Simple Stereo

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Columbia University

Topic: Camera Calibration, Module: Reconstruction II

First Principles of Computer Vision

Backward Projection: From 2D to 3D

Given a calibrated camera, can we find the 3D scene point from a single 2D image?

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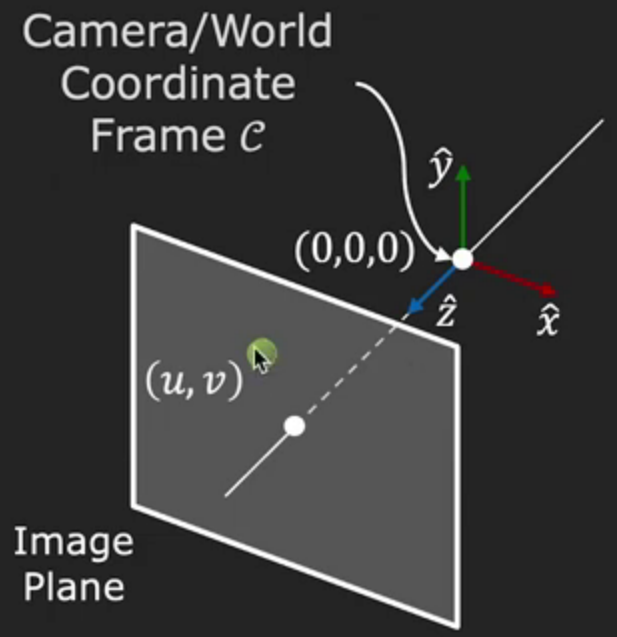

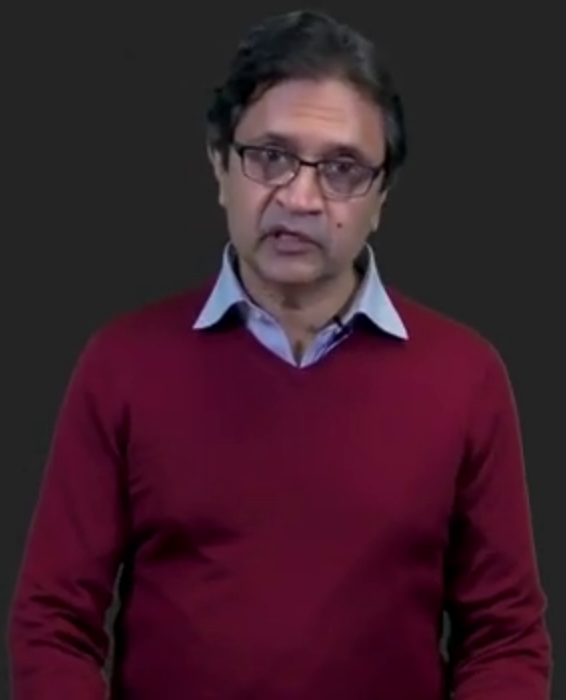

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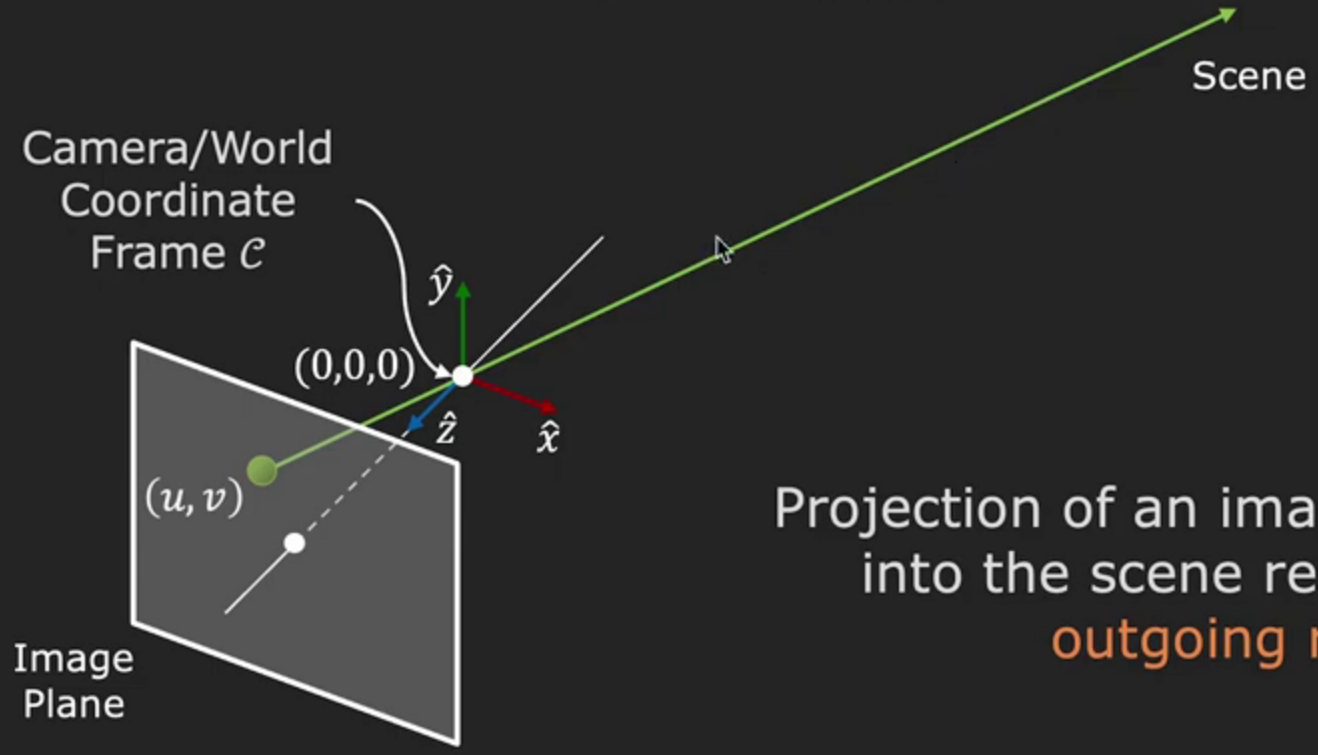
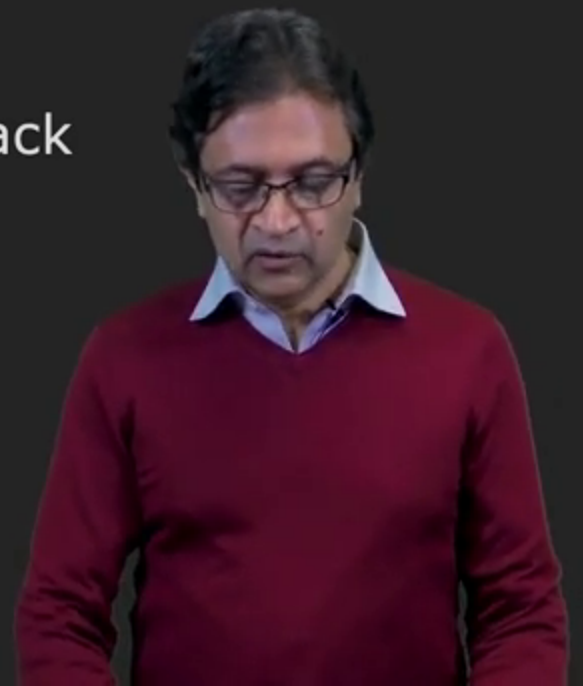
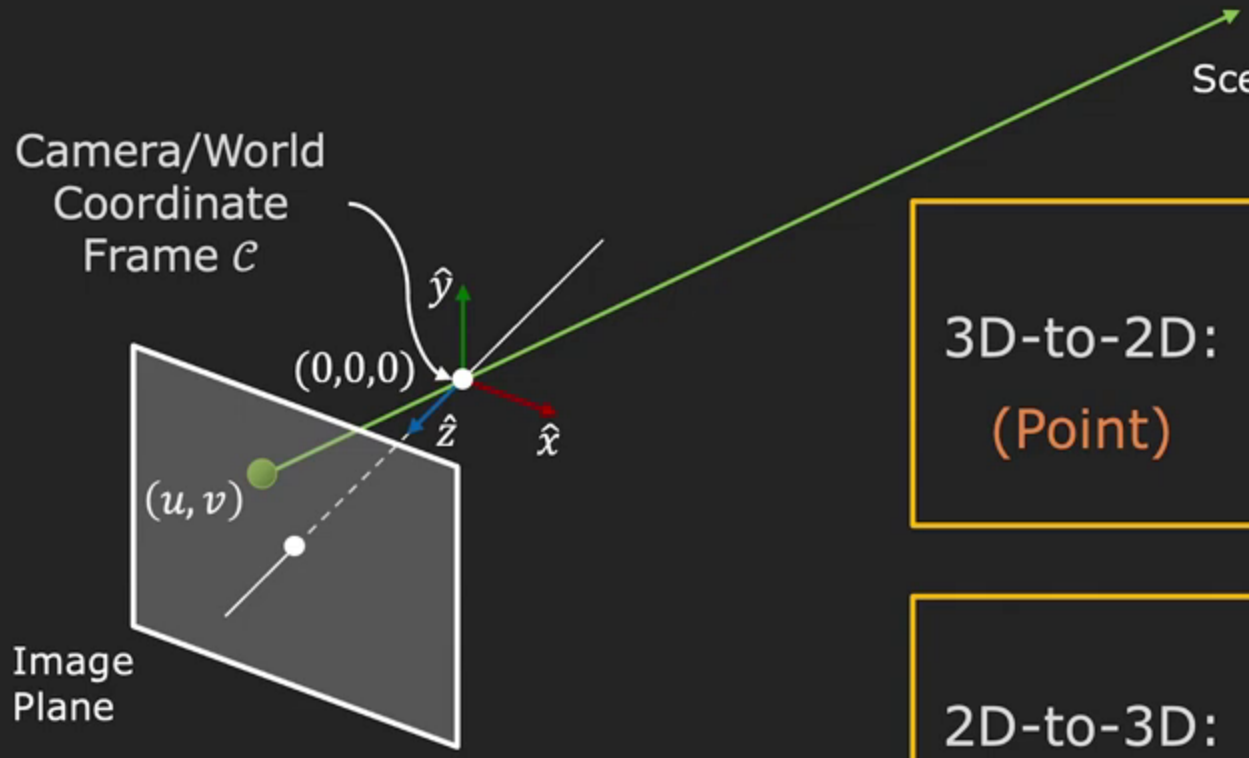


Projection of an image point back into the scene results in an outgoing ray.



Computing 2D-to-3D Outgoing Ray



3D-to-2D: (Point)

$$u = f_x \frac{x_c}{z_c} + o_x$$

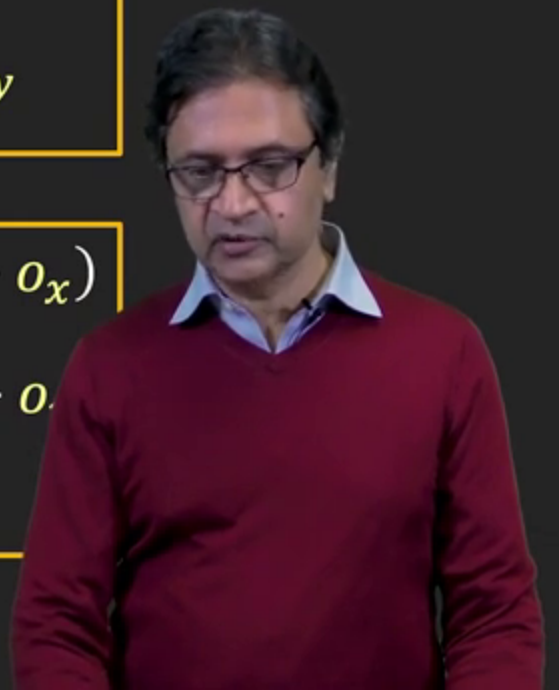
$$v = f_y \frac{y_c}{z_c} + o_y$$

2D-to-3D: (Ray)

$$x = {}^{z}/_{f_x}\,(u - o_x)$$

$$y = {}^{z}/_{f_y}\,(v - o_y)$$

$$z > 0$$



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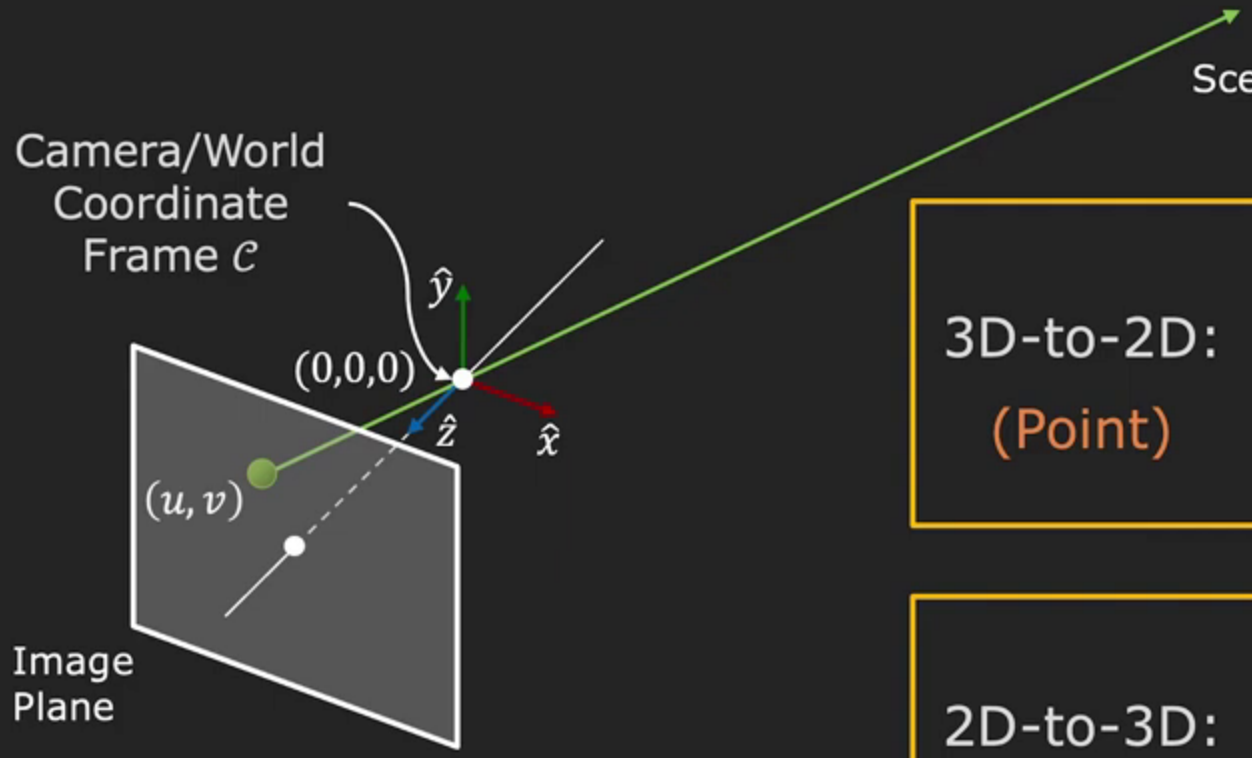

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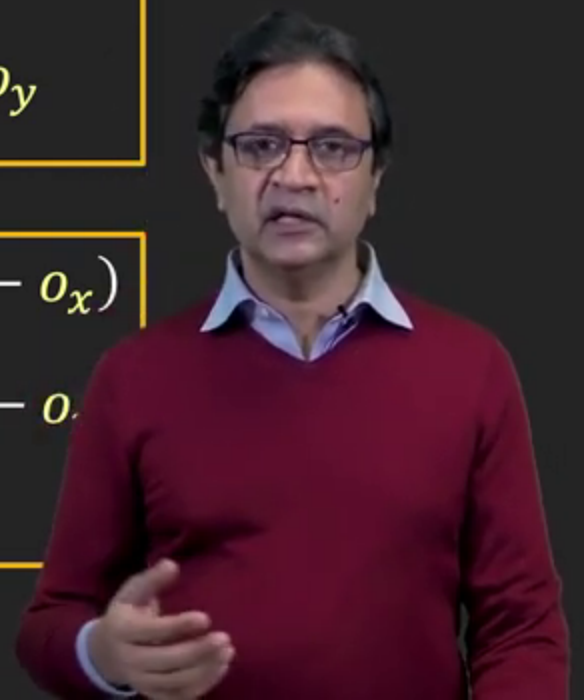
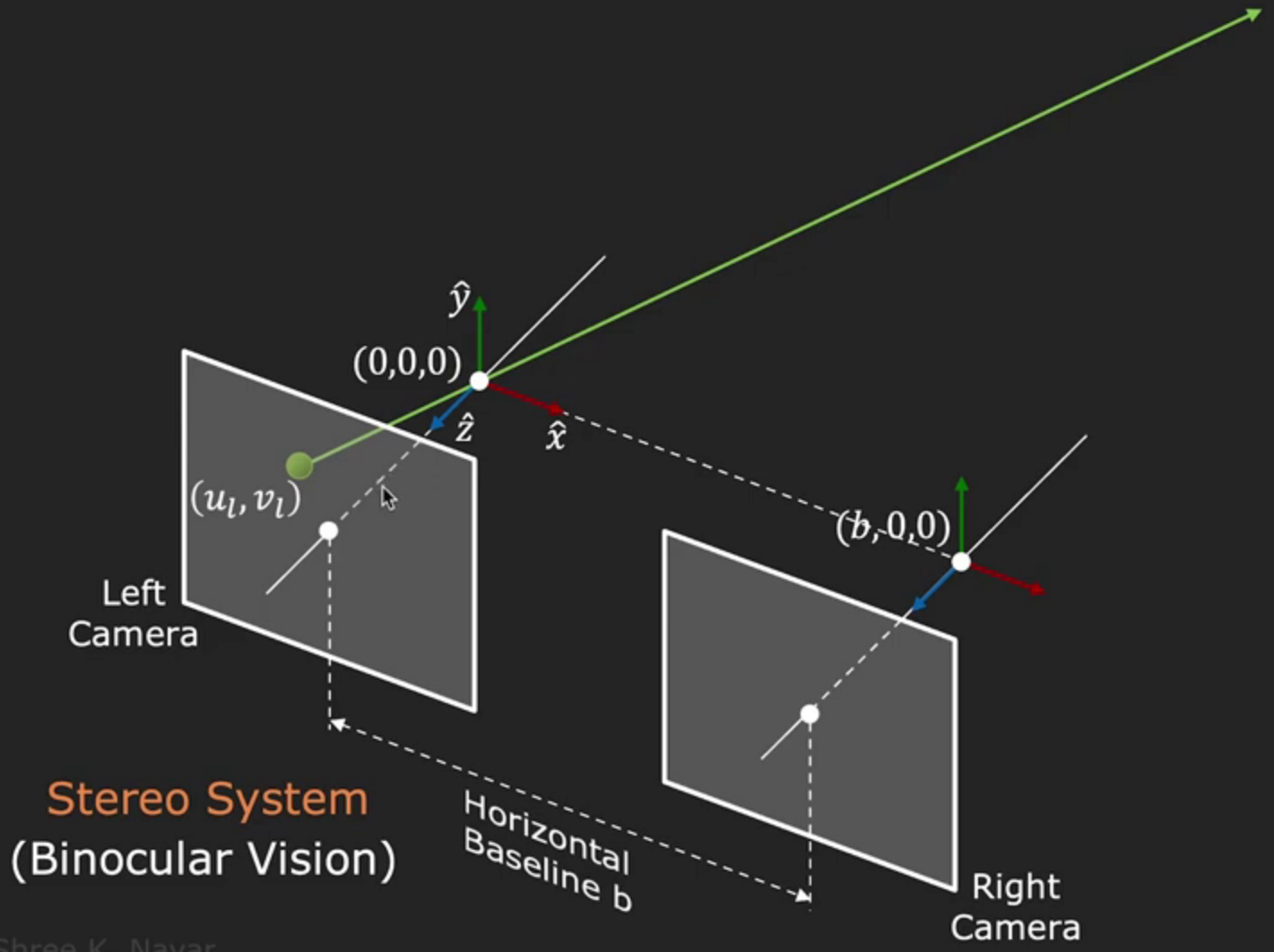


