Estimating Fundamental Matrix

Shree K. Nayar

Columbia University

Topic: Uncalibrated Stereo, Module: Reconstruction II

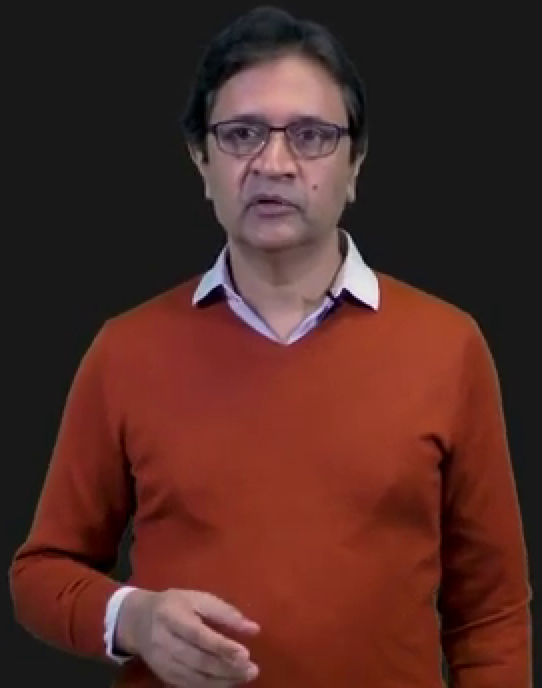
First Principles of Computer Vision

Stereo Calibration Procedure

Find a set of corresponding features in left and right images (e.g. using SIFT or hand-picked)

Left image

Right image

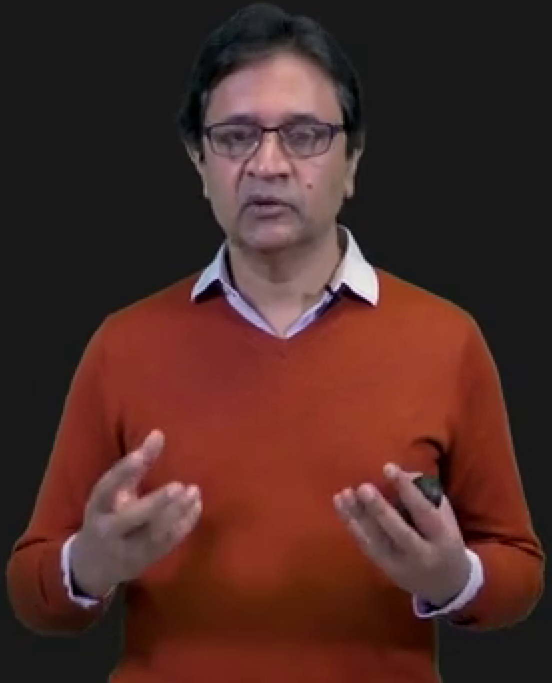


Stereo Calibration Procedure

Find a set of corresponding features in left and right images (e.g. using SIFT or hand-picked)

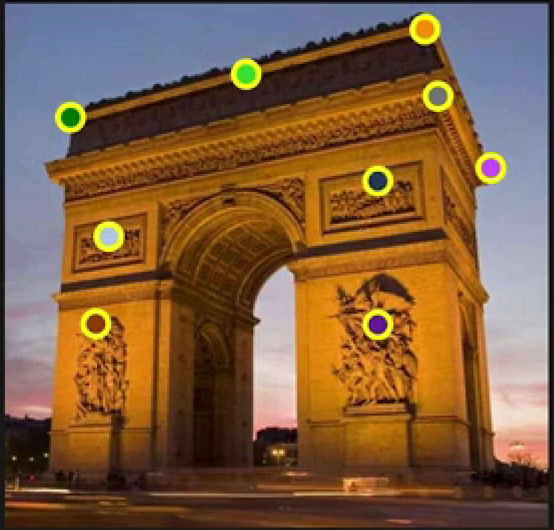
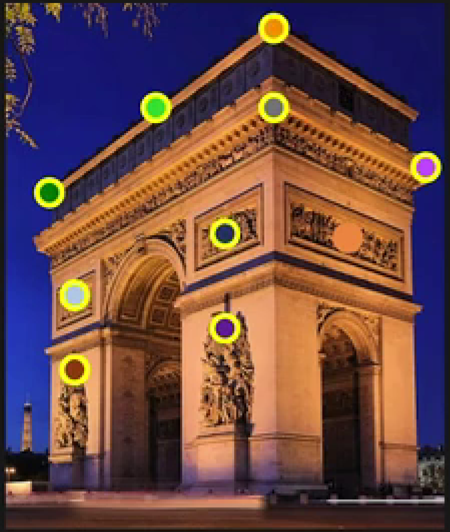
Left image

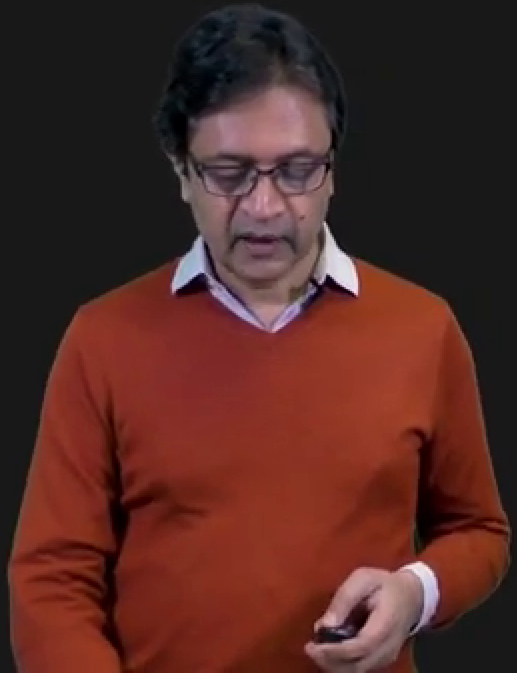
Right image



Stereo Calibration Procedure

Find a set of corresponding features in left and right images (e.g. using SIFT or hand-picked)

