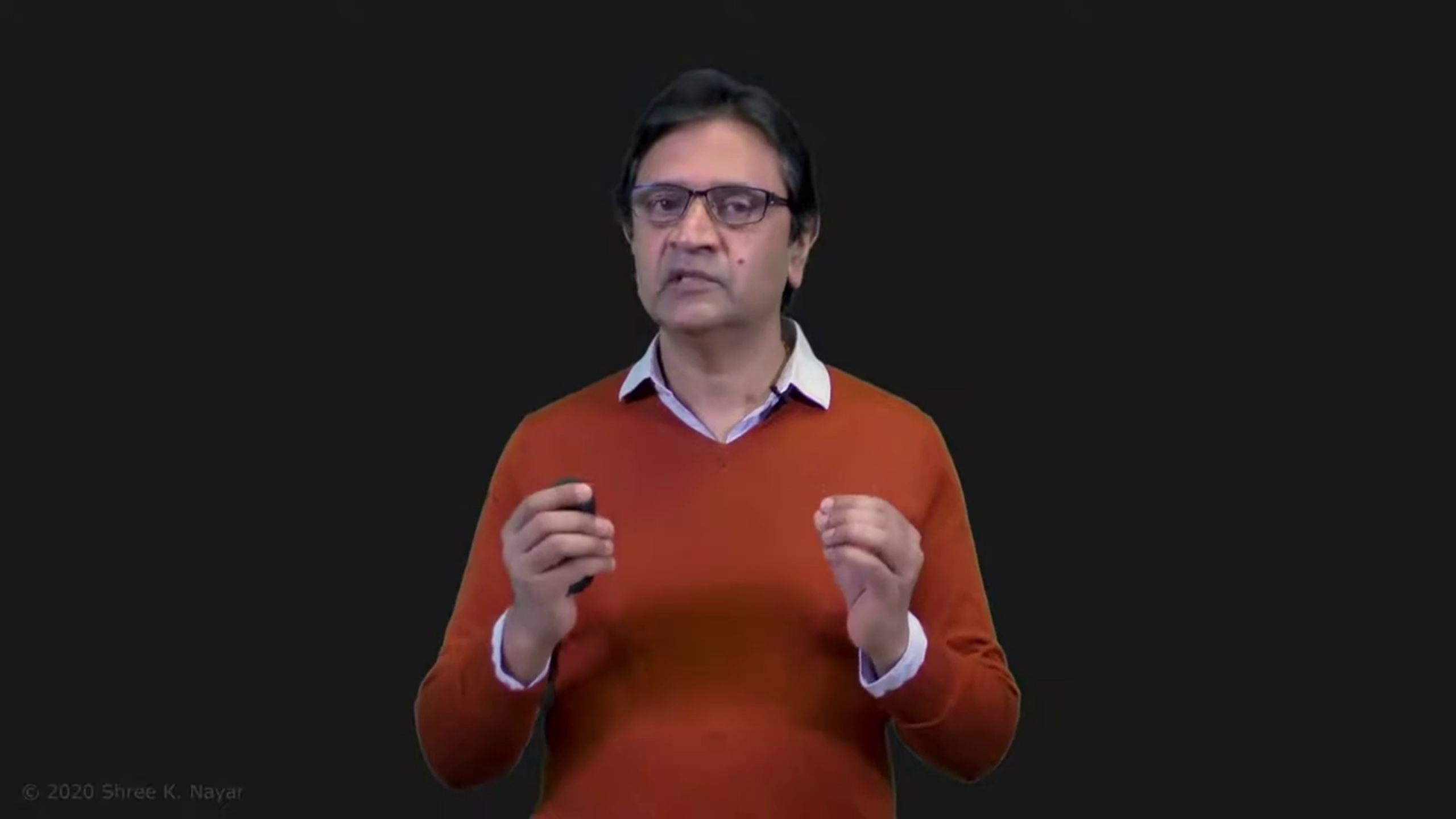
Computing Depth

Shree K. Nayar

Columbia University

Topic: Uncalibrated Stereo, Module: Reconstruction II

First Principles of Computer Vision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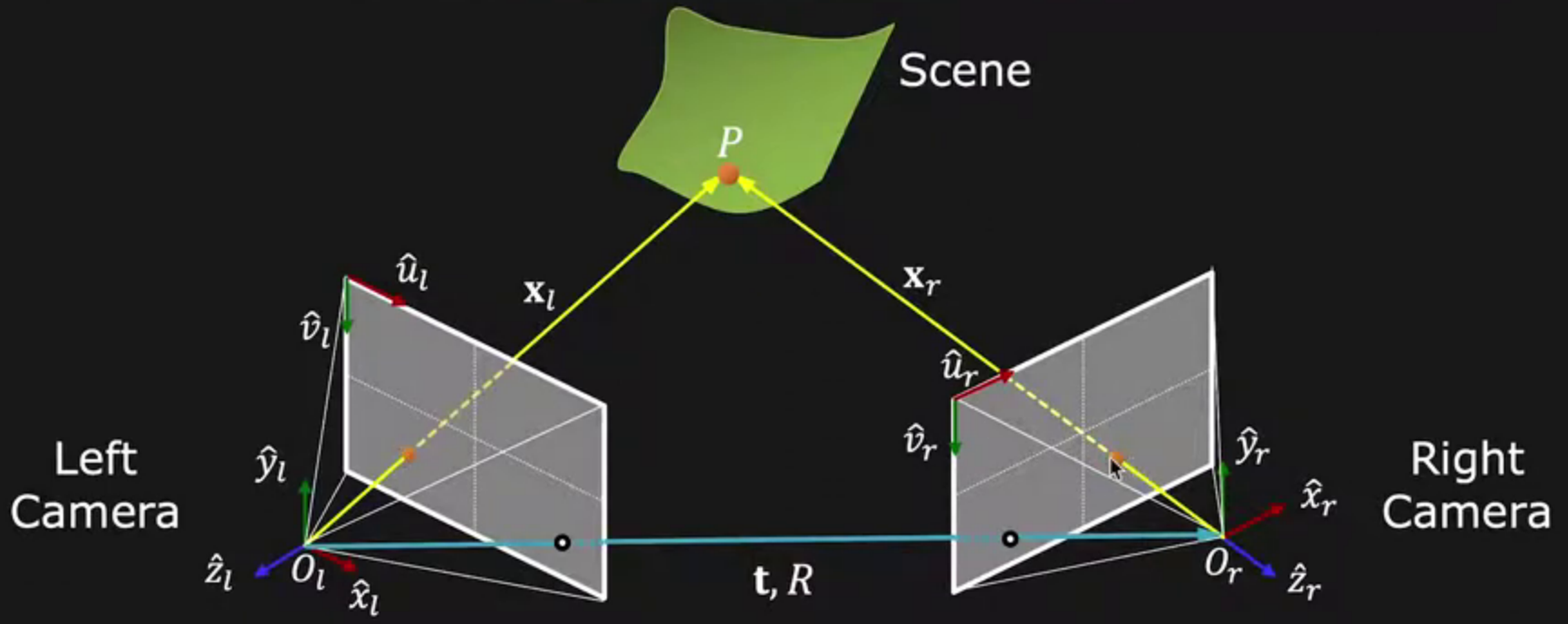
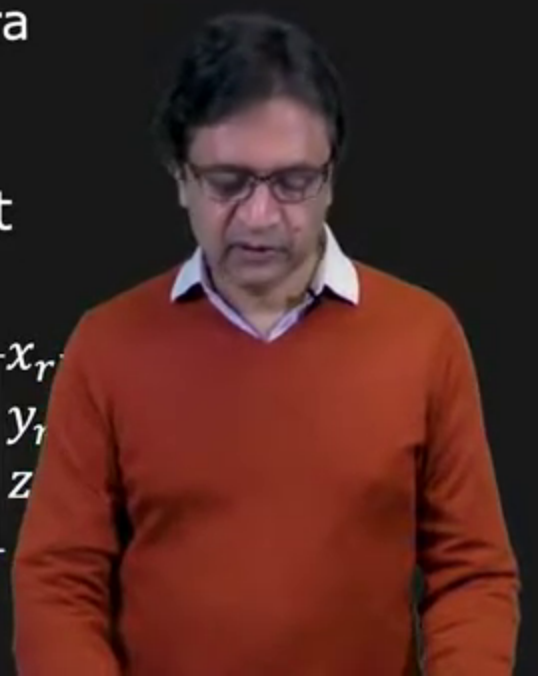
Computing Depth