Triangulation using Two Cameras



Stereo System
(Binocular Vision)

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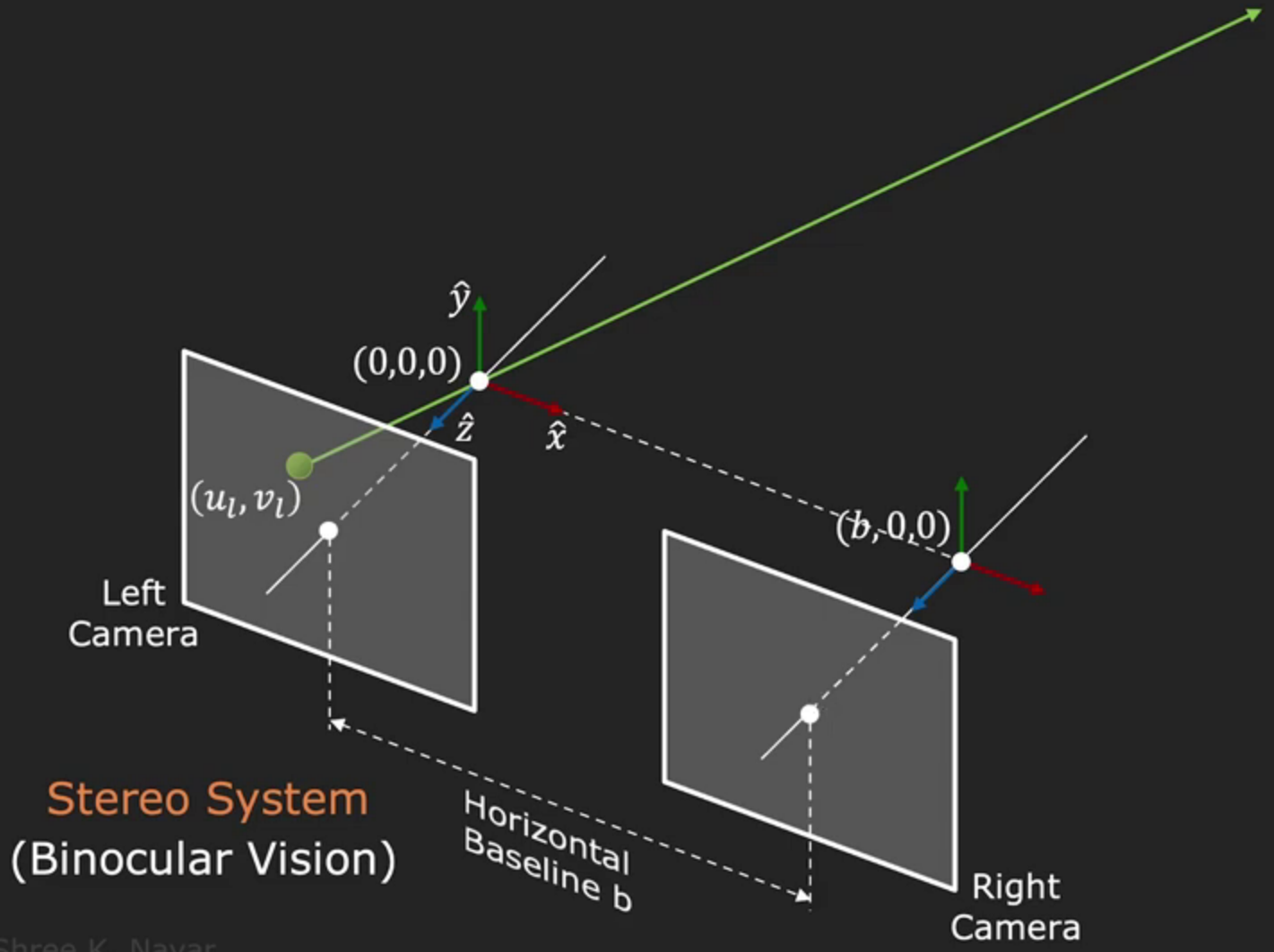

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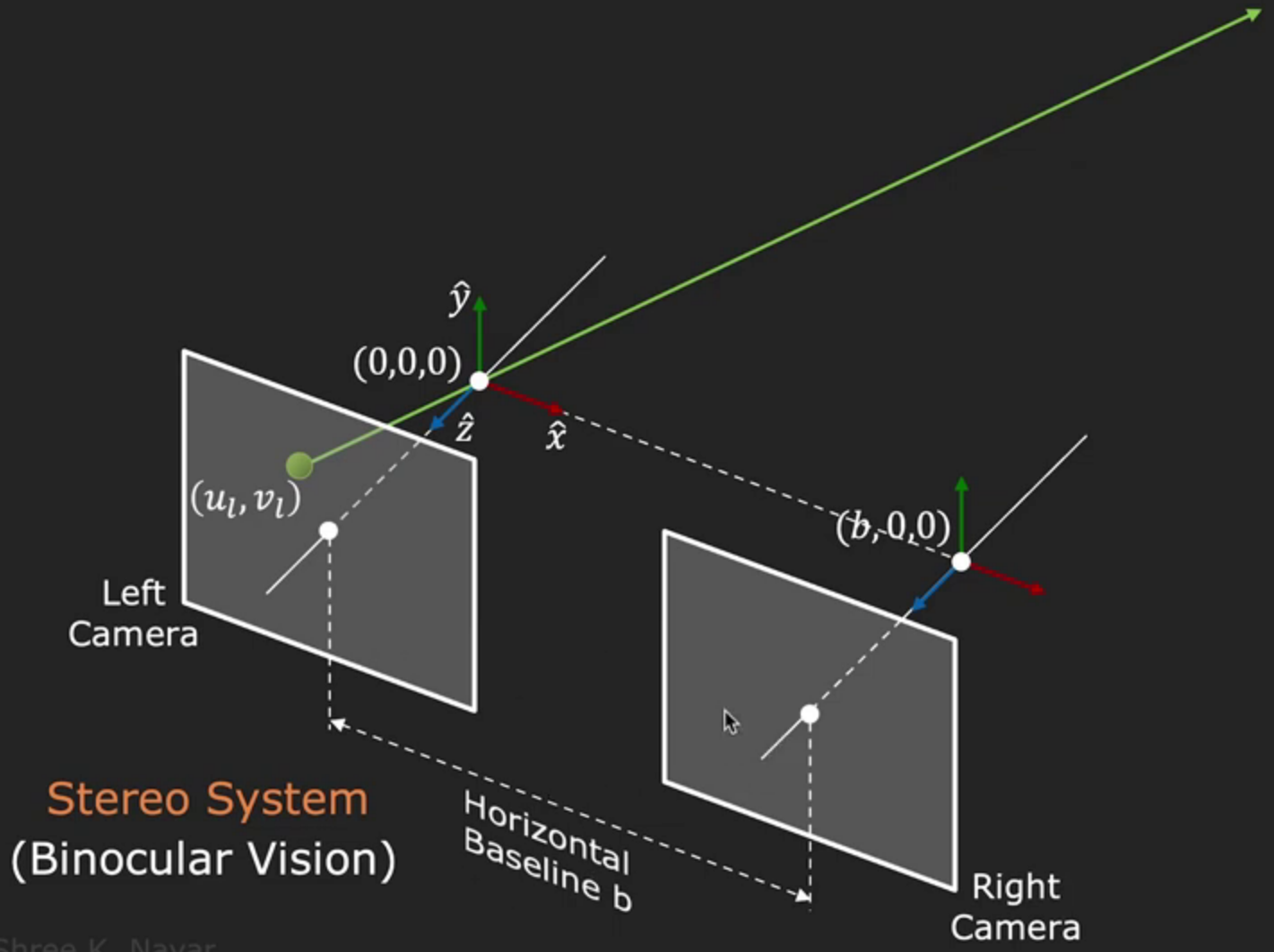

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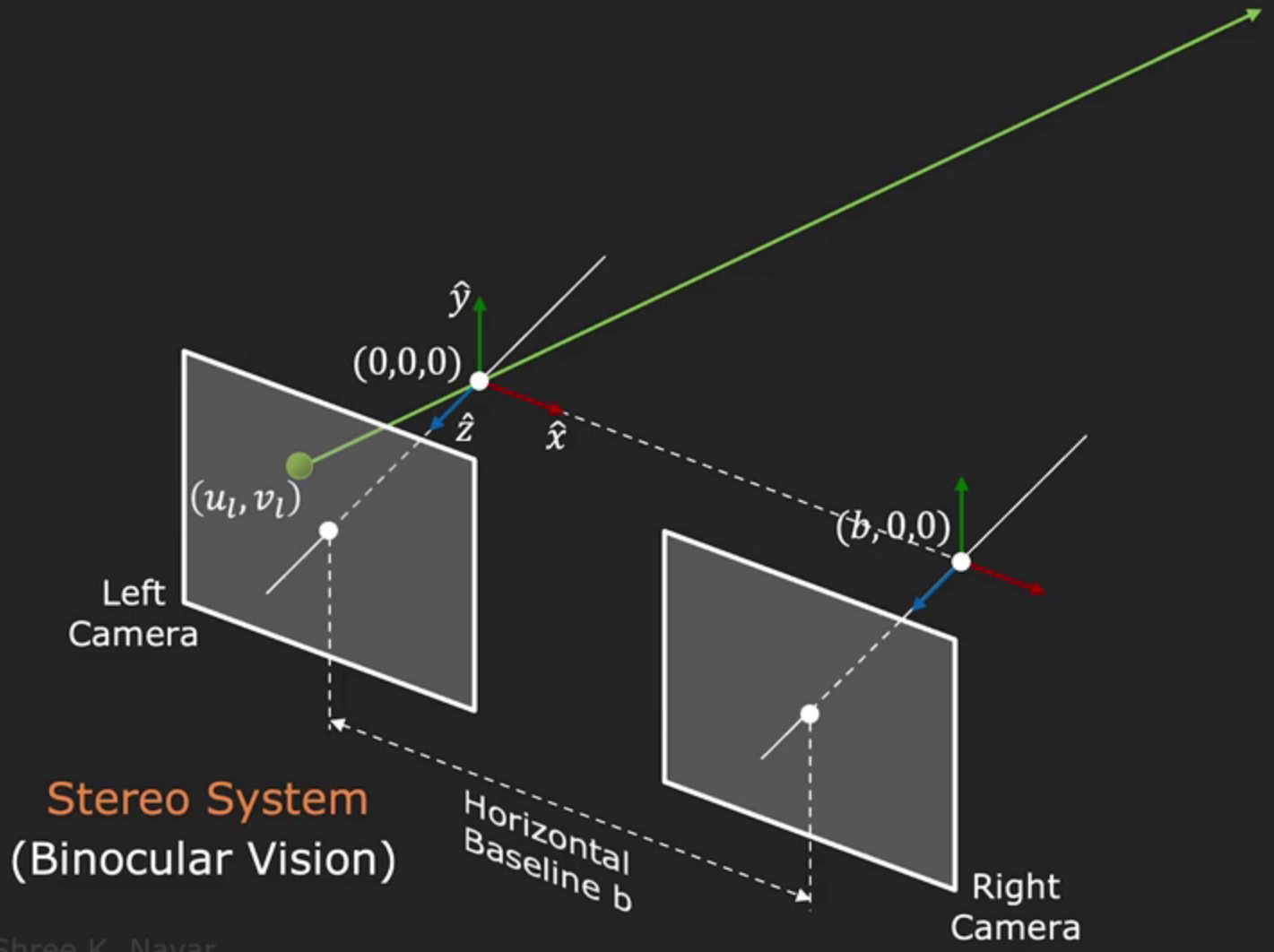

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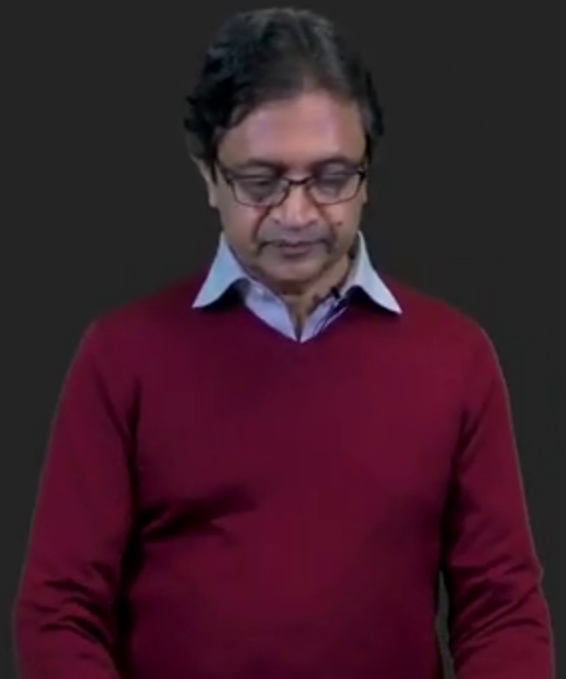

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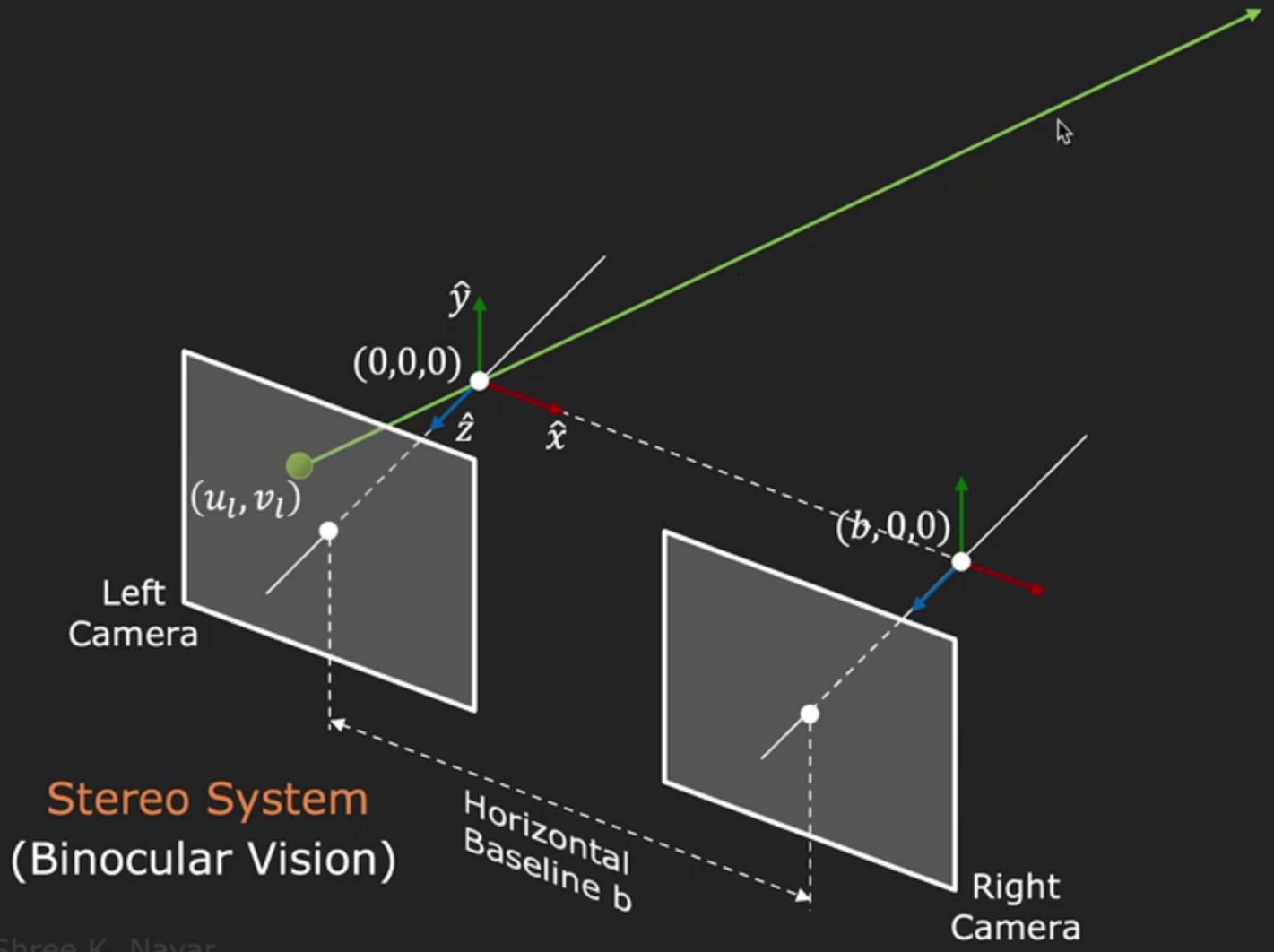

