1 Introduction

I, with the help of Devojyoti, wrote these codes to investigate the effect of linear and non-linear polarisation calibration schemes, under various conditions. In particular, I have tested the cases, where the instrumental leakage is low and high. Under each possibility, I have also investigated the effect when the source has high and low polarisation fractions. For the linear calibration scheme, I have used CASA. For the non-linear calibration scheme, I have used quartical as well as a code I wrote. While the codes were developed by me, I have discussed the details of the procedure with Devojyoti and we are in complete agreement about the results and the methodology. So, this is really a joint effort and should be treated as such. In the spirit of reproducibility, we thought that it is better to upload all codes used in this work on Github, along with a detailed description of the steps which is needed to run this code as well as the understanding we get from it.

2 Simulating visibilities

Let us denote the jones matrix corresponding to antenna i, which describes the leakage matrix, as

$$J_i^{leakage} = \begin{pmatrix} 1 & d_{xi}e^{i\phi_{xi}} \\ d_{yi}e^{i\phi_{yi}} & 1 \end{pmatrix} \tag{1}$$

Both d_{xi}s and d_{yi}s are chosen from randomly from a uniform distribution. For the low leakage case, the uniform distribution corresponding to $(0.01, 0.07)$. For the high leakage case, the limits are $(0.15, 0.25)$. ϕ_{xi} and ϕ_{yi} are drawn from a gaussian distribution with mean of 50° and sigma of 10°. We also use a crosshand phase (θ) to be 50°. Thus the combined Jones matrix for each antenna i is given by

$$J_i = \begin{pmatrix} 1 & d_{xi}e^{i\phi_{xi}} \\ d_{yi}e^{i\phi_{yi}} & 1 \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{2}$$

The visibilities for baseline corresponding to the antenna pair (p, q) for a source with Stokes parameters (I, Q, U, V) are predicted as

$$\begin{pmatrix} V_{pq,RR} & V_{pq,RL} \\ V_{pq,LR} & V_{pq,LL} \end{pmatrix} = 0.5 F J_p \begin{pmatrix} I+V & Q+iU \\ Q-iU & I-V \end{pmatrix} J_q^\dagger, \quad F \text{ is the Fourier Transform operator} \tag{3}$$

We simulate the visibilities for 5 point sources, the properties of which are given in Table 1

Source name	I	Q	U	V
S_1	10	0	0	0
S_2	50	1	5	0
S_3	50	5	1	0
S_4	50	4	15	0
S_5	50	15	-5	0

Table 1: Model properties of the sources simulated

The goal of this experiment is to investigate how each of the algorithms fare in recovering these input source properties after calibration.

The simulated MS can be generated in the following way.

1. If low leakage case, first run *simulate_leakages.py* in the low_leakages folder. Similarly,for different cases, we run the codes with same name, but located in its own corresponding folder.

2. Create a folder called CASA, quartical and self_crosshand inside the folder. If you have the github code, then the folders already exist.

3. Copy a GMRT MS (band2band3band4, just to ensure that the polarization is stated as RRLL in the MS header), such that it has only 1 channel and 1 time. Name the MS es should be as follows: 'CASA/unpolarized_source_I10.ms', 'CASA/polarized_source_I50_Q1_U5_V0.ms', 'CASA/polarized_source_I50_Q5_U1_V0.ms', 'CASA/polarized_source_I50_Q4_U 'CASA/polarized_source_I50_Q15_neg_U5_V0.ms'

4. Next run simulate_vis_polarized_all.py after changing the file names appropriately.

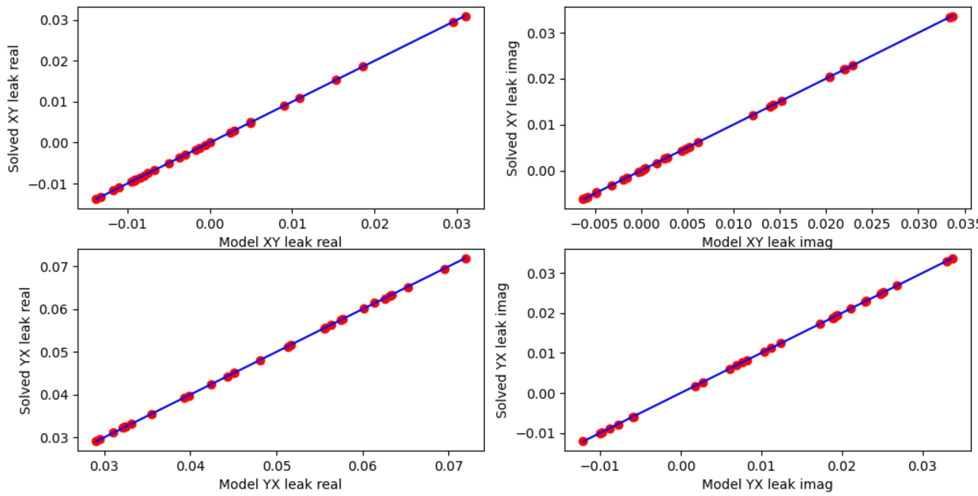


Figure 1: Shows the comparison between the relatives leakages calculated using the input parameters and the relative leakages estimated from the simulated data using CASA. Top and bottom panels show the parameters corresponding to $d_{xi}e^{i\phi_{xi}}$ and $d_{yi}e^{i\phi_{yi}}$ respectively.

