

Finding Correspondences

Shree K. Nayar

Columbia University

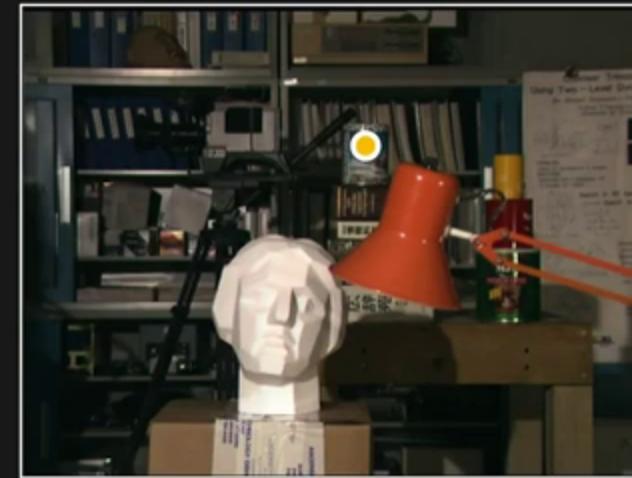
Topic: Uncalibrated Stereo, Module: Reconstruction II

First Principles of Computer Vision

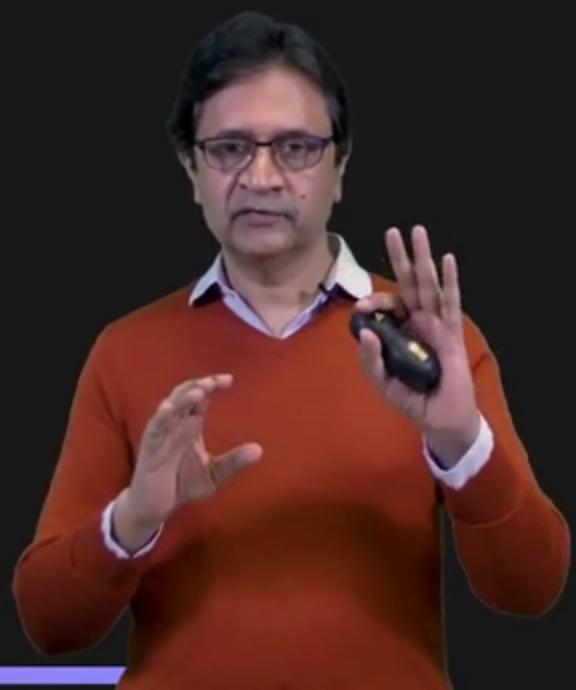
Simple Stereo: Finding Correspondences



Left Camera Image



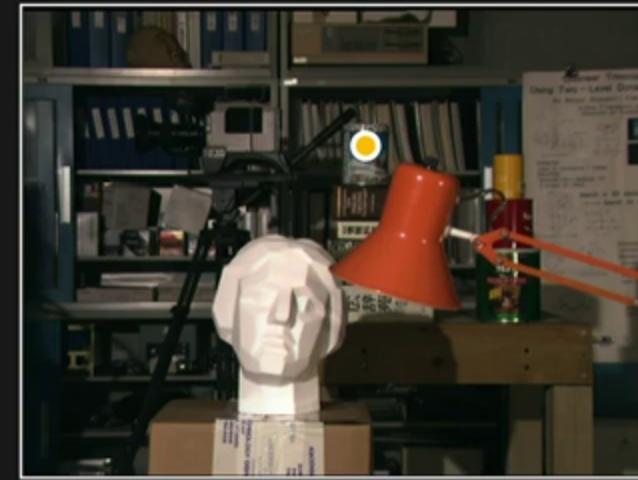
Right Camera Image



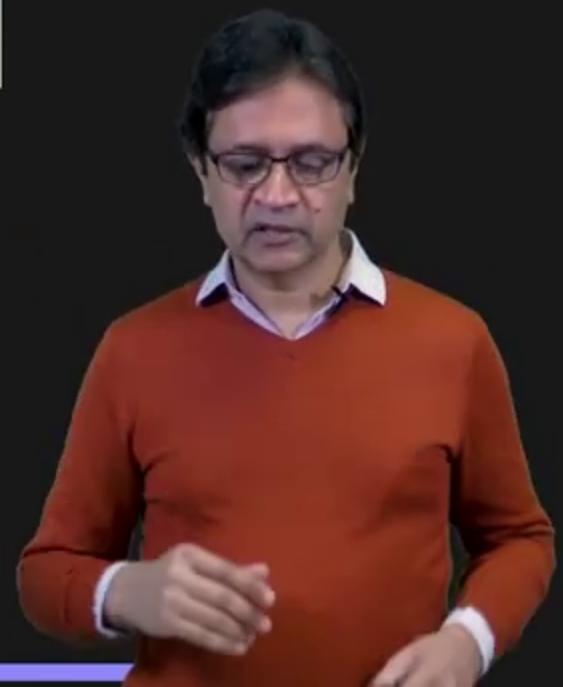
Simple Stereo: Finding Correspondences



Left Camera Image

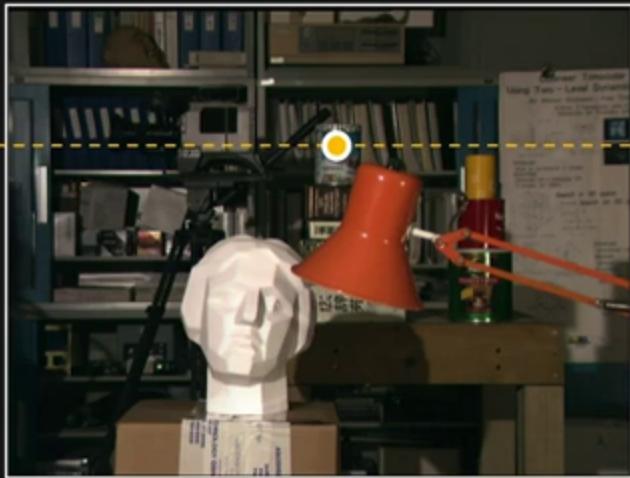


Right Camera Image

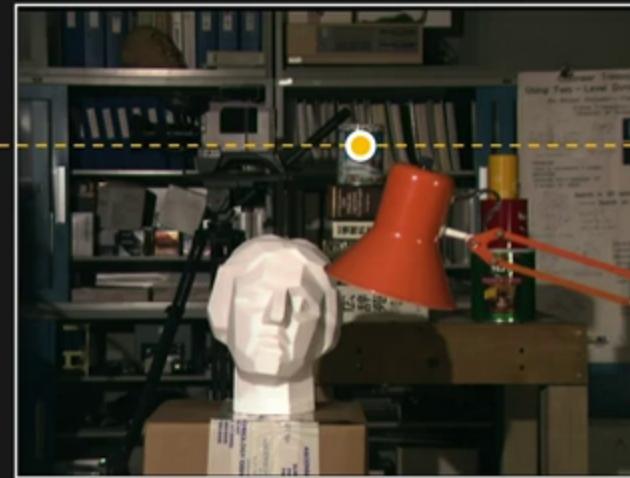


REVIEW

Simple Stereo: Finding Correspondences



Left Camera Image

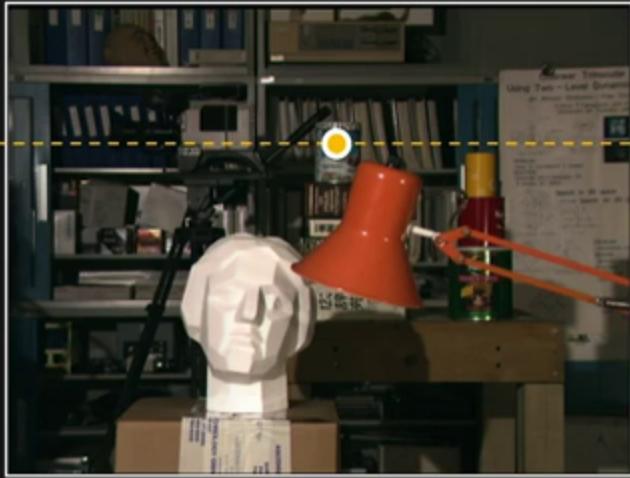


Right Camera Image

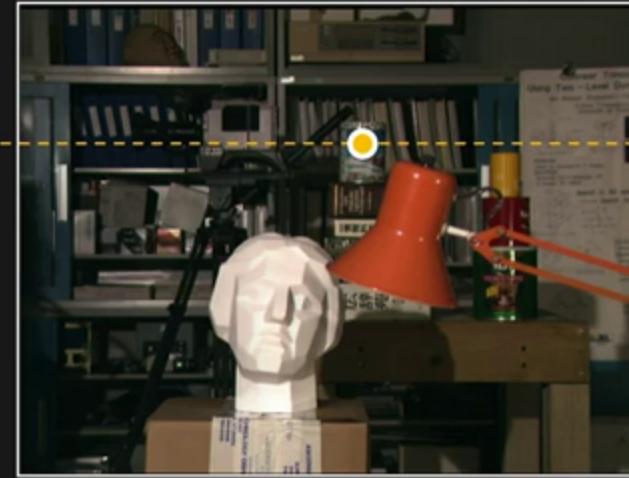
Corresponding scene points lie on the **same** horizontal scan-line.
Finding correspondence is a **1D** search.