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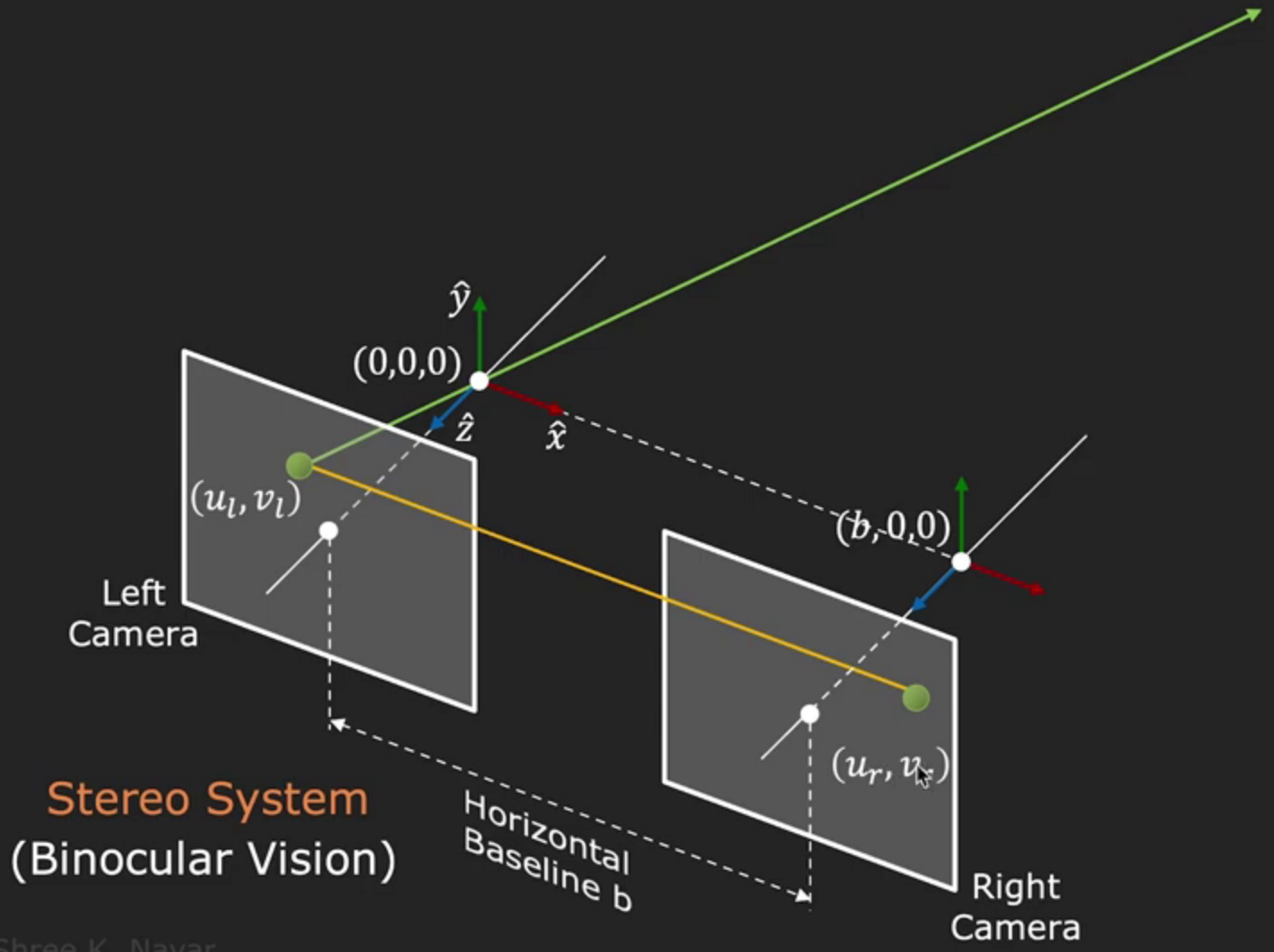

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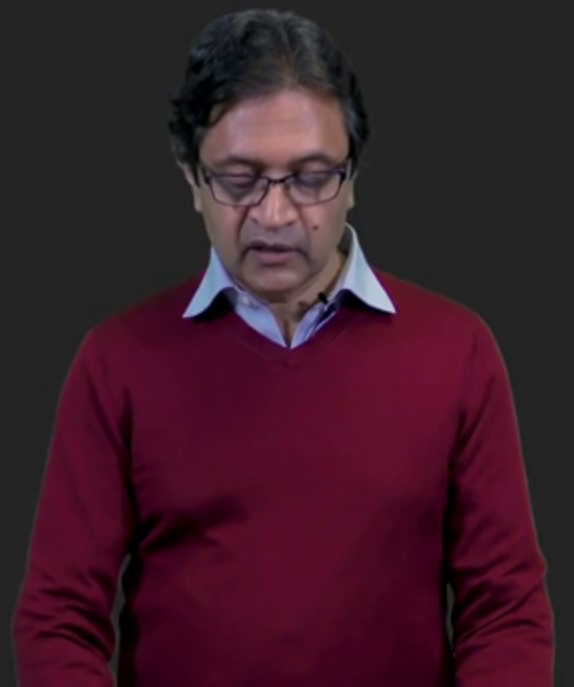

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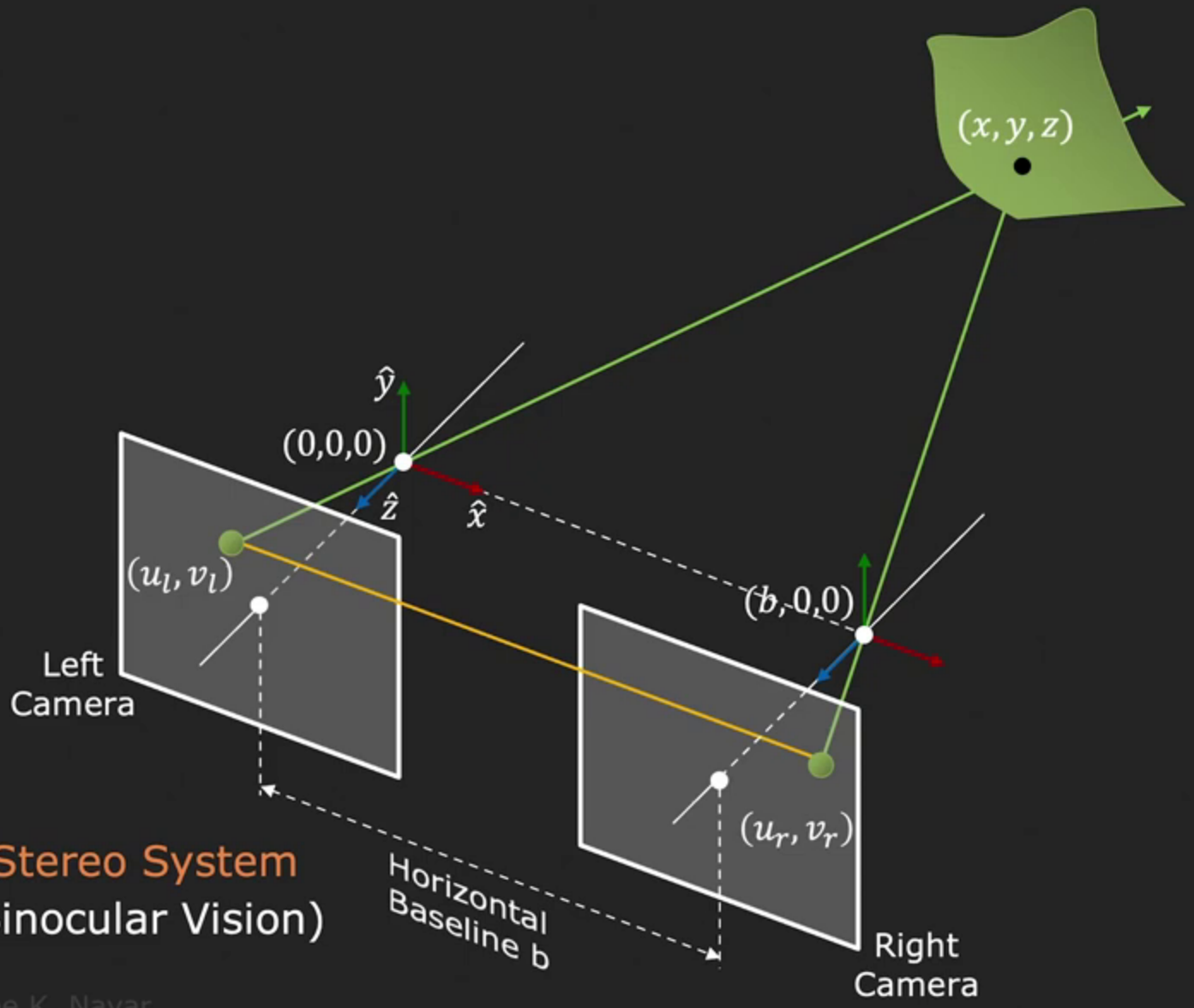

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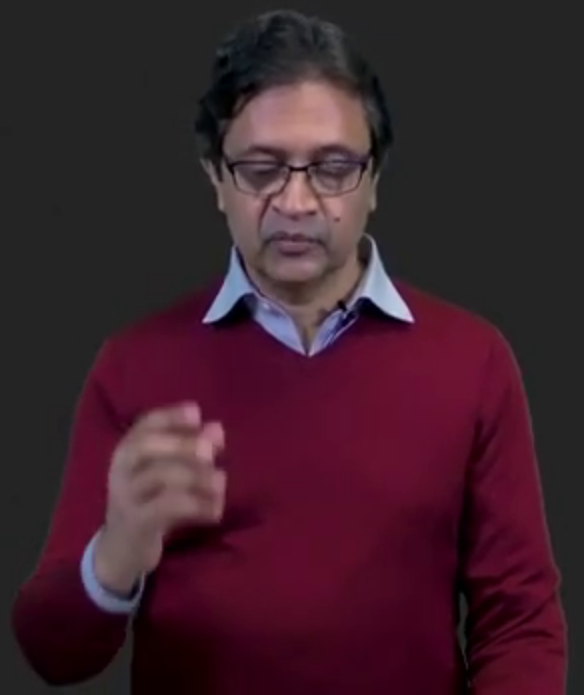

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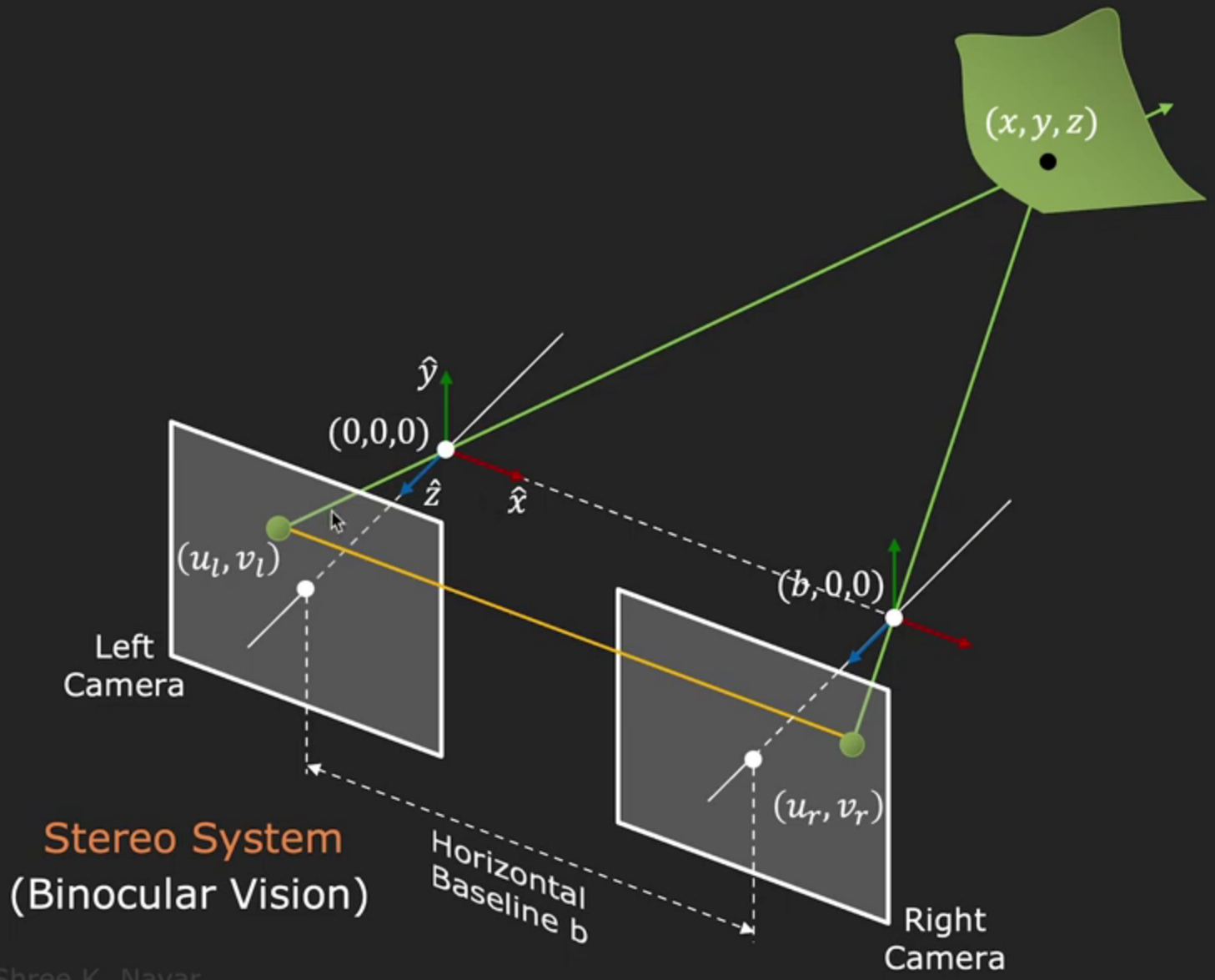

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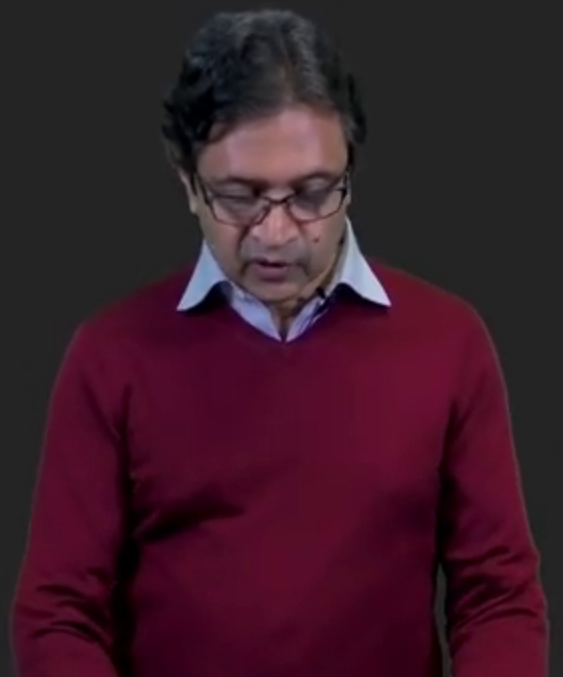

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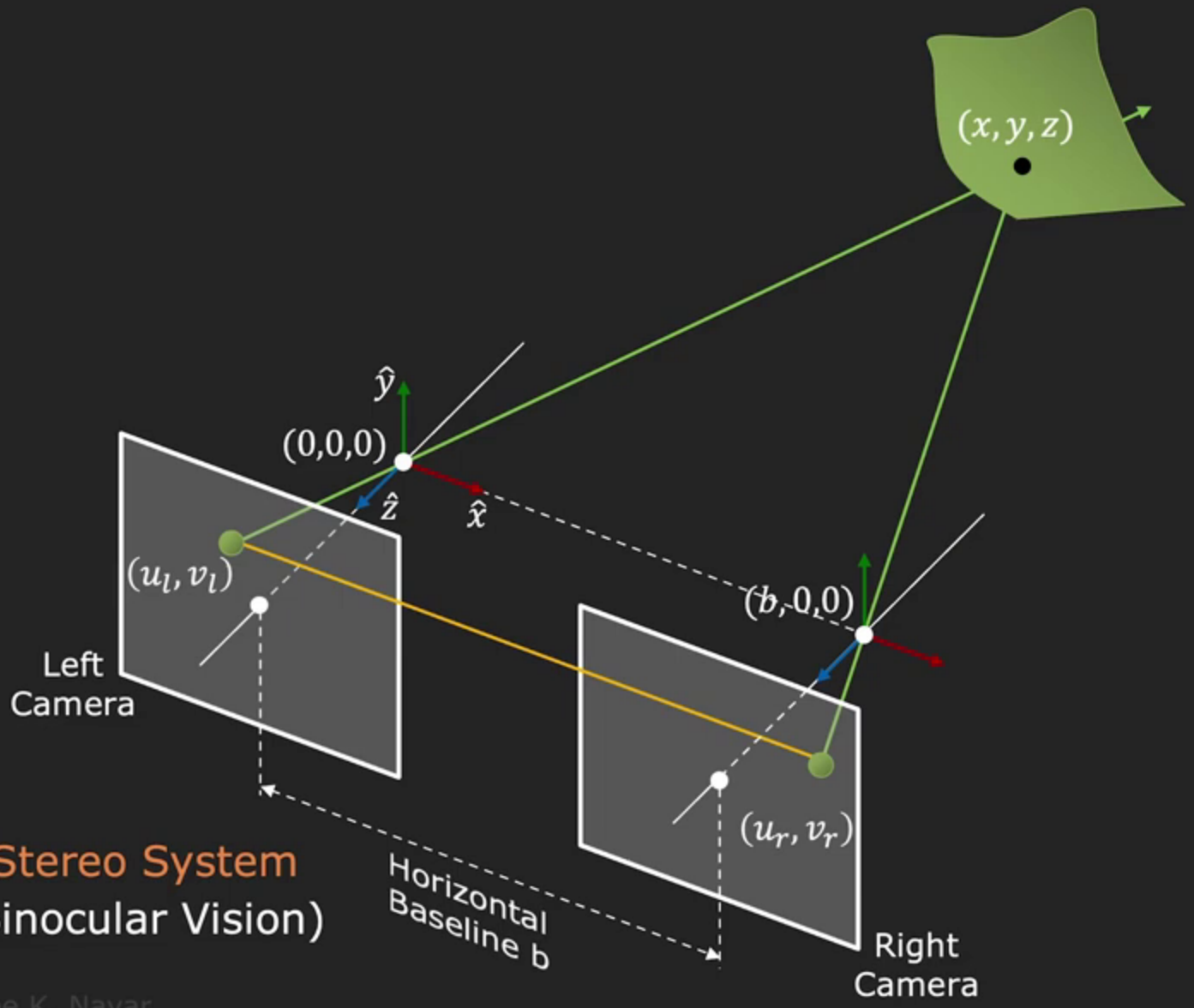

