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EXCEL in MATHEMATICS

TEACHERS' MANUAL

Book 10

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Preface

This **"Teachers' Manual of Vedanta EXCEL in MATHEMATICS BOOKS-9 and 10"** is prepared for teachers to aiming at assistance in pedagogical teaching learning activities. Its special focus intends to fulfillment the motto of text books EXCEL in MATHEMATICS approved by the Government of Nepal, Ministry of Education, CDC, Sanothimi, Bhaktapur.

EXCEL in MATHEMATICS has incorporated the applied constructivism which focuses on collaborative learning so that the learners actively participate in the learning process and construct the new knowledge. The project works given at the end of each chapter provides the ideas to connect mathematics to the real life situations. Similarly, the text book contains enough exercises for uplifting critical thinking and creation as per the optimum goal of Bloom's Taxonomy. The objective questions at the end of each area of subject content strengthen the students' knowledge level.

This manual helps the teachers to have the chapter-wise learning competencies, learning outcomes and level-wise learning objectives. Also, it helps the teachers in selecting the effective instructional materials, adopting the productive teaching activities, solving the creative problems and getting more extra objective and subjective questions which can be useful for the summative assessments.

Grateful thanks are due to all Mathematics Teachers throughout the country who encouraged and provided the feedback to me in order to prepare the new series.

Last but not least, any constructive comments, suggestions and criticisms from the teachers for the further improvements of the manual will be highly appreciated.

Authors

Contents (Class 10)

Unit 1 Set

Allocated teaching periods	8

Competency

- To find the relation between the sets and solve the related problems by demonstrating the relations on the basis of the properties of the sets

Learning Outcomes

- To solve the verbal problems related to the cardinality of sets by using Venn diagram and solve the behavioural problems by using the properties of relations of sets.

Level-wise learning objectives

S.N.	LEVELS	OBJECTIVES
1.	Knowledge (K)	- To define a set - To tell the types of sets - To state the relations between sets - To list the operations on sets - To define cardinality of sets - To tell the formulae involving two/three sets
2.	Understanding (U)	- To find the cardinality of a set. - To identify the set notation with special terminologies - To write the word problems based on cardinality relations in set notations
3.	Application (A)	- To solve the verbal problems on operations (union, intersection, difference and complement) of sets by using Venn-diagram - To solve the verbal problems on operations of sets by using formulae
4.	High Ability (HA)	- To relate the problem related to set with other areas of learning like percentage, ratio and so on. - To link various real life/ contemporary problems with sets and solve

Required Teaching Materials/ Resources

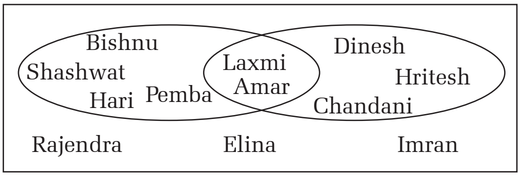
Definitions and formulae in colourful chart-paper, scissors, cello tape, different coloured markers, highlighter, models of Venn-diagrams etc.

A. Problems including two sets

Pre-knowledge: Check the pre-knowledge on cardinality of sets, relation of sets, operations of sets and Venn-diagram

Teaching Activities

1. Present models of Venn-diagrams and recall the relations of sets as warm up.
2. Write any two disjoint sets on the board and tell the students to list the elements of A, B and $A \cup B$ then n (A), n (B),n ($A \cup B$) and n (A) + n (B). Then draw the conclusion from the examples as n ($A \cup B$) = n (A) + n (B).
3. Select some students and ask whether they like coffee or tea. Denote the set of students who like coffee by C, students who like tea by T and who do not like both the drinks by $\overline{C \cup T}$. List the members of set and discuss about the relation of the sets (overlapping or disjoint). Then find the cardinality of the sets.



Use Venn diagram to show the relation of sets C and T.

For example:

If U = {Bishnu, Laxmi, Elina, Chandani, Shashwat, Rajendra, Imran, Hari, Pemba, Hritesh, Amar, Dinesh}, C = {Laxmi, Bishnu, Amar, Shashwat, Hari, Pemba}, T = {Amar, Dinesh, Laxmi, Hritesh, Chandani} and $\overline{C \cup T}$= {Rajendra, Elina, Imran}

Tell the students to find $C \cup T$, $C \cap T$, n (C), n (T), n ($C \cup T$), n ($C \cap T$) and n($\overline{C \cup T}$) from Venn-diagram then tell them to try to establish the relations of n (C), n (T), n ($C \cap T$) and n ($C \cup T$).

Finally, conclude that $C \cap T$ = {Laxmi, Amar}, $C \cup T$ = {Laxmi, Bishnu, Amar, Shashwat, Hari, Pemba, Dinesh, Hritesh, Chandani} $\therefore$ n ($C \cap T$) = 2, n ($C \cup T$) = 9.

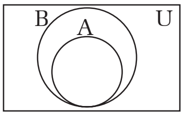
Ask to students why n ($C \cup T$) is not equal to n (C) + n (T) in this case?

Clarify that n ($C \cup T$) = n (C) + n (T) – n ($C \cap T$) by similar real life examples of overlapping sets.

Also, establish the relation of n (U), n ($C \cup T$) and n ($C \cap T$).

1. List the following formula under discussion through examples in Venn-diagrams.

Case-I: When A is a subset of B

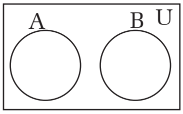


(i) n ($A \cap B$) = A

(ii) n ($A \cup B$) = n (B)

(iii) n_o (B) = n (B) – n (A)

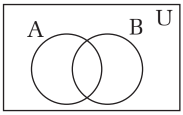
Case- II: When A and B are disjoint sets.



(i) n ($A \cap B$) = 0

(ii) n ($A \cup B$) = n (A) + n (B)

Case- III: When A and B are overlapping sets



(i) n ($A \cup B$) = n (A) + n (B) – n ($A \cap B$)

(ii) n ($A \cap B$) = n (A) + n (B) – n ($A \cup B$)

(iii) n (only A) = n_o (A) = n (A) – n ($A \cap B$)

(iv) $n\text{ (only B)} = n_o(B) = n(A) - n(A\cap B)$

(v) $n\text{ (only one) or } n\text{ (exactly one)} = n_o(A) + n_o(B)$

(vi) $n(A\cup B) = n_o(A) + n_o(B) + n(A\cap B)$

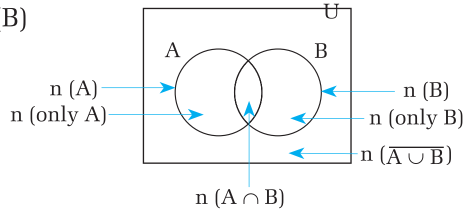
(vii) $n(A\cup B) = n_o(A) + n(B) = n(A) + n_o(B)$

(viii) $n(\overline{A\cup B}) = n(U) - n(A\cup B)$

(ix) $n(U) = n(A) + n(B) - n(A\cap B) + n(\overline{A\cup B})$

(x) $n(U) = n_o(A) + n_o(B) + n(A\cap B) + n(\overline{A\cup B})$

(xi) $n\text{ (at most one)} = n(\overline{A\cup B}) = n_o(A) + n_o(B) + n(\overline{A\cup B})$



2. Explain the useful terminologies in solving verbal problems. For examples;

(i) Number of people who like both tea and coffee / who like tea as well as coffee = $n(T\cap C)$

(ii) Number of people who like at least one drinks / either tea or coffee = $n(T\cup C)$

(iii) Number of people who like only tea = $n_o(T)$ and only coffee = $n_o(C)$

(iv) Number of people who like only one drink = $n_o(T) + n_o(C)$ or $n(C\cup T) - n(\overline{C\cap T})$

(v) Number of people who like at most one drink/ don't like both = $n(\overline{C\cap T})$

(vi) No. of people who don't like any drinks / drink neither tea nor coffee = $n(\overline{C\cup T})$

Note:

1. If the information are given in percentage, consider that n (U) = 100 or x.
2. If the data are given in fraction then suppose that n (U) = x.
3. If each people participates in at least one activity then $n(U) = n(A\cup B)$

Note:

1. Solving the verbal problems related to sets using Venn-diagram is more clear and effective rather than using formula. However, the given problems can be solved by using only formula.

Solution of selected questions from Vedanta Excel in Mathematics

1. ***In a group of students, the ratio of number of students who liked music and sports is 9: 7. Out of which 25 liked both the activities, 20 liked music only and 15 liked none of the activities.***

(i) Represent the above information in a Venn-diagram.

(ii) Find the total number of students in the group.

Solution:

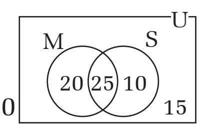
Let M and S denote the sets of students who liked music and sports respectively.

Then $n(M) = 9x$, $n(S) = 7x$ (say), $n(M\cap S) = 25$, $n_o(M) = 20$, $n(\overline{M\cup S}) = 15$

Now, $n_o(M) = n(M) - n(M\cap S)$ or, $20 = 9x - 25$ $\therefore x = 5$

$\therefore n(M) = 9x = 45$, $n(S) = 7x = 35$

(i) Illustrating the above information in the Venn-diagram



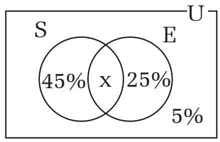
(ii) $n(U) = n(M) + n(S) - n(M\cap S) + n(\overline{M\cup S}) = 45 + 35 - 25 + 15 = 70$

Hence, the total number of students in the group was 70.

2. ***In an examination 45% students passed in Science only, 25% passed in English only and 5% students failed in both subjects. If 200 students passed in English, find the total number of students by using a Venn-diagram.***

Solution:

Let S and E denote the sets of students who passed in Science and English respectively.
Then $n(U) = 100\%$, $n_o(S) = 45\%$, $n_o(E) = 25\%$, $n(\overline{S \cup E}) = 5\%$ and $n(E) = 200$
Let $n(S \cap E) = x\%$
Now, representing the above data in a Venn-diagram

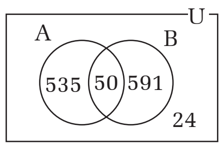


Again,
From the Venn-diagram,
$n(U) = n_o(S) + n_o(E) + n(S \cap E) + n(\overline{S \cup E})$
or, $100\% = 45\% + 25\% + x\% + 5\%$ $\quad \therefore x\% = 25\%$
Also, $n(E) = n_o(E) + n(S \cap E) = 25\% + 25\% = 50\%$
Let the total number of students $n(U) = y$
Then, by question, $n(E) = 200$ or, 50% of $y = 200$ $\quad \therefore y = 400$
Hence, the total number of students was 400.

3. ***There are 1200 students in a school. They are allowed to cast vote either only for X or for Y as their school prefect. 50 of them cast vote for both X and Y and 24 didn't cast the vote. The candidate Y won the election with majority of 56 more votes than X.***
   (i) How many students cast the vote?
   (ii) How many valid votes are received by X candidate?
   (iii) Show the result in Venn-diagram.

Solution:

Let A and B denote the sets of students who cast vote the candidates X and Y respectively.
Then, $n(U) = 1200$, $n(A \cap B) = 50$, $n(\overline{A \cup B}) = 24$, $n_o(B) = n_o(A) + 56$
Now,
(i) $n(A \cup B) = n(U) - n(\overline{A \cup B}) = 1200 - 24 = 1176$.
Hence, 1176 students cast the vote.
(ii) $n(U) = n_o(A) + n_o(B) + n(A \cap B) + n(\overline{A \cup B})$
or, $1200 = n_o(A) + n_o(A) + 56 + 50 + 24$ $\therefore n_o(A) = 535$.
Hence, the valid votes received by X candidates are 535.
(iii) Showing the above information in the Venn-diagram



4. ***Due to the heavy rain fall during a few monsoon days, 140 households were victimized throughout the country. In the first phase, Nepal government decided to provide the support of either food, shelter or both to a few victimized households. 80 households got food support, 70 got shelter and 50 household got the support of food and shelter both. The government has managed the budget of Rs 10,000 per household for food, Rs 25,000 per household for shelter and Rs 35,000 per household for food and shelter.***
   (i) Calculate the amount of budget for food only.
   (ii) Calculate the amount of budget for shelter only.
   (iii) Calculate the total amount of budget allocated.
   (iv) How many households were remained to get support in the first phase?

Solution:

Let F and S denote the sets of households who got food support and shelter respectively.

Then, n (U) = 140, n (F) = 80, n (S) = 70, n $(F \cap S)$ = 50

Now,

(i) $n_o(F) = n(F) - n(F \cap S) = 80 - 50 = 30$.

The number of households who got food support only is 30.

Since, the budget for food support per household is Rs 10,000.

Thus, the amount of budget for food only = 30 × Rs 10,000 = Rs 3,00,000.

(ii) $n_o(S) = n(S) - n(F \cap S) = 70 - 50 = 20$.

The number of households who got shelter only is 20.

Since, the budget for shelter per household is Rs 25,000.

Thus, the amount of budget for shelter only = 20 × Rs 25,000 = Rs 5,00,000.

(iii) The number of households who got food and shelter both is 50.

Since, the budget for food and shelter per household is Rs 35,000.

So, the amount of budget for food and shelter both = 50 × Rs 35,000 = Rs 17,50,000

Thus, the total amount of budget allocated = Rs 3,00,000 + Rs 5,00,000 + Rs 17,50,000

= Rs 25,50,000

(iv) $n(U) = n(F) + n(S) - n(F \cap S) + n(\overline{F \cup S})$ or, $140 = 80 + 70 - 50 + n(\overline{F \cup S})$

$\therefore n(\overline{F \cup S}) = 40$.

Hence, 40 households were remained to get support in the first phase.

5. ***In a class the ratio of number of students who passed maths but not science and those who passed science but not maths is 3:5. Also, the ratio of the number of students who passed both the subjects and those who failed both the subjects is 2:1. If 80 students passed only one subject and 100 passed at least one subjects, find,***

(i) The total number of students.

(ii) Draw a Venn-diagram to show all result.

Solution:

Let M and S denote the sets of students who passed maths and science respectively.

Then, $n_o(M) + n_o(S) = 80$, $n(M \cup S) = 100$

Let $n_o(M) = 3x$ and $n_o(S) = 5x$.

Then $n_o(M) + n_o(S) = 80$ or, $3x + 5x = 80$ or, $8x = 80$ $\therefore x = 10$

$\therefore n_o(M) = 3x = 3 \times 10 = 30$ and $n_o(S) = 5x = 5 \times 10 = 50$

Also, let $n(M \cap S) = 2y$ and $n(\overline{M \cup S}) = y$

Now, $n(M \cup S) = n_o(M) + n_o(S) + n(M \cap S)$ or, $100 = 30 + 50 + 2y$ $\therefore y = 10$

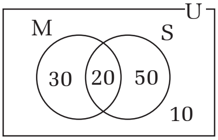
$\therefore n(M \cap S) = 2y = 2 \times 10 = 20$ and $n(\overline{M \cup S}) = 10$

(i) $n(U) = n_o(M) + n_o(S) + n(M \cap S) + n(\overline{M \cup S})$

$= 30 + 50 + 10 + 20 = 110$

Hence, the total number of students is 110.

(ii) Drawing the Venn-diagram to show above result.



Extra Questions

1. In class X Jamboree of a school, students performed their various talent shows. Out of 80 parents participating in the Jamboree; 34 preferred dance but not comedy shows, 23 preferred comedy but not dance shows and 13 parents preferred the shows other than these.
   (i) How many parents preferred both the shows?
   (ii) How many parents preferred at most one show?
   (iii) Show the above information in a Venn-diagram. [Ans: (i) 10 (ii) 70]
2. In a group of 125 students, the ratio of student who like tea to the number of students who like coffee is 4:5.If 20 of them like both the drinks and 10 of them like none of them then find:
   (i) How many of them like only one drink?
   (ii) Find the ratio of number of students who like and don't like both the drinks.
   (iii) Draw a Venn-diagram to represent the above information. [Ans: (i) 95 (ii) 2:1]
3. In a school 32 teachers like either milk or curd or both. The ratio of number of teacher who like milk to the number of teacher who like curd is 3:2 and 8 teachers like both milk and curd. Find:
   (i) How many teachers like milk only? (ii) How many teachers like curd only?
   (iii) Show the above information in a Venn-diagram. [Ans: (i) 16 (ii) 8]
4. In a group of 50 students 20 like only Math and 15 like only science. If the member of students who do not like any of the two subjects is double of the number of students who like both subjects, find the number of students who like at most one subject by using a Venn-diagram. [Ans: 45]
5. Out of 200 students in a class, 150 like football and 120 like cricket. If the number of students who like only cricket is one-third of the number of students who like football only,, then using a Venn-diagram, find the number of students who like:
   (i) Both games (ii) Football only (iii) None of these games [Ans: (i) 105 (ii) 45 (iii) 35]

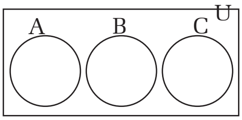
B. Problems Including Three Sets

Pre-knowledge: Cardinality relation of union of two sets, verbal problems involving two sets, set operations on three sets, Venn-diagrams of three sets

Teaching Activities

1. Create a sound learning environment through a real life problem based on three sets by the use of Venn-diagram.
2. By visualization of relations of sets in Venn-diagrams, clarify the following cardinality relations of union of three sets.

Case-I: When A, B and C are disjoint sets.



(i) $n(A \cap B \cap C) = 0$

(ii) $n(A \cup B \cup C) = n(A) + n(B) + n(C)$

Case- II: When A, B and C are overlapping subsets of a universal set U.

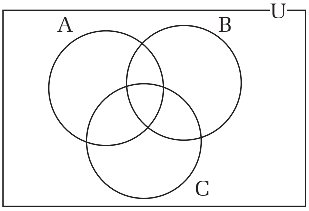
(i) $n(A B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

(ii) $n(A \cup B \cup C) = n_o(A) + n_o(B) + n_o(C) + n_o(A \cap B) + n_o(B \cap C) + n_o(C \cap A) + n(A \cap B \cap C)$

(iii) $n(A \cup B \cup C) = n_o(A) + n(A \cup B) = n_o(B) + n(B \cup C) = n_o(C) + n(C \cup A)$

(iv) $n_o(A) = n(A) - n(A \cap B) - n(C \cap A) + n(A \cap B \cap C)$

(v) $n_o(B) = n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$



(vi) $n_o(C) = n(C) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

(vii) $n_o(A) = n(A) - n_o(A \cap B) - n_o(C \cap A) - n(A \cap B \cap C)$

(viii) $n_o(B) = n(B) - n_o(A \cap B) - n_o(B \cap C) - n(A \cap B \cap C)$

(ix) $n_o(C) = n(C) - n_o(B \cap C) - n_o(C \cap A) - n(A \cap B \cap C)$

(x) $n(\overline{A \cup B \cup C}) = n(U) - n(A \cup B \cup C)$

3. Explain the following useful terminologies in solving verbal problems.

(i) Number of people who like all three activities = $n(A \cap B \cap C)$

(ii) Number of people who like at least one activity/either A or B or C = $n(A \cup B \cup C)$

(iii) Number of people who like only/exactly one activity = $n_o(A) + n_o(B) + n_o(C)$

(iv) Number of people who like only/exactly two activities

$= n_o(A \cap B) + n_o(B \cap C) + n_o(C \cap A)$

(v) Number of people who like at most one activity = $n_o(A) + n_o(B) + n_o(C) + n(\overline{A \cup B \cup C})$

(vi) Number of people who like at most two activities = $n(\overline{A \cap B \cap C})$

(vii) Number of people who don't like any activity / neither A nor B nor C = $n(\overline{A \cup B \cup C})$

Solution of selected questions from Vedanta Excel in Mathematics

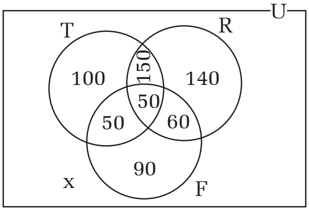
1. In a survey of 700 tourists who arrived in Nepal during 'Visit Nepal 2020', 350 preferred to go trekking, 400 preferred rafting and 250 preferred forest safari. Likewise 200 preferred trekking and rafting, 110 preferred rafting and forest safari and 100 preferred forest safari and trekking. If 50 tourists preferred all these activities, by drawing a Venn-diagram find how many tourists preferred none of these activities?

Solution

Let T, R and F denote the sets of tourists who preferred trekking, rafting and forest safari respectively.

Then, $n(U) = 700$, $n(T) = 350$, $n(R) = 400$, $n(F) = 250$, $n(T \cap R) = 200$, $n(R \cap F) = 110$, $n(F \cap T) = 100$ and $n(T \cap R \cap F) = 50$, $n(\overline{T \cup R \cup F}) = ?$

Let the number of tourists who preferred none of these activities $n(\overline{T \cup R \cup F})$ be x.



Now, illustrating the above information in a Venn-diagram

From Venn-diagram,

$n(U) = 100 + 140 + 90 + 150 + 60 + 50 + 50 + x$

or, $700 = 640 + x \therefore x = 60$

Thus, 60 tourists who preferred none of these activities

2. Of the total students in an examination, 40% students passed in Mathematics, 45% in Science and 55% in Nepali. 10% students passed in both Mathematics and

Science, 20% passed in Science and Nepali and 15% in Nepali and Mathematics.

(i) ***Find the percentage of students who passed in all the three subjects?***

(ii) ***Show the above information in a Venn-diagram.***

Solution

Let M, S and N denote the sets of students who passed in Mathematics, Science and Nepali respectively.

Then, $n(U) = 100\% = n(M \cup S \cup N)$, $n(M) = 40\%$, $n(S) = 45\%$, $n(N) = 55\%$, $n(M \cap S) = 10\%$, $n(S \cap N) = 20\%$, $n(N \cap M) = 15\%$ and $n(M \cap S \cap N) = ?$

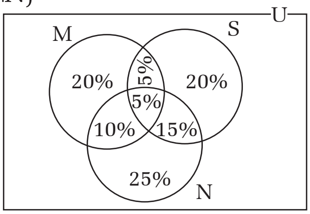
(i) Now, $n(M \cup S \cup N) = n(M) + n(S) + n(N) - n(M \cap S) - n(S \cap N) - n(N \cap M) + n(M \cap S \cap N)$

or, $100\% = 40\% + 45\% + 55\% - 10\% - 20\% - 15\% + n(M \cap S \cap N)$

$\therefore n(M \cap S \cap N) = 5\%$

Hence, the 5% students passed in all the three subjects.

(ii) Illustrating the above information in the Venn-diagram



3. ***In a survey, people were asked what types of movies they like. It was found that 75 liked Nepali movie, 60 liked English, 40 liked Hindi, 35 liked Nepali and English, 30 liked Nepali and Hindi, 20 liked Hindi and English, 10 liked all three and 25 people were found not interested in any types of movies.***

(i) ***How many people did not like only Hindi films?***

(ii) ***How many people did not like only Nepali or Hindi films?***

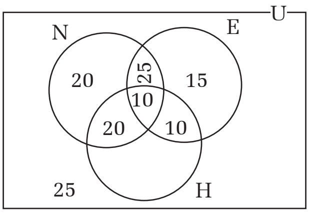
Solution:

Let N, H and E denote the sets of people who liked Nepali, Hindi and English movies respectively.

Then, $n(N) = 75$, $n(E) = 60$, $n(H) = 40$, $n(N \cap E) = 35$, $n(N \cap H) = 30$, $n(H \cap E) = 20$, $n(N \cap H \cap E) = 10$ and $n(\overline{N \cap H \cap E})$

Now,

Illustration in Venn-diagram,



From Venn-diagram,

(i) No. of people who did not like only Hindi films $= n_o(N) + n_o(E) + n(N \cap E)$

$= 20 + 15 + 25 = 60$

(ii) No. of people did not like only Nepali or Hindi films $= n_o(E) = 15$

4. ***In a group of students, 20 study Economics, 18 study History, 21 study Science, 7 study Economic only, 10 study Science only, 6 study Economics and Science only and 3 study Science and History only.***

(i) ***Represent the above information in a Venn-diagram.***

(ii) ***How many study all the subjects?***

(iii) ***How many students are there altogether?***

Solution

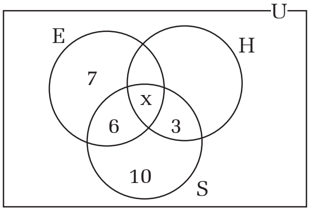
Let E, H and S denote the sets of students who study Economics, History and Science respectively.

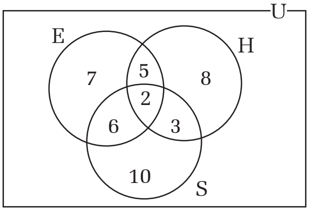
Then, n (E) = 20, n (H) = 18, n (S) = 21, n_o (E) = 7, n_o(S) = 10, n_o(E∩S) = 6 and n_o (S∩H) = 3

n (E∩H∩S) =?, n (U) = ?

Let the number of students who study all the subjects n (E∩H∩S) be x.

(i) Representing the above information in a Venn-diagram.





(ii) From Venn-diagram, n(S) = 10 + 6 + 3 + x or, x = 2

∴ The number of students who study all the subjects n (EH∩S) = x = 2

(ii) Also, the number of students who study only Economics and History

= n_o (E∩H) = n (E) – n_o (E) - n_o (E∩S) - n (E∩H∩S) = 20 – 7 – 6 – 2 = 5

Again, the number of students who study only History = n_o (H)

= n (H) – n_o (E∩H) - n_o (H∩S) - n (E∩H∩S) = 18 – 5 – 3 – 2 = 8

Hence, total number of students = n (U) = 7+8+10+5+3+6+2 = 41

Extra Questions

1. In a survey of 160 farmers of a village, it was found that 60 farmers has buffalo farming, 70 have cow farming and 80 have goat farming. Similarly, 25 have buffalo as well as cow farming, 30 have cow as well as goat farming and 15 have goat as well as buffalo farming. If 10 farmers have all these farming then by drawing a Venn-diagram find:
   (i) How many farmers have only one of these farming?
   (ii) How many farmers have exactly two of these farming?
   (iii) How many farmers have other than these farming [Ans: (i) 100, (ii) 40 (iii) 10]
2. In a survey of 2000 Indian tourists who arrived in Nepal, 65% wished to visit Pashupati, 50% wished to visited Chandragiri and 45% wished to visit Manakamana. Similarly, 35% wished to visit Pashupati and Chandragiri, 25% to Chandragiri and Manakamana and 20% to Manakamana and Pashupati. If 5% wished to visit none of these places, find the number of tourists who wished to visit all these three places. Also, show the above data in a Venn-diagram. [Ans: 300]
3. Among examinees in an examination, 40% obtained A^+ grade in Science, 45% in Math and 55% in Social Studies. Similarly, 10% obtained A^+ grade in Math and Science, 20% in Science and Social Studies and 15% in Social Studies and Math. If every student obtained A^+ grade in at least one subject.
   (i) If 300 students were surveyed, how many students did obtain A^+ grade in only one subject?
   (ii) Find the percentage of students who obtained A^+ in all the three subjects?
   (iii) Show the above information in a Venn-diagram. [Ans: 5%, (ii) 195]

Unit 2 Tax and Money Exchange

Allocated teaching periods	7

Competency

- To solve and test the behavioural arithmetic problems on daily life activities using the mathematical instruction and logical thought.

Learning Outcome

- To collect and solve the problems of value added tax and money exchange in daily life activities.

Level-wise learning objectives

S.N.	LEVELS	OBJECTIVES
1.	Knowledge (K)	- To define value added tax - To tell the current rate of VAT used in Nepal. - To state the formula for finding the VAT amount when selling price and VAT rate are given. - To relate the marked price, discount amount, VAT amount and selling price including VAT. - To state buying/selling rate of money exchange. - To recall revaluation and devaluation of currency.
2.	Understanding (U)	- To describe the VAT exempted goods or services. - To compare the price excluding and including VAT - To find the rate of discount/VAT - To covert the currencies of different countries. - To find the money with commission
3.	Application (A)	- To solve the problems on VAT. - To find profit or loss in money exchange when devaluation/ revaluation of currency is given - To solve the real life problems based on money exchange.
4.	High Ability (HA)	- To mathematize the contextual probems on VAT and solve them.

Required Teaching Materials/ Resources

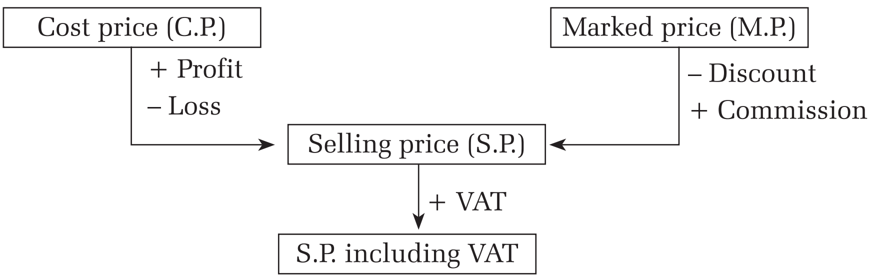
VAT bills, money exchange rates published in newspaper, chart paper, sign pen, etc.

A. Value Added Tax (VAT)

Pre-knowledge: cost price (C.P.), marked price (M.P.), selling price (S.P.) , income tax etc.

Teaching strategies

1. Recall cost price (C.P.), marked price (M.P.), selling price (S.P.) and income tax as warm up.
2. Paste/show the different types of taxes in the colourful chart paper and explain with appropriate examples.
3. List the following formulae under discussion
   - (i) Discount amount = M.P. – S. P.
   - (ii) Discount amount = Discount % of M.P.
   - (iii) Rate of discount = $\dfrac{\text{Discount amount}}{\text{M.P.}} \times 100\%$
   - (iv) S.P. = M.P. – Discount amount
   - (v) S.P. = M.P. – Discount% of M.P. = M.P. (1 – Discount %)
   - (vi) S.P. = M.P. $(1 - D_1\%)(1 - D_2\%)$ when two successive discount rates $D_1\%$ and D_2 % are given.
   - (vii) VAT amount = S.P. including VAT – S. P. excluding VAT
   - (viii) VAT amount = VAT% of S.P.
   - (ix) Rate of VAT = $\dfrac{\text{VAT amount}}{\text{S.P.}} \times 100\%$
   - (x) S.P. with VAT = S.P. + VAT amount
   - (xi) S.P. with VAT = S.P. + VAT% of S.P. = S.P. (1 + VAT %)
4. Discuss on the local tax, transportation cost and profit /loss
5. Make clear about the concepts about value added tax (VAT in various stages of sales.
6. Make clear about value added tax VAT) with flow chat as shown below.



7. Focus on group work/project work.

Note: The tax levied on purchase of goods or service is called value added tax (VAT).

Project Work

Divide the students into the groups of 3 members each and give the work:

- To collect different types of goods purchasing bills and prepare the report about marked price, discount rate/ discount amount, VAT rate/ VAT amount and selling price including VAT as mentioned in the bill. Then call to present in the class.
- To collect the electricity bill, water bill, telephone bill, T.V/ internet recharge bills of house and make a report on rebate, discount, fine, VAT, Service charge etc. and call the group to present in the class.

Solution of selected questions from Vedanta Excel in Mathematics

1. ***A trader bought some electric blenders for Rs 4,000 per piece. He sold each blender to customer for Rs 4,332 with 14% VAT. Find his profit or loss.***

Solution:

Here, Cost price (C.P.) of each blender = Rs 4,000, Rate of VAT (R) = 14%,
S.P. with VAT = Rs 4,332

Let selling price (S.P.) of the blender without VAT be Rs x.

