Rank of Observation Matrix

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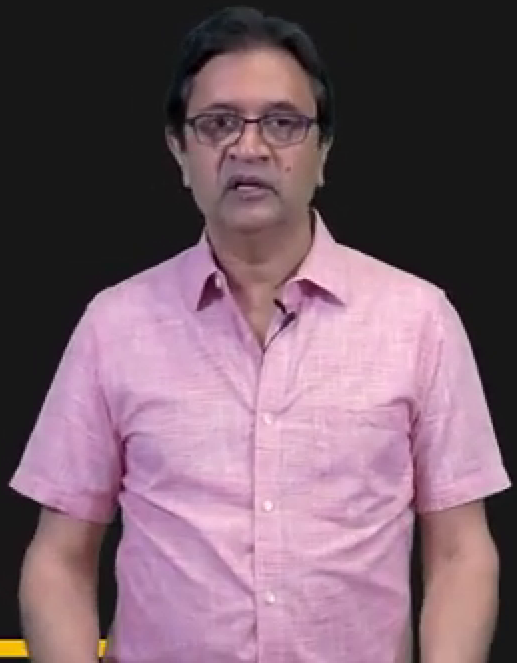
Columbia University

Topic: Structure from Motion, Module: Reconstruction II

First Principles of Computer Vision

Linear Independence of Vectors

A set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n\}$ is said to be linearly independent if no vector can be represented as a weighted linear sum of the others.



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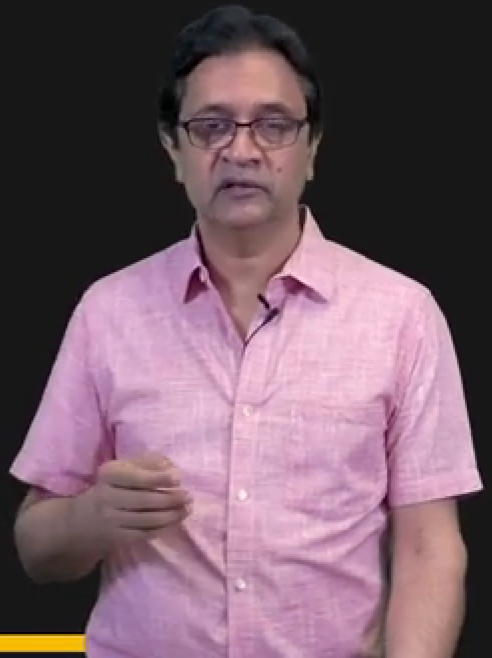

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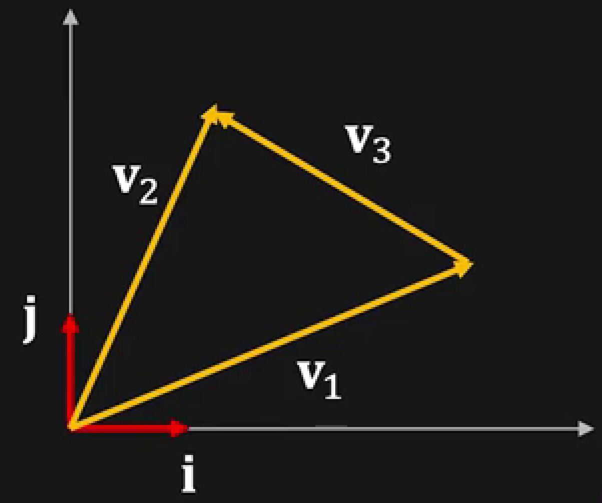

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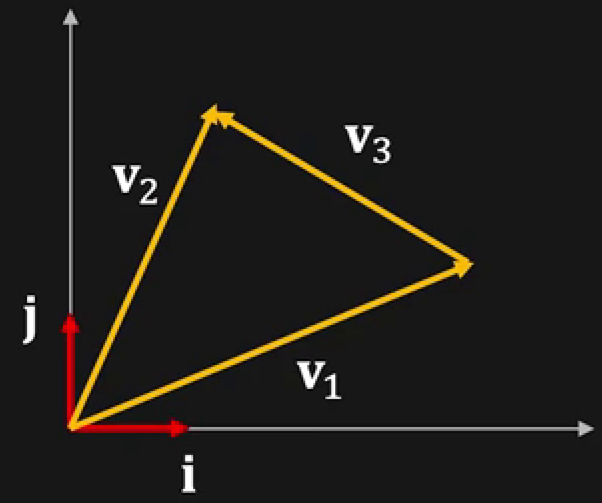

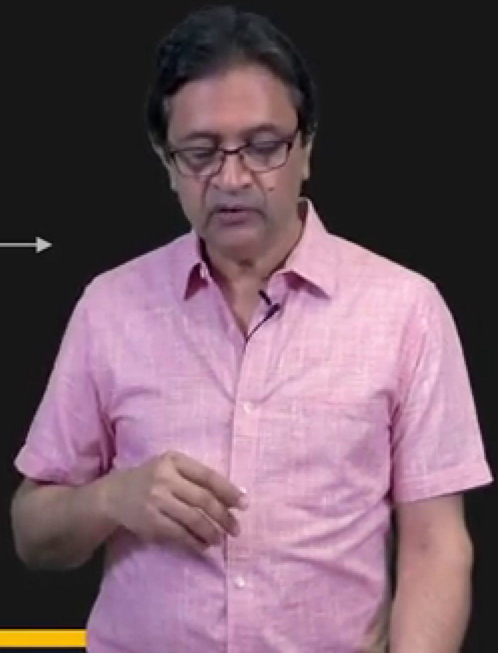

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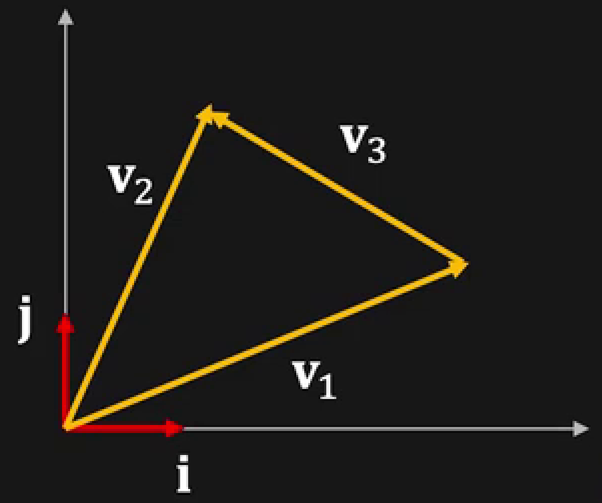

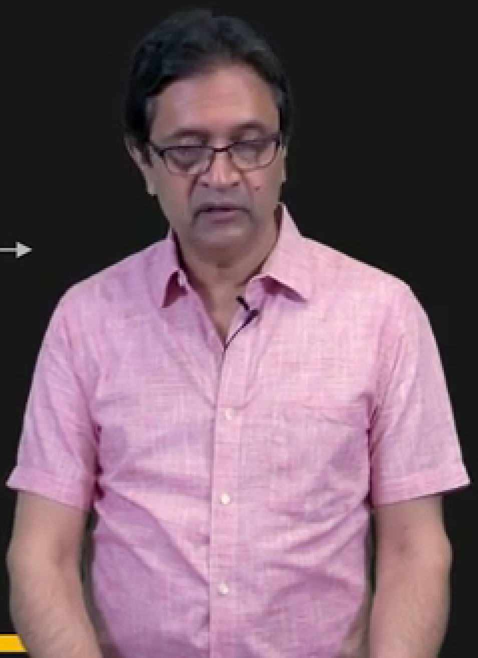

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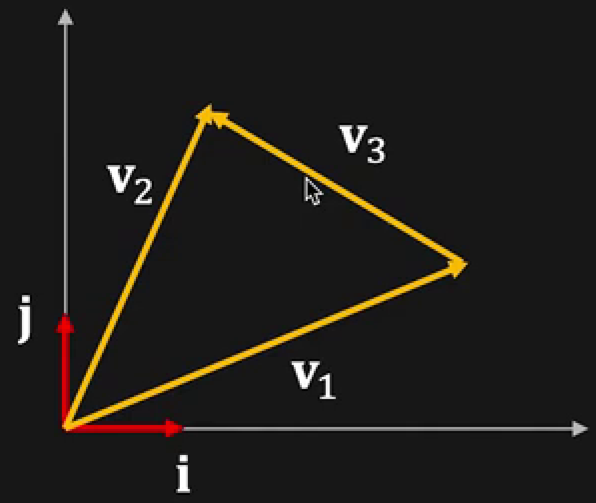