Simple Stereo: Finding Correspondences

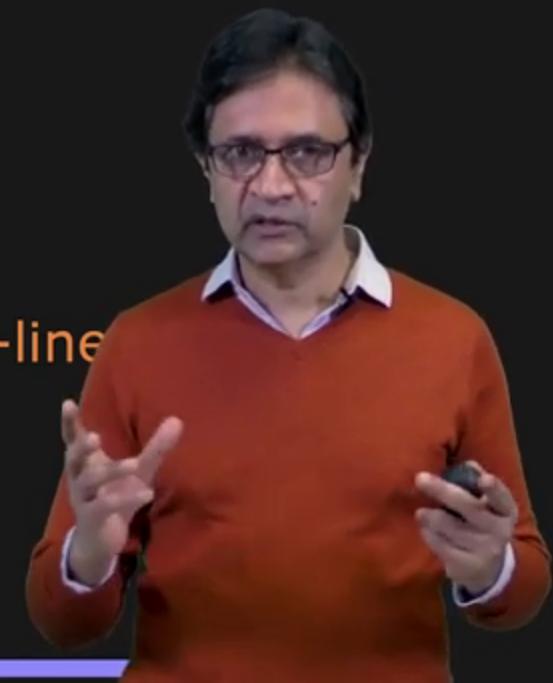


Left Camera Image

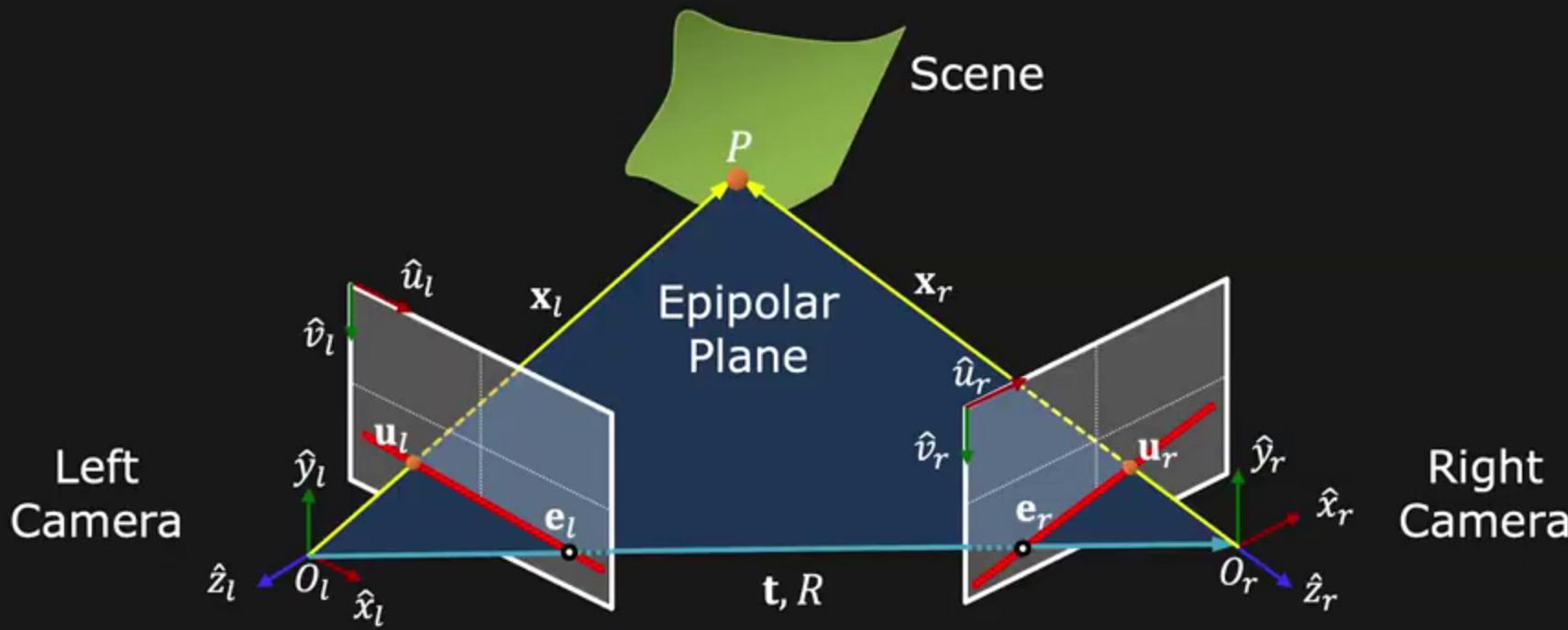


Right Camera Image

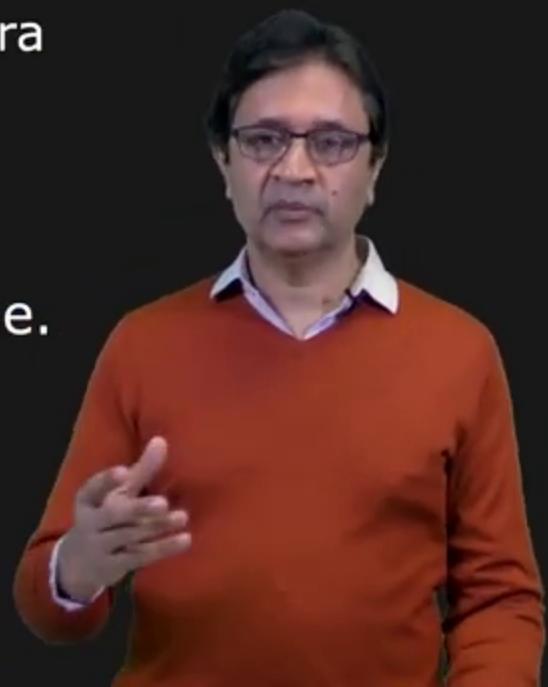
Corresponding scene points lie on the **same horizontal scan-line**.
Finding correspondence is a **1D search**.