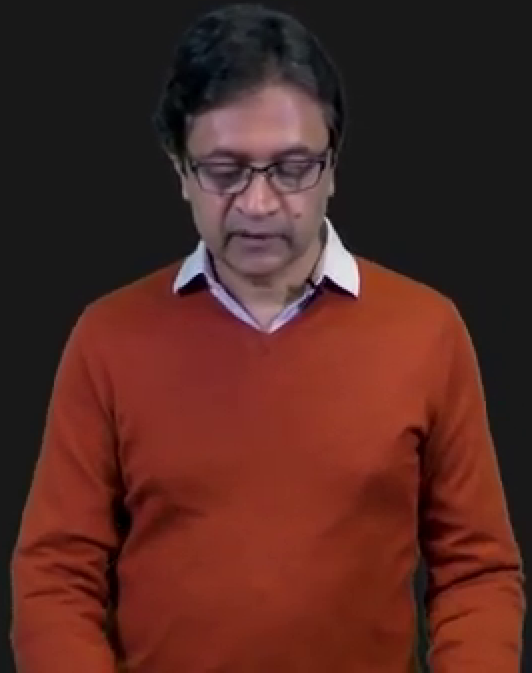
Left image	Right image
	
$(u_l^{(1)}, v_l^{(1)})$	$(u_r^{(1)}, v_r^{(1)})$
$\vdots$	$\vdots$
$(u_l^{(m)}, v_l^{(m)})$	$(u_r^{(m)}, v_r^{(m)})$



Stereo Calibration Procedure

Step A: For each correspondence i, write out epipolar constraint.

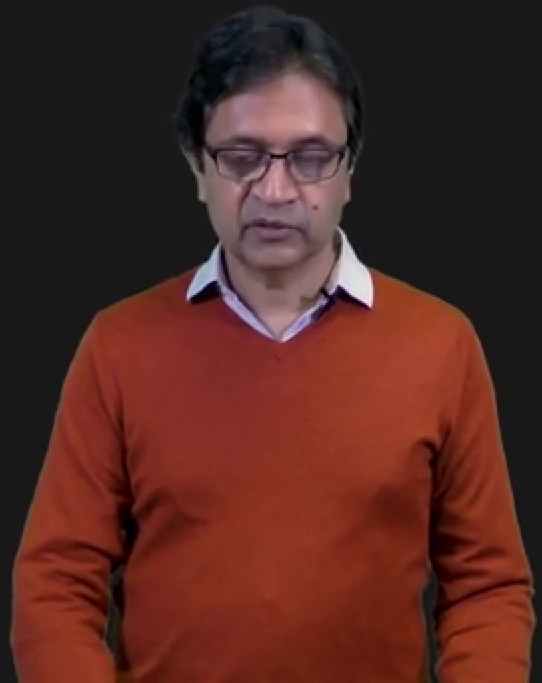
$$\begin{bmatrix} u_l^{(i)} & v_l^{(i)} & 1 \end{bmatrix} \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix} \begin{bmatrix} u_r^{(i)} \\ v_r^{(i)} \\ 1 \end{bmatrix} = 0$$



Stereo Calibration Procedure

Step A: For each correspondence i, write out epipolar constraint.

$$\underbrace{\begin{bmatrix} u_l^{(i)} & v_l^{(i)} & 1 \end{bmatrix}}_{\text{Known}} \underbrace{\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}}_{\text{Unknown}} \underbrace{\begin{bmatrix} u_r^{(i)} \\ v_r^{(i)} \\ 1 \end{bmatrix}}_{\text{Known}} = 0$$



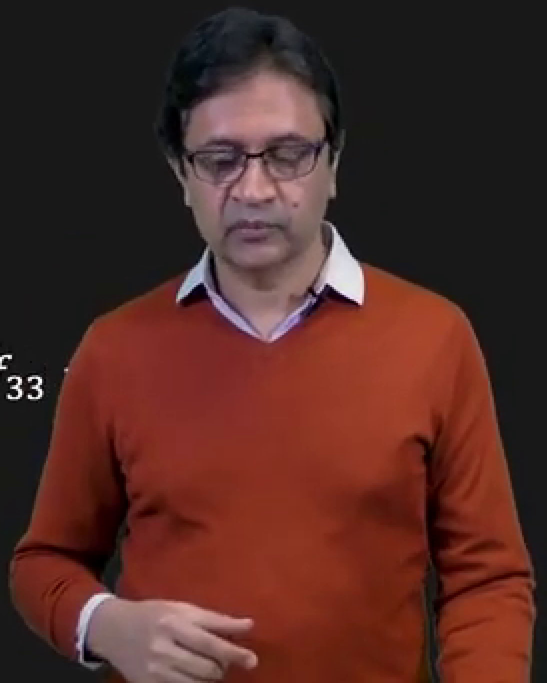
Stereo Calibration Procedure

Step A: For each correspondence i, write out epipolar constraint.