Now, S.P. with VAT = S.P. + VAT % of S.P. or, Rs 4, 332 = x + 14% of x $\therefore$x = 3800

S.P. of the blender = Rs 3800. Since C.P.> S.P

Thus, his loss = C.P.-S.P = Rs 4000-Rs 3800= Rs 200

2. ***A supplier purchased a photocopy machine for Rs 2,00,000 and spent Rs 1,500 for transportation and Rs 500 local tax. He/she sold it to a customer at 10% profit. At what price did the customer purchase the machine with 13% VAT?***

Solution

Here, <u>For supplier:</u>

Cost price (C.P.) of photocopy machine = Rs 2,00,000

Cost price (C.P.) of photocopy machine with transportation cost and local tax

= Rs 2,00,000 + Rs 1,500 + Rs 500 = Rs 2,02,000

Selling price (S.P.) of the photocopy machine at 10%

profit = Rs 2,02,000 + 10% of Rs 2,02,000 = Rs 222200

<u>For customer:</u>

C.P. of the photocopy machine with 13% VAT = Rs222200 + 13% of Rs 222200 = Rs 2,51,086

Hence, the customer purchased the machine at Rs 2,51,086

3. ***Mr. Jha purchased a bicycle costing Rs 5,600 from a dealer at 5% discount and sold at a profit of 10%. If he had sold it at 5% discount, find its marked price.***

Solution:

For dealer, M.P. of the cycle = Rs 5,600 and discount rate = 5%

Now, S.P. = M.P. – D% of M.P. = Rs 5,600 – 5% of Rs 5,600 = Rs 5,320

For Mr. Jha; C.P. = Rs 5,320, profit rate = 10%, discount rate = 5%

Now, S.P. = C.P. + P% of C.P. = Rs 5,320 + 10% of Rs 5,320 = Rs 5,852

Let the marked price of the cycle be Rs x.

Then, S.P. = M.P. – D% of M.P. = Rs x – 5% of Rs x = Rs 0.95 x

But, S.P. = Rs 5, 852 or, 0.95x = Rs 5,852 $\therefore$x = 6,160

Hence, Mr. Jha marked Rs 6,160 as the price of bicycle.

4. ***A grocer fixed the price of his goods 25% above the cost price. If he/she sold a box of noodles allowing 5% discount, find his/her profit percent.***

Solution:

Let the cost price of the box of noodles (C.P.) be Rs x. Then, M.P. = Rs x + 25% of Rs x = Rs 1.25 x

S.P. = MP – D% of MP = Rs 1.25x – 5% of Rs 1.25x = Rs 1.1875x

Now, profit percent = $\frac{\text{S.P.} - \text{C.P.}}{\text{C.P.}} \times 100\% = \frac{1.1875x - x}{x} \times 100\% = 18.75\%$

5. ***A trader fixed the price of cosmetic items 30% above the cost price. When he/she sold and item at 25% discount, there was a loss of Rs 15. Find the cost price and marked price of the item.***

Solution:

Let the cost price of the cosmetic item (C.P.) be Rs x.

Then, M.P. = Rs x + 30% of Rs x = Rs 1.3 x

S.P. = MP – D% of MP = Rs 1.3x – 25% of Rs 1.3x = Rs 0.975x

According to question

Loss amount = C.P. – S.P. or, Rs 15 = x – 0.975x or, 15 = 0.025x $\therefore$x = 600

Hence, the required cost price of the item is Rs 600 and its marked price is

Rs 1.3×600 = Rs 780

6. ***A shopkeeper sold an article at 20% discount and made a loss of Rs 90. If he had sold it at 5% discount, he would have gained Rs 90. Find the cost price and the marked price of the article.***

Solution:

Let the marked price of the article (M.P.) be Rs x.

Now, S.P. in 20% discount = M.P. – D% of M.P. = x – 20% of x = Rs 0.8x

$\therefore$C.P. = S.P. + loss = Rs (0.8x + 90) ... (i)

Again, S.P. in 5% discount = M.P. – D% of M.P. = x – 5% of x = Rs 0.95x

$\therefore$C.P. = S.P. – profit = Rs (0.95x - 90) ... (ii)

Equating equations (i) and (ii), we get

0.8x + 90 = 0.95x – 90 or, 0.15x = 180 $\therefore$x = 1200

Again, putting the value of x in equation (i), we get

C.P. = Rs (0.8×1200 + 90) = Rs 1050

Hence, the required cost price of the article is Rs 1050 and marked price is Rs 1200.

7. ***An article after allowing a discount of 20% on its marked price was sold at a gain of 20%. Had it been sold after allowing 25% discount, there would have been a gain of Rs 125. Find the marked price of the article.***

Solution:

Let the marked price of the article (M.P.) be Rs x.

Now, S.P. in 20% discount = M.P. – D% of M.P. = x – 20% of x = Rs 0.8x

$$\therefore \text{C.P.} = \frac{100\ \text{S.P.}}{100 + \text{profit\%}} = \frac{100 \times 0.8x}{100 + 20} = \frac{80x}{120} = \text{Rs } \frac{2x}{3} \quad \dots \text{(i)}$$

Again, S.P. in 25% discount = M.P. – D% of M.P. = x – 25% of x = Rs $\frac{2x}{3}$

$$\therefore \text{C.P.} = \text{S.P.} - \text{profit} = \text{Rs}\left(\frac{3x}{3} - 125\right) = \text{Rs } \frac{3x - 500}{4} \quad \dots \text{(ii)}$$

Equating equations (i) and (ii), we get

$\frac{2x}{3} = \frac{3x - 500}{4}$ or, 8x = 9x – 1500 $\therefore$ x = 1500

Hence, the required marked price of the article is Rs 1500.

8. ***A shopkeeper purchased a bicycle fro Rs 5,000 and the marked its price a certain percent above the cost price. Then he sold it at 10% discount. If a customer paid Rs 6,356.25 with 13% VAT to buy it, how many percentage is the marked price above the cost price?***

Solution:

Let the selling price of the bicycle (S.P.) be Rs x.

Now, S.P. with VAT = S.P. + VAT% of S.P. or, Rs 6,356.25 = x + 13% of x $\therefore$x = Rs 5625

Let the marked price of the bicycle (M.P.) be Rs y.

Also, S.P. = M.P. – D% of M.P. or, Rs 5625 = y – 10% of y $\therefore$y = Rs 6250

Again, difference between M.P. and C.P. = Rs 6,250 – Rs 5,000 = Rs 1,250

Hence, the marked price is above the cost price by $\dfrac{\text{Difference}}{\text{C.P.}}\times 100\%$

$$= \frac{\text{Rs } 1250}{\text{Rs } 5{,}000}\times 100\% = 25\%$$

9. ***After allowing 20% discount on the marked price of a mobile, 15% VAT was levied and sold it. If the difference between the selling price with VAT and selling price after discount is Rs 1,800, find the marked price of the mobile.***

Solution:

Let the marked price of the mobile be Rs x

Then, S.P. after 20% discount = M.P. – D% of M.P = x – 20% of x = Rs 0.8x

Again, S.P. with 15% VAT = S.P. + VAT% of S.P. = Rs0.8x + 15% of Rs 0.8x = 0.92x

According to the question,

S.P. with VAT – S.P. after discount = Rs 1,800 or, 0.92x – 0.8x = Rs 1,800 $\therefore$x = Rs 15,000

Hence, the marked price of the mobile is Rs15,000.

10. ***When an article was sold at a discount of 10%, a customer paid Rs 9,153 with 13% VAT. If 8% profit is made in this transaction by how many percent was the marked price above the cost price?***

Solution

Discount rate = 10%, VAT rate = 13%, S.P. with VAT = Rs 9,153

Profit percent = 8%

Let marked price (M.P.) of an article be Rs x.

Now, S.P. = M.P. – discount % of M.P. = x – 10% of x = Rs 0.9x

Also, S.P. with VAT = S.P. + 13% of S.P. or, Rs 9,153 = 0.9x + 13% of 0.9x $\therefore$x = 9000

Thus, M.P. of the article = Rs 9,000 and S.P. = 0.9x = 0.9×9000 = Rs 8100

Again, let C.P. of the article be Rs y then S.P. = C.P. + 8% of C.P. or, Rs 8100 = y + 8% of y $\therefore$y = 7500

Thus, C.P. of the article = Rs 7,500 and difference of M.P. and C.P. = Rs 9,000 – Rs 7,500 = Rs 1500

Again, M.P. of the article is more than C.P. by $\dfrac{\text{Difference}}{\text{C.P.}}\times 100\% = \dfrac{\text{Rs } 1500}{\text{Rs } 7500}\times 100\% = 20\%$

Note: Difference between the selling price with VAT and selling price after discount = VAT amount

11. ***In the peak season of winter days, a retailer marked the price of an electric heater as Rs 4000 and 10% discount was given to make 20% profit. But in the summer days she/he increased the discount percent to get only 12% profit from the same type of heater. How much she/he increased the discount percent?***

Solution:

Here, marked price (M.P.) of an electric heater = Rs 4000

Case – I: discount rate = 10%, profit percent = 20%

We have, S.P. = M.P. – discount % of M.P. = Rs 4000 – 10% of Rs 4000 = Rs 3600

Also, C.P. = $\left(\frac{100}{100 + \text{profit}\%}\right) \times$ S. P. = $\left(\frac{100}{100 + 20}\right) \times$ Rs 3600 = Rs 3000

Case – II: Profit percent = 12%

We have, new S.P. = $\left(\frac{100 + \text{profit}\%}{100}\right) \times$ C. P. = $\left(\frac{100 + 12}{100}\right) \times$ Rs 3000 = Rs 3360

Also, new discount = M.P. – S.P = Rs 4000 – Rs 3360 = Rs 640

New Discount rate = $\frac{\text{Discount}}{\text{M. P.}} \times 100\%$ = = $\frac{\text{Rs } 640}{\text{Rs } 4000} \times 100\% = 16\%$

Hence, the discount should be increased by 16% - 10% = 6% in summer season.

12. A retailer allowed 4% discount on her/his goods to make 20% profit and sold a refrigerator for Rs 10,848 with 13% VAT. By how much is the discount percent to be increased so that she/he can gain only 15% profit?

Solution:

Let the S.P. without VAT be Rs x.

Now, S.P. with VAT = S.P. + VAT% of S.P. or, Rs 10,848 = x + 13% of x $\therefore$x = Rs 9,600

Let the marked price of the goods (M.P.) be Rs y.

Also, S.P. = M.P. – D% of M.P. or, Rs 9,600 = y – 4% of y $\therefore$M.P. = y = Rs 10,000

Again, S.P. without VAT = C.P. + P% of C.P. or, 9,600 = C.P. + 20% of C.P. $\therefore$C.P. = Rs 8,000

Now, new profit% = 15%

$\therefore$New S.P. = C.P. + 5% of C.P. = Rs 8,000 + 15% of Rs 8,000 = Rs 9,200

And, new discount = M.P. – new S.P. = Rs 10,000 – Rs 9,200 = Rs 800

New discount percent = $\frac{\text{New discount}}{\text{M.P.}} \times 100\% = \frac{\text{Rs } 800}{\text{Rs } 10{,}000} \times 100\% = 8\%$

Hence, the discount should be increased by 8% - 4% = 4%.

13. A supplier sold a scanner machine for Rs 41, 400 with 15% VAT after allowing 10% discount on it's marked price and gained 20%. By how much the discount percent to be reduced to increase the profit by 4%?

Solution:

Let the S.P. without VAT of scanner machine be Rs x.

Now, S.P. with VAT = S.P. + VAT% of S.P. or, Rs 41,400 = x + 15% of x $\therefore$x = Rs 36,000

Let the marked price of the scanner machine (M.P.) be Rs y.

Also, S.P. = M.P. – D% of M.P. or, Rs 36,000 = y – 10% of y $\therefore$M.P. = y = Rs 40,000

Again, S.P. without VAT = C.P. + P% of C.P. or, 36,000 = C.P. + 20% of C.P. $\therefore$C.P. = Rs 30,000

Now, new profit% = 20% + 4% = 24%

$\therefore$New S.P. = C.P. + 24% of C.P. = Rs 30,000 + 25% of Rs 30,000 = Rs 37,200

And, new discount = M.P. – new S.P. = Rs 40,000 – Rs 37,200 = Rs 2,800

New discount percent = $\frac{\text{New discount}}{\text{M.P.}} \times 100\% = \frac{\text{Rs } 2{,}800}{\text{Rs } 40{,}000} \times 100\% = 7\%$

Hence, the discount should be decreased by 10% - 7% = 3%.

14. Mrs. Gurung allowed 10% discount on her fancy items to make 25% profit and sold a lady bag for Rs 5,085 with 13% VAT. Due to excessive demands of her items, she decreased the discount percent by 2%. By how much was her profit percent increased?

Solution:

Let the S.P. without VAT of lady bag be Rs x.

Now, S.P. with VAT = S.P. + VAT% of S.P. or, Rs 5,085 = x + 13% of x $\therefore$x = Rs 4,500

Let the marked price of lady bag (M.P.) be Rs y.

Also, S.P. = M.P. – D% of M.P. or, Rs 4,500 = y – 10% of y $\therefore$M.P. = y = Rs 5,000

Again, S.P. without VAT = C.P. + P% of C.P. or, 4,500 = C.P. + 25% of C.P. $\therefore$C.P. = Rs 3,600

Now, new discount% = 10% - 2% = 8%

$\therefore$New S.P. = M.P. – D% of M.P. = Rs 5,000 – 8% of Rs 5,000 = Rs 4,600

And, new profit = new S.P. –C.P. = Rs 4,600 – Rs 3,600 = Rs 1,000

New profit percent = $\frac{\text{New profit}}{\text{C.P.}} \times 100\% = \frac{\text{Rs } 1{,}000}{\text{Rs } 3{,}600} \times 100\% = 27.78\%$

Hence, the profit should be increased by 27.78% - 25% = 2.78%.

15. *Mrs. Dhital makes a profit of 50% of the cost of her investment in the transaction of her cosmetic items. She further increases her cost of investment by 25% but the selling price remains same. How much is the decrease in her profit percent?*

Solution:

Let the cost of her investment be Rs x.

Now, S.P. with 50% profit = C.P. + P% of C.P. $\therefore S.P._1$ = x + 50% of x = Rs 1.5x

Again, new C.P. = x + 25% of x = Rs1.25x and $S.P._2 = S.P._1$ = Rs 1.5x

$\therefore$New profit = Rs 1.5x – 1.25x = Rs 0.25x

Now, new profit percent = $\frac{\text{New profit}}{\text{C.P.}} \times 100\% = \frac{0.25x}{1.25x} \times 100\% = 20\%$

Hence, the profit should be decreased by 50% - 20% = 30%.

16. *A retailer hired a room in a shopping mall at Rs 45,000 rent per month and started a business of garments. He spent Rs 20,00,000 to purchase different items in the first phase and marked the price of each item 30% above the cost price. Then he allowed 10% discount on each item and sold to customers. His monthly miscellaneous expenditure was Rs 15,000 and the item of worth 10% of the investment remained as stock after two months. Find his net profit or loss percent.*

Solution:

Here, the amount of investment = Rs 20,00,000

Stocks after two months = 10% of Rs 20,00,000 = Rs 2,00,000

$\therefore$The investment excluding stocks = Rs 20,00,000 – Rs 2,00,000 = Rs 18,00,000

Now, M.P. of items = Rs 18,00,000 + 30% of Rs 18,00,000 = Rs 23,40,000, discount percent = 10%

S.P. of items = M.P. – D% of M.P. = Rs 23,40,000 – 10% of Rs 23,40,000 = Rs 21,06,000

Gross profit = Rs 21,06,000 – Rs 18,00,000 = Rs 3,06,000

Again, rent of room in 2 months = 2×Rs 45,000 = Rs 90,000

Miscellaneous expenditure in 2 months = 2×Rs 15,000 = Rs 30,000

$\therefore$Total expenditure = Rs 90,000 + Rs 30,000 = Rs 1,20,000

Now, net profit = Gross profit – total expenditure = Rs 3,06,000 – Rs 1,20,000 = Rs 1,86,000

Then, net profit percent = $\frac{\text{Net profit}}{\text{investment}} \times 100\% = \frac{1{,}86{,}000}{18{,}00{,}000} \times 100\% = 10.33\%$

Hence, the required net profit is 10.33%

17. The marked price of a calculator is Rs 1,000 and a shopkeeper allows discount which is two times of VAT rate. If the customer pays Rs 836.20, find the rates of discount and VAT.

Solution:

Here, marked price (M.P.) of a calculator = Rs 1000

Let VAT rate be x% then discount rate = 2x%

Now, S.P. = M.P. – discount % of M.P. = Rs 1000 – 2x% of Rs 1000 = Rs (1000 – 20x)

Again, S.P. with VAT = S.P. + VAT% of S.P.

or, Rs 836.20 = Rs (1000 – 20x) + x% of Rs (1000 – 20x)

or, Rs 836.20 $= \text{Rs } (1000 - 20x) + \frac{x}{100}(1000 - 20x)$

or, Rs 836.20 $= 1000 - 20x) + 10x - \frac{x^2}{5}$

or, $836.20 = 1000 - 10x - \frac{x^2}{5}$

or, $4181 = 5000 - 50x - x^2$

or, $4181 = 5000 - 50x - x^2$

or, $x^2 + 50x - 819 = 0$

or, $x^2 + 63x - 13x - 819 = 0$

or, $x(x + 63) - 13(x + 63) = 0$

or, $(x + 63)(x - 13) = 0$

Either, $x + 63 = 0 \therefore x = -63$ which is not possible because rate of VAT cannot be negative.

OR, $x - 13 = 0 \therefore x = 13$

Thus rate of VAT = x% = 13% and rate of discount = 2x% = 26%.

Extra Questions

1. If the marked price of a camara is Rs. 3200, a shopkeeper announces a discount of 8%. How much will a costomer have to pay for buying the camera with 10% VAT? Find it. [Ans: 3,238.40]
2. A shopkeeper sold his goods for Rs. 9944 after allowing 20% discount levying 13% value added tax, what was the amount of discount? [Ans: Rs 2200]
3. A tourist bought a Nepali Thjanka at 15% discount on the marked price with 13% Value Added Tax. If he got Rs. 663 back at the airport when he was leaving Nepal, find the discount amount. [Ans: Rs 900]
4. A mobile price is tagged Rs. 5,000. If a customer gets 12% discount and certain percent VAT reaches as Rs. 4,972, find out the percentage of VAT. [Ans: 13%]
5. A retailer allowed 20% discount and sold a mobile set for Rs 4,520 with 13% VAT and made a profit of 25%. By what percent is the discount to be reduced to increase the profit by 5%? [Ans: 3.2%]

B. Money Exchange

Pre-knowledge

Money exchange rates in news or newspaper.

Teaching strategies

1. With money exchange rates published in newspaper, discuss about buying and selling rates of money exchange, revaluation or devaluation of currencies and commission.
2. Discuss about importantce of money exchange.
3. Solve some problems based on money exchange under discussion.
4. Explain about chain rule and its use in money exchange.
5. Make the studnets note out the following points.

Note:

1. When a person has the foreign currency and needs Nepali rupees then the bank or money exchange centre buys the foreign currency and so uses the buying rate.
2. When a person need the foreign currency and goes to bank or money exchange centre with Nepali rupees then the bank or money exchange centre sells the foreign currency and so uses the selling rate.
3. Devaluation is the official reduction in the value in the currency of a country in compared to the value of foreign currency in a fixed exchange-rate system.
4. Revaluation is the official rise in the value of a country's currency in compared with value of other country's currency within a fixed exchange-rate system.
5. If \$1 = NPR x and Nepali currency is devaluated by y% in comparison with dollar then \$1 = NPR (x + y% of x).
6. If \$1 = NPR x and Nepali currency is revaluated by y% in comparison with dollar then \$1 = NPR (x – y% of x).

Solution of selected questions from Vedanta Excel in Mathematics

1. Manoj Regmi is going to visit Thailand for his family trip. He estimated to exchange US \$ 4,000 in a bank. If the bank charges 1.5% commission for exchanging the money, how much Nepali rupees is required for him? (US \$1 = NPR 114.50)

Solution:

Here, US \$1 = NPR 114.50 ∴ US \$4000 = 4000× NPR 114.50 = NPR 4,58,000

Now, commission = 1.5% of NPR 4,58,000 = NPR 6870

∴Required Nepali rupees with commission = NPR (4,58,000 + 6870) = NPR 4,64,870

2. Nirmal Shrestha bought some EURO (€) for NPR 3,20,000 at the exchange rate of € 1 = NPR 128 to visit a few European countries. Unfortunately because of his Visa problem, he cancelled his trip. Within a week Nepali rupees is devaluated by 2%. He again exchanged his EURO into Nepali rupees after a week. How much did he gain or loss?

Solution:

Here, € 1 = NPR 128 ∴ NRs. 3,20,000 = € $\frac{3,20,000}{128}$ = €2500

When Nepali rupees is devaluated by 2% then €1 = NPR 128+2% of NPR 128 = NPR 130.56

Again, €2500 = 2500×NPR 130.56 = NPR 3,26,400

Hence, his profit amount = NPR. 3,26,400 – NPR 3,20,000 = NPR 6400

3. A businessman exchanged Rs 5,50,000 in to US dollars at the rate of \$ 1 = NPR 110. After a few days, Nepali currency was revaluated by 10% and he exchanged his dollars into Nepali currency again. Calculate his gain or loss.

Solution:

Here, US \$ 1 = NPR 110 ∴ NRs. 5,50,000 = \$ $\frac{5,50,000}{110}$ = \$ 5000

When Nepali rupees is revaluated by 10% then \$1 = NPR 110 – 10% of NPR 110 = NPR 99

Again, \$ 5000 = 5000×NPR 99 = NPR 4,95,000

Hence, his loss amount = NPR. 5,50,000 – NPR 4,95,000 = NPR 55,000

4. Mrs. Magar wants to buy a book online. She finds a publisher in London selling the book for £ 15. This publisher is offering free transportation on the product. She finds the same book from a publisher in New York for \$ 17 with transportation fee of \$ 2. Which publisher should she buy the book from? (Exchanged rate: £ 1 = NPR 140 and \$ 1 = NPR 113.00)

Solution:

The cost of the book in London = £ 15. Since, £ 1 = NPR 140 $\therefore$ £ 15 = 15×NPR 140

= NPR 2,100

The cost of the book in New York with transportation fee = \$ 17 + \$ 2 = 19 \$

We have, 1 \$ = NPR 113 $\therefore$ 19 \$ = 19×NPR 113 = NPR 2,147

Hence, she should buy the book in London because it costs Rs 47 less in London.

5. Mr. Gurung bought 10 Tola gold in Hong Kong for KHD\$ 3800 per Tola and brought to Nepal paying 25% custom duty. If he sold the gold with 13% VAT in Nepal, how much Nepali rupees did he get? (HKD\$ 1 = NRs 14.00)

Solution

Here, cost of 1 Tola gold = HKD\$ 3800

$\therefore$ cost of 10 Tola gold = 10 ×HKD\$ 3800 = HKD\$ 38000

Now,

Cost of 10 Tola gold with 25% custom duty = HKD\$ 38000 + 25% of HKD\$ 38000

= HKD\$ 47500

Also,

Cost of the gold with 13% VAT = HKD\$ 47500 + 13% of HKD\$ 47500 = HKD\$ 53675

Again, HKD\$ 1 = NRs 14.00 $\therefore$ Cost of the gold = 53675×NRs 14 = NRs 751450

Thus, he got NRs 751450 in Nepal.

Extra Questions

1. If Rs. 500 (IC) = Rs. 800 (NC), \$ 45 = Rs. 4645.35 (NC), how many US dollars can be exchanged with RS. 567765(IC)? Find it. [Ans: \$ 8800]
2. Sulochana is going to Australia for her higher study and needs 4000 AUD and pays 2% commission to the bank. How many Nepali rupees does she require at the rate selling rate of Rs. 76? [Ans: NPR 3,10,080]
3. A business man exchanged Rs 7,00,000 into pound sterling at the rate of £1 = NPR 140. After a day, Nepali currency was devaluated by 5% in compared to the pound sterling and he exchanged his pounds into Nepali currency again. What is his gain or loss? [Ans: NPR 35,000 profit]
4. A merchant purchased 600 pieces of Nepali Pasmina at Rs 2,400 per piece. He exported them to UK with 5% export tax. If he sold them at £ 20 per piece in UK, calculate his profit or loss. (£1 = NPR 140) [Ans: 1,68,000 profit]

Unit 3 Compound Interest

5. Jasmin wishes to buy a book online. She finds a publisher in London selling the book for £15 with offering free transportation cost. She finds the same book from a publisher in New York for $ 18 with transportation cost $2. From which publisher should she buy the book? (£1 = NPR 140, $1 = NPR 113.00) [Ans: London]

Allocated teaching periods	6

Competency

- To solve and test the behavioural arithmetic problems on daily life activities using the mathematical instruction and logical thought.

Learning Outcome

- To take the information of system of calculation of compound interest from financial company and solve the related simple problems

Level-wise learning objectives

S.N.	LEVELS	OBJECTIVES
1.	Knowledge (K)	- To recall simple interest - To write the formula of finding the simple interest. - To define compound amount - To state compound interest - To tell the formula for finding yearly compound amount/ interest - To write the formula for finding the half yearly compound interest
2.	Understanding (U)	- To differentiate compound interest from simple interest - To compare yearly and half yearly compound amount / interest - To calculate the compound amount/interest on the basis of yearly and half yearly system. - To find the time when principal, annual/semi-annual compound amount/ interest and rate of interest are known. - To find the rate of interest when principal, annual/semi-annual compound amount/interest and time duration are known.
3.	Application (A)	- To calculate the difference between compound and simple interest - To find the sum when difference / sum between the simple and compound interest on a fixed sum are given - To solve real life problems based on yearly and half yearly compound interest up to 3 years and 2 years respectively. - To prepare a record on interest systems of nearby financial institutions

4.	High Ability (HA)	- To derive the formula of annual compound amount/ interest. - To compare among simple interest, yearly and half-yearly compound interest with proper reason - To evaluate the difference percentage of yearly and half yearly compound interest in the mentioned policies. - To justify the annual instalments with calculation.

Required Teaching Materials/ Resources

Bank account forms, charts of interest rates, model for balance sheet, multi-media

Pre-knowledge: principal, rate of interest, simple interest

Teaching strategies

1. Recall simple interest and ask the formulae of simple interest.
2. Explain with daily life problems about the difference between simple interest and compound interest.
3. Derive the formula C.A. $= P\left(1 + \frac{R}{100}\right)^T$ of annual compound amount through the inductive method based on simple interest as shown in the following table.
   BOOK ko page 35 ma xa
4. Encourage the students to tell the formula for finding the compound interest compounded annually as
   $\text{C.I.} = \text{C.A.} - P = P\left(1 + \frac{R}{100}\right)^T - P = P\left[\left(1 + \frac{R}{100}\right)^T - 1\right]$
5. Discuss with students on the formula C.A. $= P\left(1 + \frac{R}{2 \times 100}\right)^{2T}$ of semi-annual compound amount.
6. Ask the students to tell the formula for finding the compound interest compounded semi-annually and conclude that $\text{C.I.} = \text{C.A.} - P = P\left(1 + \frac{R}{100}\right)^T - P = P\left[\left(1 + \frac{R}{2 \times 100}\right)^{2T} - 1\right]$ by the discussion on the meaning of half year/semi-annual and rate of interest during it.
7. Make clear on the points which are mentioned below.
   (i) According to the yearly compound interest system, the interest is added to the principal at the end of every year and a new principal is obtained for the next year.
   (ii) According to the half-yearly compound interest system, the interest is added to the principal at the end of every 6 months and a new principal is obtained for the next 6 month period.
   (iii) The yearly compound interest is equal to simple interest only up to the first year on the same principal (sum) at the same rate of interest. Then after the compound interest is always more than simple interest on the same sum at the same rate for the same time duration.
   (iv) The half-yearly compound interest is equal to simple interest / annual compound interest only up to the first 6 months on the same principal (sum) at the same rate of interest. Then after the half yearly compound interest is always more than yearly compound interest / simple interest on the same sum at the same rate for the same time duration.
   Thus, simple interest < yearly compound interest < half yearly compound interest

(v) The difference between yearly compound interest and simple interest or the difference between half-yearly compound interest and yearly compound interest/simple interest determine the gain/profit or loss.

Note:

1. When P is principal, R_1%, R_2% and R_3% are rates of annual compound interest for the first, second and the third years respectively then

   (i) Annual compound amount (C.A.) for 3 years = $P\left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)\left(1 + \frac{R_3}{100}\right)$

   (ii) Annual compound interest (C.I.) for 3 years=$P\left[\left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)\left(1+ \frac{R_3}{100}\right)-1\right]$

2. When P is principal, R1%, R2% and R3% are rates semi-annual compound interest for the first, second and the third years respectively then

   (i) Semi-annual compound amount (C.A.) for 3 years

   $= P\left(1 + \frac{R_1}{200}\right)^2\left(1 + \frac{R_2}{200}\right)^2\left(1 + \frac{R_3}{200}\right)^2$

   (ii) Semi-annual compound interest (C.I.) for 3 years

   $= P\left[\left(1 + \frac{R_1}{200}\right)^2\left(1 + \frac{R_2}{200}\right)^2\left(1+ \frac{R_3}{200}\right)^2 -1\right]$

3. When principal = P, rate of annual compound interest = R% , time duration = a fraction or T years and M months then

   Annual compound amount (C.A.) = $P\left(1 + \frac{R}{100}\right)^T\left(1 + \frac{MR}{1200}\right)$

   Annual compound interest (C.I.) = $P\left[\left(1 + \frac{R}{100}\right)^T\left(1 + \frac{MR}{1200}\right) - 1\right]$

4. When principal = P, rate of semi-annual compound interest = R% , time duration = a fraction in 2T or 2T half-years and M months then

   Annual compound amount (C.A.) = $P\left(1 + \frac{R}{200}\right)^{2T}\left(1 + \frac{MR}{1200}\right)$

   Annual compound interest (C.I.) = $P\left[\left(1 + \frac{R}{200}\right)^{2T}\left(1 + \frac{MR}{1200}\right) - 1\right]$

   Give some examples related to above formula and make a sound classroom atmosphere to solve the problems in groups or individually.

Solution of selected questions from Vedanta Excel in Mathematics

1. ***Pratik borrowed Rs 1,50,000 from Bishu at the rate of 21% per year. At the end of nine months how much compound interest compounded half-yearly should he pay?***

Solution:

Here, principal (P) = Rs 1,5,00,000; time (T) = 9 months = 6 months + 3 months = $\frac{1}{2}$ year + 3 months; rate (R) = 21% p.a.

Now, half-yearly C.I.= $P\left[\left(1 + \frac{R}{200}\right)^{2T}\left(1 + \frac{MR}{1200}\right)- 1\right] = 15000\left[\left(1 + \frac{21}{200}\right)^{2T}\left(1+\frac{3\times21}{1200}\right)-1\right]$

= Rs 24451.88

Hence, he should pay Rs 24451.88 interest in nine months.