Given the intrinsic parameters, the projections of scene point on the two image sensors are:

$$\begin{bmatrix} u_l \\ v_l \\ 1 \end{bmatrix} \equiv \begin{bmatrix} f_x^{(l)} & 0 & o_x^{(l)} & 0 \\ 0 & f_y^{(l)} & o_y^{(l)} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_l \\ y_l \\ z_l \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} \equiv \begin{bmatrix} f_x^{(r)} & 0 & o_x^{(r)} & 0 \\ 0 & f_y^{(r)} & o_y^{(r)} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$



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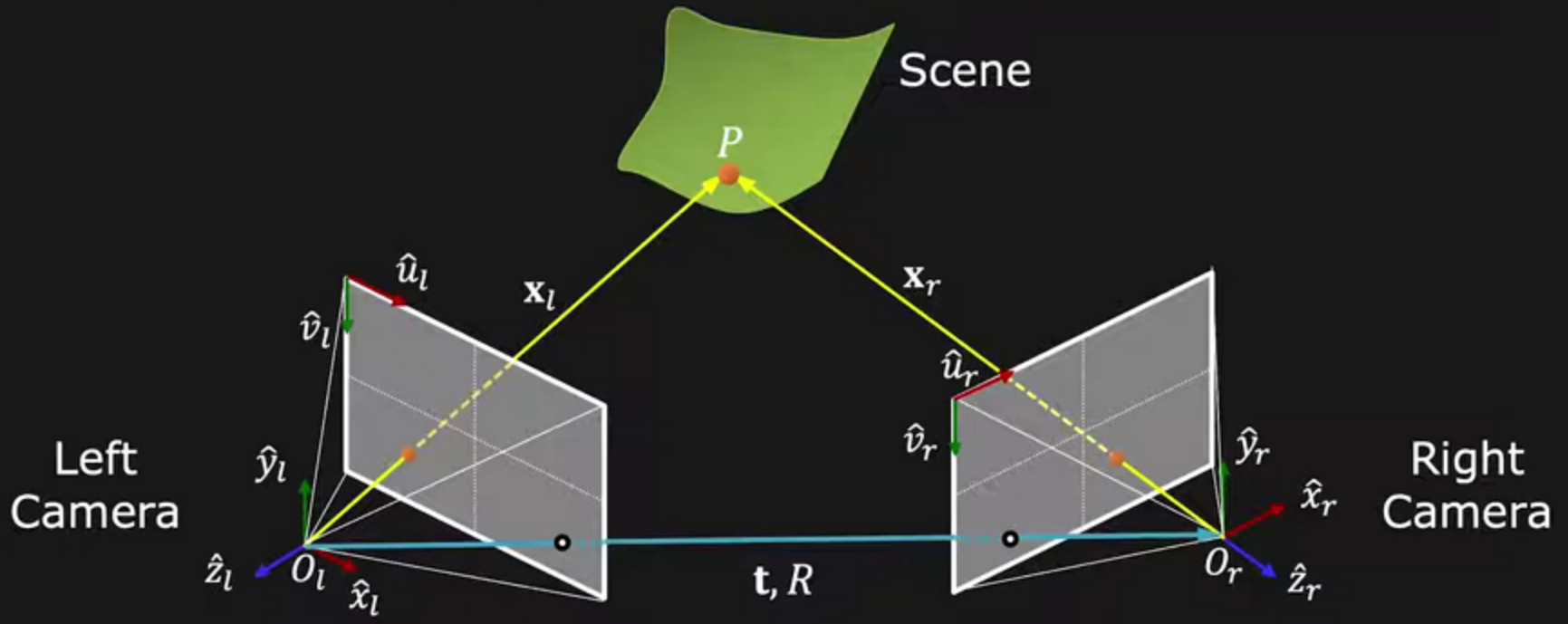

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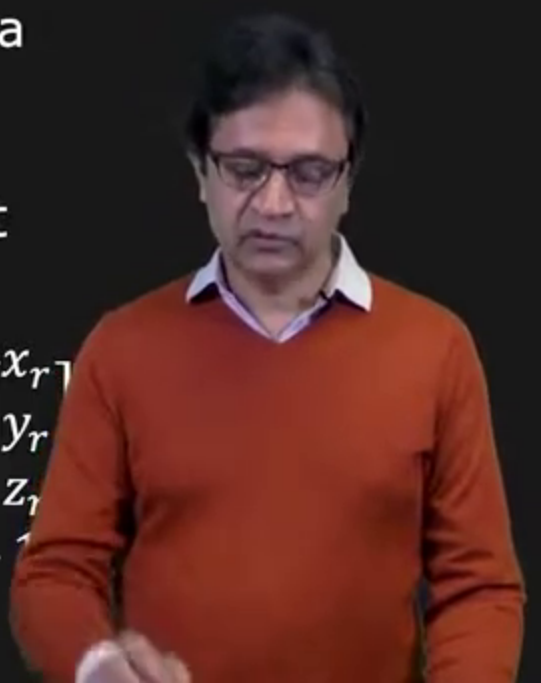