3 Low leakage

3.1 Linear analysis

Linear analysis under each test scenario can be performed by running the script named polcalibrate_CASA.py located inside the CASA folder in each test case.

In the linear regime, we can solve for relative leakages in the circular basis. Hence, for accurate comparison, we have used a reference antenna to solve for the leakages. When comparing with the input leakages, we have appropriately computed the relative leakage of each antenna with respect to the reference antenna. In Figure 1, we show the comparison between the relative input and solved leakages. The script named compare_leakage_model.py inside the CASA folder is run to produce this figure. It is evident that the relative leakages solved matches exacty with the relative input leakages. Next, we have solved for the crosshand phase using S_1. Then we apply all the solved parameters to all sources. In Figure 2, we compare the model and recovered source linear polarization properties for sources $S_2 \cdots S_5$. We observe a very tiny deviation in the recovered polarisation angle compared to the model. The script, compare_model_corrected.py, is run after using the appropriate figure names to generate this figure.

3.2 Nonlinear analysis

Nonlinear analysis, using quartucal, under each test scenario can be performed by running the bash script named analyse_quartical.sh located inside the quartical folder in each test case.

Comparing the determined leakages using quartical with the input has another issue. We note here that quartical solves a matrix of the following form when use the *leakage* form.

$$\begin{pmatrix} 1 & d_{xi}e^{i\phi_{xi}} \\ d_{yi}e^{i\phi_{yi}} & 1 \end{pmatrix}$$

As mentioned in Kansabanik et al. 2024, the optimizing equation, which uses the Frobenius norm, is degenerate with respect to a unitary matrix. This implies that the leakage matrix determined using quartical has an unknown unitary matrix[1]. multiplied with it, which makes it not comparable directly with the input leakages. To do proper comparison, we take the following approach. The approach has been mentioned in Kansabanik et al. 2024. Here, we mention the procedure for completeness purposes.

Let us assume that the leakage matrix determined by quartical for the reference antenna and a general antenna i

[1]the effect of this multiplication during calibration is discussed in detail later in the high leakage case

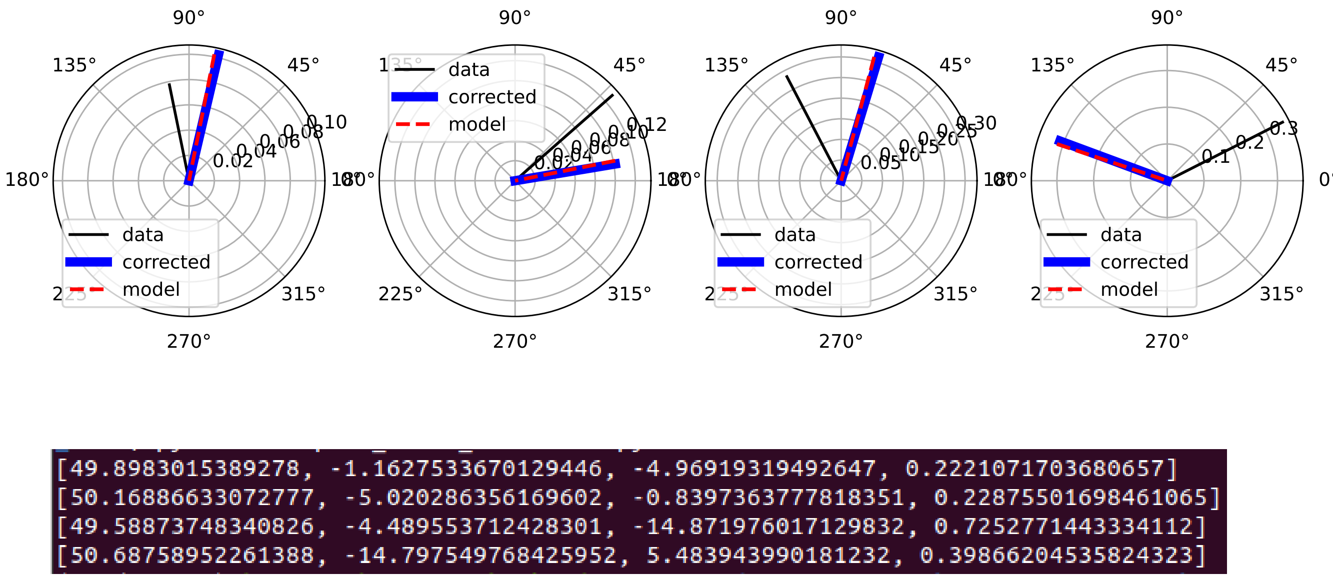


Figure 2: Top panel: Shows the linear polarization of $S_2 \cdots S_5$. Black solid, blue solid ad red dashed lines show the source properties, before clibration, after calibration and the inpit parameters. Bottom panel: Shows the I,Q,U,V of the 4 sources after calibration.

