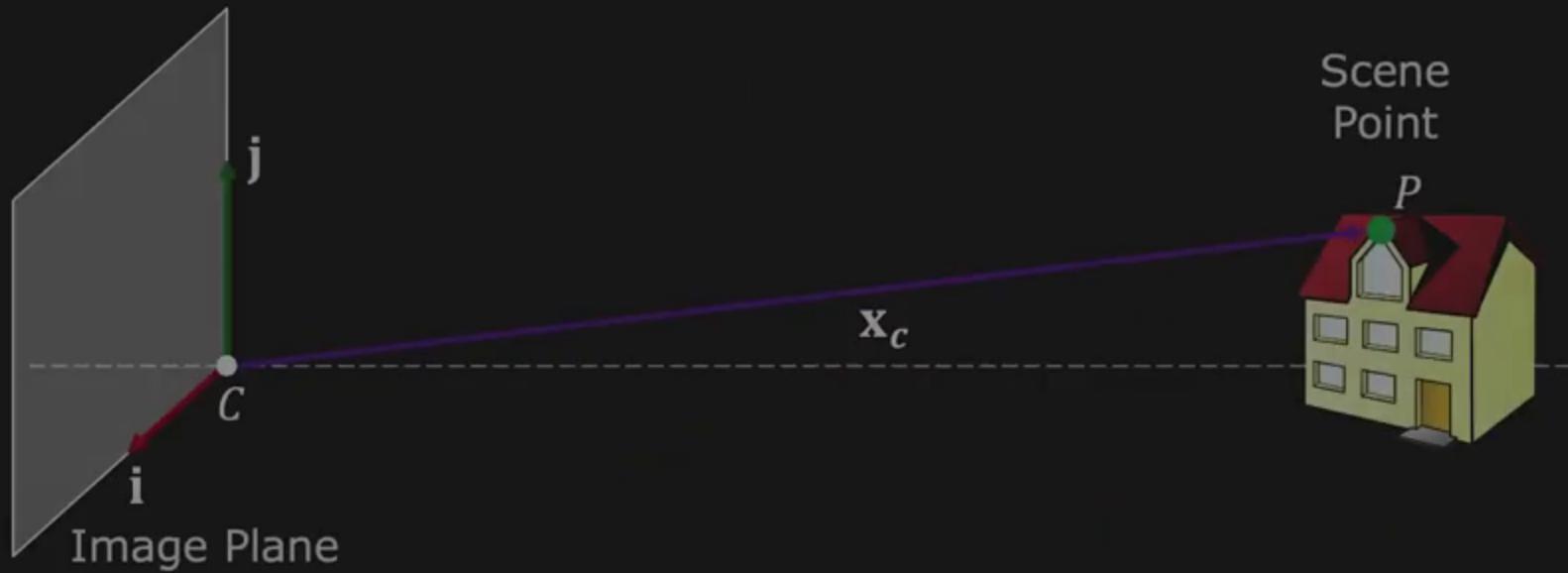
Shree K. Nayar

Columbia University

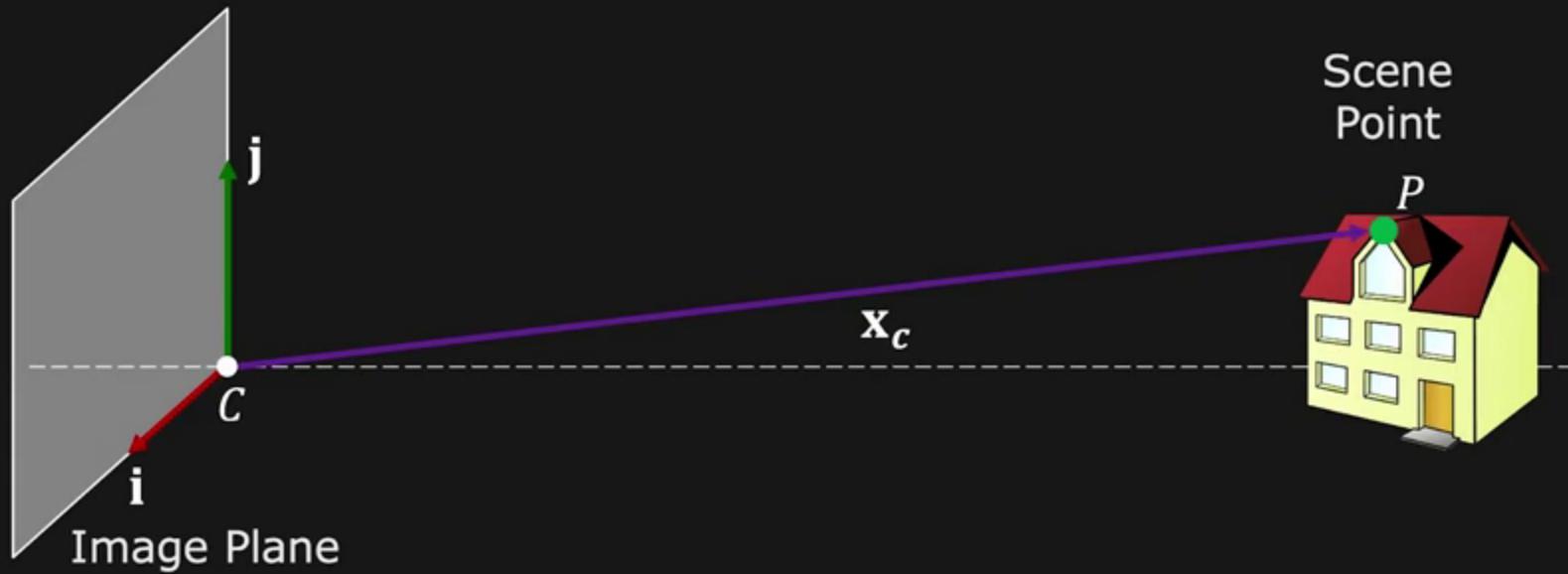
Topic: Structure from Motion, Module: Reconstruction II