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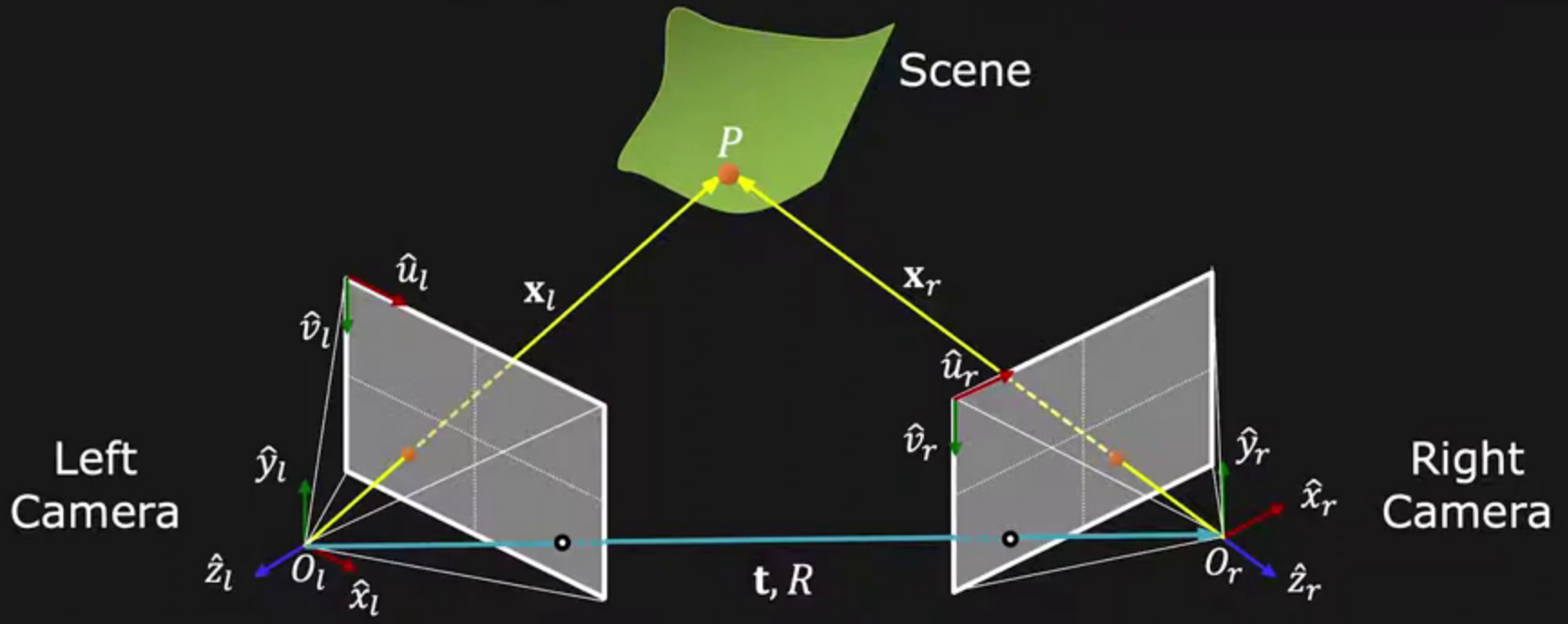

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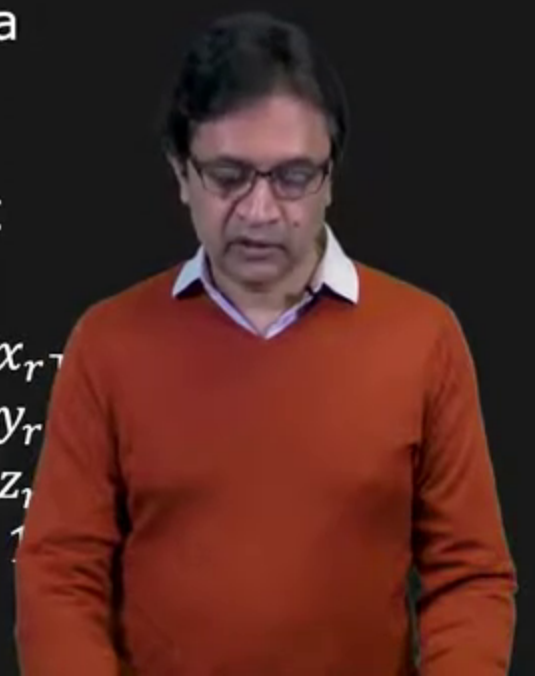

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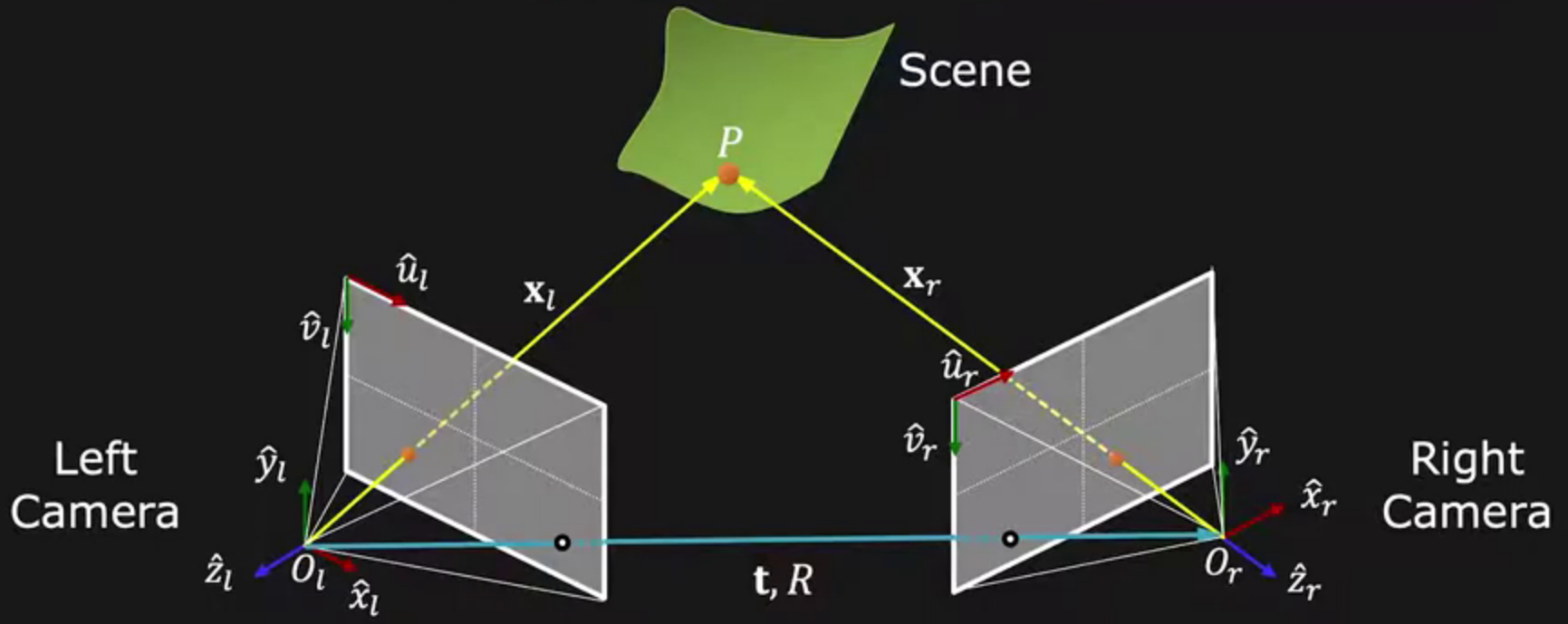