are denoted by J_{ref} and J_i respectively. The normalized Jones matrix for each antenna is then given by

$$J_{ref} = H_{ref} U_{ref} \tag{4}$$
$$J_{norm,i} = J_i J_{ref}^{-1} H_{ref} \tag{5}$$

The input $J_i^{leakage}$ is also normalised in the same manner, using the same reference antenna. Then they are compared, and the comparison is shown in Figure 3. This figure can be generated by running the script, compare_quartical_model_leak1.py, located inside the quartical folder. In Figure 4 we show the comparison between the input and recovered source parameters. The agreement is slightly better than what we find with CASA, but not sufficiently to say that CASA has performed poorly.

4 High leakage

The most general case is implemented in a folder named high_leakage_case_general.

4.1 Linear analysis

The CASA analysis is exactly similar and the codes which are run are also named same. In Figure 5, we show the comparison between the relative input and solved leakages. We find that the solved leakages match exactly with the input leakage, after appropriate normalisation. This is not surprising, considering that the cross-hand visibilities are exactly same for unpolarized source both in the linear and non-linear regime. In Figure 6, we compare the model and recovered source polarization properties for sources $S_2 \cdots S_5$. We observe large deviations in the recovered polarisation properties compared to the model. In fact, the Stokes I is different for the recovered source, in comparison to the input source model. We have verified that the I and V after calibration is same as that in the data, after corruption by the antenna Jones matrices. This demonstrates that CASA does not apply leakage corrections and crosshand corrections to the co-polar visibilities.

4.2 Nonlinear analysis: Quartical

Here, we also follow the strategy described in the linear case. We solve for the leakage matrices, and then use them to determine the crosshand phase matrix. All of the Jones matrices are determined using quartical. The analysis

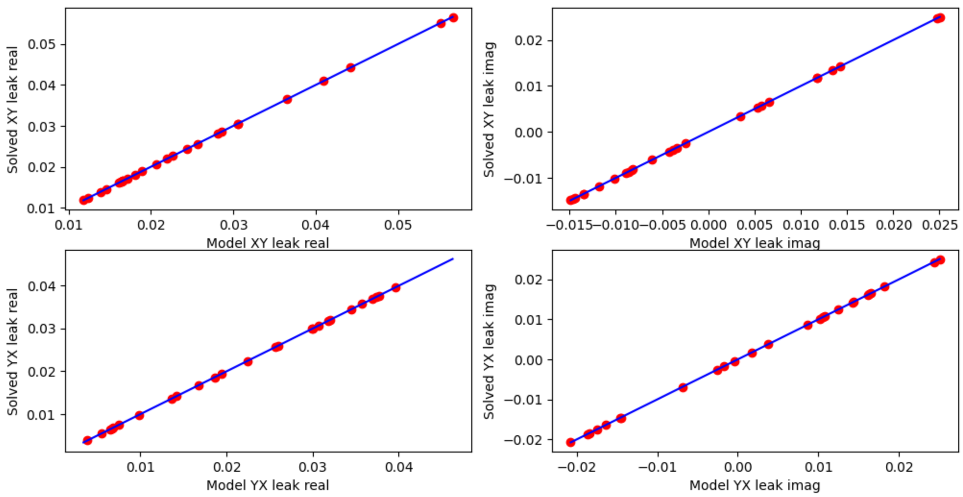
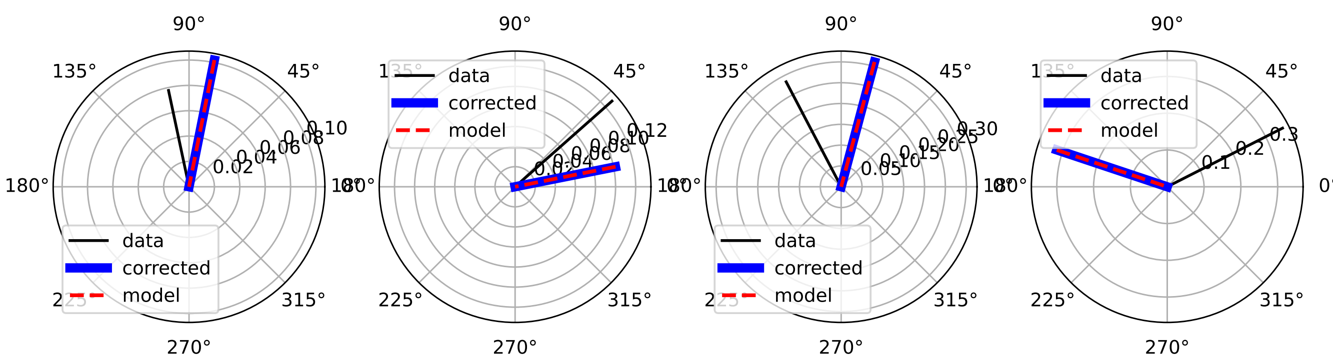


Figure 3: Shows the comparison between the J_{norm} calculated using the input parameters and the J_{norm} estimated from the simulated data using quartical. Top and bottom panels show the parameters corresponding to $d_{xi}e^{i\phi_{xi}}$ and $d_{yi}e^{i\phi_{yi}}$ respectively.