First Principles of Computer Vision

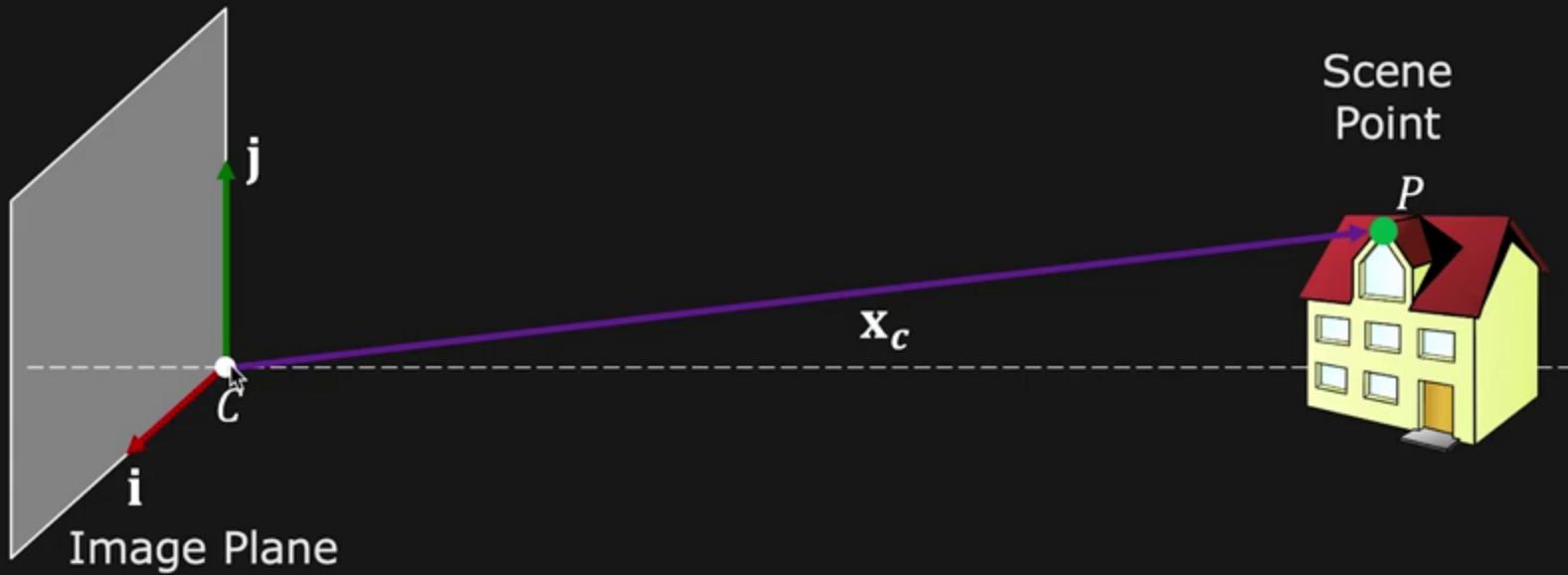
From 3D to 2D: Orthographic Projection



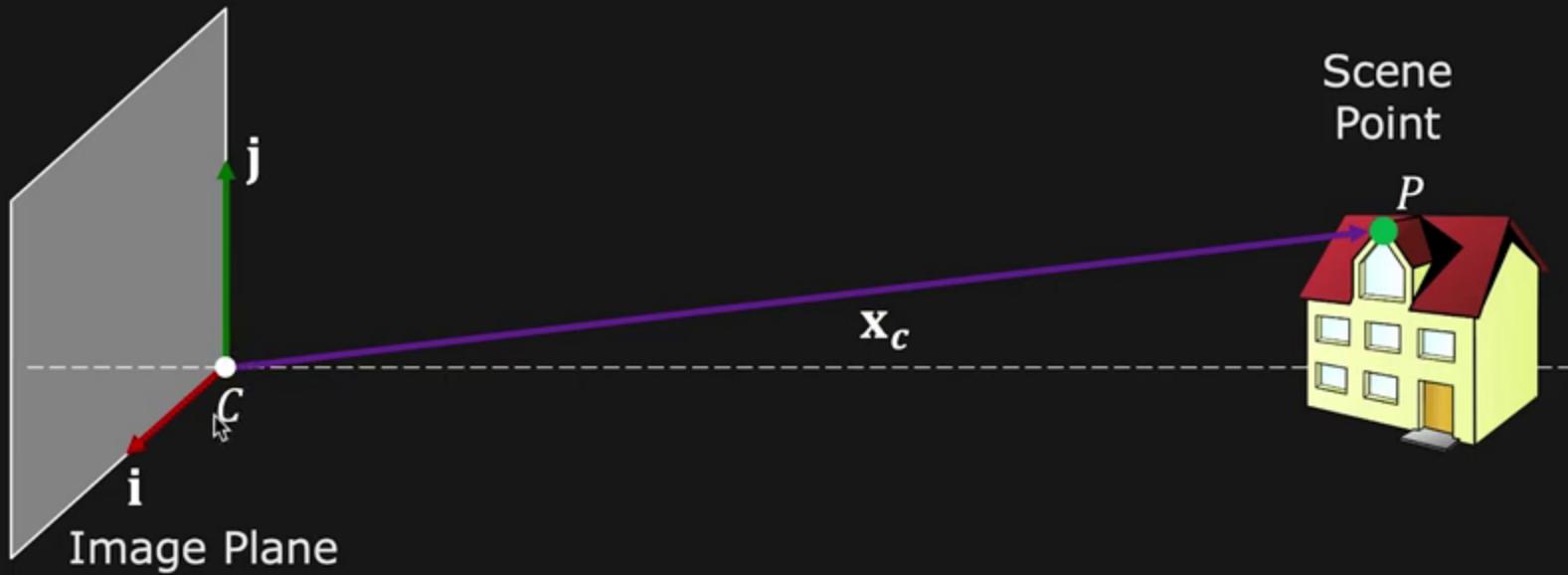
From 3D to 2D: Orthographic Projection



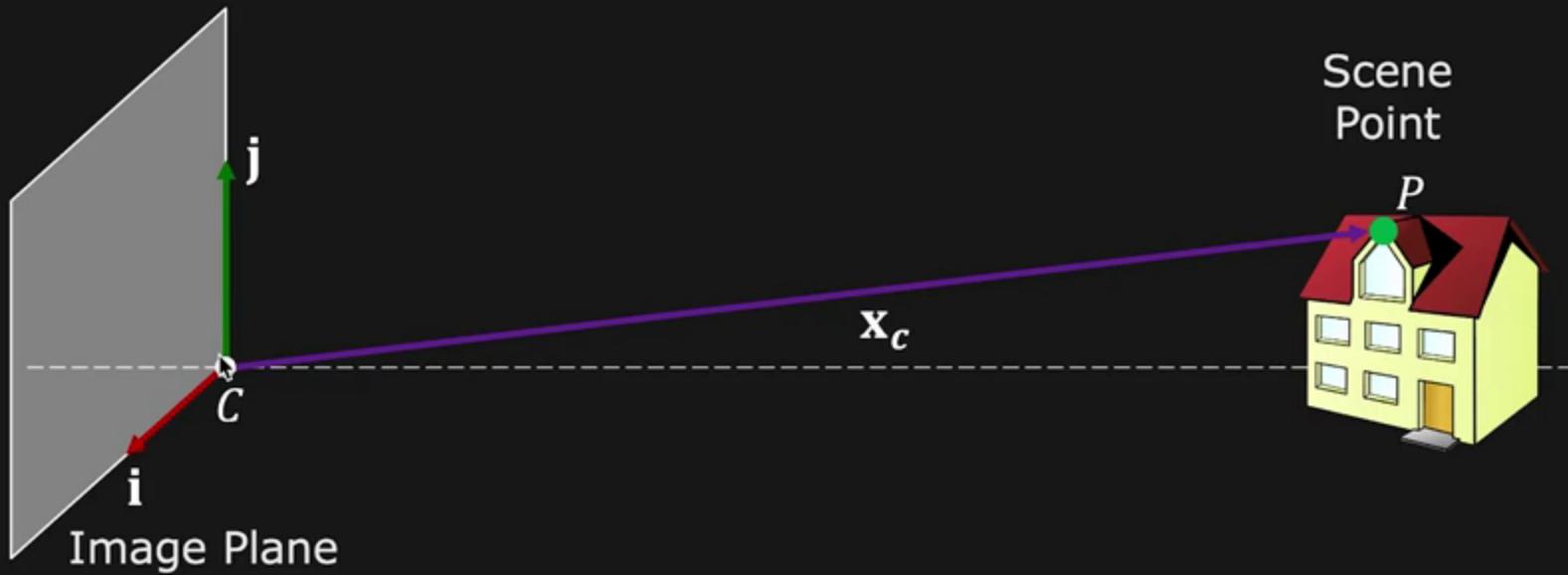
From 3D to 2D: Orthographic Projection



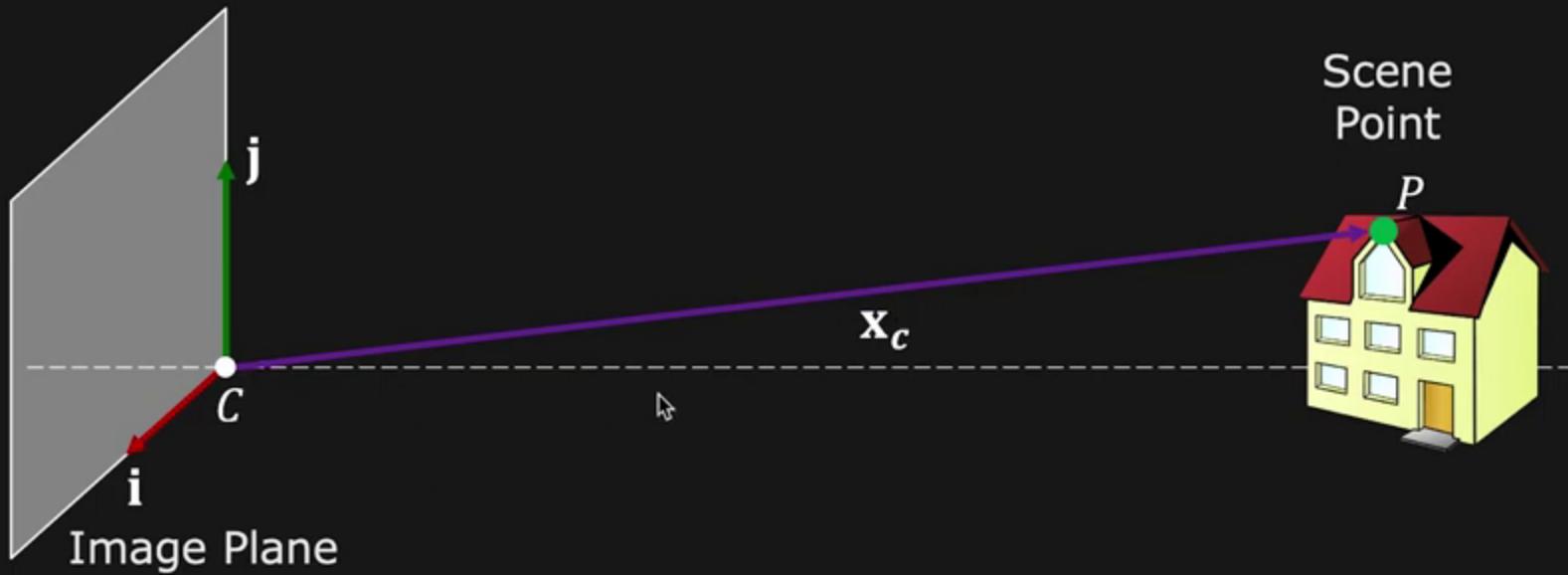
From 3D to 2D: Orthographic Projection



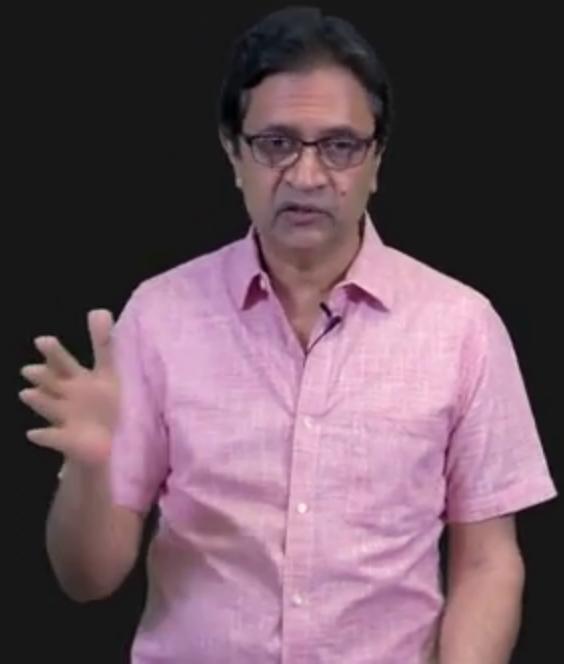
From 3D to 2D: Orthographic Projection



From 3D to 2D: Orthographic Projection



From 3D to 2D: Orthographic Projection



From 3D to 2D: Orthographic Projection

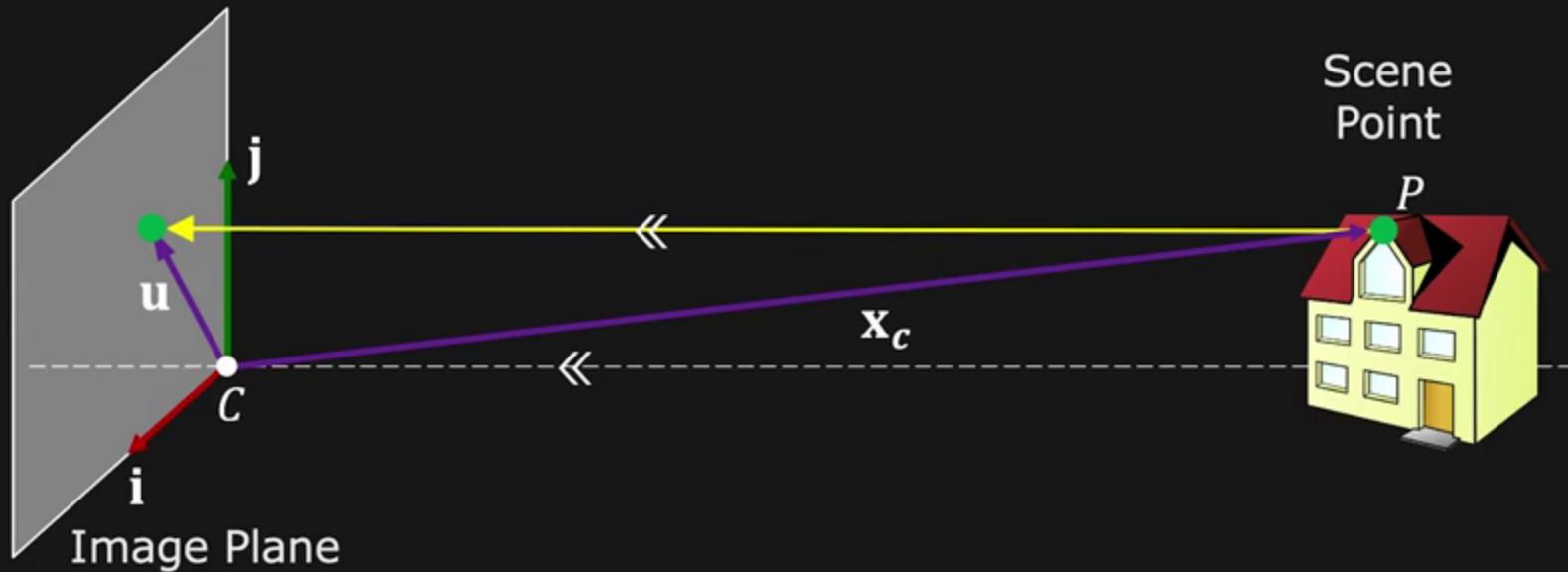


$$u = \mathbf{i} \cdot \mathbf{x}_c = \mathbf{i}^T \mathbf{x}_c$$

$$v = \mathbf{j} \cdot \mathbf{x}_c = \mathbf{j}^T \mathbf{x}_c$$



From 3D to 2D: Orthographic Projection



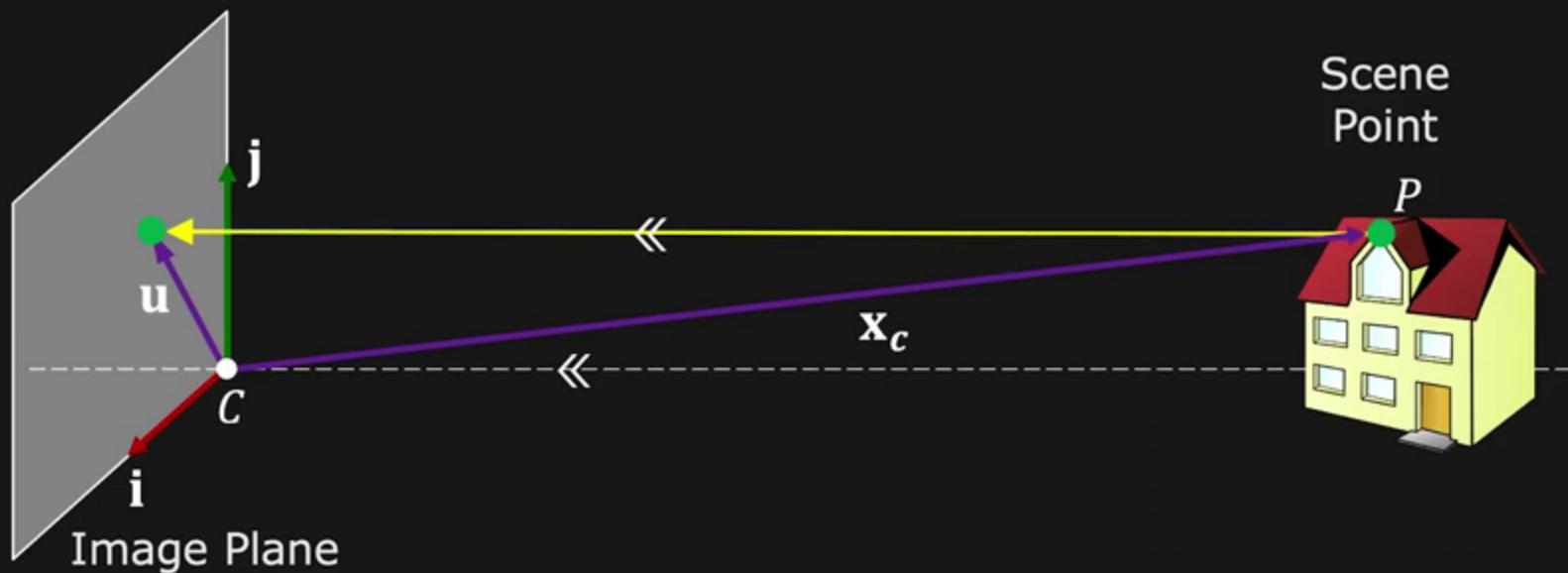
$$u = \mathbf{i} \cdot \mathbf{x}_c = \mathbf{i}^T \mathbf{x}_c$$

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Perspective cameras exhibit orthographic projection when distance of scene from camera is large compared to depth variation within scene (magnification is nearly constant).



From 3D to 2D: Orthographic Projection



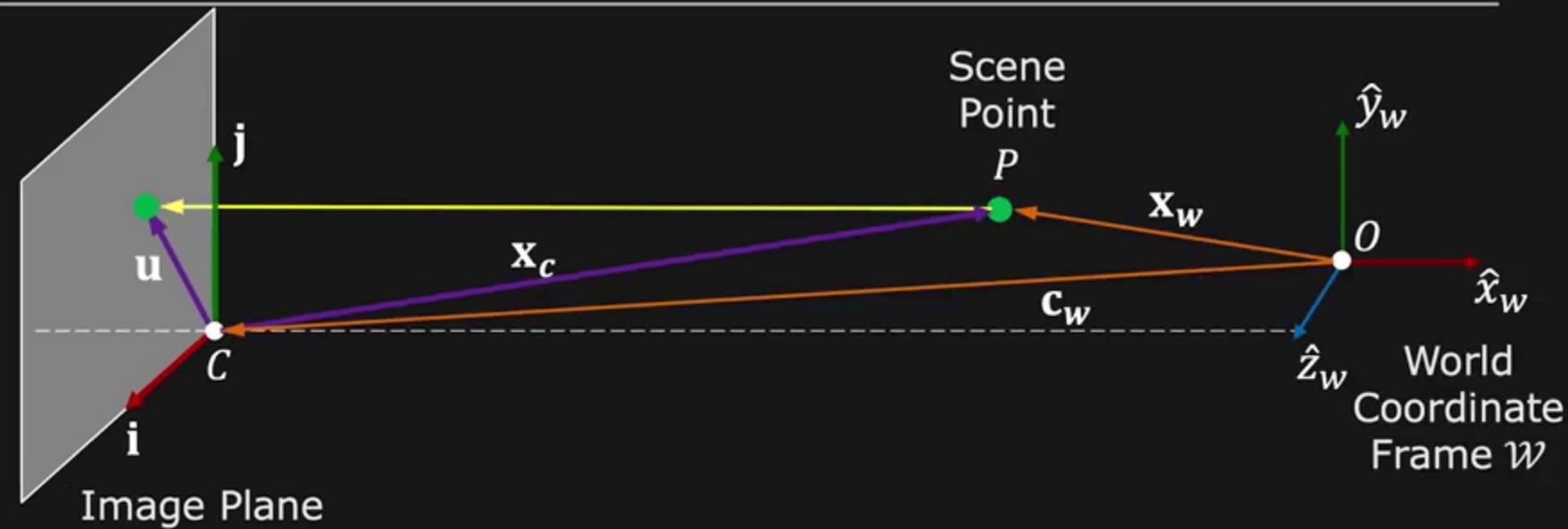
$$u = \mathbf{i} \cdot \mathbf{x}_c = \mathbf{i}^T \mathbf{x}_c$$

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From 3D to 2D: Orthographic Projection

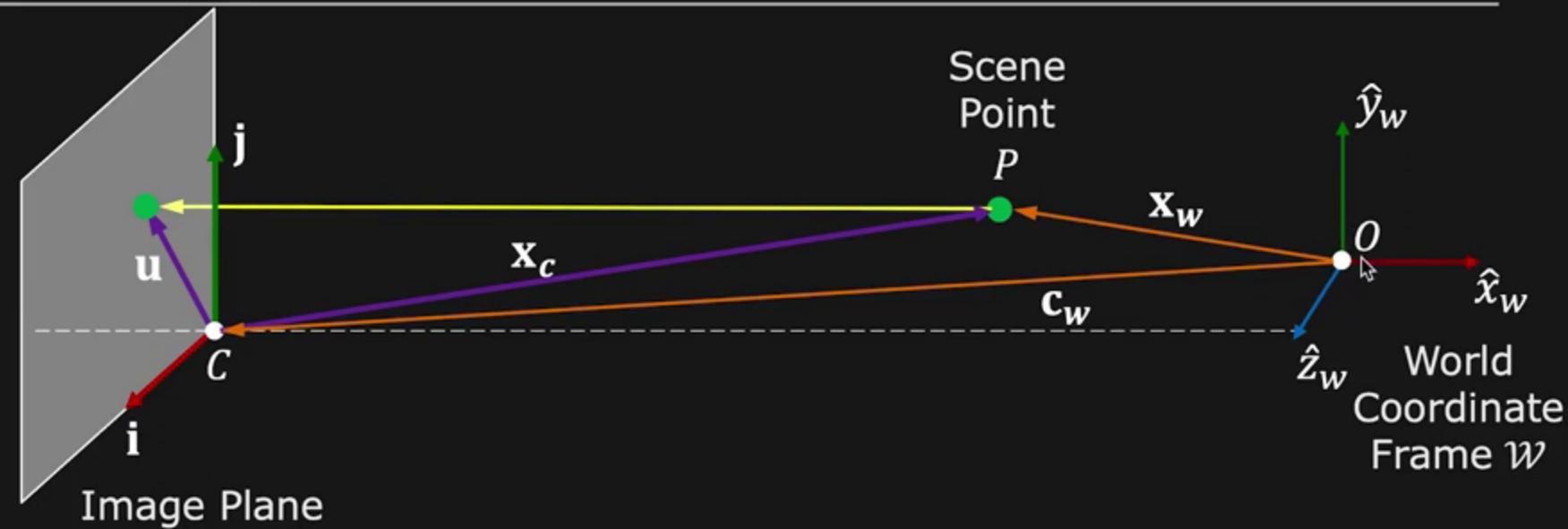


$$u = \mathbf{i}^T \mathbf{x}_c$$

$$v = \mathbf{j}^T \mathbf{x}_c$$



From 3D to 2D: Orthographic Projection

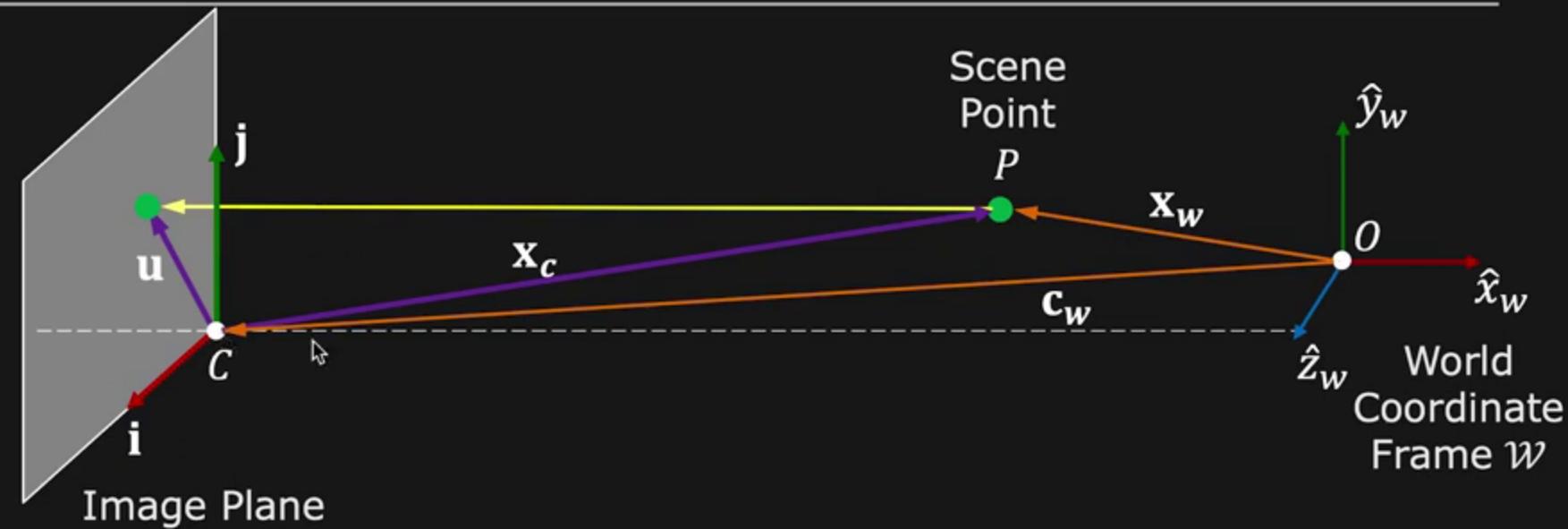


$$u = \mathbf{i}^T \mathbf{x}_c$$

$$v = \mathbf{j}^T \mathbf{x}_c$$



From 3D to 2D: Orthographic Projection

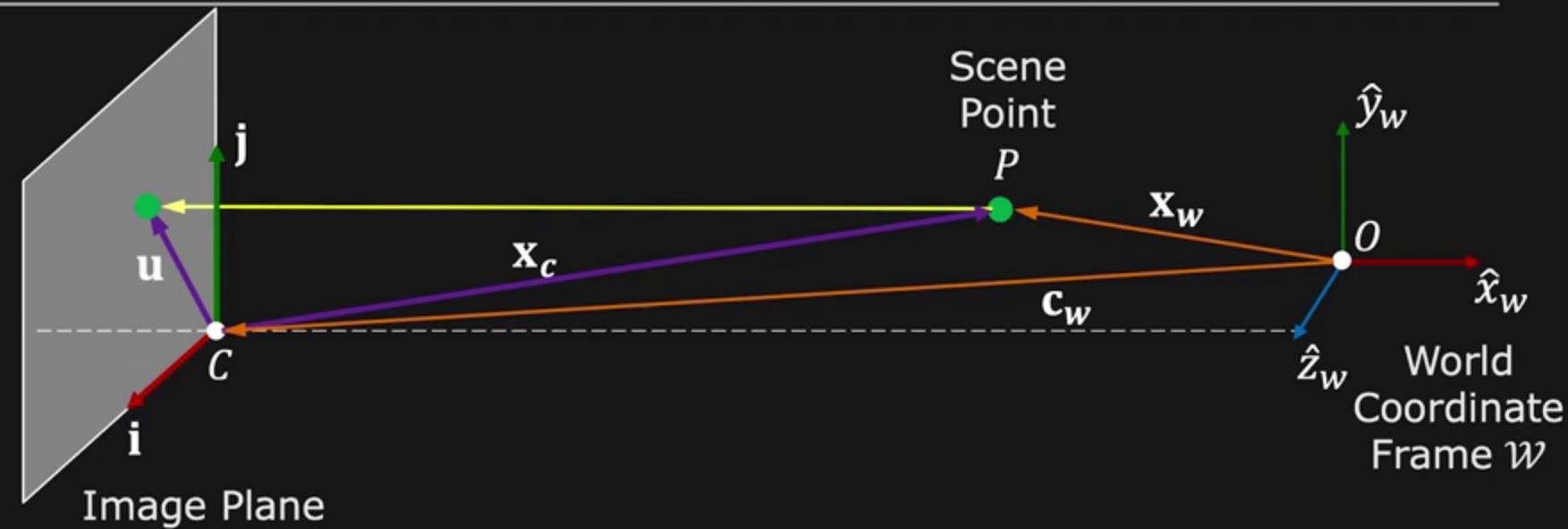


$$u = \mathbf{i}^T \mathbf{x}_c$$

$$v = \mathbf{j}^T \mathbf{x}_c$$



From 3D to 2D: Orthographic Projection

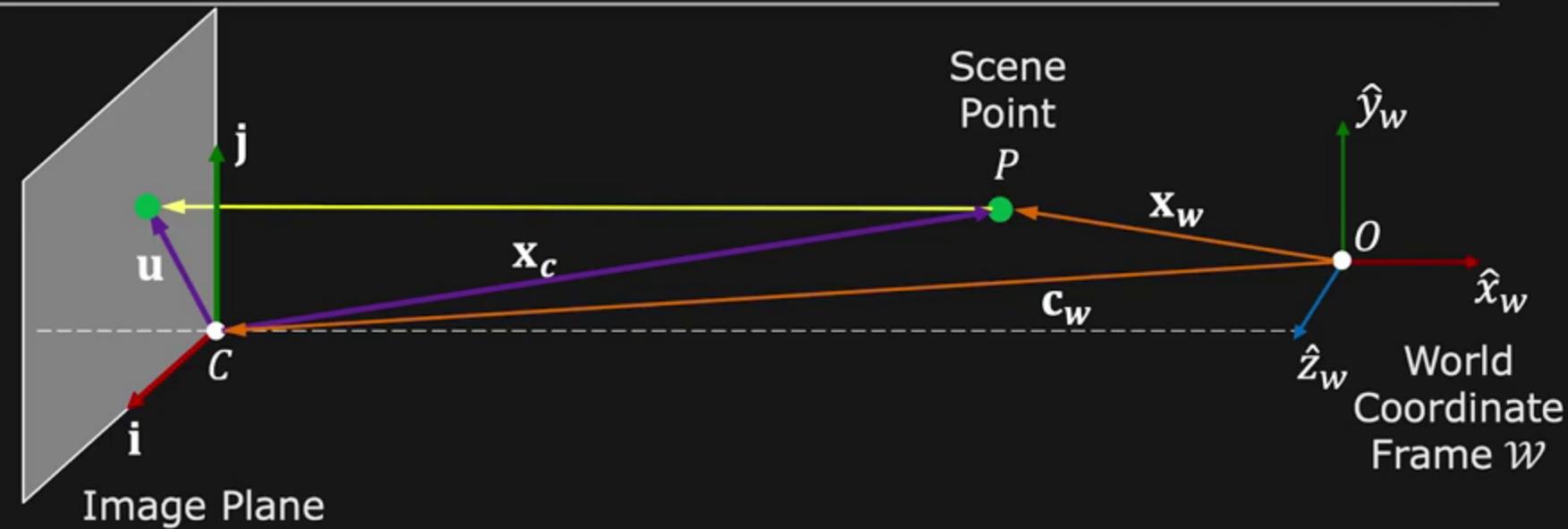


$$u = \mathbf{i}^T \mathbf{x}_c = \mathbf{i}^T (\mathbf{x}_w - \mathbf{c}_w)$$

$$v = \mathbf{j}^T \mathbf{x}_c = \mathbf{j}^T (\mathbf{x}_w - \mathbf{c}_w)$$



From 3D to 2D: Orthographic Projection

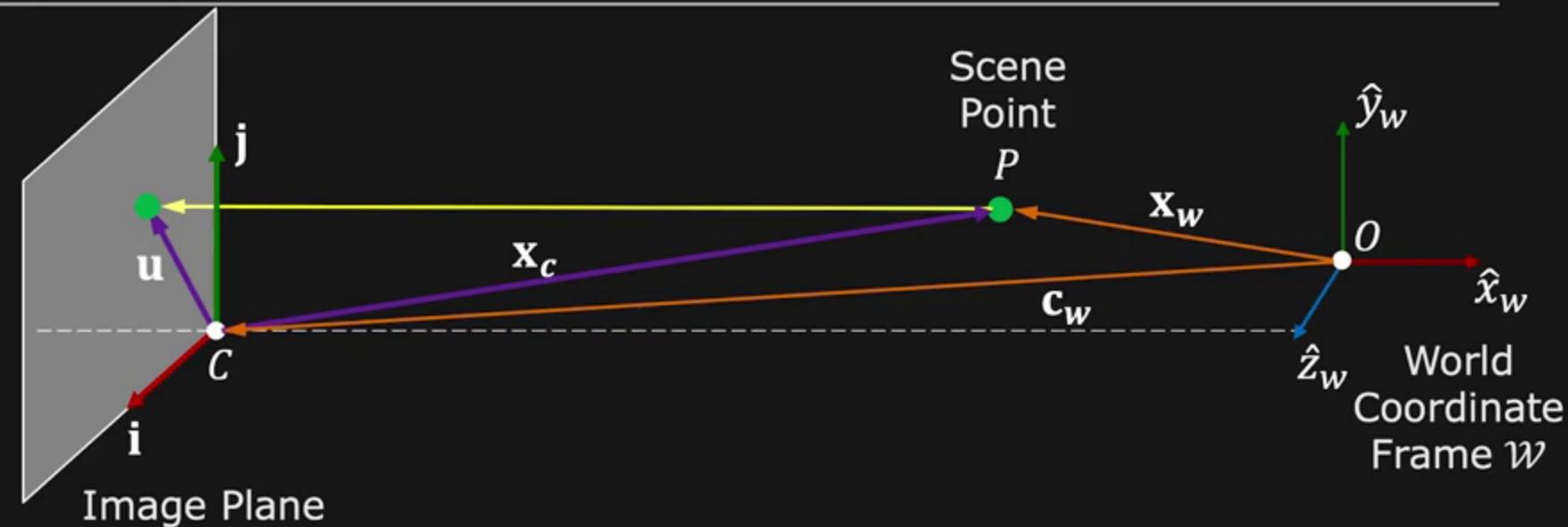


$$u = \mathbf{i}^T \mathbf{x}_c = \mathbf{i}^T (\mathbf{x}_w - \mathbf{c}_w)$$

$$v = \mathbf{j}^T \mathbf{x}_c = \mathbf{j}^T (\mathbf{x}_w - \mathbf{c}_w)$$



From 3D to 2D: Orthographic Projection



$$u = \mathbf{i}^T \mathbf{x}_c = \mathbf{i}^T (\mathbf{x}_w - \mathbf{c}_w) = \mathbf{i}^T (\underset{\downarrow}{P} - C)$$