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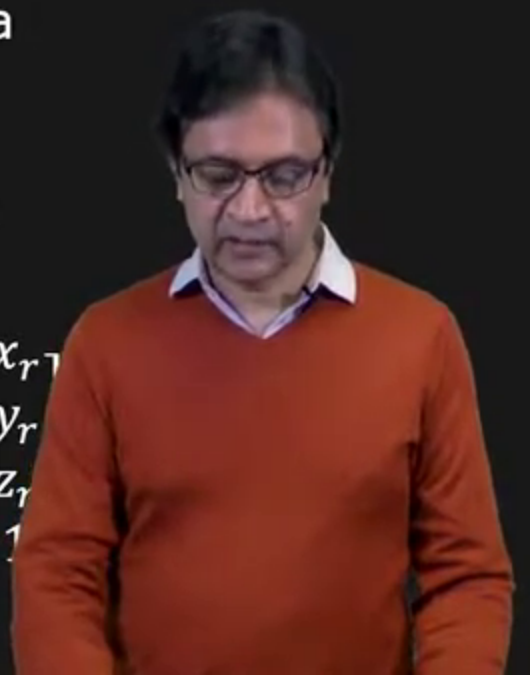


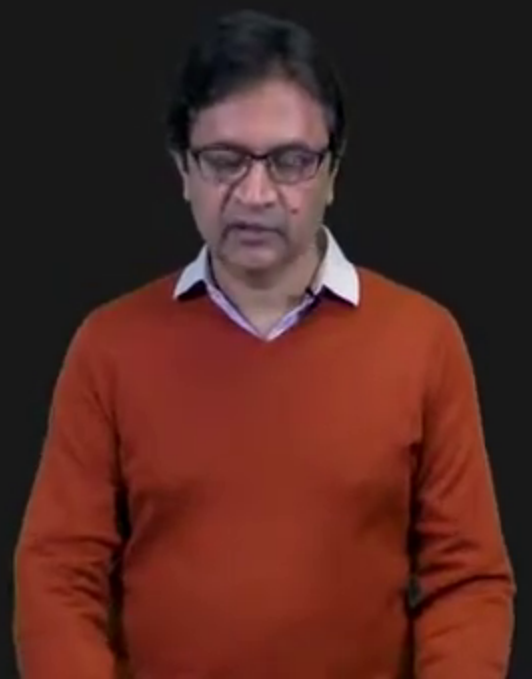
Computing Depth

Left Camera Imaging Equation

$$\begin{bmatrix} u_l \\ v_l \\ 1 \end{bmatrix} \equiv \begin{bmatrix} f_x^{(l)} & 0 & o_x^{(l)} & 0 \\ 0 & f_y^{(l)} & o_y^{(l)} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_l \\ y_l \\ z_l \\ 1 \end{bmatrix}$$

Right Camera Imaging Equation

$$\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} \equiv \begin{bmatrix} f_x^{(r)} & 0 & o_x^{(r)} & 0 \\ 0 & f_y^{(r)} & o_y^{(r)} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$



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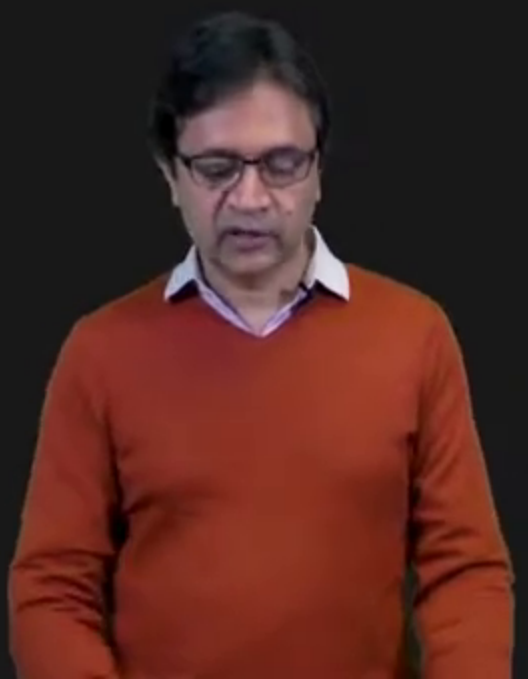

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We also know the relative position and orientation between the two cameras.

$$\begin{bmatrix} x_l \\ y_l \\ z_l \\ 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$

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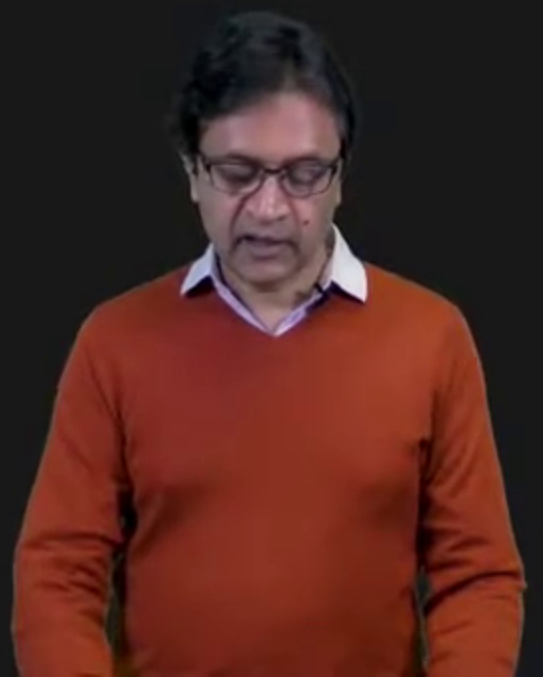

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$$\tilde{\mathbf{u}}_l = P_l \, \tilde{\mathbf{x}}_r$$

Right Camera Imaging Equation:

$$\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} \equiv \begin{bmatrix} f_x^{(r)} & 0 & o_x^{(r)} & 0 \\ 0 & f_y^{(r)} & o_y^{(r)} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$

$$\tilde{\mathbf{u}}_r = M_{int_r} \, \tilde{\mathbf{x}}_r$$

Computing Depth

Left Camera Imaging Equation:

$$\begin{bmatrix} u_l \\ v_l \\ 1 \end{bmatrix} \equiv \begin{bmatrix} f_x^{(l)} & 0 & o_x^{(l)} & 0 \\ 0 & f_y^{(l)} & o_y^{(l)} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$

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$$\tilde{\mathbf{u}}_r = M_{int_r} \, \tilde{\mathbf{x}}_r$$

Computing Depth

The imaging equations:

$$\tilde{\mathbf{u}}_r = M_r \, \tilde{\mathbf{x}}_r \qquad \tilde{\mathbf{u}}_l = P_l \, \tilde{\mathbf{x}}_r$$

$$\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} \equiv \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix} \qquad \begin{bmatrix} u_l \\ v_l \\ 1 \end{bmatrix} \equiv \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$