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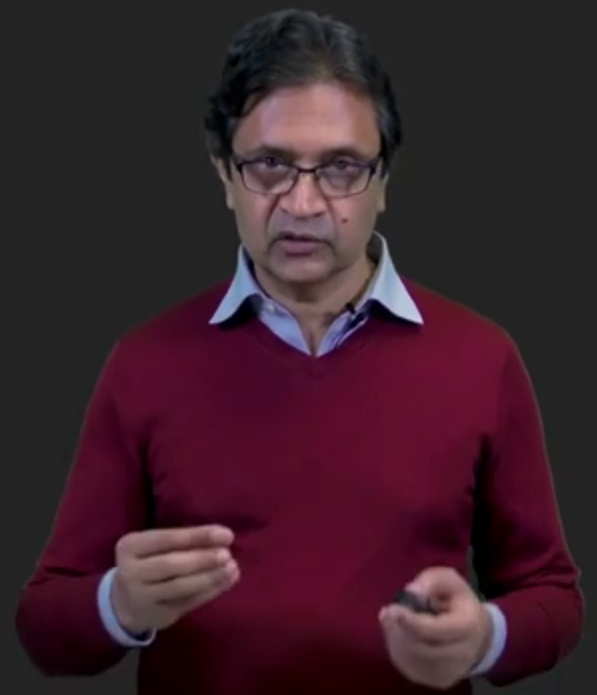

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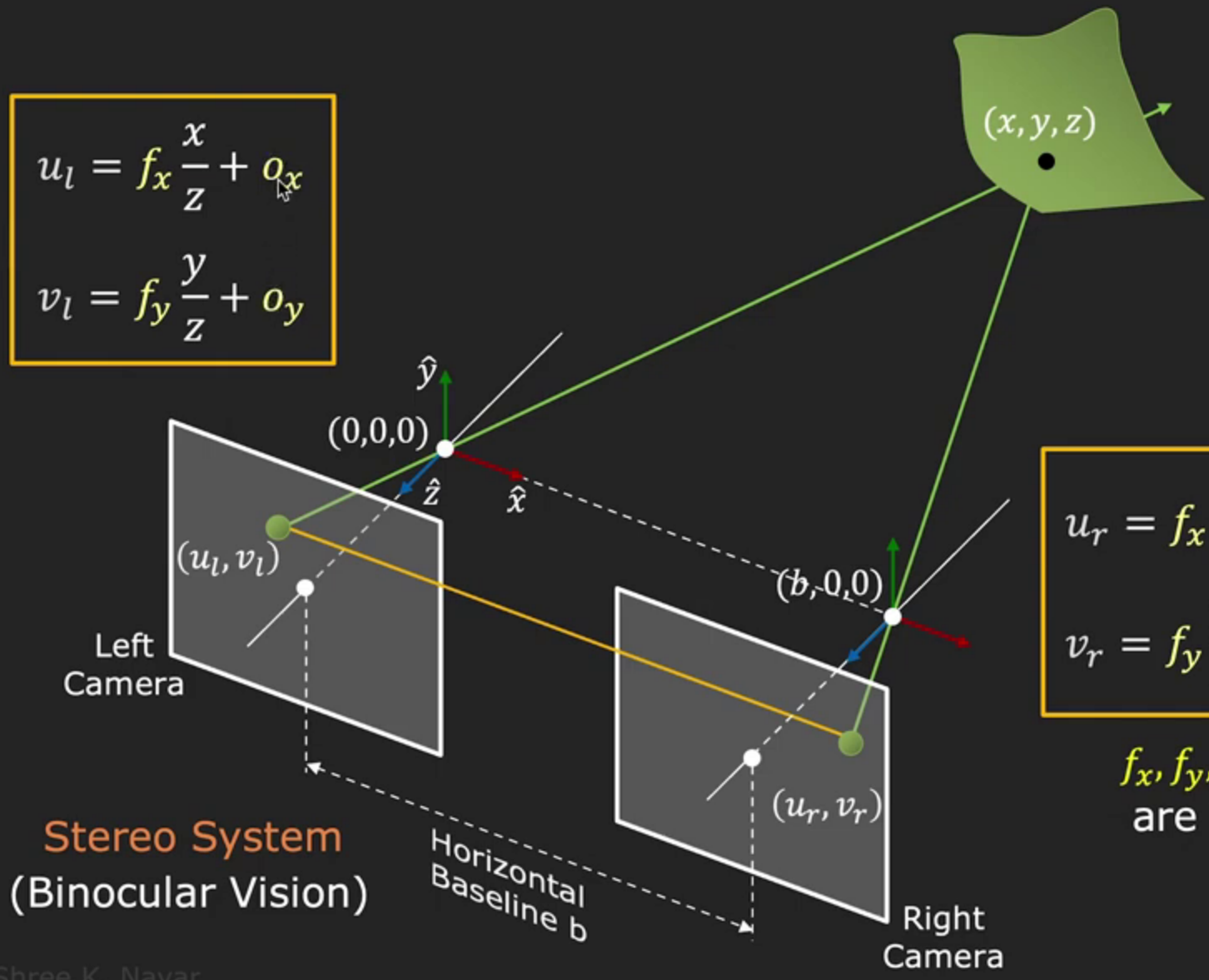
$$u_l = f_x \frac{x}{z} + o_x$$

$$v_l = f_y \frac{y}{z} + o_y$$



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$$u_r = f_x \frac{x - b}{z} + o_x$$

$$v_r = f_y \frac{y}{z} + o_y$$

f_x, f_y, b, o_x, o_y are known.

Triangulation using Two Cameras

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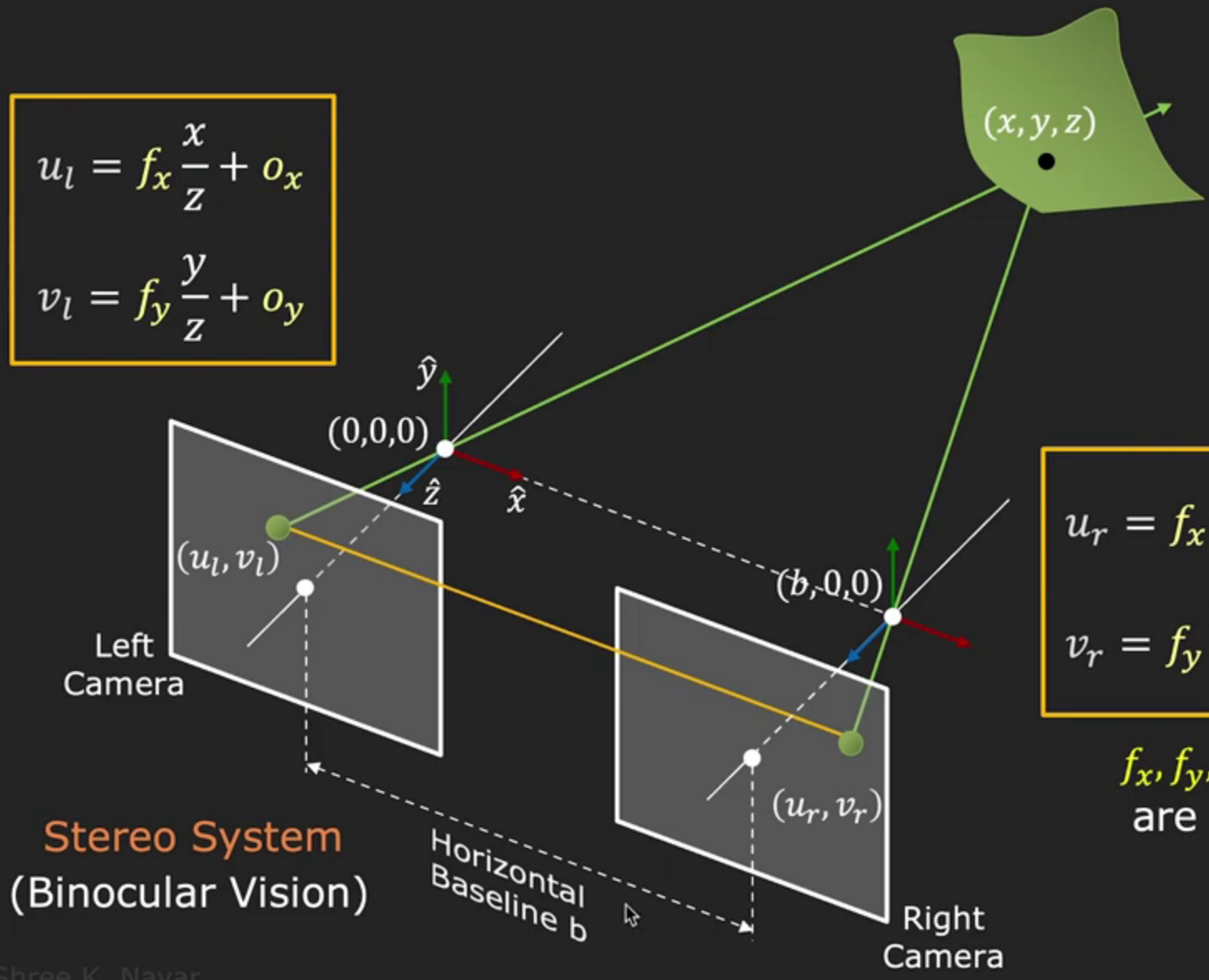

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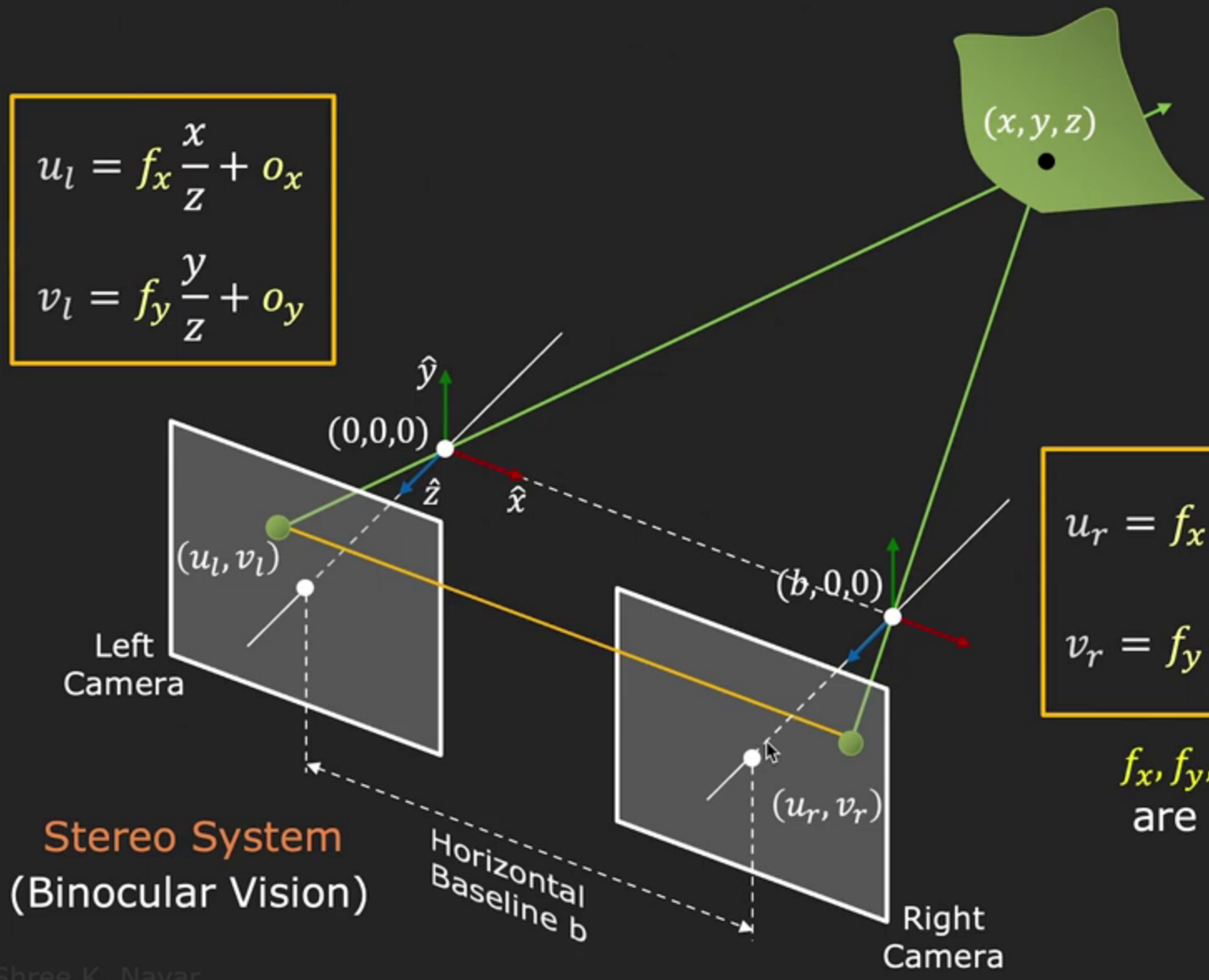

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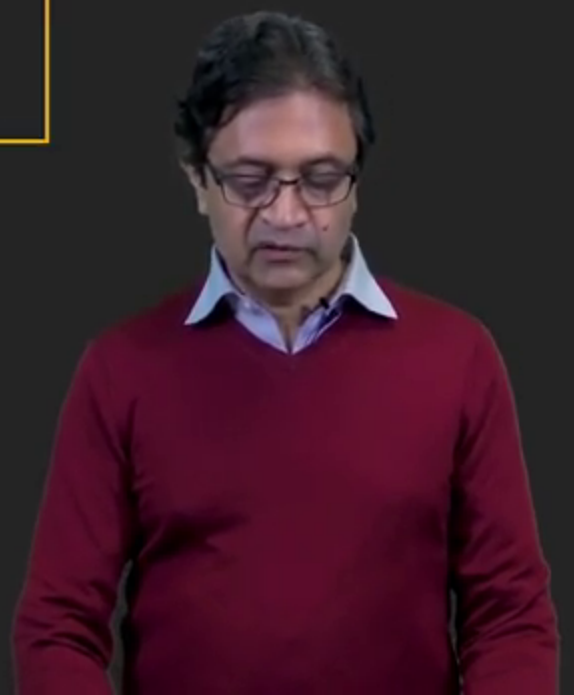
Simple Stereo: Depth and Disparity

From perspective projection:

$$(u_l, v_l) = \left(f_x \frac{x}{z} + o_x, f_y \frac{y}{z} + o_y\right) \qquad (u_r, v_r) = \left(f_x \frac{x-b}{z} + o_x, f_y \frac{y}{z} + o_y\right)$$

Solving for (x, y, z):

$$x = \frac{b(u_l - o_x)}{(u_l - u_r)} \qquad y = \frac{b f_x (v_l - o_y)}{f_y (u_l - u_r)} \qquad \boxed{z = \frac{b f_x}{(u_l - u_r)}}$$



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Depth z is inversely proportional to Disparity.

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A Simple Stereo Camera

Fujifilm FinePix REAL 3D W3

A Simple Stereo Camera

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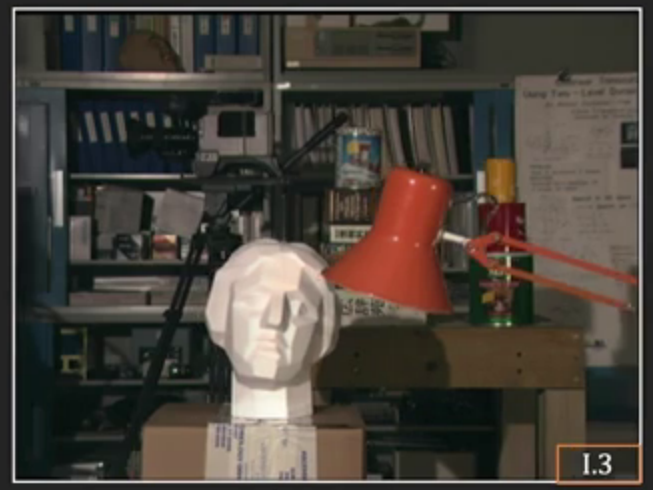
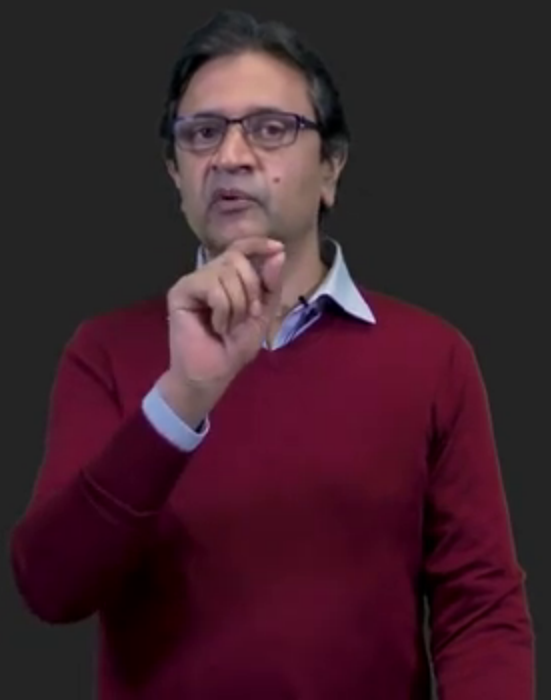
Stereo Matching: Finding Disparities

Goal: Find the disparity between left and right stereo pairs.