2. ***The difference between the annual and semi-annual compound interest on a sum of money is Rs 63 at the 10% per annum for $1\frac{1}{2}$ years. Find the sum.***

Solution:

Let time (T) = $1\frac{1}{2}$years = $\frac{3}{2}$ years = 1 year + 6 months, rate (R) = 10% p.a. and let the sum (P) be Rs x.

Now, annual C.I. $= P\left[\left(1 + \frac{R}{100}\right)^T\left(1 + \frac{MR}{1200}\right) - 1\right] = x\left[\left(1 + \frac{18}{100}\right)^1\left(1+\frac{6\times10}{1200}\right)-1\right] = 0.155x$

Again, semi-annual C.I. $= P\left[\left(1 + \frac{R}{200}\right)^{2T} - 1\right] = x\left[\left(1 + \frac{10}{200}\right)^{2\times\frac{3}{2}} - 1\right] = 0.157625X$

By question, semi-annual C.I. - Annual C.I. = Rs 63

or, $0.157625x - 0.155x$ = Rs 63

or, $0.002625x = 63$ $\therefore x = 24000$

Hence, required sum is Rs 24,000.

3. ***If a sum of money becomes Rs 1,40,450 in 1 year and Rs 1,48,877 in $1\frac{1}{2}$ years, interest being compounded semi-annually, calculate the rate of interest and the sum.***

Solution:

Let the required principal be Rs P and the rate of the interest be R% p.a.

According to the first condition:

Time duration (T) = 1 year, semi-annually C.A. = Rs 1,40,450

We have, CI $= P\left[\left(1 + \frac{R}{200}\right)^{2T}\right.$ or, Rs 1,40,450 $= P\left[\left(1 + \frac{R}{200}\right)^{2}\right.$ ... (i)

According to the second condition:

Time duration (T) $= 1\frac{1}{2}$ years $= \frac{3}{2}$ years, semi-annually C.A. = Rs 1,48,877

We have, CI $= P\left[\left(1 + \frac{R}{200}\right)^{2T}\right.$ or, Rs 1,48,877 $= P\left(1 + \frac{R}{200}\right)^{2\times\frac{3}{2}}$

or, Rs 1,48,877 $= P\left[\left(1 + \frac{R}{200}\right)^{3}\right.$ ... (ii)

Dividing equation (ii) by equation (i), we get $\frac{148877}{140450} = \frac{P\left(1 + \frac{R}{200}\right)^3}{P\left(1 + \frac{R}{200}\right)^2}$ $\therefore R = 12\%$ p.a.

Substituting the value of R in eqn (i), we get

Rs 1,40,450 $= P\left[\left(1 + \frac{12}{200}\right)^{2}\right.$ $\therefore$ P = Rs 125000

Hence, the required sum is Rs 1,25,000 and rate of interest is 12% p.a.

4. ***The compound interest calculated yearly on a certain sum of money for the second year is Rs 1,320 and for the third year is Rs 1,452. Calculate the rate of interest and the original sum of money.***

Solution:

Here, the difference between the C.I. of two successive years = Rs 1,452 – Rs 1,320 = Rs 132

$\therefore$ Rs 132 is the interest on Rs 1,320 for 1 year.

Now, rate of interest (R) $= \frac{I \times 100}{P \times T} = \frac{\text{Rs } 132 \times 100}{\text{Rs } 1,320 \times 1} = 10\%$ p.a.

C.A. for the first year $= P\left(1 + \frac{R}{100}\right)^T = P\left(1 + \frac{10}{100}\right)^1 = 1.1\,P$

Since, C.A. of the 1st year is the principal for the 2nd year. So, principal for the 2nd year = 1.1P

Again, C.I. in the second year $= 1.1P\left[\left(1 + \frac{R}{100}\right)^T - 1\right] = 1.1P\left[\left(1 + \frac{10}{100}\right)^1 - 1\right] = 0.11P$

By question, 0.11P = Rs 1,320 $\therefore$ P = 12000

Hence, the rate of interest is 10% p.a. and the original principal is Rs 12,000.

5. ***The compound compounded in 1 year and 2 years are Rs 450 an Rs 945 respectively. Find the rate of interest compounded yearly and the sum.***

Solution:

Let the required principal be Rs P and the rate of the interest be R% p.a.

According to the first condition:

Time duration (T) = 1 year, yearly C.I. = Rs 450

We have, CI = $P\left[\left(1+\frac{R}{100}\right)^{T}-1\right]$ or, Rs 450 = $P\left[\left(1+\frac{R}{100}\right)^{1}-1\right]$ ... (i)

According to the second condition:

Time duration (T) = 2 years, yearly C.I. = Rs 945

We have, CI = $P\left[\left(1+\frac{R}{100}\right)^{T}-1\right]$ or, Rs 945 = $P\left[\left(1+\frac{R}{100}\right)^{2}-1\right]$ ... (ii)

Dividing equation (ii) by equation (i), we get

or, $\frac{\text{Rs } 945}{\text{Rs } 450} = \frac{P\left[\left(1+\frac{R}{100}\right)^{2}-1\right]}{P\left[\left(1+\frac{R}{100}\right)-1\right]}$

or, $2.1 = 2 + \frac{R}{100}$ $\therefore R = 10$

Substituting the value of R in equation (i), we get

Rs 450 = $P\left[\left(1+\frac{10}{100}\right)^{1}-1\right]$ $\therefore P$ = Rs 4500

Hence, the required sum is Rs 4,500 and rate of interest is 10% p.a.

6. ***Krishna lent altogether Rs 10,000 to Gopal and Radha for 2 years. Gopal agrees to pay simple interest at 12% p.a. and Radha agrees to pay compound interest at the rate of 9% p.a. If Radha paid Rs 596.70 more than Gopal as interest, find how much did Krishna lend to each.***

Solution:

Let the money lent to Gopal (P_1) = Rs x. Then, the money lent to Radha (P_2) = Rs (10,000 – x)

For Gopal

Principal (P_1) = Rs x, rate of simple interest (R) = 12 % p.a. and time (T) = 2 years.

$\therefore S.I. = \frac{P_1 \times T \times R}{100} = \frac{x \times 2 \times 12}{100} = 0.24x$

For Radha;

Principal (P_2) = Rs (10000 – x), rate of compound interest (R) = 9 % p.a. and time (T) = 2 years.

$\therefore C.I. = P_2\left[\left(1+\frac{R}{100}\right)^{T}-1\right] = (10000-x)\left[\left(1+\frac{9}{100}\right)^{2}-1\right] = 0.1881(10000-x) = 1881-0.1881x$

According to question, C.I. – S.I. = Rs 596.70 or, (1881 – 0.1881x) – 0.24x = Rs 596.70

or, 1284.3 = 0.4281x $\therefore x = 3000$

So, money lent to Gopal = x = Rs 3,000.

The money lent to Radha = 10000 – x = 10000 – 3000 = Rs 7,000

7. ***Suntali deposited Rs 9,000 altogether in her saving account and fixed deposit account in a bank. Saving account gives here 5% p.a. interest compounded annually and fixed deposit account gives 10% p.a. interest compounded half-yearly. If she got Rs 160 more interest from fixed deposit account at the end of 1 year, find how much money did she deposit in her each account?***

Solution:

Let the money deposited in saving account (P_1) = Rs x. Then, the money deposit to fixed account (P_2) = Rs (9,000 – x)

For saving account

Principal (P_1) = Rs x, rate of annual compound interest (R) = 5 % p.a. and time (T) = 1 year.

$\therefore C.I._1 = P_1\left[\left(1+\frac{R}{100}\right)^T - 1\right] = x\left[\left(1+\frac{5}{100}\right)^1 - 1\right] = 0.05x$

For fixed deposit account;

Principal (P_2) = Rs (9000 – x), rate of half-yearly compound interest (R) = 10 % p.a. and time (T) = 1 year.

$\therefore C.I._2 = P_2\left[\left(1+\frac{R}{200}\right)^{2T} - 1\right] = (9000-x)\left[\left(1+\frac{10}{200}\right)^2 - 1\right] = (9000-x)0.1025 = 922.5 - 0.1025x$

According to question, $C.I._2 - C.I._1$ = Rs 160 or, 922.5 – 0.1025x – 0.05x = Rs 160

or, 762.5 = 1=0.1525x $\therefore$ x = 5000

So, sum deposited in saving account = x = Rs 5,000.

The sum deposited in fixed deposit account = 9000 – x = 9000 – 5000 = Rs 4,000

8. ***A bank has fixed the rate of interest 10% p.a. semi-annually compound interest in account X and 12% p.a. compound interest in account Y. If you are going to deposit Rs 50,000 for 2 years, in which account will you deposit and why? Give your reason with calculation.***

Solution:

For account X

Principal (P) = Rs 50,000, rate (R) = 10% p.a., time (T) = 2 years, semi-annual C.I. =?

We have, half yearly C.I. = $P\left[\left(1+\frac{R}{200}\right)^{2T} - 1\right]$ = Rs 50,000$\left[\left(1+\frac{10}{200}\right)^{2\times2} - 1\right]$ = Rs 10,775.31

For account Y

Principal (P) = Rs 50,000, rate (R) = 12% p.a., time (T) = 2 years, annual C.I. =?

We have, yearly C.I. = $P\left[\left(1+\frac{R}{100}\right)^{T} - 1\right]$ = Rs 50,000$\left[\left(1+\frac{12}{100}\right)^{2} - 1\right]$ = Rs 12,720

Again, half yearly C.I. – yearly C.I. = Rs 12,720 – Rs 10,775.31 = Rs 1,944.69

Hence, I would deposit in account Y because it gives Rs 1,944.69 more interest.

9. ***A person deposited Rs 80,000 in a bank at the rate of 12% p.a. interest compounded semi-annually for 2 years. After one year, the bank revised its policy to pay the interest compounded annually at the same rate. What is the percentage difference between the interests of the second year due to the revised policy? Give reason with calculation.***

Solution:

For the first year

Principal (P) = Rs 80,000, rate (R) = 12% p.a., time (T) = 1 year, semi-annual C.I. =?

We have, half yearly C.I. = $P\left[\left(1+\frac{R}{200}\right)^{2T} - 1\right]$ = Rs 80,000$\left[\left(1+\frac{12}{200}\right)^{2\times1} - 1\right]$ = Rs 9,888

For the second year

Principal (P) = Rs 80,000 + Rs 9,888 = Rs 89,888, rate(R) = 12% p.a., time (T) = 1 year, annual C.I. =?

We have, yearly C.I. = $P\left[\left(1+\frac{R}{100}\right)^{T} - 1\right]$ = Rs 89,888$\left[\left(1+\frac{12}{100}\right)^{1} - 1\right]$ = Rs 10786.56

Again, semi-yearly C.I. = $P\left[\left(1+\frac{R}{200}\right)^{2T} - 1\right]$ = Rs 89,888$\left[\left(1+\frac{12}{200}\right)^{2} - 1\right]$ = Rs 11110.16

$\therefore$Half yearly C.I. – yearly C.I. = Rs 11110.16– Rs 10786.56 = Rs 323.60

Again, the difference of interest in percent = $\dfrac{323.6}{11110.16} \times 100\% = 2.91\ \%$

Hence, due to the policy revised, the interest in the second year is decreased by 2.91% because the interest is calculated annually.

10. A housewife deposited Rs 10,000 on saving account at 5% p.a. interest compounded yearly and another sum on fixed deposit account at 8% p.a. interest compounded half yearly. After one year, the interest on fixed deposit account was Rs 152.80 more than the interest on the saving account; find the total amount of money in her two accounts at the end of the year.

Solution:

<u>For the saving account</u>

Principal (P) = Rs 10,000, rate (R) = 5% p.a., time (T) = 1 year, yearly C.I. =?

We have, yearly C.I. = $P\left[\left(1 + \dfrac{R}{100}\right)^{T} - 1\right]$ = Rs $10,000\left[\left(1 + \dfrac{5}{100}\right)^{1} - 1\right]$ = Rs 500

<u>For the fixed deposit account</u>

Let principal (P) = Rs x, rate(R) = 8% p.a., time (T) = 1 year, half-yearly C.I. =?

We have, half yearly C.I.=$P\left[\left(1 + \dfrac{R}{200}\right)^{2T} - 1\right] = x\left[\left(1 + \dfrac{8}{200}\right)^{2} - 1\right]$ = Rs 0.0816x

By question, interest on fixed deposit – interest on saving account = Rs 152.80

or, 0.0816x – 500 = 152.80 $\quad\therefore$x = 8000

Again, amount in saving account = Rs 10,000 + Rs 500 = Rs 10,500 and amount on fixed deposit = Rs 8,000 + 0.0816×Rs 8000 = Rs 8,652.80

Hence, the total amount of money in two account at the end of the year is Rs 10,500 + Rs 8,652.80 = Rs 19,152.80

Extra Questions

1. According to yearly compound interest, the sum becomes Rs 3240 in 3 years and Rs 3888 in 4 years. How much is the amount more than the principal in 2 years? [Ans: Rs 1875, 20%, Rs 825]
2. A sum of Rs 150000 amounts to Rs 2,62,500 at a certain rate of simple interest in 5 years. Find the sum of money that amounts to Rs 1,98,375 at the same rate of compound interest in 2 years. [Ans: Rs 1,50,000]
3. A bank has fixed the rate of interest 10% p.a. semi-annually compound interest in account M and 12% per annum annually compound interest in account N. If you are going to deposit Rs 80,000 for 2 years in the same bank, in which account will you deposit and why? Give your reason with calculation. [Ans: account N]

Unit 4 Population Growth and Compound Depreciation

Allocated teaching periods	7

Competency

- To solve and test the behavioural arithmetic problems on daily life activities using the mathematical instruction and logical thought.

Learning Outcome

- To solve the simple problems related to population growth and compound depreciation

Level-wise learning objectives

S.N.	LEVELS	OBJECTIVES
1.	Knowledge (K)	- To define population growth - To recall the formula of annual compound amount - To write the formula of finding the population after T years - To define compound depreciation - To tell the formula for finding the depreciated value after T years
2.	Understanding (U)	- To calculate the population after T years when population growth rate is given - To find the rate of population growth rate. - To calculate the time duration when initial population, population after T years and population growth rate are given - To find out the value of item after T years when annual depreciation rate is given - To find out the original value before T years when annual depreciation rate and the depreciated value are given - To find the depreciation rate and time interval
3.	Application (A)	- To solve the problems based on initial population, population after T years, no. of deaths, no. of in-migrants and no. of out-migrants. - To solve the real life problems based on depreciation
4.	High Ability (HA)	- To relate the behavioural problems with compound appreciation and depreciation

Required Teaching Materials/ Resources

Formulae chart, multi-media

A. Population growth

Pre-knowledge: annual compound amount, annual compound depreciation

Teaching strategies

1. Ask about population of the students' village or city or municipality whether it is increasing or decreasing, increment of number of students of a school, numbers of vehicles etc.
2. Make clear that the population of a certain places increases in a certain annual rate and the population after T years can be calculated as the annual compound amount.

   If P_o = initial/previous population, T = time duration, R% = annual population growth rate and P_T = population after T years then

   $P_T = P_o\left(1 + \frac{R}{100}\right)^T$

   Increased population $= P_T - P_o = P_o\left(1 + \frac{R}{100}\right)^T - p_o = P_o\left[\left(1 + \frac{R}{100}\right)^T - 1\right]$
3. Give a simple problem and assist to solve the problem. Furthermore, check whether the formula works effectively or not by finding the total population of each year up to 2/3 years and answer what is obtained by using formula.
4. Discuss in group about value of an article/ land, height of plant, number of bacteria instead of population.
5. Encourage the students to tell the formula for finding the population after T years in the following cases and conclude the result.

Case-1:

When P is initial population, R_1%, R_2% and R_3% are rates of annual population growth for the first, second and the third years respectively then

Population after 3 years $(P_T) = P\left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)\left(1 + \frac{R_3}{100}\right)$

Increased population in 3 years $= P\left[\left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)\left(1 + \frac{R_3}{100}\right)^2 - 1\right]$

Case-2:

If P_o = initial/previous/original population, T = time duration, R% = annual population growth rate, no. of deaths during the given time period, M_{in} = no. of in-migrants and M_{out} = no. of out-migrants at the end of given time of period then

Actual population after T years $(P_T) = P_o\left(1 + \frac{R}{100}\right)^T$ - No. of deaths $+ M_{in} - M_{out}$

Note:

1. Population growth rate per year = annual birth rate – annual deathrate
2. Population growth rate per year = annual birth rate + annual immigration rate
3. Time duration (for example) from beginning of 2076 B.S. to beginning of 2077 B.S. = from end of 2075 B.S. to the beginning of 2077 B.S. = 1 year but from the beginning of 2076 B.S. to the end of 2077 B.S. = 2 years

Project Work

Give the following project works in groups or individual

- To visit school administration, collect the number of students of the school before 2 years and at present then find the growth rate of students.
- To visit available website and search the rate of growth of population of various country in the world and compare their growth rate of population of those countries.

Solution of selected questions from Vedanta Excel in Mathematics

1. ***The population of a town increases every year by 10%. At the end of two years, if 5,800 people were added by migration and the total population of the town became 30,000, what was the population of the town in the beginning?***

Solution

Population at the end of 2 years (P_T) = 30,0000, time (T) = 2 years, population growth rate (R) = 10% p.a., no. of in-migrants (M_{in}) = 5800, population in the beginning (P_o) = ?

We have, $P_T = P_o\left(1 + \frac{R}{100}\right)^T + M_{in}$

or, $30,000 = P_o\left(1 + \frac{R}{100}\right)^2 + 5800$

or, $24,200 = 1.21P_o \therefore P_o = 20,000$

Hence, the population of the town in the beginning was 20,000.

2. ***In the beginning of 2074 B.S., the population of a town was 1,00,000 and the rate of growth of population is 2% every year. If 8,000 people migrated there from different places in the beginning of 2075 B.S., what will be the population of the town in beginning of 2077 B.S.?***

Solution

Here, Population in the beginning of 2074 B.S. (P_o) = 1,00,000

Population growth rate (R) = 10% p.a.

Number of in-migrants in the beginning of 075 B.S. (M_{in}) = 8,000, time (T) = 1 year

Population in the beginning of 2075 B.S. (P_T) =?

We have, $P_T = P_o\left(1 + \frac{R}{100}\right)^T + M_{in} = 1,00,000\left(1 + \frac{2}{100}\right)^1 + 8,000 = 1,10,000$

Again, population in the beginning of 2075 B.S. (P_o) = 1,10,000

Population growth rate (R) = 10% p.a.

Population in the beginning of 2077 B.S. (P_T) =?, time (T) = 2 years

We have, $P_T = P_o\left(1 + \frac{R}{100}\right)^T = 1,10,000\left(1 + \frac{2}{100}\right)^2 = 1,10,000 = 114444$

Hence, the population of the town in beginning of 2077 B.S. will be 114444.

Extra Questions

1. The population of a municipality increases every year by 5%. At the end of two years, if 198 people were added by migration and the total population of the municipality became 9,900, what was the population of the municipality in the beginning? [Ans: 8800]

2. The population of a village increases every year by 5%. At the end of two years, if 1,025 people migrated to other places and the population of the village remained 10,000, what was the population of the village in the beginning? [Ans: 10,000]

3. In the beginning of 2017 A.D., the population of a metropolitan city was 2,00,000 and the rate of growth of population is 2% every year. If 16,000 people migrated there from different places in the beginning of 2018 B.S., what will be the population of the town at the end of 2018 A.D.? [Ans: 2,28,888]

B. Depreciation

Pre-knowledge: annual compound amount, population growth

Teaching strategies

1. Give the real life examples based on depreciation. Like, the value of furniture, vehicles, machinery items, building etc. decreases for some time.
2. Make clear that the value of machines or article lower at a certain rate till a period of time. The process of lowering the value of an item at a certain rate every year is known as compound depreciation.
3. Establish the formulae for compound depreciation relating with the formulae of population growth.

   If P_o = initial/previous/original value, T = time period for depreciation, R% = rate of depreciation and P_T = value after T years then

   $$P_T = \left(1 + \frac{R}{100}\right)^T$$

   Depreciated amount $= Po - P_T = P_o P_o \left(1 + \frac{R}{100}\right)^T = P_o\left[1 - \left(1 + \frac{R}{100}\right)^T\right]$

6. Encourage the students to discover the formula for finding the value after T years when P_o is initial value, R_1%, R_2% and R_3% are rates of annual depreciation for the first, second and the third years respectively then

   $$P_T = P_o\left(1 - \frac{R_1}{100}\right)\left(1 - \frac{R_2}{100}\right)\left(1 - \frac{R_3}{100}\right)$$

   Depreciated amount $= P\left[1 - \left(1 + \frac{R_1}{100}\right)\left(1 + \frac{R_2}{100}\right)\left(1 + \frac{R_3}{100}\right)\right]$

Solution of selected questions from Vedanta Excel in Mathematics

1. Mr. Ghising bought a tripper for Rs 28,08,000. He used it for transporting construction materials and earned Rs 7,80,000 in 2 years. If he sold it at 15% p.a. compound depreciation, find his profit or loss.

Solution:

Here, the original cost of tripper (P_o) = Rs 28,08,000, time (T) = 2 years

The rate of compound depreciation (R) = 15% p.a.

Now, depreciated cost $(P_T) = P_o\left(1 - \frac{R}{100}\right)^T = \text{Rs } 28,08,000\left(1 - \frac{15}{100}\right)^2 = \text{Rs } 20,28,780$

Again, profit in 2 years = Rs 7,80,000

The value of tripper with profit = Rs 20,28,780 + Rs 7,80,000 = Rs 28,08,780

Since, the original cost of tripper = Rs 28,08,000

$\therefore$ Profit = Rs 28,08,780 – Rs 28,08,000 = Rs 780

Hence, his profit is Rs 780.

Extra Questions

1. Mr. Tamang bought a bulldozer for Rs 90,00,000. He used it for road construction and earned Rs 15,00,000 in 3 years. If he sold it at 5% p.a. compound depreciation, find his profit or loss. [Ans: Rs 2,16,375 profit]
2. Mr Dhurmus bought a tractor and used it for 3 years and earned Rs 2,25,000. If he sold it at 10% p.a. compound depreciation and made a profit of Rs 41,330, at what price was the tractor bought? Find. [Ans: Rs 7,77,000]
3. A man buys a piece of land for Rs 800000 in a rural municipality and immediately invests certain amount for building a house on it. If the value of land increases every year by 20% and that of the house decreases every year by 20% so that their values becomes equal in 3 years, what was the cost of the house 3 years ago? [Ans: Rs 27,00,000]

Unit 5 Mensuration (I): Area of Plane Surface

Allocated teaching periods	7

Competency

- To solve the problems related to area of triangle (right angled, equilateral, isosceles, scalene), quadrilateral (rectangle, square, parallelogram, rhombus, trapezium)

Learning Outcomes

- To solve the problems related to area of plane surfaces like triangle (right angled, equilateral, isosceles, scalene), quadrilateral (rectangle, square, parallelogram, rhombus, trapezium

Level-wise learning objectives

S.N.	LEVELS	OBJECTIVES
1.	Knowledge (K)	- To define plane surface - To state area of plane surface - To tell the formula to find the area of triangles (right angled, equilateral, isosceles, scalene) - To write Heron's formula for finding the area of triangle
2.	Understanding (U)	- To find the perimeter of triangle - To find the area of triangles using Heron's formula
3.	Application (A)	- To find the area of triangle when the ratio of sides and perimeter are given - To estimate the cost of specific purpose based on area of triangle
4.	High Ability (HA)	- To relate the problem related area of triangle and the cost/ rent for some more extent - To link various real life/ contemporary problems with area of triangles and solve

Required Teaching Materials/ Resources

Graph-board, paper, scissors, rubber-band, geo-board, colourful chart-paper with definitions and formulae

Teaching learning strategies

Pre-knowledge: Perimeter of plane and area of plane figures like rectangle, square etc.

Teaching Activities

1. Discuss upon the perimeter of place surface as the total of the length its boundary.Recall the formulae to find the perimeter of triangle, rectangle, square etc and units of perimeter.
2. Using rubber-band in the geo-board or drawing the plane figure on the graph-board, clarify about area of plane figures.

3. Discuss upon the area of place surface as amount of space inside its boundary. Recall the formula to find the area of triangle, rectangle, square, parallelogram etc. and units of area.
Prove the formulae to find the area of equilateral triangle = $\frac{\sqrt{3}}{4}$ (side)^2 and isosceles triangle = $\frac{b}{4}\sqrt{4a^2 - b^2}$ where 'a' is the length of each of two equal sides and 'b' the base. Discuss and prove the Heron's formula to find the area of scalene triangle in terms of three sides as $\sqrt{s(s-a)(s-b)(s-c)}$ where a, b and c are the lengths of sides of the triangle and's' the semi-perimeter.
4. Give the project work to the students to write the area formula of triangles with clear figures in the colourful chart-paper and present in the classroom.

Solution of selected questions from Vedanta Excel in Mathematics

1. ***The area of isosceles triangle is 240 cm^2. If the base of the triangle is 20 cm, find the length of its equal sides.***

Solution:

Here, the area of the isosceles triangle (A) = 240 cm^2
length of base (b) = 20 cm, equal sides (a) = ?

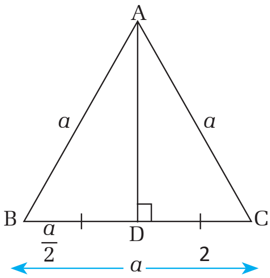
Now, Area of an isosceles triangle (A) = $\frac{b}{4}\sqrt{4a^2 - b^2}$

or, $240 = \frac{20}{4}\sqrt{4a^2 - 20^2} = 48\sqrt{4a^2 - 400}$

On squaring bothe sides, we get
$2304 = 4a^2$ or, $a^2 = 676$ $\therefore$ a = 26
Hence, the length of equal sides are 26 cm and 26 cm

2. ***Derive that the area of an equilateral triangle is $\frac{\sqrt{3}}{4}$(side)^2.***

Solution:



Let ABC be an equilateral triangle in which
AB = BC = CA = a units.
Construction: AD $\perp$ BC is drawn.

Now, BD = CD = $\frac{1}{2}$ BC = $\frac{a}{2}$ units. [The height of equilateral triangle bisects the base]

In ΔABD; AD $= \sqrt{AB^2 - BD^2}$ [By Pythagoras theorem]

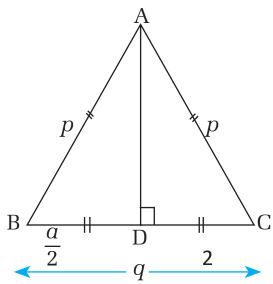
$= \sqrt{a^2 - \left(\frac{a}{2}\right)^2} = \frac{1}{2}\sqrt{3}$ a unit

$\therefore$ Area of equilateral ΔABC = $\frac{1}{2}$ BC $\times$ AD $= \frac{1}{2} \times a \times \frac{1}{2}\sqrt{3}a = \frac{\sqrt{3}}{4}a^2$

Hence, the area of the equilateral triangle is $\frac{\sqrt{3}}{4}$(side)^2.

3. ***Derive that the area of an isosceles triangle is $\frac{1}{4}q\sqrt{4p^2 - q^2}$ where p is the lenght of each of two equal sides and q is the length of the base.***

Solution:



Let ABC is an isosceles triangle in which
AB = CA = p and BC = q.
Construction: AD $\perp$ BC is drawn.

Now, $BD = CD = \frac{1}{2} BC = \frac{1}{2}q$

In ΔABD; $AD = \sqrt{AB^2 - BD^2}$ [By Pythagoras theorem]

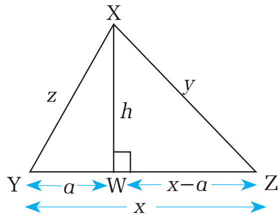
$= \sqrt{p^2 - \left(\frac{q}{2}\right)^2} = \frac{1}{2}\sqrt{4p^2 - q^2}$

Again area of $\Delta ABC = \frac{1}{2} \times BC \times AD = \frac{1}{2} \times q \times \frac{1}{2}\sqrt{4p^2 - q^2}$

$= \frac{1}{4}q\sqrt{4p^2 - q^2}$ sq.unit

4. ***If x, y, z are the three sides and s is the semi - perimeter of a triangle derive that its area is $\sqrt{s(s - x)(s - y)(s - z)}$***

Solution:



Let XYZ be a triangle in which YZ = x, ZX = y and XY = z.

$\therefore$ Perimeter (P) = YZ + ZX + XY

or $2s = x + y + z$

$\therefore s = \frac{x + y + z}{2}$

Let $XW \perp YZ$ be drawn such that XW = h = height of ΔXYZ,

YW = a. $\therefore$ WZ = x – a.

Now, in right angled ΔXYW, $XW^2 = XY^2 - YW^2$ or, $h^2 = z^2 - a^2$ ... (i)

Also, in right angled ΔXWZ; $XW^2 = XZ^2 = WZ^2$ or, $h^2 = y^2 - (x - a)^2$ ... (ii)

From (i) and (ii), we get, $z^2 - a^2 = y^2 - (x - a)^2$

or, $z^2 - \cancel{a^2} = y^2 - x^2 + 2ax - \cancel{a^2}$

$\therefore a = \frac{x^2 - y^2 + z^2}{2x}$ ... (iii)

Substituting the value of a from (iii) in (i), we get

$h^2 = z^2 - \left(\frac{x^2 - y^2 + z^2}{2x}\right)^2 = \left(z + \frac{x^2 - y^2 + z^2}{2x}\right)\left(z - \frac{x^2 - y^2 + z^2}{2x}\right)$

$= \left(\frac{2xz + x^2 - y^2 + z^2}{2x}\right)\left(\frac{2xz - x^2 + y^2 - z^2}{2x}\right)$

$= \left[\frac{(x + z)^2 - y^2}{2x}\right]\left[\frac{y^2 - (x^2 - 2xz + z^2)}{2x}\right]$

$= \left[\frac{(x + z)^2 - y^2}{2x}\right]\left[\frac{y^2 - (x - z)^2}{2x}\right]$

$= \frac{(x + z + y)(x + z - y)(y + x - z)(y - x + z)}{4x^2}$

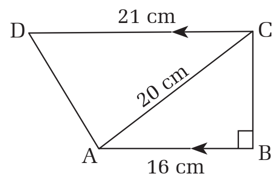
$= \frac{2s(2s - 2y)(2s - 2z)(2s - 2x)}{4x^2}$ [$\therefore x + y + z = 2s$]

$\therefore h = \sqrt{\frac{16s(s - y)(s - z)(s - x)}{4x^2}} = \frac{2}{x}\sqrt{s(s - x)(s - y)(s - z)}$

Again,

Area of $\Delta ABC = \frac{1}{2} YZ \times XW = \frac{1}{2} \times \cancel{x} \times \frac{2}{\cancel{x}}\sqrt{s(s - x)(s - y)(s - z)}$

$= \sqrt{s(s - x)(s - y)(s - z)}$ sq.unit



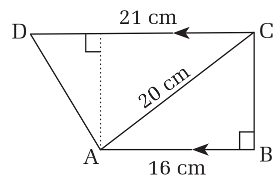
5. ***The adjoining figure is a trapezium ABCD. Find***

i) Its area ***ii) the length of sides BC and AD.***

Solution:

Here in trap. ABCD; CD // BA, AB = 16 cm, DC = 21 cm and

AC = 20 cm, ∠ABC = 90°.