$$\underbrace{\begin{bmatrix} u_l^{(i)} & v_l^{(i)} & 1 \end{bmatrix}}_{\text{Known}} \underbrace{\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}}_{\text{Unknown}} \underbrace{\begin{bmatrix} u_r^{(i)} \\ v_r^{(i)} \\ 1 \end{bmatrix}}_{\text{Known}} = 0$$

Expand the matrix to get linear equation:

$$\left(f_{11}u_r^{(i)} + f_{12}v_r^{(i)} + f_{13}\right)u_l^{(i)} + \left(f_{21}u_r^{(i)} + f_{22}v_r^{(i)} + f_{23}\right)v_l^{(i)} + f_{31}u_r^{(i)} + f_{32}v_r^{(i)} + f_{33}$$



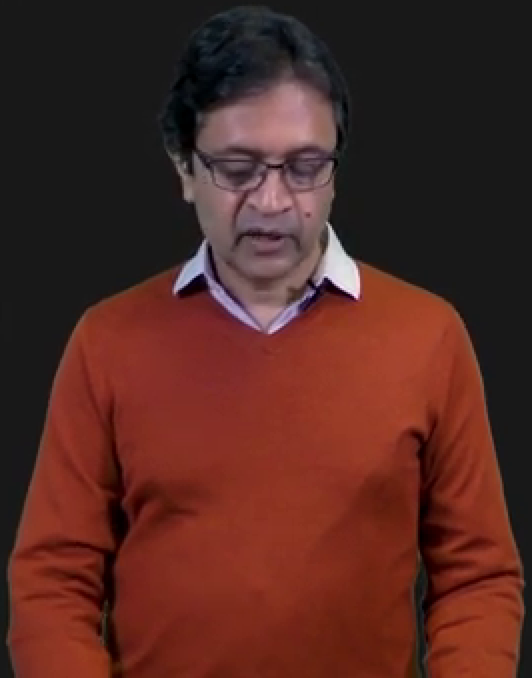
Stereo Calibration Procedure

Step B: Rearrange terms to form a linear system.

$$\underbrace{\begin{bmatrix} u_l^{(1)}u_r^{(1)} & u_l^{(1)}v_r^{(1)} & u_l^{(1)} & v_l^{(1)}u_r^{(1)} & v_l^{(1)}v_r^{(1)} & v_l^{(1)} & u_r^{(1)} & v_r^{(1)} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_l^{(i)}u_r^{(i)} & u_l^{(i)}v_r^{(i)} & u_l^{(i)} & v_l^{(i)}u_r^{(i)} & v_l^{(i)}v_r^{(i)} & v_l^{(i)} & u_l^{(i)} & u_r^{(i)} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_l^{(m)}u_r^{(m)} & u_l^{(m)}v_r^{(m)} & u_l^{(m)} & v_l^{(m)}u_r^{(m)} & v_l^{(m)}v_r^{(m)} & v_l^{(m)} & u_l^{(m)} & u_r^{(m)} & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} f_{11} \\ f_{21} \\ f_{31} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix}}_{\mathbf{f}} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

A
(Known)

$\mathbf{f}$
(Unknown)



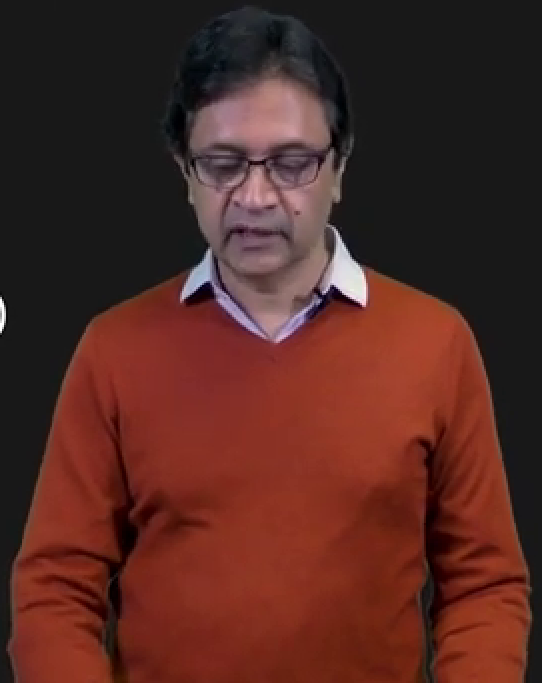
Stereo Calibration Procedure

Step B: Rearrange terms to form a linear system.

$$\boxed{\begin{bmatrix} u_l^{(1)}u_r^{(1)} & u_l^{(1)}v_r^{(1)} & u_l^{(1)} & v_l^{(1)}u_r^{(1)} & v_l^{(1)}v_r^{(1)} & v_l^{(1)} & u_r^{(1)} & v_r^{(1)} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_l^{(i)}u_r^{(i)} & u_l^{(i)}v_r^{(i)} & u_l^{(i)} & v_l^{(i)}u_r^{(i)} & v_l^{(i)}v_r^{(i)} & v_l^{(i)} & u_l^{(i)} & u_r^{(i)} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_l^{(m)}u_r^{(m)} & u_l^{(m)}v_r^{(m)} & u_l^{(m)} & v_l^{(m)}u_r^{(m)} & v_l^{(m)}v_r^{(m)} & v_l^{(m)} & u_l^{(m)} & u_r^{(m)} & 1 \end{bmatrix}} \boxed{\begin{bmatrix} f_{11} \\ f_{21} \\ f_{31} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix}} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