Known Unknown Known Unknown

Computing Depth

The imaging equations:

$$\tilde{\mathbf{u}}_r = M_r \, \tilde{\mathbf{x}}_r$$

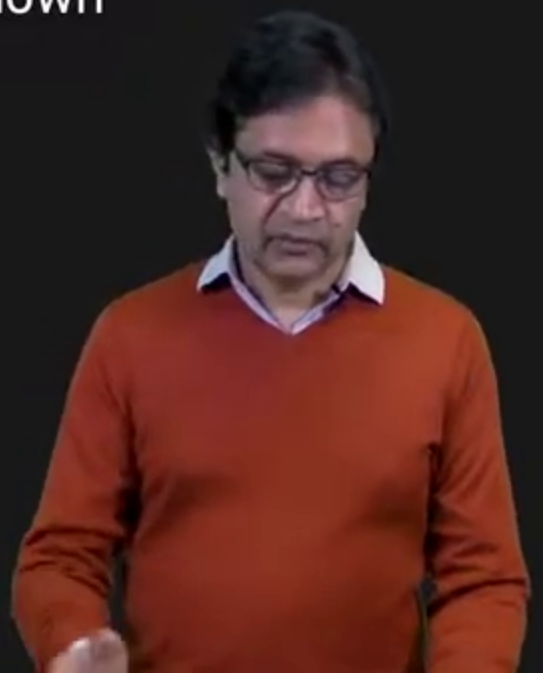
$$\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} \equiv \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$

Known Unknown

$$\tilde{\mathbf{u}}_l = P_l \, \tilde{\mathbf{x}}_r$$

$$\begin{bmatrix} u_l \\ v_l \\ 1 \end{bmatrix} \equiv \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$

Known Unknown



Computing Depth

The imaging equations:

$$\tilde{\mathbf{u}}_r = M_r \, \tilde{\mathbf{x}}_r \qquad \tilde{\mathbf{u}}_l = P_l \, \tilde{\mathbf{x}}_r$$

$$\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} \equiv \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix} \qquad \begin{bmatrix} u_l \\ v_l \\ 1 \end{bmatrix} \equiv \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \\ 1 \end{bmatrix}$$

Known Unknown Known Unknown

Rearranging the terms:

$$\begin{bmatrix} u_r m_{31} - m_{11} & u_r m_{32} - m_{12} & u_r m_{33} - m_{13} \\ v_r m_{31} - m_{21} & v_r m_{32} - m_{22} & v_r m_{33} - m_{23} \\ u_l p_{31} - p_{11} & u_l p_{32} - p_{12} & u_l p_{33} - p_{13} \\ v_l p_{31} - p_{21} & v_l p_{32} - p_{22} & v_l p_{33} - p_{23} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = \begin{bmatrix} m_{14} - m_{34} \\ m_{24} - m_{34} \\ p_{14} - p_{34} \\ p_{24} - p_{34} \end{bmatrix}$$

Computing Depth: Least Squares Solution

$$\begin{bmatrix} u_r m_{31} - m_{11} & u_r m_{32} - m_{12} & u_r m_{33} - m_{13} \\ v_r m_{31} - m_{21} & v_r m_{32} - m_{22} & v_r m_{33} - m_{23} \\ u_l p_{31} - p_{11} & u_l p_{32} - p_{12} & u_l p_{33} - p_{13} \\ v_l p_{31} - p_{21} & v_l p_{32} - p_{22} & v_l p_{33} - p_{23} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = \begin{bmatrix} m_{14} - m_{34} \\ m_{24} - m_{34} \\ p_{14} - p_{34} \\ p_{24} - p_{34} \end{bmatrix}$$

$A_{4\times3}$ (Known)

$\mathbf{x}_r$ (Unknown)

$\mathbf{b}_{4\times1}$ (Known)

Computing Depth: Least Squares Solution

$$\begin{bmatrix} u_r m_{31} - m_{11} & u_r m_{32} - m_{12} & u_r m_{33} - m_{13} \\ v_r m_{31} - m_{21} & v_r m_{32} - m_{22} & v_r m_{33} - m_{23} \\ u_l p_{31} - p_{11} & u_l p_{32} - p_{12} & u_l p_{33} - p_{13} \\ v_l p_{31} - p_{21} & v_l p_{32} - p_{22} & v_l p_{33} - p_{23} \end{bmatrix} \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = \begin{bmatrix} m_{14} - m_{34} \\ m_{24} - m_{34} \\ p_{14} - p_{34} \\ p_{24} - p_{34} \end{bmatrix}$$

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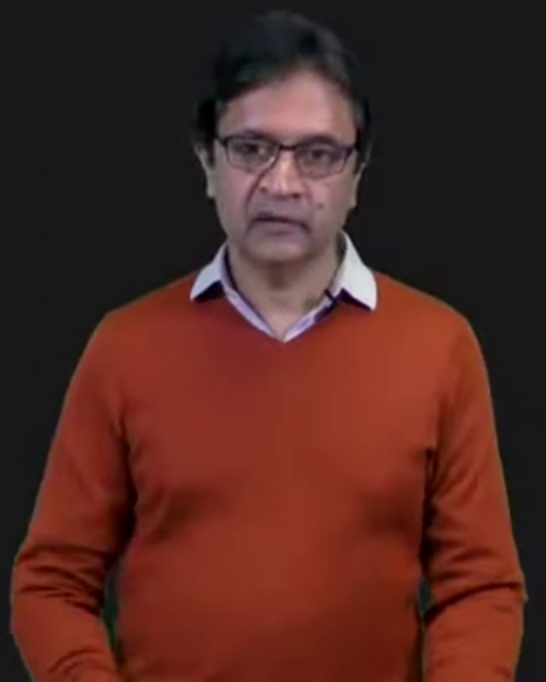

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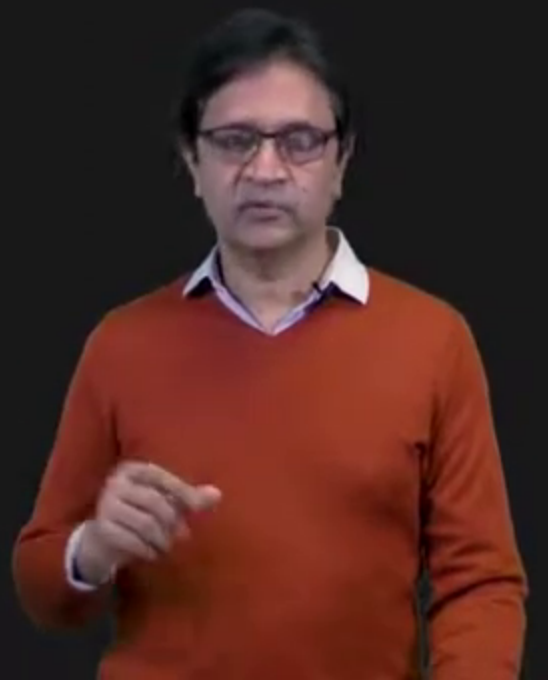
$A_{4\times3}$ (Known) $\quad \mathbf{x_r}$ (Unknown) $\quad \mathbf{b}_{4\times1}$ (Known)

Find least squares solution using pseudo-inverse:

$$A\mathbf{x_r} = \mathbf{b}$$

$$A^T A\mathbf{x_r} = A^T\mathbf{b}$$

$$\boxed{\mathbf{x_r} = (A^T A)^{-1} A^T \mathbf{b}}$$



Computing Depth: Least Squares Solution

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$A_{4\times3}$ (Known) $\quad \mathbf{x_r}$ (Unknown) $\quad \mathbf{b}_{4\times1}$ (Known)