Now, in right angled ΔABC, BC $= \sqrt{AC^2 - AB^2}$

$= \sqrt{(20cm)^2 - (20cm)^2} = \sqrt{144cm^2} = 12$ cm

i) Area of trap. ABCD $= \frac{1}{2} \times$ BC (DC + AB)

$= \frac{1}{2} \times 12$ cm (21 cm + 16 cm)

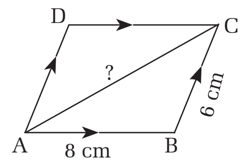
$= 6$ cm × 37 cm $= 222$ cm^2

ii) Construction: AE ⊥ DC is drawn where E is on DC.
Then, AE = BC = 12 cm, EC = AB 16 cm ∴ DE = DC – EC = 21 cm – 16 cm = 5 cm

In right angled ΔAED; AD $= \sqrt{AE^2 - DF^2} = \sqrt{(12cm)^2 - (5cm)^2} = 13$ cm

∴ Required length of BC is 12 cm and that of AD is 5 cm.

6. ***Two adjacent sides of a parallelogram are 8 cm and 6 cm and its area is 448 cm². Find the length of its diagonal .***



Solution:

Here, area of ▱ABCD = 48cm², AD = 8 cm and BC = 6 cm.

Now, area of ΔABC $= \frac{1}{2}$ area of ▱ABCD

[Diagonal bisects the parm,]

$= \frac{1}{2} \times 48$ cm^2 $= 24$ cm^2

Also,

In ΔABC, AB(c) = 8 cm, BC(a) = 6 cm and AC(b) = ?

∴ semi - perimeter (S) $= \frac{a+b+c}{2} = \frac{b + 14 \text{ cm}}{2}$

Again, Area of ΔABC $= \sqrt{s(s-a)(s-b)(s-c)}$

or, $24 = \sqrt{\left(\frac{b+14}{2}\right)\left(\frac{b+14}{2} - 6\right)\left(\frac{b+14}{2} - b\right)\left(\frac{b+14}{2} - 8\right)}$

or, $24 = \sqrt{\left(\frac{b+14}{2}\right)\left(\frac{b+2}{2}\right)\left(\frac{14-b}{2}\right)\left(\frac{b-2}{2}\right)} = \sqrt{\frac{(196-b^2)(b^2-4)}{16}}$

on squaring Both sides, we get

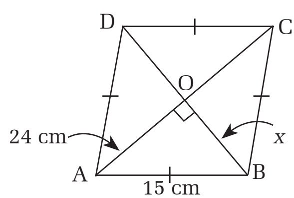
$576 = \frac{1}{16}(196 - b^2)(b^2 - 4)$

or, $9216 = 196b^2 - 784 - b^4 + 4b^2 = 200b^2 - 784 - b^4$

or, $b^4 - 200b^2 + 10000 = 0$ or, $(b^2 - 100)^2 = 0$ or, $b^2 = 100$ ∴ $b = 10$

Hence, the length of diagonal AC (b) = 10 cm.

7. ***A garden is in the shape of a rhombus whose each side is 15 m and its one of two diagonals is 24 cm. Find the area of the garden.***



Solution:

Here, in rhombus side AB = 15m, diagonal AC = 24 m

Since, the diagonals of the rhombus bisect to each other at a right angle.

∴ OA $= \frac{1}{2}$ AC $= \frac{1}{2} \times 24$ m $= 12$ m

In right angled ΔAOB, OB $= \sqrt{AB^2 - OA^2} = \sqrt{(15\text{ m})^2 - (12\text{ m})^2}$
$= 9$ m.

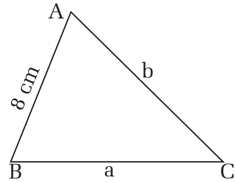
$\therefore BD = 2 \times OB \;= 2 \times 9 \text{ m} \;= 18 \text{ m}$

Now, Area of rhombus ABCD $= \frac{1}{2} \times AC \times BD \;= \frac{1}{2} \times 24 \text{ m} \times 18 \text{ m} \;= 216\text{m}^2$

Hence, the area of the rhombus ABCD is 216 m^2.

8. ***The perimeter of a triangular garden is 18 m. If its area is $\sqrt{135}$ m² and one of the three sides is 8 m, find the remaining two sides.***

Solution:



Let the unknown sides of the triangle be a and b.

Here perimeter of the triangle = 18 m

$\therefore$ semi-perimeter (s) $= \frac{18\text{m}}{2} = 9\text{m}$

Also, $a + b + c = 18$ m or, $a + b + 8$ m $= 18$ m $\therefore b = (10 - a)$m ... (i)

Now, the area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

or, $\sqrt{135} = \sqrt{9(9-a)(9-10+a)(9-8)}$

or, $\sqrt{135} = \sqrt{9 \times (9-a) \times (a-1) \times 1}$

On squaring both sides, we get

$135 = 9(9a - 9 - a^2 + a)$

or, $15 = 10a - 9 - a^2$ or, $a^2 - 10a + 24 = 0$

or, $a^2 - 6a - 4a + 24 = 0$ or, $(a-6)(a-4) = 0$ $\therefore$ 6 or 4

When a = 6, from (i); $b = (10 - 6) = 4$

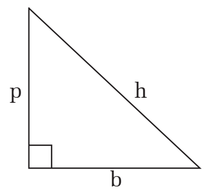
When a= 4, from (i); $b = (10 - 4) = 6$

Hence the required remaining sides of triangle are 6 m and 4 m or 4 m and 6 m

9. ***The perimeter of a right angled triangle is 24 cm and its area is 24 cm². Find the sides of the triangle.***

Solution:

Let p, b and h be the perpendicular base and hypotenuse of the right angled triangle.



Then perimeter of the triangle = 24 cm

or, $p + b + h = 24$ cm or, $p + b = 24 - h$

Squaring on both side, we get

$(p + b)^2 = (24 - h)^2$ or, $p^2 + 2pb + b^2 = 576 - 48h + h^2$

or, $h^2 + 2pb = 576 - 48h + h^2$ $[\therefore p^2 + b^2 = h^2]$

or, $2pb + 48h = 576$ ... (i)

Also, area of $\Delta = \frac{1}{2}$ b × por, $24 = \frac{1}{2}$ pb $\therefore pb = 48$ ... (ii)

Putting the value of pb in equation (i) we get

$2 \times 48 + 48h = 576$ $\therefore h = 10$

Again $p + b + h = 24$ or, $p + b + 10 = 24$

or, $p + b = 14$ $\therefore b = 14 - p$ ..... (iii)

Putting the value of b in equn (ii), we get

$p(14 - p) = 48$ or, $14p - p^2 = 48$ or, $p^2 - 14p + 48 = 0$

or, $p^2 - 8p - 6p + 48 = 0$ or, $(p - 8)(p - 6) = 0$ $\therefore p = 8$ or 6.

When p = 8, from (iii), $b = 14 - 8 = 6$

When p = b, from (iii), $b = 14 - 6 = 8$

Hence, the required sides of the triangle are 8 cm, 6 cm, 10 cm or 6 cm, 8 cm, 10 cm.

10. ***An umbrella is made up of 6 isosceles triangular pieces of cloths. The measurement of the base of each triangular piece is 28 cm and two equal sides are 50 cm each. If the rate of cost of the cloth Rs 0.50 per sq.cm, find the cost of the cloths required to make the umbrella.***

Solution:

Here, base of each triangular piece (b) = 28 cm

Two equal sides of each triangular piece (a) = 50 cm

Now, the area of each isosceles triangular piece of cloth

$$= \frac{1}{4} b\sqrt{4a^2 - b^2} = \frac{1}{4} \times 28\sqrt{4(50)^2 - 28^2} = 672 \text{ cm}^2$$

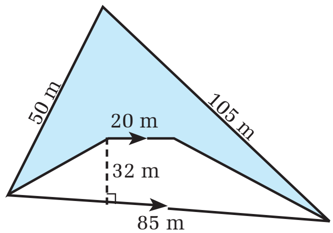
$\therefore$ The area of 6 isosceles triangular piece of cloth (A) = 6 × 672 cm² = 4030 cm²

Again, the rate of cost of cloth (R) = Rs 0.50 per sq.cm

$\therefore$ Total cost of the cloth (T) = Area of cloth (A) × rate (R) = 4032 × Rs 0.50 = Rs 2016

Hence, the cost of the cloth required to make the umbrella is Rs 2016.

11. ***A real state agent divided the adjoining triangular land into two 'kitta' in which one is in the shape of a trapezium. If he sold the shaded 'kitta' at Rs 2,10,000 per 'aana', how much did the buyer pay for it ? (1 aana = 31.79 sq. m)***



Solution:

Here in the triangular piece of land, sides are:

a = 50 m, b = 85 m and c = 105 m

Now, semi-perimeter (s) $= \frac{a + b + c}{2} = \frac{50 \text{ m} + 85 \text{ m} + 105 \text{ m}}{2} = 120$ m

$\therefore$ Area of triangular land $(A_1) = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{120(120-50)(120-85)(120-105)}$

$= \sqrt{120 \times 70 \times 35 \times 15} = 2100$ sq m

Also, Area of trapezium shaped piece of land $(A_2) = \frac{1}{2} h(a + b)$

$= \frac{1}{2} \times 32(20 + 85) = 1680$ sq.m²

$\therefore$ Area of shaded 'kitta' $(A) = A_1 - A_2 = 2100 - 1680 = 420$ m²

We have, 1 aana = 31.79 m²

$\therefore$ 420 m² $= \frac{420}{31.79}$ aana = 13.2117018 aana

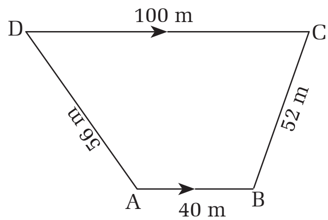
Again, the rate of cost of land (R) = Rs 2,10,000 per aana.

$\therefore$ Total cost of shaded 'kitta' = Area of land (A) × rate (R)

= 13.211708 × Rs 2,10,000 = Rs 2774457.38

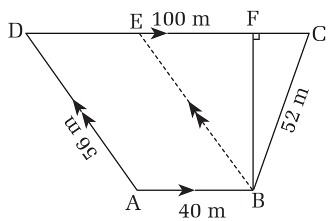
Hence, the cost of shaded 'kitta' is Rs 2774457.38

12. ***ABCD is a field in the shape of a trapezium whose parallel sides are 100 m and 40 m and the non - parallel sides are 56 m and 52 m. Find the cost of ploughing the field at Rs 7.50 per sq.m***



Solution:

Here, in trapezium ABCD; DC // AB; AB = 40 m, CD = 100 m, BC = 52 m and AD = 56 m



Construction : BE // AD to meet CD at E and BF ⊥ CD at F are drawn.

Now, AB = DE = 40 m, AD = BE = 56 m

EC = DC – DE = 100 m – 40 m = 60 m

In ΔBCE, semi- perimeter (s) $= \frac{a+b+c}{2} = \frac{52\text{ m} + 60\text{ m} + 56\text{ m}}{2} = 84\text{ m}$

$\therefore$ Area of $\Delta BCE = \sqrt{84(84-52)(84-60)(84-56)} = 1344\text{ m}^2$

Also, the area of $\Delta BCE = \frac{1}{2} \times EC \times BF$

or, $1344 = \frac{1}{2} \times 60 \times BF \quad \therefore BF = 44.8\text{m}$

Again

Area of trap. ABCD(A) $= \frac{1}{2} BF (DC + AB) = \frac{1}{2} \times 44.8 (100 + 40) = 3136\text{ m}^2$

Next,

Rate of ploughing the field (R) $=$ Rs 7.50 Per m^2

$\therefore$ Total cost of ploughing the field (T) $=$ area of field (A) $\times$ Rate (R) $= 3136 \times$ Rs 7.50

$=$ Rs 23,520

Hence, the cost of ploughing the field is Rs 23,520

Extra Questions

1. The length equal sides of an isosceles triangular plotare 17 feet each. If the area of the plot is 120square feet, find the length of its base. [Ans: 16 feet or 30 feet]
2. The perimeter of aright angled triangle is 24 cm and its area is 24 cm^2, find the sides of the triangle. [Ans: 6 cm, 8 cm and 10 cm]
3. An umbrella is made up by stitching 10 triangular pieces of cloth of two different colours, each piece measuring 15 cm, 41 cm and 28 cm. much cloth is required for the umbrella? If the rate of cost of the cloth is 44 paisa per sq. cm, find the total cost of the cloth to make the umbrella. [Ans: 1260 cm^2, Rs 554.40]
4. The sides of a triangular field are 20 ft., 507 ft. and 493 ft. find the perimeter and area of the field. Estimate the cost of fencing it with 4 rounds by the wire at Rs 6.25 per ft. and panting the vegetable at Rs 20 per square feet.
   [Ans: 1020 ft, 3570 sq.ft.; Rs 25,500, Rs 71,400]

Unit 6 Mensuration (II): Cylinder and Sphere

Allocated teaching periods	10

Competency

- Solving the problems related to surface area and volume of regular solid cylinder, sphere and hemisphere

Learning Outcomes

- To solve the problems related to surface area and volume of regular solid cylindrical, spherical and hemispherical objects.

Level-wise learning objectives

S.N.	LEVELS	OBJECTIVES
1.	Knowledge (K)	- To define cylinder - To tell the formula of finding the volume, curved surface area and total surface area of cylinder - To recall the definition of great circle of the sphere - To list the formula of finding the surface area and volume of sphere and hemisphere
2.	Understanding (U)	- To find the volume of cylinder - To calculate the total surface area of cylinder - To find the volume and surface area of sphere - To find the volume and surface area of hemisphere
3.	Application (A)	- To find the volume and total surface area of combined solid made up of cylinder and hemisphere - To determine the unknown dimension of cylinder when its volume or capacity or surface area is given - To find the unknown dimension of sphere and hemisphere when its volume or surface area is given - To estimate the cost
4.	High Ability (HA)	- To relate the volumes when a metallic sphere is melted and recast in to a cylinder or vice versa. - To link various real life/ contemporary problems with area and volume of cylinder and sphere - To relate the curved surface areas of cylinder and sphere. - To construct the solid cylinder from the cardboard under the given measurements.

Required Teaching Materials/ Resources

Graph-board, pencil, scissors,volleyball, marbles, models of cylinder, models of sphere and hemisphere, definitions and formulae in colourful chart-paper, ICT tools, etc.

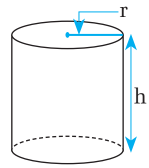
Teaching learning strategies

Pre-knowledge: Perimeter and area of circle.

A. CYLINDER

Teaching Activities

1. Ask the students about the real life examples cylindrical, spherical and hemispherical objects.
2. Discuss upon the definition, curved surface area and total surface area of cylinder by presenting the models of cylinder or folding the rectangular sheet of paper in the cylindrical form.



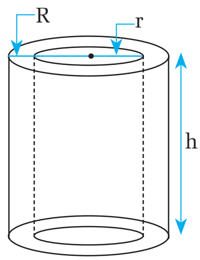
3. Explain the formulae of perimeter and area of circular base, volume and curved/total surface area of the cylinder
   - Volume (V) = Area of circular base (A) $\times$ length (h) = $\pi r^2 h$
   - Curved surface area (C.S.A) = Perimeter of circular base $\times$height (h) = $2\pi rh$
   - Total surface area (T.S.A.) = C.S.A. + 2A = $2\pi rh + 2\pi r^2 = 2r(r + h)$
4. Explain the formulae of volume and curved/total surface area of the hollow cylinder
   - Volume (V) = Area of annular base (A) $\times$ height (h) = $\pi (R^2 - r^2)h$

     $= \pi h (R+ r) (R - r)$
   - Curved surface area (C.S.A) = internal C.S.A. + external C.S.A.

     $= 2rh + 2\pi Rh = 2\pi h(R + r)$
   - Total surface area (T.S.A.) = C.S.A. + area of 2 annular base

     $= 2\pi h(R + r) + 2\pi (R+ r) (R - r)$

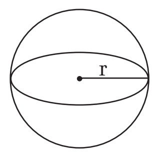
     $= 2\pi(R + r) (h + R - r)$



B. SPHERE AND HEMISPHERE

Teaching Activities

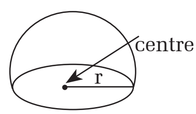
1. With manipulative models, discuss on curved surface, total surface of sphere and hemisphere.
2. Use measuring cylinder and sphere of equal radii and height to find the volume formula of sphere
3. Use thread and two hemispheres of equal radii to explain about the surface area of the sphere.
4. To motivate the students to find the following formulae under discussion



(i) Sphere

- Circumference of the great circle = $2\pi r$
- Area of great circle = πr^2 Surface area of sphere = $4\pi r^2$

Volume of sphere = $\frac{4}{3}\pi r^3$



(ii) Hemisphere

- Curved surface area of hemisphere = $2\pi r^2$
- Total surface area of hemisphere = $2\pi r^2 + r^2 = 3\pi r^2$
- Volume of hemisphere = $\frac{2}{3}\pi r^3$
- Give the project work to the students to write the formulae of surface are and volume of the cylinder, sphere and hemisphere with clear figures in the colourful chart-paper and present in the classroom.

Solution of selected questions from Vedanta Excel in Mathematics

1. ***A cylindrical water tank contains 4,62,000 litres of water and its radius is 3.5 m, find the height of the tank.***

Solution:

Here,

Volume of the cylindrical water tank (V) $= \frac{462000}{1000}$ m3 $[\because 1000\, l = 1m^3]$

$= 462\ m^3$

radius (r) $= 3.5$ m

height (h) $= ?$

Now

Volume (V) $= \pi r^2 h$

or, $462 = \frac{22}{7} \times (3.5)^2 \times h \quad \therefore h = 12$ m

Hence the height of the tank is 12 m.

2. ***If the perimeter of the great circle is π cm, find the volume of the hemisphere.***

Solution:

Here, perimeter of the great circle (C) $= \pi$ cm

or, $2\pi r = \pi$ cm $\quad \therefore r = \frac{1}{2}$ cm

Now,

Volume of hemisphere (V) $= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times \left(\frac{1}{2}\right)^3$

$= \frac{2}{3}\pi \times \frac{1}{8} = \frac{\pi}{12}\ cm^3$

3. ***The total surface area of a hemisphere is 243πcm², find its volume.***

Solution:

TSA of hemisphere $= 243\pi\ cm^2$

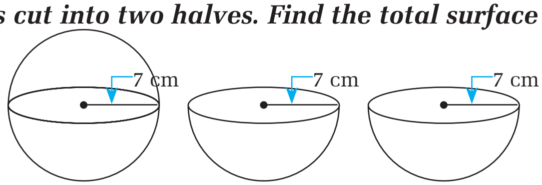
or, $3\pi r^2 = 243\pi\ cm^2$

$r^2 = 81 \quad \therefore r = 9$ cm

Now,

Volume (V) $= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi \times 9^3 = 486\pi\ cm^3$

4. ***A solid metallic sphere of radius 7 cm is cut into two halves. Find the total surface area of the two hemispheres so formed.***



Solution:

T.S.A of a hemisphere $= 3\pi r^2$

$= 3 \times \frac{22}{7} \times 7^2 = 462\ cm^2$

$\therefore$ T.S.A. of two hemisphere $= 2 \times 462\ cm^2 = 924\ cm^2$

5. ***The area of curved surface area of a solid cylinder is $\frac{2}{3}$ of the total surface area. If the total surface area is 924 cm², find the volume of the cylinder.***

Solution:

Let r and h be the radius of base and height of a cylinder respectively.

Now

Curved surface area $= \frac{2}{3}$ of total surface area

or, $\cancel{2\pi r}h = \frac{2}{3} \times \cancel{2\pi r}(r + h)$

or, $3h = 2r + 2h$

$\therefore h = 2r$ ..... (i)

Also, total sruface area $= 924$ cm2

or, $2\pi r(r + h) = 924$

or, $2 \times \frac{22}{7} \times r\ (r + 2r) = 924$ [From (i); $h = 2r$]

or, $\frac{22r}{7} \times 3r = 324$

or, $r^2 = 49$ $\therefore r = 7$

$\therefore r = 7$ and $h = 2r = 2 \times 7$ cm $= 14$ cm

Again, volume of the cylinder (V) $= \pi r^2 h = \frac{22}{7} \times 7^2 \times 14 = 2156$ cm^3

6. ***By how many times the volume of a cylinder increase when its diameter is doubled.***

Solution:

Let V_1 be the volume of the cylinder when its radius is x units and height y units.

$\therefore V_1 = \pi r^2 h = \pi x^2 y$ cu.unit ... (i)

When the radius is doubled, then radius of new cylinder is 2x units and height is y units.

Let V_2 be the volume of new cylinder

$\therefore V_2 = \pi r^2 h = \pi (2x)^2 \times y = 4\pi x^2 y$ cu.units $= 4 \times \pi x^2 y$ cu.unit $= 4V_1$ [From ... (i)]

Hence the volume become 4 times that of original cylinder when the radius is doubled.

7. ***A hollow cylindrical metallic pipe is 21 cm long. If the external and internal diameters of the pipe are 12 cm and 8 cm respectively, find the volume of metal used in making the pipe.***

Solution:

Here, the external diameter of the pipe (D) $= 12$ cm

$\therefore$ external radius (R) $= \frac{22}{7}$ cm $= 6$ cm

the internal diameter of the pipe (d) $= 8$ cm

$\therefore$ internal radius (r) $= \frac{8}{2}$ cm $= 4$ cm

height of the pipe (h) $= 21$ cm

Now, Volume of the material (V) $= \pi h(R + r)(R - r) = \frac{22}{7} + 21\ (6 + 4)\ (6 - 4) = 1320$

Hence, the volume of the material used in making the pipe is 1320 cm^3.

8. ***The internal radius of a cylindrical bucket of height 50 cm is 21 cm. It is filled with water completely. If the water is poured into a rectangular vessel with interval length 63 cm and breadth 44 cm and it is completely filled with water, find the height of the vessel.***

Solution:

Here, the internal radius of the bucket (r) $= 21$ cm

The height of the bucket (h) $= 50$ cm

Now, the interval volume of bucket (V) $= \pi r^2 h = \frac{22}{7} \times 21 \times 21 \times 50 = 69{,}300$ cm^3

$\therefore$ Volume of water = interval volume of bucket $= 69{,}300$ cm^3

Again, Volume of the rectangular vessel = Volume of water

or, $l \times b \times h = 69{,}300$

or, $63 \times 44 \times h = 69{,}300$

or, $h = \frac{69{,}300}{2{,}772} = 25$ cm

9. ***An iron pipe of internal diameter 2.8 cm and uniform thickness 1 mm is melted and a solid cylindrical rod of the same length is formed. Find the diameter of the rod.***

Solution:

Here, the internal diameter of an iron pipe (d) = 2.8 cm

So, its internal radius (r) $= \frac{2.8}{2}$ cm $= 1.4$ cm

thickness of pipe $= 1$ mm $= \frac{1}{10}$ cm $= 0.1$ cm

$\therefore$ the external radius of the pipe (R) $= 1.4$ cm $+ 0.1$ cm $= 1.5$ cm

Length of pipe = h cm (say)

Now, Volume of the material (V) $= \pi h(R + r)(R - r)$

$= \pi h(1.5 + 1.4)(1.5 - 1.4) = \pi h \times 2.9 \times 0.1 = 0.29\pi h$

Again, Volume of cylindrical rod = Volume of material used to make the pipe

or, $\pi r^2 h = 0.29\pi h$ [Length of pipe and rod are equal]

or, $r^2 = 0.29$ $\therefore$ r = 0.54 cm and d = 2r = 2 × 0.5 cm = 1.08 cm

Hence; the diameter of the rod is 1.08 cm.

10. ***A solid iron sphere of diameter 42 cm is dropped into a cylindrical drum partly filled with water. If the radius of the drum is 1.4 m. by how much will the surface of the water be raised ?***

Solution:

Here, the diameter of an iron sphere (d) = 42 cm $\therefore$ radius (r) $= \frac{42}{2}$ cm $= 21$ cm

Now, volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 21 \times 21 \times 21 = 12348\ \pi\ cm^3$

Let height of water surface raised in the drum be h cm.

Then, volume of water displaced by the sphere = Volume of sphere

or, $\cancel{\pi} r^2 h = 12348\ \cancel{\pi}$

or, $140 \times 140\ h = 12348$ = 0.63 cm = 0.63 × 10 mm = 6.3 mm

Hence. the surface of water is raised by 6.3 mm.

11. ***A cylindrical jar of radius 6 cm contains water. How many iron solid spheres each of radius 1.5 cm are required to immerse into the jar to raise the level of water by 2 cm.***

Solution:

Here, radius of a cylindrical jar (r) = 6 cm

height of water level raised by iron spheres (h) = 2 cm

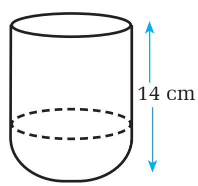
Now, Volume of water displaced by the iron sphere (V_1) $= \pi r^2 h = \pi \times 62 \times 2 = 72\pi\ cm^3$

Also, radius of each iron sphere (r_1) = 1.5 cm

$\therefore$ Volume of each iron sphere (V_2) $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 1.5 \times 1.5 \times 1.5 = 4.5\pi\ cm^3$

$\therefore$ Required number of iron spheres (N) $= \dfrac{\text{Volume of displaced water } (V_1)}{\text{Volume of each sphere } (V_2)} = \dfrac{72\ \pi\ cm^3}{4.5\ \pi\ cm^3} = 16$

12. ***Given figure in a solid composed of a cylinder with hemisphere at one end. If the total surface area and height of the solid are 770 sq.cm and 14 cm respectively, find the height of the cylinder.***



Solution:

Let r and h be the common radius of base and height of the cylinder respectively.

Then, height of hemisphere = radius of hemisphere = r

Now, total height of combined solid = r + h = 14 cm

$\therefore$ h - (14 – r) cm ... (i)

Also

T.S.A. of solid = C.S.A of hemisphere + C.S.A of cylinder + Area of base

or, $770 = 2\pi r^2 + 2\pi rh + \pi r^2$

or, $770 = \pi r\,(2r + 2h + r)$

or, $770 = \frac{22}{7} r\,[3r + 2(14 - r)]$ [From (i)]

or, $245 = r\,(r + 28)$

or, $r^2 + 28r - 245 = 0$

or, $r^2 + 35r - 7r - 245 = 0$

or, $r(r + 35) - 7(r + 35) = 0$

or, $(r + 35)\,(r - 7) = 0$

Either, $r + 35 = 0 \;\therefore\; r = -35$ which is impossible as the radius can not be negative

or, $r - 7 = 0 \;\therefore\; r = 7$ and $h = (14 - 7)$ cm $= 7$ cm

Hence, the height of the cylinder is 7 cm.

13. ***A combined solid made up of a cylinder of radius 3 cm and length h cm and a hemisphere with the same radius as the cylinder has volume 792 cm³. Find the value of h.***

Solution:

Here, radius of cylinder = radius of hemisphere (r) = 3 cm

height of cylinder = h cm

Volume of combined solid = 792 cm^3

Now, volume of combined solid = Volume of cylindr + Volume of hemisphere

or, $792\text{ cm}^3 = \pi r^2 + \frac{2}{3}\pi r^3$

or, $792 = \pi r^2(h + \frac{2}{3} r)$

or, $792 = \frac{22}{7} \times 3 \times 3\,(h + \frac{2}{3} \times 3)$ or, $\frac{792 \times 7}{22 \times 9} = (h + 2)$

or, $28 = h + 2$

or, $h = 26$

Hence, the height of the cylindrical part is 26 cm.

14. ***A roller of diameter 112 cm and length 150 cm takes 550 complete revolution to level a compound. Calculate the cost of levelling the compound at Rs 9 per square meter.***

Solution:

Here, diameter of the roller = 112 cm

$\therefore$ Radius of the roller (r) $= \frac{112}{100}$ cm $= 56$ cm $= \frac{56}{100} = 0.56$ m

length of the roller (h) = 150 cm $= \frac{150}{100}$ cm $= 1.5$ m

Now, area of covered by the roller in 1 resolution = C.S.A. of the roller

$= 2\pi rh = 2 \times \frac{22}{7} \times 0.56 \times 1.5 = 5.28\text{ m}^2$

$\therefore$ Area covered by the roller in 550 revolution = 550 × 5.28 m2 = 2904 m^2

$\therefore$ Area of the compound = 290 m^2

Again, rate of levelling the compound (R) = Rs 9 per m^2

$\therefore$ Cost of levelling the compound (T) = A × R = 2904 × 9 = Rs 26,136

Hence, the required cost of levelling the compound is Rs 26,136.

15. ***The external and internal radii of hollow cylindrical metallic vessel 56 cm long are 10.5 cm and 10.1 cm respectively. Find the cost of metal contained by the vessel at Rs 2 per cubic cm. Also, find the cost of polishing its outer surface at 20 paisa per square cm.***

Solution:

Here, the internal radius of teh cylindrical Vessel (r) = 10.1 cm
the external radius of the vesel (R) = 10.5 cm
the length of the vessel (h) = 56 cm

Now, Volume of metal contained by the vessel (V) $= \pi h(R + r)(R - r)$

$= \frac{22}{7} \times 56\ (10.5 + 10.1)\ (10.5 - 10.1)$

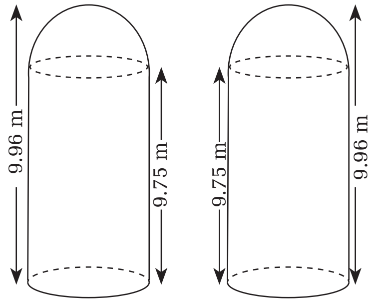
$= 22 \times 8 \times 20.6 \times 0.4 \quad = 1450.24\ cm^3$

$\therefore$ Cost of metal contained by the vessel (T) = V × R = 1450.24 × Rs 2 = Rs 2900.48

Again, the external curved surface area (A) $= 2\pi Rh = 2 \times \frac{22}{7} \times 10.5 \times 56 \quad = 3696\ cm^2$

$\therefore$ Cost of polishing the outer surface (T) = A × R = 3696 × 20 paisa = 73920 paisa
= Rs 739.20

16. ***A gate has two cylindrical pillars with a hemispherical end on a top of each pillar. The height of each pillar is 9.96 cm and the height of each cylinder is 9.75 m. Find the cost of colouring the surface of both pillars at Rs 500 per sq.m.***



Solution:

Here, For a pillar height of cylindrical part (h) = 9.75 m
height of hemispherical part = radius of base (r) = 9.96 m – 9.75 m = 0.21 m

Now, C.S.A of a pillar = C.S.A of hemisphere + C.S.A of cylinder

$= 2\pi r2 + 2\pi rh = 2\pi r(r + h) = 2 \times \frac{22}{7} \times 0.21(0.21 + 9.75) = 13.1472\ m^2$

$\therefore$ Total surface area of two pillars = $2 \times 13.1472\ m^2 = 26.2944\ m^2$

$\therefore$ Cost of colouring the surface of the both pillar (T) = A × R = 26.2944 × Rs 500
= Rs 13,147.20

Extra Questions

1. The total surface and curved surface area of a cylindrical can are $748\ cm^2$ and $440\ cm^2$ respectively. Find its volume. [Ans: $1540\ cm^3$]
2. Some circular plates of same size, each of diameter 14 cm and thickness 5 mm are placed one over other to form a right cylinder having a volume of $7700\ cm^3$, find the number of plates. [Ans: 100]
3. The sum of the height and radius of base of a cylinder 14 cm. If the curved surface area of the cylinder is 231 square centimetres, find the volume of the cylinder when its height is less than the radius of base. [Ans: $1212.75\ cm^3$]
4. The height and the total surface area of a solid log composed of a cylinder and a hemisphere at one end are 19 cm and $2288\ cm^2$ respectively, find the he radius of its base. [Ans: Rs 55,440]
5. A roller of diameter 1.4 m and length 2m takes 600 complete revolutions to level a playground. Calculate the cost of gravelling the ground at Rs 10.50 per square meter.
6. A water-well is composed of 28 identical cement rings with 1.4m diameter and 0.2m height.