A
(Known)

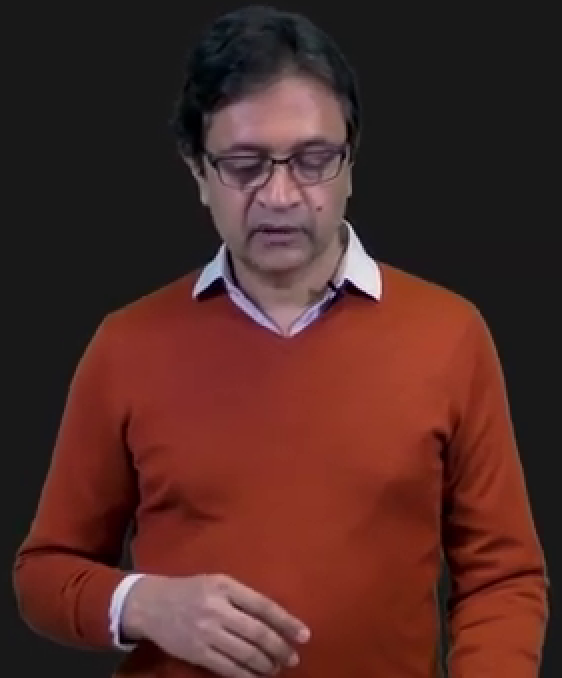
$\mathbf{f}$
(Unknown)



The Tale of Missing Scale

Fundamental matrix acts on homogenous coordinates.

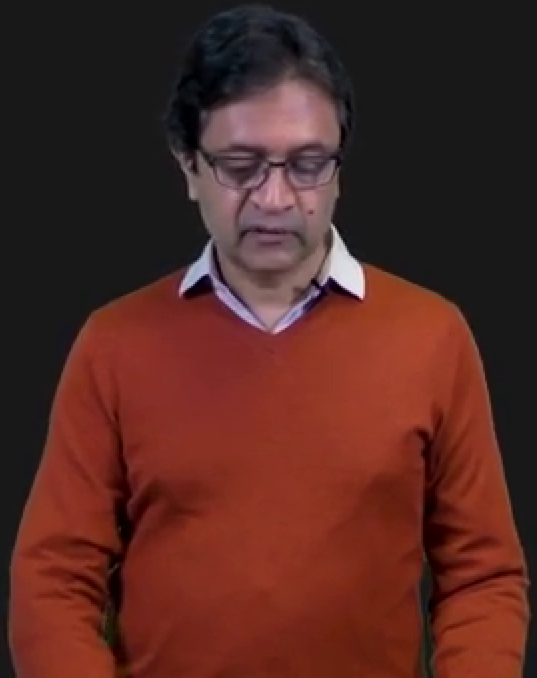
$$[u_l \quad v_l \quad 1]\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0 = [u_l \quad v_l \quad 1]\begin{bmatrix} kf_{11} & kf_{12} & kf_{13} \\ kf_{21} & kf_{22} & kf_{23} \\ kf_{31} & kf_{32} & kf_{33} \end{bmatrix}\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix}$$



The Tale of Missing Scale

Fundamental matrix acts on homogenous coordinates.

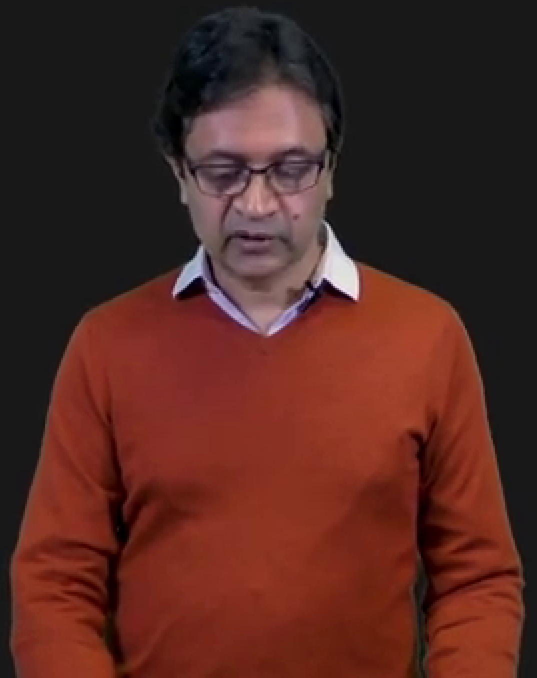
$$[u_l \quad v_l \quad 1]\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0 = [u_l \quad v_l \quad 1]\begin{bmatrix} kf_{11} & kf_{12} & kf_{13} \\ kf_{21} & kf_{22} & kf_{23} \\ kf_{31} & kf_{32} & kf_{33} \end{bmatrix}\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix}$$



The Tale of Missing Scale

Fundamental matrix acts on homogenous coordinates.

$$[u_l \quad v_l \quad 1]\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0 = [u_l \quad v_l \quad 1]\begin{bmatrix} kf_{11} & kf_{12} & kf_{13} \\ kf_{21} & kf_{22} & kf_{23} \\ kf_{31} & kf_{32} & kf_{33} \end{bmatrix}\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix}$$