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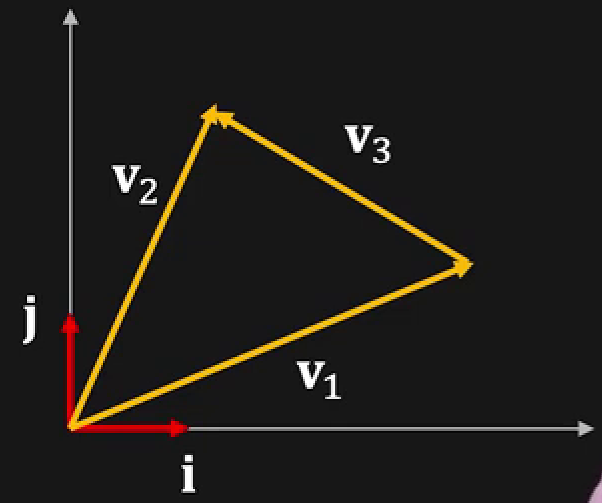
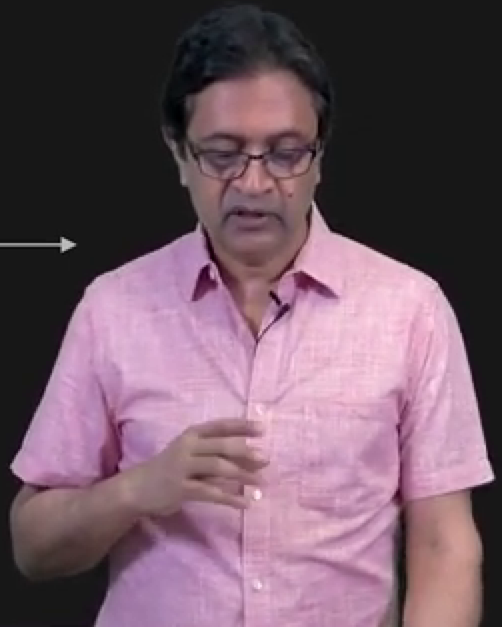


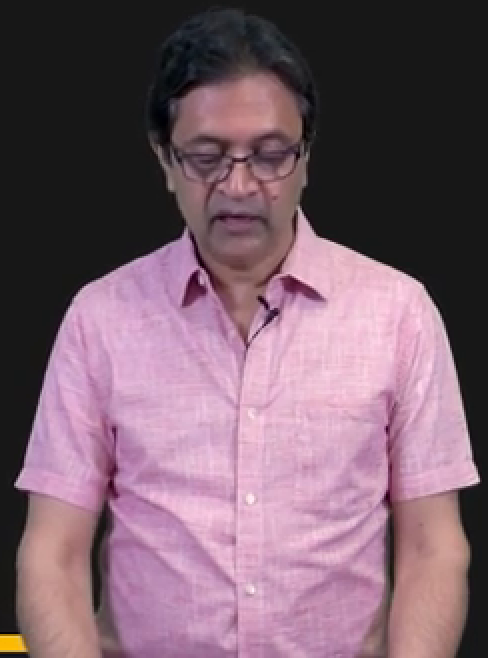


Rank of a Matrix

Column Rank: The number of linearly independent columns of the matrix.

Row Rank: The number of linearly independent rows of the matrix.

$$\begin{array}{cc} m & \begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \\ & n \end{array} = \begin{bmatrix} \mathbf{c}_1 & \mathbf{c}_2 & \dots & \mathbf{c}_n \end{bmatrix} = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \vdots \\ \mathbf{r}_m^T \end{bmatrix}$$



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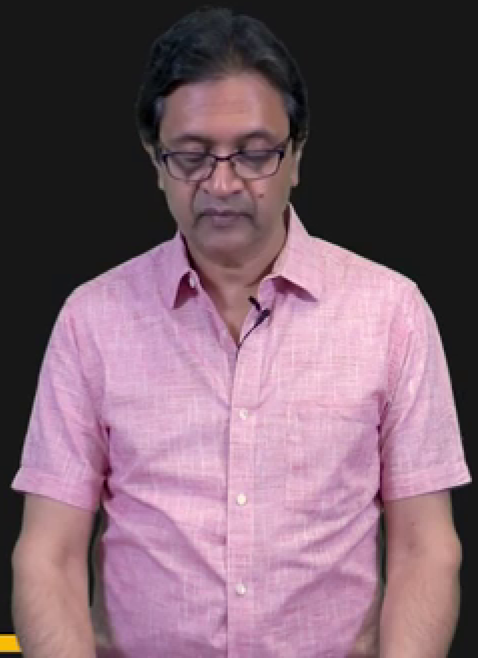

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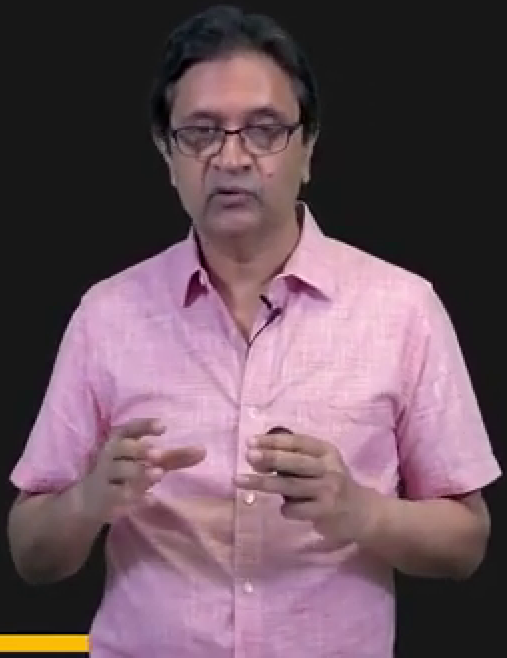

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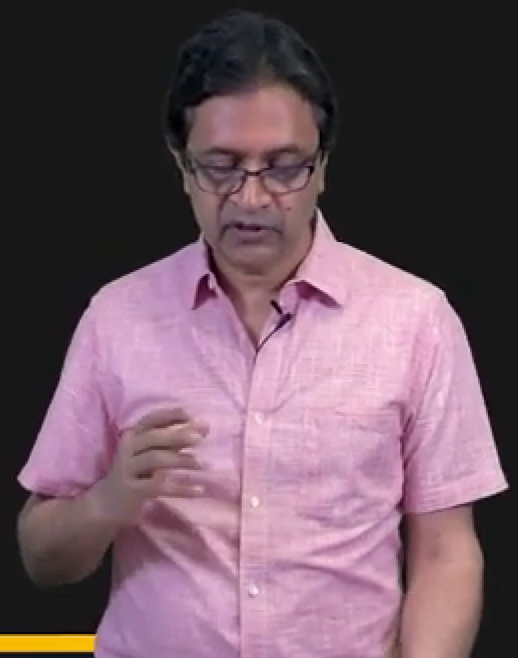

