Digital Image Processing Assignment Week 2

1. In formula $g(x,y) = T[f(x,y)]$, T is the
   a) Transformed image
   b) Transformation vector
   c) Transformation theorem
   d) Transformation function

   Solution: T is transformation function because it transforms f(x, y) into g(x, y).

2. Speech signal has a bandwidth of 4 KHz. If every sample is digitized using 8 bits and the digital speech is to be transmitted over a communication channel, what is the minimum bandwidth requirement of the channel?
   a) 16 Kbps
   b) 32 Kbps
   c) 64 Kbps
   d) 128 Kbps

   Solution: First we need to convert continuous signal into discrete signal by the process of sampling and then digitize using digital bits. For minimum bandwidth we sample at Nyquist rate. Therefore, minimum bandwidth will be 2 x 4 x 8=64Kbps.

3. Compute the Euclidean Distance (D_1), City-block Distance (D_2) and Chessboard distance (D_3) for points p and q, where p and q be (3, 2, 3) and (2, 3, 7) respectively. Give answer in the form (D_1, D_2, D_3).
   a) $(3\sqrt{2}, 3, 4)$
   b) $(3\sqrt{2}, 6, 4)$
   c) $(3\sqrt{2}, 4, 3)$
   d) $(3\sqrt{3}, 2, 3)$

   Solution:

   Euclidean distance (D1) =$\sqrt{(x-u)^2 + (y-v)^2 + (z-w)^2}$ =$3\sqrt{2}$
   City block distance (D2) =|x-u|+|y-v| +|z-w|= 6
   Chessboard distance (D3) =max (|x-s|+|y-t|+|z-w|) = 4

4. Consider the following two images

$$f1 = \begin{pmatrix} 100 & 150 & 70 \\ 50 & 150 & 175 \\ 200 & 50 & 150 \end{pmatrix} \text{ and } f2 = \begin{pmatrix} 50 & 50 & 25 \\ 45 & 55 & 50 \\ 50 & 50 & 75 \end{pmatrix}$$

The addition and subtraction of images are given by $f1 + f2$ and $f1 - f2$. Assume both the images are of the 8-bit integer type.

a. $f1 + f2 = \begin{pmatrix} 51 & 153 & 132 \\ 50 & 70 & 230 \\ 255 & 100 & 225 \end{pmatrix}$ and $f1 - f2 = \begin{pmatrix} -49 & -147 & -118 \\ -40 & -40 & 20 \\ 0 & 0 & 75 \end{pmatrix}$

b. $f1 + f2 = \begin{pmatrix} 51 & 153 & 132 \\ 50 & 70 & 255 \\ 255 & 100 & 225 \end{pmatrix}$ and $f1 - f2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 20 \\ 0 & 0 & 75 \end{pmatrix}$

c. $f1 + f2 = \begin{pmatrix} 150 & 200 & 95 \\ 95 & 205 & 225 \\ 250 & 100 & 225 \end{pmatrix}$ and $f1 - f2 = \begin{pmatrix} 50 & 100 & 45 \\ 5 & 95 & 125 \\ 150 & 0 & 75 \end{pmatrix}$

d. $f1 + f2 = \begin{pmatrix} 51 & 153 & 132 \\ 50 & 70 & 230 \\ 255 & 100 & 225 \end{pmatrix}$ and $f1 - f2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 20 \\ 0 & 0 & 75 \end{pmatrix}$

Solution:

As we performing simple pixel to pixel addition and subtraction therefore,

$$f1 + f2 = \begin{pmatrix} 150 & 200 & 95 \\ 95 & 205 & 225 \\ 250 & 100 & 225 \end{pmatrix} \text{ And } f1 - f2 = \begin{pmatrix} 50 & 100 & 45 \\ 5 & 95 & 125 \\ 150 & 0 & 75 \end{pmatrix}$$

5. Consider an image point $[2,2]^T$ in a continuous image. Rotation of the image point around the origin by 45 ° in anticlockwise direction around origin is given by :
   a) $[0, 2]^T$
   b) $[1, 2.8]^T$
   c) $[1, 1.8]^T$
   d) $[0, 2.8]^T$

   Solution: For this we can use rotation matrix in homogenous system for anticlockwise rotation by an angle of 45 °

$$\begin{pmatrix} \cos 45 & -\sin 45 & 0 \\ \sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{pmatrix} * [2, \quad 2, \quad 1]^T = [0, 2.828, 1]^T$$