(i) If the labour cost for digging out the well is Rs. 500 per cubic metre and the cost of a cement ring is Rs 2500, estimate the total cost of well construction.

(ii) If the water level is found up to 20 rings from the bottom, find how many litres of water were in the well? Find it. [Ans: Rs 74, 312, 6,160 lt.]

Unit 7 Mensuration (II): Prism and Pyramid

Allocated teaching periods	11

Competency

- Solving the problems related to surface area and volume of regular solid objects (prism, pyramid, cylinder, sphere, hemisphere and cone)

Learning Outcomes

- To solve the problems related to surface area and volume of regular solid objects (prism, pyramid, cylinder, sphere, hemisphere and cone)

Level-wise learning objectives

S.N.	LEVELS	OBJECTIVES
1.	Knowledge (K)	- To define prism - To recall the area formula of triangles - To tell the formula of finding the volume, lateral surface area and total surface area of triangular prism - To define pyramid - To relate the side of base (a), height (h) and slant height (l) of square based pyramid - To tell the formula of finding the volume and total surface area of square based pyramid - To label radius (r), height (h) and slant height (l) of cone - To relate the radius of base (r), height (h) and slant height (l) of cone - To list the formula of finding the volume , C.S.A. and T.S.A. of cone
2.	Understanding (U)	- To find the volume of triangular prism - To find the area of rectangular faces of prism - To calculate the area of total surface area of prism - To find the volume, LSA and TSA of square based pyramid - To find the volume, LSA and TSA of cone with the given measurements
3.	Application (A)	- To apply the related formula to solve the problems of triangular prism - To find the CSA, TSA and volume of combine d solid figure composed of cone and hemisphere, cylinder and cone, pyramid and cuboid etc.
4.	High Ability (HA)	- To estimate the cost and quantity related the solid objects (gate, compound wall, well etc.)

Required Teaching Materials/ Resources

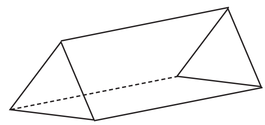
Pencil, scissors, models of triangular prisms, nets of triangular prism, models of cone and square based pyramid, model of combined solids, colourful chart-paper with definitions and formulae etc.

Teaching learning strategies

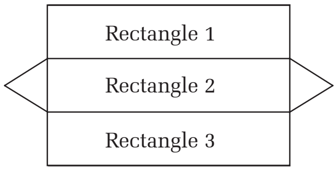
Pre-knowledge: Perimeter and area of triangle, rectangle and circle.

A. PRISM

Teaching Activities



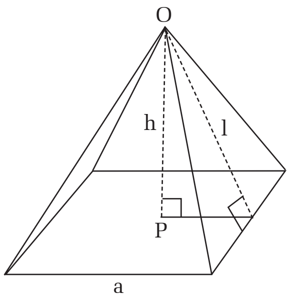
1. With manipulative models of prisms, discuss upon the prism and its base and lateral surface.
2. Discuss with models on volume of prism as the product of cross-sectional area and the height i.e., $V = A \times h$

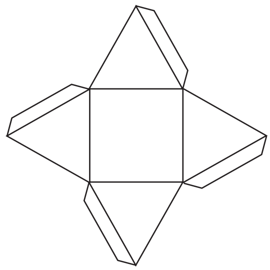


3. Make clear on area of rectangular faces or lateral surface area (L.S.A) as the product perimeter of base and height i.e., $L.S.A = (a + b + c) \times h = P \times h$ and the total surface area $= L.S.A + 2A = P \times h + 2A$ by using models or nets of prism.

B. PYRAMID

Teaching Activities



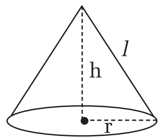


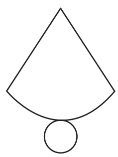
1. With manipulative models of prisms, discuss upon the pyramid and its base and lateral surface.
   Explain the relation among a, h and l as $l^2 = \left(\frac{a}{2}\right)^2 + h^2$
2. List the following formulae:
   Area of base (A) $= a^2$
   Volume of pyramid (V) $= \frac{1}{3}$ area of base (A) $\times$ height (h) $= \frac{1}{3} a^2h$
   Area triangular faces or lateral surface area (L.S.A) $== 4\frac{1}{2}$ al $= 2al$
   Total surface area (T.S.A.) $= a^2 + 2al$

C. CONE

Teaching Activities

1. Using manipulative models of cone describe about the cone and its base and curved surface.





2. Explain the relation among r, h and l as $l^2 = r^2 + h^2$
3. List the following formulae:
   - Area of base (A) $= \pi r^2$
   - Volume of pyramid (V) $= \frac{1}{3}$ area of base (A) $\times$ height (h) $= \frac{1}{3} \pi r^2h$
   - Curved surface area (C.S.A) $= = \pi rl$
   - Total surface area (T.S.A.) = Area of base (A) + C.S.A. $= \pi r^2 + \pi rl = \pi r (r + l)$
4. Using manipulative models of combined solid ask and explain the volume and surface area of the solid.

Solution of selected questions from Vedanta Excel in Mathematics

1. ***The area of cross section of a triangular prism is 126 cm² and its volume is 5040 cm³. If the perimeter of the base is 54 cm. find its***
   i) lateral surface area (ii) total surface area

Solution:

Here, Crsos - section area (A) $= 126\text{ cm}^2$

Volume (V) $= 5040\text{ cm}^3$

Perimeter of bae (P) $= 54\text{ cm}$

Now, Volume (V) = cross - section area (A) × height (h)

or, $5040\text{ cm}^3 = 126\text{ cm}^2 \times h \quad \therefore h = 40\text{ cm}$

i) L.S.A. of triangular prism = Perimeter of base (P) × height (h)

$= 54\text{ cm} \times 40\text{ cm} = 2160\text{ cm}^2$

ii) T.S.A of triangular prism $= \text{L.S.A} \times 2A = 2160\text{ cm}^2 + 2 \times 126\text{ cm}^2 = 2412\text{ cm}^2$

2. ***The volume of an isosceles right angled triangular prism is 1000 cm³. If the length of the prism is 20 cm, find (i) its lateral surface area (ii) its total surface area***

Solution:

Here, the triangular base is isosceles right angled triangle. Let the equal sides of base be x cm.

Now, area of base (A) $= \frac{1}{2} b \times p = \frac{1}{2} \times x \times x = \frac{x^2}{2}\text{ cm}^2$

We have, volume (V) = Area of base (A) × height (h)

or, $1000\text{ cm}^3 = \frac{x^2}{2} \times \cancel{20}10 \quad \therefore x = 10$

$\therefore p = b = 10\text{ cm}$ and $h = \sqrt{p^2 + b^2} = \sqrt{10^2 + 10^2} = 10\sqrt{2}$

Perimeter of base (P) $= (p + b + h)$

$= (10\text{ cm} + 10\text{ cm} + 10\sqrt{2}\text{ cm}) = (20 + 10\sqrt{2})\text{ cm} = 34.14\text{ cm}$

i) L.S.A. of the prism $= P \times h = 34.14\text{ cm} \times 20\text{ cm} = 682.8\text{ cm}^2$

ii) T.S.A. of the prism $= P \times h + 2A$

$= 34.34 \times 20 + \cancel{2} \times \frac{1}{\cancel{2}} \times b \times p = 682.8\text{ cm}^2 + 10 \times 10 = 782.8\text{ cm}^2$

3. ***The area and perimeter of the base of a triangular prism are 24 cm² and 24 cm respectively. If the total surface area of the prism is 408 cm², find its lateral surface area and volume.***

Solution:

Here, in the triangular prism; area of base (A) $= 24\text{ cm}^2$

perimeter of base (P) $= 24\text{ cm}$

T.S.A. $= 408\text{ cm}^2$

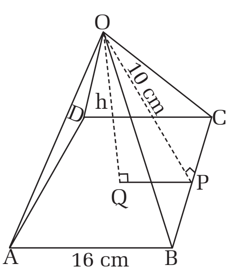
Now, T.S.A of triangular prism $= P \times h + 2A$

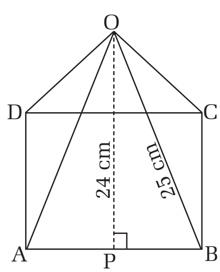
or, $408 = 24 \times h + 2 \times 24$ or, $h = 15\text{ cm}$

Again, L.S.A. of prism $= P \times h = 24\text{ cm} \times 15\text{ cm} = 360\text{ cm}^2$

Volume of prism $= A \times h = 24\text{ cm}^2 \times 15\text{ cm} = 360\text{ cm}^2$

4. ***Find the total surface area and volume of the given square - based pyramids.***





Solution:

a) Here, length of side of square base (a) $= 16\text{ cm}$

slant height (l) $= 10$ cm

Now, total surface area of the pyramid $= a^2 + 2al = 16^2 + 2 \times 10 \times 10 = 156$ cm^2

Let OQ be the height of the pyramid

Then, $PQ = \frac{1}{2} AB = \frac{1}{2} \times 16$ cm $= 8$ cm

In right angled ΔPOQ, $OQ = \sqrt{OP^2 - PQ^2} = \sqrt{10^2 - 8^2} = 6$ cm

Also, Volume of the pyramid (V) $= \frac{1}{3} a^2h = \frac{1}{3} \times 16 \times 16 \times 6 = 512$ cm^3

Alternative method:

We have, $\left(\frac{a}{2}\right)^2 + h^2 = l2$ or, $\left(\frac{16}{2}\right)^2 + h^2 = 10^2 \therefore h = 6$ cm

$\therefore$ Volume (V) $= \frac{1}{3}a^2h = \frac{1}{2} \times 16 \times 16 \times 6 = 512$ cm^3

b) Here, slant height (OP) $= 24$ cm, edge (OB) $= 25$ cm

Now, in right angled ΔDPB; $PB = \sqrt{OB^2 - OP^2} = \sqrt{25^2 - 24^2} = 7$ cm

$\therefore$ AB(a) $= 2 \times PB = 2 \times 7$ cm $= 14$ cm

Also, T.S.A. of pyramid $= a^2 + 2al = 14^2 + 2 \times 14 \times 24 = 868$ cm^2

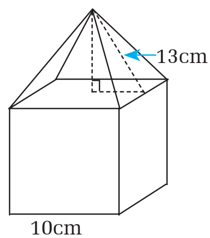
We have, $(\frac{a}{2})^2 + h^2 = l^2$ or, $(\frac{14}{2})^2 + h2 = 24^2$

or, $h^2 = 576 - 49 = 527 \therefore h = 22.96$ cm

Again, Volume of the pyramid (V) $= \frac{1}{3}a^2h = \frac{1}{3} \times 14 \times 14 \times 22.96 = 1500.05$ cm^3

5. *The adjoining solid object is formed with the combination of a pyramid and a square - based prism. If the volume of the solid object is 900 cu.cm, find the height of the prism.*

Solution:



Here, the given solid object is formed with the combination of a pyramid and a square - based prism

For uppermost pyramid:

Length of side of squared base (a) $= 10$ cm

slant height (l) $= 13$ cm, height (h) $= ?$

We have $\left(\frac{a}{2}\right)^2 + h^2 = l^2$ or, $\left(\frac{10}{2}\right)^2 + h^2 = 13^2$

or, $h^2 = 169 - 25 = 144 \therefore h = 12$ cm

$\therefore$ Volume of pyramid $(V_1) = \frac{1}{3}a^2h = \frac{1}{\cancel{3}} \times 10 \times 10 \times \cancel{12}^{4} = 400$ cm^3

For lowermost pyramid:

Length $(l_1) = 10$ cm, breadth $(b_1) = 10$ cm, height $(h_1) = ?$

$\therefore$ Volume $(V_2) = l_1 \times b_1 \times h_1 = 10 \times 10 \times h_1 = 100h_1$ cm^3

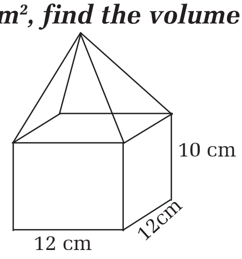
Again, Volume of solid (V) = Volume of pyramid (V_1) + Volume of prism (V_2)

or, $900 = 400 + 100h_1$ or, $h_1 = 5$ cm

Hence, the height of the prism is 5 cm

6. *The solid given alongside is the combination of a square based pyramid and a square -based prism. If the total surface area of the solid is 86cm², find the volume of the solid.*

Solution:



Here

For uppermost square-based pyramid;

length of side of base (a) $= 12$ cm

slant height $= l$ cm

$\therefore$ (L.S.A.)$_1 = 2al = 2 \times 12 \times l = 24l$ cm^2

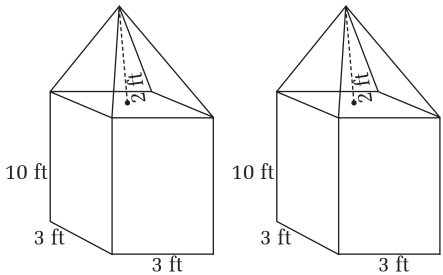
For lower most prism;

Length $(l_1) = 12$ cm, breadth $(b_1) = 12$ cm, height $(h_1) = 10$ cm

$\therefore$ $(L.S.A)_2 = 2h(l + b) \; = 2 \times 10(12 + 12) \; = 480 \text{ cm}^2$
Area of base (A) $= l^2, \; = 12 \text{ cm} \times 12 \text{ cm} \; = 144 \text{ cm}^2$
Now, total surface area of the solid $= (L.S.A.)_1 + (L.S.A.)_2 + A$
or, $864 \text{ cm}^2 = 24l + 480 \text{ cm}^2 + 144 \text{ cm}^2$
or, $24l = 240 \;\; \therefore l = 10$ cm
Also, height of pyramid (h) $= \sqrt{l^2 - \left(\frac{a}{2}\right)^2} \; = \sqrt{10^2 - \left(\frac{12}{2}\right)^2} \; = 8$ cm
Again, volume of the solid (V) = Volume of pyramid + Volume of prism

$$= \frac{1}{3} a^2h + l_1 \times b_1 \times h_1$$
$$= \frac{1}{\cancel{3}} \times \cancel{12}^{4} \times 8 + 12 \times 12 \times 10 = 1824 \text{ cm}^3$$

7. ***The gate lof stadium has two pillars, each of height 10 ft. with four visible lateral faces and 3 ft. × 3 ft. bases. The top of each pillar has combined pyramid of height 2 ft. If the combined structures of both pillars and pyramid are painted at the rate Rs 80 per sq.ft, calculate the cost of painting.***

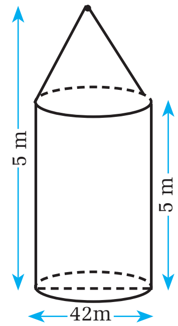


Solution:
Here,
For square - based pyramid;
length of side of base (a) = 3 ft
height (h) = 2ft
We have slane height $(l) = \sqrt{\left(\frac{a}{2}\right)^2 + h^2} \; = \sqrt{\left(\frac{3}{2}\right)^2 + 2^2} \; = \sqrt{2.25 + 4} \; = \sqrt{6.25} \; = 2.5$ ft
$\therefore$ Lateral surface area of pyramid $(L.S.A.)_1 = 2al \; = 2 \times 3 \times 2.5 \; = 15$ sq.ft
Also, for cuboid; length(l_1) = 3 ft, breadth(b_1) = 3 ft and height(h_1) = 10 ft
$\therefore$ Lateral surface area of cuboid $(L.S.A.)_2 = 2h(l + b) \; = 2 \times 10(3 + 3) \; = 120$ sq.ft
Now, total surface area of each combined structure $= (L.S.A)_1 + (L.S.A)_2$
$= 15 + 120 = 135$ sq.ft
$\therefore$ Total surface area of both combined structure $= 2 \times 135 \; = 270$
Again, rate of painting the structure (R) = Rs 80 per sq.ft
$\therefore$ Cost of painting $= A \times R = 270 \times$ Rs 80 = Rs 21,600

8. ***150 people can be accommodated inside a conical tent. If 3m² space and 10 m³ air is required for each person, find the height of the tent.***

Solution:
Here,
Space required for 1 person on the ground $= 3 \text{ m}^2$
$\therefore$ Space required for 150 people on the ground $= 150 \times 3\text{m}^2 \; = 450 \text{ m}^2$
$\therefore$ Area of base of conical tent (A) $= 450 \text{ m}^2$
Also,
Volume of air required to breathe for 1 person $= 10 \text{ m}^3$
Volume of air required to breathe for 150 people $= 150 \times 10 \text{ m}^3 \; = 1500 \text{ m}^3$
$\therefore$ Volume of conical tent (V) = Volume of air required for 150 people $= 150 \text{ m}^3$
Now, volume of conical tent (V) $= \frac{1}{3} A \times h$
or, $1500 = \frac{1}{\cancel{3}} \times \cancel{450}^{150} \times h$
$\therefore h = 10$
Hence the height of the tent is 10 m

9. ***A tent of height 33 m is in the form of a sight circular cylinder of diameter 42 m and height 5 m surmounted by a right circular cone of the same diameter. Find the total area of cloths required to cover the tent. Also, find the cost of cloths at Rs 125 per sq.m.***



Solution:

Here,

For conical Part

Diameter of base = 42 m $\therefore$ radius (r) = $\frac{42}{2}$m = 21 m

Height (h) = 33 m – 5 m = 28 m

We have, slant height (l) = $\sqrt{r^2 - h^2}$ = $\sqrt{21^2 - 28^2}$ = 35 cm

$\therefore$ curved surface area $(C.S.A)_1$ = Nl = $\frac{22}{7} \times 21 \times 35$ = 2310m^2

For cylindrical part; radius(r) = 21 m and height (h_1) = 5m

Now, the total area of required cloth (A) = $(C.S.A.)_1 + (C.S.A)_2$

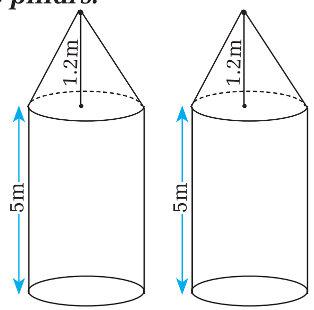
2310 m^2 + 660 m^2 = 2970 m^2

Again, rate of cloths (R) = Rs 125 per m^2

$\therefore$ Cost of required cloths (T) = A × R = 2970 × Rs 125 = Rs 3,71,250

10. ***The gate of a stadium has two cylindrical pillars, each of height 5 m and the circumference of each base is 22 m. The top of each pillar has a combined cone of height 1.2m. If the combined structures of both pillars and cones are painted at the rate of Rs 100 per sq.m, find teh total cost of painting the pillars.***

Solution:



Here, circumference of circular base = 22 m

or, $2\pi r = 22$ m

or, $2 \times \frac{22}{7} \times r = 22$ m

or, $\therefore$ r = 3.5 m

Height of cylinder (h) = 5 m

Also, height of cone (h_1) = 1.2 m, radius (r) = 3.5 m

$\therefore$ Slant height (l) = $\sqrt{r^2 + h_1^2}$ = $\sqrt{3.5^2 + 1.2^2}$ = 3.7 cm

Now,the surface area of a tructure = C.S.A of cone + C.S.A. of cylinder

$= \pi r l + 2\pi r h = \pi r(l + 2h)$

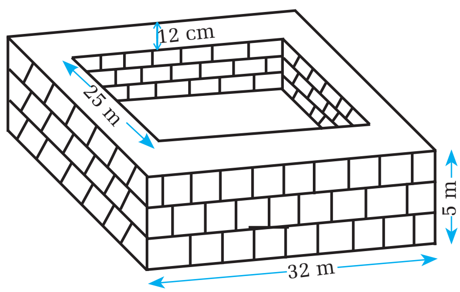
$= \frac{22}{7} \times 3.5\ (3.7 + 2 \times 5) = 150.7$ m^2

$\therefore$ The surface area of two structures (A) = 2 × 150.7 m^2 = 301.4 m^2

Again, rate of painting the corbined structures (R) = Rs 100 per sq.m

$\therefore$ Cost of painting the pillar (T) = A × R = 301.4 × Rs 100 = Rs 30,140

11. ***The adjoining figure is a rectangular compound wall of a house. Estimate the cost of building the wall by using bricks of size 20 cm × 8 cm × 5 cm. The rate of the cost of the brick is Rs 1,750 per 1,000 bricks. Labourer wages is Rs 750 per labourer per day and 3 labourers are required for 4 days.***



Solution:

Here, the outer length of the wall (l) $= 32$ m
the inner breadth of the wall $= 25$ m
the thickness of the wall (d) $= 12$ cm $= 0.12$ m
$\therefore$ the outer breadth of the wall (b) $= 25 \text{ m} + 2d = 25 \text{ m} + 2 \times 0.12 \text{ m} = 25.24$ m
the height of the wall (h) $= 5$ m

Now, the cross-sectional area of the wall (A) $= 2d(l + b - 2d)$
$= 2 \times 0.12\,(32 + 25.24 - 2 \times 0.12)$
$= 13.68 \text{ m}^2$

$\therefore$ Volume of the wall (V) $=$ cross - sectional area (A) $\times$ height(h)
$= 13.68 \text{ m}^2 \times 5 \text{ m} = 68.4 \text{ cm}^3$
$= 68.4 \times 100 \times 100 \times 100 \text{ cm}^3$
$= 68400000 \text{ cm}^3$

Also Volume of each brick (v) $= 20 \text{ cm} \times 8 \text{ cm} \times 5 \text{ cm} = 800 \text{ cm}^3$

$\therefore$ Number of bricks required (N) $= \dfrac{\text{Volume of wall (V)}}{\text{Volume of each brick (v)}} = \dfrac{68400000 \text{ cm}^3}{800 \text{ cm}^3}$
$= 85{,}500$

Again, the cost of 1,000 bricks $=$ Rs 1,750
the cost of 1 brick $=$ Rs $\dfrac{1{,}750}{1000}$
$\therefore$ the cost of 85,500 brick $=$ Rs$\dfrac{1{,}750}{1000} \times 85{,}500 =$ Rs 1,49,625

Also, labour wages for 3 labourers per day $= 3 \times$ Rs 750 $=$ Rs 2,250
$\therefore$ labourer wages for 3 labourers for 4 days $= 4 \times$ Rs 2,250 $=$ Rs 9,000

Hence, the total estimation of building the compound wall $=$ Rs 1,49,625 + Rs 9,000
$=$ Rs 1,58,625

Extra Questions

1. Mr. Rai builds a square based tent 24 feet high for the accommodation of the guests for his daughter's marriage ceremony. If each edge of the tent is 25 feet and each people requires 16 cu feet of air to breathe. How many people can be arranged inside the tent?
[Ans: 49]

2. The roof of a temple is a square based pyramid. The vertical height of the roof is 3 m and slant height is 5m. How much zinc plate is required to cover the roof of a temple? Estimate the cost of zinc plates required at the rate of Rs 110 per sq. meter.
[Ans: 80 sq. m, Rs 8800]

3. Khushbu wants to fill the ice-cream cones with chocolate cream making hemispherical tops. For this, she brings some cones each of having base diameter 4.2 cm and slant height 7.5 cm and a can of cream with capacity 800 cm^3, for how many cones can the cream be enough? [Ans: 15]

Unit 8 Highest Common Factor (H.C.F.) and Lowest Common Multiples (L.C.M.)

Allocated teaching periods	6

Competency

- To find the H.C.F. and L.C.M. of algebraic expressions by factorization method
- To solve the contextual problems related to H.C.F. and L.C.M.

Learning Outcomes

- To find the H.C.F. and L.C.M. of algebraic expressions by factorization method

Level-wise learning objectives

S.N.	LEVELS	OBJECTIVES
1.	Knowledge (K)	- To define H.C.F. of given expressions - To define L.C.M. of given expressions
2.	Understanding (U)	- To factorise the simple expressions and determine the H.C.F. and L.C.M.
3.	Application (A)	- To find the H.C.F. and L.C.M.
4.	High Ability (HA)	- To relate the contextual problems in to H.C.F. and L.C.M. and solve them

Required Teaching Materials/ Resources

Charts with formulae, models of Venn-diagram, audio-video materials etc

Pre-knowledge: Factorisation of algebraic expressions

Teaching Activities

1. Warm up the class with recalling highest common factor and lowest common multiples of the numbers as learnt in the arithmetic in basic level.
2. Recall the following formulae

S.N.	Expressions	Factorised form	Expanded form
1.	$(a+b)^2$	$(a+b)(a+b)$	$a^2+2ab+b^2$ or $(a-b)^2+4ab$
2.	$(a-b)^2$	$(a-b)(a-b)$	$a^2-2ab+b^2$ or $(a+b)^2-4ab$
3.	a^2-b^2	$(a+b)(a-b)$	
4.	a^2+b^2		$(a+b)^2-2ab$ or $(a-b)^2+2ab$
5.	$(a+b)^3$	$(a+b)(a+b)(a+b)$	$a^3+3a^2b+3ab^2+b^3$ or $a^3+b^3+3ab(a+b)$
6.	$(a-b)^3$	$(a-b)(a-b)(a-b)$	$a^3-3a^2b+3ab^2-b^3$ or $a^3-b^3-3ab(a-b)$

7.	$a^3 + b^3$	$(a + b)(a^2 - ab + b^2)$	$(a + b)^3 - 3ab(a + b)$
8.	$a^3 - b^3$	$(a - b)(a^2 + ab + b^2)$	$(a - b)^3 + 3ab(a - b)$

3. Divide the students into four five groups and give some real life problems related to factorisation

Group A: A students distributed $x^2 + 5x + 6$ chocolates equally among some of his/her friends, find the possible number of his/her friends and the number of chocolates received by each

Group B: The area of a rectangular floor is $(3x^2 + 12xy)$ m^2, find the length and breadth of the room.

Group C: The area of a rectangular plot is $(x^2 - 9y^2)$ ft^2, find the length and breadth of the plot.

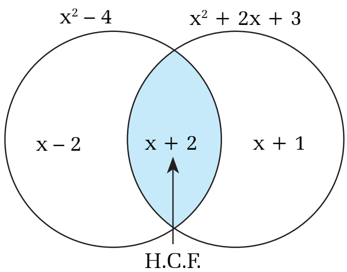
Group D: The volume of a rectangular room is $(x^3 - 8y^3)$ m^3, find the height and base area of the room.

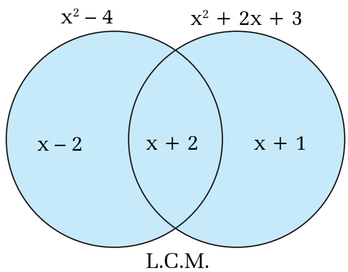
4. Discuss on following process of the factorisations

(i) Taking common (ii) Using proper formula

(iii) Splitting the middle terms (iv) Completing the perfect squares

5. Explain H.C.F. and L.C.M. with proper examples by using the models of intersection and union of sets in Venn-diagrams. For example show the H.C.F. and L.C.M. of $x^2 - 4$ and $x^2 + 2x + 3$ using chart paper





From Venn-diagram, H.C.F. = $x + 2$ and L.C.M. = $(x + 2)(x - 2)(x + 1)$

Divide the students and ask to find the H.C.F. and L.C.M. of similar expressions by showing in the Venn-diagram in chart paper.

6. Define H.C.F. as the algebraic term/s having highest degree and largest coefficient that divides all the given expressions without remainders

7. Define L.C.M. as the algebraic term/s having lowest degree and least coefficient which is exactly divisible by the given expressions.

8. While factorising the expressions, raise the questions like how?, why?, what comes then? in every steps

9. Guide the students to find the highest common factor (H.C.F.) and lowest common multiples (L.C.M.) of the expressions from the book

10. Call the students in front of the class to solve the related problems on the board.

11. Give some contextual problems related to the lesson and make discussion in the group.

Solution of selected questions from Vedanta Excel in Mathematics

1. *Find the HCF of $16x^4 - 4x^2 - 4x - 1$ and $8x^3 - 1$*

Solution:

Here, the 1st expression $= 16x^4 - 4x^2 - 4x - 1$

$= 16x^4 - (4x^2 + 4x + 1) = 16x^4 - [(2x)^2 + 2.2x.1 + (1)^2]$

$= (4x^2)^2 - (2x + 1)^2 = (4x^2 + 2x + 1)(4x^2 - 2x - 1)$

The 2nd expression $= 8x^3 - 1 = (2x)^3 - (1)^3 = (2x - 1)(4x^2 + 2x + 1)$

$\therefore$ H.C.F $= 4x^2 + 2x + 1$

2. *Find the HCF of $9x^2 - 4y^2 - 8yz - 4z^2$, $4z^2 - 4y^2 - 9x^2 - 12xy$, $9x^2 + 12xz + 4z^2 - 4y^2$*

Solution:

Here, the 1st expression $= 9x^2 - 4y^2 - 8yz - 4z^2 = 9x^2 - (4y^2 + 8yz + 4z^2)$

$= 9x^2 - [(2y)^2 + 2.2y.2z + (2z)^2] = (3x)^2 - (2y + 2z)^2 = (3x + 2y + 2z)(3x - 2y - 2z)$

the 2nd expression $= 4z^2 - 4y^2 - 9x^2 - 12xy = 4z^2 - (4y^2 + 9x^2 + 12xy)$

$= 4z^2 - [(2y)^2 + 2.2y.3x + (3x)^2] = (2z)^2 - (2y + 3x)^2$

$= (2z + 2y + 3x)(2z - 2y - 3x) = (3x + 2y + 2z)(2z - 2y - 3x)$

the 3rd expression $= 9x^2 + 12xz + 4z^2 - 4y^2$

$= (3x)^2 + 2.3x.2z + (2z)^2 - (2y)^2 = (3x + 2z)^2 - (2y)^2 = (3x + 2y + 2z)(3x - 2y + 2z)$

$\therefore$ H.C.F $= (3x + 2y + 2z)$

3. *Find the H.C.F and L.C.M of $x^2 - 10x - 11 + 12y - y^2$, $x^2 - y^2 + 22y - 121$*

Solution:

Here, the 1st expression $= x^2 - 10x - 11 + 12y - y^2$

$= x^2 - 2.x.5 + 5^2 - 5^2 - 11 + 12y - y^2$

$= (x - 5)^2 - y^2 + 12y - 36 = (x - 5)^2 - (y^2 - 12y + 36) = (x - 5)^2 - (y^2 - 2.y.6 + 6^2)$

$= (x - 5)^2 - (y - 6)^2 = (x - 5 + y - 6)[(x - 5) - (y - 6)] = (x + y - 11)(x - y + 1)$

the 2nd expression $= x^2 - x^2 + 22y - 121 = x^2 - (y^2 - 22y + 121)$

$= x^2 - (y^2 - 2.y.11 + 11^2) = x^2 - (y - 11)^2 = (x + y - 11)(x - y + 11)$

$\therefore$ H.C.F $= (x + y - 11)$ and L.C.M $= (x + y - 11)(x - y + 1)(x - y + 11)$

4. *Find the H.C.F and L.C.M of $1 + 2x - 6x^3 - 9x^4$, $9x^4 + 2x^2 + 1$*

Solution:

Here, 1st expression $= 1 + 2x - 6x^3 - 9x^4$

$= 1 - 9x^4 + 2x - 6x^3 = (1)^2 - (3x^2)^2 + 2x(1 - 3x^2)$

$= (1 + 3x^2)(1 - 3x^2) + 2x(1 - 3x^2) = (1 - 3x^2)(1 + 3x^2 + 2x) = (1 - 3x^2)(1 + 2x + 3x^2)$

2nd expression $= 9x^4 + 2x^2 + 1 = (3x^2)^2 + (1)^2 + 2x^2 = (3x^2 + 1)^2 - 6x^2 + 2x^2$

$= (3x^2 + 1)^2 - (2x)^2 = (3x^2 + 2x + 1)(3x^2 - 2x + 1)$

$\therefore$ H.C.F $= 3x^2 + 2x + 1$ and L.C.M $= (1 - 3x^2)(3x^2 + 2x + 1)(3x^2 - 2x + 1)$

$= (1 - 3x^2)(9x^4 + 2x^2 + 1)$

Extra questions for practice

A. *Find the H.C.F and L.C.M of the following expressions*

1. $x^3 + 1 + 2x^2 + 2x$, $x^3 - 1$, $x^4 + x^2 + 1$ [Ans: $x^2 + x + 1$, L.C.M. $= (x^2 - 1)(x^4 + x + 1)$]
2. $(a + b)^2 - 4ab$, $a^3 - b^3$, $a^2 + ab - b^2$
   [Ans: H.C.F. $= (a - b)$, L.C.M. $= (a + b)^2 (a + 2b)(a^2 + ab + b^2)$]
3. $16x^4 - 4x^2 - 4x - 1$, $8x^3 - 1$, $16x^4 + 4x^2 + 1$
   [Ans: H.C.F. $= 4x^2 + 2x + 1$, L.C.M. $= (8x^3 - 1)(4x^2 - 2x + 1)$]
4. $p^4 + 10p^2 + 169$, $p^3 + p(p + 13) + 3p^2$, $3p2 + 4(3p + 5) + 19$
   [Ans: H.C.F. $= p^2 + 4p + 13$, L.C.M. $= 3p(p^4 + 10p^2 + 169)$]
5. $x^4 + (2b^2 - a^2)x^2 + b^4$, $x^4 + 2ax^3 + a^2x^2 - b^4$
   [Ans: H.C.F. $= x^2 + ax + b^2$, L.C.M. $= (x^2 + ax + b^2)(x^2 - ax + b^2)(x^2 + ax - b^2)$]

Unit 9 Simplification of Rational Expressions

Allocated teaching periods	5

Competency

- To simplify the algebraic fractions

Learning Outcomes

- To simplify the problems related to rational expressions

Level-wise learning objectives

S.N.	LEVELS	OBJECTIVES
1.	Knowledge (K)	- To define algebraic rational expressions - To tell the condition under which the expression become the rational expression
2.	Understanding (U)	- To simplify the simple rational expressions
3.	Application (A)	- To simplify the algebraic rational expressions
4.	High Ability (HA)	- To prove the rational expressions under the given conditions

Required Teaching Materials/ Resources

Chart paper, scissors, colourful markers, ICT tools, audio-video materials etc.