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[50.038868099451065, 15.001575043797494, -5.015838436037303, 0.39674518704414297]
```

Figure 4: Top panel: Shows the linear polarization of $S_2 \cdots S_5$. Black solid, blue solid ad red dashed lines show the source properties, before clibration, after calibration and the inpit parameters. Bottom panel: Shows the I,Q,U,V of the 4 sources after calibration. Calibration is done using quartical.

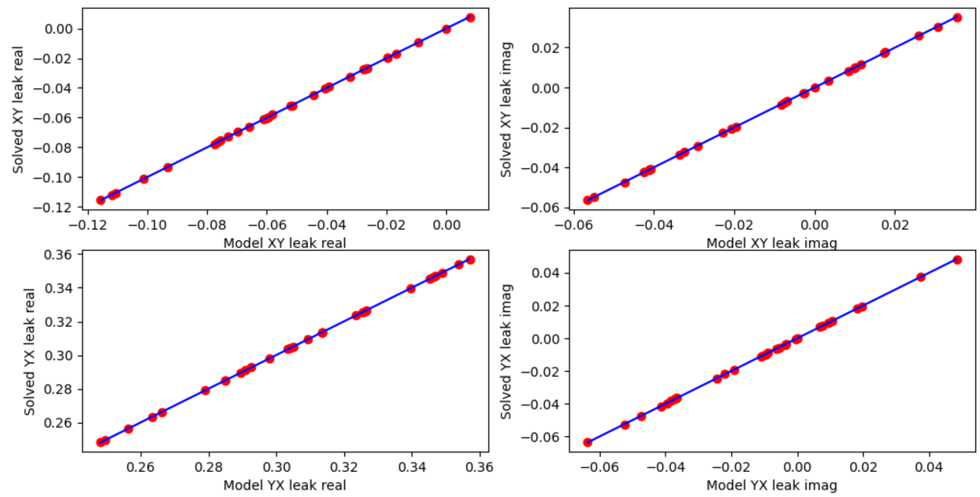
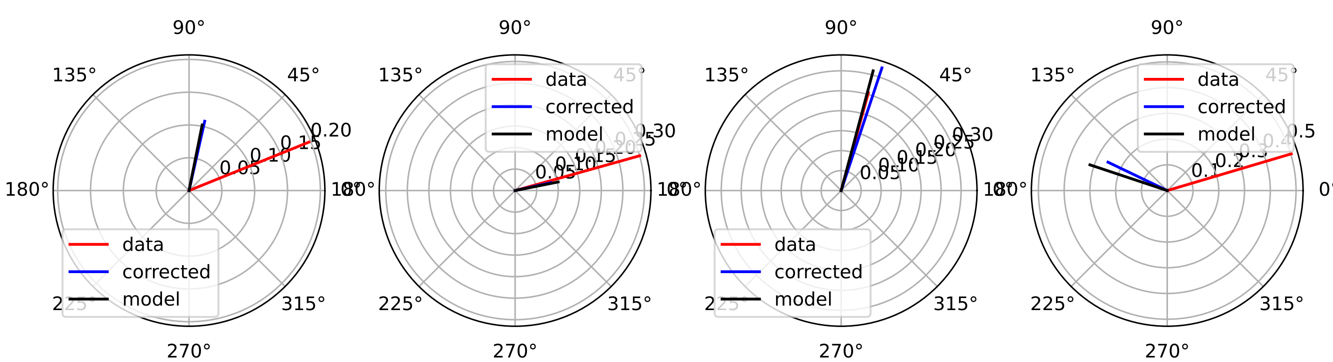


Figure 5: Shows the comparison between the relatives leakages calculated using the input parameters and the relative leakages estimated from the simulated data using CASA. Top and bottom panels show the parameters corresponding to $d_{xi}e^{i\phi_{xi}}$ and $d_{yi}e^{i\phi_{yi}}$ respectively.

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[49.70745687813594, -5.068158581065035, -15.273808608658012, 3.891132587125931]
[55.38156587447243, -12.733582663262027, 6.109436315229569, 2.558318846253144]
```

Figure 6: Top panel: Shows the linear polarization of $S_2 \cdots S_5$. Black solid, blue solid ad red dashed lines show the source properties, before clibration, after calibration and the inpit parameters. Bottom panel: Shows the I,Q,U,V of the 4 sources after calibration. Calibration has been done using CASA.