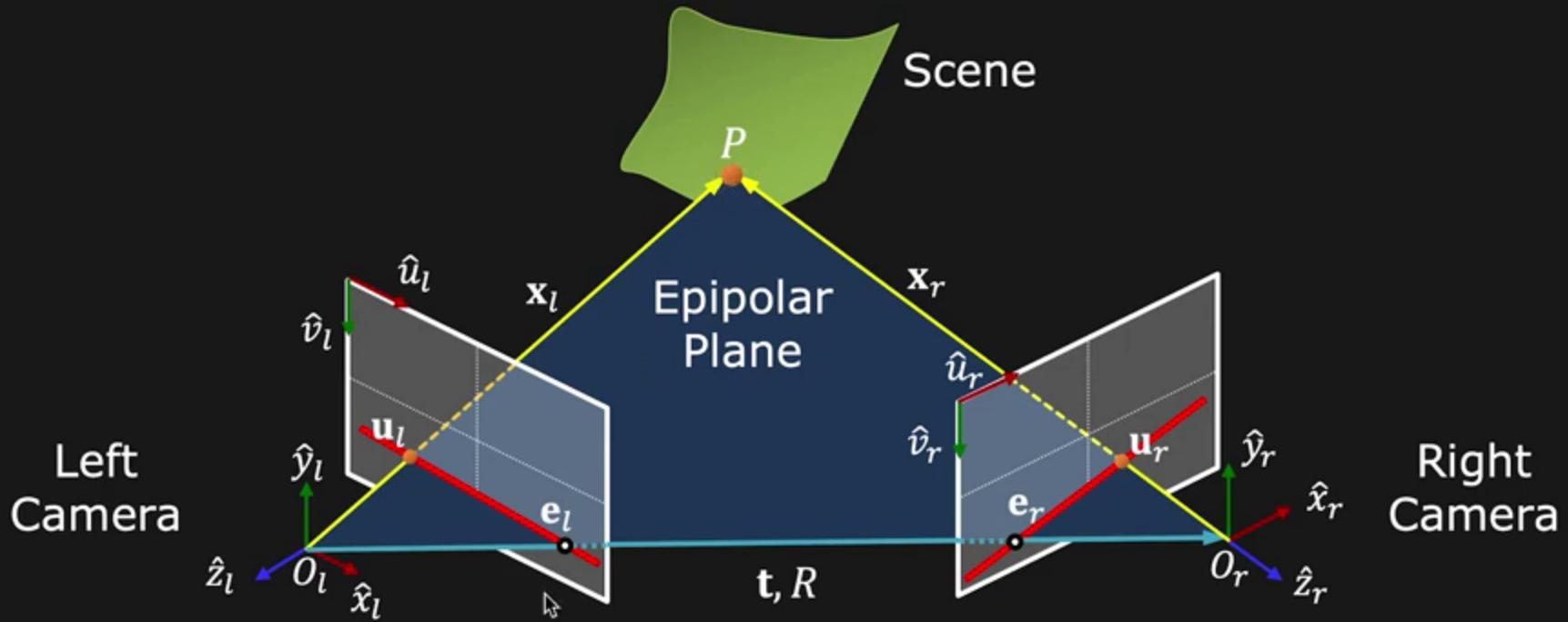
Epipolar Geometry: Epipolar Line



Epipolar Line: Intersection of image plane and epipolar plane.



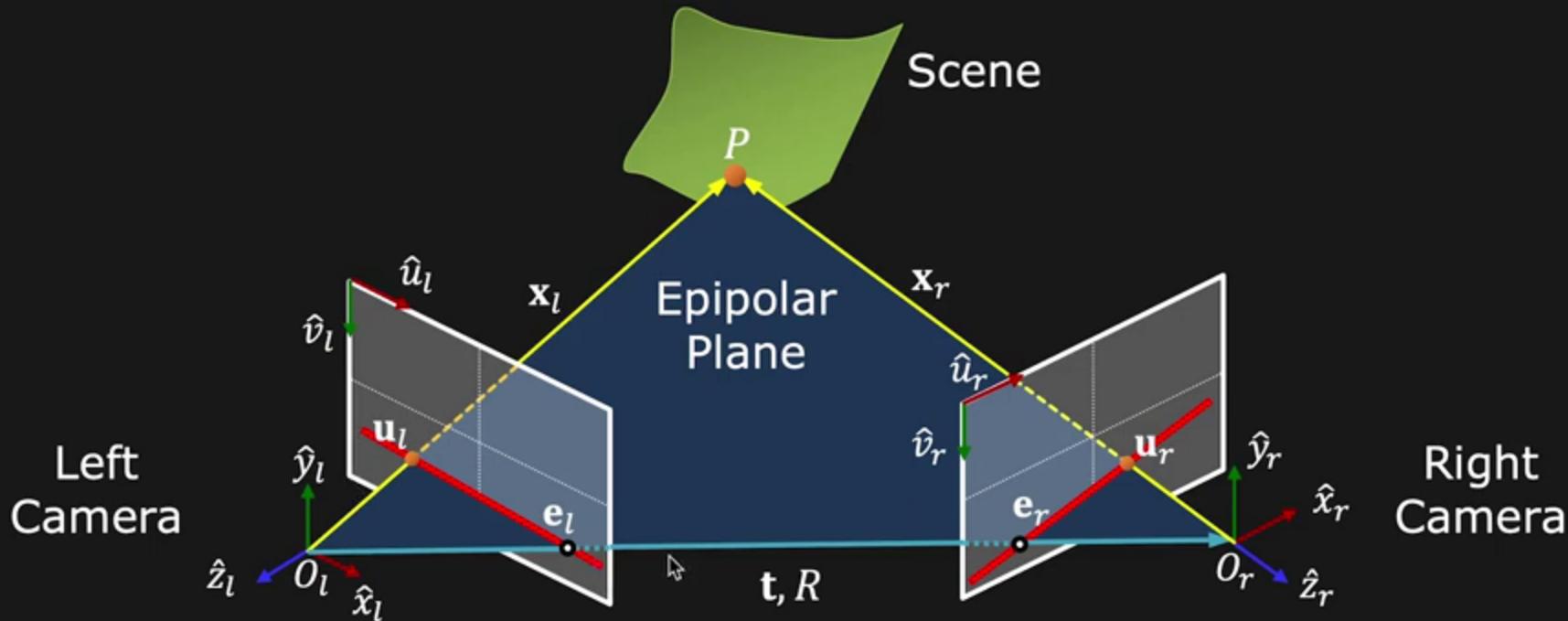
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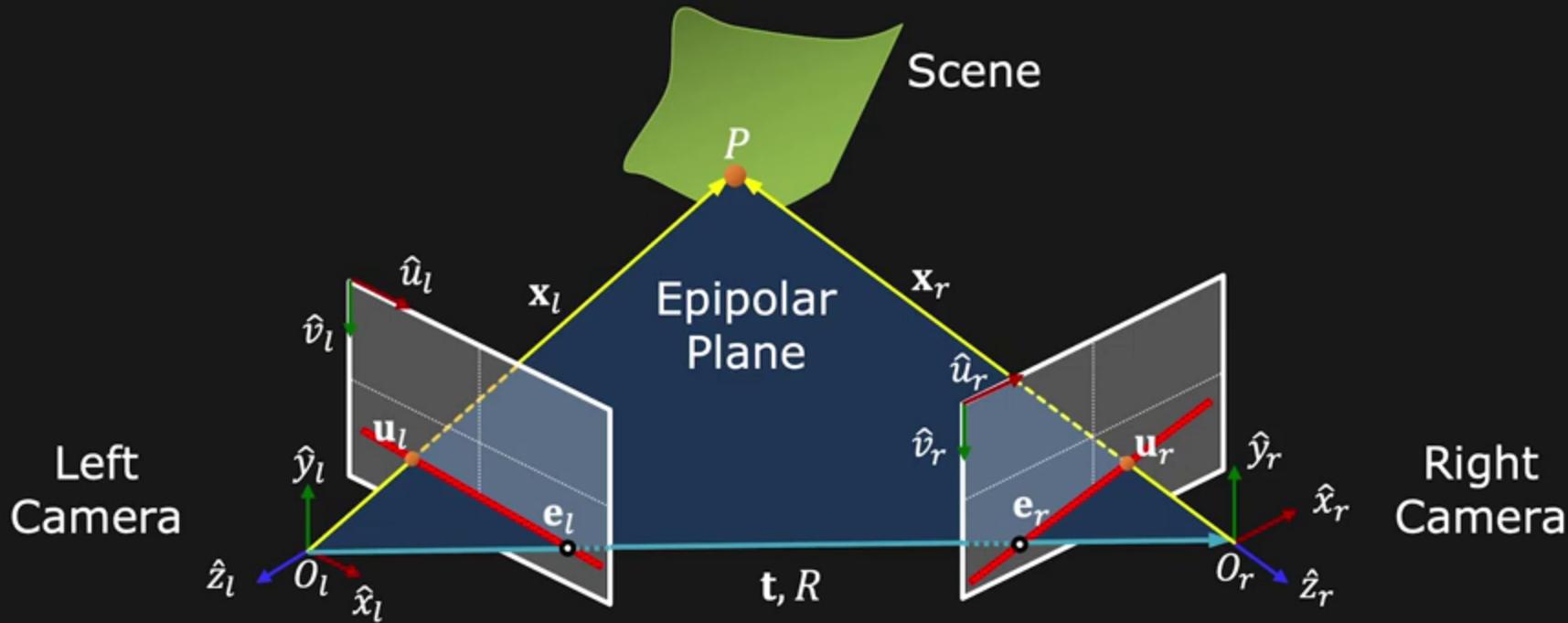
Epipolar Geometry: Epipolar Line