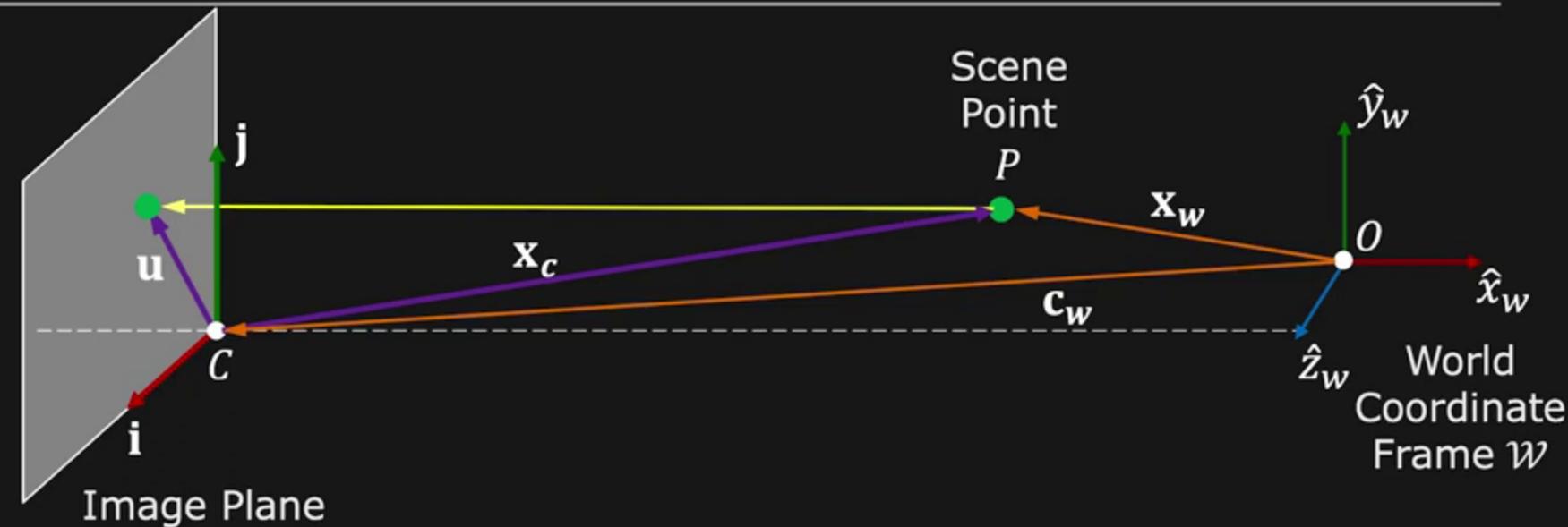
$$v = \mathbf{j}^T \mathbf{x}_c = \mathbf{j}^T (\mathbf{x}_w - \mathbf{c}_w) = \mathbf{j}^T (\underset{\downarrow}{P} - C)$$

$$u = \mathbf{i}^T (P - C)$$

$$v = \mathbf{j}^T (P - C)$$



From 3D to 2D: Orthographic Projection



$$u = \mathbf{i}^T \mathbf{x}_c = \mathbf{i}^T (\mathbf{x}_w - \mathbf{c}_w) = \mathbf{i}^T (P - C)$$

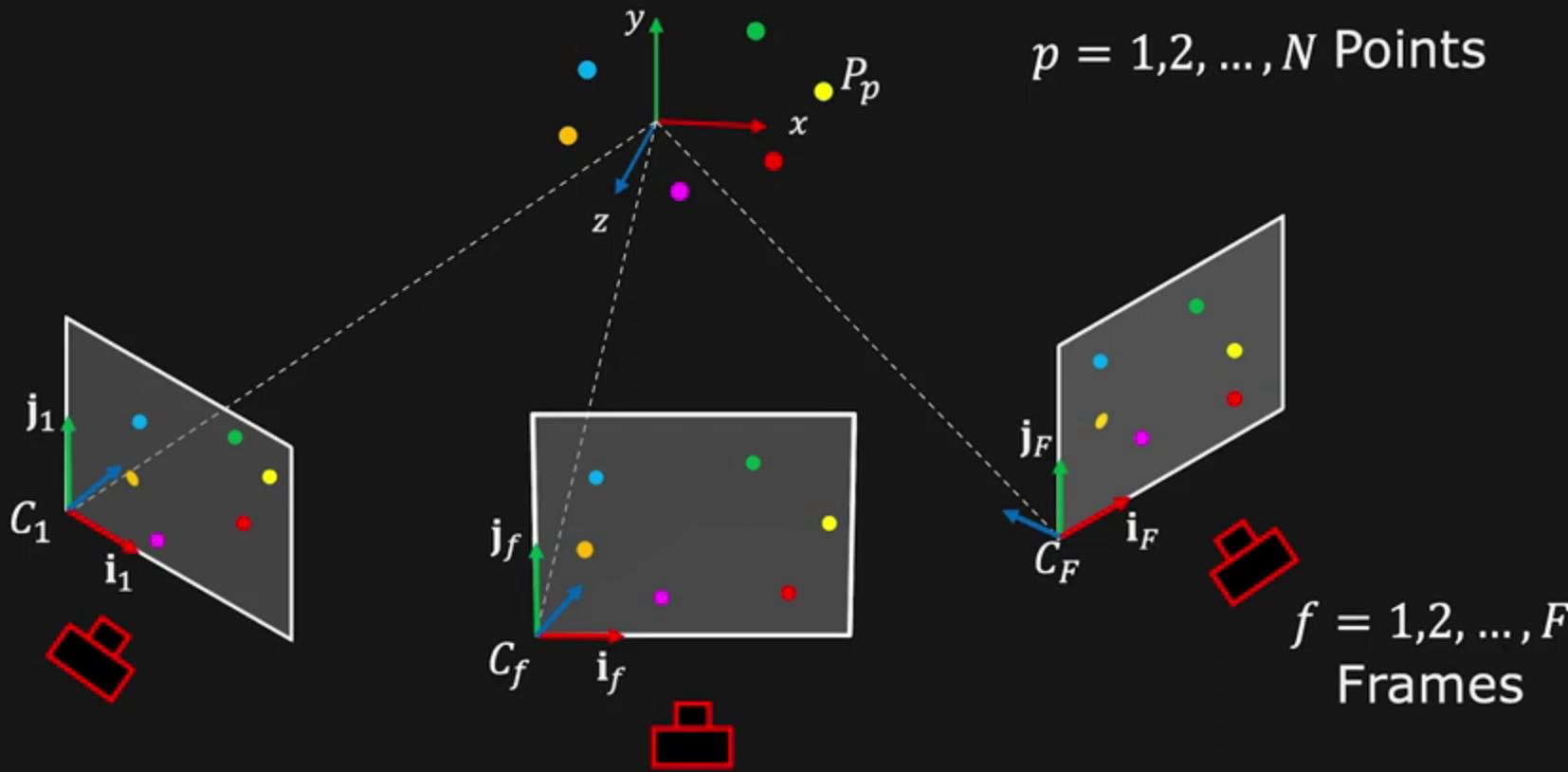
$$v = \mathbf{j}^T \mathbf{x}_c = \mathbf{j}^T (\mathbf{x}_w - \mathbf{c}_w) = \mathbf{j}^T (P - C)$$

$$u = \mathbf{i}^T (P - C)$$

$$v = \mathbf{j}^T (P - C)$$



Orthographic SFM



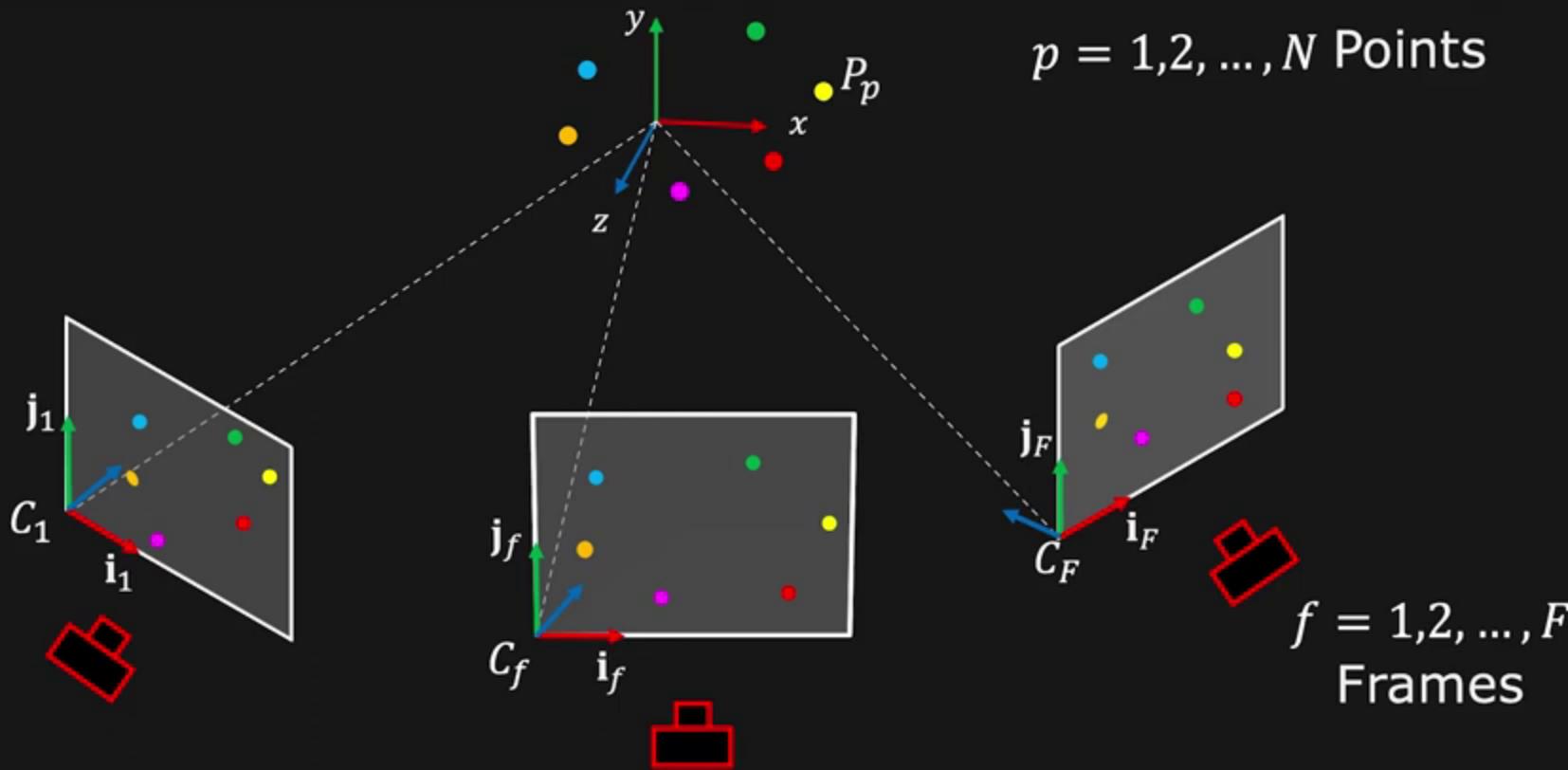
Given corresponding image points (2D) $(u_{f,p}, v_{f,p})$

Find scene points $\{P_p\}$.

Camera Positions $\{C_f\}$, camera orientations $\{(\mathbf{i}_f, \mathbf{j}_f)\}$ are unknown.



Orthographic SFM



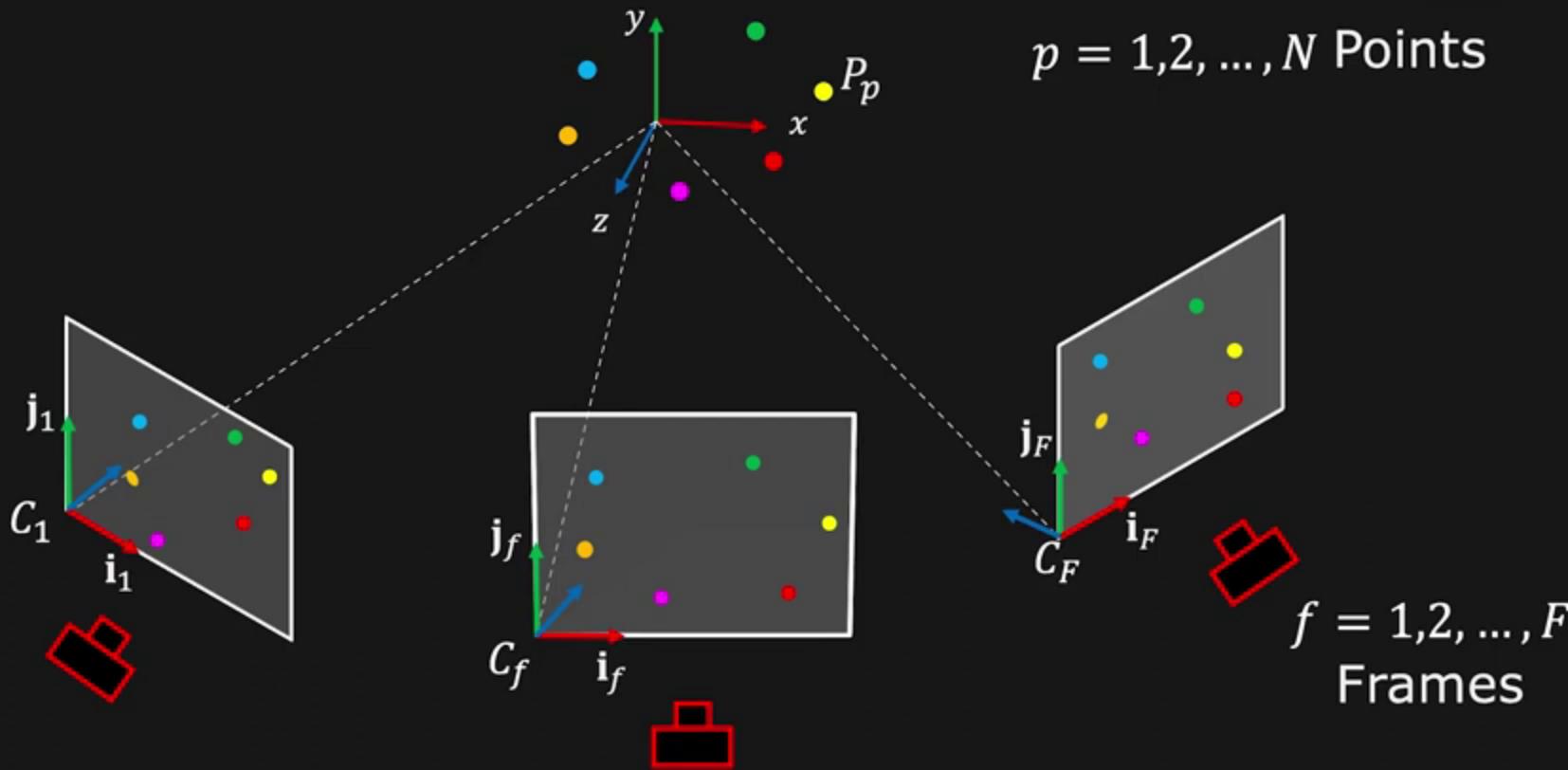
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Orthographic SFM



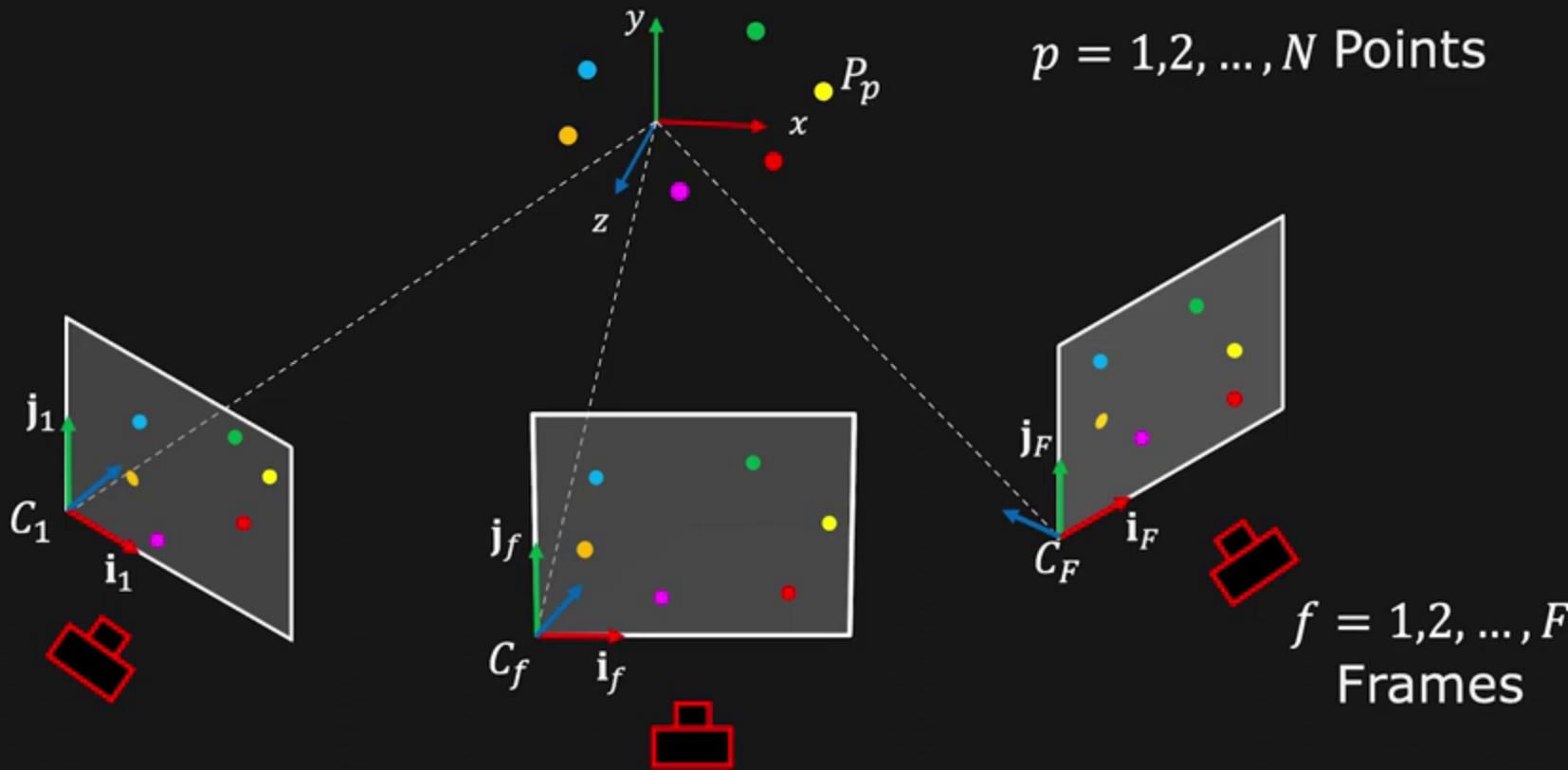
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Orthographic SFM



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Find scene points $\{P_p\}$.

Camera Positions $\{C_f\}$, camera orientations $\{(\mathbf{i}_f, \mathbf{j}_f)\}$ are unknown.