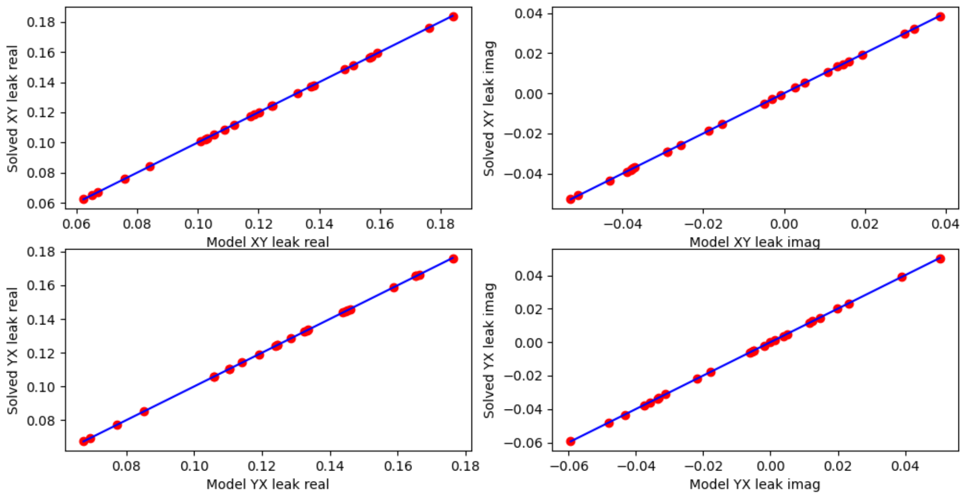


Figure 7: Shows the comparison between the J_{norm} calculated using the input parameters and the J_{norm} estimated from the simulated data using quartical. Top and bottom panels show the parameters corresponding to $d_{xi}e^{i\phi_{xi}}$ and $d_{yi}e^{i\phi_{yi}}$ respectively.

code is again named analyse_quartical.sh, located in a folder named quartical_leakage. The comparison between the leakages after appropriate normalization is shown in Figure 7. This figure can be generated by running the script, compare_quartical_model_leak1.py, located inside the quartical folder. In Figure 8 we show the comparison between the input and recovered source parameters. The agreement is slightly better than what we find with CASA, in the sense that Stokes I is well matched now. However, Stokes V is roughly of the same order as that determined in the CASA calibration. The question is hence, why even when we are using Quartical, which is a non-linear solver, we can not recovering the input parameters.

The answer to this question is quite non-trivial, and has to do the "unitary matrix" degeneracy we mentioned earlier. Remember that we had created a Jones matrix $J_i^{leakage}$ for each antenna. We thought that this matrix will only cause mixing between the Stokes pairs (I,Q), (I,U) and (I,V). However, as we show below, this is not correct.

The $J_i^{leakage}$ of each antenna can be written as

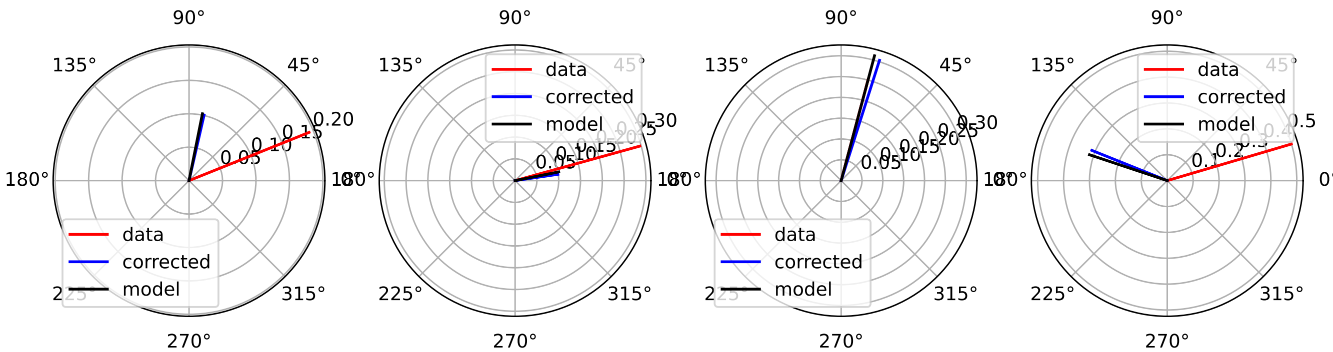
$$J_i^{leakage} = J_i^{leakage}\left(J_{ref}^{leakage}\right)^{-1} H_{ref}U_{ref} \tag{6}$$

$$J_{ref}^{leakage} = H_{ref}U_{ref} \tag{7}$$

H_{ref}, U_{ref} are Hermitian and unitary matrices respectively. This suggests that $J_i^{lekaage}$ of each antenna i has a common unitary matrix multiplied with it, and the unitary matrix multiplied depends on the jones matrix of the reference antenna. This unitary matrix is will cause rotation of the Stokes vector in the Q, U, V space, even in absence of any crosshand phase term. Hence, solving for just the crosshand phase will not correct for the entire polrotation term. The reason why Stokes I was ok, was because, the non-linear calibration procedure, has uniuqely determined the polconversion matrix and hence Stokes I is ok when we use Quartical. The solution to this issue is to solve for the full general polrotation matrix. This is done in the next subsection. However, we use the antenna leakages determined using Quartical. We first completely remove the effect of the unitary matrix of the reference antenna and store those in a file. The file is named quartical_ant_leaks.txt and can be generated by running get_quartical_leak.py . The created txt file is copied from the quartical folder to the folder named self_crosshand.