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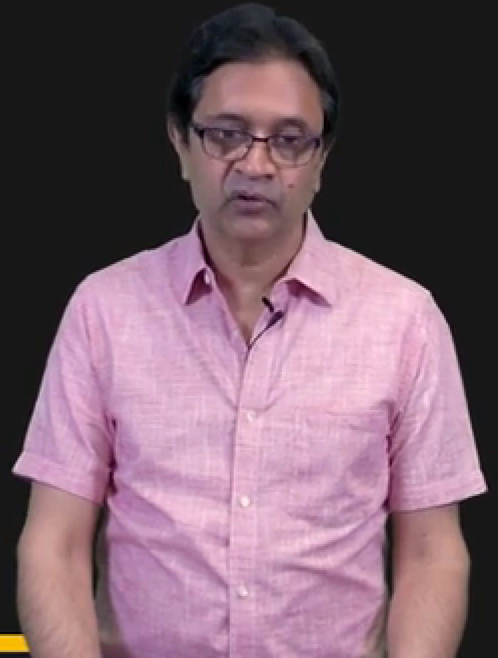
$$m \left[\begin{array}{ccc} & & \\ & A & \\ & & \end{array} \right]_{n} = [\mathbf{c}_1 \quad \mathbf{c}_2 \quad \dots \quad \mathbf{c}_n] = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \vdots \\ \mathbf{r}_m^T \end{bmatrix}$$

$$ColumnRank(A) \leq n \qquad RowRank(A) \leq m$$



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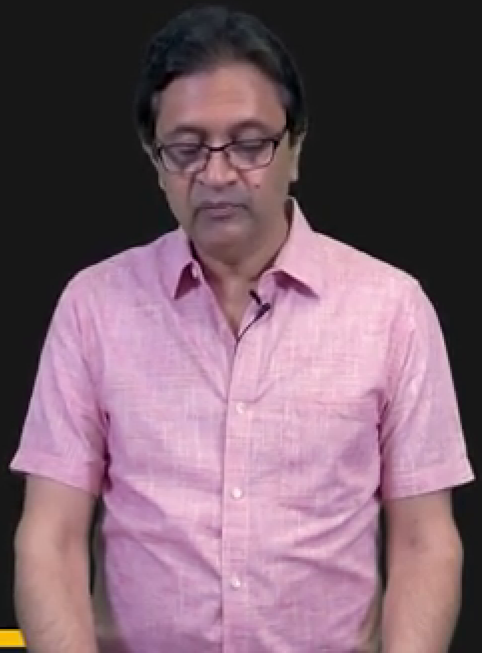
$$ColumnRank(A) \leq n \qquad\qquad RowRank(A) \leq m$$

$$\boxed{\begin{gathered} ColumnRank(A) = RowRank(A) = Rank(A) \\ Rank(A) \leq \min(m, n) \end{gathered}}$$

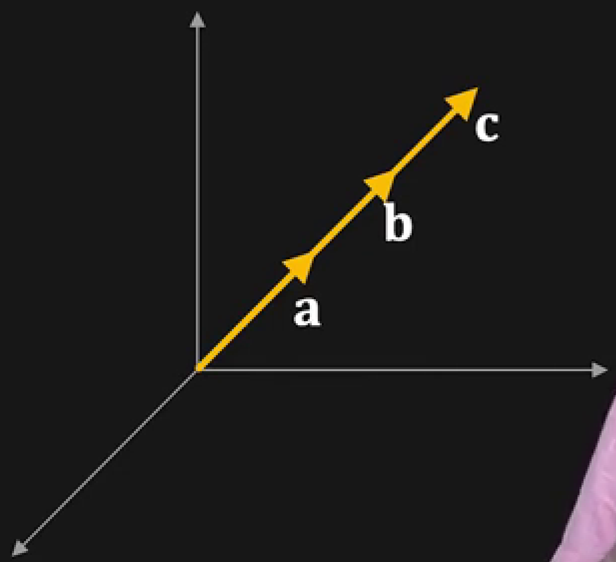


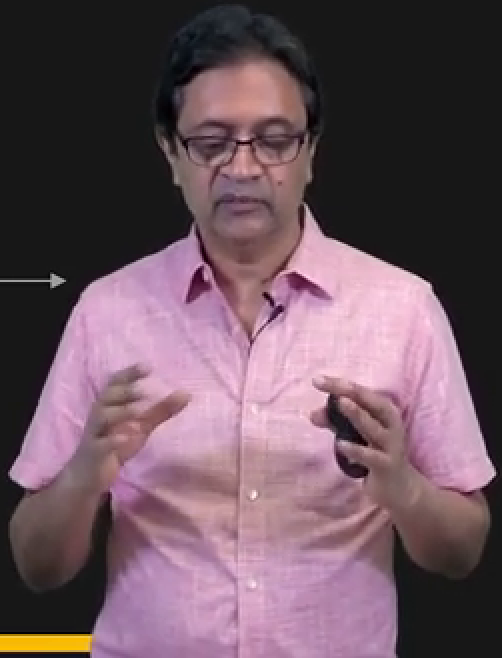
Geometric Meaning of Matrix Rank

Rank is the dimensionality of the space spanned by column or row vectors of the matrix.

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} = [\mathbf{a} \quad \mathbf{b} \quad \mathbf{c}]$$

$$Rank(A) = 1$$





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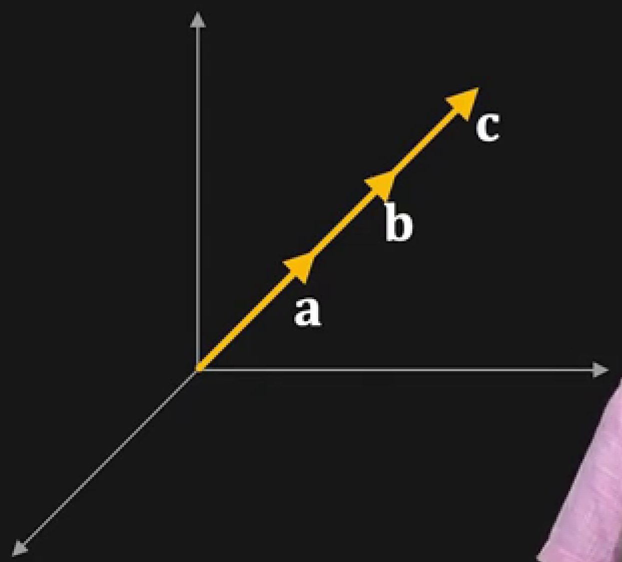

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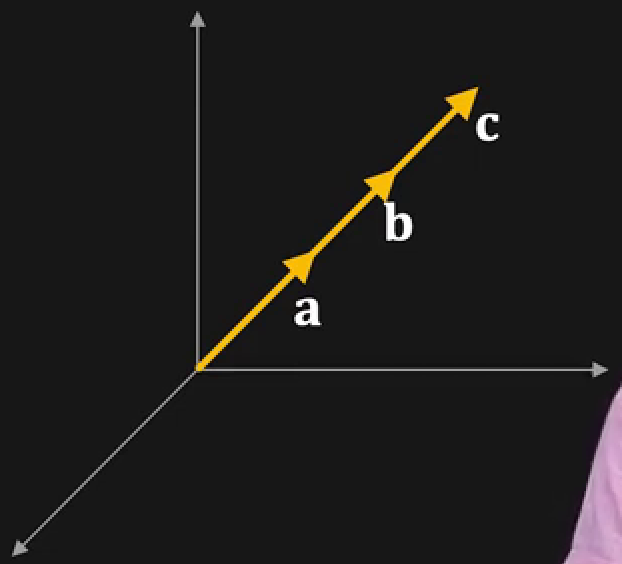