Orthographic SFM

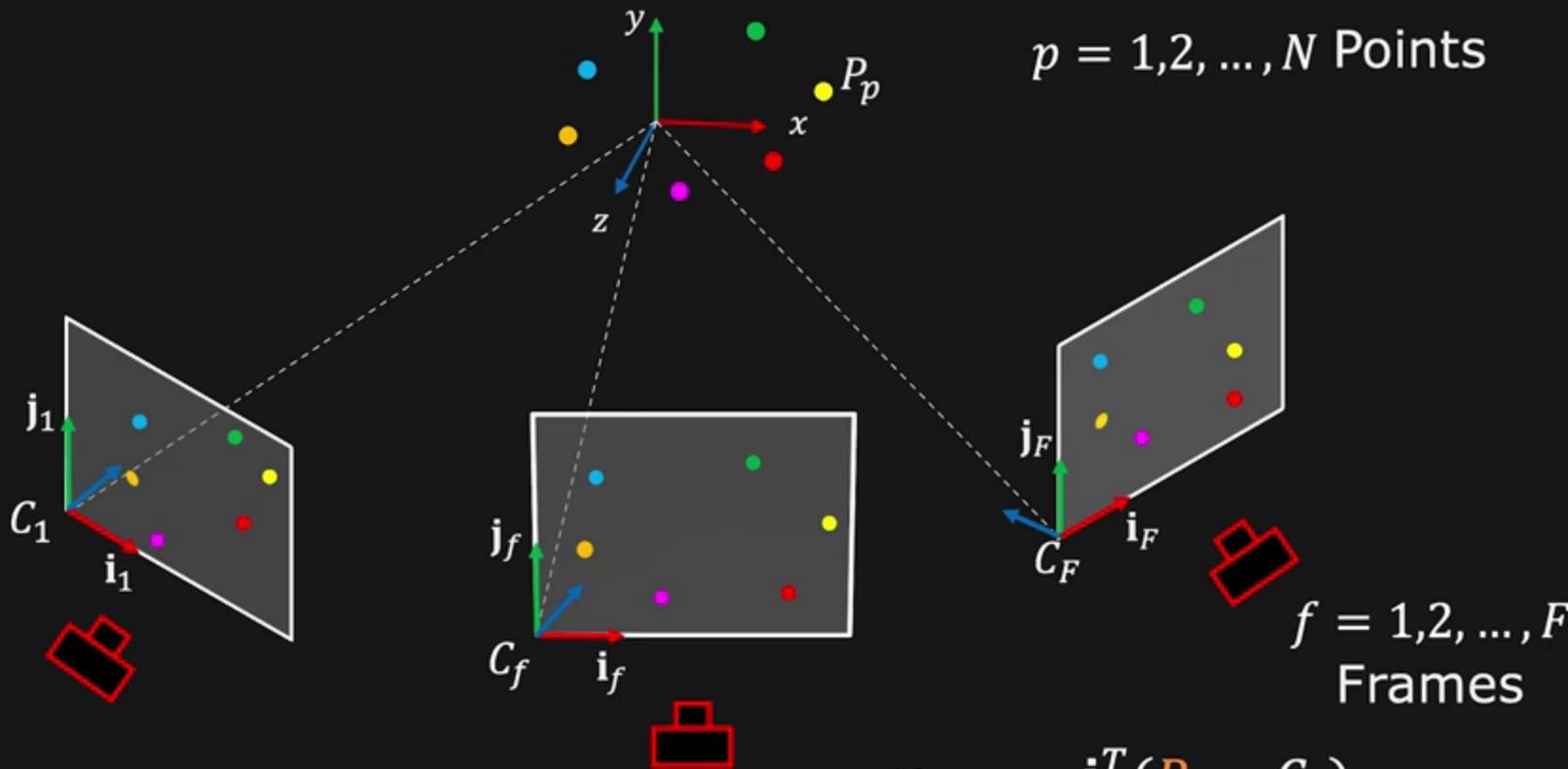
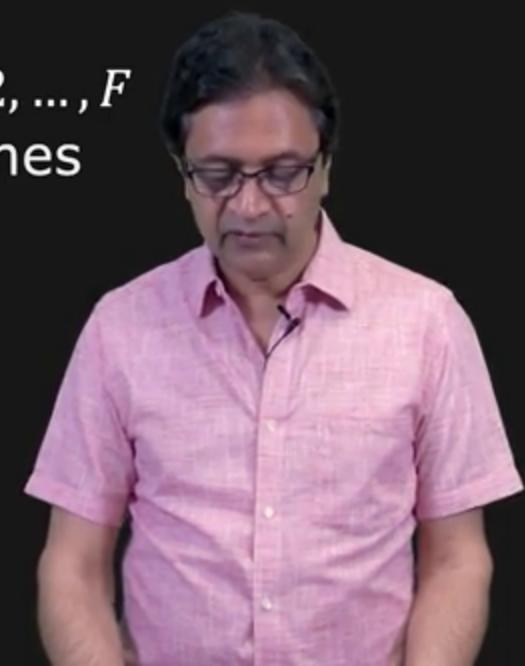


Image of point P_p in camera frame f :

$$u_{f,p} = \mathbf{i}_f^T (\mathbf{P}_p - \mathbf{C}_f)$$

$$v_{f,p} = \mathbf{j}_f^T (\mathbf{P}_p - \mathbf{C}_f)$$



Orthographic SFM

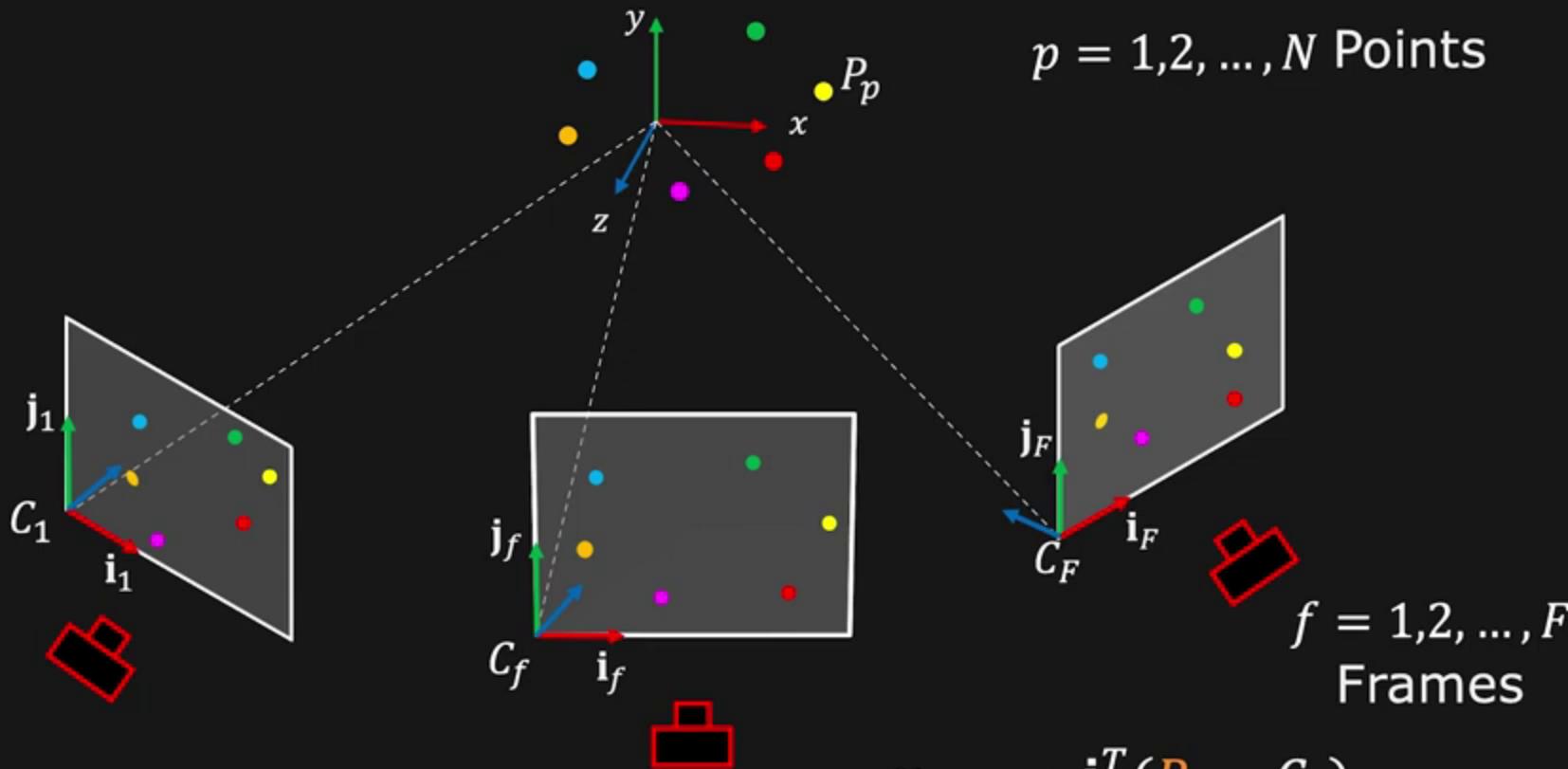
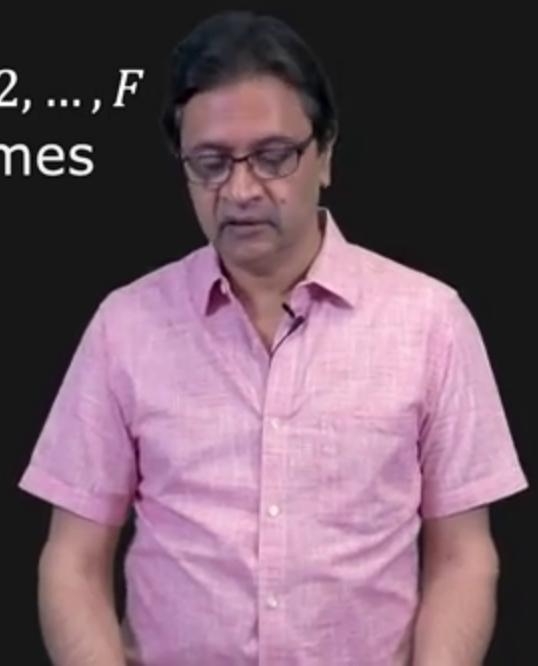


Image of point P_p in camera frame f :

$$u_{f,p} = \mathbf{i}_f^T (\mathbf{P}_p - \mathbf{C}_{f\triangleright})$$

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Orthographic SFM

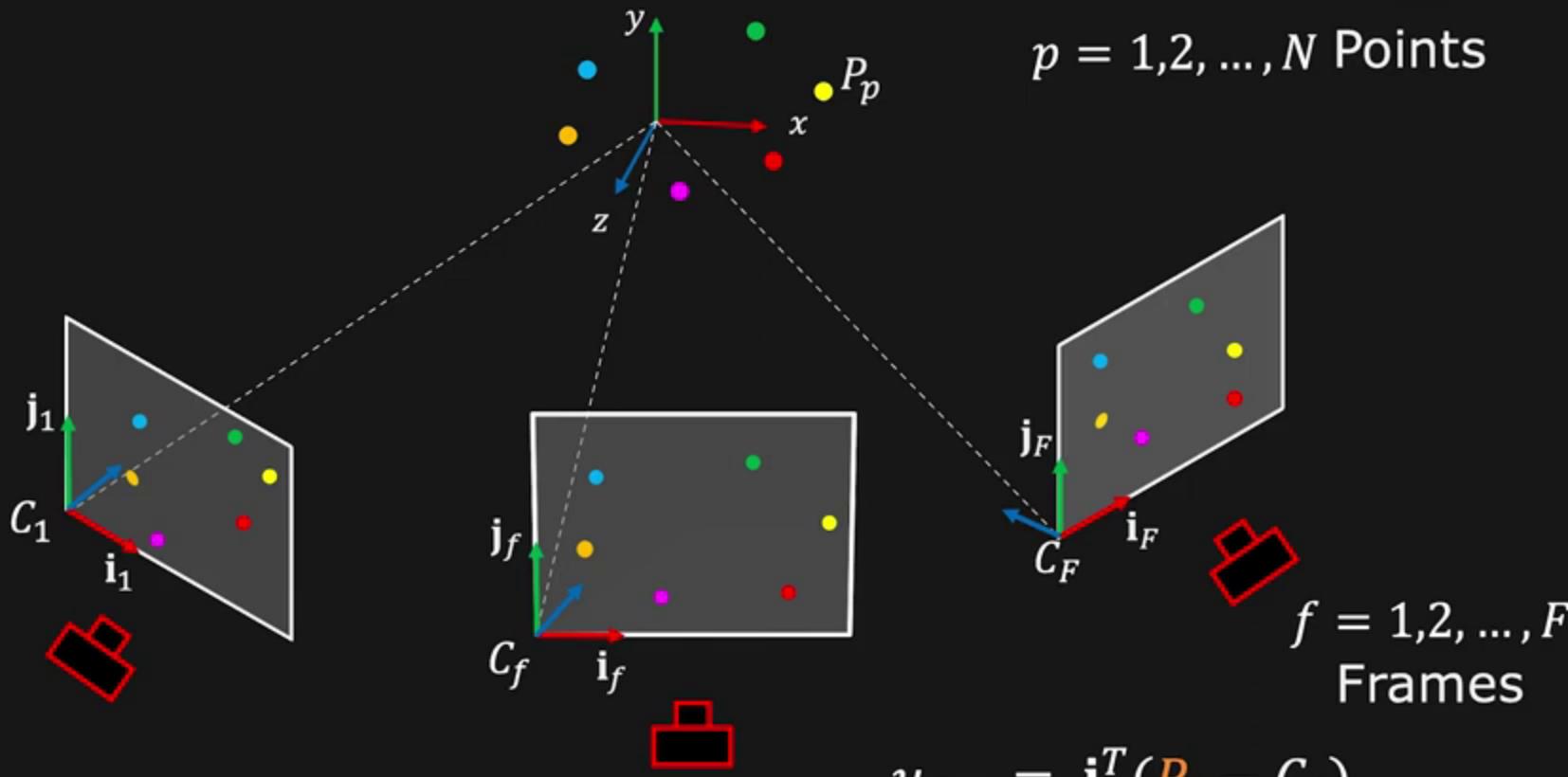
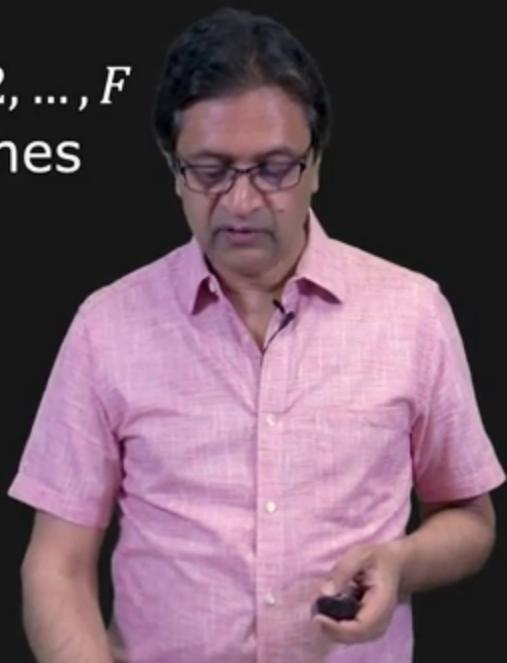


Image of point P_p in camera frame f :

$$u_{f,p} = \mathbf{i}_f^T (\mathbf{P}_p - \mathbf{C}_f)$$

$$v_{f,p} = \mathbf{j}_f^T (\mathbf{P}_p - \mathbf{C}_f)$$

Known



Orthographic SFM

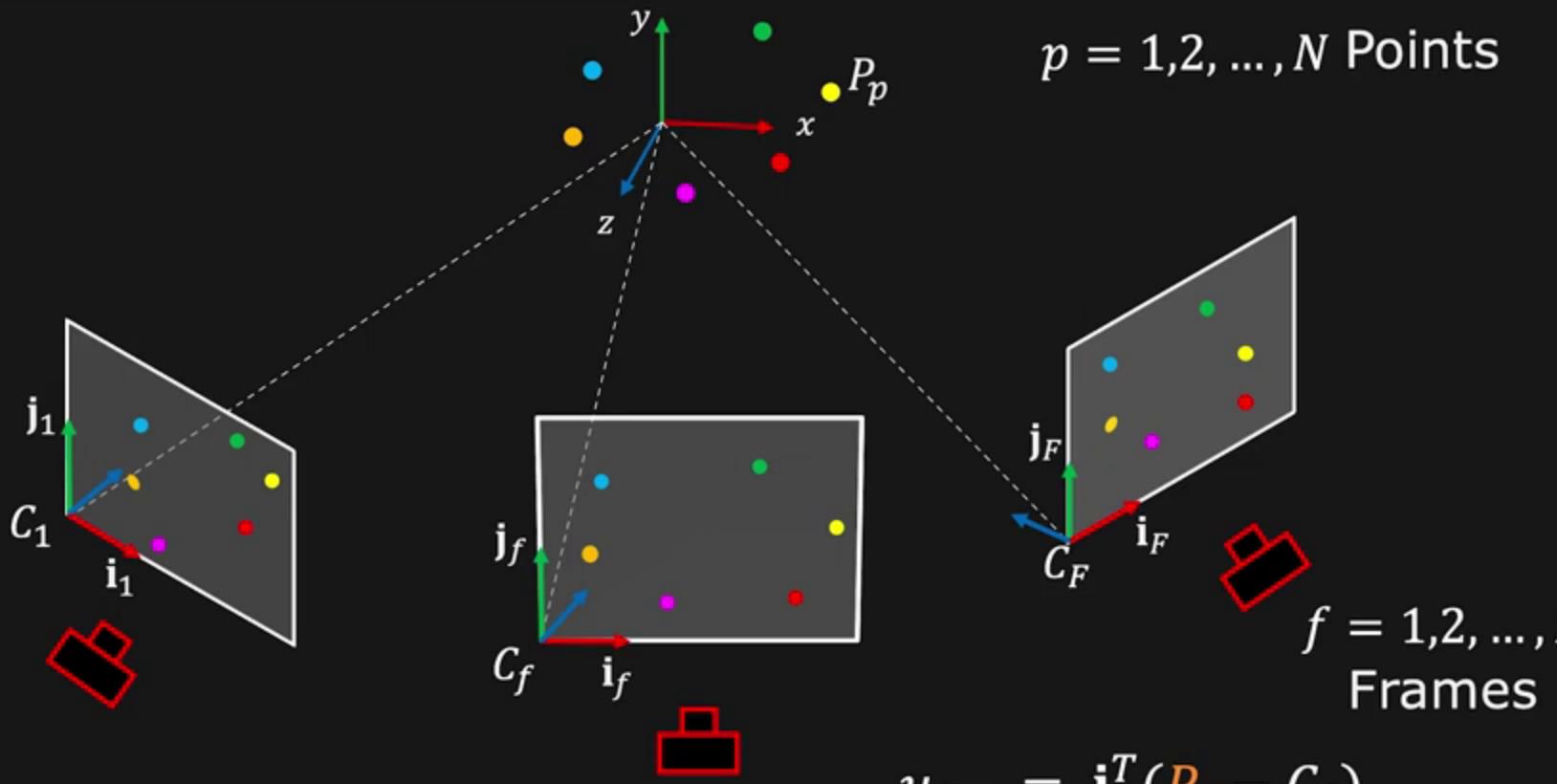
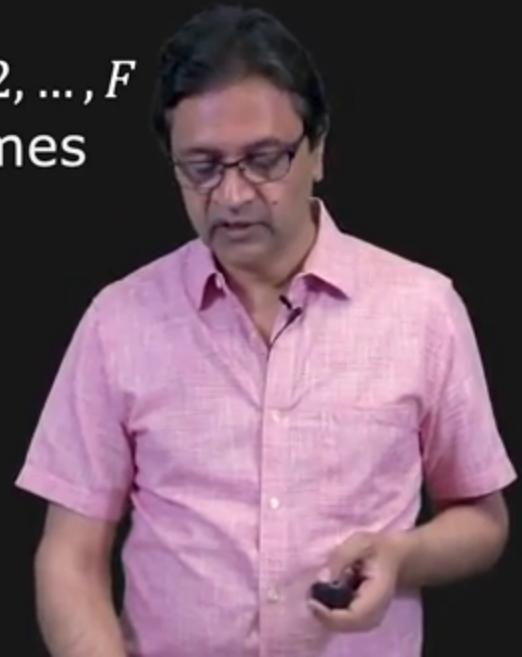


Image of point P_p in camera frame f :

$$\frac{u_{f,p}}{\text{Known}} = \mathbf{i}_f^T (\mathbf{P}_p - \mathbf{C}_f)$$
$$\frac{v_{f,p}}{\text{Unknown}} = \mathbf{j}_f^T (\mathbf{P}_p - \mathbf{C}_f)$$



