Digital Image Processing Assignment Week 4- Solutions

1. What is the tool used in tasks such as resolution enhancement, image in-painting, image warping, etc.?
   a) Sampling
   b) Filters
   c) Interpolation
   d) None of the above

   Solution: - Resolution enhancement, image in-painting, image warping, etc., requires some sort of missing information, therefore we can use image interpolation in these tasks.

2. Which of the following B-spline is not symmetric and hardly used?
   a) Constant
   b) Cubic
   c) Linear
   d) Quadratic

   Solution: - Region of support for quadratic b-spline is 3 and it is asymmetric. Therefore quadratic b-spline is hardly used.

3. What are the desirable properties of an interpolation function?

   I. Finite Region of Support.
   II. Smooth Interpolation.
   III. Shift Invariant

   a) Only I and II.
   b) Only II and III.
   c) All I, II and III.
   d) None of I, II and III.

   Solution:- I, II and III are desirable properties of B-spline functions.

4. _____ B-spline function has maximum region of support.
   a) Quadratic
   b) Cubic
   c) Linear
   d) Constant

   Solution:- Among given options, cubic has maximum region of support of 4.

5. For an image, the type of Interpolation in which the intensity for an unknown location is found by assigning the intensity of the nearest pixel is _____.
   a) Bicubic interpolation
   b) Cubic interpolation
   c) Constant interpolation
   d) Bilinear interpolation

   Solution:- Region of support for constant interpolation is one. So because of this it will look for a single location which is nearest to it.

6. For an image, the type of Interpolation where the intensity of the four neighboring pixels is used to obtain intensity a new location is called _____.
   a) Cubic interpolation
   b) Bicubic interpolation
   c) Nearest neighbor interpolation
   d) Bilinear interpolation

   Solution:- Region of support for linear b-spline is two. But when we go for two dimensional interpolation as in case of images, it will consider two pixels in horizontal direction and two pixels in vertical direction.

7. Zooming of an image can be viewed as _____.
   a) Critical sampling
   b) Under sampling
   c) Over sampling
   d) Nyquist sampling

8. For the one-dimensional function f(x), given below. Using modified cubic interpolation, find out the value at location x = 6.3.

x	1	2	3	4	5	6	7	8	9	10
f(x)	1.5	2.5	3	2.5	3	2.4	2	2.5	1	2.4

   a) 2.43
   b) 1.34
   c) 1.95
   d) 2.30

Using cubic B-Spline function

$$B_{0,4}(t) = \begin{cases} \frac{t^3}{6} & ; \quad 0 \le t < 1 \\ \frac{-3t^3+12t^2-12t+4}{6} & ; \quad 1 \le t < 2 \\ \frac{3t^3-24t^2+60t-44}{6} & ; \quad 2 \le t < 3 \\ \frac{(4-t)^3}{6} & ; \quad 3 \le t < 4 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Since we have to find/interpolate $f(6.3)$, we have to shift this function by 4.3 (peak of bic-cubic lie on 6.3) to get modified B-spline function.

$B_{0,4}(t-4.3)$

$$= \begin{cases} \frac{(t-4.3)^3}{6} & ; \quad 4.3 \le t < 5.3 \\ \frac{-3(t-4.3)^3+12(t-4.3)^2-12(t-4.3)+4}{6} & ; \quad 5.3 \le t < 6.3 \\ \frac{+3(t-4.3)^3-24(t-4.3)^2+60(t-4.3)-44}{6} & ; \quad 6.3 \le t < 7.3 \\ \frac{[4-(t-4.3)]^3}{6} & 7.3 \le t < 8.3 \\ 0 & \text{otherwise.} \end{cases}$$

Substituting the values $t = 5, 6, 7, 8$ in proper intervals, we get the coefficient that has to multiplied with intensity at $t = 5, 6, 7, 8$:

$$B_{4.3,4}(t) = \begin{cases} \frac{(5-4.3)^3}{6} & ; \quad 4.3 \le t < 5.3 \\ \frac{-3(6-4.3)^3 + 12(6-4.3)^2 - 12(6-4.3) + 4}{6} & ; \quad 5.3 \le t < 6.3 \\ \frac{3(7-4.3)^3 - 24(7-4.3)^2 + 60(7-4.3) - 44}{6} & ; \quad 6.3 \le t < 7.3 \\ \frac{[4-(8-4.3)]^3}{6} & ; \quad 7.3 \le t < 8.3 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

$$B_{4.3,4}(t) = \begin{cases} 0.0572 & ; \quad 4.3 \le t \le 5.3 \\ 0.592 & ; \quad 5.3 \le t < 6.3 \\ 0.3482 & ; \quad 6.3 \le t < 7.3 \\ 0.0045 & ; \quad 7.3 \le t \le 8.3 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