4.3 Nonlinear analysis: Determining the full polrotation matrix

For solving the full polrotation matrix, we need to use at least 2 polarised sources. For this purpose, we use S_2 and S_3. The code to solve the 3 angles is named solve_crosshand_phase.py . The phases are applied to the data using the script named generate_corrected_vis.py . by running the script, compare_quartical_model_leak1.py, located inside the quartical folder. In Figure 9 we show the comparison between the input and recovered source parameters. It is evident that all Stokes parameters have been recovered with a much better accuracy. However, in spite of this, we find that

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[50.93223669528962, 4.681695224344731, 14.7222655788064, 3.026615834236143]
[50.9105604171753, 14.814082783460618, -5.9512815117836, -0.3002300262451172]
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Figure 8: Top panel: Shows the linear polarization of $S_2 \cdots S_5$. Black solid, blue solid ad red dashed lines show the source properties, before clibration, after calibration and the inpit parameters. Bottom panel: Shows the I,Q,U,V of the 4 sources after calibration. Calibration is done using quartical.

Stokes V is still non-zero. This we think is due to numerical errors and convergence parameters chosen in Quartical and our solver used for determining the polrotation matrix.

4.4 Does determing the diagonal gains help

In this section, we try to understand, if it is possible to solve the Stokes V issue, if we were general observers, who had no understanding of how the data was generated. Observers typically does not assume that the diagonal gain is 0. This implies that for quartical, we shal solve for a general matrix using S_1. We also run a gaincal before polcal with S_1. For the purpose of this, scripts with similar names are present in folders named CASA_gaincal and quartical In both cases, we find that the estimated amplitude and phase gains are very similar. We show the determined antenna gain terms in Figure 10. The amplitude gains deviation from 1 are about 1–2%. The phase deviations are also of same order. We otain similar numbers when we use CASA gaincal task as well. We repeat the analysis by applying these parameters, and we find that while CASA results have not changed appreciably, for the non-linear case, Stokes V has improved significantly. In Figure 11, we show the recovered polarization properties from CASA analysis, and the procedure we mentioned in Section 4.3. We find that while the non-linear analysis has now recovered all Stokes parameters with an accuracy better than 1%, there are quite large errors in the CASA analysis. Observers can solve the spurious Stokes V issue, by running CASA gaincal for each of the calibrator, and then assuming that the gains are varying with time. The determined gain terms are in reality a function of the source polarization properties and the antenna leakages and hence interpolation schemes based on time-variability will not work.

4.5 How to ensure that the only polrotation term is crosshand phase

In the previous subsections, we have discussed why even though we have added the crosshand phase, we needed to solve for all the polrotation terms. We also explained how the leakage matrices itself contained another unitary matrix, which is, in general, as all three rotation terms. The crosshand phase applied is thus, added on top of this unitary matrix. However, to close the loop, and demnstrate the validity of this understanding, it is important to simulate a set of antenna leakages, for which the polrotation term is zero. This simulation is done in the folder named high_leakage_case_only_crosshand . The visibility generation scripts are similarly named and used as in other folders. The main thing which we have implemented in ensuring that the only polrotation term is the crosshand phase, we have chosen the leakage of one antenna to be a Hermitian matrix. This matrix is used as the reference antenna in all cases. However, even in this instance, we cannot just use Quartical to calibrate the data. This is because, the antenna leakage matrices solved by Quartical, itself will have an unknown unitary matrix multiplies due to the inherent degeneracies

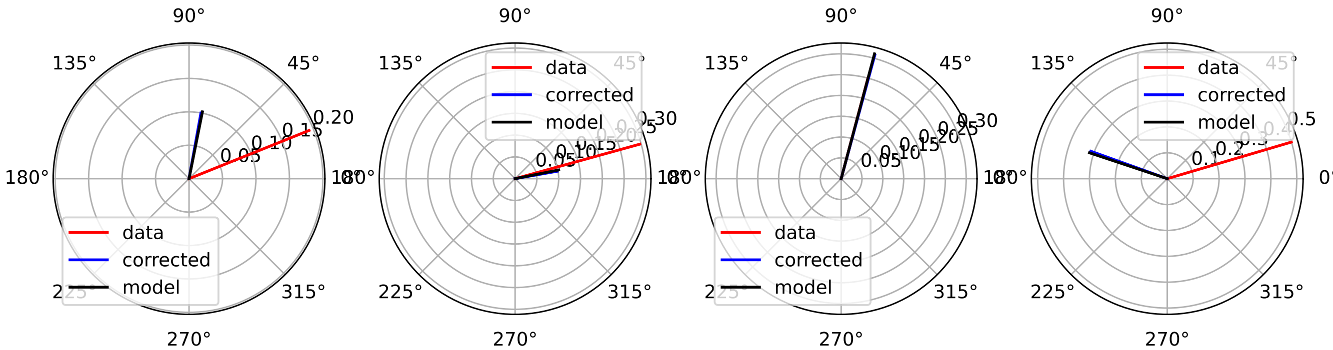


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Figure 9: Top panel: Shows the linear polarization of $S_2 \cdots S_5$. Black solid, blue solid ad red dashed lines show the source properties, before clibration, after calibration and the inpit parameters. Bottom panel: Shows the I,Q,U,V of the 4 sources after calibration. Calibration is done using normalized antenna leakaged solved using Quartical and then solving for the full polrotation matrix.