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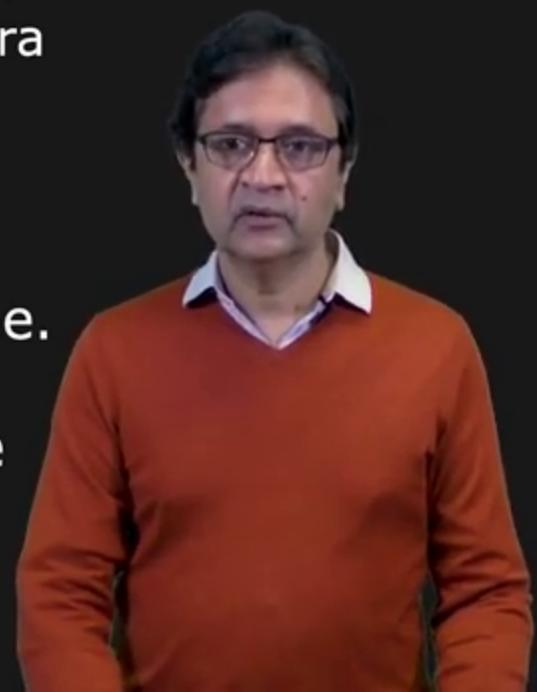


Epipolar Geometry: Epipolar Line

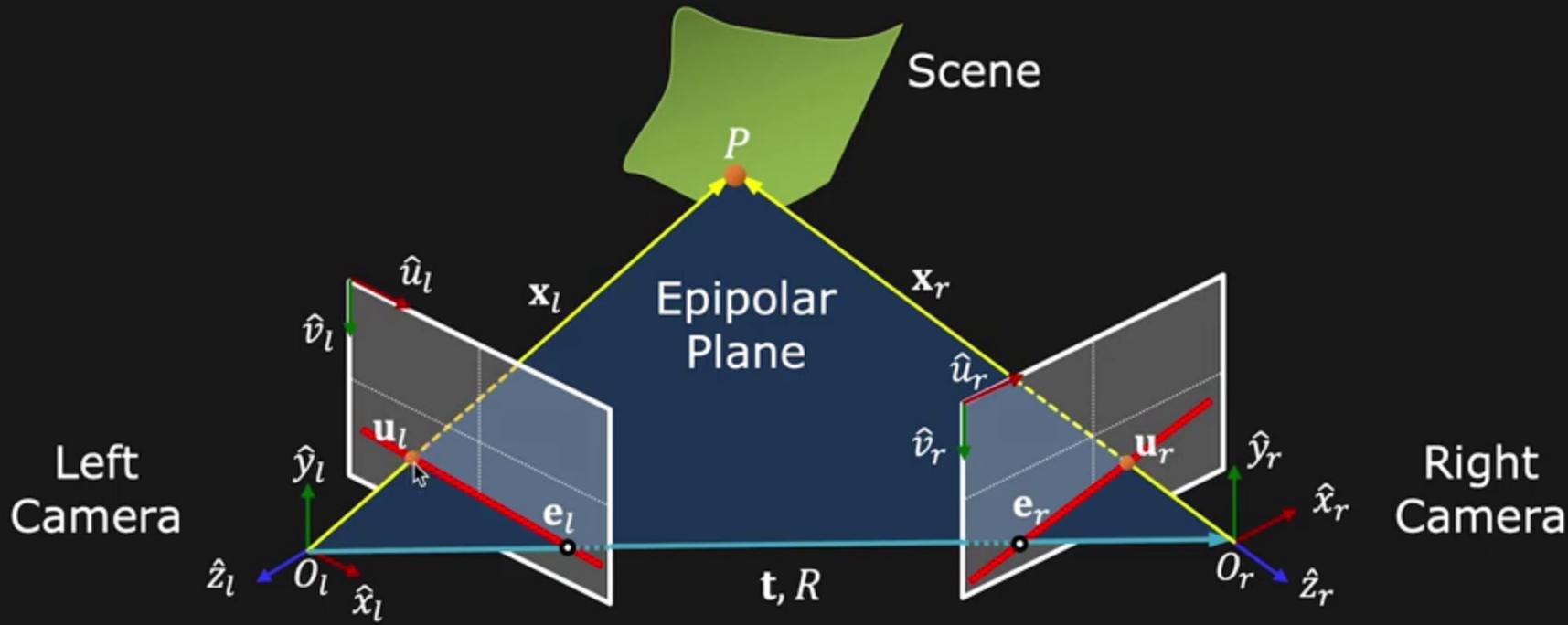


Epipolar Line: Intersection of image plane and epipolar plane.

Every scene point has two corresponding epipolar lines, one each on the two image planes.



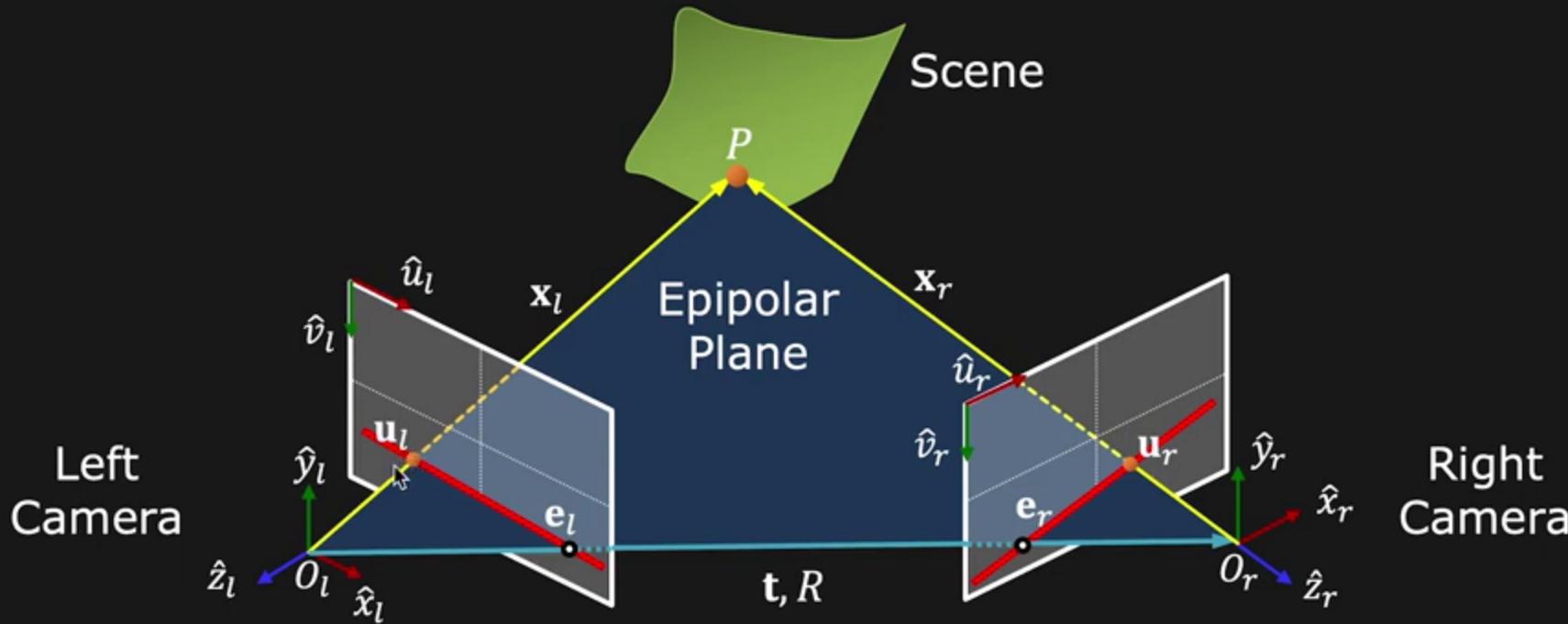
Epipolar Geometry: Epipolar Line



Given a point in one image, the corresponding point in the other image must lie on the epipolar line.