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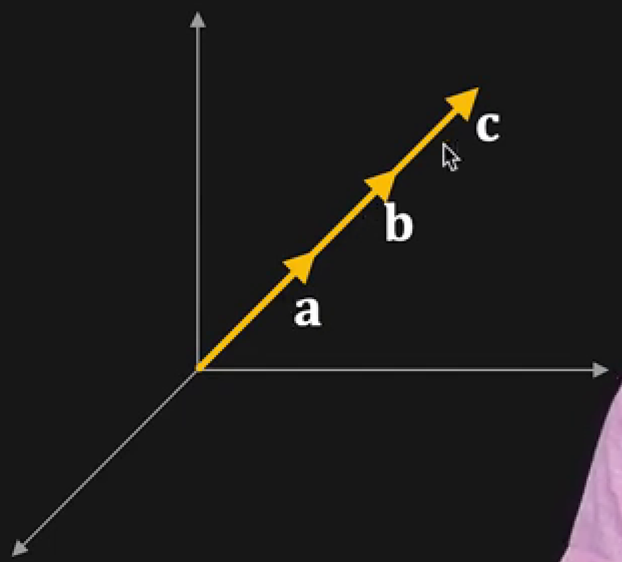

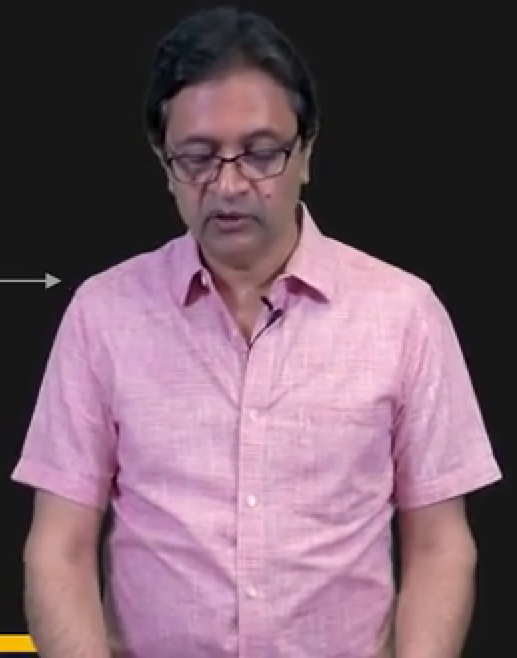

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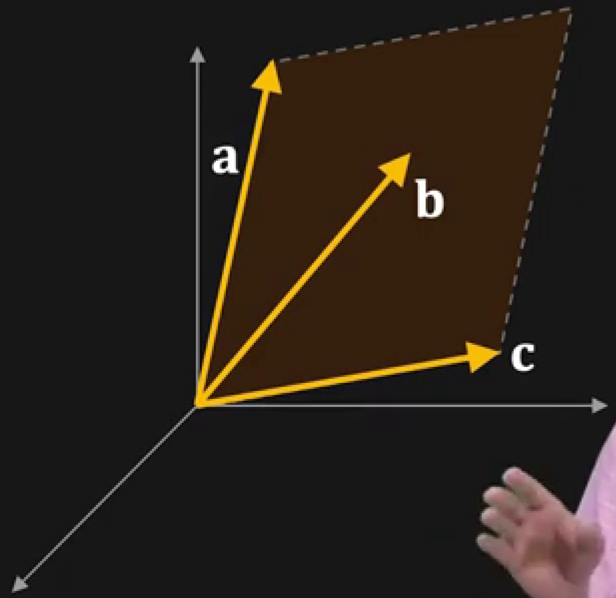
$$Rank(A) = 2$$



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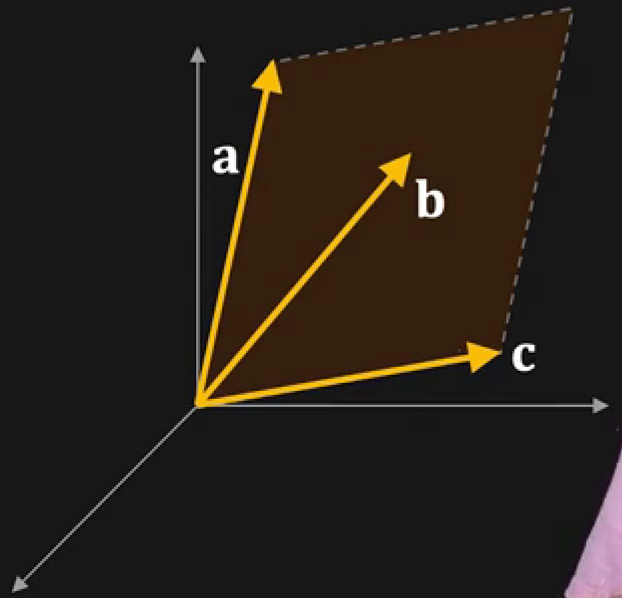

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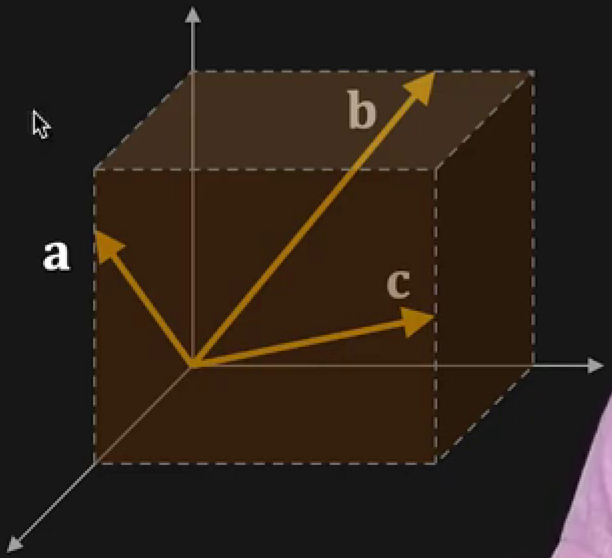
$$Rank(A) = 3 \iff$$