Left/Right Camera Images



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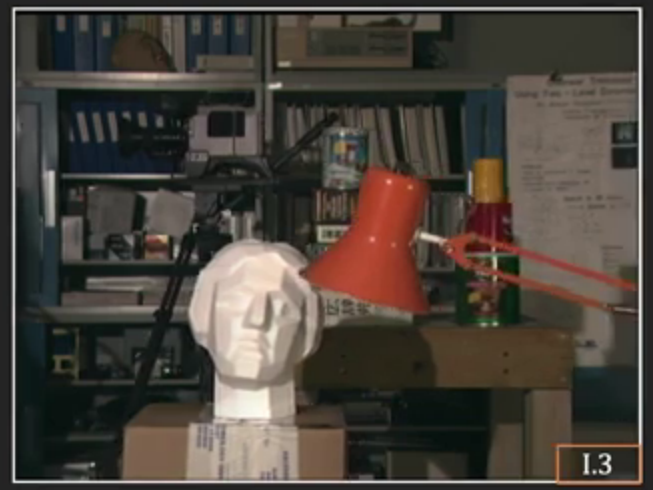

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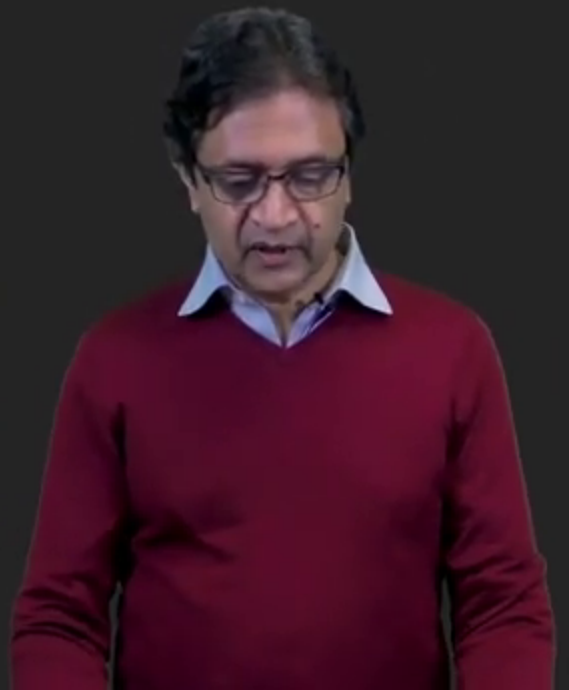

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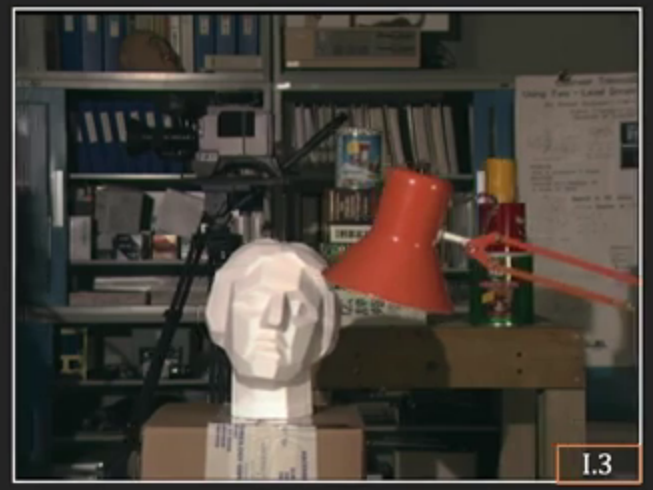
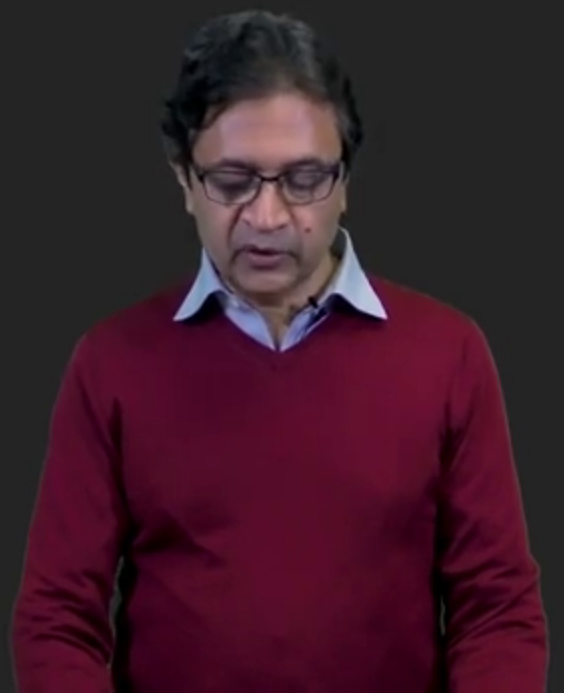


Left/Right Camera Images

Disparity Map (Ground Truth)



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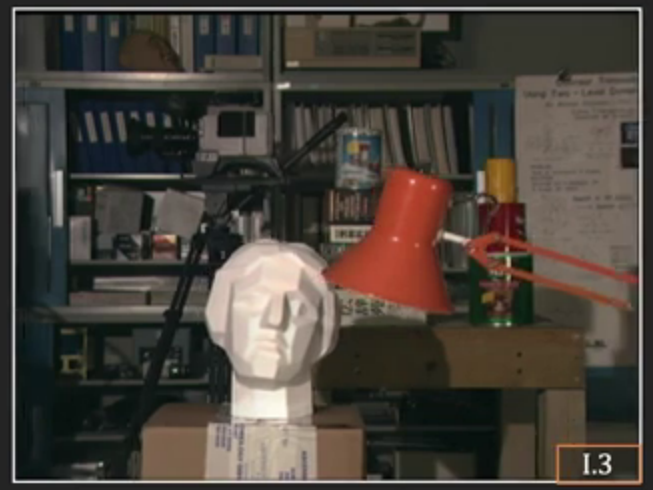

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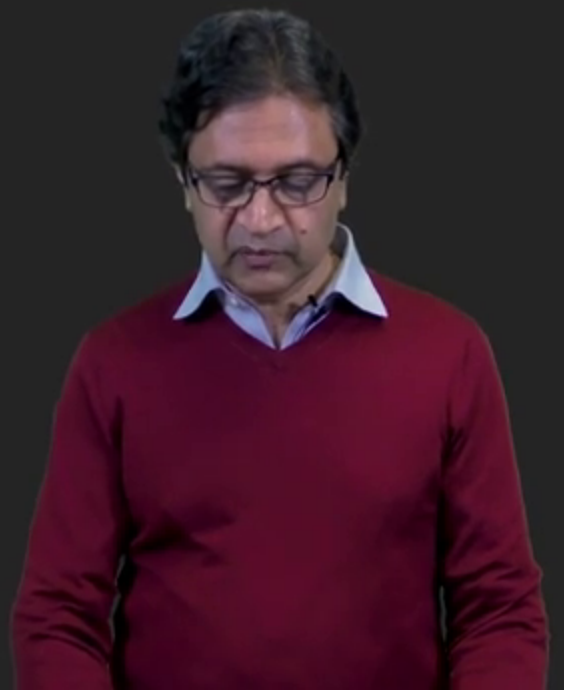
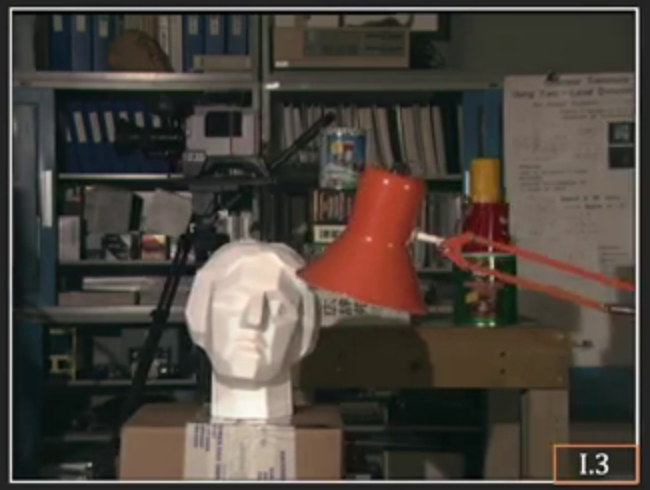
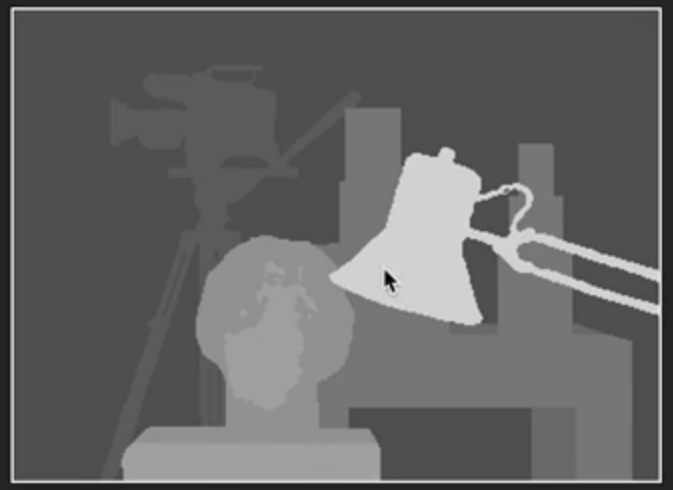


Stereo Matching: Finding Disparities

Goal: Find the disparity between left and right stereo pairs.



Left/Right Camera Images



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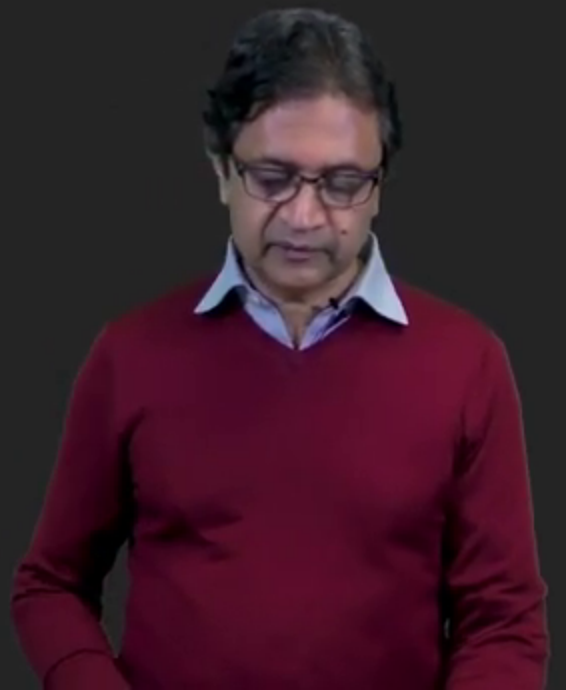

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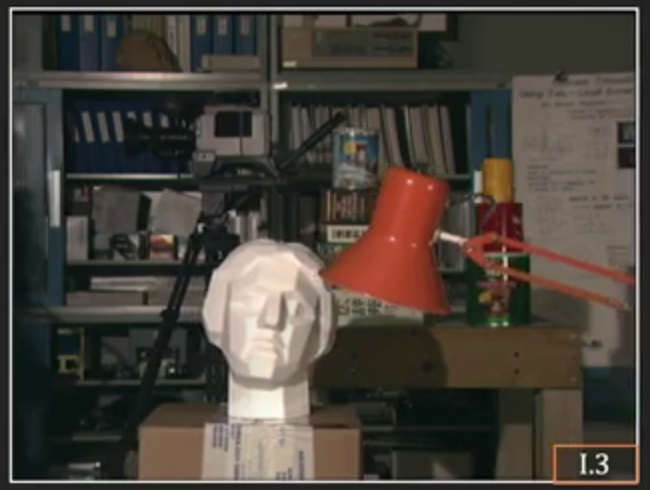

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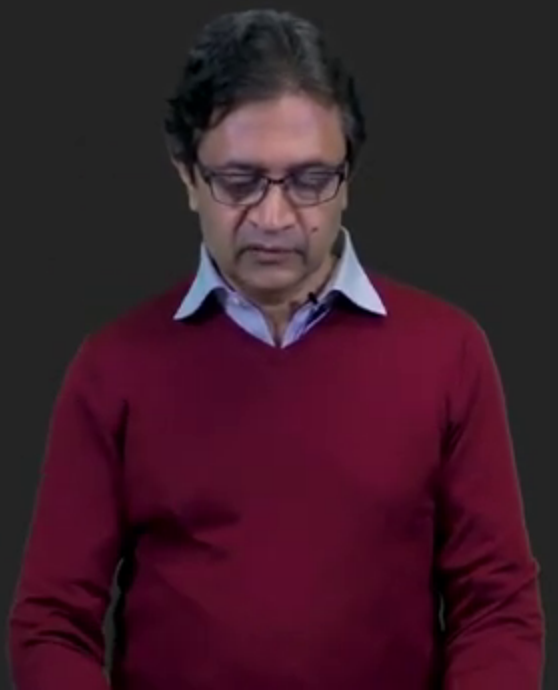
Disparity Map (Ground Truth)

From perspective projection: $v_l = v_r = f_y \frac{y}{z} + o_y$



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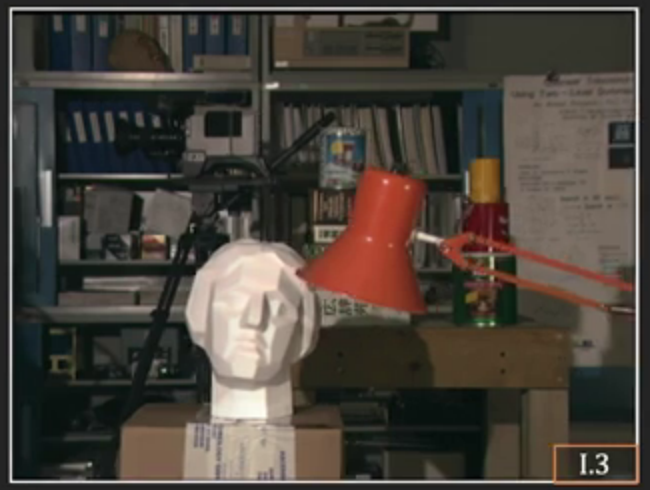

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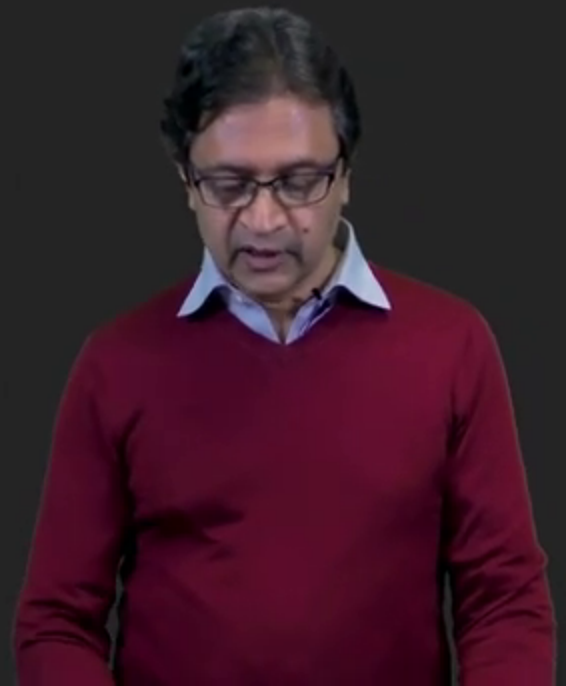

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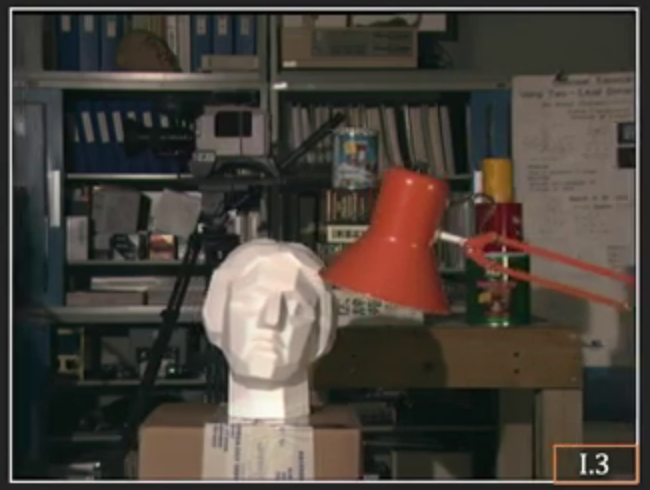
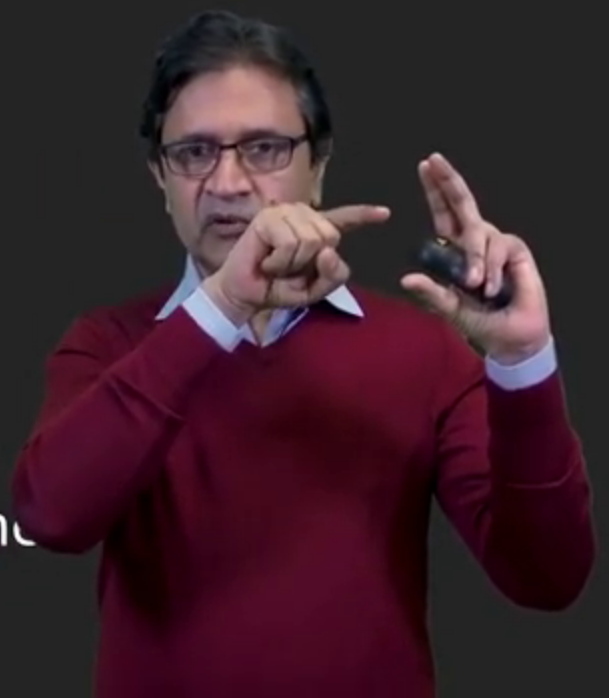


Left/Right Camera Images

Disparity Map (Ground Truth)

From perspective projection: $v_l = v_r = f_y \frac{y}{z} + o_y$

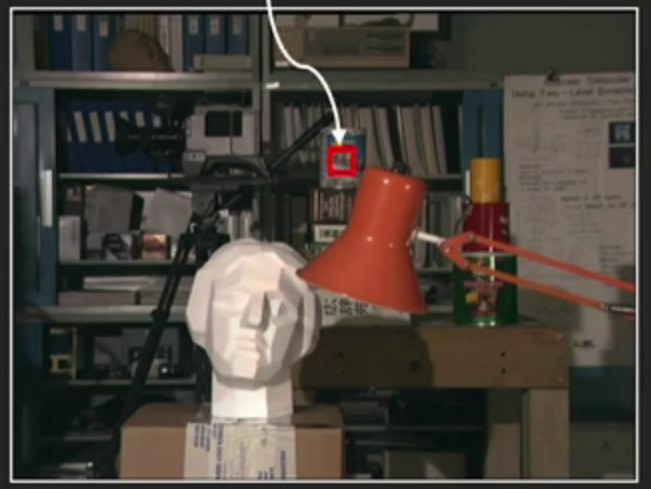
Corresponding scene points lie on the same horizontal scan line