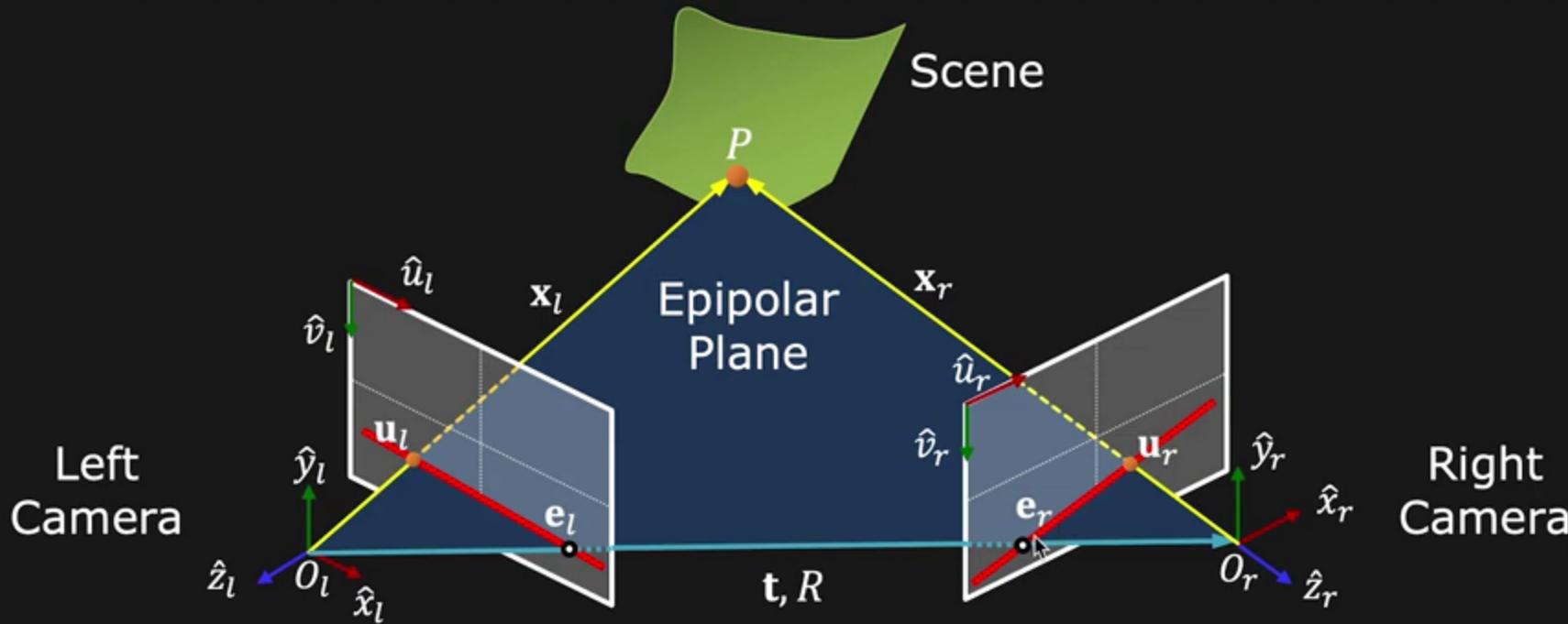
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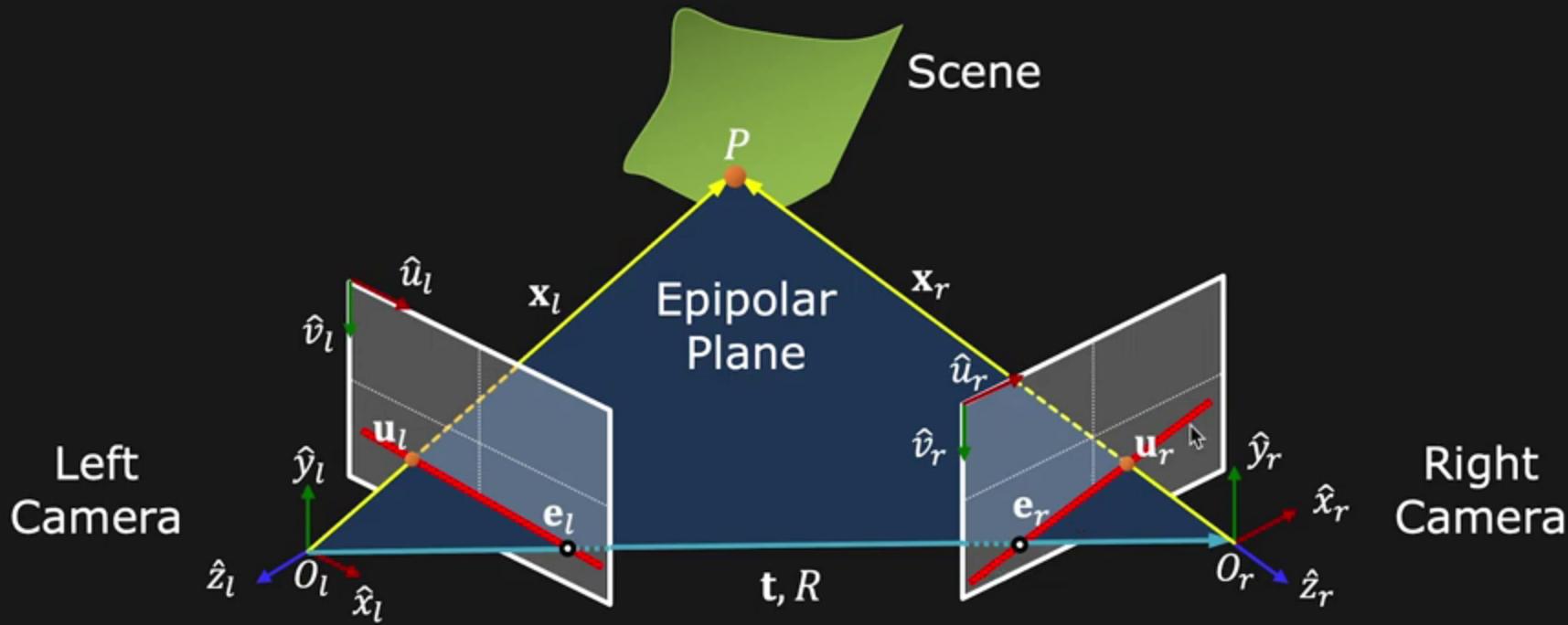
Epipolar Geometry: Epipolar Line



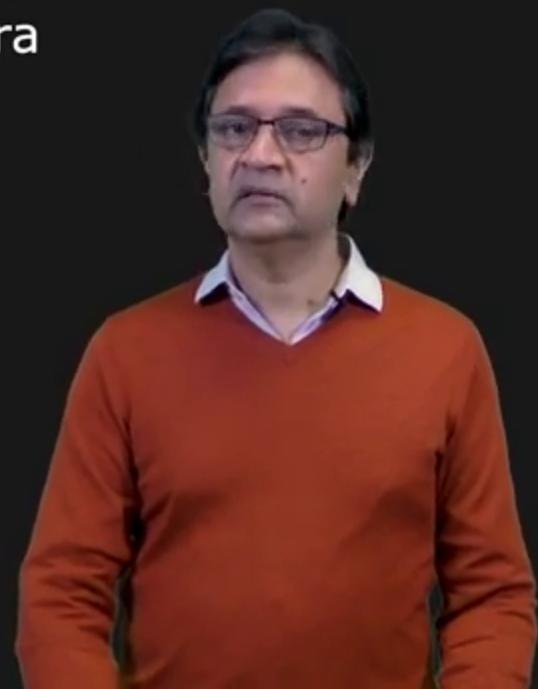
Given a point in one image, the corresponding point in the other image must lie on the epipolar line.