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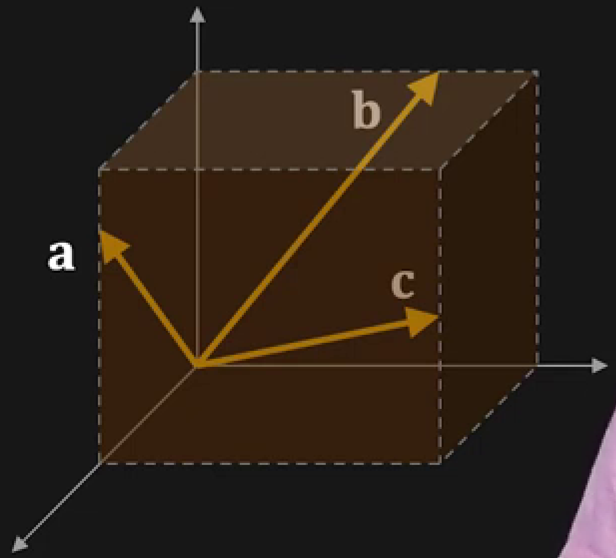
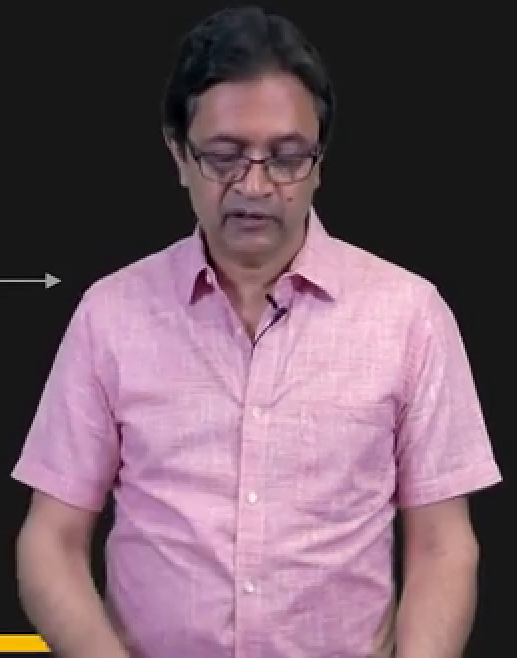
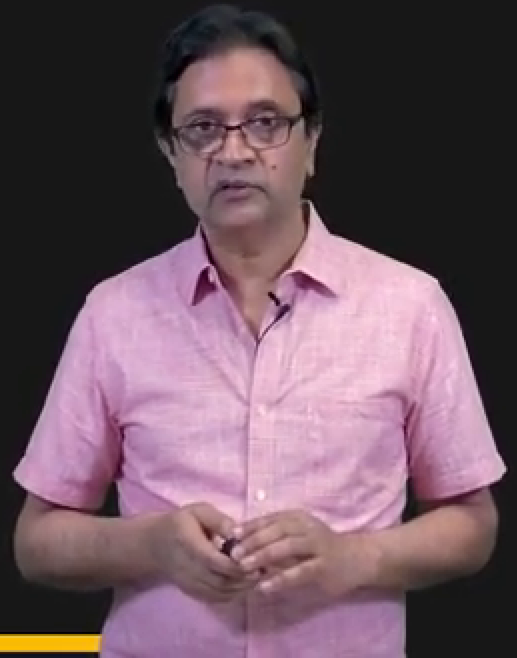




Important Properties of Matrix Rank

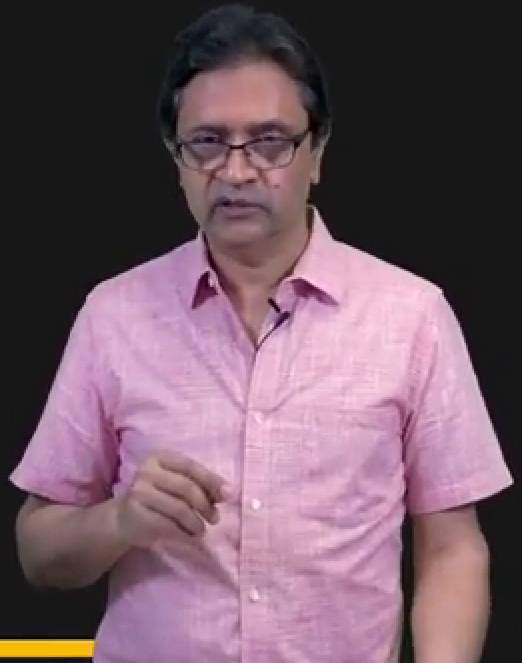
- $Rank(A^T) = Rank(A)$



Important Properties of Matrix Rank

- $Rank(A^T) = Rank(A)$

- $$Rank(A_{m\times n}B_{n\times p}) = \min\left(Rank(A_{m\times n}), Rank(B_{n\times p})\right)$$
  $$\leq \min(m, n, p)$$



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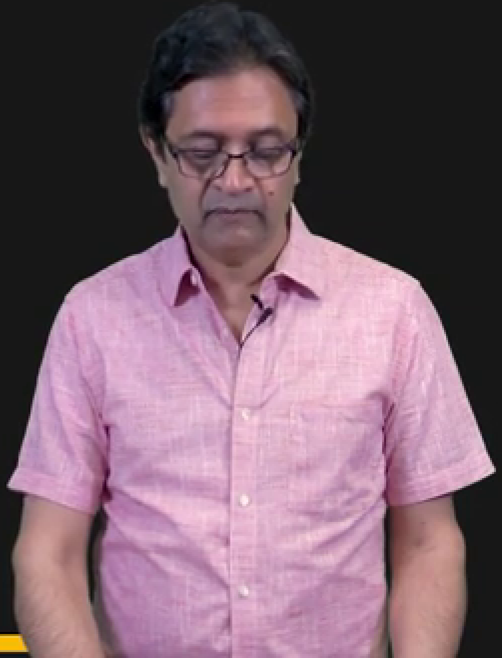

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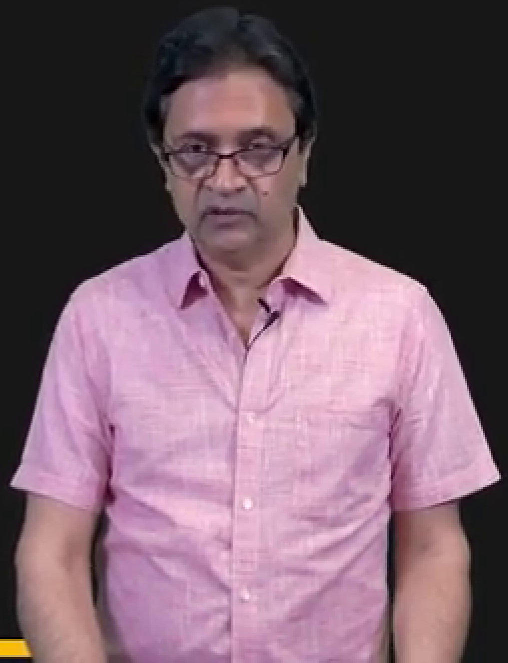

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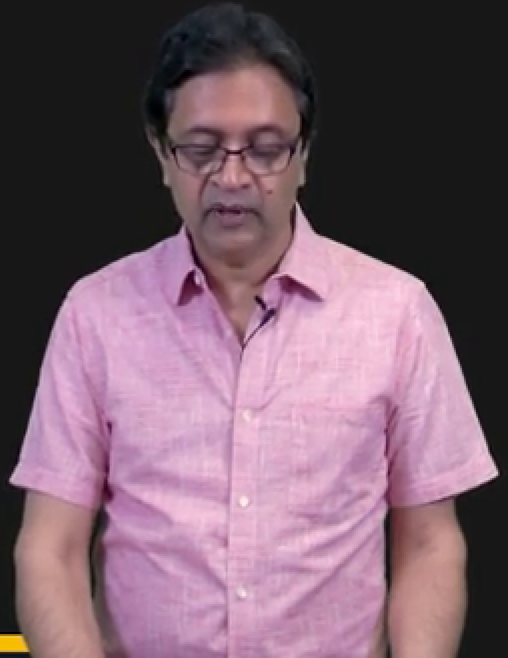

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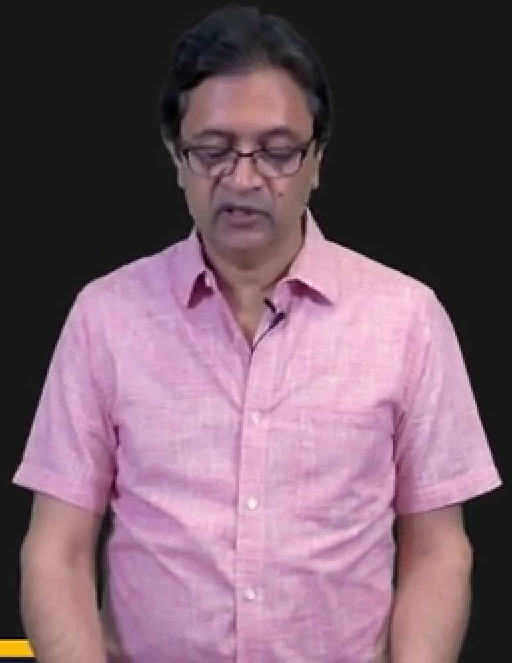
  $$\leq \min(m, n, p)$$