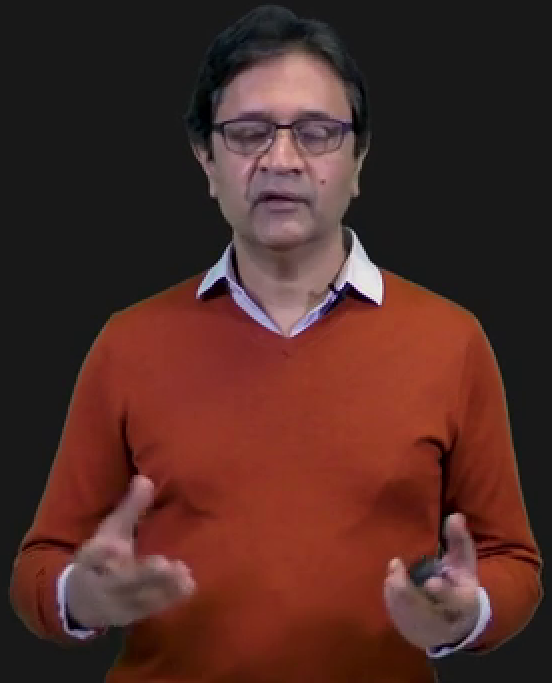


The Tale of Missing Scale

Fundamental matrix acts on homogenous coordinates.

$$[u_l \quad v_l \quad 1]\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0 = [u_l \quad v_l \quad 1]\begin{bmatrix} kf_{11} & kf_{12} & kf_{13} \\ kf_{21} & kf_{22} & kf_{23} \\ kf_{31} & kf_{32} & kf_{33} \end{bmatrix}\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix}$$

Fundamental Matrix F and kF describe the same epipolar geometry. That is, F is defined only up to a scale.

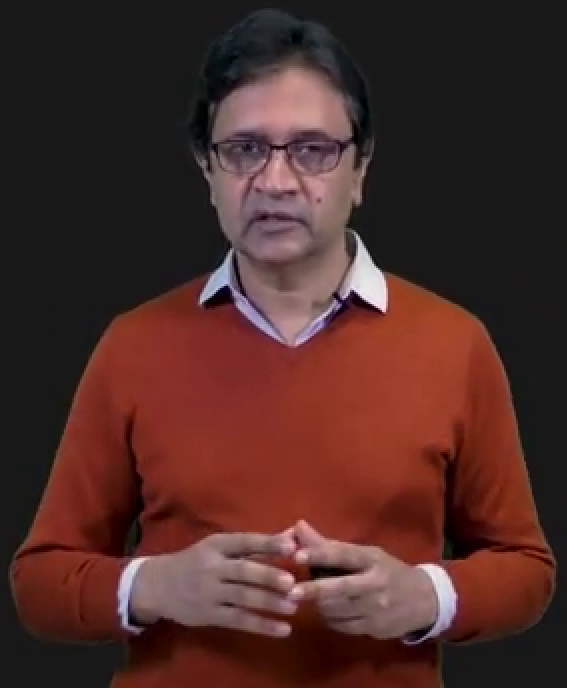


The Tale of Missing Scale

Fundamental matrix acts on homogenous coordinates.

$$[u_l \quad v_l \quad 1]\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0 = [u_l \quad v_l \quad 1]\begin{bmatrix} kf_{11} & kf_{12} & kf_{13} \\ kf_{21} & kf_{22} & kf_{23} \\ kf_{31} & kf_{32} & kf_{33} \end{bmatrix}\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix}$$

Fundamental Matrix F and kF describe the same epipolar geometry. That is, F is defined only up to a scale.



The Tale of Missing Scale

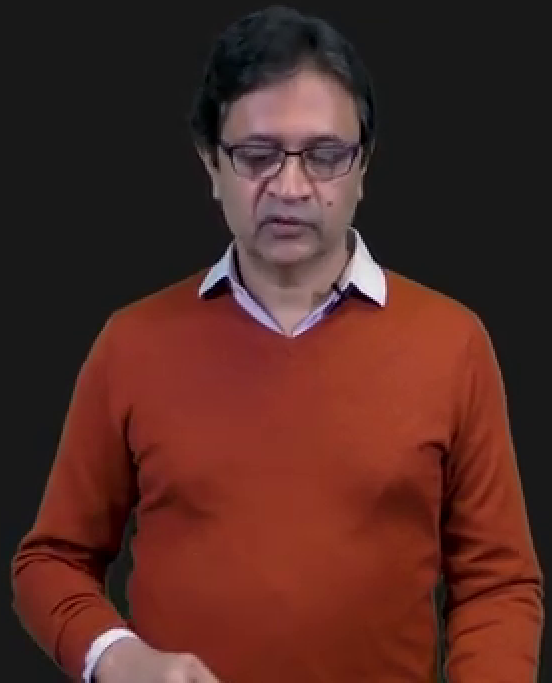
Fundamental matrix acts on homogenous coordinates.

$$[u_l \quad v_l \quad 1]\begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix} = 0 = [u_l \quad v_l \quad 1]\begin{bmatrix} kf_{11} & kf_{12} & kf_{13} \\ kf_{21} & kf_{22} & kf_{23} \\ kf_{31} & kf_{32} & kf_{33} \end{bmatrix}\begin{bmatrix} u_r \\ v_r \\ 1 \end{bmatrix}$$

Fundamental Matrix F and kF describe the same epipolar geometry. That is, F is defined only up to a scale.