Pre-knowledge: L.C.M. of expressions

Teaching Activities

1. Recall the lowest common multiples (L.C.M.) of the algebraic expressions
2. Explain about the existence of rational expressions
3. Simplify the expressions and give the same type problems as class-work in groups

Solution of selected questions from Vedanta Excel in Mathematics

1. *Simplify* $\dfrac{a^2-(b-c)^2}{(a+c)^2-b^2}+\dfrac{b^2-(a-c)^2}{(a+b)^2-c^2}+\dfrac{c^2-(a-b)^2}{(b+c)^2-a^2}$

Solution:

$$\frac{a^2-(b-c)^2}{(a+c)^2-b^2}+\frac{b^2-(a-c)^2}{(a+b)^2-c^2}+\frac{c^2-(a-b)^2}{(b+c)^2-a^2}$$

$$=\frac{(a+b-c)\cancel{(a-b+c)}}{(a+c+b)\cancel{(a+c-b)}}+\frac{\cancel{(b+a-c)}(b-a+c)}{(a+b+c)\cancel{(a+b-c)}}+\frac{(c+a-b)\cancel{(c-a+b)}}{(b+c+a)\cancel{(b+c-a)}}$$

$$=\frac{a+b-c+b-a+c+c+a-b}{a+b+c}=\frac{a+b+c}{a+b+c}=1$$

2. *Simplify* $\dfrac{x+1}{2x^3-4x^2}+\dfrac{x-1}{2x^3+4x^2}-\dfrac{1}{x^2-4}$

Solution:

$$\frac{x+1}{2x^3-4x^2}+\frac{x-1}{2x^3+4x^2}-\frac{1}{x^2-4}=\frac{x+1}{2x^2(x-2)}+\frac{x-1}{2x^2(x+2)}-\frac{1}{x^2-4}$$

$$=\frac{(x+1)(x+2)+(x-1)(x-2)}{2x^2(x-2)(x+1)}-\frac{1}{(x+2)(x-2)}=\frac{x^2+2x+x+2+x^2-2x-x-2-2x^2}{2x^2(x+2)(x-2)}=0$$

3. Simplify $\dfrac{3x-1}{9x^2-3x+1} - \dfrac{3x+1}{9x^2+3x+1} + \dfrac{54x^3}{81x^4+9x^2+1}$

Solution:

$$\frac{3x-1}{9x^2-3x+1} - \frac{3x+1}{9x^2+3x+1} + \frac{54x^3}{81x^4+9x^2+1}$$

$$= \frac{(3x-1)(9x^2+3x+1)-(3x+1)(9x^2-3x+1)}{(9x^2-3x+1)(9x^2+3x+1)} + \frac{54x^3}{(9x^2)^2+(1)^2+9x^2}$$

$$= \frac{(3x)^3-(1)^3-[(3x)^3+(1)^3]}{(9x^2-3x+1)(9x^2+3x+1)} + \frac{54x^3}{(9x^2+1)^2-(3x)^2}$$

$$= \frac{\cancel{27x^3}-1-\cancel{27x^3}-1}{(9x^2-3x+1)(9x^2+3x+1)} + \frac{54x^3}{(9x^2+3x+1)(9x^2-3x+1)}$$

$$= \frac{54x^3-2}{(9x^2+3x+1)(9x^2-3x+1)} = \frac{2[(3x)^3-(1)^3]}{(9x^2+3x+1)(9x^2-3x+1)}$$

$$= \frac{2(3x-1)(9x^2+3x+1)}{(9x^2+3x+1)(9x^2-3x+1)} = \frac{2(3x-1)}{9x^2-3x+1)}$$

4. Simplify: $\dfrac{1}{x-5} - \dfrac{1}{x-3} + \dfrac{1}{x+5} + \dfrac{1}{x+3}$

Solution:

$$\frac{1}{x-5} - \frac{1}{x-3} + \frac{1}{x+5} + \frac{1}{x+3}$$

$$= \frac{1}{x-5} - \frac{1}{x+5} - \left(\frac{1}{x-3} + \frac{1}{x+3}\right) = \frac{x+5+x-5}{(x+5)(x-5)} - \frac{x+3+x-3}{(x+3)(x-3)}$$

$$= \frac{2x}{x^2-25} - \frac{2x}{x^2-9} = \frac{2x^3-18x-2x^3+50x}{(x^2-25)(x^2-9)} = \frac{32x}{(x^2-25)(x^2-9)} = \frac{32}{(x^2-9)(x^2-25)}$$

5. Simplify: $1 + \dfrac{b}{a-b} + \dfrac{2ab}{a^2+b^2} - \dfrac{a}{a+b} + \dfrac{4a^3b}{a^4+b^4}$

Solution:

$$1 + \frac{b}{a-b} + \frac{2ab}{a^2+b^2} - \frac{a}{a+b} + \frac{4a^3b}{a^4+b^4} = \frac{a-b+b}{a-b} - \frac{a}{a+b} + \frac{2ab}{a^2+b^2} + \frac{4a^3b}{a^4+b^4}$$

$$= \frac{\cancel{a^2}+ab-\cancel{a^2}+ab}{(a-b)(a+b)} + \frac{2ab}{a^2+b^2} + \frac{4a^3b}{a^4+b^4} = \frac{2a^3b+\cancel{2ab^3}+2a^3b-\cancel{2ab^3}}{(a^2-b^2)(a^2+b^2)} + \frac{4a^3b}{a^4+b^4}$$

$$= \frac{4a^7b+\cancel{4a^3b^5}+4a^7b-\cancel{4a^3b^5}}{(a^4-b^4)(a^4+b^4)} = \frac{8a^7b}{a^8-b^8}$$

Extra questions for practice

1. $\dfrac{a^2+b^2}{ab} - \dfrac{a^2}{b(a+b)} - \dfrac{b^2}{a(a+b)}$ [Ans: 1]

2. $\dfrac{a-x}{a^2-ax+x^2} + \dfrac{a+x}{a^2+ax+x^2} - \dfrac{2x^2}{a^4+a^2x^2+x^4}$ [Ans: $\dfrac{2(a-x)}{a^2-ax+x^2}$]

3. $\dfrac{p+q}{(p-r)(q-r)} + \dfrac{q+r}{(q-p)(r-p)} + \dfrac{r+p}{(r-q)(p-q)}$ [Ans: 0]

4. $\dfrac{a+b}{(a+b)^2-c^2} + \dfrac{b-c}{a^2-(b-c)^2} + \dfrac{c+a}{b^2-(c+a)^2}$ [Ans: 0]

5. $\left(\dfrac{1}{x}+\dfrac{1}{y}\right)(x+y-z) + \left(\dfrac{1}{y}+\dfrac{1}{z}\right)(y+z-x) + \left(\dfrac{1}{z}+\dfrac{1}{x}\right)(z+x-y)$ [Ans: 6]

Unit 10 Indices

Allocated teaching periods	6

Competency

- To simplify the expressions involving indices and solve the exponential equations

Learning Outcomes

- To simplify the expressions by using the laws of indices related to negative and fractional power
- To solve the exponential quadratic equations

Level-wise learning objectives

S.N.	LEVELS	OBJECTIVES
1.	Knowledge (K)	- To identify coefficient, base and power of expression - To recall the laws of indices - To express the product/quotient of expressions having same base in terms of single base
2.	Understanding (U)	- To evaluate the numerical problems by using laws of indices - To simplify/prove the simple given expressions - To solve the exponential equations
3.	Application (A)	- To simplify/prove the given rational expressions (involving roots as well) by applying the laws of indices - To solve the exponential equations of the quadratic form
4.	High Ability (HA)	- To prove the rational expression under the given condition/s. - To prepare the report about the use of indices

Required Teaching Materials/ Resources
Chart papers with laws of indices, scissors, ruler, glue-stick and computer/projector if possible

Pre-knowledge: Laws of indices, basic operations

A. Indices

Teaching Activities

1. Give the practical examples of use of laws of indices.

For example

(i) The cost of 1 kg of apple is Rs 125. Find the cost of 5 kg of apples by using the product law of indices. For, $5 \times 125 = 5^1 \times 5^3 = 5^{1+3} = 5^4 = 625$

(ii) Divide 64 copies are equally among 4 friends by using the quotient law of indices. For, $\frac{64}{4} = \frac{2^6}{2^2} = 2^{6-2} = 2^4 = 16$

2. Recall the following laws of indices by presenting in chart paper with proper examples

(i)	**Product law**	$a^m \times a^n = a^{m+n}$, where 'm' and 'n' are positive integers
(ii)	**Quotient law**	$a^m \div a^n = a^{m-n}$ when $m > n$ $a^m \div a^n = \frac{1}{a^{n-m}}$ when $m < n$
(iii)	**Power law of indices**	$(a^m)^n = a^{m \times n}$, $(ab)^m = a^m b^m$, $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
(iv)	**Law of negative index**	$a^{-m} = \frac{1}{a^m}$ or $a^m = \frac{1}{a^{-m}}$
(v)	**Law of zero index**	$a^\circ = 1, b^\circ = 1, x^\circ = 1$ and so on
(vi)	**Root law of indices**	$\sqrt[n]{a^m} = a^{\frac{m}{n}}$

3. Discuss, give the way of solving the various problems and involve the students in solving the problems from exercise
4. Under given condition, prove the expressions and give the same type problems to the students and tell them to prove in the class.
5. Call the students randomly to solve the problems on the board in order to make them confident to solve the problems

Solution of selected questions from Vedanta Excel in Mathematics

1. *Simplify:* $\left(a^{\frac{1}{x-y}}\right)^{\frac{1}{x-z}} . \left(a^{\frac{1}{y-z}}\right)^{\frac{1}{y-x}} . \left(a^{\frac{1}{z-x}}\right)^{\frac{1}{z-y}}$

Solution:

$$\left(a^{\frac{1}{x-y}}\right)^{\frac{1}{x-z}} . \left(a^{\frac{1}{y-z}}\right)^{\frac{1}{y-x}} . \left(a^{\frac{1}{z-x}}\right)^{\frac{1}{z-y}}$$

$$= a^{\frac{1}{(x-y)(x-z)}} . a^{\frac{1}{(y-z)(y-x)}} . a^{\frac{1}{(z-x)(z-y)}}$$

$$= a^{\frac{1}{(x-y)(x-z)} + \frac{1}{(y-z)(y-x)} + \frac{1}{(z-x)(z-y)}} = a^{-\frac{1}{(x-y)(z-x)} - \frac{1}{(y-z)(x-y)} - \frac{1}{(z-x)(y-z)}}$$

$$= a^{\frac{-(y-z)-(z-x)-(x-y)}{(x-y)(y-z)(z-x)}} = a^{\frac{-y+z-z+x-x+y}{(x-y)(y-z)(z-x)}} = a^0 = 1$$

2. *Simplify:* $\dfrac{p + (pq^2)^{\frac{1}{3}} + (p^2q)^{\frac{1}{3}}}{p-q} \times \left(1 - \dfrac{q^{\frac{1}{3}}}{p^{\frac{1}{3}}}\right)$

Solution:

Here, $\dfrac{p + (pq^2)^{\frac{1}{3}} + (p^2q)^{\frac{1}{3}}}{p-q} \times \left(1 - \dfrac{q^{\frac{1}{3}}}{p^{\frac{1}{3}}}\right)$

$$= \frac{p + p^{\frac{1}{3}}q^{\frac{2}{3}} + p^{\frac{2}{3}}q^{\frac{1}{3}}}{p-q} \times \left(\frac{p^{\frac{1}{3}} - q^{\frac{1}{3}}}{p^{\frac{1}{3}}}\right)$$

$$= \frac{\cancel{p^{\frac{1}{3}}}\left(p^{\frac{2}{3}} + q^{\frac{2}{3}} + p^{\frac{1}{3}}q^{\frac{1}{3}}\right)}{p-q} \times \left(\frac{p^{\frac{1}{3}} - q^{\frac{1}{3}}}{\cancel{p^{\frac{1}{3}}}}\right)$$

$$= \frac{\left(p^{\frac{1}{3}} - q^{\frac{1}{3}}\right)\left[\left(p^{\frac{1}{3}}\right)^2 + p^{\frac{1}{3}}.q^{\frac{1}{3}} + \left(q^{\frac{1}{3}}\right)^2\right]}{p-q} = \frac{\left(p^{\frac{1}{3}}\right)^3 - \left(q^{\frac{1}{3}}\right)^3}{p-q} = \frac{p-q}{p-q} = 1$$

3. *Simplify:* $\dfrac{\left(a^2 - \frac{1}{b^2}\right)^a \times \left(a - \frac{1}{b}\right)^{b-a}}{\left(b^2 - \frac{1}{a^2}\right)^b \times \left(b + \frac{1}{a}\right)^{a-b}}$

Solution:

Here, $$\frac{\left(a^2 - \frac{1}{b^2}\right)^a \times \left(a - \frac{1}{b}\right)^{b-a}}{\left(b^2 - \frac{1}{a^2}\right)^b \times \left(b + \frac{1}{a}\right)^{a-b}} = \frac{\left(a + \frac{1}{b}\right)^a \times \left(a - \frac{1}{b}\right)^a \times \left(a - \frac{1}{b}\right)^{b-a}}{\left(b - \frac{1}{a}\right)^b \times \left(b - \frac{1}{a}\right)^b \times \left(b + \frac{1}{a}\right)^{a-b}}$$

$$= \frac{\left(a + \frac{1}{b}\right)^a \times \left(a - \frac{1}{b}\right)^{a+b-a}}{\left(b + \frac{1}{a}\right)^{b+a-b} \times \left(b - \frac{1}{a}\right)^b} = \left(\frac{a + \frac{1}{b}}{b + \frac{1}{a}}\right)^a \times \left(\frac{a - \frac{1}{b}}{b - \frac{1}{a}}\right)^b$$

$$= \left(\frac{ab+1}{b} \times \frac{a}{ab+1}\right)^a \times \left(\frac{ab-1}{b} \times \frac{a}{ab-1}\right)^b = \left(\frac{a}{b}\right)^a \times \left(\frac{a}{b}\right)^b = \left(\frac{a}{b}\right)^{a+b}$$

4. *If $a + b + c = 0$, show that* $\dfrac{1}{1 + x^a + x^{-b}} + \dfrac{1}{1 + x^b + x^{-c}} + \dfrac{1}{1 + x^c + x^{-a}} = 1$

Solution:

Here, $a + b + c = 0$, $\quad \therefore a + b = -c$

Now L.H.S $= \dfrac{1}{1 + x^a + x^{-b}} + \dfrac{1}{1 + x^b + x^{-c}} + \dfrac{1}{1 + x^c + x^{-a}}$

$$= \frac{x^b}{x^b(1 + x^a + x^{-b})} + \frac{1}{1 + x^b + x^{a+b}} + \frac{x^{a+b}}{x^{a+b}(1 + x^c + x^{-a})}$$

$$= \frac{x^b}{x^b + x^{a+b} + 1} + \frac{1}{1 + x^b + x^{a+b}} + \frac{x^{a+b}}{x^{a+b} + x^{a+b+c} + x^b}$$

$$= \frac{x^b}{1 + x^{a+b} + x^b} + \frac{1}{1 + x^{a+b} + x^b} + \frac{x^{a+b}}{x^{a+b} + x^0 + x^b}$$

$$= \frac{x^b}{1 + x^{a+b} + x^b} + \frac{1}{1 + x^{a+b} + x^b} + \frac{x^{a+b}}{x^{a+b} + 1 + x^b} = \frac{x^{a+b} + x^b + 1}{x^{a+b} + x^b + 1} = \text{RHS} \quad \textit{Proved.}$$

5. *If $abc + 1 = 0$, prove that* $\dfrac{1}{1 - a - b^{-1}} + \dfrac{1}{1 - b - c^{-1}} + \dfrac{1}{1 - c - a^{-1}} = 1$

Solution:

Here, $abc + 1 = 0$ or $abc = -1$ $\quad \therefore ab = \dfrac{-1}{c} = -c^{-1}$

Now, L.H.S $= \dfrac{1}{1 - a - b^{-1}} + \dfrac{1}{1 - b - c^{-1}} + \dfrac{1}{1 - c - a^{-1}}$

$$= \frac{b}{b(1 - a - b^{-1})} + \frac{1}{1 - b + ab} + \frac{ab}{ab(1 - c - a^{-1})}$$

$$= \frac{b}{b - ab - 1} + \frac{1}{1 - b + ab} + \frac{ab}{ab - abc - b}$$

$$= -\frac{b}{1-b+ab} + \frac{1}{1-b+ab} + \frac{ab}{ab+1-b} = \frac{ab-b+1}{ab-b+1} = \text{R.H.S } \textit{proved}$$

6. *Simplify:* $\dfrac{x^{\frac{2}{3}}}{x^{\frac{1}{3}}-1} + \dfrac{x^{\frac{2}{3}}}{x^{\frac{1}{3}}+1} - \dfrac{1}{x^{\frac{1}{3}}+1} - \dfrac{1}{x^{\frac{1}{3}}-1}$

Solution:

Here, $= \dfrac{x^{\frac{2}{3}}}{x^{\frac{1}{3}}-1} + \dfrac{x^{\frac{2}{3}}}{x^{\frac{1}{3}}+1} - \dfrac{1}{x^{\frac{1}{3}}+1} - \dfrac{1}{x^{\frac{1}{3}}-1}$

$$= \frac{x^{\frac{2}{3}}-1}{x^{\frac{1}{3}}-1} + \frac{x^{\frac{2}{3}}-1}{x^{\frac{1}{3}}+1} - \frac{\left(x^{\frac{1}{3}}\right)^2-(1)^2}{x^{\frac{1}{3}}-1} - \frac{\left(x^{\frac{1}{3}}\right)^2-(1)^2}{x^{\frac{1}{3}}+1}$$

$$= \frac{\left(x^{\frac{1}{3}}+1\right)\cancel{\left(x^{\frac{1}{3}}-1\right)}}{\cancel{x^{\frac{1}{3}}-1}} + \frac{\cancel{\left(x^{\frac{1}{3}}+1\right)}\left(x^{\frac{1}{3}}-1\right)}{\cancel{x^{\frac{1}{3}}+1}} = x^{\frac{1}{3}}+1+x^{\frac{1}{3}}-1 = 2x^{\frac{1}{3}}$$

7. *Simplify:* $\dfrac{p^2}{(p-y)^y} - \dfrac{2p}{(p-y)^{y-1}} + \dfrac{1}{(p-y)^{y-z}}$

Solution:

Here, $\dfrac{p^2}{(p-y)^y} - \dfrac{2p}{(p-y)^{y-1}} + \dfrac{1}{(p-y)^{y-z}}$

$$= \frac{p^2}{(p-y)^y} - \frac{2p\,(p-y)}{(p-y)^y} + \frac{(p-y)^2}{(p-y)^y} = \frac{p^2-2p(p-y)+(p-y)^2}{(p-y)^y} = \frac{\{p-(p-y)\}^2}{(p-y)^y} = \frac{y^2}{(p-y)^y}$$

B. Exponential Equations

Teaching Activities

1. Ask the laws of indices
2. Discuss upon the exponential equations like $2^x = 8$, x =? etc.
3. With more examples, list he following ideas
   (i) If $a^x = a^p$ then $x = p$ (ii) If $x^n = k^n$ then $x = k$ (iii) If $a^x = 1$ then $x = 0$
4. Solved some equations and give same type of equations to solve in the class or at home
5. Discuss upon the problems given in the exercise

Solution of selected questions from Vedanta Excel in Mathematics

8. *If $x^a.x^b = (x^a)^b$, prove that $\frac{a}{b} + \frac{b}{x} = ab - 2$.*

Solution:

Here, $x^a \,.\, x^b = (x^a)^b$

or, $x^{a+b} = x^{ab}$

or, $a + b = ab$

(squaring on both sides, we get $(a+b)^2 = (ab)^2$

or, $a^2 + 2ab + b^2 = a^2b^2$

or, $a^2 + b^2 = a^2b^2 - 2ab$

Dividing both sides by ab, we get

$$\frac{a^2+b^2}{ab} = \frac{a^2b^2-2ab}{ab}$$

or, $\dfrac{a}{b} + \dfrac{b}{x} = ab - 2$

9. If $x^2 + 2 = 2^{\frac{2}{3}} + 2^{\frac{-2}{3}}$, show that $2x(x^2 + 3) = 3$

Solution:

Here, $x^2 + 2 = 2^{\frac{2}{3}} + 2^{\frac{-2}{3}}$

or, $x^2 = 2^{\frac{2}{3}} - 2 + 2^{\frac{-2}{3}}$

or, $x^2 = \left(2^{\frac{1}{3}}\right)^2 - 2 \times 2^{\frac{1}{3}} \times 2^{\frac{-1}{3}} + \left(2^{\frac{-1}{3}}\right)^2$

or, $x^2 = \left(2^{\frac{1}{3}} - 2^{\frac{-1}{3}}\right)^2$

or, $x = 2^{\frac{1}{3}} - 2^{\frac{-1}{3}}$

Cubing on both side. We get

$x^3 = \left(2^{\frac{1}{3}} - 2^{\frac{-1}{3}}\right)^3$

or, $x^3 = \left(2^{\frac{1}{3}}\right)^3 - \left(2^{\frac{-1}{3}}\right)^3 - 3 \times 2^{\frac{1}{3}} \times 2^{\frac{-1}{3}} \left(2^{\frac{1}{3}} - 2^{\frac{-1}{3}}\right)$ $[\therefore (a - b)^3 = a^3 - b^3 - 3ab(a - b)]$

or, $x^3 = 2 - 2^{-1} - 3 \times 1 \times x$ $[\therefore 2^{\frac{-1}{3}} - 2^{\frac{-1}{3}} = x$

or, $x^3 = 2 - \frac{1}{2} - 3x$

or, $2x^3 = 4 - 1 - 6x$

or, $2x^3 + 6x = 3$

$\therefore 2x(x^2 + 3) = 3$ *Proved*

10. If $2^x = 3^y = 6^{-z}$, show that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

Solution:

Let $2^x = 3^y = 6^{-z} = k$ then $2^x = k$ $\therefore 2 = k^{\frac{1}{x}}$, $3^y = k$ $\therefore 3 = k^{\frac{1}{y}}$ and $6^{-z} = k$ $\therefore 6 = k^{\frac{-1}{z}}$

Now, $2 \times 3 = 6$

or, $k^{\frac{1}{x}} \times k^{\frac{1}{y}} = k^{\frac{-1}{z}}$ or, $k^{\frac{1}{x} + \frac{1}{y}} = k^{\frac{-1}{z}}$ or, $\frac{1}{x} + \frac{1}{y} = \frac{-1}{z}$ $\therefore \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 0$

11. Solve: $2^{3x-5} a^{x-2} = 2^{x-2} a^{1-x}$

Solution:

Here, $2^{3x-5} a^{x-2} = 2^{x-2} a^{1-x}$

or, $\frac{2^{3x-5} a^{x-2}}{2^{x-2} a^{1-x}} = 1$ or, $2^{3x-5-x+2} a^{x-2-1+x} = 1$

or, $2^{2x-3} a^{2x-3} = 1$ or, $(2a)^{2x-3} = (2a)^0$

or, $2x - 3 = 0$ $\therefore x = \frac{3}{2}$

12. Solve: $5^{1-x} + 5^{x-1} = \frac{26}{5}$

Solution:

$5^{1-x} + 5^{x-1} = \frac{26}{5}$

or, $\frac{5}{5x} + \frac{5x}{5} = \frac{26}{5}$ ..... eq^n (i)

Let's $5^x = a$ then eq^n (i) becomes $\frac{5}{a} + \frac{a}{5} = \frac{26}{5}$

or, $\frac{25 + a^2}{5a} = \frac{26}{5}$ or, $a^2 - 26a + 25 = 0$

or, $(a - 1)(a - 25) = 0$

Either $a - 1 = 0$ or, $a - 25 = 0$

or, $a = 1$ or, $a = 25$

or, $5^x = 5^0$ or, $5^x = 5^2$

$\therefore x = 0$ $\therefore x = 2$

Hence, x = 0, 2

13. If $x^y = y^x$, prove that $\left(\frac{x}{y}\right)^{\frac{x}{y}} = x^{\frac{x}{y}-1}$

Solution:

Here, $x^y = y^x$, $\therefore x = x^{\frac{y}{x}}$

Now, L.H.S $= \left(\frac{x}{y}\right)^{\frac{x}{y}} = \left(\frac{x}{x^{\frac{y}{x}}}\right)^{\frac{x}{y}} = \left(x^{1-\frac{y}{x}}\right)^{\frac{x}{y}} = x^{\frac{x}{y}-1} =$ R.H.S

Extra questions for practice

1. ***Simplify.***

a. $(1 - x^{m-n})^{-1} + (1 - x^{n-m})^{-1}$

b. $\dfrac{11^{n+2} - 55.11^{n-1}}{11^n \times 116}$

c. $\dfrac{9^{p+2} + 10 \times 9^p}{9^{p+1} \times 11 - 8 \times 9^p}$

d. $\sqrt[3]{(x+y)^{-8}} \times (x+y)^{\frac{2}{3}}$

2. ***Simplify.***

a. $\sqrt[\frac{a+b}{c}]{\frac{x^{a^2}}{x^{b^2}}} \times \sqrt[\frac{b+c}{a}]{\frac{x^{b^2}}{x^{c^2}}} \times \sqrt[\frac{c+a}{b}]{\frac{x^{c^2}}{x^{a^2}}}$

b. $\dfrac{m + (m^2n)^{\frac{1}{3}} + (mn^2)^{\frac{1}{3}}}{m - n} \times \left(1 - \dfrac{n^{\frac{1}{3}}}{m^{\frac{1}{3}}}\right)$

c. $\left(x^{\frac{1}{a-b}}\right)^{\frac{1}{a-c}} \times \left(x^{\frac{1}{b-c}}\right)^{\frac{1}{b-a}} \times \left(x^{\frac{1}{c-a}}\right)^{\frac{1}{c-b}}$

d. $\dfrac{\left(x^2 - \frac{1}{y^2}\right)^x \times \left(x - \frac{1}{y}\right)^{y-x}}{\left(y^2 - \frac{1}{x^2}\right)^y \times \left(y + \frac{1}{x}\right)^{x-y}}$

3. ***Simplify.***

a) If $p + q + r = 0$, show that $\dfrac{1}{1 + x^p + x^{-q}} + \dfrac{1}{1 + x^q + x^{-r}} + \dfrac{1}{1 + x^r + x^{-p}} = 1$

b) If $xyz = 1$, show that $\dfrac{1}{1 + x + y^{-1}} + \dfrac{1}{1 + y + z^{-1}} + \dfrac{1}{1 + z + x^{-1}} = 1.$

c) If a + b + c = p, prove that $\dfrac{x^{2a}}{x^{2a} + x^{p-b} + x^{p-c}} + \dfrac{x^{2b}}{x^{2b} + x^{p-c} + x^{p-a}} + \dfrac{x^{2c}}{x^{2c} + x^{p-a} + x^{p-b}} = 1$

d) If $x^2 + 2 = 3^{\frac{2}{3}} + 3^{\frac{-2}{3}}$, show that $3x(x^2 + 3) = 8$

4. ***Solve***

a. $7^x + 7^{x+1} = 56$

b. $10^{2x-3} = 0.001$

c. $2^{z+3} \times 3^{x+4} = 18$

d. $6^{2x-3} . a^{x-2} = 6^{5x-4} . a^{4x-3}$

5. ***Solve***

a. $4^x + 4^{-x} = 16\frac{1}{16}$

b. $7^x + \dfrac{343}{7^x} = 56$

c. $2^{x-2} + 2^{3-a} = 3$

d. $5.4^{x+1} - 16^x = 64$

6.

a. If $x^{\frac{1}{p}} = y^{\frac{1}{q}} = z^{\frac{1}{r}}$ and $xyz = 1$, prove that $p + q + r = 0$

b. If $a^p . a^q = (a^p)^q$, prove that: $(x^{q-2})^p \times (x^{p-2})^q = 1$

c. If $x = a^p.b^{q+r}$, $y = a^q.b^{r+p}$ and $z = a^r.b^{p+q}$, prove that $x^{q-r}\, y^{r-p}\, z^{p-q} = 1.$

d. If $3^x = 7^y = 21^{-z}$, show that: $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = 0$

Answer key:

1. a) 1 b) 1 c) 1 d) $\dfrac{1}{(x+y)^2}$ 2. a) 1 b) 1 c) 1 d) $\left(\dfrac{x}{y}\right)^{x+y}$

4. a) 1 b) 0 c) –2 d) $\dfrac{1}{3}$ 5. a) ±2 b) 1, 2 c) 2, 3 d) 1, 2

Unit 11 Surds

Allocated teaching periods	5

Competency

To discover and present the simplifications of algebraic relations and solve the problems

Learning Outcomes

- To solve the problems related to radical and surds using four fundamental operations

Level-wise learning objectives

S.N.	LEVELS	OBJECTIVES
1.	Knowledge (K)	- To define surd - To tell the order of the surd - To state the rationalizing factor or conjugate of the surd
2.	Understanding (U)	- To arrange the surds in ascending or descending order of magnitudes - To simplify the surds by using the four fundamental operations (+, - , × and ÷) - To rationalize the denominator - To solve the simple equations involving surds
3.	Application (A)	- To simplify the rational surds by rationalizing the denominators - To solve the equations involving surds
4.	High Ability (HA)	- To explain the laws of surds with examples

Required Teaching Materials/ Resources

Colourful chart paper, colourful markers, tape, scissors etc.