- $Rank(AA^T) = Rank(A^TA) = Rank(A^T) = Rank(A)$



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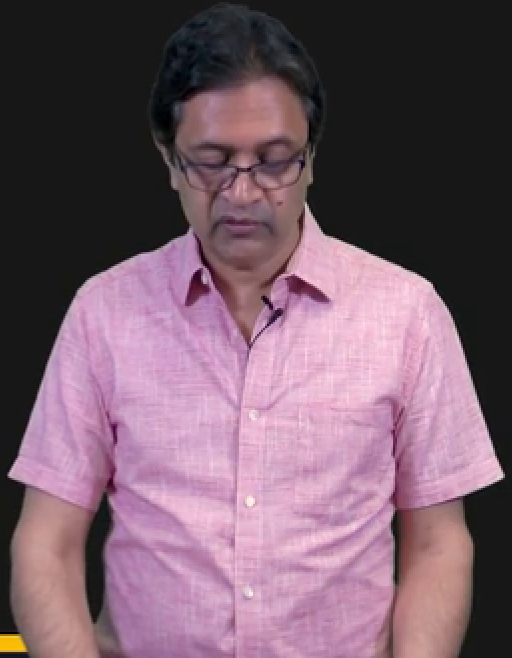
- $Rank(AA^T) = Rank(A^T A) = Rank(A^T) = Rank(A)$

- $A_{m\times m}$ is invertible iff $Rank(A_{m\times m}) = m$



...Back to Observation Matrix W

$$
\begin{array}{rc}
 & \begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{array}{r} \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \end{array} &
\begin{bmatrix}
\tilde{u}_{1,1} & \tilde{u}_{1,2} & \cdots & \tilde{u}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{u}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{u}_{F,1} & \tilde{u}_{F,2} & \cdots & \tilde{u}_{F,N} \\
\tilde{v}_{1,1} & \tilde{v}_{1,2} & \cdots & \tilde{v}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{v}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{v}_{F,1} & \tilde{v}_{F,2} & \cdots & \tilde{v}_{F,N}
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
\mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \\ \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T
\end{bmatrix}
\begin{array}{c}
\begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{bmatrix} P_1 & P_2 & \cdots & P_N \end{bmatrix}
\end{array}
$$

$S_{3\times N}$

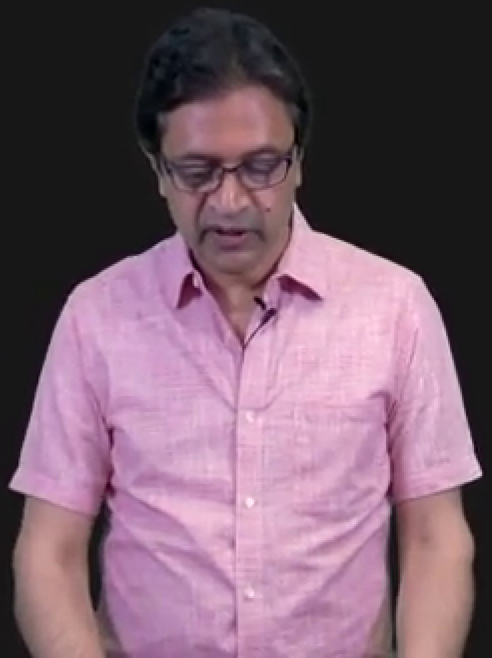
Scene Structure
(Unknown)

$W_{2F\times N}$

Centroid-Subtracted
Feature Points (Known)

$M_{2F\times 3}$

Camera Motion
(Unknown)



...Back to Observation Matrix W

$$
\begin{array}{l} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \end{array}
\begin{array}{c}
\begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{bmatrix}
\tilde{u}_{1,1} & \tilde{u}_{1,2} & \cdots & \tilde{u}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{u}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{u}_{F,1} & \tilde{u}_{F,2} & \cdots & \tilde{u}_{F,N} \\
\tilde{v}_{1,1} & \tilde{v}_{1,2} & \cdots & \tilde{v}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{v}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{v}_{F,1} & \tilde{v}_{F,2} & \cdots & \tilde{v}_{F,N}
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
\mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \\ \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T
\end{bmatrix}
\begin{array}{c}
\begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{bmatrix} P_1 & P_2 & \cdots & P_N \end{bmatrix}
\end{array}
$$

$S_{3\times N}$

Scene Structure
(Unknown)

$W_{2F\times N}$

Centroid-Subtracted
Feature Points (Known)

$M_{2F\times 3}$

Camera Motion
(Unknown)

...Back to Observation Matrix W

$$
\begin{array}{rc}
 & \begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{array}{r} \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \end{array} &
\begin{bmatrix}
\tilde{u}_{1,1} & \tilde{u}_{1,2} & \cdots & \tilde{u}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{u}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{u}_{F,1} & \tilde{u}_{F,2} & \cdots & \tilde{u}_{F,N} \\
\tilde{v}_{1,1} & \tilde{v}_{1,2} & \cdots & \tilde{v}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{v}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{v}_{F,1} & \tilde{v}_{F,2} & \cdots & \tilde{v}_{F,N}
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
\mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \\ \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T
\end{bmatrix}
\begin{array}{c}
\begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{bmatrix} P_1 & P_2 & \cdots & P_N \end{bmatrix}
\end{array}
$$

$W_{2F\times N}$

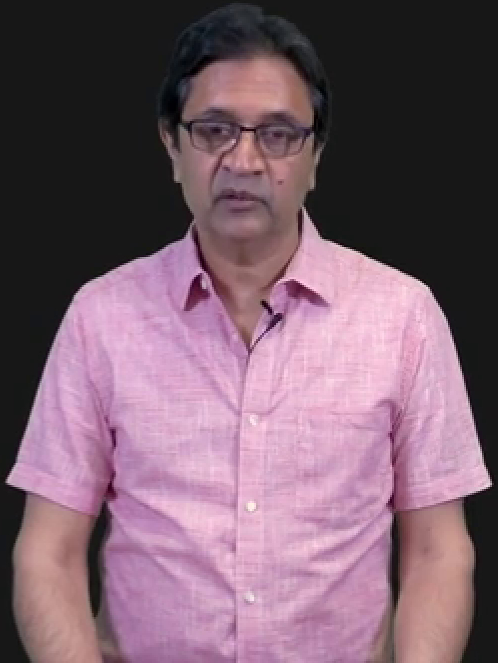
Centroid-Subtracted Feature Points (Known)

$M_{2F\times 3}$

Camera Motion (Unknown)

$S_{3\times N}$

Scene Structure (Unknown)



...Back to Observation Matrix W

$$
\begin{array}{cc}
 & \begin{array}{cccc}\text{Point 1} & \text{Point 2} & & \text{Point N}\end{array} \\
\begin{array}{c}\text{Image 1}\\ \text{Image 2}\\ \\ \text{Image F}\\ \text{Image 1}\\ \text{Image 2}\\ \\ \text{Image F}\end{array} &
\begin{bmatrix}
\tilde{u}_{1,1} & \tilde{u}_{1,2} & \cdots & \tilde{u}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{u}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{u}_{F,1} & \tilde{u}_{F,2} & \cdots & \tilde{u}_{F,N} \\
\tilde{v}_{1,1} & \tilde{v}_{1,2} & \cdots & \tilde{v}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{v}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{v}_{F,1} & \tilde{v}_{F,2} & \cdots & \tilde{v}_{F,N}
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
\mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \\ \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T
\end{bmatrix}
\begin{array}{cccc}
\text{Point 1} & \text{Point 2} & & \text{Point N} \\
[P_1 & P_2 & \cdots & P_N]
\end{array}
$$