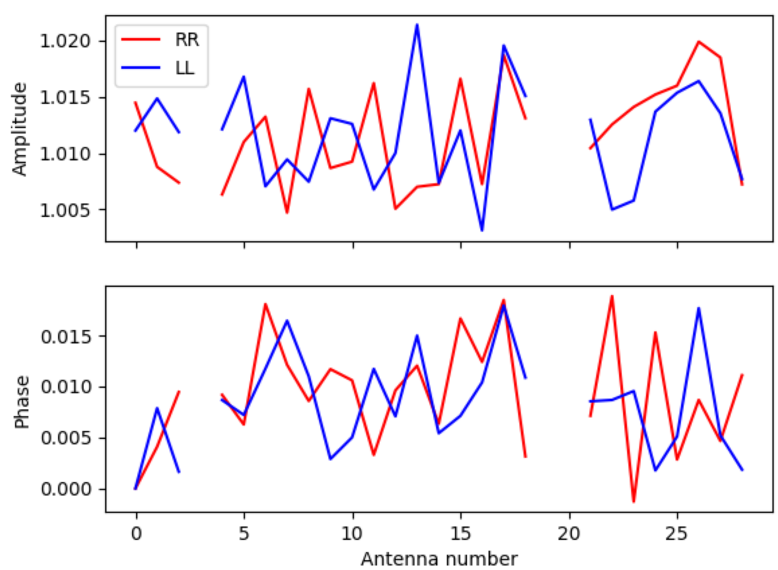

Figure 10: Shows the amplitude and phase of the diagonal gain terms of the Jones matrix determined using quartical, when we solve for the general matrix. This is same as the what is solved in CASA, in the linear regime.

```
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[47.850743149760106, -4.174162381055348, -14.560377900756663, 3.7309313336668524]
[53.21408007151721, -12.942756688153302, 5.533361271915274, 2.451773434283048]
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[49.999999839067456, 1.1102094439789654, 4.975942489504815, -0.08619864583015513]
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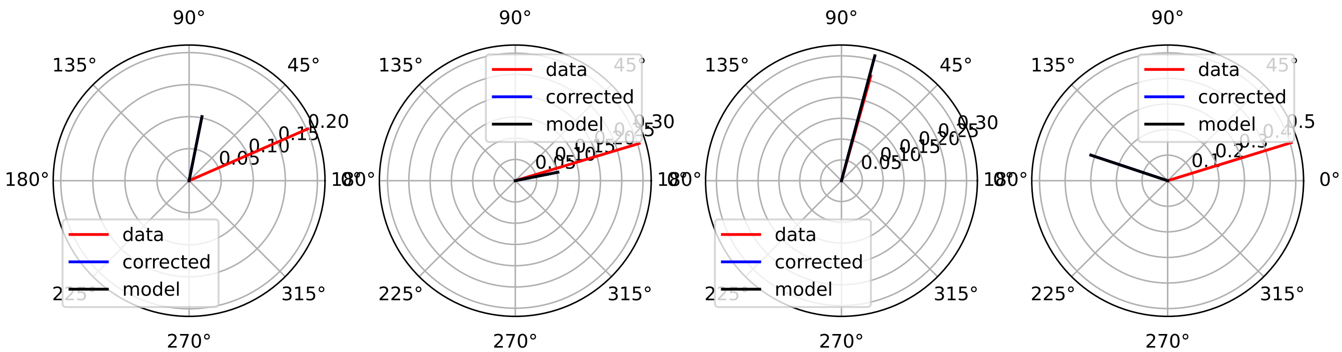
```

Figure 11: Shows the I,Q,U,V of the 4 sources after calibration. Top panel: Calibration is done using CASA. Bottom panel: Calibration is done using normalized antenna leakaged solved using Quartical and then solving for the full polrotation matrix.

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```

Figure 12: Top panel: Shows the linear polarization of $S_2 \cdots S_5$. Black solid, blue solid ad red dashed lines show the source properties, before clibration, after calibration and the inpit parameters. Bottom panel: Shows the I,Q,U,V of the 4 sources after calibration. Calibration is done using normalized antenna leakaged solved using Quartical and then solving only for the full crosshand phase.

of the Frobenius norm. Hence we have extracted the $J_{norm,i}$ for each antenna, and then solved just for the crosshand phase. The crosshand phase solving script is present in a folder again named as self_crosshand. The name of the script is solve_crosshand_phase.py . It can be seen that we have only solved for the crosshand phase and the crosshand phase comes out to be 50.0013°, which is exactly same as the crosshand phase we have used in the simulation. Next we apply the solved parameters and generate the corrected visibilities. In Figure 12 we show the comparison between the input and recovered source parameters. It is evident that all Stokes parameters have been recovered with extremely high accuracy.