Pre-knowledge: Different sets of numbers like set of natural numbers, whole numbers, integers, rational numbers etc

A. Surds

Teaching Activities

1. Recall the sets of numbers like set of natural numbers, whole numbers, integers, rational numbers and irrational number with examples by showing in chart paper
2. Discuss about rational number with the following properties
   (i) Rational numbers can be expressed in the form $\frac{p}{q}$ where p and q are integers, q ≠ 0. For example $0 = \frac{0}{\text{any non zero integer}}$, $\frac{2}{3}$ etc.
   (ii) When a rational number is expressed in decimal, then the decimal part may be terminated or non-terminating but recurring. For example $\frac{1}{2} = 0.5$, $\frac{5}{8} = 0.625$ etc are terminating decimals, $\frac{2}{3} = 0.6666...$, $\frac{5}{6} = 0.8333...$ etc are non-terminating but recurring decimal.
3. To the contrary, discuss about the irrational numbers. For examples $\sqrt{2}$, $\sqrt[4]{6}$ etc
4. With examples, note that the decimal part of irrational numbers is neither terminate nor non-terminating recurring
5. Define surd as the irrational number whose exact root cannot be found. For example $\sqrt{2}, \sqrt{3}, \sqrt[3]{6}, \sqrt[4]{7}$ etc.

6. Discuss about the parts of a surd as shown below

Radical sign

$$\text{Order} \rightarrow \sqrt[n]{a} \leftarrow \text{Radicand}$$

7. Explain the parts of surd $\sqrt[n]{a}$ as follows
   (i) The order 'n' of the surd is a natural number
   (ii) The radicand 'a' is a positive rational number
8. Discuss about the laws of surds

Laws of surds	Examples
(i) $\sqrt[n]{a} = a^{\frac{1}{n}}$, So, $\left(\sqrt[n]{a}\right)^n = a$	$\sqrt[3]{2} = 2^{\frac{1}{3}}$, $\left(\sqrt[3]{5}\right)^3 = 5^{\frac{3}{3}} = 5$
(ii) $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$	$\sqrt{3 \times 5} = \sqrt{3} \times \sqrt{5}$
(iii) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$	$\sqrt[4]{\frac{4}{5}} = \frac{\sqrt[4]{4}}{\sqrt[4]{5}}$
(iv) $\sqrt[n]{a} + \sqrt[n]{a} = 2\sqrt[n]{a}$	$\sqrt[3]{5} + \sqrt[3]{5} = 2\sqrt[3]{5}$
(v) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[n]{\sqrt[m]{a}} = \sqrt[mn]{a}$	$\sqrt{\sqrt[3]{64}} = \sqrt{4} = 2 = \sqrt[2\times3]{64} = \sqrt[6]{64}$

9. Explain about the pure surds, mixed surd, like surds, unlike surds, operations of surds
10. Divide the students into 5 groups and give the following activities and call for presentation in the class

    Group A: Find the orders of $\sqrt[3]{4}$ and $\sqrt[p]{a}$

    Group B: Simplify $\sqrt{8} + \sqrt{18} - \sqrt{32}$

    Group C: Simplify $(\sqrt{5} - \sqrt{3})^2$

    Group D: Simplify $(3\sqrt{2} + 2\sqrt{3})(3\sqrt{2} - 2\sqrt{3})$

    Group E: Simplify $\frac{\sqrt{a^2 - b^2}}{\sqrt{a - b}}$
11. Explain about rationalization and conjugate with examples
12. Divide the students into 5 groups and give the following activities and call for presentation in the class

    Group A: Rationalize the denominator and simplify $\frac{\sqrt{2} - 1}{\sqrt{2} + 1}$

    Group B: Rationalize the denominator and simplify $\frac{\sqrt{3} - 1}{\sqrt{3} + 1}$

    Group C: Rationalize the denominator and simplify $\frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$

    Group D: Rationalize the denominator and simplify $\frac{\sqrt{5} + \sqrt{2}}{\sqrt{5} - \sqrt{2}}$

Group E: Rationalize the denominator and simplify $\dfrac{2\sqrt{3} - 3\sqrt{2}}{2\sqrt{3} + 3\sqrt{2}}$

13. Explain the process of simplification of the form $\dfrac{3\sqrt{2}}{\sqrt{3} + \sqrt{6}} + \dfrac{\sqrt{6}}{\sqrt{2} + \sqrt{3}} - \dfrac{4\sqrt{3}}{\sqrt{6} + \sqrt{2}}$

B. Simple surd equations

Teaching Activities

1. Recall the order and operations of surds
2. Explain about the equation involving surds with examples
3. Ask about the process of expressing the surds into rational number
4. Give some examples with discussion and divide the students into 5 groups and give the following activities and call for presentation in the class

Group A: Solve $\sqrt{3x - 2} - 1 = 4$

Group B: Solve $\sqrt{x + 9} + \sqrt{x} = 9$

Group C: Solve $\sqrt[3]{4X + 1} = \sqrt{25}$

Group D: Solve $\dfrac{x - 9}{\sqrt{x} + 3} = 2$

Group E: Solve $\dfrac{\sqrt{y} + 5}{\sqrt{y} - 5} = 3$

5. Discuss about the roots and extraneous roots of the equation
6. With discussion, solve the equations like $\dfrac{x - 1}{\sqrt{x} + 1} = 4 + \dfrac{\sqrt{x} - 1}{2}$ and give same type of equation from textbook as class-work or homework
7. Solve the equations like $\dfrac{\sqrt{x} + \sqrt{2}}{\sqrt{x} - \sqrt{2}} + \dfrac{\sqrt{x} - \sqrt{2}}{\sqrt{x} + \sqrt{2}} = 6$ and give same type of equation from textbook as class-work or homework

Solution of selected questions from Vedanta Excel in Mathematics

1. *Simplify:*

a. $\boldsymbol{3\sqrt{2} + \sqrt[4]{2500} - \sqrt[4]{64} + 6\sqrt{8}}$

Solution:

Here, $3\sqrt{2} + \sqrt[4]{2500} - \sqrt[4]{64} + 6\sqrt{8} = 3\sqrt{2} + \sqrt[4]{5^4 \times 2^2} - \sqrt[4]{2^4 \times 2^2} + 6\sqrt{2^2 \times 2}$

$= 3\sqrt{2} + 5\sqrt{2^2} - 2\sqrt{2^2} + 6 \times 2\sqrt{2} = 3\sqrt{2} + 5\sqrt{2} - 2\sqrt{2} + 12\sqrt{2} = 18\sqrt{2}$

b. $\boldsymbol{\dfrac{3x - 16}{4 + \sqrt{3x}}}$

Solution:

$$= \frac{3x - 16}{4 + \sqrt{3x}} = \frac{(\sqrt{3x})^2 - (4)^2}{4 + \sqrt{3x}} = \frac{(\sqrt{3x} + 4)(\sqrt{3x} - 4)}{(4 + \sqrt{3x})} = \sqrt{3x} - 4$$

c. $\boldsymbol{\dfrac{3\sqrt[3]{81} - 3\sqrt[3]{24} + 2\sqrt[3]{375}}{13\sqrt[3]{192}}} = \dfrac{3\sqrt[3]{81} - 3\sqrt[3]{24} + 2\sqrt[3]{375}}{13\sqrt[3]{192}}$

$$= \frac{3 \times 3\sqrt[3]{3} - 3 \times 2\sqrt[3]{3} + 2 \times 5\sqrt[3]{3}}{13 \times 4\sqrt[3]{3}} = \frac{13\sqrt[3]{3}}{54\sqrt[3]{3}} = \frac{1}{4}$$

2. *Rationalize the denominator and simplify:* $\dfrac{\sqrt{a+b}-\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}}$

Solution:

$$\frac{\sqrt{a+b}-\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}} = \frac{\sqrt{a+b}-\sqrt{a-b}}{\sqrt{a+b}+\sqrt{a-b}} \times \frac{\sqrt{a+b}-\sqrt{a-b}}{\sqrt{a+b}-\sqrt{a-b}} = \frac{(\sqrt{a+b}-\sqrt{a-b})^2}{(\sqrt{a+b})^2+(\sqrt{a-b})^2}$$

$$= \frac{(\sqrt{a+b})^2 - 2\sqrt{a+b}\,.\,\sqrt{a-b} + (\sqrt{a-b})^2}{a+b-(a-b)} = \frac{2a-2\sqrt{a^2-b^2}}{2b} = \frac{a-\sqrt{a^2-b^2}}{b}$$

3. *Simplify:* $\dfrac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}} - \dfrac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}}$

Solution:

Here, $\dfrac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}} - \dfrac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}} = \dfrac{(x+\sqrt{x^2-1})^2}{(x-\sqrt{x^2-1})} - \dfrac{(x-\sqrt{x^2-1})^2}{(x+\sqrt{x^2-1})}$

$$= \frac{x^2+2.x\sqrt{x^2-1}+(\sqrt{x^2-1})^2-x^2+2x\sqrt{x^2-1}-(\sqrt{x^2-1})^2}{x^2-(\sqrt{x^2-1})^2} = \frac{4x\sqrt{x^2-1})}{1} = 4x\sqrt{x^2-1})$$

4. *Simplify:* $\dfrac{5\sqrt{2}}{\sqrt{5}(\sqrt{2}+1)} - \dfrac{8\sqrt{5}}{\sqrt{10}+\sqrt{2}} + \dfrac{3\sqrt{10}}{\sqrt{2}+\sqrt{5}}$

Solution:

Here, $\dfrac{5\sqrt{2}}{\sqrt{5}(\sqrt{2}+1)} - \dfrac{8\sqrt{5}}{\sqrt{10}+\sqrt{2}} + \dfrac{3\sqrt{10}}{\sqrt{2}+\sqrt{5}}$

$$= \frac{5\sqrt{2}}{\sqrt{10}+\sqrt{5}} \times \frac{\sqrt{10}-\sqrt{5}}{\sqrt{10}-\sqrt{5}} - \frac{8\sqrt{5}}{\sqrt{10}+\sqrt{2}} \times \frac{\sqrt{10}-\sqrt{2}}{\sqrt{10}-\sqrt{2}} + \frac{3\sqrt{10}}{\sqrt{2}+\sqrt{5}} \times \frac{\sqrt{2}-\sqrt{5}}{\sqrt{2}-\sqrt{5}}$$

$$= \frac{5(\sqrt{20}-\sqrt{10})}{10-5} - \frac{8(\sqrt{50}-\sqrt{10})}{10-2} + \frac{3(\sqrt{20}-\sqrt{50})}{2-5}$$

$$= \frac{\cancel{5}(\sqrt{20}-\sqrt{10})}{\cancel{5}} - \frac{\cancel{8}(\sqrt{50}-\sqrt{10})}{\cancel{8}} + \frac{\cancel{3}(\sqrt{20}-\sqrt{50})}{\cancel{3}}$$

$$= \sqrt{20}-\sqrt{10}-\sqrt{50}+\sqrt{10}-\sqrt{20}+\sqrt{50} = 0$$

5. *Simplify:* $\dfrac{1+x}{1+\sqrt{1+x}} + \dfrac{1-x}{1-\sqrt{1+x}}$

Solution:

Here, $= \dfrac{1+x}{1+\sqrt{1+x}} + \dfrac{1-x}{1-\sqrt{1+x}}$

$$= \frac{(1+x)(1-\sqrt{1+x})+(1-x)(1+\sqrt{1+x})}{(1+\sqrt{1+x})(1-\sqrt{1+x})}$$

$$= \frac{1-\cancel{\sqrt{1+x}}+\cancel{x}-x\sqrt{1+x}+1+\cancel{\sqrt{1+x}}-\cancel{x}-x\sqrt{1+x}}{1-1-x}$$

$$= \frac{2-2x\sqrt{1+x}}{-x} \quad = \frac{2(x\sqrt{1+x}-1)}{x}$$

6. *Simplify:* $\dfrac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}} + \dfrac{\sqrt{x^2+2}-\sqrt{x^2-2}}{\sqrt{x^2+2}+\sqrt{x^2-2}}$

Solution:

Here, $\dfrac{\sqrt{x^2+2}+\sqrt{x^2-2}}{\sqrt{x^2+2}-\sqrt{x^2-2}} + \dfrac{\sqrt{x^2+2}-\sqrt{x^2-2}}{\sqrt{x^2+2}+\sqrt{x^2-2}}$

$$= \frac{(\sqrt{x^2+2} + \sqrt{x^2-2})^2 + (\sqrt{x^2+2} - \sqrt{x^2-2})^2}{(\sqrt{x^2+2})^2 - (\sqrt{x^2-2})^2}$$

$$= \frac{x^2 + \cancel{2} + 2\sqrt{\cancel{x^4-4}} + x^2 - \cancel{2} + x^2 + \cancel{2} - 2\sqrt{\cancel{x^4-4}} + x^2 - \cancel{2}}{4} = \frac{4x^2}{4} = x^2$$

7. *Solve:* $\sqrt{x+5} + \sqrt{x+12} = \sqrt{2x+41}$

Solution:

Here, $\sqrt{x+5} + \sqrt{x+12} = \sqrt{2x+41}$

On squaring both sides, we get

or, $(\sqrt{x+5} + \sqrt{x+12})^2 = (\sqrt{2x+41})^2$

or, $x + 5 + 2\sqrt{x+5}\,.\sqrt{x+12} + x + 12 = 2x + 41$

or, $17 + 2\sqrt{x^2+17x+60} = 41$

or, $2\sqrt{x^2+17x+60} = 24$

or, $\sqrt{x^2+17x+60} = 12$

On squaring both sides, we get

$x^2 + 17x + 60 = 144$

or, $x^2 + 17x - 84 = 0$

or, $x^2 + 21x - 4x - 84 = 0$

or $(x + 21)(x - 4)$

Either $x + 21 = 0 \quad \therefore x = -21$

or, $x - 4 \quad \therefore 4$

Substituting $x = -21$ in the given equation

$\sqrt{-21+5} + \sqrt{-21+12} = \sqrt{2(-21)+41}$

or, $\sqrt{-16} + \sqrt{-9} = \sqrt{-1}$

or, $4\sqrt{-1} + 3\sqrt{-1} = \sqrt{-1}$

or, $7\sqrt{-1} = \sqrt{-1}$ (False)

Substituting $x = 4$ is the given equation

$\sqrt{4+5} + \sqrt{4+12} = \sqrt{2 \times 4 + 41}$

or, $3 + 4 = 7$ (True)

Hence, required value of x is 4.

8. *Solve:* $\sqrt{x} + \sqrt{x - \sqrt{1-x}} = 1$

Solution:

Here, $\sqrt{x} + \sqrt{x - \sqrt{1-x}} = 1$

Squaring on both sides

or, $(\sqrt{x - \sqrt{1-x}})^2 = (1 - \sqrt{x})^2$

or, $x - \sqrt{1-x} = 1 - 2\sqrt{x} + x$

or, $2\sqrt{x} - 1 = \sqrt{1-x}$

or, $(2\sqrt{x} - 1)^2 = \sqrt{1-x}$ [squaring on both sides]

or, $4x - 4\sqrt{x} + 1 = 1 - x$

or, $5x = 4\sqrt{x}$

or, $25x^2 = 16x$ [Squaring on both sides]

or, $x(25x - 16) = 0$

Either $x = 0$ or, $25x - 16 = 0$ $\therefore x = \frac{16}{25}$

Checking:

Substituting x = 0 in the given equation

$$\sqrt{0} + \sqrt{0 - \sqrt{1 - 0}} = 1$$

or, $\sqrt{-1} = 1$ (False)

Substituting $x = \frac{16}{25}$ in the given equation

or, $\sqrt{\frac{16}{25}} + \sqrt{\frac{16}{25} - \sqrt{1 - \frac{16}{25}}}$

or, $\frac{4}{5} + \sqrt{\frac{16}{25} - \frac{3}{5}} = 1$

or, $\frac{4}{5} + \frac{1}{5} = 1$ or, $1 = 0$

Hence, the required value of x is 1.

9. Solve : $\boldsymbol{\frac{7x - 36}{6 + \sqrt{7x}} = 9 - \frac{\sqrt{7x} - 11}{3}}$

Solution:

$$\frac{7x - 36}{6 + \sqrt{7x}} = 9 - \frac{\sqrt{7x} - 11}{3}$$

or, $\frac{(\sqrt{7x} + 6)(\sqrt{7x} - 6)}{(6 + \sqrt{7x})} = \frac{27 - \sqrt{7x} + 11}{3}$

or, $\sqrt{7x} - 6 = \frac{38 - 5\sqrt{7x}}{3}$

or, $3 - 18 = 38 - 5\sqrt{7x}$

or, $8\sqrt{7x} = 56$

or, $(\sqrt{7x})2 = (7)2$ [Squaring on both sides]

$\therefore x = 7$

Checking: Putting x = 7 in the given equation

$$\frac{7 \times 7 - 36}{6 + \sqrt{7 \times 7}} = 9 - \frac{5\sqrt{7 \times 7} - 11}{3}$$

or, $\frac{13}{13} = 9 - \frac{24}{3}$

or, $1 = 1$ (True)

Hence, the required value of x is 7.

10. Solve: $\boldsymbol{\frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} - \sqrt{a}} = 4\sqrt{2} + \frac{\sqrt{x} - \sqrt{a}}{\sqrt{x} + \sqrt{a}}}$

Solution:

Here, $\dfrac{\sqrt{x}+\sqrt{a}}{\sqrt{x}-\sqrt{a}} - \dfrac{\sqrt{x}-\sqrt{a}}{\sqrt{x}+\sqrt{a}} = 4\sqrt{2}$

or, $\dfrac{x + 2\sqrt{ax} + a - x + 2\sqrt{ax} - a}{x - a} = 4\sqrt{2}$

or, $\dfrac{\sqrt{ax}}{x - a} = \sqrt{2}$

or, $\sqrt{2}(x - a) = \sqrt{ax}$

or, $\{\sqrt{2}(x - a)\}^2 = (\sqrt{ax})^2$ [Squaring on both sides]

or, $2(x^2 - 2ax + a^2) = ax$

or, $2x^2 - 4ax - ax + 2a^2 = 0$

or, $2x(x - 2a) - a(x - 2a) = 0$

or, $(3 - 2a)\ (2x - a) = 0$

Either $x - 2a = 0$ $\therefore x = 2a$

or, $2x - a = 0$ $\therefore x = \dfrac{a}{2}$

Extra questions for practice

1. *Simplify:*

a. $\sqrt{5} - \sqrt{45} + \sqrt{125}$ b. $\sqrt[3]{128} + 2\sqrt[3]{54} - 2\sqrt[3]{250}$ c. $\dfrac{\sqrt[4]{81x^4} - \sqrt[3]{8x^3}}{\sqrt{x^2}}$

2. *Simplify:*

a. $\dfrac{\sqrt{x+y} - \sqrt{x-y}}{\sqrt{x+y} + \sqrt{x-y}}$ b. $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} + \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{3}}$ c. $\dfrac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}} - \dfrac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}} + \dfrac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$

3. *Solve:*

a. $\sqrt{x+8} - 2 = \sqrt{x}$ b. $\sqrt{3x+1} = \sqrt[3]{8}$ c. $8 + \sqrt[3]{4x-7} = 13$

4. *Solve:*

a. $\dfrac{5x-4}{\sqrt{5x}+2} = 4 - \dfrac{\sqrt{5x}-3}{2}$ b. $\dfrac{\sqrt{x}+2}{\sqrt{x}+40} = \dfrac{3\sqrt{x}-4}{15+3\sqrt{x}}$

c. $\sqrt{x+2} + \sqrt{x+7} = \dfrac{15}{\sqrt{x+7}}$ d. $\dfrac{\sqrt{x}-\sqrt{2}}{\sqrt{x}+\sqrt{2}} + \dfrac{\sqrt{x}+\sqrt{2}}{\sqrt{x}-\sqrt{2}} = 6$

Answer key:

1. a) $3\sqrt{5}$ b) 0 c) 1 2. a) $\dfrac{x+\sqrt{x^2-y^2}}{y}$ b) 10 c) 0

3. a) 1 b) 1 c) 33 4. a) 5 b) 4 c) 2 d) 16

Unit 12 Equations

Allocated teaching periods	9

Competency

- To search the algebraic equations and solve

Learning Outcomes

- To solve the simultaneous equations of two variables
- To solve the quadratic equations of two variables

Level-wise learning objectives

S.N.	LEVELS	OBJECTIVES
1.	Knowledge (K)	- To define simultaneous equations - To tell the ages after x years ago or x years later when present age is given - To state the two digit number formed by the digits x at tens and y at ones places - To represent the fraction
2.	Understanding (U)	- To solve the simultaneous equations - To find the value of two different items - To identify the numbers under the given conditions
3.	Application (A)	- To find the present ages of two individual under the given conditions - To find out the two digit numbers under the given conditions - To find the fraction under the given conditions
4.	High Ability (HA)	- To make the simultaneous equations related to the context and solve the problems

Required Teaching Materials/ Resources

Chart-paper, glue-stick, colourful markers etc

Pre-knowledge: Methods of solving the simultaneous equations, quadratic equations

A. **Linear equations**

Teaching Activities

1. Discuss about the simultaneous equations with real life examples
2. Recall the methods of solving simultaneous equations
3. Explain with discussion the important terminologies used in making equations
   (a) Terminologies based fundamental operations

(i) Addition: added, sum, exceed, altogether, all, more than, increased etc
(ii) Subtraction: subtracted, difference, left away, shorter than, less than, decreased by, younger/cheaper than etc
(iii) Multiplication: product, times, multiplied by etc
(iv) Division: divided by, each, share, quotient etc.

(b) Terminologies based on mensuration
(i) If the length and breadth of a rectangle are x meter and y meter respectively then perimeter = 2 (x + y) and area = xy
(ii) If the perpendicular, base and hypotenuse of a right angled triangle are p, b and h respectively then $p^2 + b^2 = h^2$

(c) Terminologies based on ages: If the present age of an individual is x years
(i) Age before/ago: (x – t) years
(ii) Age after/ hence/ later : (x + t) years

(d) Terminologies based on consecutive numbers
(i) Consecutive numbers: x, x+1, x+2, ... etc
(ii) Consecutive odd/even numbers: x, x+2, x+4, ... etc

(e) Terminologies based on two-digit number
(i) Two digit number = 10x + y, reversed number = 10y + x
(ii) Sum of digits = x + y, product of digits = xy

(f) Terminologies based on fraction
If x and y are the numerator and denominator of a fraction, the fraction = x/y

4. Divide the students into groups and give difference questions to make the corresponding simultaneous equations and solve

Solution of selected questions from Vedanta Excel in Mathematics

1. ***If three times the sum of two numbers is 42 and five times their difference is 20, find the numbers.***

Solution:

Let the greater number be x and the smaller one be y.

Then, From the first given condition;

$$3(x+y) = 42$$
or, $x+y = 14$
$\therefore\ x = 14 - y$ ....(i)

From the second given condition

$$5(x-y) = 20$$
$\therefore\ x - y = 4$

Now, substituting the value of x from equation (i) in equation (ii), we get

$$14 - y - y = 4$$
or $-2y = -10$
$\therefore\ y = 5$

Again, substituting the value of y in equation (i), we get

$x = 14 - 5 = 9$

Hence, the required number are 9 and 5.

2. ***The total cost of a watch and a radio is Rs 500. If the watch is cheaper than the radio by Rs 150, find their cost.***

Solution:

Let the cost of a watch be Rs x and that of a radio be Rs y.

From the first given condition;

$x + y = 500$ .... (i)

From the second given condition;

$x = y - 150$ ... (ii)

Now, substituting the value of x from equ^n (ii) in equ^n (i), we get

$y - 150 + y = 500$

or, $2y = 650$

or, $y = 325$

Substituting the value of y in equation (ii), we get

$x = 325 - 150 = 175$

Hence, the cost of the watch is Rs 175 and that of the radio is Rs 325.

3. ***The cost of tickets of a comedian show of 'Gaijatra' is Rs 150 for an adult and Rs 50 for a child. If a family paid Rs 550 for 5 tickets, how many tickets were purchased in each category?***

Solution:

Let the number of tickets purchased for adult be x and for children be y.

Then,

From the first given condition;

$x + y = 5 \therefore y = 5 - x$ .... (i)

From the second given conditon;

$150x + 50y = 550$

or, $50(3x + y) = 550$

$\therefore$ $3x + y = 11$

Now, Substituting the value of y from equ^n (i) in equ^n (ii), we get

$3x + 5 - x = 11$

or, $2x = 6$ $\therefore x = 3$

Again substituting the value of x in equ^n (i), we get

$y = 5 - 3 = 2$

Hence the number of tickets purchased for adult is 3 and that for children is 2.

4. ***The numerator of a fraction is 5 less than its denominator. If 1 is added to each, its value becomes $\frac{1}{2}$. Find the original fraction.***

Solution:

Let the numerator and denominator of the original fraction be x and y respectively. Then the required fraction is $\frac{x}{y}$.

From the first given condition;

$x = y - 5$ .... (i)

From the second given condition;

$$\frac{x + y}{y + 1} = \frac{1}{2}$$

or, $2x - y = -1$ .... (ii)

Now substituting the value of x from equation (i) in equation (ii), we get

$2(y - 5) - y = -1$ $\therefore y = 9$

Again, putting the value of y in equation (i), we get

$x = 9 - 5 = 4$

Hence, the required fraction is $\frac{4}{9}$

5. ***The sum of present ages of a son and his son is 40 years. If they both live on till son becomes as old as the father is now, the sum of their ages will be 96 years. Find their present ages.***

Solution:

Let the present age of the father be x years and that of his son be y years.

From the first given condition; $x + y = 40$ $\therefore y = 40 - x$ ..... (i)

From the second given condition; $[x + (x - y)] + [y + (x - y)] = 90$

$\therefore$ $3x - y = 96$ .... (ii)

Now, substituting the value of y from equation (i) in equation (ii), we get

$3x - (40 - x) = 96$ $\therefore x = 34$

Again, putting the value of x in equation (i), we get

$y = 40 - 34 = 6$

Hence, the present age of the father is 34 years and that of his son is 6 years.

Note:

If the present ages of the father and son are x years and y years respectively, then

(i) *$(x - y)$ years hence, the son will be as old as his father is now.*

i,e age of son $(x - y)$ hence $[y + (x - y)] = x$ = present age of father

(ii) *$(x - y)$ years ages the father was as old as his son is now.*

i.e age of father $(x - y)$ years ago $= x - (x - y) = y$ = present age of his son.

6. ***The sum of present ages of a father and his son is 80 years. When the father age was equal to the present age of the son, the sum of their ages was 40 years. Find their present ages.***

Solution:

Let the present age of the father be x years and that of his son be y years.

From the first given condition; $x + y = 80$ $\therefore x = 80 - y$ .... (i)

From the second given condition; $[x - (x - y)] + [y - (x - y)] = 40$

or, $3y - x = 40$

Now, putting the value of x in equation (ii) from equation (i), we get

$3y - (80 - y) = 40$ $\therefore y = 30$

Again, putting the value of y in equation (i), we get

$x = 80 - 30 = 50$

Hence, the present ages of the father and his son are 50 years and 30 years respectively.

7. ***When the age of Rita was equal to the present age of shova; she was thrice as old as shova was. If the sum of their ages is 40 years, find their ages.***

Solution:

Let the present ages of Rita and Shova be x years and y years respectively.

From the first given equation,

$[x - (x - y)] = 3[y - (x - y)]$

or, $y = 6y - 3x$

or, $3x = 5y$

$\therefore$ $x = \frac{5y}{3}$ ..... (i)

From the second given equation;

$x + y = 40$ ..... (ii)

Now, putting the value of x in equation (ii) from (i), we get

$\frac{5y}{3} + y = 40$ $\therefore y = 15$

Again, putting the value of y in equation (i), we get

$x = \frac{5 + 15}{3} = 25$

Hence, the present ages of the Rita and Shova are 25 years and 15 years respectively.

8. *In 2010, Salim was two times as old as Kedar was. If the ratio of their ages will be 5:3 in 2015. Find the years of birth.*

Solution:

Let the ages of Salim and Kedar in 2010 be x and y years respectively.

From the first given condition,

$x = 2y$ ..... (i)

From the second given condition,

in 2015, i.e, after (2015 -2010) = 5 years, $\frac{x+5}{y+5} = \frac{5}{3}$

or, $3x - 5y = 10$ .... (ii)

Now, putting the value of x in equation (ii) from (i), we get

$x \times 2y - 5y = 10 \;\therefore\; y = 10$

Again, putting the value of y in equation (i), we get

$x = 2 \times 10 = 20$

$\therefore$ In 2010, the ages of Salim and Kedar were 20 years and 10 years respectively.

Hence, the birth year of Salim = 2010 – 20 = 1990 and

the birth year of Kedar = 2010 – 10 = 2000

9. *A number of two-digits exceeds four times the sum of its digits by 3. If 36 is added to the number the digits are reversed. Find the number.*

Solution:

Let the digits at tens and ones place of a number be x and y respectively. Then, the number $= 10x + y$.

From the first given condition;

$10 + y = 4(x + y) + 3$

or, $6x = 3y + 3$

$\therefore$ $2x - y = 1$ .... (i)

From the second given condition;

$10x + y + 36 = 10y + x$

or, $9x = 9y - 36$

$\therefore$ $x = y - 4$ ...... (ii)

Now, putting the value of x in equation (i) from (ii), we get

$2(y - 4) - y = 1 \quad \therefore y = 9$

Again, putting the value of y in equation (ii), we get

$x = 9 - 4 = 5$

Hence, the required number $= 10x + y = 10 \times 5 + 9 = 59$

10. *A number consists of two digits. If the number formed by reversing its digits is added to it, the sum is 143 and if the same number is subtracted from it the remainder is 9. Find the number.*

Solution:

Let the digits at tens and ones place of a number be x and y respectively. Then, the number $= 10x + y$.