$W_{2F\times N}$

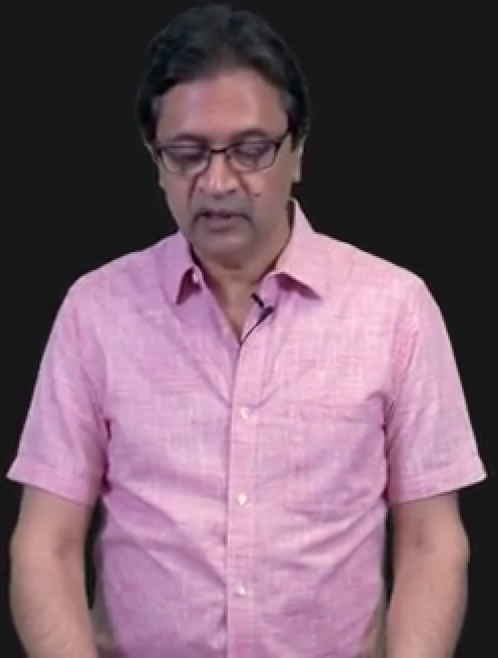
Centroid-Subtracted Feature Points (Known)

$M_{2F\times 3}$

Camera Motion (Unknown)

$S_{3\times N}$

Scene Structure (Unknown)



...Back to Observation Matrix W

$$
\begin{array}{rc}
 & \begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{array}{r} \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \\ \text{Image 1} \\ \text{Image 2} \\ \\ \text{Image F} \end{array} &
\begin{bmatrix}
\tilde{u}_{1,1} & \tilde{u}_{1,2} & \cdots & \tilde{u}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{u}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{u}_{F,1} & \tilde{u}_{F,2} & \cdots & \tilde{u}_{F,N} \\
\tilde{v}_{1,1} & \tilde{v}_{1,2} & \cdots & \tilde{v}_{1,N} \\
\tilde{u}_{2,1} & \tilde{u}_{2,2} & \cdots & \tilde{v}_{2,N} \\
\vdots & \vdots & \vdots & \vdots \\
\tilde{v}_{F,1} & \tilde{v}_{F,2} & \cdots & \tilde{v}_{F,N}
\end{bmatrix}
\end{array}
=
\begin{bmatrix}
\mathbf{i}_1^T \\ \mathbf{i}_2^T \\ \vdots \\ \mathbf{i}_F^T \\ \mathbf{j}_1^T \\ \mathbf{j}_2^T \\ \vdots \\ \mathbf{j}_F^T
\end{bmatrix}
\begin{array}{c}
\begin{array}{cccc} \text{Point 1} & \text{Point 2} & & \text{Point N} \end{array} \\
\begin{bmatrix} P_1 & P_2 & \cdots & P_N \end{bmatrix}
\end{array}
$$

$S_{3\times N}$

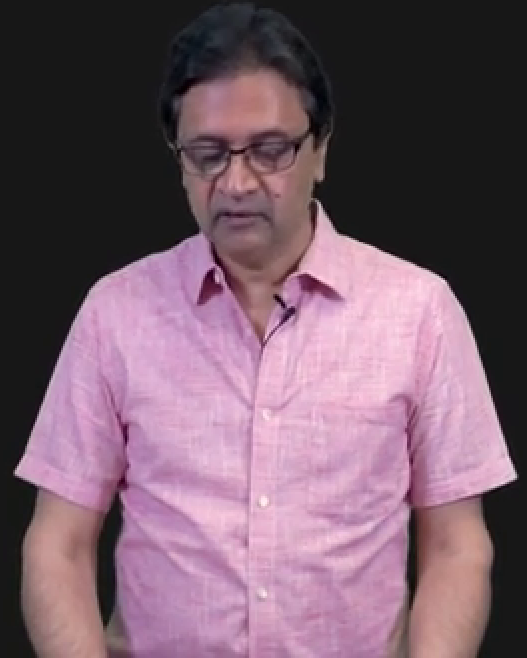
Scene Structure
(Unknown)

$W_{2F\times N}$

Centroid-Subtracted
Feature Points (Known)

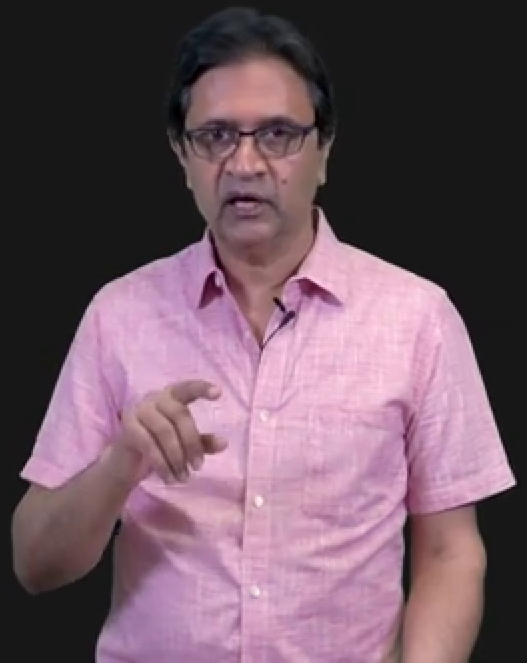
$M_{2F\times 3}$

Camera Motion
(Unknown)



Rank of Observation Matrix

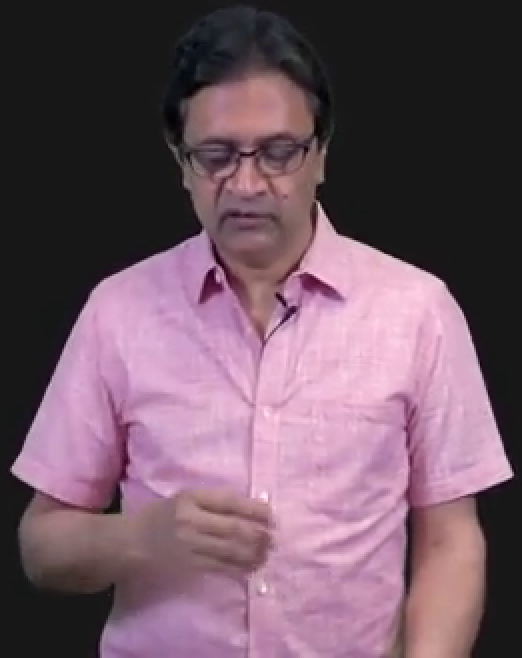
$$\underset{2F\times N}{W} = \underset{2F\times 3}{M} \times \underset{3\times N}{S}$$



[Tomasi 1992]

Rank of Observation Matrix

$$\underset{2F\times N}{W} = \underset{2F\times 3}{M} \times \underset{3\times N}{S}$$



[Tomasi 1992]

Rank of Observation Matrix

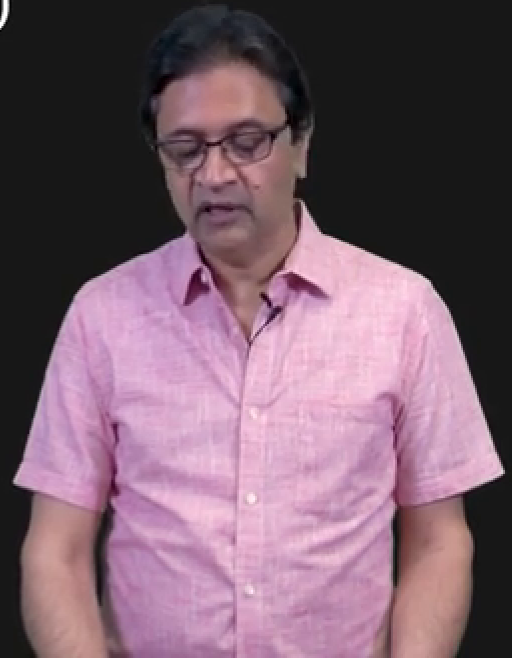
$$\underset{2F\times N}{W} = \underset{2F\times 3}{M} \times \underset{3\times N}{S}$$