Now $f(6.3) = \sum p_i B_{i-4.3,4}(t)$

$= 0.0572 \times 3 + 0.592 \times 2.4 + 0.3482 \times 2 + 0.0045 \times 2.5$

$\simeq 2.30$

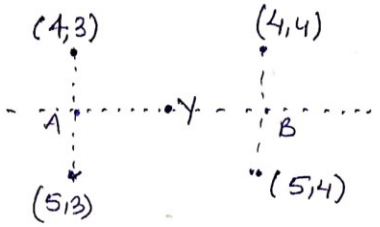
9. A 9 x 9 image is as given below. Using a modified linear interpolation, find out the intensity at location (4.3, 3.6).

Y / x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
0	20	22	15	18	25	32	16	18	20
1	22	25	28	25	35	32	20	17	15
2	45	35	16	23	44	11	18	24	18
3	35	32	26	46	24	13	25	24	25
4	32	45	23	34	45	43	42	24	16
5	34	45	42	36	45	34	25	27	19
6	43	46	47	34	13	13	24	17	20
7	43	54	34	22	21	35	27	43	29
8	32	43	26	25	27	36	23	56	34

a) 32.4
b) 37.6
c) 40.8
d) 33.3

Since we are doing bilinear interpolation, (interpolation in 2-D). So first we do interpolation along one-axis then along other axis. Lets we do interpolation first along Columns, As we using bilinear interpolation, so the spread or point of support would be one pixel to the left and one pixel to the right

Step1:



← Region of support

A → (4.3, 3)

B → (4.3, 4)

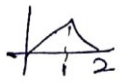
Y → (4.3, 3.6)

first we interpolate A and B, and then in the second step we interpolate Y using A and B.

A (4.3, 3)

Using ~~Bilinear~~ Linear interpolation

$$B_{0,1}(t) = \begin{cases} t & ; \quad 0 \le t < 1 \\ 2-t & ; \quad 1 \le t < 2 \end{cases}$$



For modified linear interpolation we will shift this by 3.3 to have peak at $t = 4.3$.

$$B_{3.3,1}(t) = \begin{cases} t-3.3 & 3.3 \le t < 4.3 \\ 2-(t-3.3) & 4.3 \le t \le 5.3 \\ 0 & \text{otherwise} \end{cases}$$

Now putting t in appropriate ranges, coefficients would be

$$B(t) = \begin{cases} 0.7 & 3.3 \le t < 4.3 \\ 0.3 & 4.3 \le t < 5.3 \\ 0 & \text{otherwise} \end{cases}$$

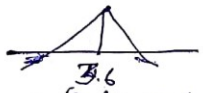
Note :- Similar coefficients while interpolating for point B.

Now $f(4.3, 3) = \sum p_i B_i(t)$

$f(4.3, 3) = 0.7 \times 34 + 0.3 \times 36 = 34.6$

Similarly we get B $\Rightarrow$ $f(4.3, 4) = 0.7 \times 45 + 0.3 \times 45 = 45$.

Now we interpolate along horizontal direction using A and B to get Y.



→ Again we have to shift linear interpolation by 2.6 to have peak at 3.6,

$$B(t) = \begin{cases} t - 2.6 & ; \ 2.6 \le t < 3.6 \\ 2 - (t - 2.6) & ; \ 3.6 \le t < 4.6 \\ 0 & ; \ \text{otherwise} \end{cases}$$

$$B(t) = \begin{cases} 0.4 & ; \ 2.6 \le t < 3.6 \\ 0.6 & ; \ 3.6 \le t < 4.6 \\ 0 & ; \ \text{otherwise} \end{cases}$$

$f(4.3, 3.6) = 34.6 \times 0.4 + 45 \times 0.6$

≈ 40.8

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10. Process of using known intensity values to estimate unknown intensity value is called _____.

a) Interpolation
b) Sampling
c) Interchange
d) None of these

Solution:- Process of using known intensity values to estimate unknown intensity value is called Interpolation.