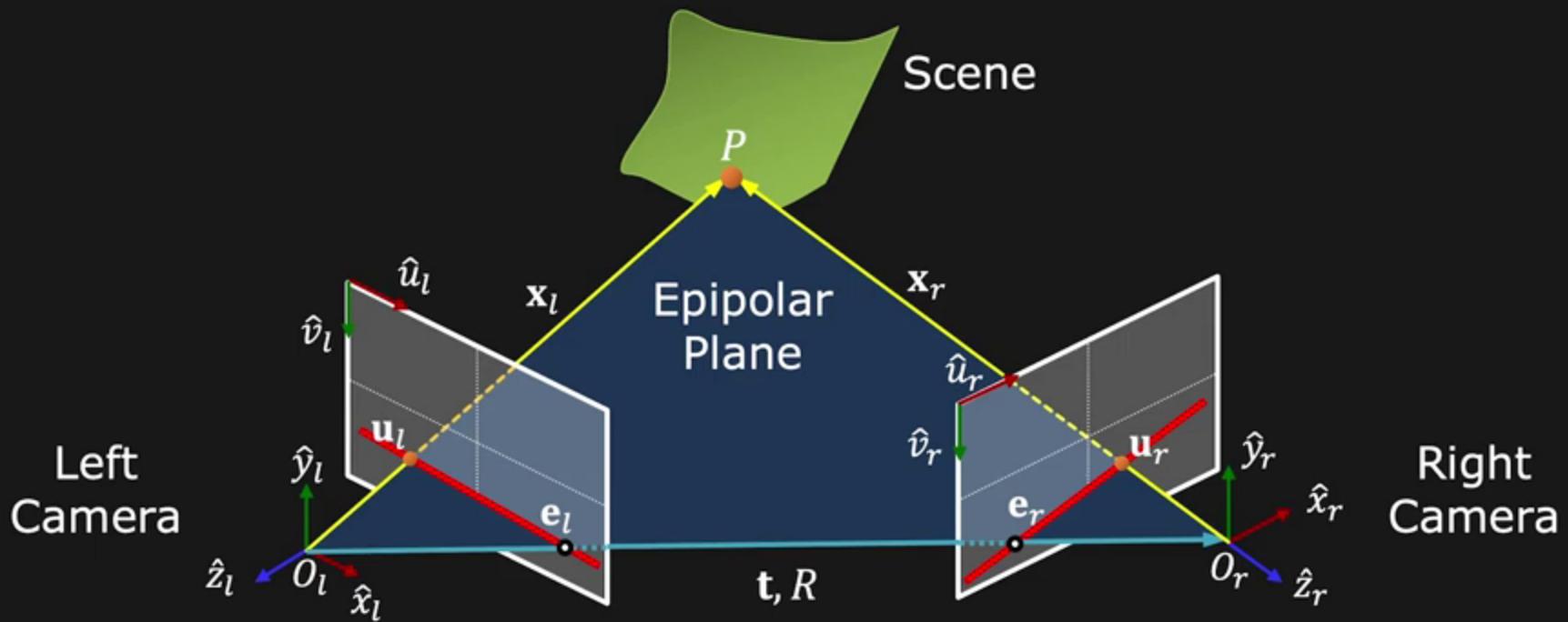
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Epipolar Geometry: Epipolar Line



Given a point in one image, the corresponding point in the other image must lie on the epipolar line.

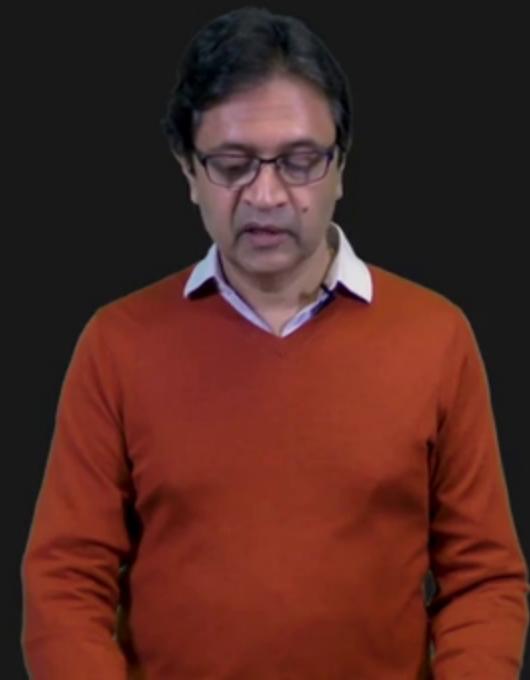
Finding correspondence reduces to a 1D search.



Finding Epipolar Lines

Given: Fundamental matrix F and point on left image (u_l, v_l)

Find: Equation of Epipolar line in the right image



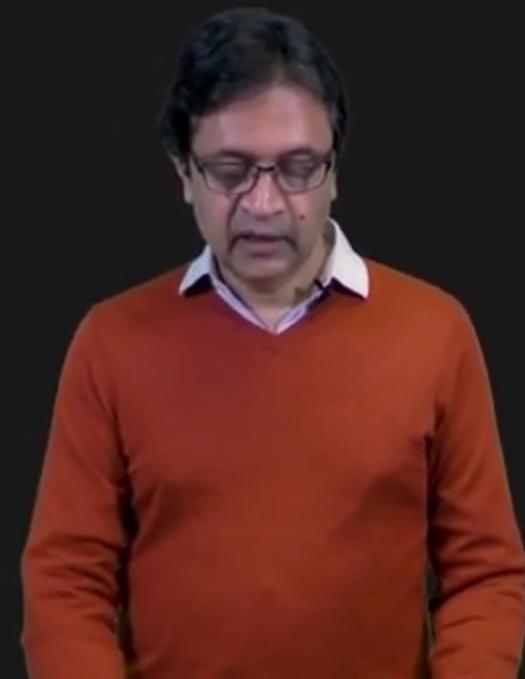
Finding Epipolar Lines

Given: Fundamental matrix F and point on left image (u_l, v_l)

Find: Equation of Epipolar line in the right image

Epipolar Constraint Equation:

$$[u_l \ v_l \ 1] \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0$$



Finding Epipolar Lines

Given: Fundamental matrix F and point on left image (u_l, v_l)

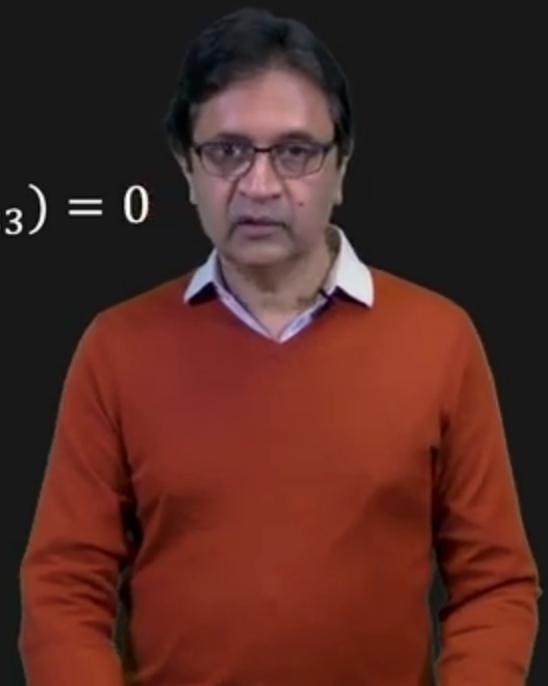
Find: Equation of Epipolar line in the right image

Epipolar Constraint Equation:

$$[u_l \ v_l \ 1] \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0$$

Expanding the matrix equation gives:

$$(f_{11}u_l + f_{21}v_l + f_{31})u_r + (f_{12}u_l + f_{22}v_l + f_{32})v_r + (f_{13}u_l + f_{23}v_l + f_{33}) = 0$$



Finding Epipolar Lines

Given: Fundamental matrix F and point on left image (u_l, v_l)

Find: Equation of Epipolar line in the right image

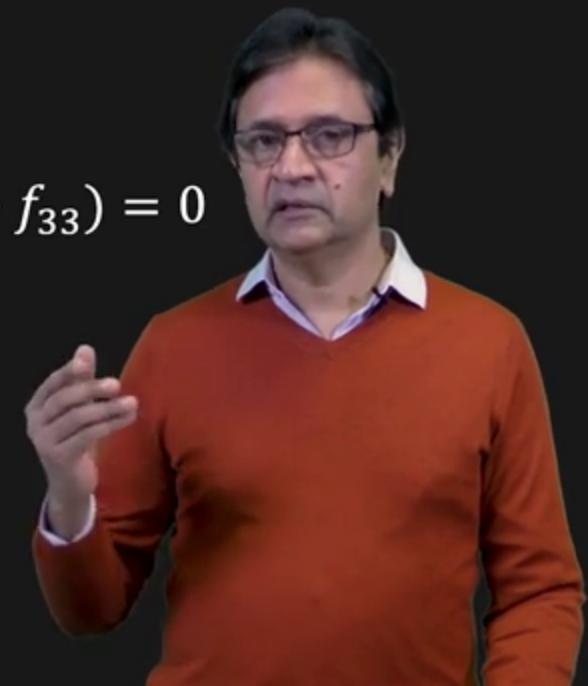
Epipolar Constraint Equation:

$$[u_l \ v_l \ 1] \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0$$

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Equation for right epipolar line: $a_l u_r + b_l v_r + c_l = 0$