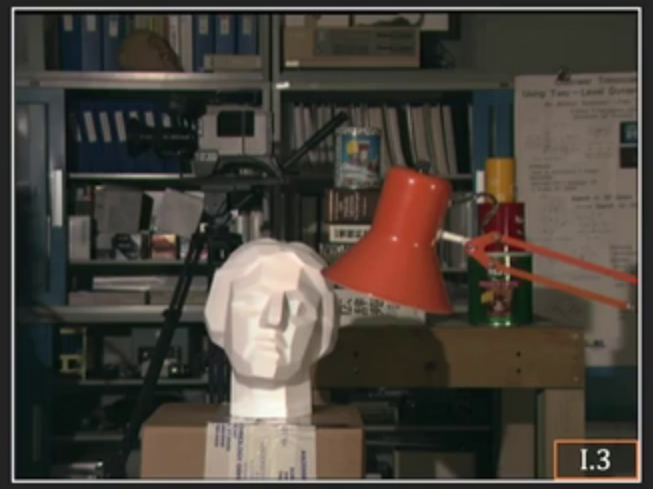
Window Based Methods

Determine Disparity using Template Matching

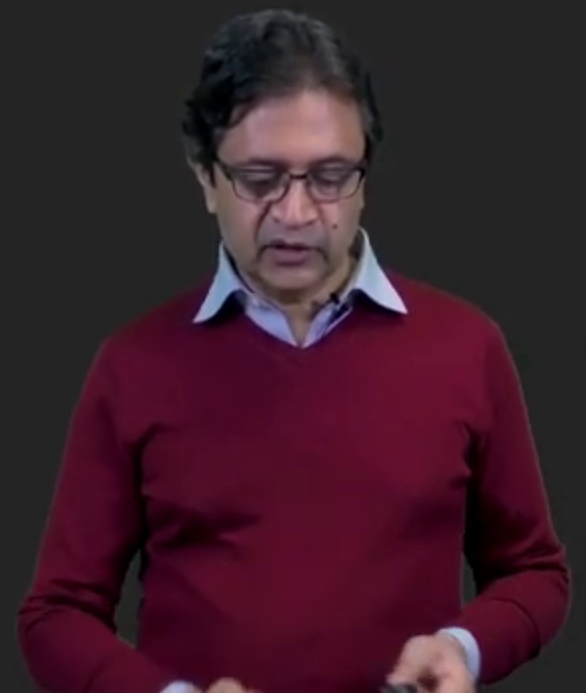
Template Window T



Left Camera Image E_l



Right Camera Image E_r



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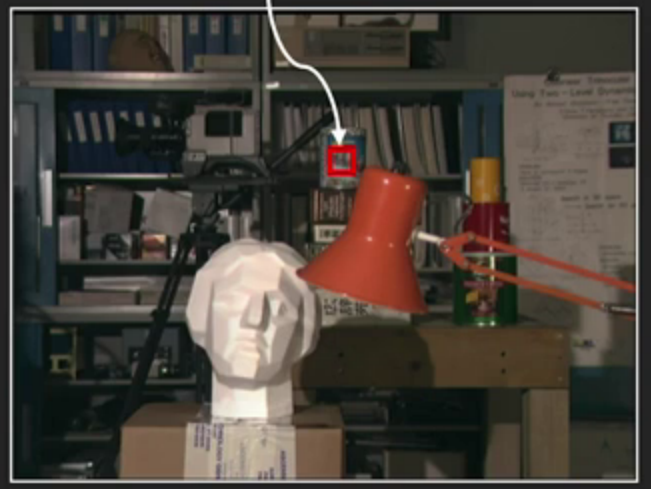

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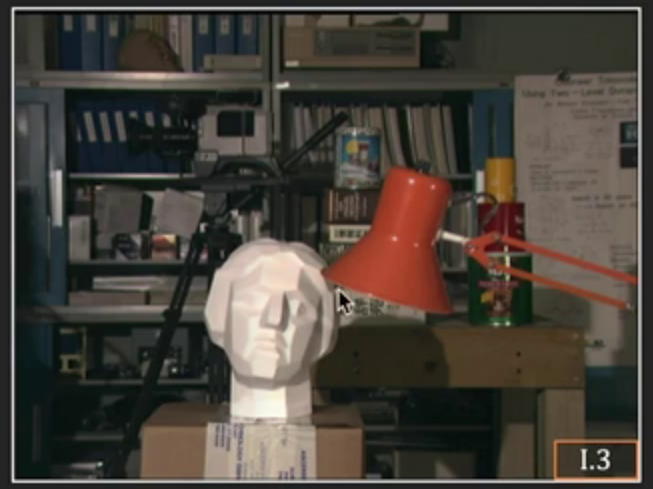
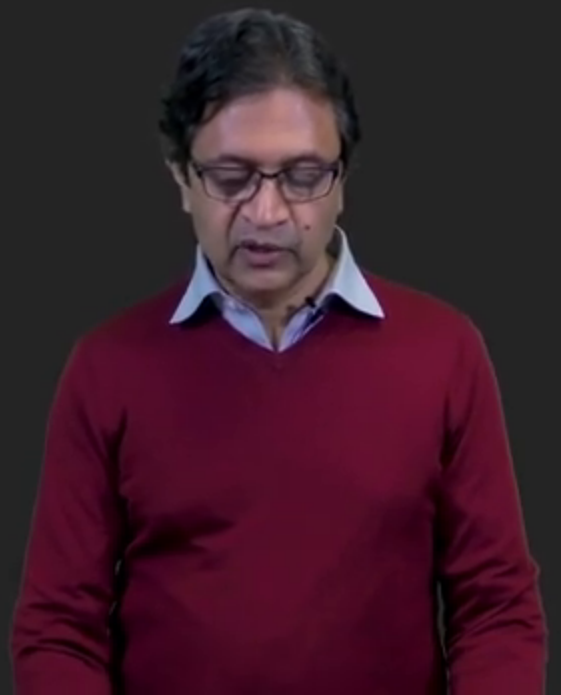


Right Camera Image E_r



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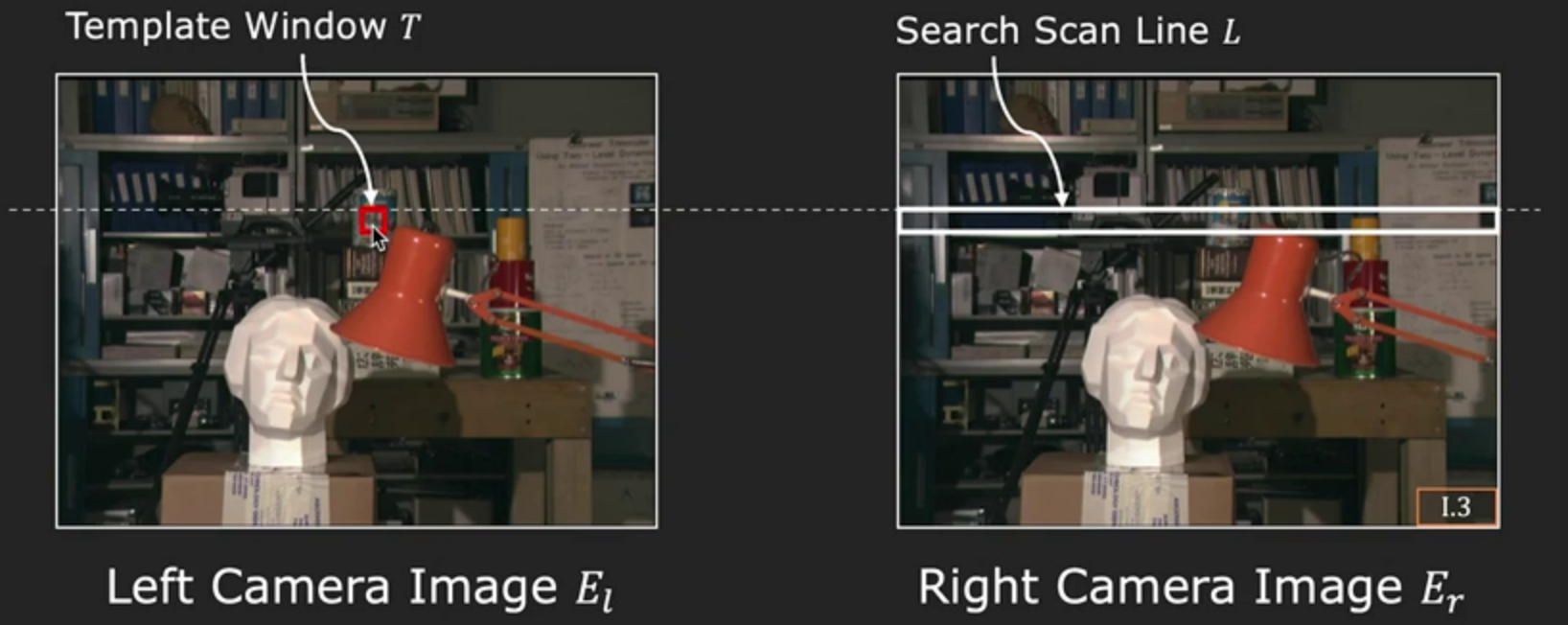

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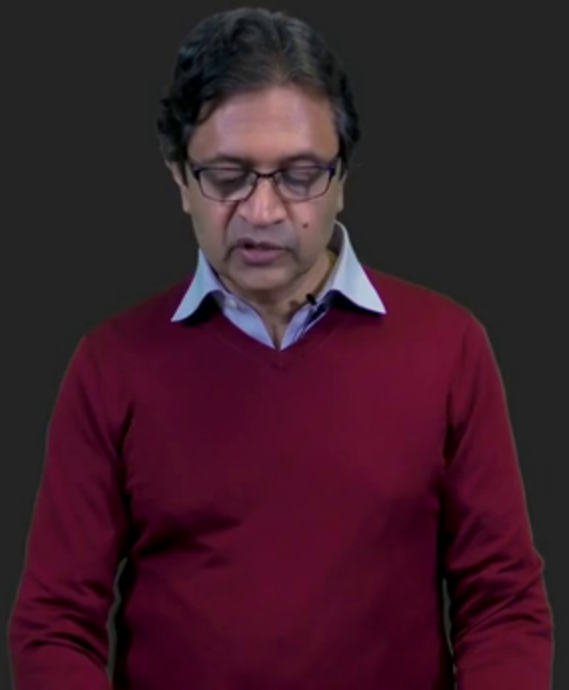

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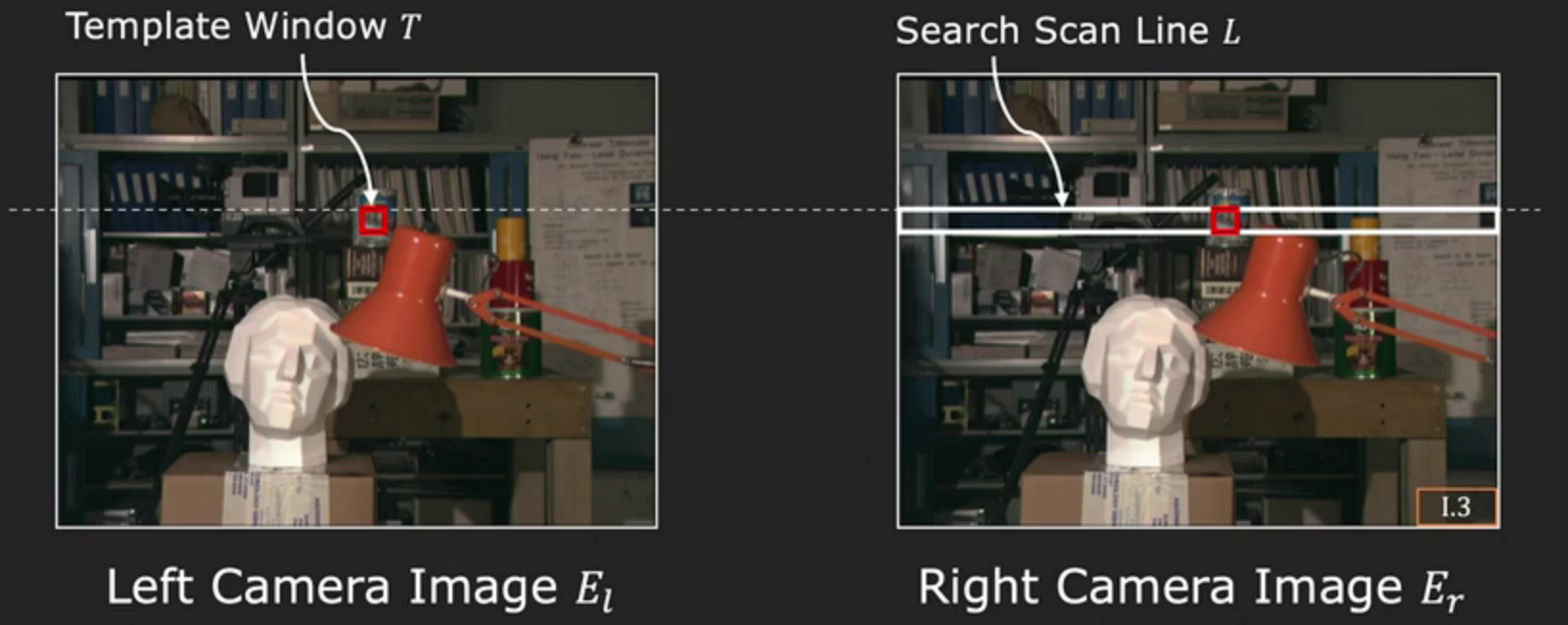

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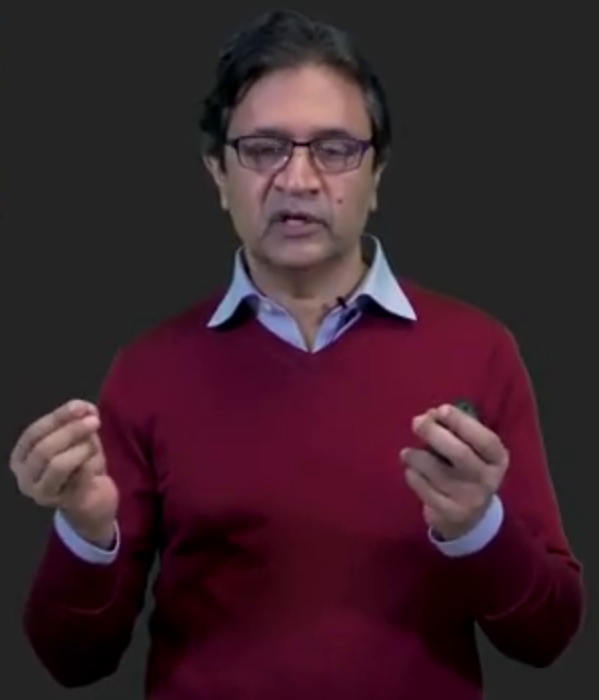

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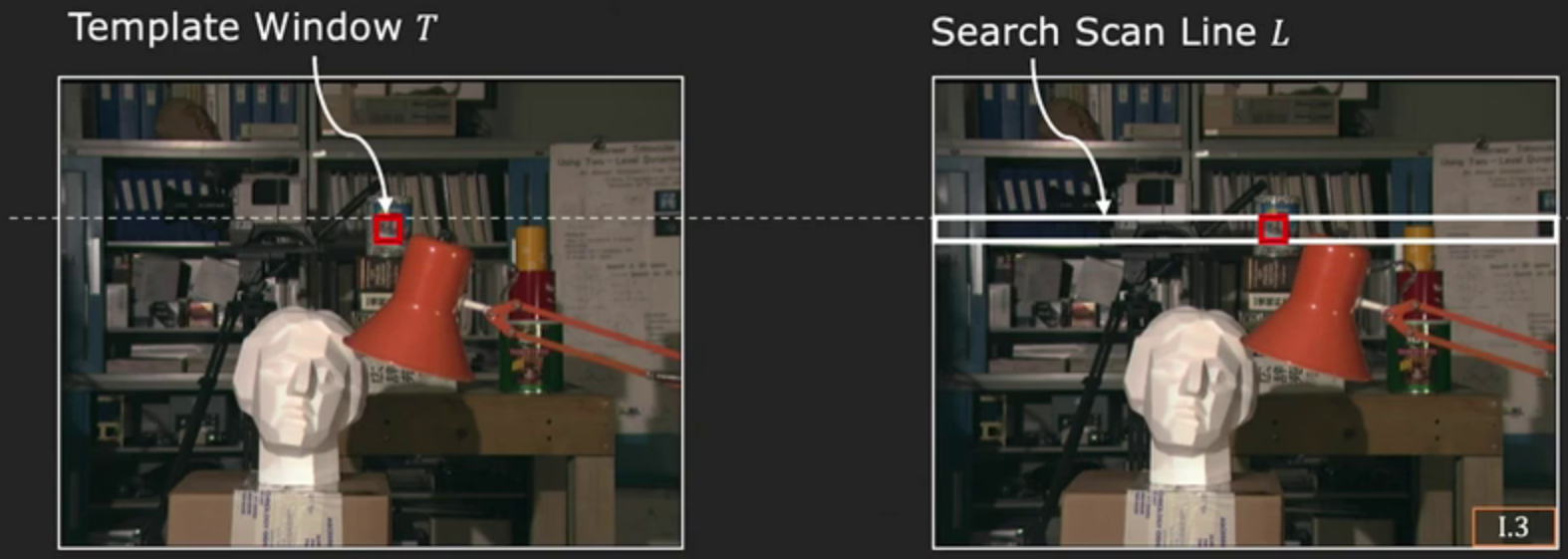

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Disparity: $d = u_l - u_r$

Depth: $z = \dfrac{b f_x}{(u_l - u_r)}$

Similarity Metrics for Template Matching

Find pixel $(k, l) \in L$ with Minimum Sum of Absolute Differences:

$$SAD(k,l) = \sum_{(i,j)\in T} |E_l(i,j) - E_r(i+k, j+l)|$$

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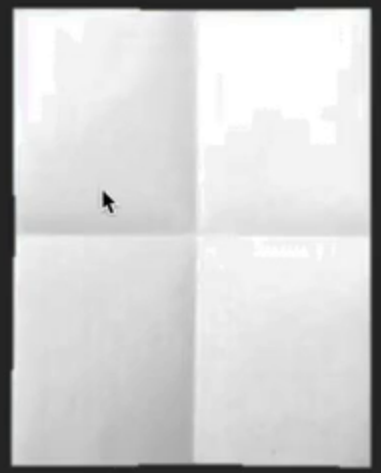

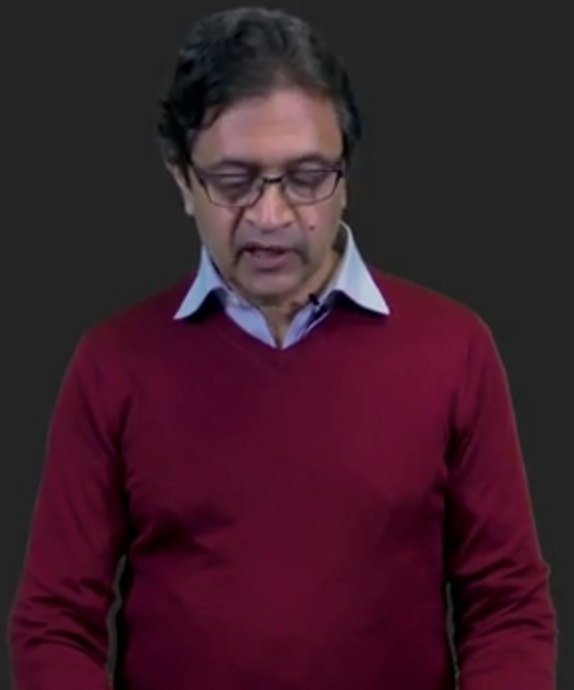

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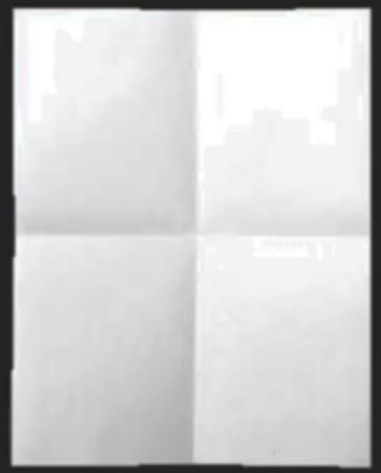