Find least squares solution using pseudo-inverse:

$$A\mathbf{x_r} = \mathbf{b}$$

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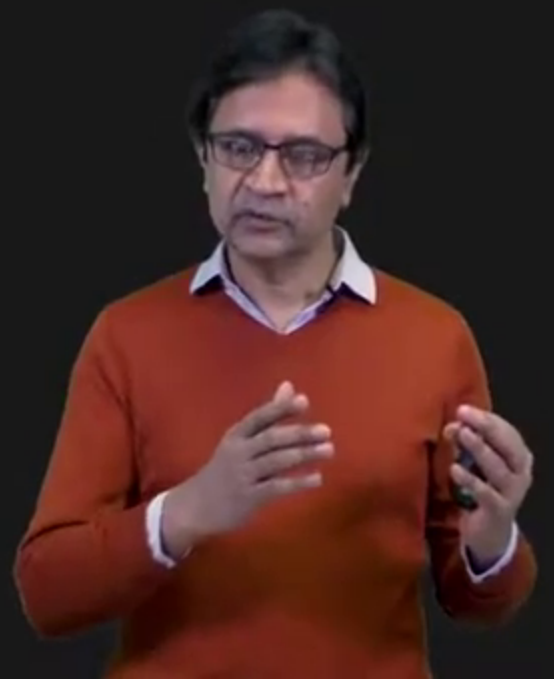
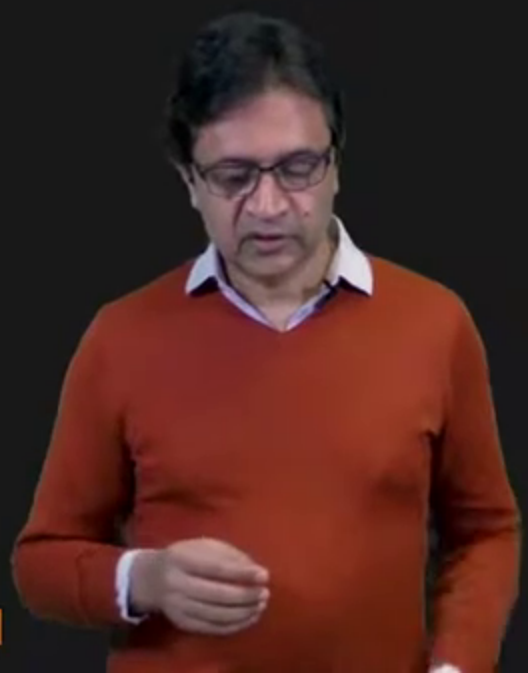


3D Reconstruction with Internet Images

St. Peter's Basilica (1275 Images)

[Snavely 2006]



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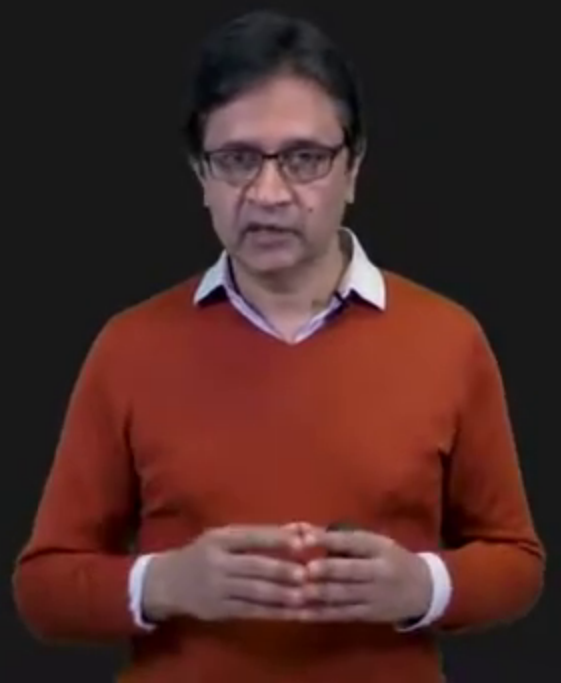

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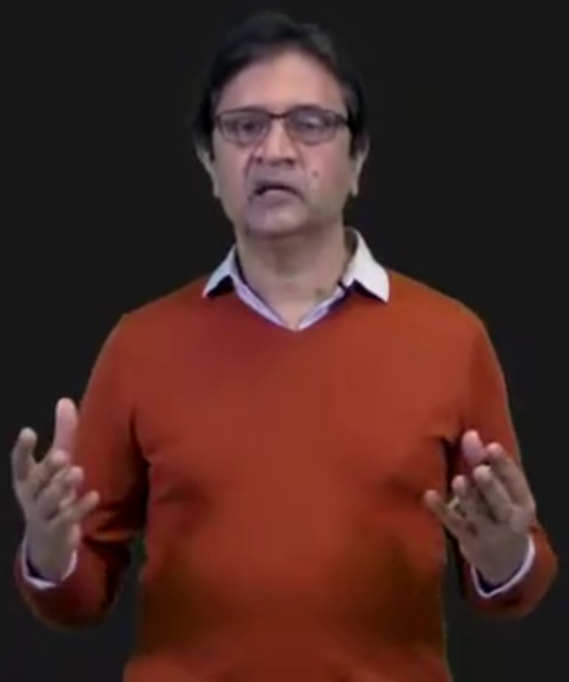

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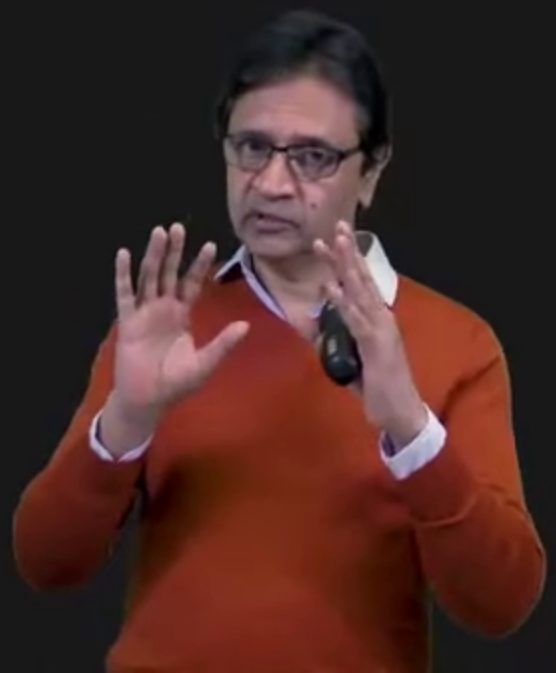
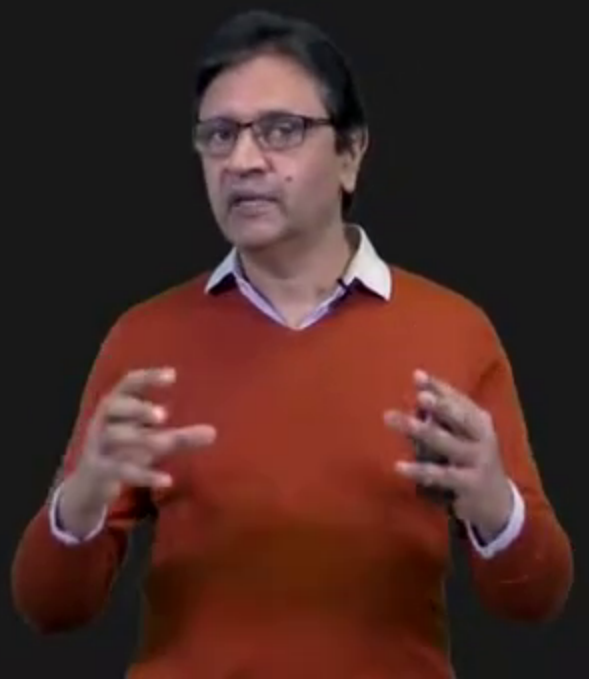