Finding Epipolar Lines

Given: Fundamental matrix F and point on left image (u_l, v_l)

Find: Equation of Epipolar line in the right image

Epipolar Constraint Equation:

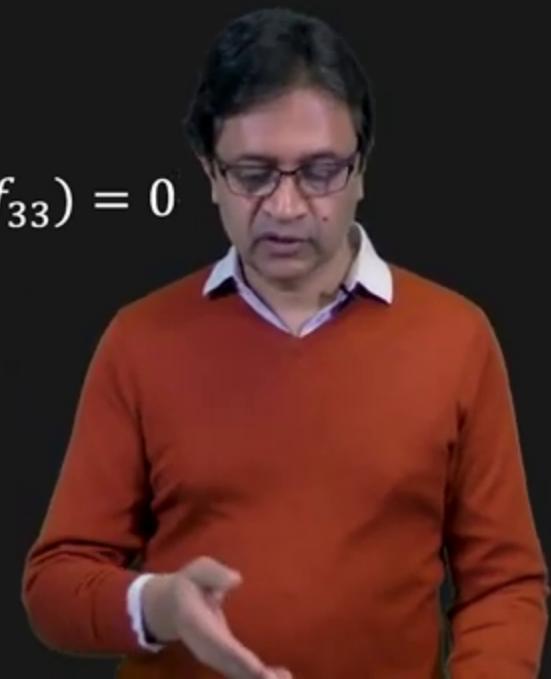
$$[u_l \ v_l \ 1] \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0$$

Expanding the matrix equation gives:

$$(f_{11}u_l + f_{21}v_l + f_{31})u_r + (f_{12}u_l + f_{22}v_l + f_{32})v_r + (f_{13}u_l + f_{23}v_l + f_{33}) = 0$$

Equation for right epipolar line: $a_l u_r + b_l v_r + c_l = 0$

Similarly we can calculate epipolar line in left image for a point in right image.



Finding Epipolar Lines: Example

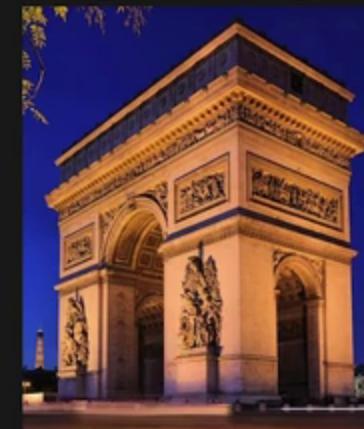
Given the Fundamental matrix,

$$F = \begin{bmatrix} -0.003 & -0.028 & 13.19 \\ -0.003 & -0.008 & -29.2 \\ 2.97 & 56.38 & -9999 \end{bmatrix}$$

Left Image



Right Image



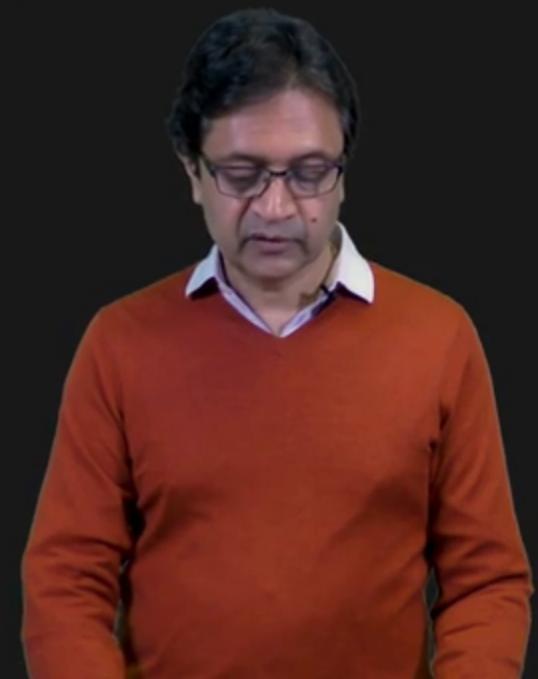
Finding Epipolar Lines: Example

Given the Fundamental matrix,

$$F = \begin{bmatrix} -0.003 & -0.028 & 13.19 \\ -0.003 & -0.008 & -29.2 \\ 2.97 & 56.38 & -9999 \end{bmatrix}$$

and the left image point

$$\tilde{u}_l = \begin{bmatrix} 343 \\ 221 \\ 1 \end{bmatrix}$$



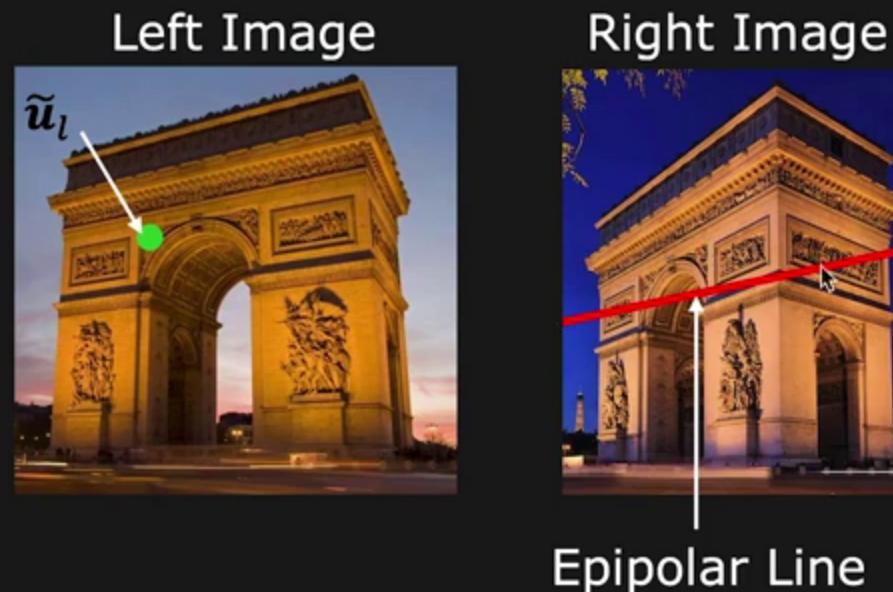
Finding Epipolar Lines: Example

Given the Fundamental matrix,

$$F = \begin{bmatrix} -.003 & -.028 & 13.19 \\ -.003 & -.008 & -29.2 \\ 2.97 & 56.38 & -9999 \end{bmatrix}$$