Orthographic SFM

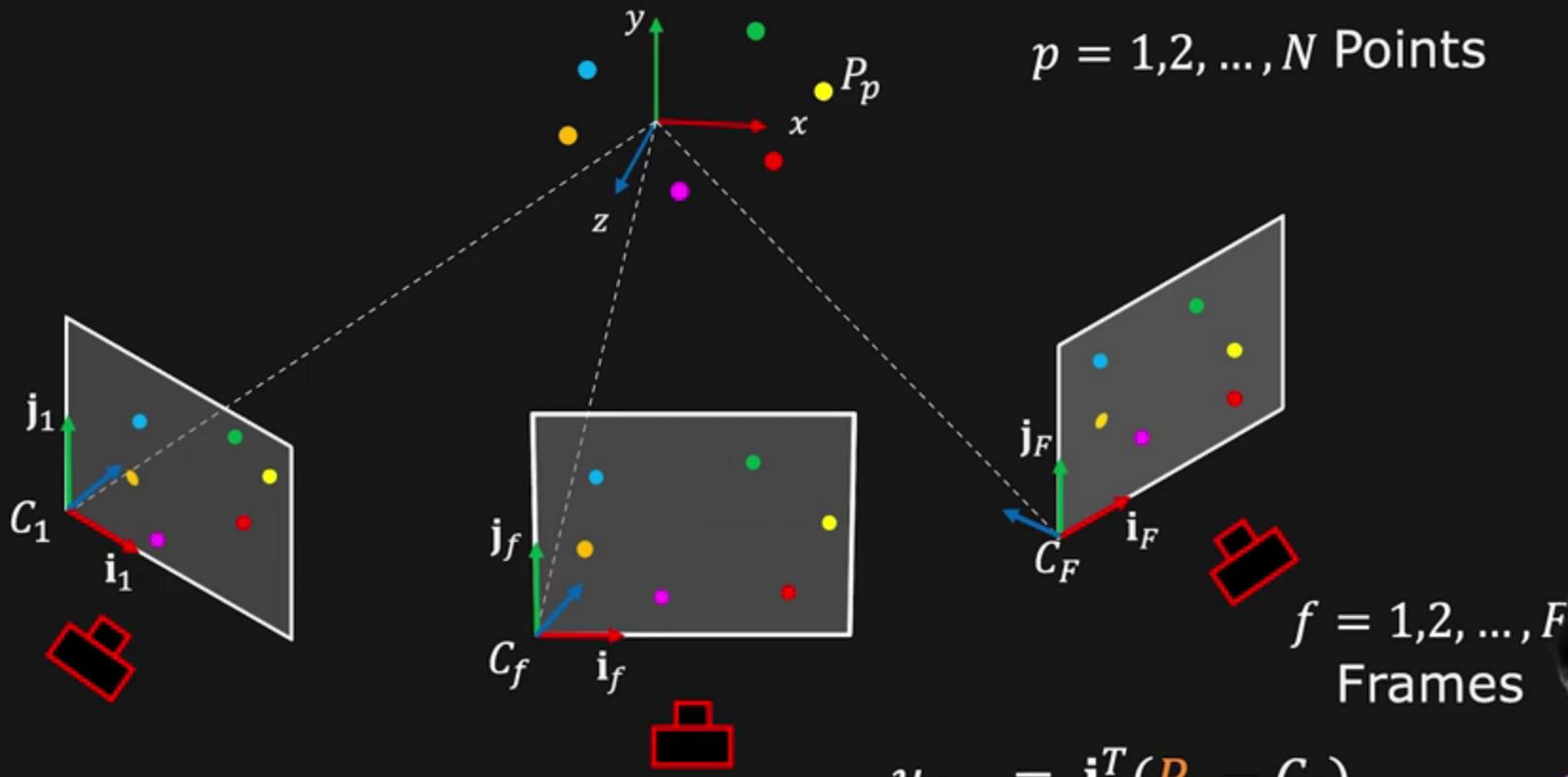
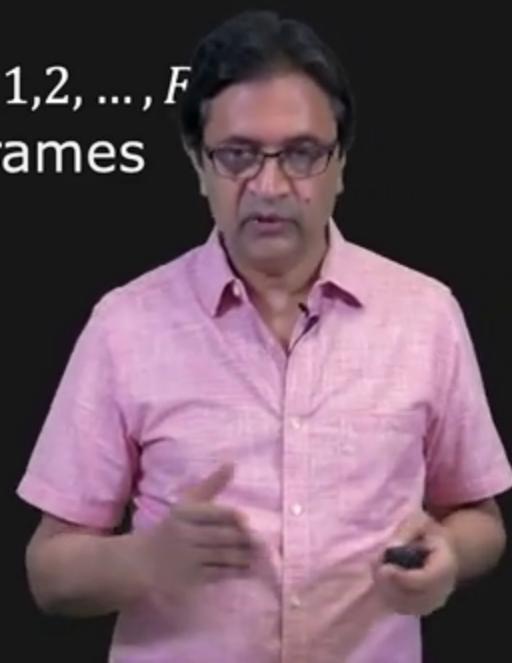


Image of point P_p in camera frame f :

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Orthographic SFM

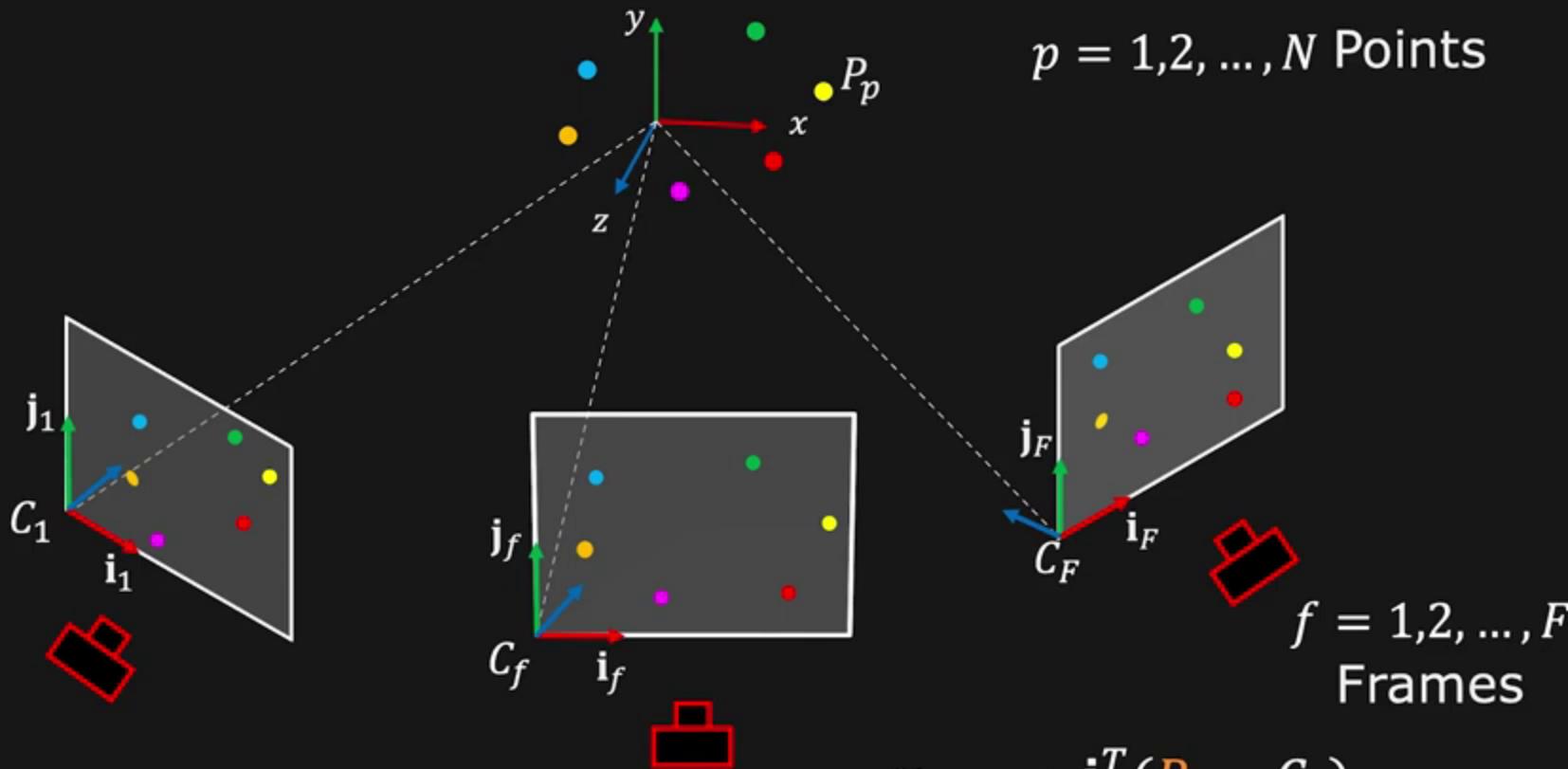
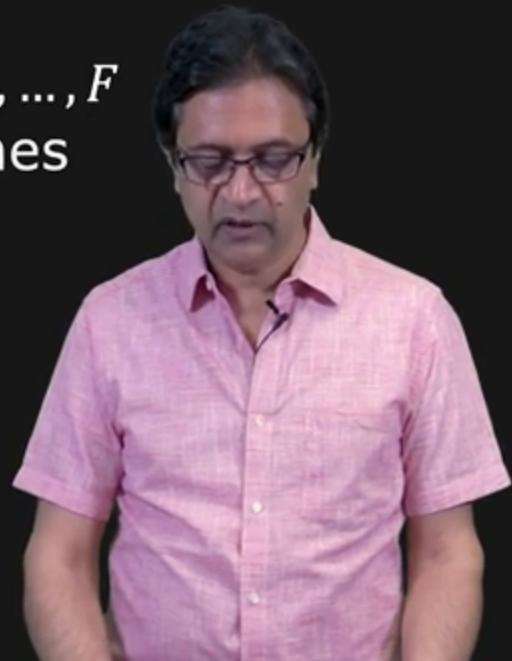


Image of point P_p in camera frame f :

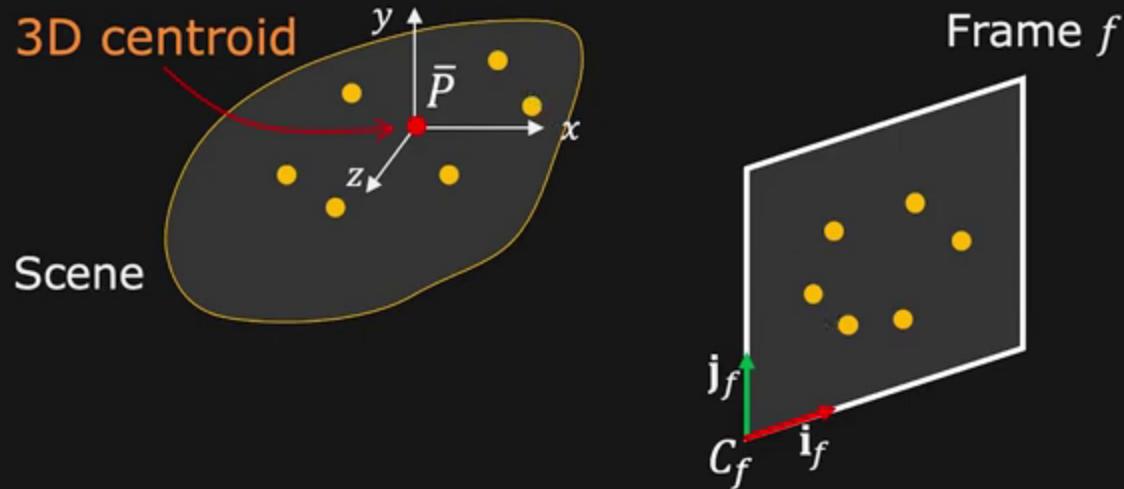
$$u_{f,p} = i_f^T (P_p - C_f)$$

$$\frac{v_{f,p}}{\text{Known}} = \frac{i_f^T (P_p - C_f)}{\text{Unknown}}$$

We can remove C_f from equations to simplify SFM problem.



Centering Trick

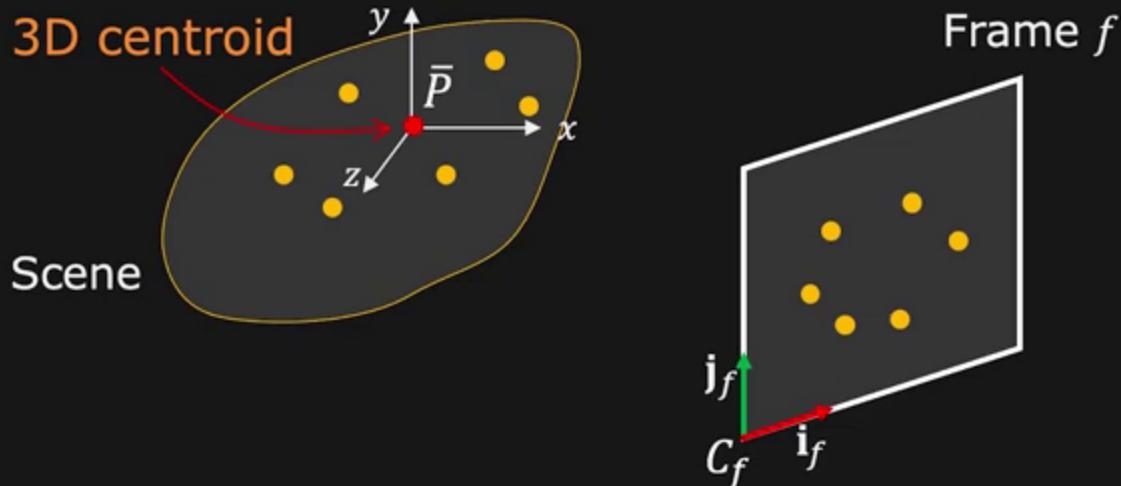


Assume origin of world at centroid of scene points:

$$\frac{1}{N} \sum_{p=1}^N P_p = \bar{P} = \mathbf{0}$$



Centering Trick

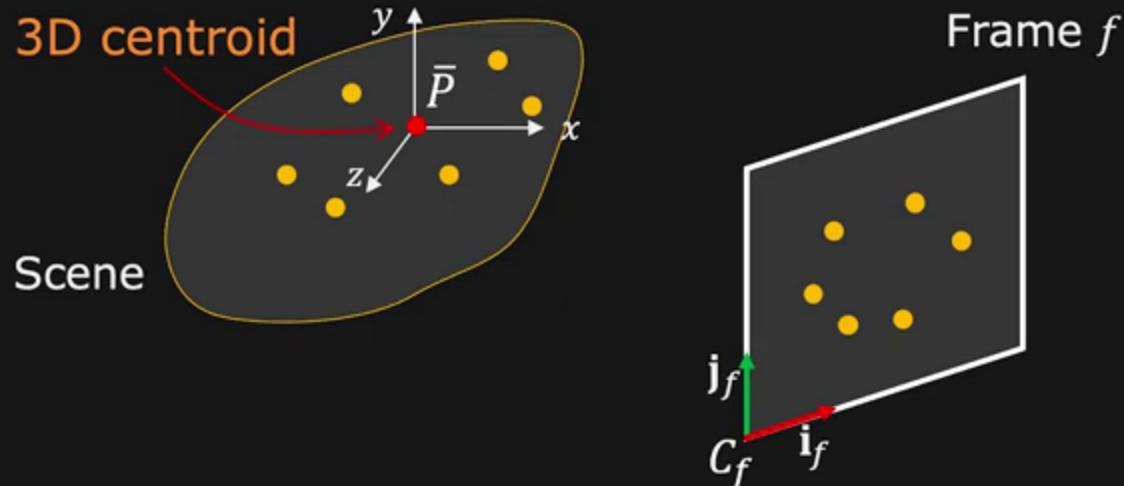


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Centering Trick

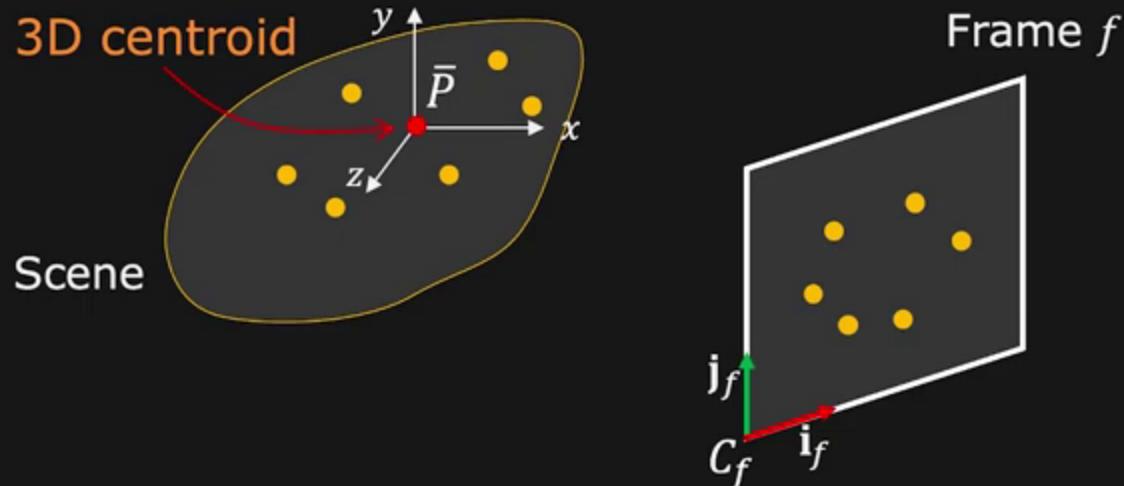


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Centering Trick



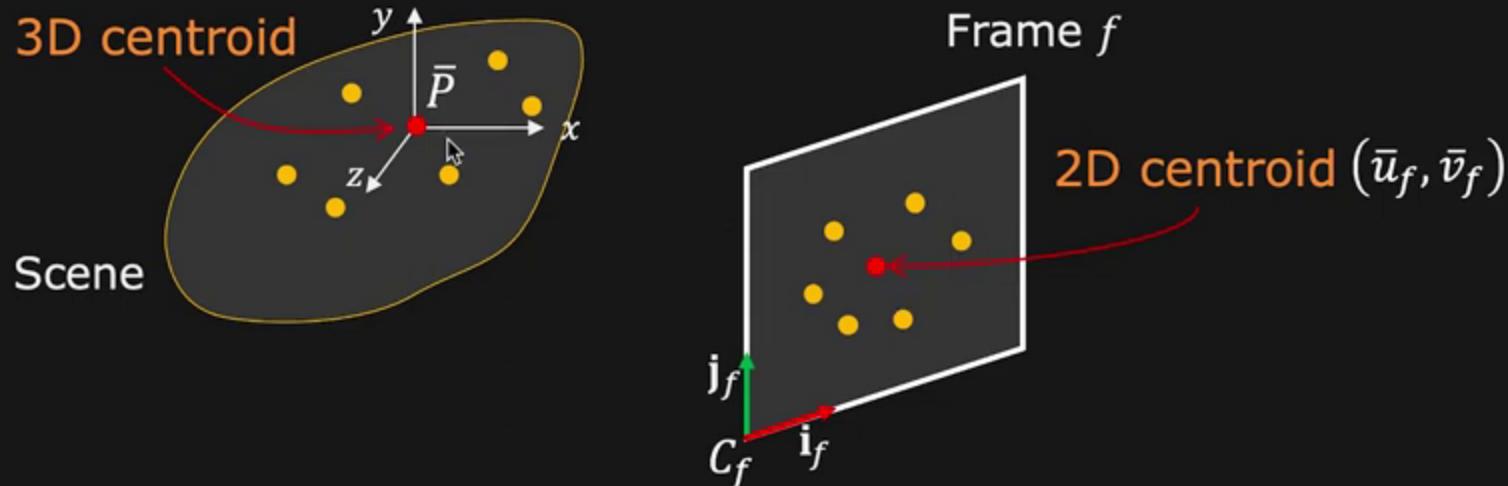
Assume origin of world at centroid of scene points:

$$\frac{1}{N} \sum_{p=1}^N P_p = \bar{P} = \mathbf{0}$$

We will compute scene points w.r.t their centroid!