3D Reconstruction with Internet Images

St. Peter's Basilica (1275 Images)

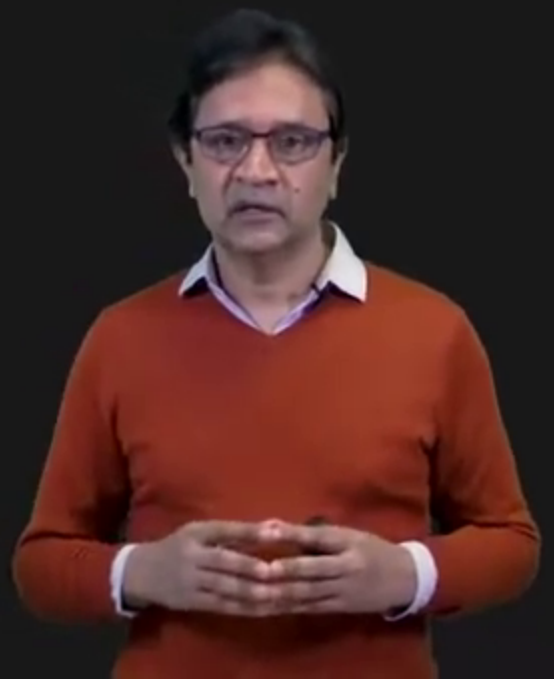
[Snavely 2006]



3D Reconstruction with Internet Images

St. Peter's Basilica (1275 Images)

[Snavely 2006]



3D Reconstruction with Internet Images

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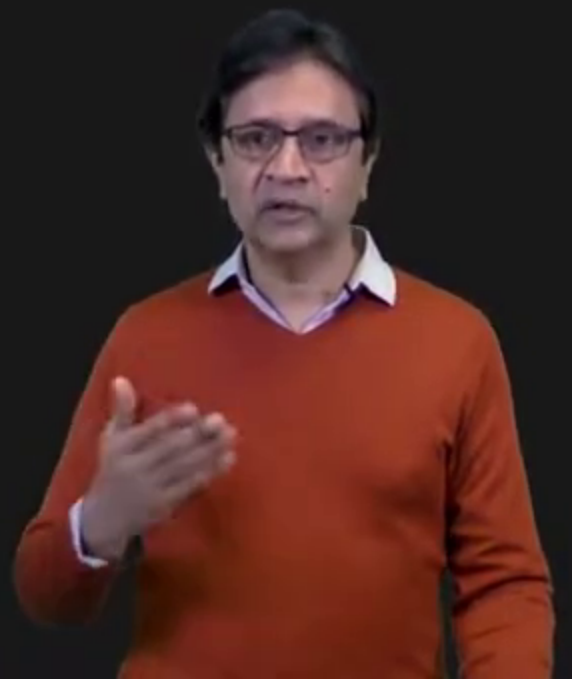
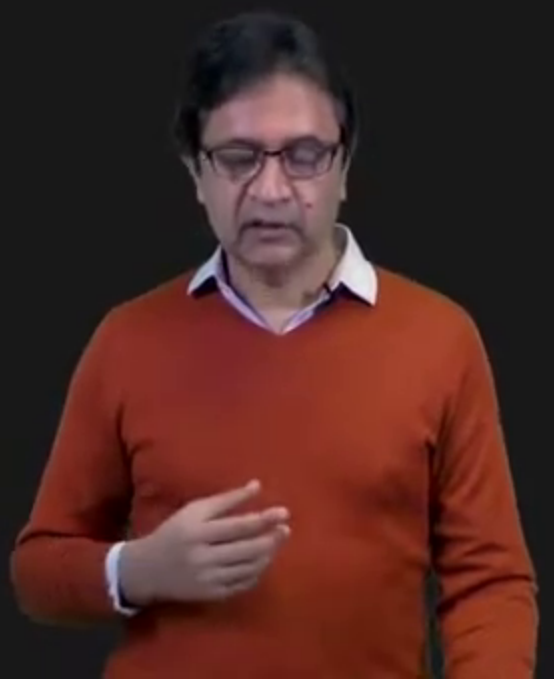


3D Reconstruction with Internet Images

St. Peter's Basilica (1275 Images)

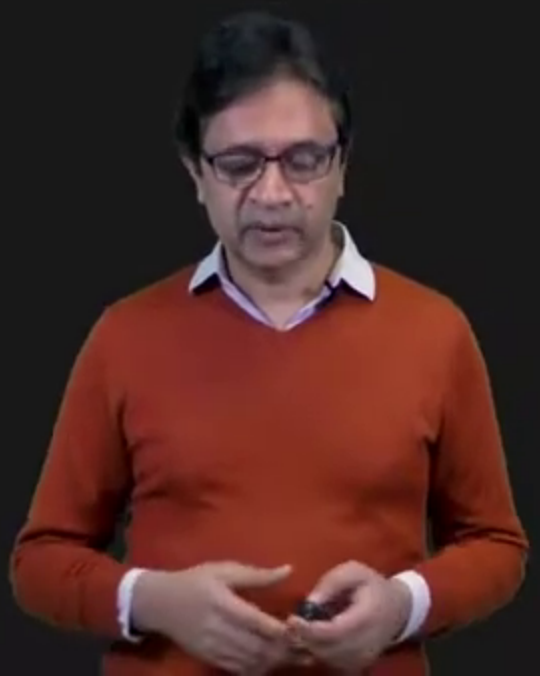
[Snavely 2006]



3D Reconstruction with Internet Images

Piazza San Marco (13709 Images)

[Furukawa 2010]



3D Reconstruction with Internet Images

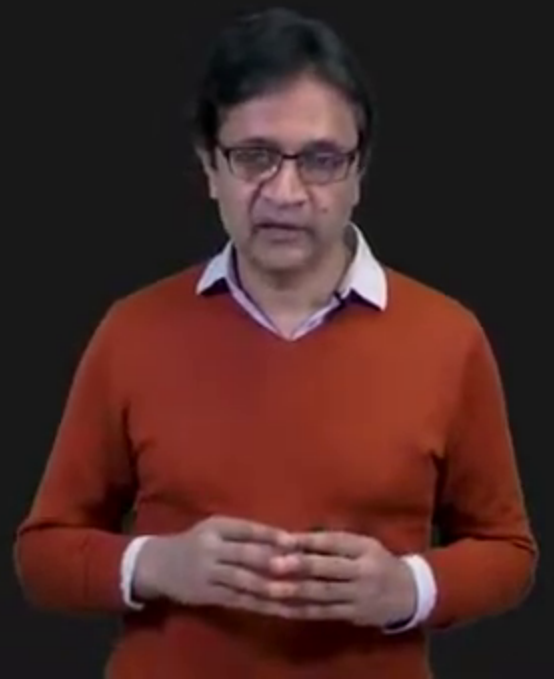
Piazza San Marco (13709 Images)