We know:

$$Rank(MS) \leq Rank(M) \qquad Rank(MS) \leq Rank(S)$$

$$\Rightarrow \quad Rank(MS) \leq \min(3, 2F) \qquad Rank(MS) \leq \min(3, N)$$

$$\Rightarrow \quad Rank(W) = Rank(MS) \leq \min(3, N, 2F)$$



[Tomasi 1992]

Rank of Observation Matrix

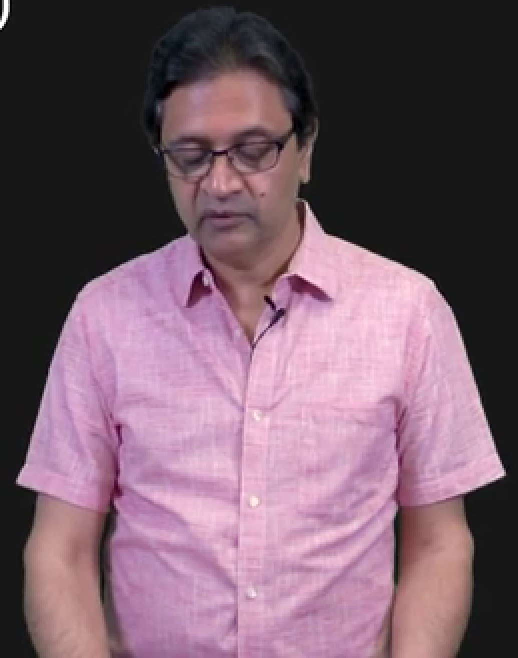
$$\underset{2F\times N}{W} = \underset{2F\times 3}{M} \times \underset{3\times N}{S}$$

We know:

$$Rank(MS) \le Rank(M) \qquad Rank(MS) \le Rank(S)$$

$$\Rightarrow \quad Rank(MS) \le \min(3,2F) \qquad Rank(MS) \le \min(3,N)$$

$$\Rightarrow \quad Rank(W) = Rank(MS) \le \min(3,N,2F)$$



[Tomasi 1992]

Rank of Observation Matrix

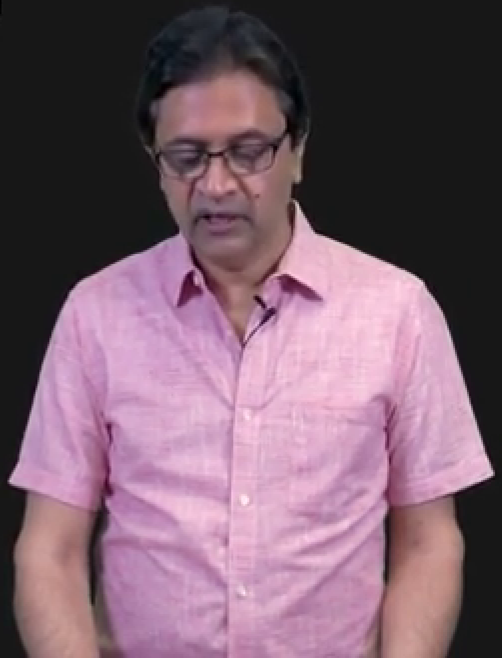
$$\underset{2F\times N}{W} = \underset{2F\times 3}{M} \times \underset{3\times N}{S}$$

We know:

$$Rank(MS) \le Rank(M) \qquad Rank(MS) \le Rank(S)$$

$$\Rightarrow \quad Rank(MS) \le \min(3, 2F) \qquad Rank(MS) \le \min(3, N)$$

$$\Rightarrow \quad Rank(W) = Rank(MS) \le \min(3, N, 2F)$$



[Tomasi 1992]

Rank of Observation Matrix

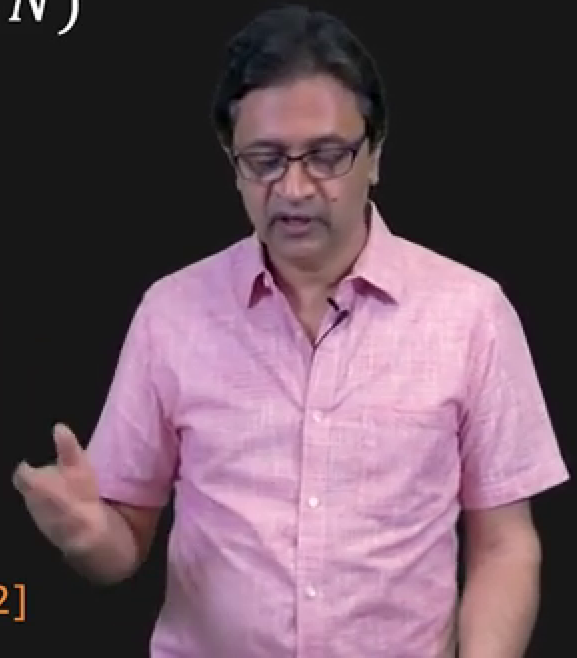
$$\underset{2F\times N}{W} = \underset{2F\times 3}{M} \times \underset{3\times N}{S}$$

We know:

$$Rank(MS) \le Rank(M) \qquad Rank(MS) \le Rank(S)$$

$$\Rightarrow \quad Rank(MS) \le \min(3,2F) \qquad Rank(MS) \le \min(3,N)$$

$$\Rightarrow \quad Rank(W) = Rank(MS) \le \min(3,N,2F)$$



[Tomasi 1992]

Rank of Observation Matrix

$$\underset{2F\times N}{W} = \underset{2F\times 3}{M} \times \underset{3\times N}{S}$$

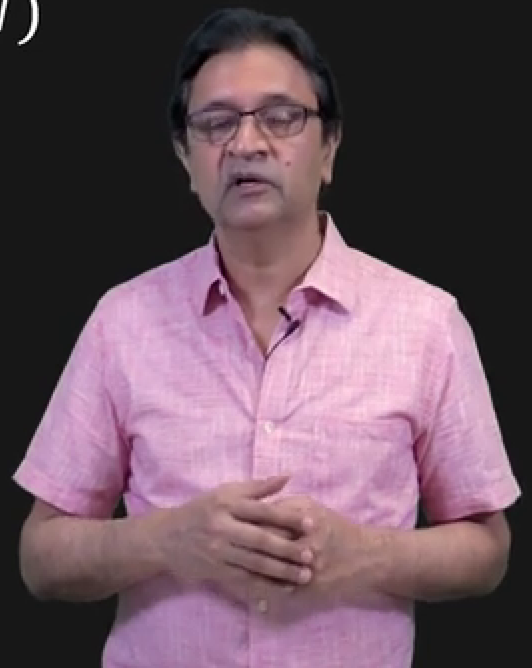
We know:

$$Rank(MS) \le Rank(M) \qquad Rank(MS) \le Rank(S)$$

$$\Rightarrow \quad Rank(MS) \le \min(3, 2F) \qquad Rank(MS) \le \min(3, N)$$

$$\Rightarrow \quad Rank(W) = Rank(MS) \le \min(3, N, 2F)$$

Rank Theorem: $Rank(W) \le 3$



[Tomasi 1992]