Centering Trick

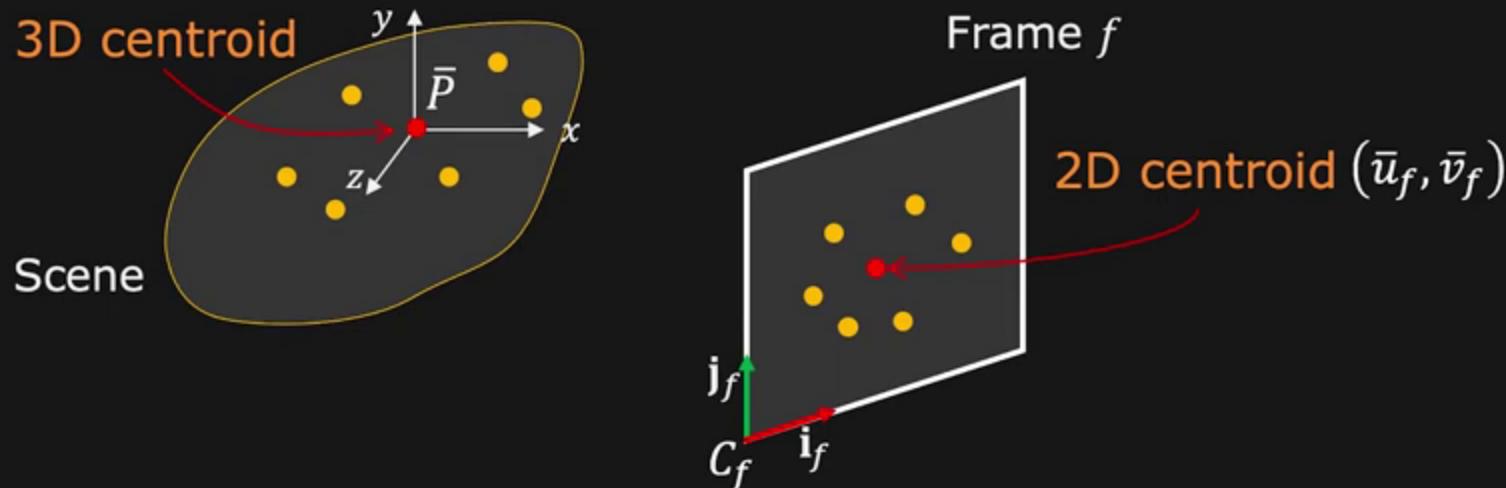


Centroid (\bar{u}_f, \bar{v}_f) of the image points in frame *f*:

$$\bar{u}_f = \frac{1}{N} \sum_{p=1}^N u_{f,p}$$



Centering Trick

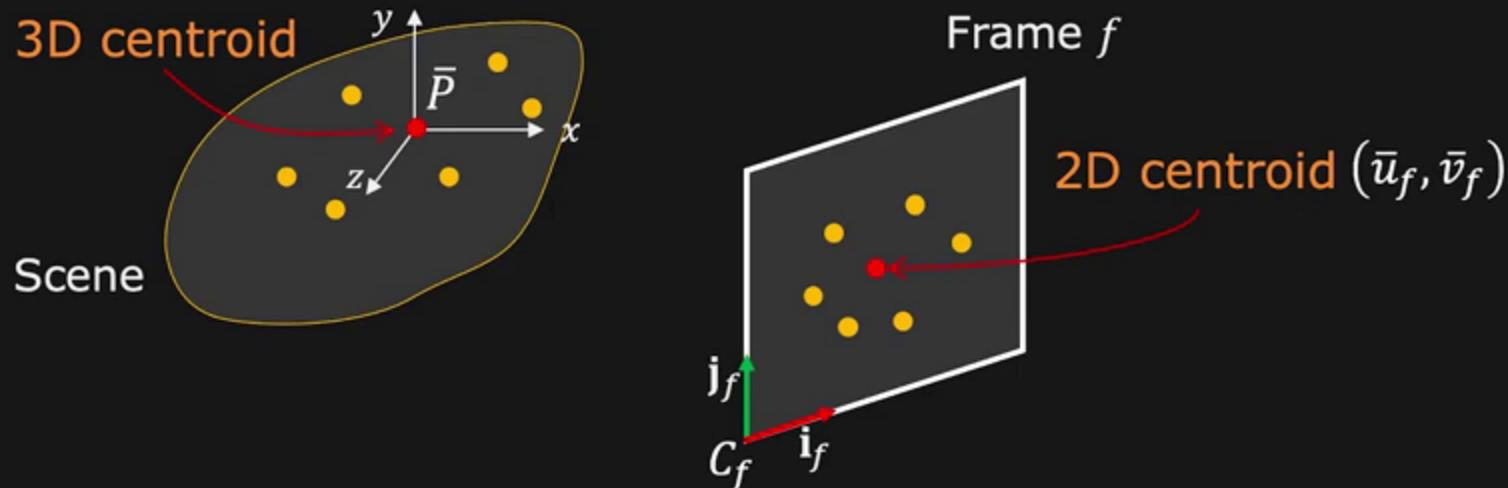


Centroid (\bar{u}_f, \bar{v}_f) of the image points in frame f :

$$\bar{u}_f = \frac{1}{N} \sum_{p=1}^N u_{f,p}$$



Centering Trick

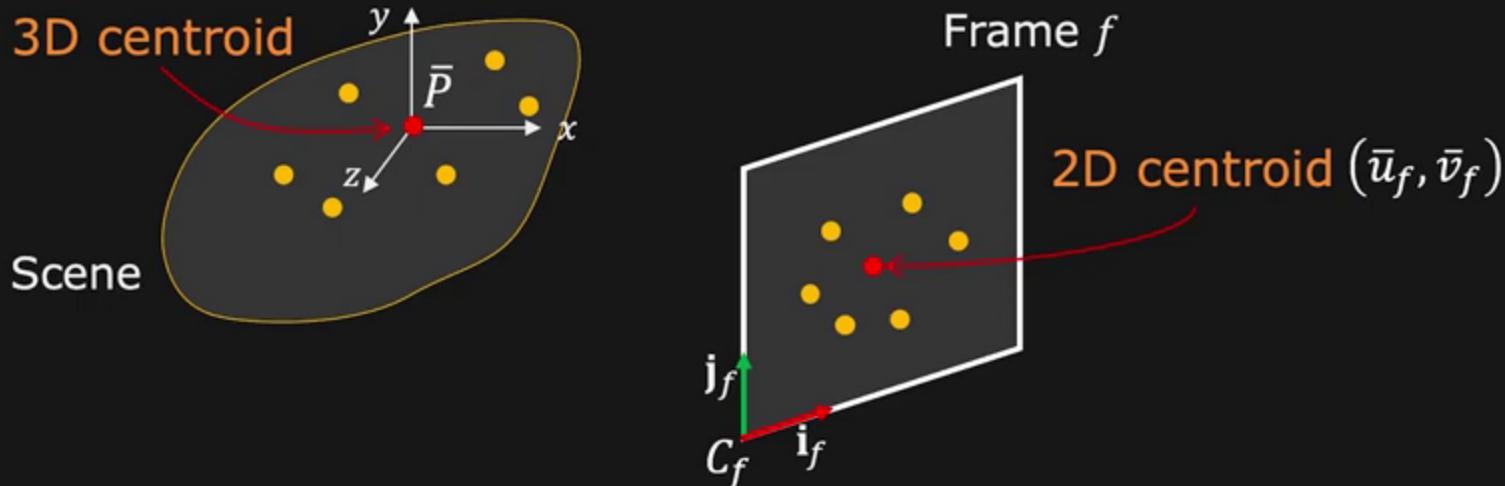


Centroid (\bar{u}_f, \bar{v}_f) of the image points in frame f :

$$\bar{u}_f = \frac{1}{N} \sum_{p=1}^N u_{f,p} = \frac{1}{N} \sum_{p=1}^N \mathbf{i}_f^T (P_p - C_f)$$



Centering Trick



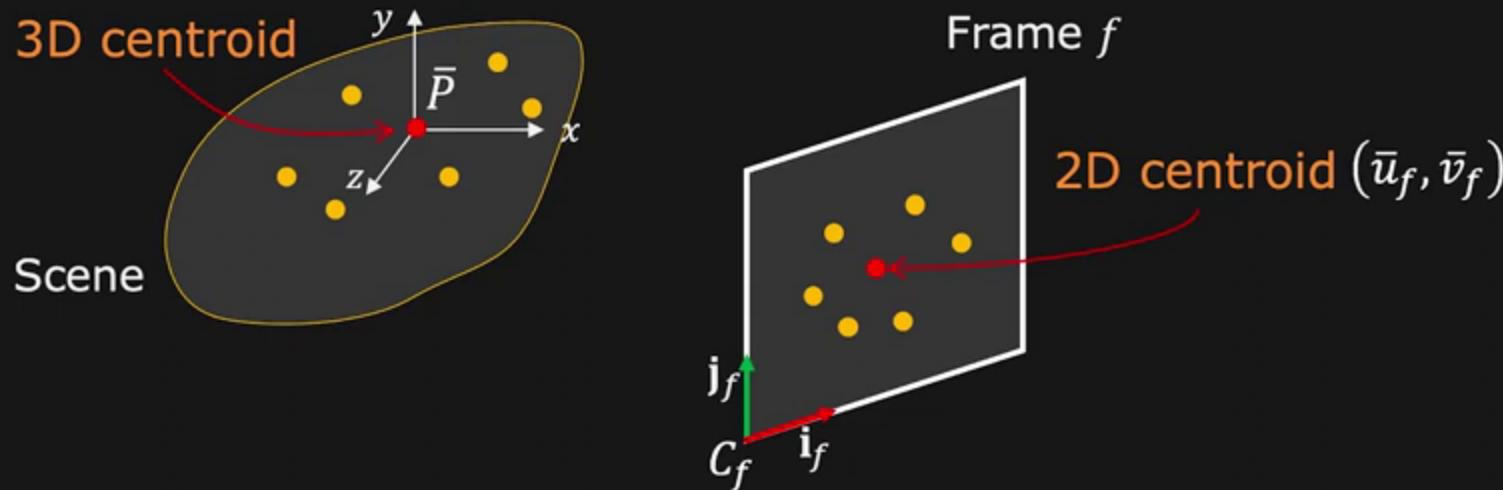
Centroid (\bar{u}_f, \bar{v}_f) of the image points in frame f :

$$\bar{u}_f = \frac{1}{N} \sum_{p=1}^N u_{f,p} = \frac{1}{N} \sum_{p=1}^N \mathbf{i}_f^T (P_p - C_f)$$

$$\bar{u}_f = \frac{1}{N} \mathbf{i}_f^T \sum_{p=1}^N P_p - \frac{1}{N} \sum_{p=1}^N \mathbf{i}_f^T C_f$$



Centering Trick



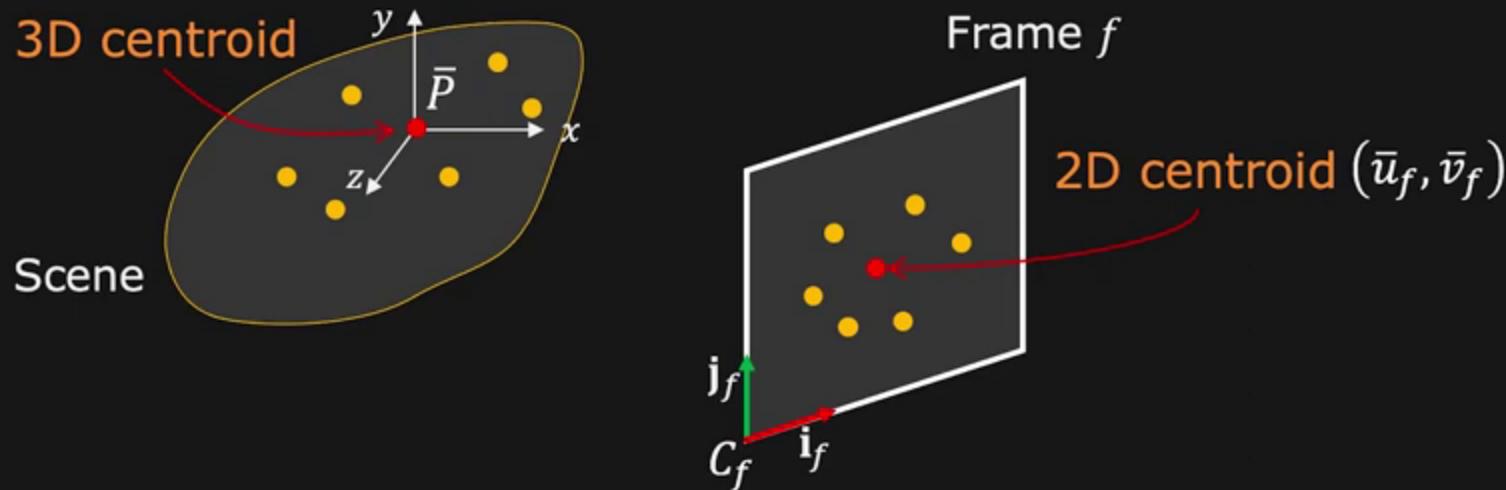
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Centering Trick



Centroid (\bar{u}_f, \bar{v}_f) of the image points in frame f :

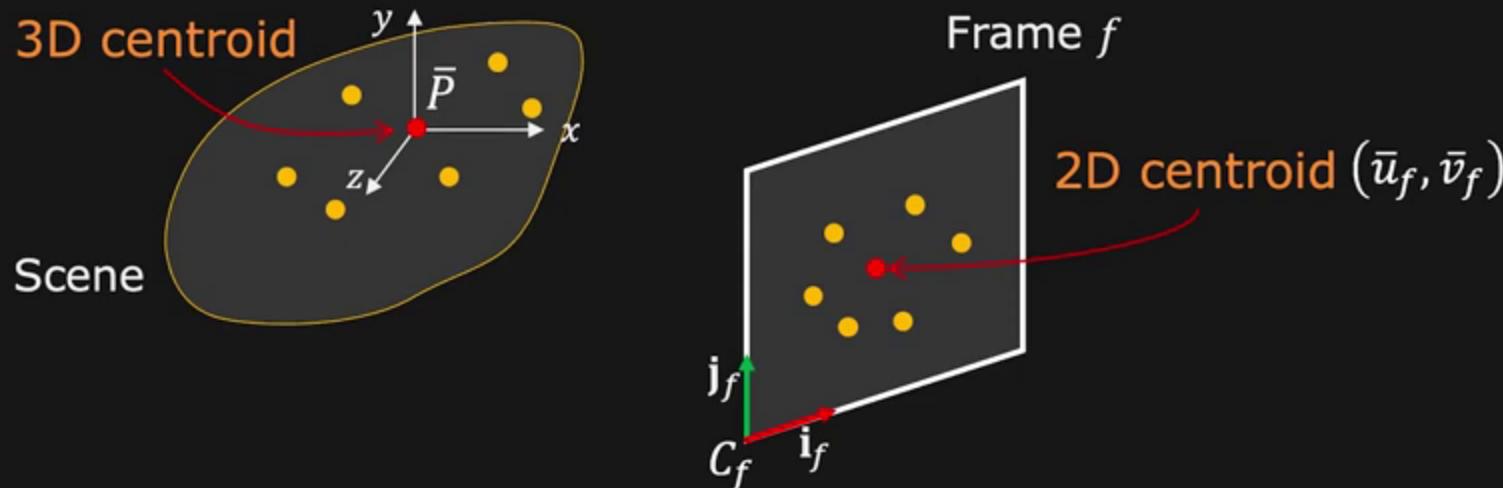
$$\bar{u}_f = \frac{1}{N} \sum_{p=1}^N u_{f,p} = \frac{1}{N} \sum_{p=1}^N \mathbf{i}_f^T (P_p - C_f)$$

$$\bar{u}_f = \frac{1}{N} \mathbf{i}_f^T \cancel{\sum_{p=1}^N P_p} - \frac{1}{N} \sum_{p=1}^N \mathbf{i}_f^T C_f$$

$$\boxed{\bar{u}_f = -\mathbf{i}_f^T C_f}$$



Centering Trick



Centroid (\bar{u}_f, \bar{v}_f) of the image points in frame f :

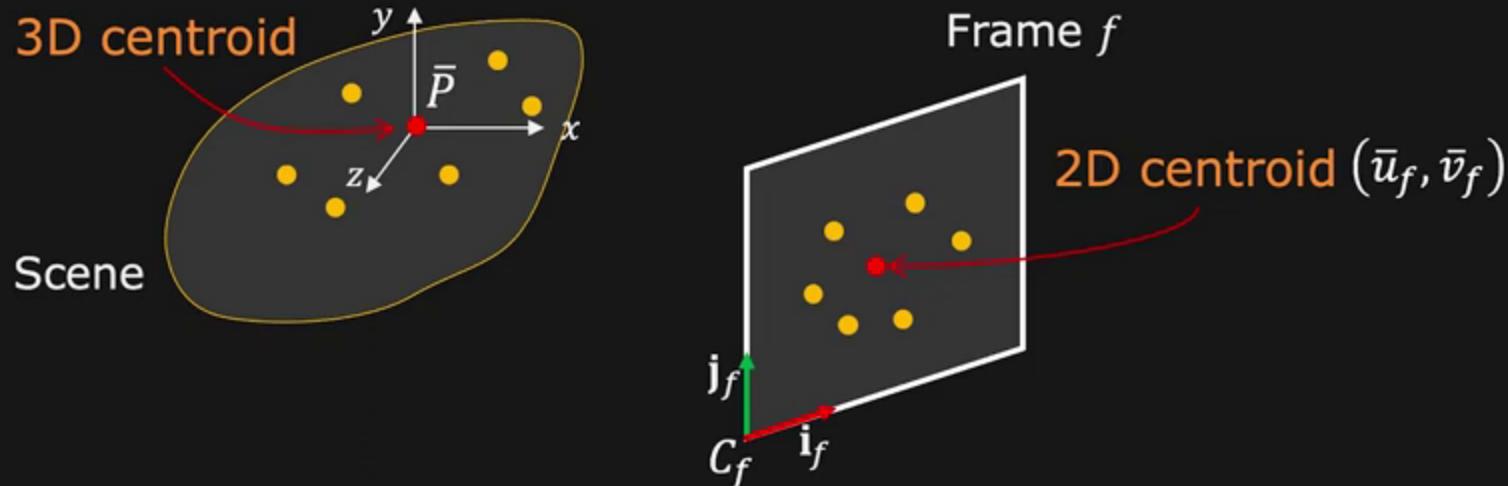
$$\bar{u}_f = \frac{1}{N} \sum_{p=1}^N u_{f,p} = \frac{1}{N} \sum_{p=1}^N \mathbf{i}_f^T (P_p - C_f)$$

$$\bar{u}_f = \frac{1}{N} \mathbf{i}_f^T \cancel{\sum_{p=1}^N P_p} - \frac{1}{N} \sum_{p=1}^N \mathbf{i}_f^T C_f$$

$$\hat{u}_f = -\mathbf{i}_f^T C_f$$



Centering Trick



Centroid (\bar{u}_f, \bar{v}_f) of the image points in frame f :

$$\bar{u}_f = \frac{1}{N} \sum_{p=1}^N u_{f,p} = \frac{1}{N} \sum_{p=1}^N \mathbf{i}_f^T (P_p - C_f)$$

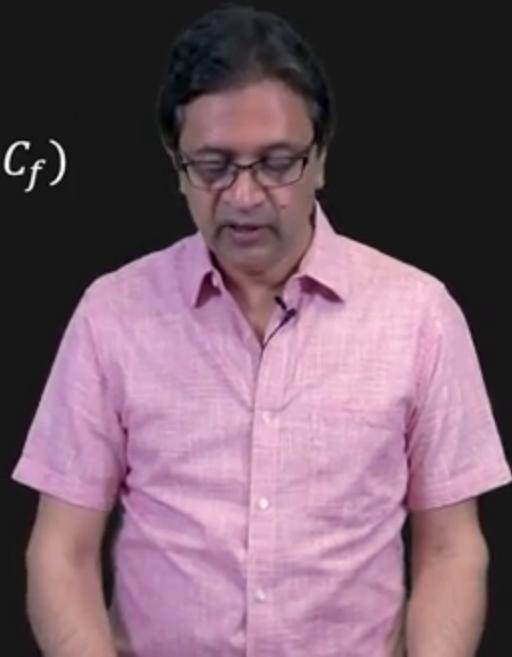
~~$$\bar{u}_f = \frac{1}{N} \mathbf{i}_f^T \sum_{p=1}^N P_p - \frac{1}{N} \sum_{p=1}^N \mathbf{i}_f^T C_f$$~~

$$\boxed{\bar{u}_f = -\mathbf{i}_f^T C_f}$$

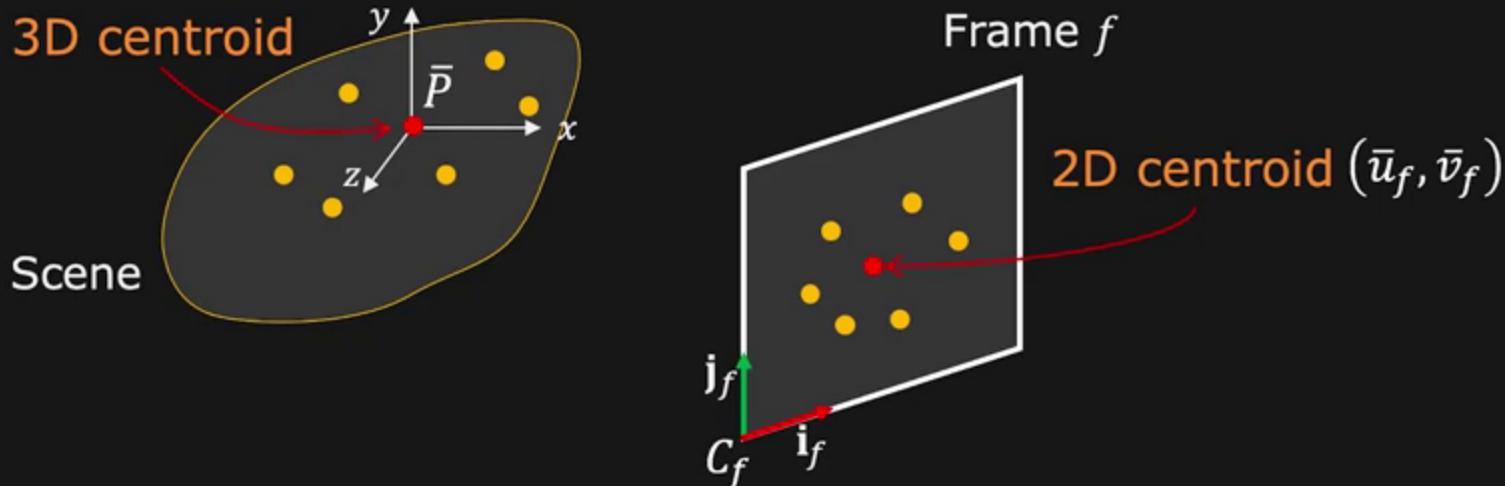
$$\bar{v}_f = \frac{1}{N} \sum_{p=1}^N v_{f,p} = \frac{1}{N} \sum_{p=1}^N \mathbf{j}_f^T (P_p - C_f)$$