[Furukawa 2010]

3D Reconstruction with Internet Images

Piazza San Marco (13709 Images)

[Furukawa 2010]



3D Reconstruction with Internet Images

Piazza San Marco (13709 Images)

[Furukawa 2010]

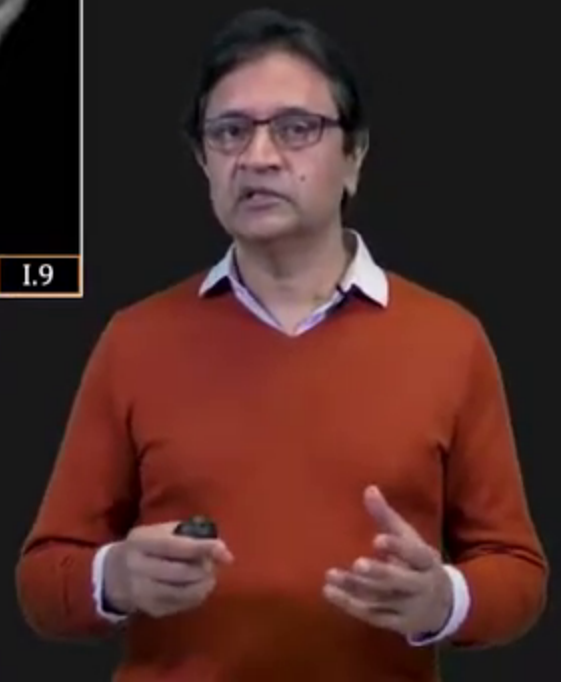
Active Stereo Results

Left Image

Right Image

3D Structure

[Zhang 2003]



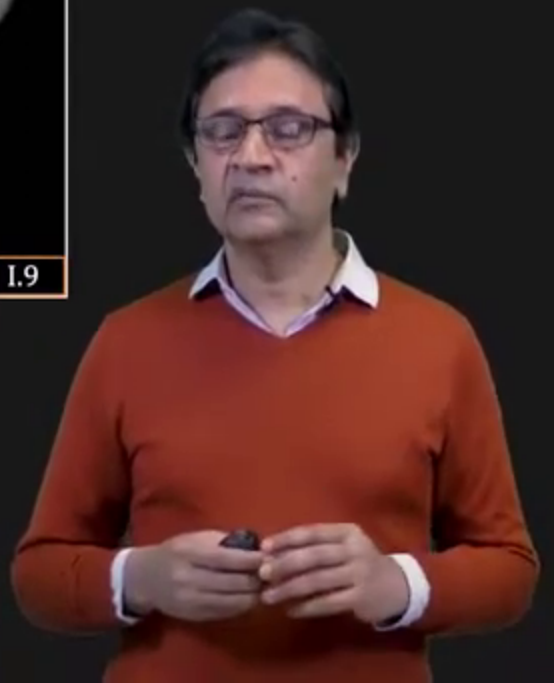
Active Stereo Results

Left Image

Right Image

3D Structure

[Zhang 2003]



Active Stereo Results

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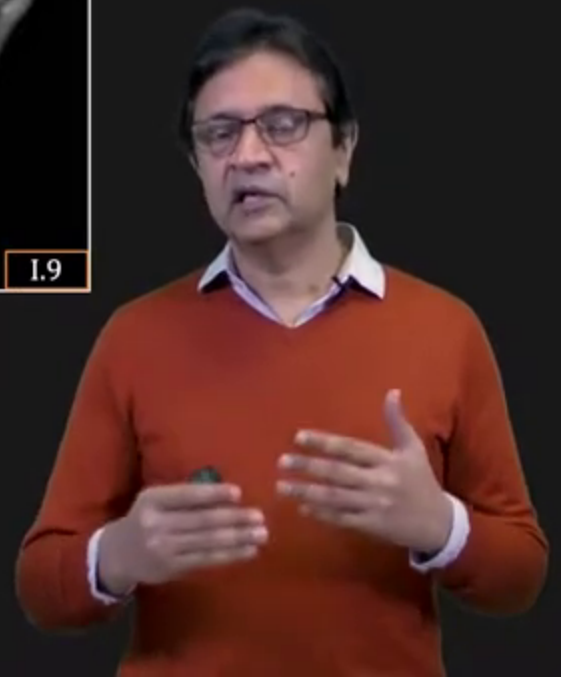
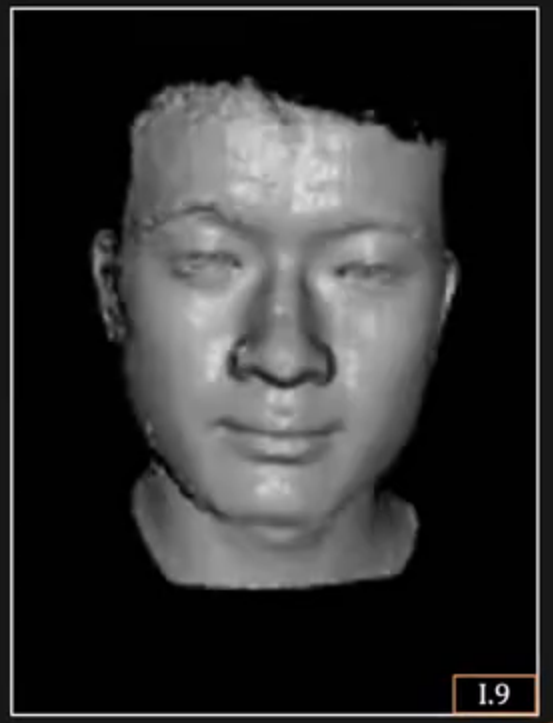
[Zhang 2003]



Active Stereo Results

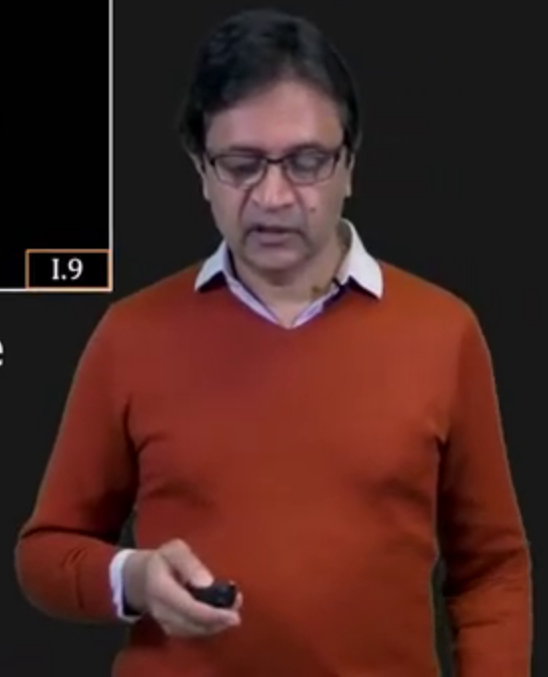
Left Image

Right Image



3D Structure

[Zhang 2003]



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[Zhang 2003]