Therefore the point after rotation will be $[0, 2.8]^T$

6. In the following figure which of the option is true?

		Q	
	P		

1) $Q \in N_4(P)$ 2) $Q \in N_8(P)$ 3) $Q \in N_D(P)$

a) only 1
b) only 3
c) both 2 and 3
d) all 1,2,3

Solution: By definition of connectivity, P and Q are 8 connected as well as diagonally connected

7. Consider an 3-D point $[2, 1, 1]^T$. Perform a scaling operation (S) in both x-axis and y-axis by 5 units.

a) $[3, 3, 1]^T$
b) $[4, 4, 1]^T$
c) $[10, 5, 1]^T$
d) None of the above

Solution:

Since we are scaling in only x-axis and y-axis by 5 units, therefore after scaling we get, $[10, 5, 1]^T$

8. The output of an image sensor takes values between 0.0 and 10.0. If it is quantized by a uniform quantizer with 256 levels, what will be the transition and reconstruction levels?

a	$t_k = \frac{10(k+1)}{256}, k = 1, \dots, 257$ $r_k = t_k + \frac{5}{256}, k = 1, \dots, 256$
b	$t_k = \frac{10(k)}{256}, k = 1, \dots, 257$ $r_k = t_k + \frac{5}{256}, k = 1, \dots, 256$
c	$t_k = \frac{10(k)}{256}, k = 1, \dots, 257$ $r_k = t_k + \frac{5}{256}, k = 1, \dots, 256$
d	$t_k = \frac{10(k-1)}{256}, k = 1, \dots, 257$ $r_k = t_k + \frac{5}{256}, k = 1, \dots, 256$

Solution: Using the uniform quantizer design, transition and reconstruction levels can be defined as

$$t_k = \frac{10(k-1)}{256}, k = 1, \dots, 257$$

$$r_k = t_k + \frac{5}{256}, k = 1, \dots, 256$$

9. Find the value of logical operation XOR for the binary images A and B (A XOR B). Assume 1 to be foreground and 0 to be background pixels.

Image-A

0	0	0	0	0	0	0	0
0	0	1	1	1	0	0	0
0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Image-B

0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0
0	0	0	0	1	1	1	0
0	0	0	0	1	1	1	0
0	0	0	0	0	0	0	0

a

0	0	0	0	0	0	0	0
0	0	1	0	0	1	1	0
0	0	1	1	0	1	1	0
0	0	0	0	1	1	1	0
0	0	0	0	0	0	0	0

b

0	0	0	0	0	0	0	0
0	0	0	1	1	0	0	0
0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

c

0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0
0	0	1	1	1	1	1	0
0	0	0	0	1	1	1	0
0	0	0	0	0	0	0	0

d

0	0	0	0	0	0	0	0
0	0	1	0	0	1	1	0
0	0	1	1	1	1	1	0
0	0	0	0	1	1	1	0
0	0	0	0	0	0	0	0

Following the truth table for XOR operation

X Y Z
----
0 0 1
0 1 0
1 0 0
1 1 1

XOR between

0	0	0	0	0	0	0	0
0	0	1	1	1	0	0	0
0	0	1	1	1	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

and

0	0	0	0	0	0	0	0
0	0	0	1	1	1	1	0
0	0	0	0	1	1	1	0
0	0	0	0	1	1	1	0
0	0	0	0	0	0	0	0

is

0	0	0	0	0	0	0	0
0	0	1	0	0	1	1	0
0	0	1	1	0	1	1	0
0	0	0	0	1	1	1	0
0	0	0	0	0	0	0	0

10. Find the 2D convolution of the given matrices. (discard padded positions so that final answer will be a 3x3 matrix)

Input

5	8	3
3	2	1
0	9	5

Kernel

-1	-2	-1
0	0	0
1	2	1

a

-8	8	-4
9	1	-5
8	8	4

b

-8	-8	-4
9	1	-5
8	8	4

c

-8	-8	-4
9	2	3
8	8	4

d

4	4	-8
9	1	-5
-8	8	4

Solution:

For convolution, we have to do either time reversal of input image or the kernel. Let's perform time reversal of kernel and move the kernel on the input image.

Input image = $\begin{matrix} 5 & 8 & 3 \\ 3 & 2 & 1 \\ 0 & 9 & 5 \end{matrix}$ and time reversed kernel= $\begin{matrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{matrix}$

Let I be the input image therefore,

I(1,1)=5x0+8x0+3x(-2)+2x(-1)=-8

I(1,2)=5x0+8x0+3x0 +(-1)x3+(-2)x2+(-1)x1=-8,

Hence we get

-8	-8	-4
9	1	-5
8	8	4