~~$$\bar{v}_f = \frac{1}{N} \mathbf{j}_f^T \sum_{p=1}^N P_p - \frac{1}{N} \sum_{p=1}^N \mathbf{j}_f^T C_f$$~~

$$\boxed{\bar{v}_f = -\mathbf{j}_f^T C_f}$$



Centering Trick



Centroid (\bar{u}_f, \bar{v}_f) of the image points in frame f :

$$\bar{u}_f = \frac{1}{N} \sum_{p=1}^N u_{f,p} = \frac{1}{N} \sum_{p=1}^N \mathbf{i}_f^T (P_p - C_f)$$

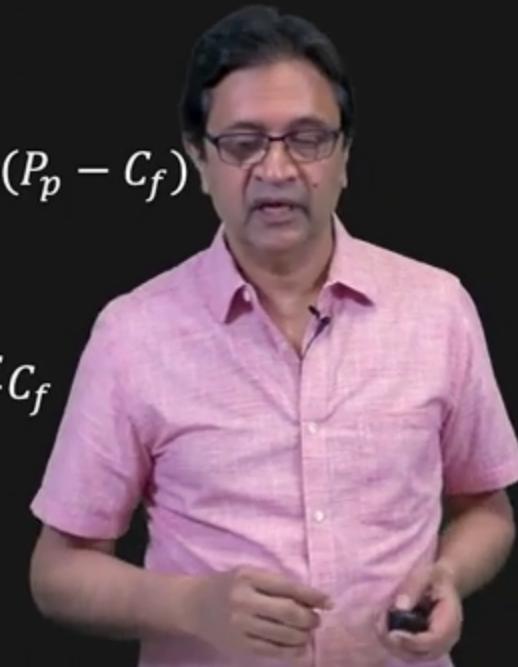
$$\bar{u}_f = \frac{1}{N} \mathbf{i}_f^T \cancel{\sum_{p=1}^N P_p} - \frac{1}{N} \sum_{p=1}^N \mathbf{i}_f^T C_f$$

$$\boxed{\bar{u}_f = -\mathbf{i}_f^T C_f}$$

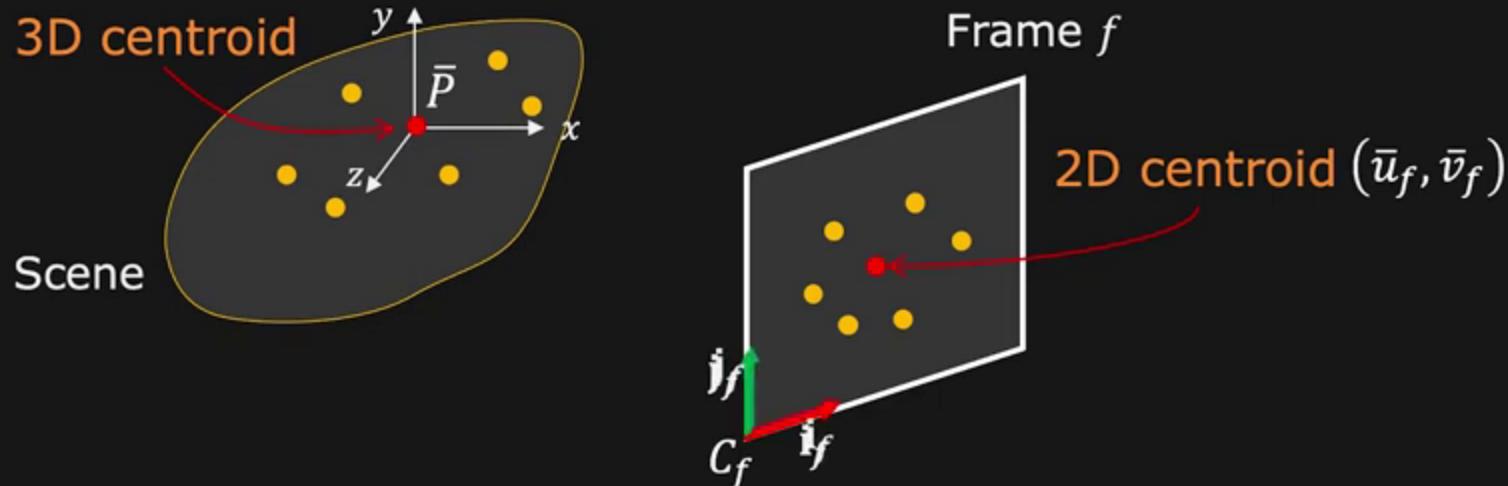
$$\bar{v}_f = \frac{1}{N} \sum_{p=1}^N v_{f,p} = \frac{1}{N} \sum_{p=1}^N \mathbf{j}_f^T (P_p - C_f)$$

$$\bar{v}_f = \frac{1}{N} \mathbf{j}_f^T \cancel{\sum_{p=1}^N P_p} - \frac{1}{N} \sum_{p=1}^N \mathbf{j}_f^T C_f$$

$$\boxed{\bar{v}_f = -\mathbf{j}_f^T C_f}$$



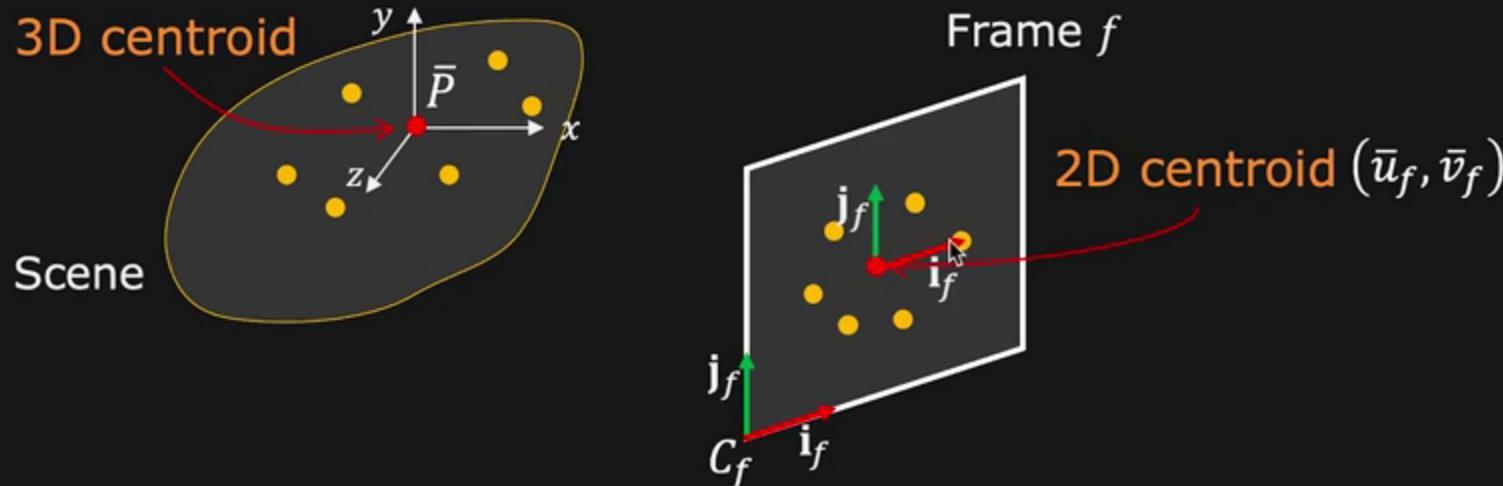
Centering Trick



Shift camera origin to the centroid (\bar{u}_f, \bar{v}_f) .



Centering Trick



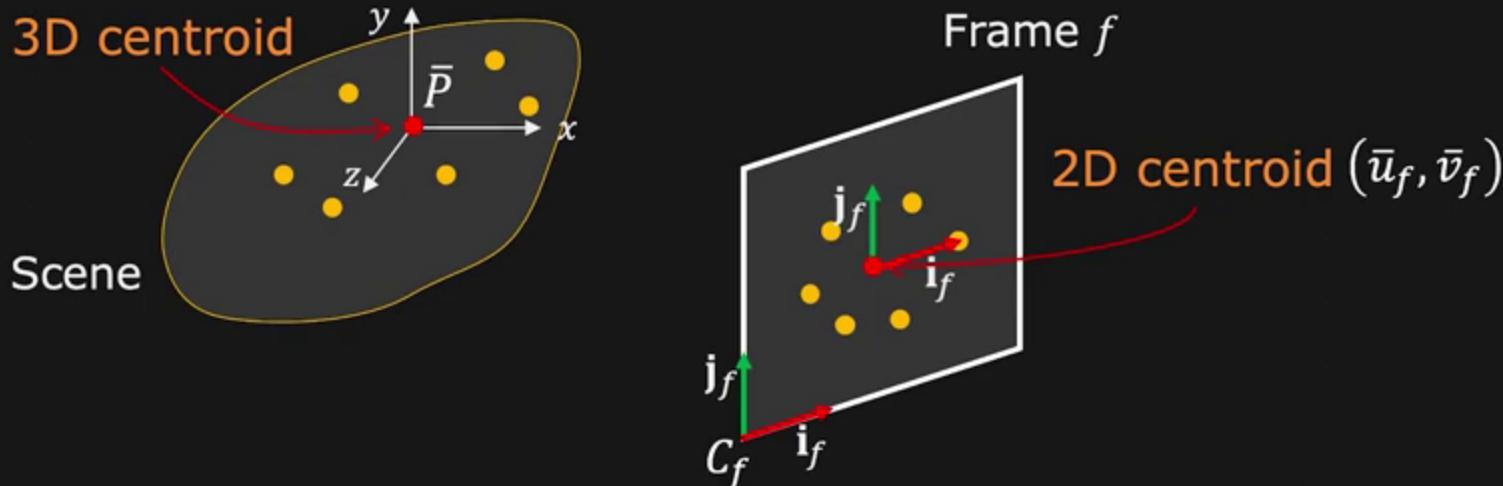
Shift camera origin to the centroid (\bar{u}_f, \bar{v}_f) .

Image points w.r.t. (\bar{u}_f, \bar{v}_f) :

$$\tilde{u}_{f,p} = u_{f,p} - \bar{u}_f$$



Centering Trick



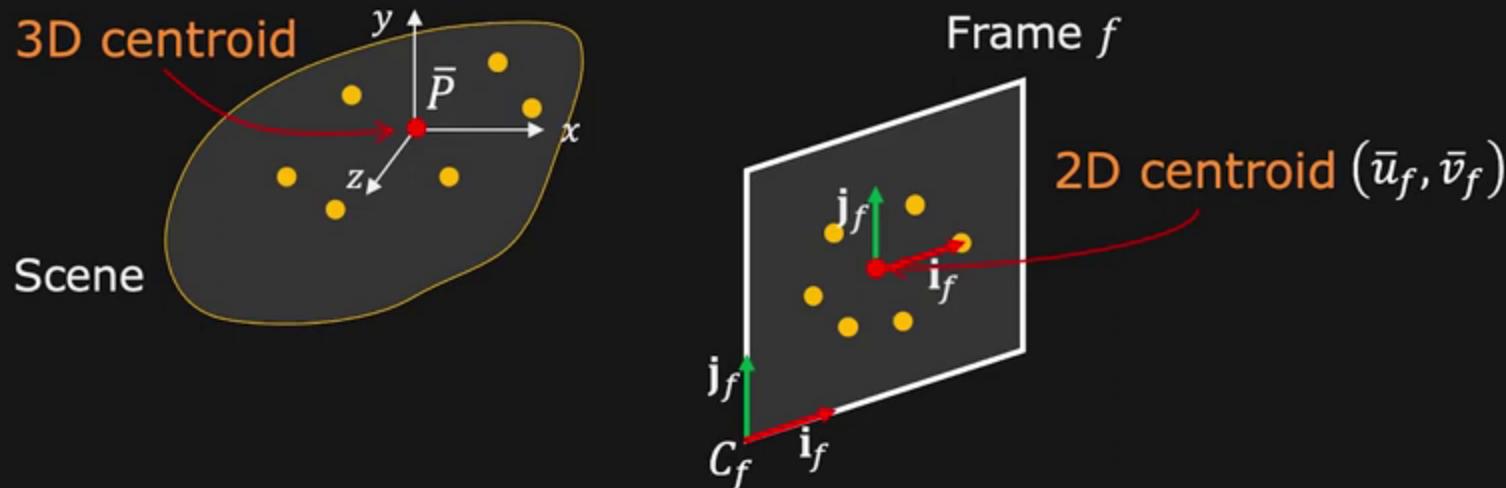
Shift camera origin to the centroid (\bar{u}_f, \bar{v}_f) .

Image points w.r.t. (\bar{u}_f, \bar{v}_f) :

$$\begin{aligned}\tilde{u}_{f,p} &= u_{f,p} - \bar{u}_f \\ &= \mathbf{i}_f^T (P_p - C_f) - \mathbf{i}_f^T C_f\end{aligned}$$



Centering Trick



Shift camera origin to the centroid (\bar{u}_f, \bar{v}_f) .

Image points w.r.t. (\bar{u}_f, \bar{v}_f) :

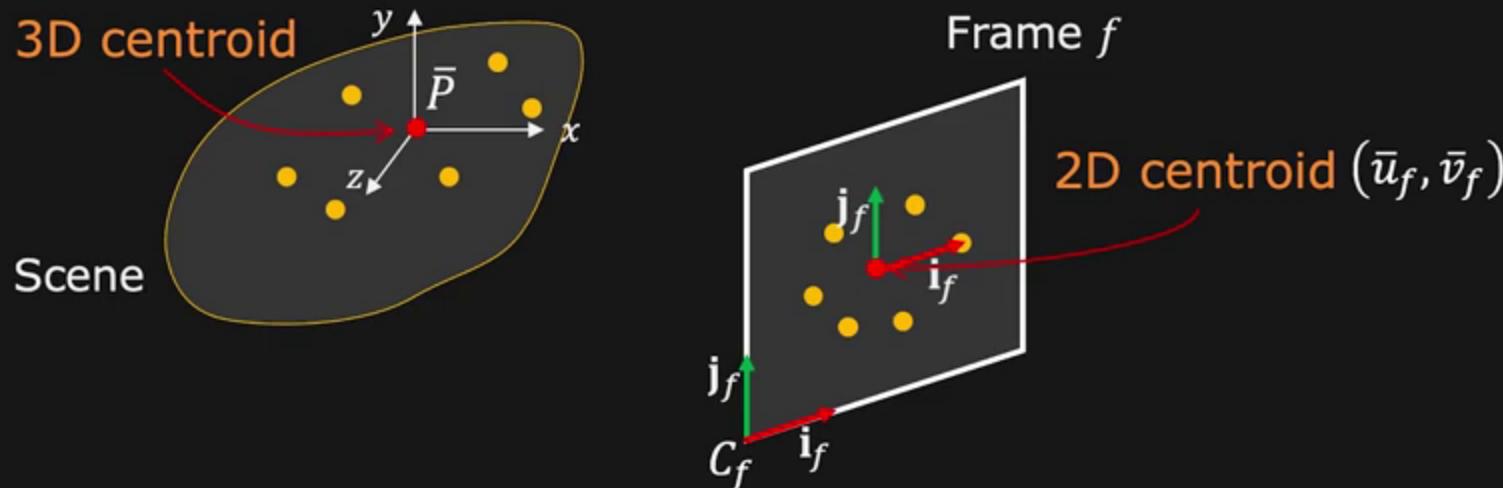
$$\tilde{u}_{f,p} = u_{f,p} - \bar{u}_f$$

$$= \mathbf{i}_f^T (P_p - C_f) - \mathbf{i}_f^T C_f$$

$$\boxed{\tilde{u}_{f,p} = \mathbf{i}_f^T P_p}$$



Centering Trick



Shift camera origin to the centroid (\bar{u}_f, \bar{v}_f) .

Image points w.r.t. (\bar{u}_f, \bar{v}_f) :

$$\begin{aligned}\tilde{u}_{f,p} &= u_{f,p} - \bar{u}_f \\ &= \mathbf{i}_f^T (P_p - C_f) - \mathbf{i}_f^T C_f\end{aligned}$$

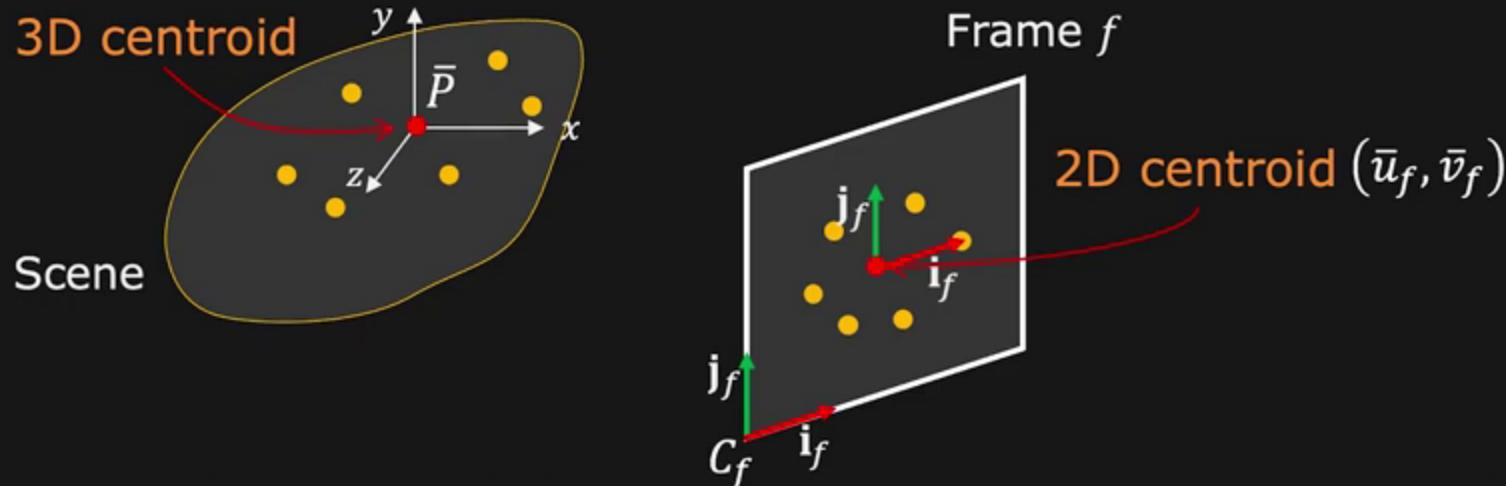
$$\boxed{\tilde{u}_{f,p} = \mathbf{i}_f^T P_p}$$

$$\begin{aligned}\tilde{v}_{f,p} &= v_{f,p} - \bar{v}_f \\ &= \mathbf{j}_f^T (P_p - C_f) - \mathbf{j}_f^T C_f\end{aligned}$$

$$\boxed{\tilde{v}_{f,p} = \mathbf{j}_f^T P_p}$$



Centering Trick



Shift camera origin to the centroid (\bar{u}_f, \bar{v}_f) .

Image points w.r.t. (\bar{u}_f, \bar{v}_f) :

$$\tilde{u}_{f,p} = u_{f,p} - \bar{u}_f$$

$$\tilde{v}_{f,p} = v_{f,p} - \bar{v}_f$$

$$= \mathbf{i}_f^T (P_p - C_f) - \mathbf{i}_f^T C_f$$

$$= \mathbf{j}_f^T (P_p - C_f) - \mathbf{j}_f^T C_f$$

$$\boxed{\tilde{u}_{f,p} = \mathbf{i}_f^T P_p}$$

$$\boxed{\tilde{v}_{f,p} = \mathbf{j}_f^T P_p}$$

Camera locations C_f now removed from equations.