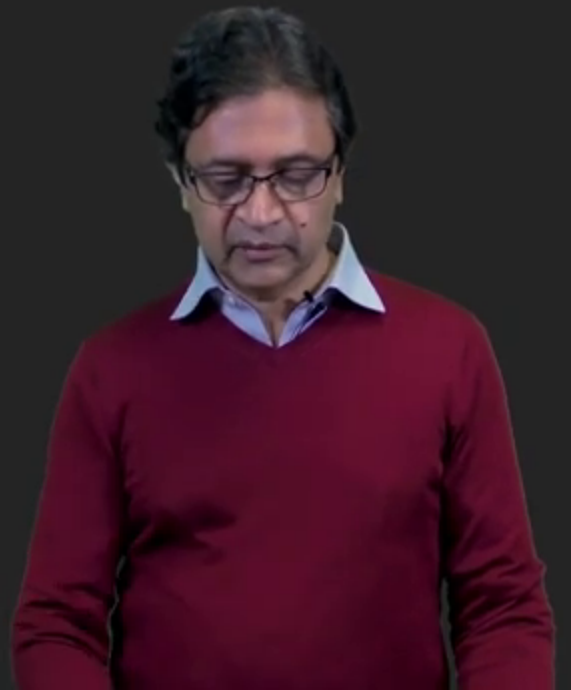

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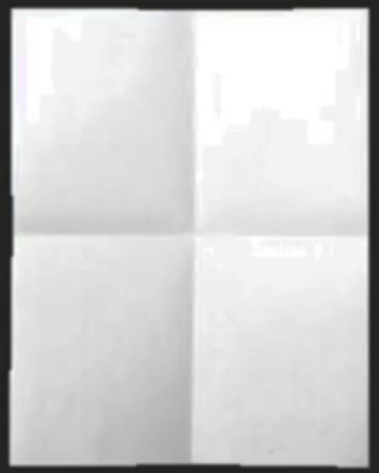

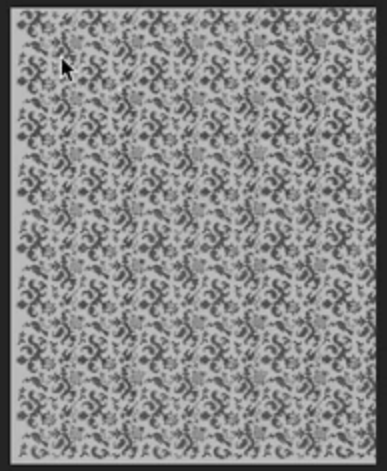

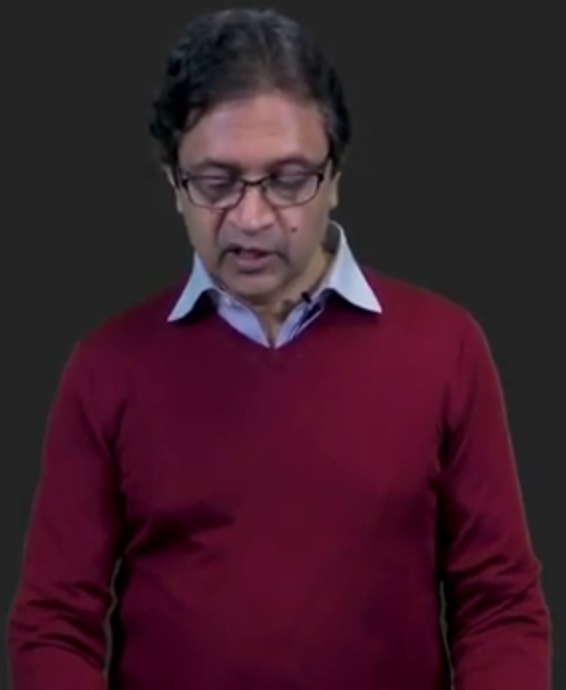

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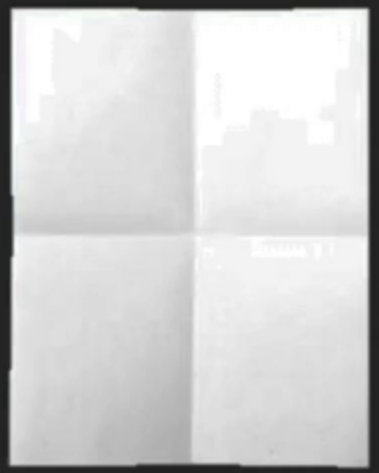

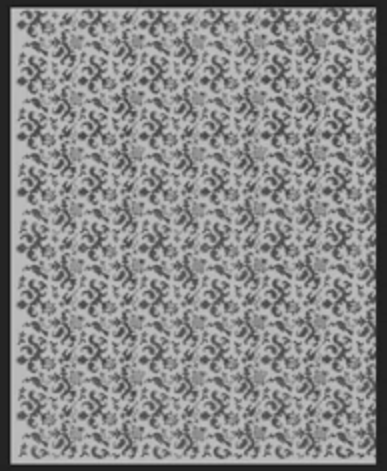

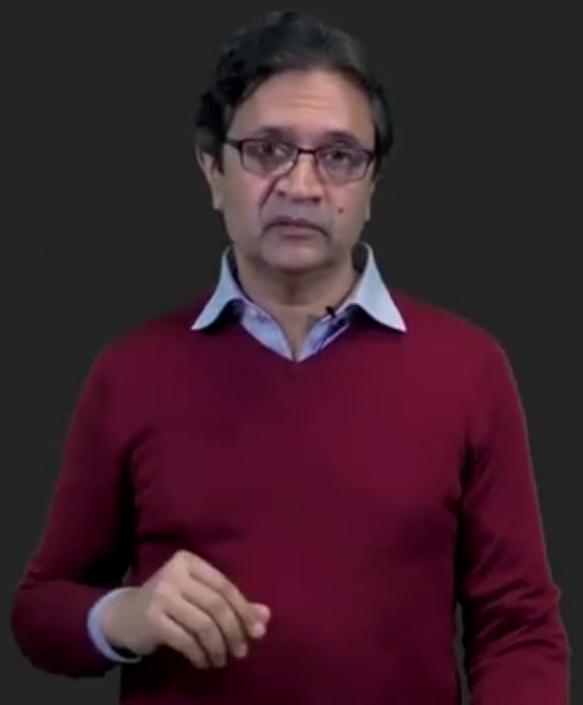

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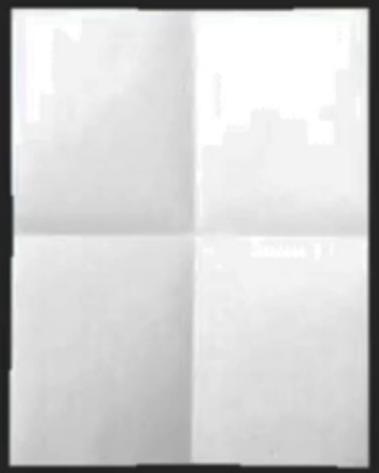

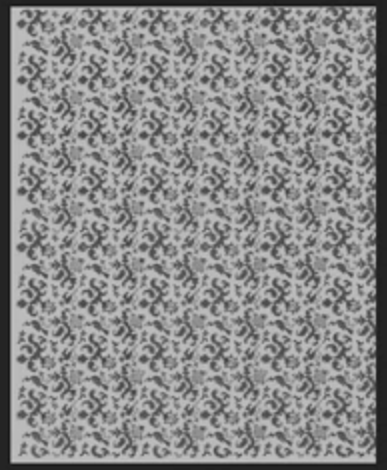

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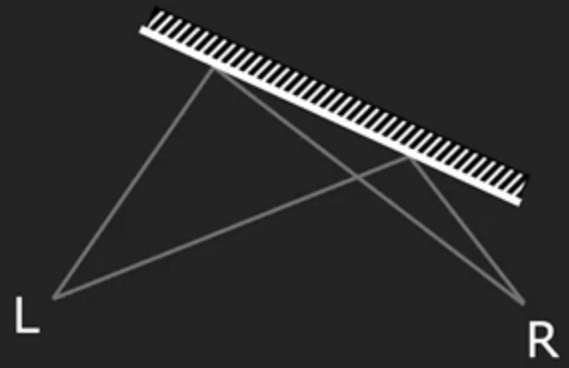

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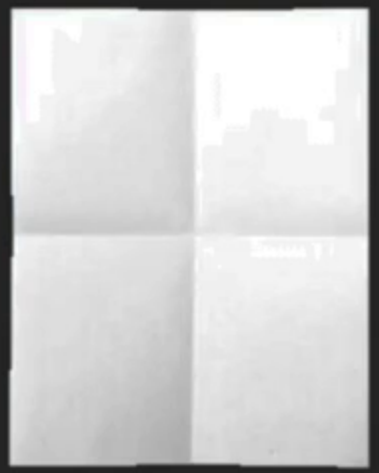

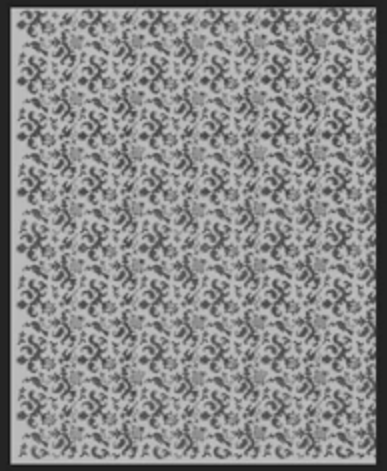

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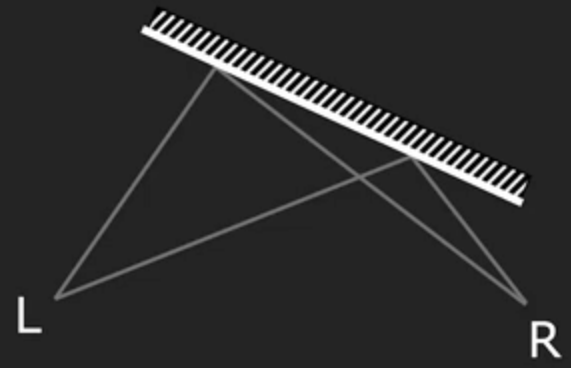

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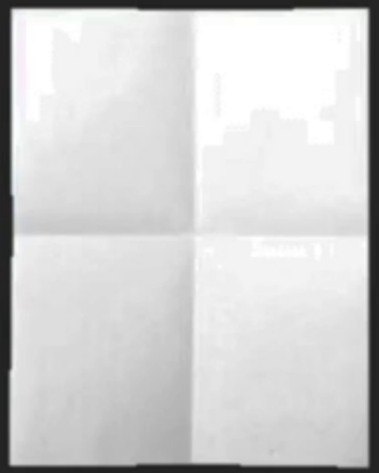

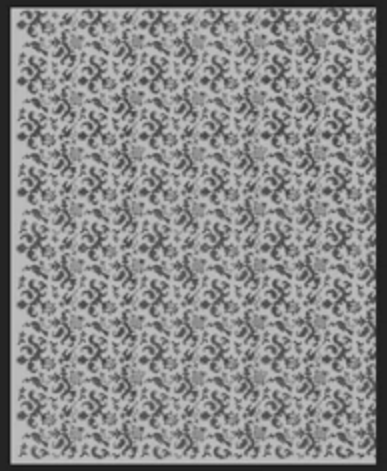

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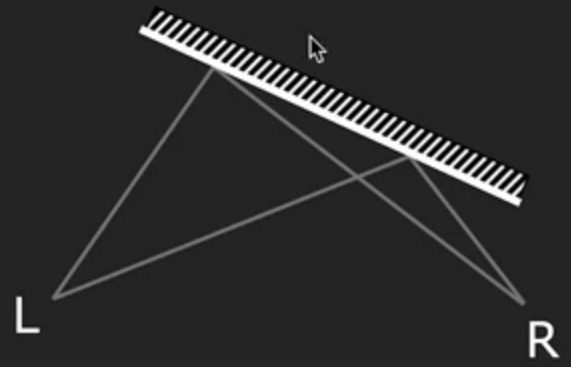

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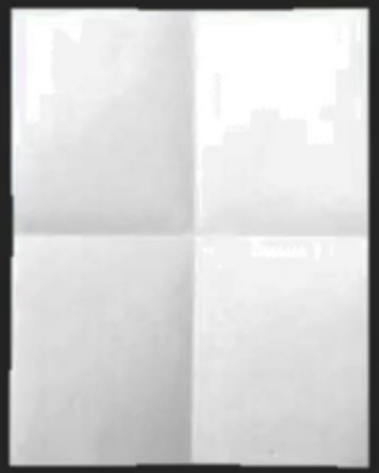

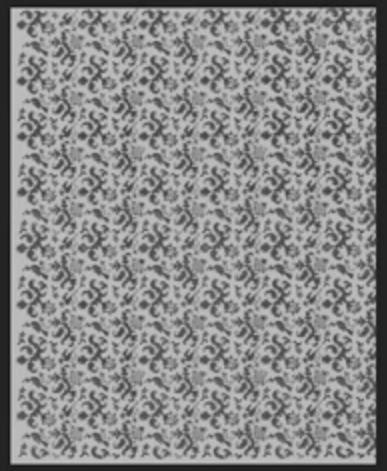

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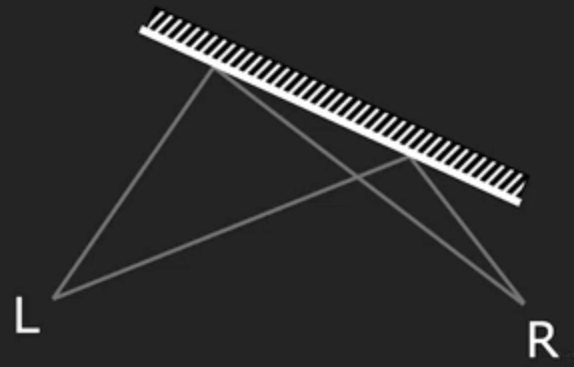

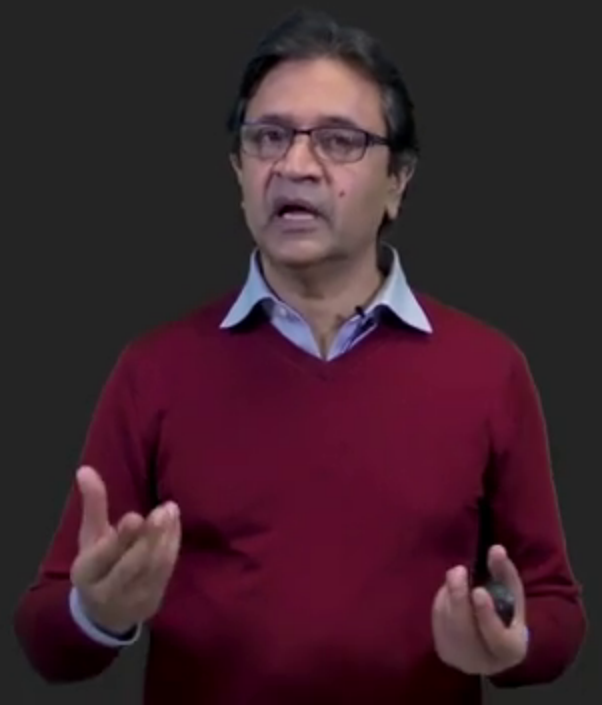

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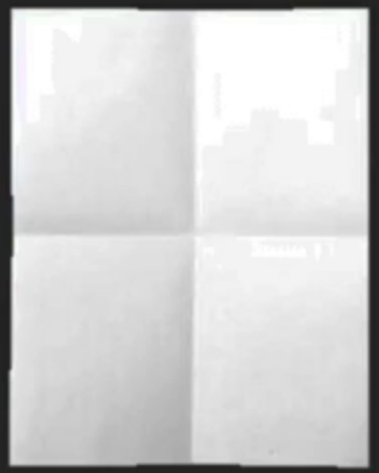

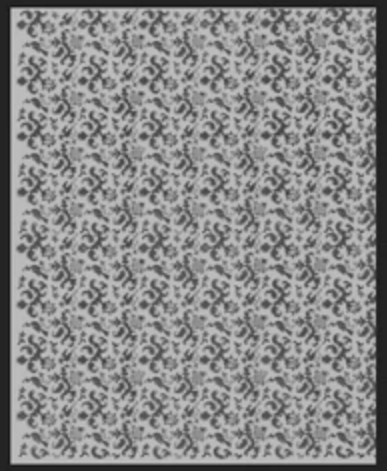

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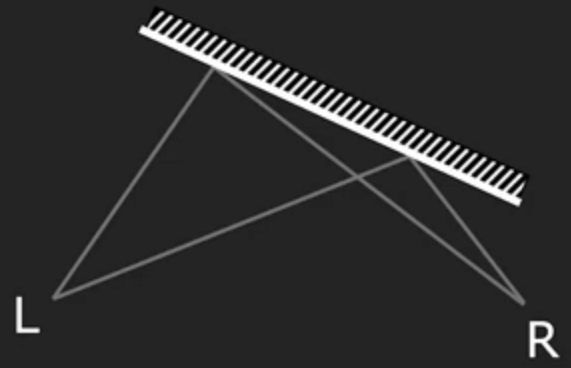
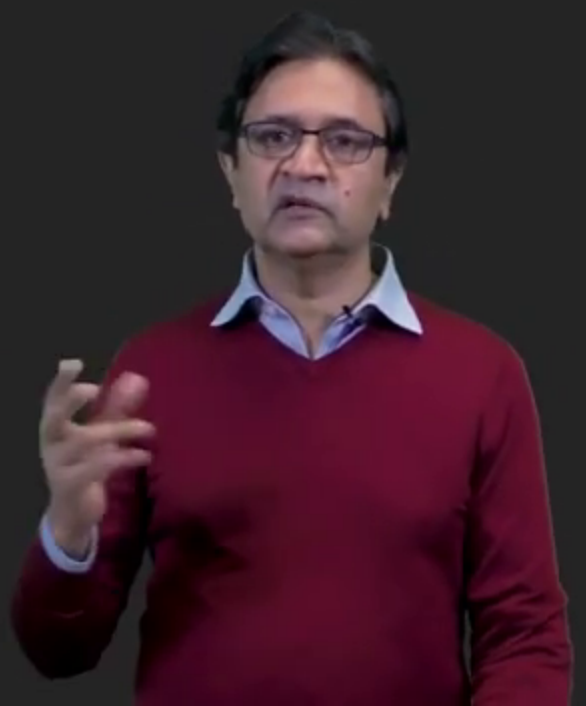


How Large Should Window Be?