5 Pitfalls in arrays which have large leakage

We have demonstrated in Figure 6 that the linear equations (implemented in CASA) are unable to recover the source polarization properties after calibration. We have mentioned earlier, that the errors in the recovered I and V can be "fixed" by assuming that the antenna gains are different for different sources. Similarly, the difference in polarisation angle can be fixed by solving crosshand phase for each source separately. While this will create issues on the target field(s), the calibrator fields will be 'perfectly" calibrated, it will create 3 key issues.

1. The antenna gains retrieved by the CASA gaincal/bandpass tasks will become dependent on the source polarisation properties.

2. The crosshand phase estimated by the above procedure also becomes dependent on the source polarisation dependent.

3. It becomes practically impossible to apply on the estimated parameters on the target field.

It is even more concerning that in arrays with large leakages, the recovered parameters after using Quartical, are also erroneous. And again, these issues can be attributed incorrectly as a source-dependent crosshand phase. Thus we have demonstrated here, using a controlled simulated antenna parameters, these issues can also arise due to the inherent properties of the non-linear equations and the algorithms used for determining the parameters. We have also explained why Quartical, in spite of being a non-linear solver, is unable to recover the source parameters exactly. Additionally, we have implemented software which, in the limited case of these simulations presented here, are able to recover the source properties to a high accuracy.

6 Mapping hybridizers which convert the linear basis observations to circular basis to this formalism

Hybridizer converts the observations taken with linear feeds to circular basis. The way this is done is combining the observations in the two channels, after shifting one of the two channels by 90° phase. Whie this is the intention, due to inaccuracies in the hardware, the output polarization is not exactly circularly polarized, but more elliptically polarized. To understand the effect of this inaccuracy, let us write the full measurement equations by incorporating Jones matrices corresponding to the hybridizer. The Jones matrix of the hybridizer is given by $J_{hybrid,i}$

$$\begin{pmatrix} \tilde{E}_{R,i} \\ \tilde{E}_{L,i} \end{pmatrix} = \begin{pmatrix} 1 & i \\ 1 & fe^{i(-\pi/2+\phi_{hybrid,i})} \end{pmatrix} \begin{pmatrix} E_{X,i} \\ E_{Y,i} \end{pmatrix} = J_{hybrid,i} \begin{pmatrix} E_{X,i} \\ E_{Y,i} \end{pmatrix} \tag{8}$$

$E_{X,i}$ and $E_{Y,i}$ are the electric field in the linear basis in antenna i. f and $\phi_{hybrid,i}$ are the error in the amplitude ratio, and phase difference between the two channels. $E_{R,i}$ and $E_{L,i}$ are the electric field vector in the RL basis. We have used the $\tilde{E}$ sign to denote the fact that the output observations are actually elliptically polarised, although we treat it as if they are observed in true circular basis.

Now let us write the Jones chain including the effect of the hybridizer.

$$\begin{aligned}
\begin{pmatrix} V_{pq,RR} & V_{pq,RL} \\ V_{pq,LR} & V_{pq,LL} \end{pmatrix} =& 0.5F J_{hybrid,p} J_p \begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix} J_q^\dagger J_{hybrid,q}^\dagger \\
=& 0.5F J_{hybrid,p} J_p \begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix} J_q^\dagger J_{hybrid,q}^\dagger \\
=& 0.5F J_{hybrid,p} J_p \left(J_{hybrid}^{ideal}\right)^{-1} J_{hybrid}^{ideal} \begin{pmatrix} I+Q & U+iV \\ U-iV & I-Q \end{pmatrix} \left(J_{hybrid}^{ideal}\right)^\dagger \left(J_{hybrid}^{ideal}\right) \dagger^{-1} J_q^\dagger J_{hybrid,q}^\dagger \\
=& 0.5F J_{hybrid,p} J_p \left(J_{hybrid}^{ideal}\right)^{-1} \begin{pmatrix} I+V & Q+iU \\ Q-iU & I-V \end{pmatrix} \left(J_{hybrid}^{ideal}\right) \dagger^{-1} J_q^\dagger J_{hybrid,q}^\dagger \\
=& 0.5F \tilde{J}_p \begin{pmatrix} I+V & Q+iU \\ Q-iU & I-V \end{pmatrix} \tilde{J}_q^\dagger
\end{aligned} \tag{9}$$

$$\tilde{J}_p = J_{hybrid,p} J_p \left(J_{hybrid}^{ideal}\right)^{-1} \tag{10}$$

$$J_{hybrid}^{ideal} = \begin{pmatrix} 1 & i \\ 1 & -i \end{pmatrix} \tag{11}$$

Thus, we have converted the effect of the non-ideal hybridizer into an effective Jones matrix, which we can calibrate using the non-linear equations as shown earlier.