Observation Matrix W

$$\tilde{u}_{f,p} = \mathbf{i}_f^T P_p$$

$$\tilde{v}_{f,p} = \mathbf{j}_f^T P_p$$



$$\begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$



Observation Matrix W

$$\tilde{u}_{f,p} = \mathbf{i}_f^T P_p$$

$$\tilde{v}_{f,p} = \mathbf{j}_f^T P_p$$



$$\begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$



Observation Matrix W

$$\tilde{u}_{f,p} = \mathbf{i}_f^T P_p$$

$$\tilde{v}_{f,p} = \mathbf{j}_f^T P_p$$



$$\begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$



Observation Matrix W

$$\begin{aligned}\tilde{u}_{f,p} &= \mathbf{i}_f^T P_p \\ \tilde{v}_{f,p} &= \mathbf{j}_f^T P_p\end{aligned}\quad \Rightarrow \quad \begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

$$\begin{array}{c} \text{Point 1} \quad \text{Point 2} \quad \quad \text{Point N} \\ \hline \text{Image 1} \quad \left[\begin{array}{cccc} \tilde{u}_{1,1} & \tilde{u}_{1,2} & \dots & \tilde{u}_{1,N} \end{array} \right] \quad \left[\begin{array}{c} \mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \end{array} \right] \\ \text{Image 2} \quad \left[\begin{array}{cccc} \tilde{u}_{2,1} & \tilde{u}_{2,2} & \dots & \tilde{u}_{2,N} \end{array} \right] \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \text{Image F} \quad \left[\begin{array}{cccc} \tilde{u}_{F,1} & \tilde{u}_{F,2} & \dots & \tilde{u}_{F,N} \end{array} \right] \\ \hline \text{Image 1} \quad \left[\begin{array}{cccc} \tilde{v}_{1,1} & \tilde{v}_{1,2} & \dots & \tilde{v}_{1,N} \end{array} \right] \\ \text{Image 2} \quad \left[\begin{array}{cccc} \tilde{v}_{2,1} & \tilde{v}_{2,2} & \dots & \tilde{v}_{2,N} \end{array} \right] \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \text{Image F} \quad \left[\begin{array}{cccc} \tilde{v}_{F,1} & \tilde{v}_{F,2} & \dots & \tilde{v}_{F,N} \end{array} \right] \end{array} = \left[\begin{array}{c} \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T \end{array} \right] \begin{array}{c} \text{Point 1} \quad \text{Point 2} \quad \quad \text{Point N} \\ \hline [P_1 \quad P_2 \quad \dots \quad P_N] \\ S_{3 \times N} \\ \text{Scene Structure} \\ (\text{Unknown}) \end{array}$$

$W_{2F \times N}$ $M_{2F \times 3}$

Centroid-Subtracted
Feature Points (Known)

Camera Motion
(Unknown)



Observation Matrix W

$$\begin{aligned}\tilde{u}_{f,p} &= \mathbf{i}_f^T P_p \\ \tilde{v}_{f,p} &= \mathbf{j}_f^T P_p\end{aligned}\quad \Rightarrow \quad \begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

$$\begin{array}{c} \text{Point 1} \quad \text{Point 2} \quad \quad \text{Point N} \\ \hline \text{Image 1} \quad \left[\begin{array}{cccc} \tilde{u}_{1,1} & \tilde{u}_{1,2} & \dots & \tilde{u}_{1,N} \end{array} \right] \quad \left[\begin{array}{c} \mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \end{array} \right] \\ \text{Image 2} \quad \left[\begin{array}{cccc} \tilde{u}_{2,1} & \tilde{u}_{2,2} & \dots & \tilde{u}_{2,N} \end{array} \right] \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \text{Image F} \quad \left[\begin{array}{cccc} \tilde{u}_{F,1} & \tilde{u}_{F,2} & \dots & \tilde{u}_{F,N} \end{array} \right] \\ \hline \text{Image 1} \quad \left[\begin{array}{cccc} \tilde{v}_{1,1} & \tilde{v}_{1,2} & \dots & \tilde{v}_{1,N} \end{array} \right] \\ \text{Image 2} \quad \left[\begin{array}{cccc} \tilde{v}_{2,1} & \tilde{v}_{2,2} & \dots & \tilde{v}_{2,N} \end{array} \right] \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \text{Image F} \quad \left[\begin{array}{cccc} \tilde{v}_{F,1} & \tilde{v}_{F,2} & \dots & \tilde{v}_{F,N} \end{array} \right] \end{array} = \left[\begin{array}{c} \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T \end{array} \right] \begin{array}{c} \text{Point 1} \quad \text{Point 2} \quad \quad \text{Point N} \\ \hline [P_1 \quad P_2 \quad \dots \quad P_N] \\ S_{3 \times N} \\ \text{Scene Structure} \\ (\text{Unknown}) \end{array}$$

$W_{2F \times N}$ $M_{2F \times 3}$

Centroid-Subtracted
Feature Points (Known)

Camera Motion
(Unknown)



Observation Matrix W

$$\begin{aligned}\tilde{u}_{f,p} &= \mathbf{i}_f^T P_p \\ \tilde{v}_{f,p} &= \mathbf{j}_f^T P_p\end{aligned}$$



$$\begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

$$\begin{array}{c} \text{Point 1} \quad \text{Point 2} \quad \quad \quad \text{Point N} \\ \hline \text{Image 1} \quad \left[\begin{array}{cccc} \tilde{u}_{1,1} & \tilde{u}_{1,2} & \dots & \tilde{u}_{1,N} \end{array} \right] \quad \quad \quad \left[\begin{array}{c} \mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \end{array} \right] \\ \text{Image 2} \quad \left[\begin{array}{cccc} \tilde{u}_{2,1} & \tilde{u}_{2,2} & \dots & \tilde{u}_{2,N} \end{array} \right] \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \text{Image F} \quad \left[\begin{array}{cccc} \tilde{u}_{F,1} & \tilde{u}_{F,2} & \dots & \tilde{u}_{F,N} \end{array} \right] \\ \hline \text{Image 1} \quad \left[\begin{array}{cccc} \tilde{v}_{1,1} & \tilde{v}_{1,2} & \dots & \tilde{v}_{1,N} \end{array} \right] \\ \text{Image 2} \quad \left[\begin{array}{cccc} \tilde{v}_{2,1} & \tilde{v}_{2,2} & \dots & \tilde{v}_{2,N} \end{array} \right] \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \text{Image F} \quad \left[\begin{array}{cccc} \tilde{v}_{F,1} & \tilde{v}_{F,2} & \dots & \tilde{v}_{F,N} \end{array} \right] \end{array} = \left[\begin{array}{c} \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T \end{array} \right] \begin{array}{c} \text{Point 1} \quad \text{Point 2} \quad \quad \quad \text{Point N} \\ \hline [P_1 \quad P_2 \quad \dots \quad P_N] \\ S_{3 \times N} \\ \text{Scene Structure} \\ (\text{Unknown}) \end{array}$$

$W_{2F \times N}$ $M_{2F \times 3}$

Centroid-Subtracted
Feature Points (Known)

Camera Motion
(Unknown)



Observation Matrix W

$$\begin{aligned}\tilde{u}_{f,p} &= \mathbf{i}_f^T P_p \\ \tilde{v}_{f,p} &= \mathbf{j}_f^T P_p\end{aligned}$$



$$\begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

$$\begin{array}{c} \text{Point 1} \quad \text{Point 2} \quad \dots \quad \text{Point N} \\ \hline \text{Image 1} \quad \left[\begin{array}{cccc} \tilde{u}_{1,1} & \tilde{u}_{1,2} & \dots & \tilde{u}_{1,N} \end{array} \right] = \begin{bmatrix} \mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \end{bmatrix} [P_1 \quad P_2 \quad \dots \quad P_N] \\ \text{Image 2} \quad \left[\begin{array}{cccc} \tilde{u}_{2,1} & \tilde{u}_{2,2} & \dots & \tilde{u}_{2,N} \end{array} \right] \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \text{Image F} \quad \left[\begin{array}{cccc} \tilde{u}_{F,1} & \tilde{u}_{F,2} & \dots & \tilde{u}_{F,N} \end{array} \right] \\ \hline \text{Image 1} \quad \left[\begin{array}{cccc} \tilde{v}_{1,1} & \tilde{v}_{1,2} & \dots & \tilde{v}_{1,N} \end{array} \right] \\ \text{Image 2} \quad \left[\begin{array}{cccc} \tilde{v}_{2,1} & \tilde{v}_{2,2} & \dots & \tilde{v}_{2,N} \end{array} \right] \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \text{Image F} \quad \left[\begin{array}{cccc} \tilde{v}_{F,1} & \tilde{v}_{F,2} & \dots & \tilde{v}_{F,N} \end{array} \right] \end{array} = \begin{bmatrix} \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T \end{bmatrix} S_{3 \times N}$$

$W_{2F \times N}$ $M_{2F \times 3}$

Centroid-Subtracted
Feature Points (Known) Camera Motion
(Unknown)



Observation Matrix W

$$\begin{aligned}\tilde{u}_{f,p} &= \mathbf{i}_f^T P_p \\ \tilde{v}_{f,p} &= \mathbf{j}_f^T P_p\end{aligned}\quad \Rightarrow \quad \begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

$$\begin{array}{c} \text{Point 1} \quad \text{Point 2} \quad \quad \text{Point N} \\ \hline \text{Image 1} \quad \left[\begin{array}{cccc} \tilde{u}_{1,1} & \tilde{u}_{1,2} & \dots & \tilde{u}_{1,N} \end{array} \right] \quad \left[\begin{array}{c} \mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \end{array} \right] \\ \text{Image 2} \quad \left[\begin{array}{cccc} \tilde{u}_{2,1} & \tilde{u}_{2,2} & \dots & \tilde{u}_{2,N} \end{array} \right] \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \text{Image F} \quad \left[\begin{array}{cccc} \tilde{u}_{F,1} & \tilde{u}_{F,2} & \dots & \tilde{u}_{F,N} \end{array} \right] \\ \hline \text{Image 1} \quad \left[\begin{array}{cccc} \tilde{v}_{1,1} & \tilde{v}_{1,2} & \dots & \tilde{v}_{1,N} \end{array} \right] \\ \text{Image 2} \quad \left[\begin{array}{cccc} \tilde{v}_{2,1} & \tilde{v}_{2,2} & \dots & \tilde{v}_{2,N} \end{array} \right] \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \text{Image F} \quad \left[\begin{array}{cccc} \tilde{v}_{F,1} & \tilde{v}_{F,2} & \dots & \tilde{v}_{F,N} \end{array} \right] \end{array} = \left[\begin{array}{c} \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T \end{array} \right] \begin{array}{c} \text{Point 1} \quad \text{Point 2} \quad \quad \text{Point N} \\ \hline [P_1 \quad P_2 \quad \dots \quad P_N] \\ S_{3 \times N} \\ \text{Scene Structure} \\ (\text{Unknown}) \end{array}$$

$W_{2F \times N}$ $M_{2F \times 3}$

Centroid-Subtracted
Feature Points (Known)

Camera Motion
(Unknown)



Observation Matrix W

$$\begin{aligned}\tilde{u}_{f,p} &= \mathbf{i}_f^T P_p \\ \tilde{v}_{f,p} &= \mathbf{j}_f^T P_p\end{aligned}$$



$$\begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

$$\begin{array}{c} \text{Point 1} \quad \text{Point 2} \quad \quad \text{Point N} \\ \hline \text{Image 1} \quad \left[\begin{array}{cccc} \tilde{u}_{1,1} & \tilde{u}_{1,2} & \dots & \tilde{u}_{1,N} \end{array} \right] \quad \left[\begin{array}{c} \mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \end{array} \right] \\ \text{Image 2} \quad \left[\begin{array}{cccc} \tilde{u}_{2,1} & \tilde{u}_{2,2} & \dots & \tilde{u}_{2,N} \end{array} \right] \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \text{Image F} \quad \left[\begin{array}{cccc} \tilde{u}_{F,1} & \tilde{u}_{F,2} & \dots & \tilde{u}_{F,N} \end{array} \right] \\ \hline \text{Image 1} \quad \left[\begin{array}{cccc} \tilde{v}_{1,1} & \tilde{v}_{1,2} & \dots & \tilde{v}_{1,N} \end{array} \right] \\ \text{Image 2} \quad \left[\begin{array}{cccc} \tilde{v}_{2,1} & \tilde{v}_{2,2} & \dots & \tilde{v}_{2,N} \end{array} \right] \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \text{Image F} \quad \left[\begin{array}{cccc} \tilde{v}_{F,1} & \tilde{v}_{F,2} & \dots & \tilde{v}_{F,N} \end{array} \right] \end{array} = \left[\begin{array}{c} \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T \end{array} \right] \left[\begin{array}{cccc} P_1 & P_2 & \dots & P_N \end{array} \right]$$

$W_{2F \times N}$ $M_{2F \times 3}$

Centroid-Subtracted Feature Points (Known) Camera Motion (Unknown)

$S_{3 \times N}$
Scene Structure (Unknown)



Observation Matrix W

$$\begin{aligned}\tilde{u}_{f,p} &= \mathbf{i}_f^T P_p \\ \tilde{v}_{f,p} &= \mathbf{j}_f^T P_p\end{aligned}$$



$$\begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

$$\begin{array}{c} \text{Point 1} \quad \text{Point 2} \quad \quad \text{Point N} \\ \hline \text{Image 1} \quad \left[\begin{array}{cccc} \tilde{u}_{1,1} & \tilde{u}_{1,2} & \dots & \tilde{u}_{1,N} \end{array} \right] \quad \left[\begin{array}{c} \mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \end{array} \right] \\ \text{Image 2} \quad \left[\begin{array}{cccc} \tilde{u}_{2,1} & \tilde{u}_{2,2} & \dots & \tilde{u}_{2,N} \end{array} \right] = \left[\begin{array}{c} \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T \end{array} \right] \quad \begin{array}{c} \text{Point 1} \quad \text{Point 2} \quad \quad \text{Point N} \\ [P_1 \quad P_2 \quad \dots \quad P_N] \end{array} \\ \vdots \\ \text{Image F} \quad \left[\begin{array}{cccc} \tilde{u}_{F,1} & \tilde{u}_{F,2} & \dots & \tilde{u}_{F,N} \end{array} \right] \\ \hline \text{Image 1} \quad \left[\begin{array}{cccc} \tilde{v}_{1,1} & \tilde{v}_{1,2} & \dots & \tilde{v}_{1,N} \end{array} \right] \\ \text{Image 2} \quad \left[\begin{array}{cccc} \tilde{v}_{2,1} & \tilde{v}_{2,2} & \dots & \tilde{v}_{2,N} \end{array} \right] \\ \vdots \\ \text{Image F} \quad \left[\begin{array}{cccc} \tilde{v}_{F,1} & \tilde{v}_{F,2} & \dots & \tilde{v}_{F,N} \end{array} \right] \end{array} = S_{3 \times N}$$

$W_{2F \times N}$ $M_{2F \times 3}$

Centroid-Subtracted
Feature Points (Known)

Camera Motion
(Unknown)



Observation Matrix W

$$\begin{aligned}\tilde{u}_{f,p} &= \mathbf{i}_f^T P_p \\ \tilde{v}_{f,p} &= \mathbf{j}_f^T P_p\end{aligned}$$



$$\begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

$$\begin{array}{c} \text{Point 1} \quad \text{Point 2} \quad \quad \text{Point N} \\ \hline \text{Image 1} \quad \left[\begin{array}{cccc} \tilde{u}_{1,1} & \tilde{u}_{1,2} & \dots & \tilde{u}_{1,N} \end{array} \right] \quad \left[\begin{array}{c} \mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \end{array} \right] \\ \text{Image 2} \quad \left[\begin{array}{cccc} \tilde{u}_{2,1} & \tilde{u}_{2,2} & \dots & \tilde{u}_{2,N} \end{array} \right] \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \text{Image F} \quad \left[\begin{array}{cccc} \tilde{u}_{F,1} & \tilde{u}_{F,2} & \dots & \tilde{u}_{F,N} \end{array} \right] \\ \hline \text{Image 1} \quad \left[\begin{array}{cccc} \tilde{v}_{1,1} & \tilde{v}_{1,2} & \dots & \tilde{v}_{1,N} \end{array} \right] \\ \text{Image 2} \quad \left[\begin{array}{cccc} \tilde{v}_{2,1} & \tilde{v}_{2,2} & \dots & \tilde{v}_{2,N} \end{array} \right] \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \text{Image F} \quad \left[\begin{array}{cccc} \tilde{v}_{F,1} & \tilde{v}_{F,2} & \dots & \tilde{v}_{F,N} \end{array} \right] \end{array} = \left[\begin{array}{c} \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T \end{array} \right] \begin{array}{c} \text{Point 1} \quad \text{Point 2} \quad \quad \text{Point N} \\ \hline [P_1 \quad P_2 \quad \dots \quad P_N] \\ S_{3 \times N} \\ \text{Scene Structure} \\ (\text{Unknown}) \end{array}$$

$W_{2F \times N}$ $M_{2F \times 3}$

Centroid-Subtracted
Feature Points (Known)

Camera Motion
(Unknown)



Observation Matrix W

$$\begin{aligned}\tilde{u}_{f,p} &= \mathbf{i}_f^T P_p \\ \tilde{v}_{f,p} &= \mathbf{j}_f^T P_p\end{aligned}$$



$$\begin{bmatrix} \tilde{u}_{f,p} \\ \tilde{v}_{f,p} \end{bmatrix} = \begin{bmatrix} \mathbf{i}_f^T \\ \mathbf{j}_f^T \end{bmatrix} P_p$$

$$\begin{array}{c} \text{Point 1} \quad \text{Point 2} \quad \quad \text{Point N} \\ \hline \text{Image 1} \quad \left[\begin{array}{cccc} \tilde{u}_{1,1} & \tilde{u}_{1,2} & \dots & \tilde{u}_{1,N} \end{array} \right] \quad \left[\begin{array}{c} \mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \end{array} \right] \\ \text{Image 2} \quad \left[\begin{array}{cccc} \tilde{u}_{2,1} & \tilde{u}_{2,2} & \dots & \tilde{u}_{2,N} \end{array} \right] \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \text{Image F} \quad \left[\begin{array}{cccc} \tilde{u}_{F,1} & \tilde{u}_{F,2} & \dots & \tilde{u}_{F,N} \end{array} \right] \\ \hline \text{Image 1} \quad \left[\begin{array}{cccc} \tilde{v}_{1,1} & \tilde{v}_{1,2} & \dots & \tilde{v}_{1,N} \end{array} \right] \\ \text{Image 2} \quad \left[\begin{array}{cccc} \tilde{v}_{2,1} & \tilde{v}_{2,2} & \dots & \tilde{v}_{2,N} \end{array} \right] \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \text{Image F} \quad \left[\begin{array}{cccc} \tilde{v}_{F,1} & \tilde{v}_{F,2} & \dots & \tilde{v}_{F,N} \end{array} \right] \end{array} = \left[\begin{array}{c} \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T \end{array} \right] \left[\begin{array}{cccc} P_1 & P_2 & \dots & P_N \end{array} \right]$$

$W_{2F \times N}$ $M_{2F \times 3}$

Centroid-Subtracted Feature Points (Known) Camera Motion (Unknown)

$S_{3 \times N}$
Scene Structure (Unknown)



Observation Matrix W

$$\begin{array}{c}
 \text{Point 1} \quad \text{Point 2} \quad \dots \quad \text{Point N} \\
 \text{Image 1} \quad \tilde{u}_{1,1} \quad \tilde{u}_{1,2} \quad \dots \quad \tilde{u}_{1,N} \quad \left[\begin{array}{c} \mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \end{array} \right] \\
 \text{Image 2} \quad \tilde{u}_{2,1} \quad \tilde{u}_{2,2} \quad \dots \quad \tilde{u}_{2,N} \\
 \vdots \quad \vdots \quad \vdots \quad \vdots \\
 \text{Image F} \quad \tilde{u}_{F,1} \quad \tilde{u}_{F,2} \quad \dots \quad \tilde{u}_{F,N} \\
 \text{Image 1} \quad \tilde{v}_{1,1} \quad \tilde{v}_{1,2} \quad \dots \quad \tilde{v}_{1,N} \quad [P_1 \quad P_2 \quad \dots \quad P_N] \\
 \text{Image 2} \quad \tilde{v}_{2,1} \quad \tilde{v}_{2,2} \quad \dots \quad \tilde{v}_{2,N} \\
 \vdots \quad \vdots \quad \vdots \quad \vdots \\
 \text{Image F} \quad \tilde{v}_{F,1} \quad \tilde{v}_{F,2} \quad \dots \quad \tilde{v}_{F,N} \quad \left[\begin{array}{c} \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T \end{array} \right] \\
 \text{Centroid-Subtracted Feature Points (Known)} \quad W_{2F \times N} \\
 \text{Scene Structure (Unknown)} \quad S_{3 \times N} \\
 \text{Camera Motion (Unknown)} \quad M_{2F \times 3}
 \end{array}$$

Can we find M and S from W ?



Observation Matrix W

$$\begin{array}{c} \text{Point 1} \quad \text{Point 2} \quad \dots \quad \text{Point N} \\ \hline \text{Image 1} \quad \tilde{u}_{1,1} \quad \tilde{u}_{1,2} \quad \dots \quad \tilde{u}_{1,N} \\ \text{Image 2} \quad \tilde{u}_{2,1} \quad \tilde{u}_{2,2} \quad \dots \quad \tilde{u}_{2,N} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \text{Image F} \quad \tilde{u}_{F,1} \quad \tilde{u}_{F,2} \quad \dots \quad \tilde{u}_{F,N} \\ \hline \text{Image 1} \quad \tilde{v}_{1,1} \quad \tilde{v}_{1,2} \quad \dots \quad \tilde{v}_{1,N} \\ \text{Image 2} \quad \tilde{u}_{2,1} \quad \tilde{u}_{2,2} \quad \dots \quad \tilde{v}_{2,N} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ \text{Image F} \quad \tilde{v}_{F,1} \quad \tilde{v}_{F,2} \quad \dots \quad \tilde{v}_{F,N} \end{array} = \begin{bmatrix} \mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \\ \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T \end{bmatrix} \begin{array}{c} \text{Point 1} \quad \text{Point 2} \quad \dots \quad \text{Point N} \\ \hline [P_1 \quad P_2 \quad \dots \quad P_N] \\ S_{3 \times N} \\ \text{Scene Structure} \\ (\text{Unknown}) \end{array}$$

$W_{2F \times N}$ $M_{2F \times 3}$

Centroid-Subtracted
Feature Points (Known) Camera Motion
 (Unknown)

Can we find M and S from W ?