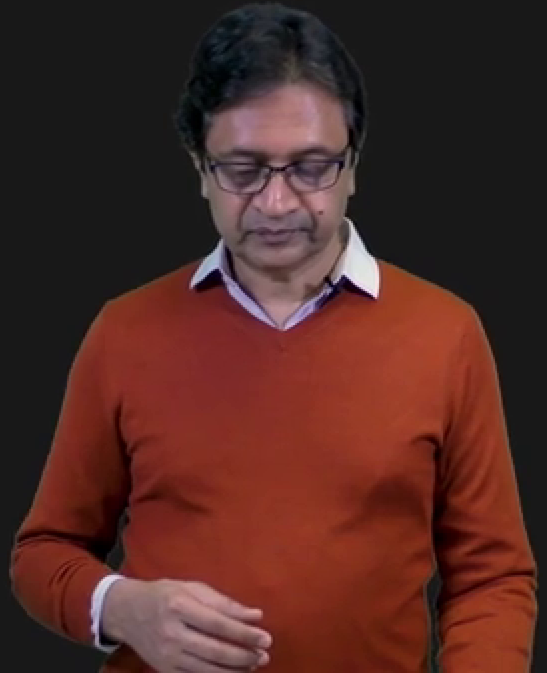
Set Fundamental Matrix to some arbitrary scale.

$$\boxed{\|\mathbf{f}\|^2 = 1}$$



Solving for F

Step C: Find least squares solution for fundamental matrix F.



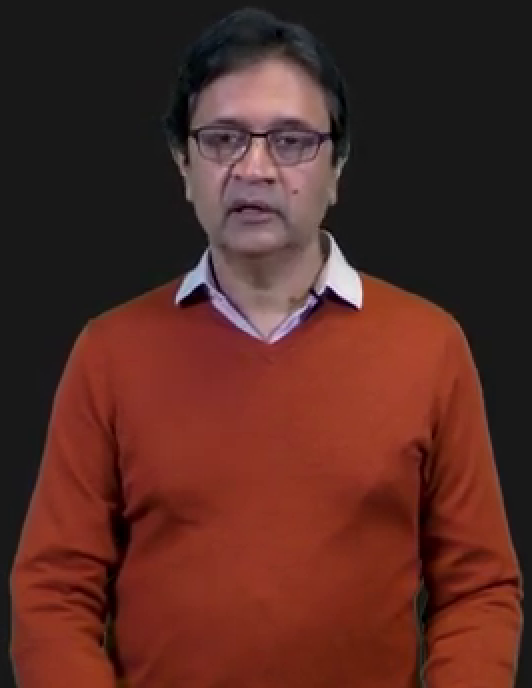
Solving for F

Step C: Find least squares solution for fundamental matrix F.

We want $A\mathbf{f}$ as close to 0 as possible and $\|\mathbf{f}\|^2 = 1$:

$$\boxed{\min_{\mathbf{f}} \|A\mathbf{f}\|^2 \quad \text{such that } \|\mathbf{f}\|^2 = 1}$$

Constrained linear least squares problem



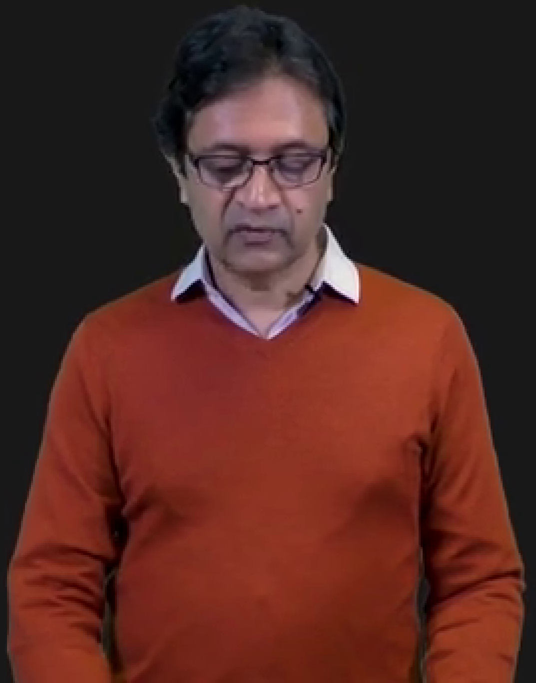
Solving for F

Step C: Find least squares solution for fundamental matrix F.

We want $A\mathbf{f}$ as close to 0 as possible and $||\mathbf{f}||^2 = 1$:

$$\boxed{\min_{\mathbf{f}} \; ||A\mathbf{f}||^2 \quad \text{such that } ||\mathbf{f}||^2 = 1}$$

Constrained linear least squares problem



Solving for F

Step C: Find least squares solution for fundamental matrix F.