Window size = 5 pixels
(Sensitive to noise)

Window size = 30 pixels
(Poor localization)



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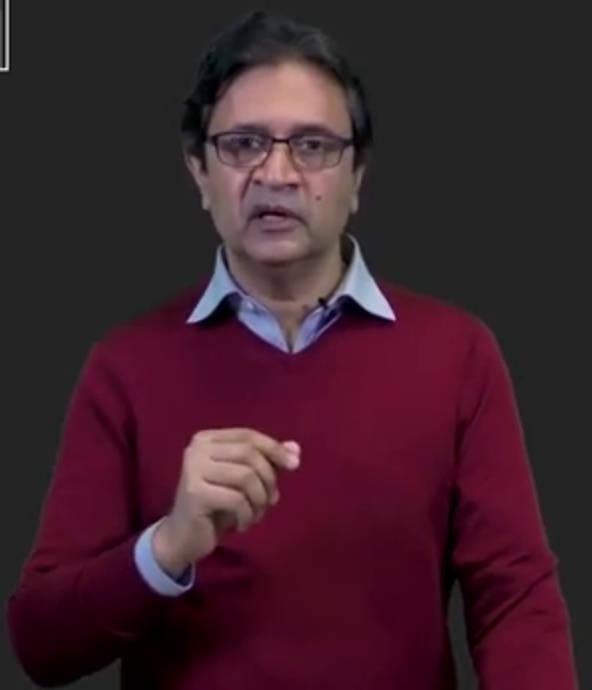

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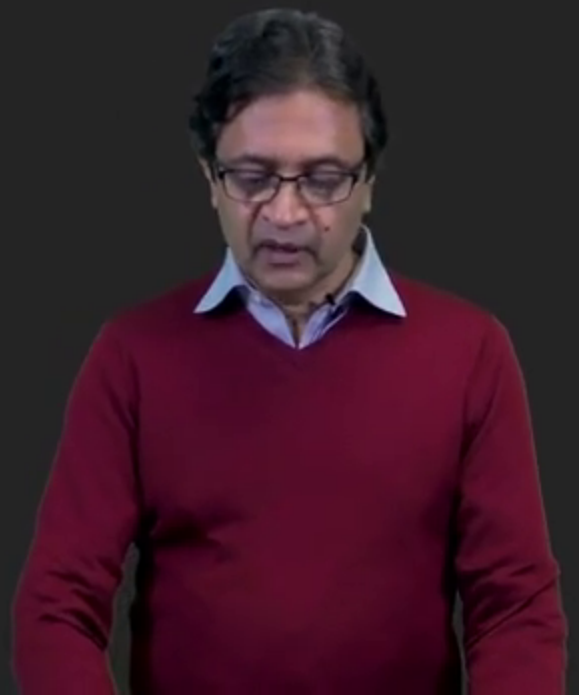

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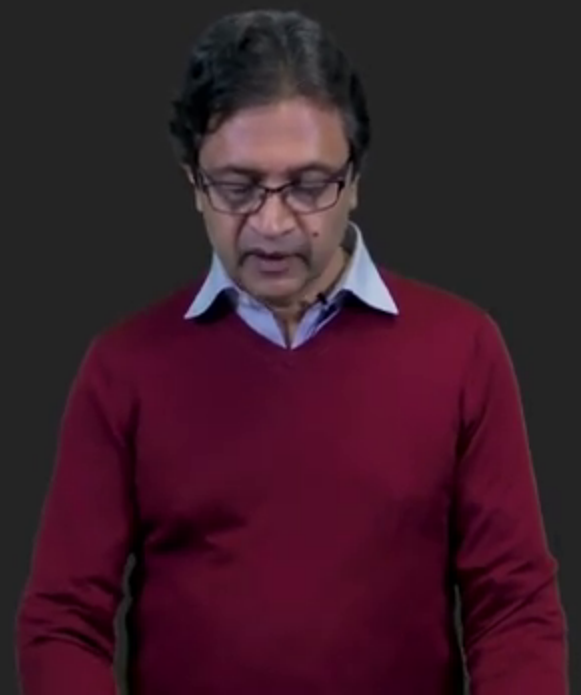
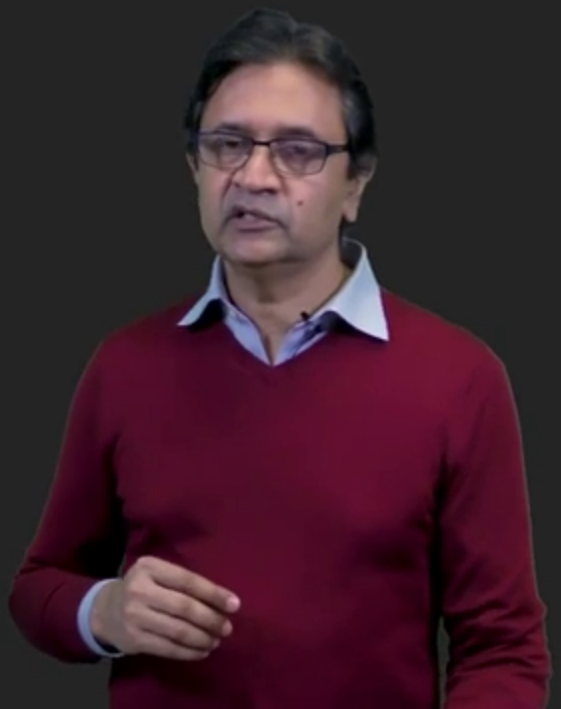


How Large Should Window Be?

Window size = 5 pixels
(Sensitive to noise)

Window size = 30 pixels
(Poor localization)



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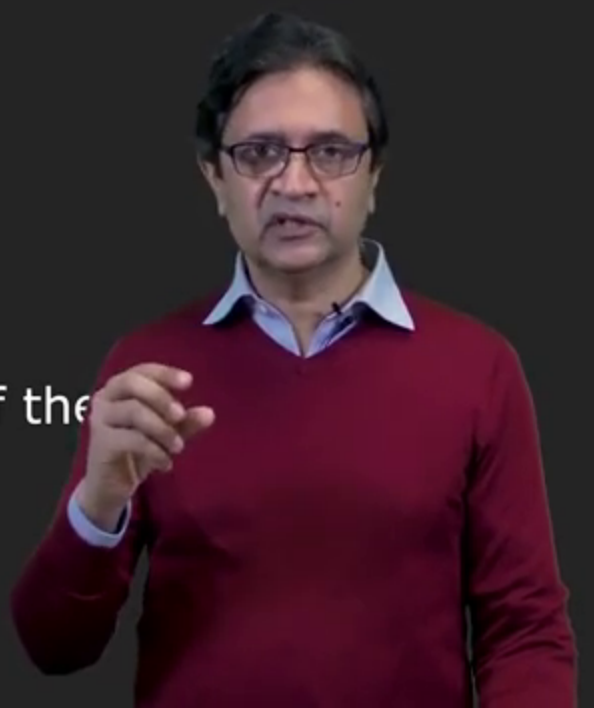
Window size = 30 pixels
(Poor localization)

Adaptive Window Method Solution: For each point, match using windows of multiple sizes and use the disparity that is a result of the best similarity measure (minimize SSD per pixel).



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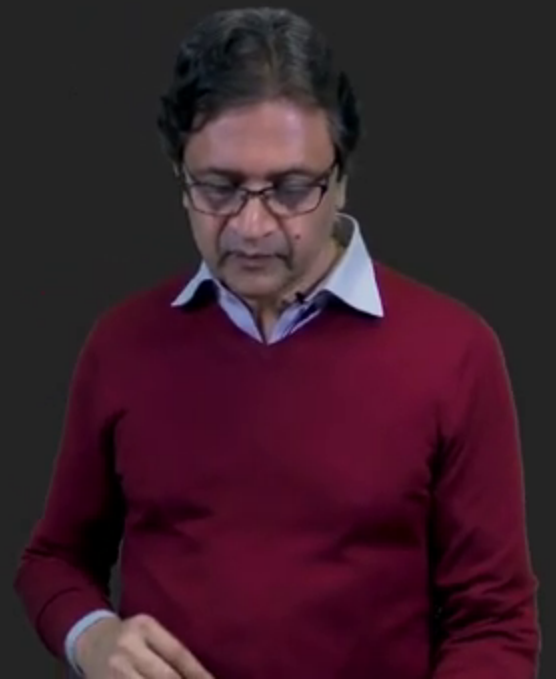
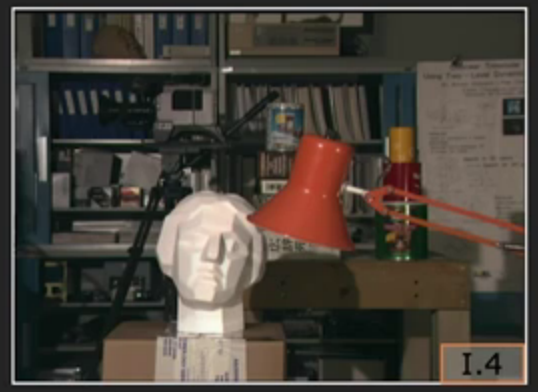
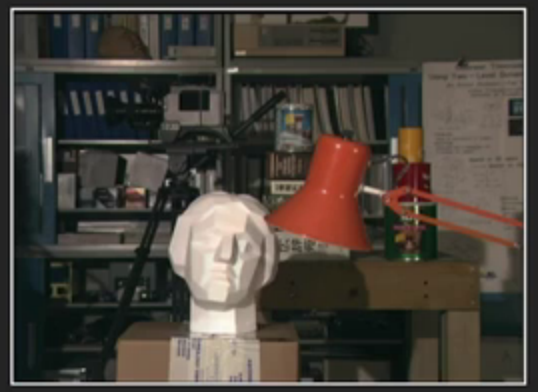


Window Based Methods: Results



Left Image



Right Image

Ground Truth

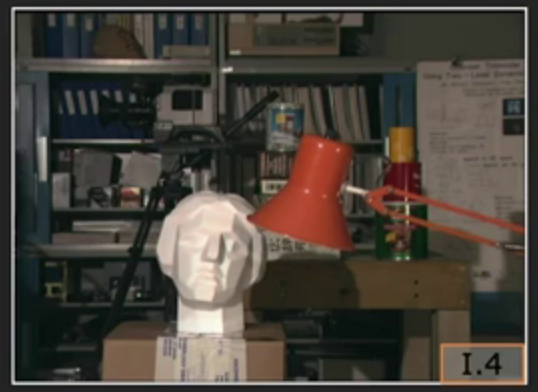
SSD (Window size=21)

SSD – Adaptive Window

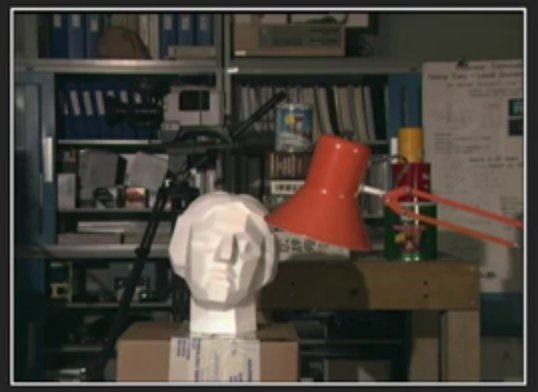
State of the Art

http://vision.middlebury.edu/stereo

Window Based Methods: Results

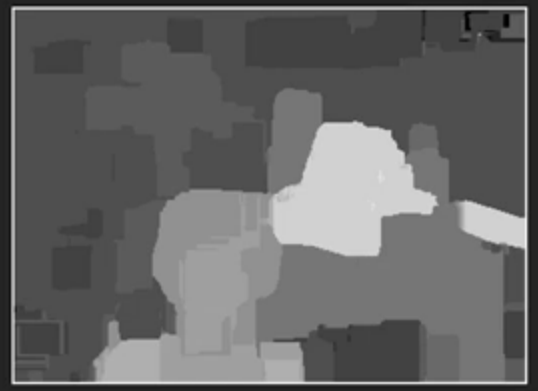


Left Image



Right Image

Ground Truth



SSD (Window size=21)

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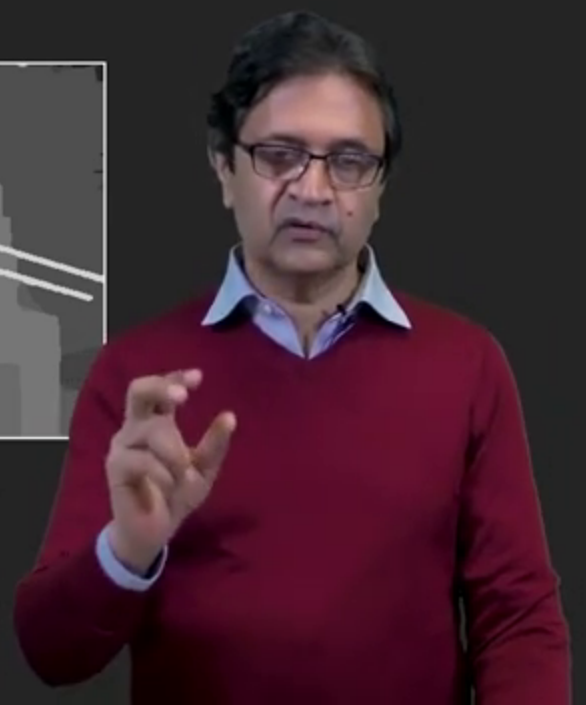
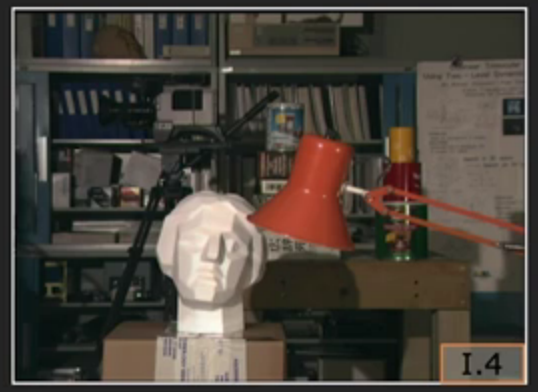
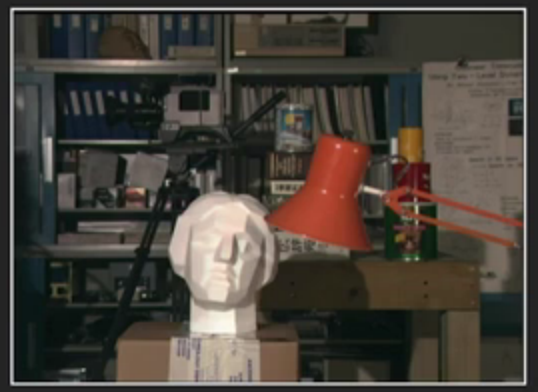


Window Based Methods: Results

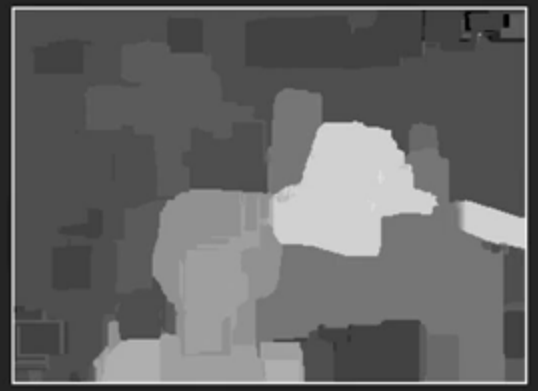


Left Image

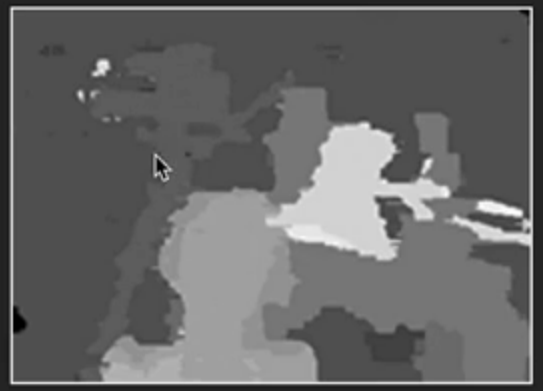


Right Image

Ground Truth



SSD (Window size=21)



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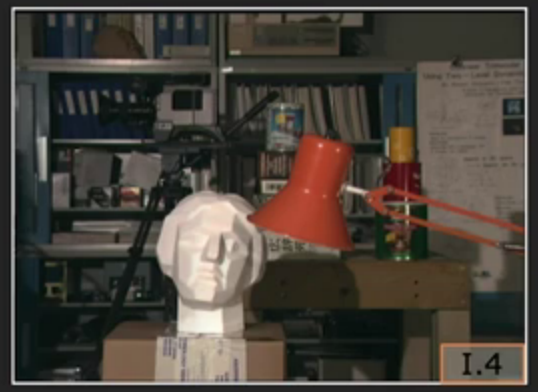
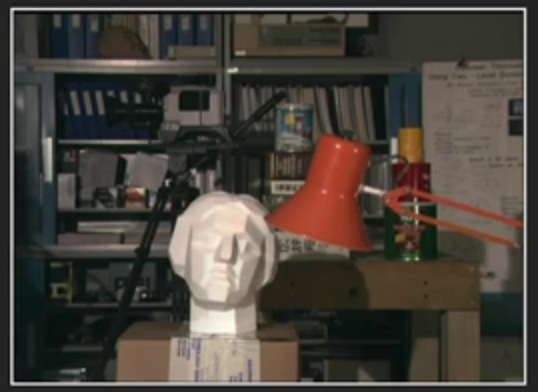
Window Based Methods: Results

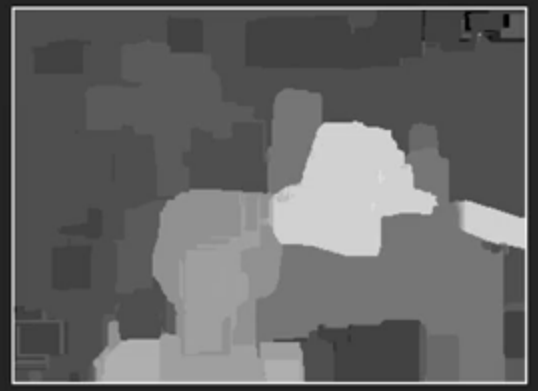


Left Image



Right Image

Ground Truth



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