and the **left** image point

$$\tilde{u}_l = \begin{bmatrix} 343 \\ 221 \\ 1 \end{bmatrix}$$



The equation for the epipolar line in the **right** image is

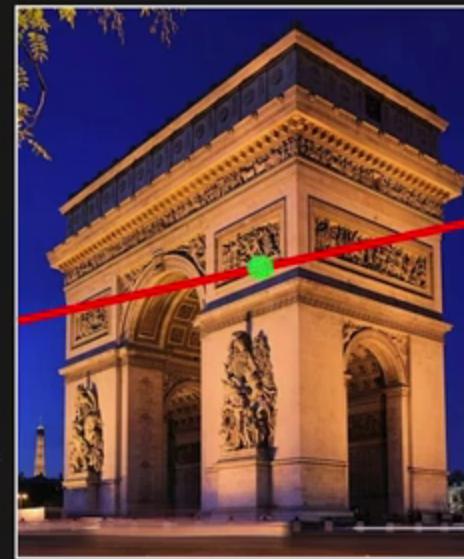
$$.03u_r + .99v_r - 265 = 0$$



Finding Correspondence



Left Image



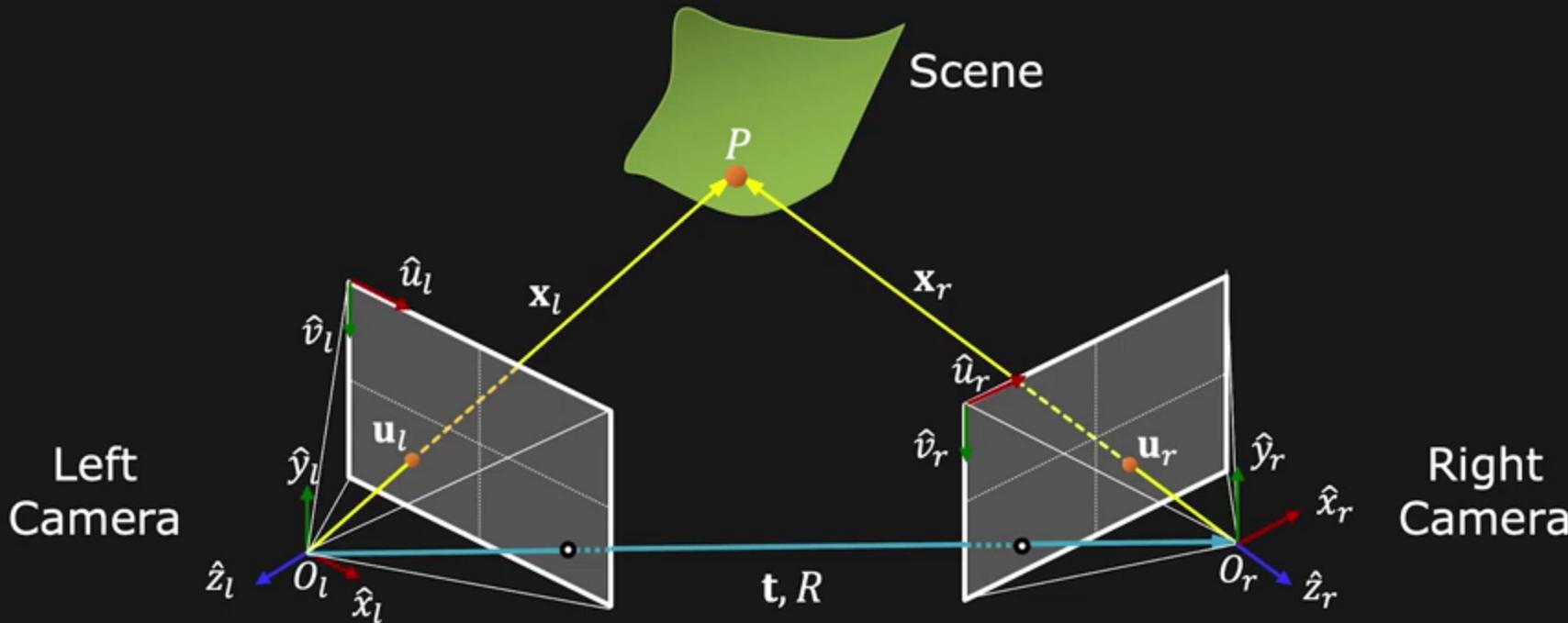
Right Image

Epipolar
Line

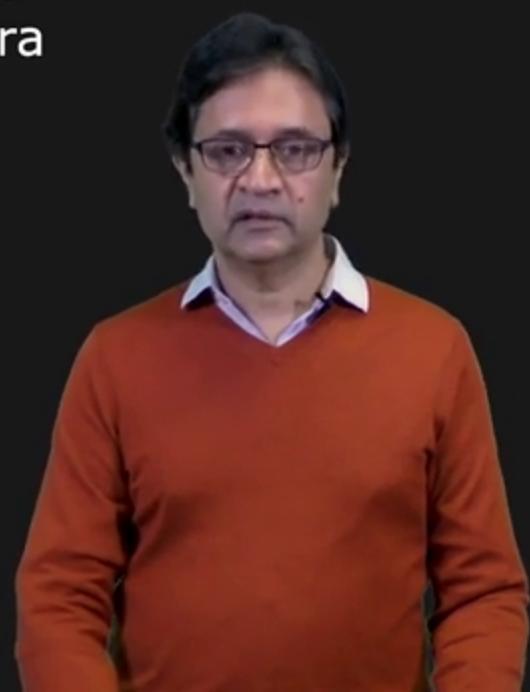
Corresponding scene points lie on the epipolar lines.
Finding correspondence is a 1D search.



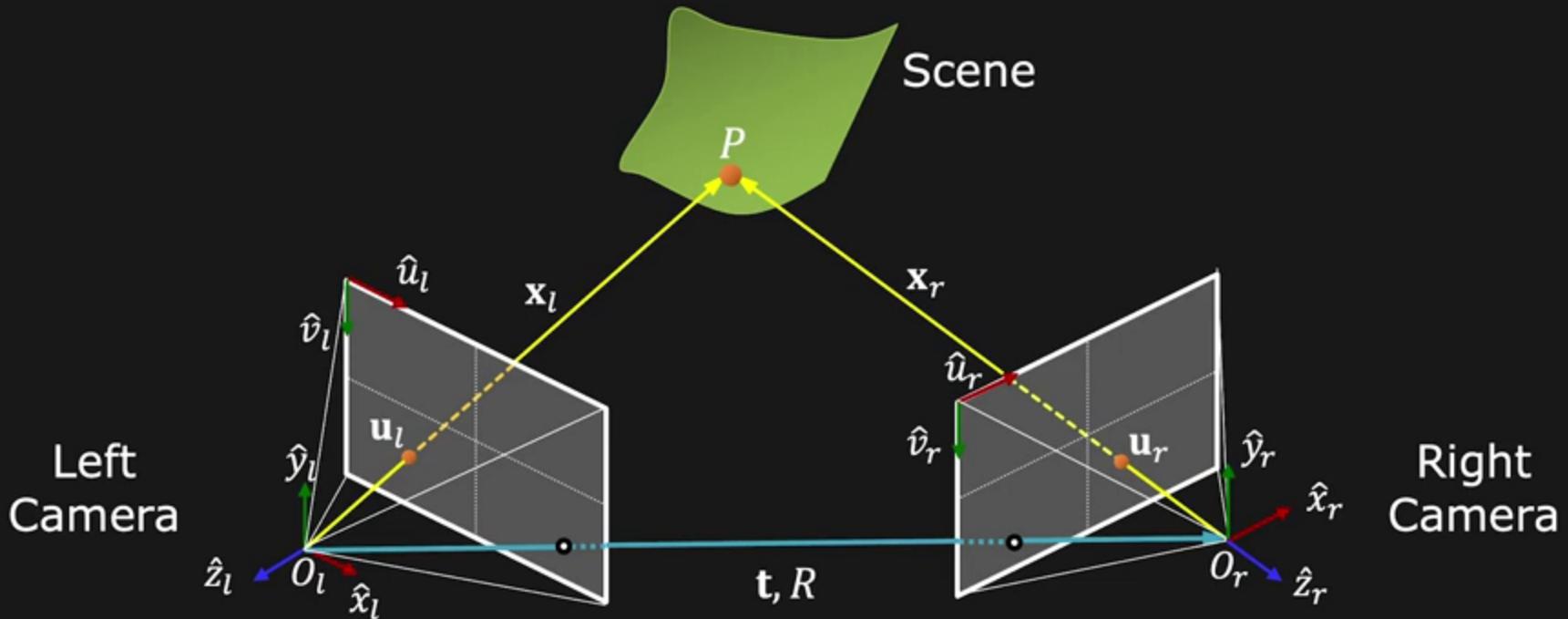
Uncalibrated Stereo



- ✓ 1. Assume Camera Matrix K is known for each camera
- ✓ 2. Find a few Reliable Corresponding Points
- ✓ 3. Find Relative Camera Position t and Orientation R
- ✓ 4. Find Dense Correspondence
- 5. Compute Depth using Triangulation



Uncalibrated Stereo



- ✓ 1. Assume Camera Matrix K is known for each camera
- ✓ 2. Find a few Reliable Corresponding Points
- ✓ 3. Find Relative Camera Position t and Orientation R
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