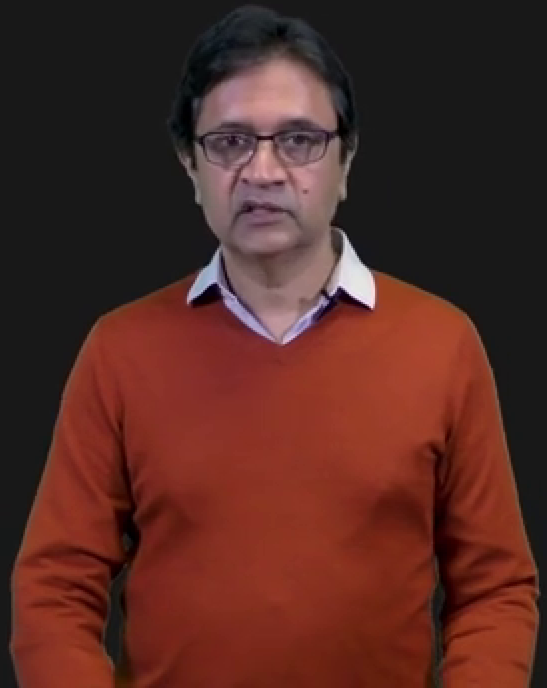
We want $A\mathbf{f}$ as close to 0 as possible and $||\mathbf{f}||^2 = 1$:

$$\boxed{\min_{\mathbf{f}} \; ||A\mathbf{f}||^2 \quad \text{such that } ||\mathbf{f}||^2 = 1}$$

Constrained linear least squares problem

Like solving Projection Matrix during Camera Calibration.

Or, Homography Matrix for Image Stitching.



Solving for F

Step C: Find least squares solution for fundamental matrix F.

We want $A\mathbf{f}$ as close to 0 as possible and $||\mathbf{f}||^2 = 1$:

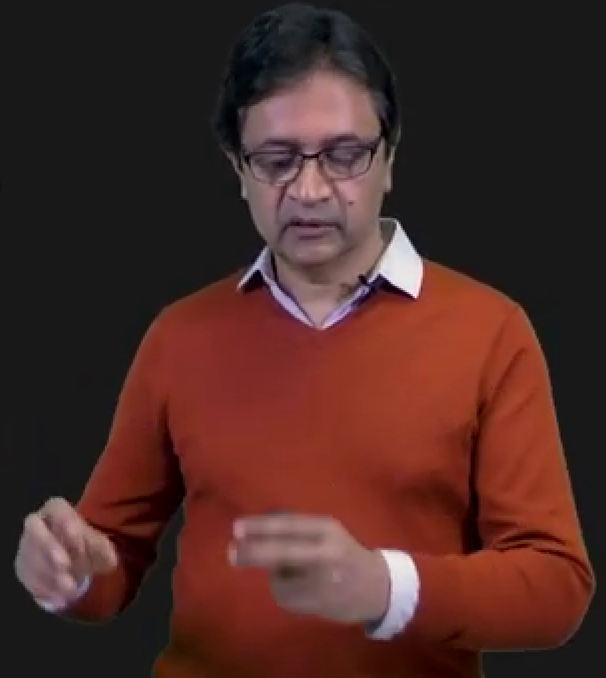
$$\boxed{\min_{\mathbf{f}} ||A\mathbf{f}||^2 \quad \text{such that } ||\mathbf{f}||^2 = 1}$$

Constrained linear least squares problem

Like solving Projection Matrix during Camera Calibration.

Or, Homography Matrix for Image Stitching.

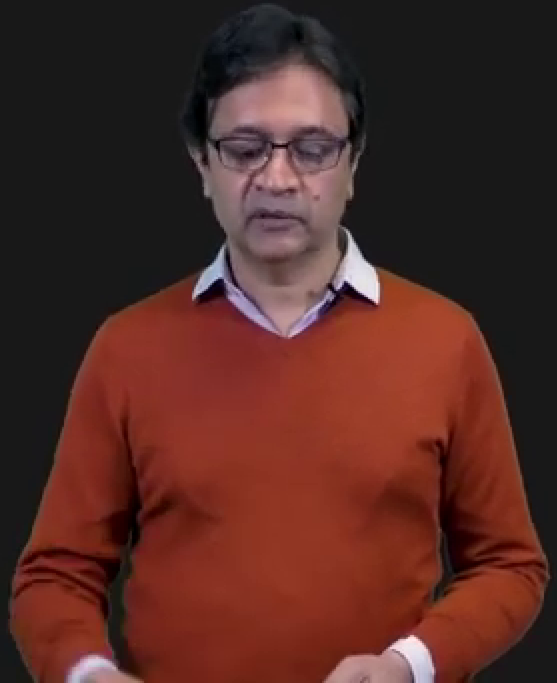
Rearrange solution $\mathbf{f}$ to form the fundamental matrix F.



Extracting Rotation and Translation

Step D: Compute essential matrix E from known left and right intrinsic camera matrices and fundamental matrix F.

$$\boxed{E = K_l^T F K_r}$$



Extracting Rotation and Translation

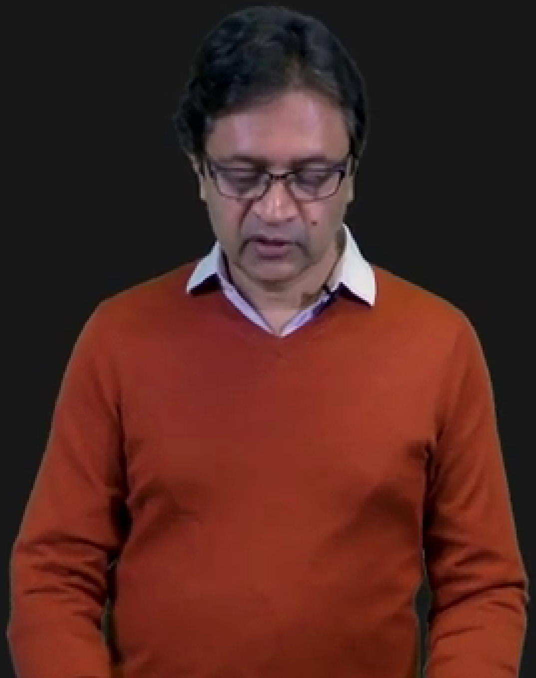
Step D: Compute essential matrix E from known left and right intrinsic camera matrices and fundamental matrix F.

$$\boxed{E = K_l^T F K_r}$$

Step E: Extract R and $\mathbf{t}$ from E.

$$\boxed{E = T_{\times} R}$$

(Using Singular Value Decomposition)



Extracting Rotation and Translation

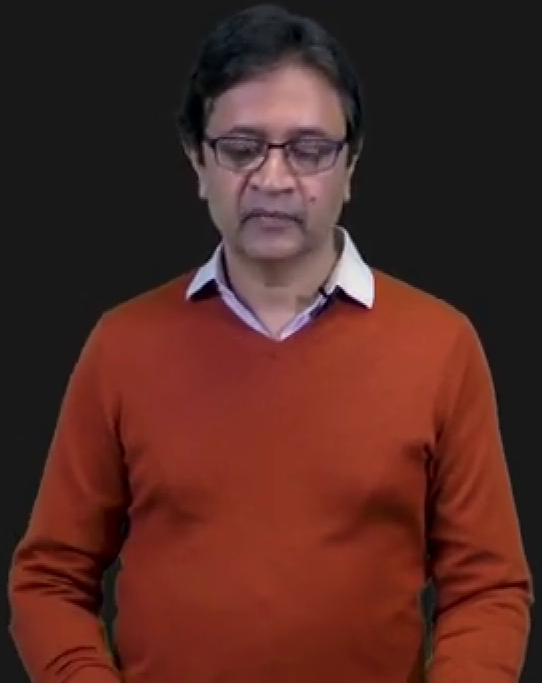
Step D: Compute essential matrix E from known left and right intrinsic camera matrices and fundamental matrix F.

$$\boxed{E = K_l^T F K_r}$$

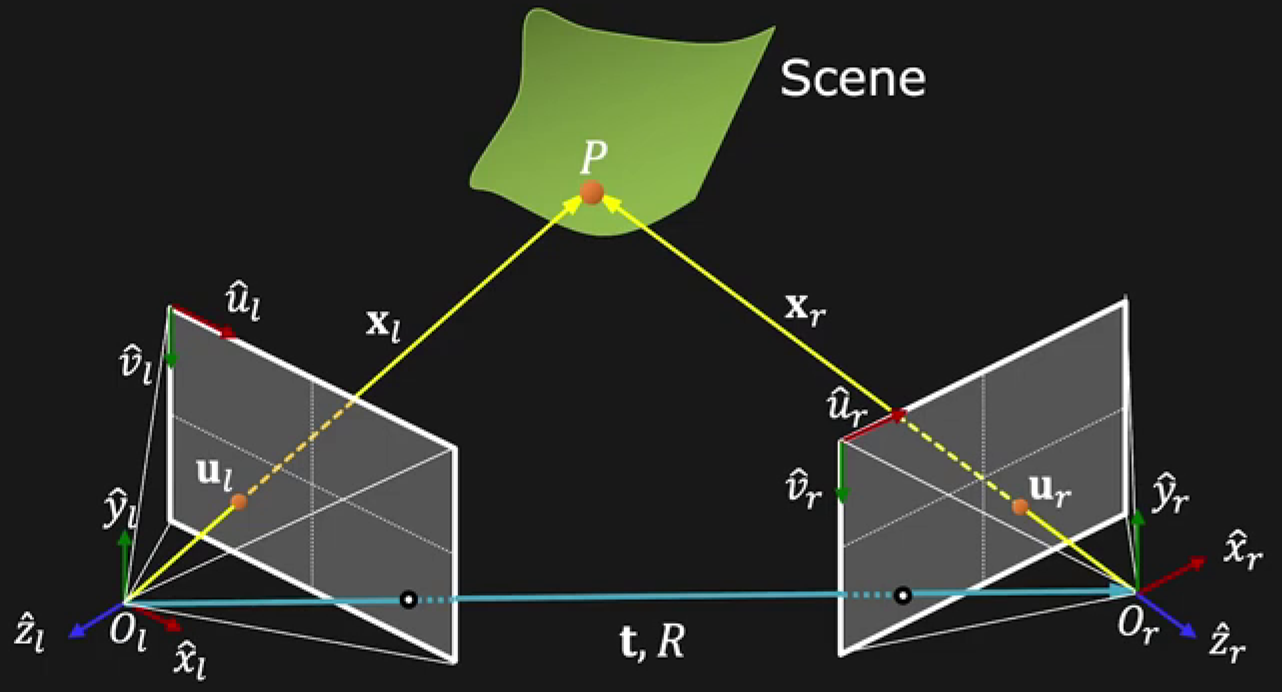
Step E: Extract R and $\mathbf{t}$ from E.

$$\boxed{E = T_\times R}$$

(Using Singular Value Decomposition)



Uncalibrated Stereo



- ✔ 1. Assume Camera Matrix K is known for each camera
- ✔ 2. Find a few Reliable Corresponding Points
- ✔ 3. Find Relative Camera Position $\mathbf{t}$ and Orientation R
- 4. Find Dense Correspondence
- 5. Compute Depth using Triangulation