From the first given condition,

$(10x + y) + (10y + x) = 143$

or, $11(x + y) = 143$

or, $x + y = 13$ ..... (i)

From the second given condition

$(10x + y) - (10y + x) = 9$

or, $9x - 9y = 9$

$\therefore \quad x - y = 1$ .... (ii)

Now, adding equation (i) and equation (ii), we get

$$\begin{array}{rcl} x + y & = & 13 \\ -\ x \pm y & = & 1 \\ \hline 2x & = & 14 \end{array}$$

or

$\therefore \quad x = 7$

Again, putting the value of x in equation (i), we get

$7 + y = 13$

$\therefore \quad y = 6$

Hence, the required number is $10 \times 7 + 6 = 76$

11. ***Five times the sum of the digit of a two digit number is 9 less the number formed by reversing its digits. If four times the value of the digit at ones place is equal to half of the place value of the digit at tens place, find the number.***

Solution:

Let the digit at tens and ones places of a two digit number be x and y respectively. Then, the numbers $= 10x + y$

From the first given condition,

$5(x + y) = 10y + x - 9$

or, $4x - 5y = -9$ ...... (i)

From the second given condition,

$4y = \frac{\cancel{10}^{5}x}{\cancel{2}}$ [$\therefore$ Place value of x is $10x$]

$\therefore \quad y = \frac{5x}{4}$ .... (ii)

Now, putting the value of y from equation (ii) in equation (i), we get

$4x - 5 \times \frac{5x}{4} = -9 \quad \therefore x = 4$

Again, putting the value of x in equation (ii), we get

$y = \frac{5 \times 4}{4} = 5$

Hence, the required number is $10 \times 4 + 5 = 45$

12. ***In a city, the taxi charges consists of two types of charges: a fixed together with the charges for the distance covered. If a person travels 10 km, he pays Rs 180 and for travelling 12 km, he pays Rs 210. Find the fixed charges and the rate of charge per km.***

Solution:

Let the fixed charge of the taxi be Rs x and the rate of charge per km be Rs y.

Then,

From the first given condition, $x + 10y = 180$ ... (i)

From the second given condition, $x + 12y = 210$ .... (ii)

Now, Subtracting equation (i) from equation (ii), we get

$$\begin{array}{rcl} x + 12y & = & 210 \\ x + 10y & = & 180 \\ (-)\quad (-) & & (-) \\ \hline 2y & = & 30 \end{array} \qquad \therefore y = 15$$

or

Again, putting the value of y in equation (i), we get

$x + 10 \times 15 = 180 \qquad \therefore x = 30$

Hence, the fixed charge is Rs 30 and rate of charge is Rs 15 per km.

13. *A lending library has a fixed charge for the first four days and an additional charge for each day thereafter. Dorje paid Rs 50 for a book kept for 9 days, while Kajal paid Rs 35 for the book she kept for 6 days. Find the fixed charge and the charge for each extra day.*

Solution:

Let the fixed charge be Rs x and the additional charge for each extra days be Rs y.
Then,
From the first given condition,
$x + 5y = 50$ [$\therefore$ Extra days $(9 - 4) = 5$ days]
$\therefore = 50 - 5y$ .... (i)
From the second given condition
$x + 2y = 35$ [$\therefore$ $6 - 4 = 2$ extra days]
$50 - 5y + 2y = 35$
or, $-3y = -15$
$\therefore$ $y = 5$
Again, putting the value of y in equation (i), we get $x = 50 - 5 \times 5 = 25$
Hence, the fixed charge is Rs 25 and charge for each extra day is Rs 5.

14. *When Gopal gives Rs 10 from his money to Laxmi her money becomes double than that of remaining money of Gopal. When Laxmi gives Rs 10 to Gopal, his money is still Rs 10 less than the remaining money of Laxmi. Find their original sum of money.*

Solution:

Let Gopal and Laxmi have Rs x and Rs y respectively.
From the first given condition,
$2(x - 10) = y + 10$
or, $2x - 30 = y$ ..... (i)
From the second given condition,
$(y - 10) - 10 = x + 10$
$\therefore$ $y - x = 30$ ..... (ii)
Now, putting the value of y from equation (i) in equation (ii), we get $2x - 30 - x = 30$
$\therefore$ $x = 60$
Again, putting the value of x in equation (i), we get $y = 2 \times 60 - 30 = 90$
Hence, Gopal had Rs 60 and Laxmi had Rs 90.

15. *Janak started his bicycle Journey from Kohalpur to Dhangadi at 6:00 a.m. with an average speed of 20 km/hr. Two hours later Ganesh also started his journey from Kohalpur to Dhangadhi with an average speed of 30 km/hr. At what time would they meet each other if both of their maintain non - stop journey.*

Solution:

Suppose, Janak meets Ganesh after x hours
But, Ganesh meets Janak after $(x - 2)$ hours
Now, the distance travelled by Janak in x hours = $20x$ km
Also, the distance travelled by Ganesh in $(x - 2)$ hours = $30(x - 2)$ km
When they travelled the equal distance, they meet each other.
So, $20x = 30(x - 2)$
or, $2x = 3x - 6$
$\therefore x = 6$
Since, Janak started his journey at 6:00 am, they meet at 12:00 noon.

16. ***Two bused were coming from two different places Situated just in the opposite direction. The average speed of one bus is 5 km/hr more than that of another one and they had started their journey in the same time. If the distance, between the places is 500 km and they meet after 4 hours, find their speed.***

Solution:

Let the speed of the faster bus be x km/hr
and the speed of the slower bus be y km/hr
From the first given condition,

$x - y = 5$ ...... (i)

From the second given condition,

$4x + 4y = 500$

or, $4(x + y) = 500$

$\therefore$ $x + y = 125$ ..... (ii)

Adding equation (i) and (ii), we get

$$\begin{array}{rcccl} & x & - & y & = 5 \\ & x & + & y & = 125 \\ \hline \text{or} & & 2x & & = 130 \end{array}$$

$\therefore$ $x = 65$

Now, putting the value of x in equation (i), we get

$65 - y = 5$

$\therefore$ $y = 60$

Hence, the speed of the faster bus is 65 km/hr and the slower bus is 60 km/hr.

17. ***When the length of a rectangular field is reduced by 5 m and breadth is increased by 3 m, its area gets reduced by 9 sq.m. If the length is increased by 3 m and breadth by 2 m the area increases by 67 sq.m. Find the length and breadth of the field.***

Solution:

Let the length and breadth of the rectangular field be x m and y m respectively. Then area field (A) = $l \times b$

From the first given condition,

$(l - 5)(b + 3) = lb - 9$

or, $\cancel{lb} + 3l - 5b - 15 = \cancel{lb} - 9$

$\therefore$ $3l - 5\text{b} = 6$ ...... (i)

From the second given condition,

$(l + 3)(b + 2) = lb + 67$

or, $\cancel{lb} + 2l + 3b + 6 = \cancel{lb} + 67$

or, $+ 2l + 3b = 61$

or, $b = \dfrac{61 \times 2l}{3}$ ..... (ii)

Now, putting the value of b in equation (i) from (ii), we get

$3l - 5(\dfrac{61 - 2l}{3}) = 6$

or, $9l - 305 + 10l = 18$

or, $19l = 323$

$\therefore$ $l = 17$

Again, putting the value of l in equation (ii), we get

$b = \dfrac{61 - 2 \times 17}{3} = \dfrac{61 \times 34}{3} = 9$

Hence, the length and breadth of the field are 17 m and 9 m respectively.

Extra questions for practice

1. A year hence, a father will be 5 times as old as his son. Two years ago, he was three times as old as his son will be four hence. Find their present ages. [Ans: 25 years, 5 years]

2. The age of two girls are in the ratio 5:7. Eight years ago, their ages were in the ratio of 7:13. Find their present ages. [Ans: 15 years, 212 years]

B. Quadratic equations

Teaching Activities

1. Make a proper discussion on the quadratic equations with real life examples
2. Recall the methods of solving quadratic equations
   (i) Factorisation method (ii) Using formulae
3. With discussion, make the equations and solve the problems
4. Make the groups of students and encourage to make the correct equations and solve the equation to get the required solution
5. Make contextual problems based on quadratic equations and ask to students to solve them

Solution of selected questions from Vedanta Excel in Mathematics

1. The area of rectangular field is 720 sq. mm. and perimeter is 108 m. By what percent is the longer side of the field to be decreased to make it a square? Why?

Solution:

Let the length and breadth of the rectangular filed be l m and b m respectively.

From the first condition,

$l \times b = 720$ … (i)

From the first condition,

$2(l + b) = 108$ or, $l + b = 54$ $\therefore l = 54 - b$ … (ii)

Now, substituting the value of l from equation (ii) in equation (i), we get

$(54 - b) \times b = 720$ or, $b^2 - 54b + 720 = 0$ or, $(b - 24)(b - 30) = 0$

Either, $b = 24$ or, $b = 30$ (Considering that the breadth is shorter than length)

Putting the Value of b in equation (ii), we get $l = 54 - 24 = 30$

Hence, length (l) = 30 m and breadth (b) = 24 m

Difference between the length and breadth = 30 m – 24 m = 6 m

Thus, to make the filed in to a square, the longer side is to be decreased by $\frac{6}{30} \times 100\% = 20\%$

2. A piece of cloth costs Rs 800. If the piece was 5m long and rate of cost of cloth per meter was Rs 8 less, the cost h of the piece would have remained unchanged. How long is the piece and what is the rate of cost of the cloth?

Solution:

Let the length of piece of the cloth be x m and the rate of cost of the cloth Rs y per meter.

From the first condition,

$xy = 800$ … (i)

From the first condition,

$(x + 5)(y - 8) = 800$

or, $xy - 8x + 5y - 40 = 800$

or, $800 - 8x + 5y - 40 = 800$ (From (i), $xy = 800$)

or, $y = \frac{40 + 8x}{5}$ (ii)

Now, substituting the value of y from equation (ii) in equation (i), we get

$x\left(\frac{40 + 8x}{5}\right)$ or, $x^2 + 5x - 500 = 0$ or, $(x + 25)(x - 20) = 0$

Either, $x = -25$ or, 20 But the length cannot be negative. So, length = 20 m

Again, putting the value of x in equation (i), we get $y = 40$

Hence, the length of the piece of cloth is 20 m and its rate of cost is Rs 40 per meter.

3. ***Mrs Magar bought a certain kilograms of vegetables for Rs 120 last week. This week, the rate of cost of vegetable decreases by Rs 20 per kg and she can buy 1 kg more vegetables for the same amount of money. By what percentage is the rate of cost of vegetables decreased?***

Solution:

Let the rate of cost of vegetable last week be Rs x per kg and the quantity of vegetable that can be bought for Rs 120 be y kg.

From the first condition,

$xy = 120$ … (i)

From the first condition,

$(x - 20)(y + 1) = 120$

or, $xy + x - 20y - 20 = 120$

or, $120 + x - 20y - 20 = 120$ (From (i), $xy = 120$)

or, $x = 20(y + 1)$ ... (ii)

Now, substituting the value of x from equation (ii) in equation (i), we get

or, $20(y + 1)y = 120$ or, $y^2 + y - 6 = 0$ or, $(y + 3)(y - 2) = 0$

Either, $y = -3$ or, 2 But the quantity of vegetable cannot be negative.

So, $y = 2$

Again, putting the value of y in equation (i), we get $x = 60$

Hence, rate of cost of vegetable last week was Rs 60 per kg.

Again, percentage of decreased amount of rate of cost $= \frac{20}{60} \times 100\% = 3\frac{1}{3}$

Thus, the rate of cost of vegetable is decreased $3\frac{1}{3}$ by in this week.

Extra questions

1. A two digit number is three times the product of digits and four times the sum of digits. Finds the number. [Ans: 24]
2. Bhawani was 25 years old when her daughter was born. Now, the product of their ages is 600. Find their present ages. [Ans: 40 years, 15 years]
3. Rs120 is equally distributed among certain number of students. If there were 3 students more, each would have received Rs 2 less. Find the number of students. [Ans: 10]

Unit 13 Area of Triangles and Quadrilaterals

Allocated teaching periods	12

Competency

- To prove logically and experimentally that the properties on the area of triangle and quadrilateral

Learning Outcomes

- To verify/prove the interrelationship between the areas of parallelograms standing on the same base and between the same parallels.
- To verify/prove the interrelationship between the areas of triangle and parallelogram standing on the same base and between the same parallels.
- To show the interrelationship between the areas of triangles standing on the same base and between the same parallels.

Level-wise learning objectives

S.N.	LEVELS	OBJECTIVES
1.	Knowledge (K)	- To tell the diagonal property of parallelogram - To tell the median property of triangle - To write the interrelationship between the areas of parallelograms standing on the same base and between the same parallels - To state the interrelationship between the areas of triangle and parallelogram standing on the same base and between the same parallels - To recall the types of parallelogram - To state the interrelationship between the areas of triangles standing on the same base and between the same parallels
2.	Understanding (U)	- To area of triangle or parallelogram based on the properties of area of triangle and parallelogram
3.	Application (A)	- To verify/prove the theorems on properties of area of triangle and parallelogram
4.	High Ability (HA)	- To prove theorems by drawing required diagrams/ figures

Required Teaching Materials/ Resources

Geo-board, graph board, circular shaped colourful chart-paper, scale, scissors, pencil, , marker, thread, rubber-band ICT tools etc

Pre-knowledge: Parts of circle

Teaching Activities

1. Recall the properties of parallelogram
2. Recall the formula to find the area of parallelograms and triangles
3. Draw the parallelogram on the graph board and tell to students to draw the parallelogram on the graph paper and discuss about the area of parallelogram by counting the squares

4. Divide the students in to groups and tell to draw the parallelograms under the following cases on the graph paper
   (i) Parallelograms standing on the same base and between the same parallels
   (ii) Parallelograms standing on different bases but lying between the same parallels
   (iii) Parallelograms standing on the same base but not lying between the same parallels

   After discussion about the areas of the parallelograms among the members of the groups, ask the group leader about the conclusion on the area.

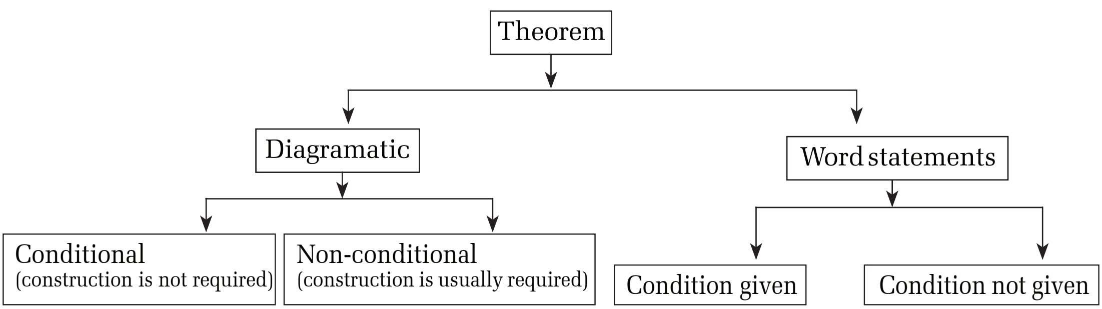
   At last, with discussion draw out the conclusion and state the theorem
5. If possible visualize the above experiments using geo-gebra tool
6. Ask the criterion of congruency of triangles
7. Verify experimentally/prove logically the theorem with discussion
8. Draw a parallelogram and a triangle under the following cases on the graph paper
   (i) Parallelogram and triangle standing on the same base and between the same parallels
   (ii) Parallelogram and triangle standing on different bases but lying between the same parallels
   (iii) Parallelogram and triangle standing on the same base but not lying between the same parallels

   After discussion about the areas of the parallelogram and triangle among the members of the groups, ask the group leader about the relationship between the area of parallelogram and the triangle.

   At last, with discussion draw out the conclusion and state the theorem
9. Ask the statement of the previous theorem
10. Verify experimentally/prove logically the theorem with discussion
11. Draw triangles under the following cases on the graph paper
    (iv) Triangles standing on the same base and between the same parallels
    (v) Triangles standing on different bases but lying between the same parallels
    (vi) Triangles standing on the same base but not lying between the same parallels

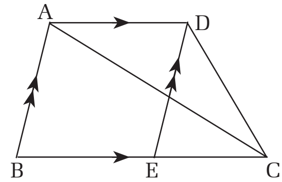
    Let the students make to explore the relationship between the areas of triangles

    At last, with discussion draw out the conclusion and state the theorem
12. Ask the statement of the previous theorems
13. Verify experimentally/prove logically the theorem with discussion
14. Recall the properties on area of parallelogram and triangle and discuss upon the problems given in exercise
15. Discuss about the nature of questions asked as higher ability level as shown below



Solution of selected questions from Vedanta Excel in Mathematics

1. ***In the figure, the area of trapezium ABCD is 100 sq.cm. and the area of ΔADC is 40 sq.cm. Find the area of ΔDEC.***



Solution:

Here, Area of trapezium ABCD = 100 cm^2

Area of ΔADC = 40 cm^2

Area of ΔDEC = ?

Now,

i) Area of ▱ ABED = 2 area of ΔADC [Both are standing on the same base AD and between AD//BC]

2×40 cm^2 = 80cm^2

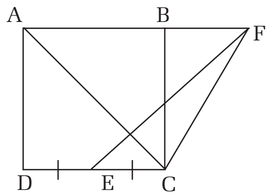
ii) Area of trapezium ABCD = Area of (▱ ABED + ΔDEC) [By whole part axion]

or, 100 cm^2 = 80 cm^2 + area of ΔDEC

∴ Area of ΔDEC = 20 cm^2

2. ***In the given figure, ABCD is a square whose perimeter is 40 cm and AB is produced to the point F. If F is the mid-point of DC, find the area of ΔEFC.***

Solution:



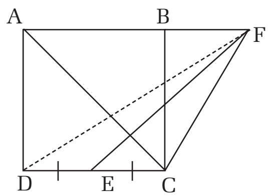
Construction : D and F are joined

i) Perimeter of square ABCD = 40 cm

or, $4l$ = 40 cm

∴ $4l$ = 10 cm

ii) Area of square ABCD = l^2 = $(10 \text{ cm})^2$ = 100 cm^2



iii) Area of ΔDCF $= \frac{1}{2} \times$ area of square ABCD

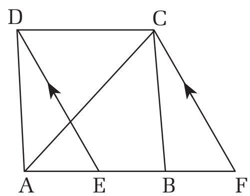
$= \frac{1}{2} \times 100$ cm^2 = 50 cm^2

iv) Area of ΔEFC $= \frac{1}{2} \times$ Area of ΔDCF [Median EF bisects the ΔDCF]

$= \frac{1}{2} \times 50$ cm^2 = 25 cm^2

So, area of ΔEFC = 25 cm^2

3. ***In the adjoining figure, AE//DC, ED//FC and ABCD is a square. If AC = $5\sqrt{2}$ cm, find the area of parallelogram DEFC.***



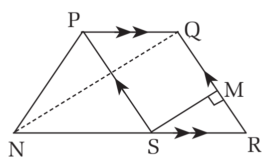
Solution:

i) Area of square ABCD $= \frac{1}{2}(ac)^2 = \frac{1}{2} \times (5\sqrt{2}\text{ cm})^2$ = 25 cm^2

ii) Area of parallelogram DEFC= Area of square ABCD = 25 cm^2

Hence, area of parallelogram DEFC = 25 cm^2

4. ***In the adjoining figure, PS = 5 cm and SM = 8 cm. Calculate the area of ΔPQN.***



Solution:

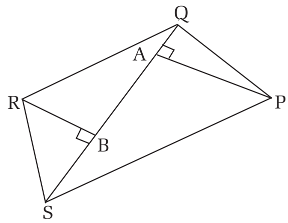
i) Area of ▱ PQRS = QR × SM

= 5 cm × 8 cm = 40 cm^2 [∵ QR = PS]

ii) Area of ΔPQN $= \frac{1}{2} \times$ Area of ▱ PQRS

$= \frac{1}{2} \times 40$ cm^2 = 20 cm^2

5. ***Find the area of quadrilateral PQRS given in the adjoining figure in which 3 RB = 2 PA = QS = 12 cm.***



Solution:

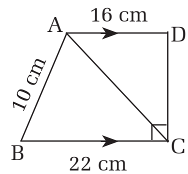
i) 3 RB = 2 PA = QS = 12 cm

$\therefore$ RB = 4 cm, PA = 6 cm and QS = 12 cm

ii) Area of quadrilateral PQRS $= \frac{1}{2} \times d(p_1 + p_2)$

$= \frac{1}{2} \times$ QS (RB + PA) $= \frac{1}{2} \times 12$ cm (4 cm + 6 cm) = 60 cm^2

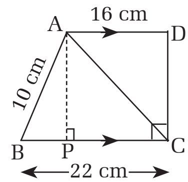
6. ***In the given figure, ABCD is a trapezium. If AB = 10 cm, BC = 22 cm, AD = 16 cm, AD//BC and DC⊥BC, calculate the area of ΔADC.***



Solution:

Construction: AP⊥BC is drawn where P is on BC.

$\therefore$ PC = AD = 16 cm, BP = 22 – 16 cm = 6 cm



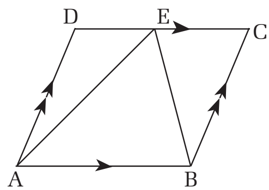
i) In right angled ΔABP; AP $= \sqrt{AB^2 - BP^2} = \sqrt{10^2 - 6^2}$ = 8 cm

ii) Area of ΔADC $= \frac{1}{2}$ AD × DC $\left[\Delta = \frac{1}{2} b \times h\right]$

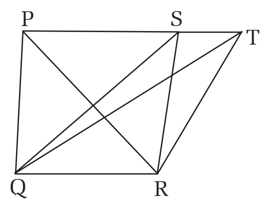
$= \frac{1}{2} \times 16 \text{ cm} \times 8 \text{ cm} = 64 \text{ cm}^2$

Extra question

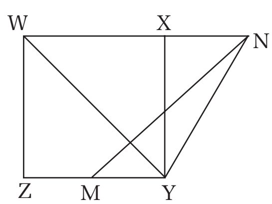
1. In the given figure, ABCD is a parallelogram. If the area of ΔABE = 25 cm^2, find the area of ΔBCE + ΔADE. [Ans: 25 cm^2]



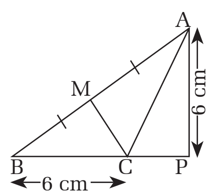
2. In the given figure, PQRS is a rhombus in which the diagonal PR = 16 cm and diagonal QS = 14 cm. If RS is produced to T, find the area of ΔQRT. [Ans: 56 cm^2]



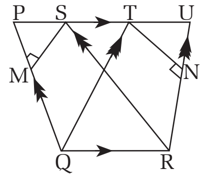
3. In the given figures, WXYZ is a square whose side WX is produced to N. If M is mid-point of YZ and area of ΔMYN is 25 cm^2, find the perimeter of square WXYZ. [Ans: 40 cm]



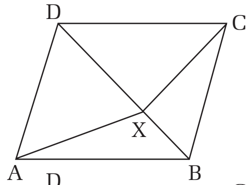
4. In the adjoining figures, M is mid-point of side AB of ΔABC, AP⊥BC. If BC = 8 cm and AP = 6 cm, find the area of ΔABC. [Ans: 12 cm^2]



5. In the adjoining figures, PU//QR, PQ//SR, TQ//UR, SM⊥PQ and TN⊥UR. If SM = 3 cm, PQ = 8 cm and TN = 4 cm, find the length of TQ. [Ans: 6 cm]

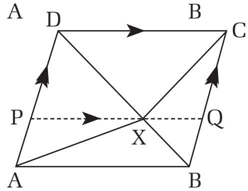


Solution of selected questions from Vedanta Excel in Mathematics



1. In the given figure, ABCD is a parallelogram. x is any point within it. Prove that the sum of area of ΔXCD and ΔXAB is equal to half of the area of ▱ ABCD.

Solution:



Given: In ▱ ABCD; x is any point within it.

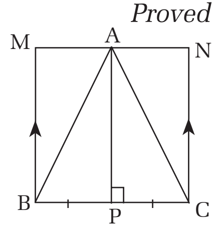
To prove: ΔXCD + ΔXAB = $\frac{1}{2}$ × Area of ▱ ABCD

Construction: PQ//AB//DC is drawn where P is on DA and Q is on BC.

Proof

	Statements		Reasons
1	DPQC and PQBA are parallelogram.	1.	By construction
2.	ΔXCD = $\frac{1}{2}$ ▱ DPQC	2.	Both are standing on the same base and between the same parallels
3.	ΔXAB = $\frac{1}{2}$ ▱ PQBA	3.	Same as (2)
4.	ΔXCD + ΔXAB = $\frac{1}{2}$ (▱ DPQC + ▱ PQBA)	4.	on adding (2) and (3)
5.	ΔXCD + ΔXAB = $\frac{1}{2}$ ▱ ABCD	5.	From (4), whole part axiom

Proved



2. In the given figure, ΔABC and parallelogram MBCN are on the same base BC and between the same parallels MN and BC. Prove that area of ΔABC = area of rectangle APCN.

Solution:

Given: ΔABC and parallelogram MBCN are on the same base BC and between the same parallels MN and BC. P is mid-point of BC and AP ⊥ BC.

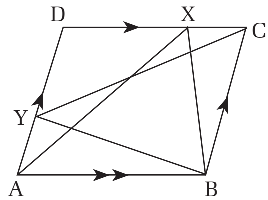
To Prove: Area of ΔABC = Area of rectangle APCN.

Proof

	Statements		Reasons
1.	ΔAPC = $\frac{1}{2}$ ΔABC	1.	Median AP bisects ΔABC
2.	ΔAPC = $\frac{1}{2}$ Rectangle APCN	2.	Diagonal AC bisects the rectangle
3.	ΔABC = rectangle APCN	3.	From (1) and (2)

Proved

3. *In the given parallelogram ABCD, X and Y are any points on CD and AD respectively. Prove that, area of ΔAXB = area of ΔBYC.*



Solution:

Given: In ▱ ABCD; X and Y are any point on CD and AD respectively.

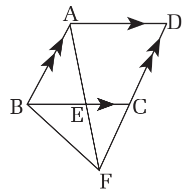
To Prove: Area of ΔAXB = Area of ΔBYC

Proof

	Statements		Reasons
1.	Area of ΔAXB = $\frac{1}{2}$ × Area of ▱ ABCD	1.	Both are standing on the same base AB and between DC // AB
2.	Area of ΔBYC = $\frac{1}{2}$ × Area of ▱ ABCD	2.	Both are standing on the same base BC and between AD // BC
3.	Area of ΔAXB = Area of ΔBYC	3.	From (1) and (2)

Proved

4. *In the adjoining parallelogram ABCD, A is joined to any point E on BC. AE and DC produced meet at F. Prove that area of ΔBEF = area of ΔCDE.*



Solution:

Given: In ▱ ABCD; E is any point on BC. AE and DC produced meet at F.

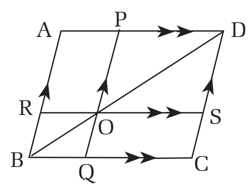
To prove: Area of ΔBEF = Area of ΔCDE

Proof

	Statements		Reasons
1.	Area of ΔAED = $\frac{1}{2}$ × Area of ▱ ABCD	1.	Both are standing on AD and between AD // BC
2.	Area of (ΔABE + ΔCDE) = $\frac{1}{2}$ × Area of ▱ ABCD	2.	Remaing part of ▱ ABCD
3.	Area of ΔABF = $\frac{1}{2}$ × Area of ▱ ABCD	3.	Both are standing on AB and between AB // DF
4.	Area of ΔABF = Area of (ΔABE + ΔBEF)	4.	Whole part axion
5.	Area of (ΔABE + ΔCDE) = Area of (ΔABE + ΔBEF)	5.	From (2), (3) and (4)
6.	Area of ΔCDE = Area of ΔBEF	6.	From (5)

Proved

5. *In the adjoining parallelogram ABCD, PQ // AB and RS // BC. Prove that area of ▱ ROPA = Area of ▱ QCSO.*



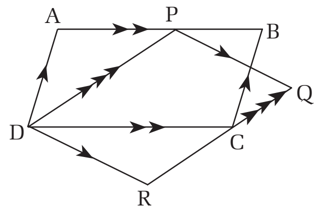
Solution:

Proof

	Statements		Reasons
1.	Area of ΔABD = Area of ΔBCD	1.	Diagonal BD bisects ▱ ABCD
2.	Area of ΔROB = Area of ΔBOQ	2.	Diagonal OB bisects ▱ RBQO
3.	Area of ΔPOD = Area of ΔSOD	3.	Diagonal OD bisects ▱ POSD
4.	Area of (ΔABD – ΔROB – ΔPOD) = Area of (ΔBCD – ΔBOQ – ΔSOD)	4.	Subtracting (2) and (3) from (1)
5.	Area of ▱ ROPA = Area of ▱ QCSO	5.	Remaining part of whole

Proved

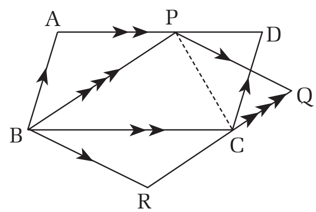
6. ***In the given diagram, ABCD and PQRD are two parallelograms. Prove that ▱ABCD = ▱ PQRD.***



Solution:

Construction: P and C are joined

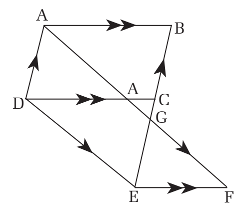
Proof



	Statements		Reasons
1.	ΔBPC = $\frac{1}{2}$ ▱ ABCD	1.	Both are standing on BC and between AD // BC
2.	ΔBPC = $\frac{1}{2}$ ▱ PQRD	2.	Both are standing on PB and between PB // QR
3.	▱ ABCD = ▱ PQRD	3.	From (1) and (2)

Proved

7. ***In the given figure, if AB // DC // EF, AD // BE and AF // DE, prove that parm DEFH = parm. ABCD.***



Solution:

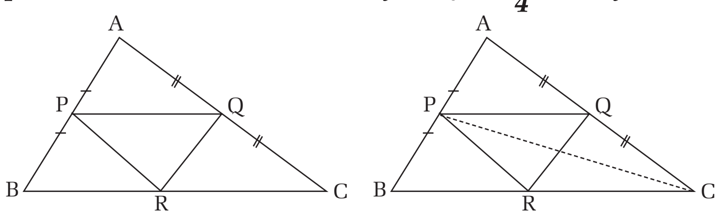
Proof

	Statements		Reasons
1.	▱ ABCD = ▱ AGED	1.	Both are standing on AD and between AD // BE
2.	▱ AGED = ▱ DEFG	2.	Both are standing on DE and between DE // AF
3.	▱ DEFG = ▱ ABCD	3.	From (1) and (2)

Proved

8. ***In the adjoining ΔABC, P and Q are the mid-point of the sides AB and AC respectively and R be any point on BC. Prove that area of ΔPQR = $\frac{1}{4}$ area of ΔABC.***

Solution:



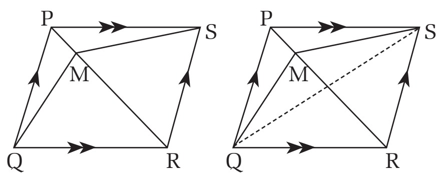
Construction:

P and C are joined.

Proof

	Statements		Reasons
1.	PQ // BC	1.	P and Q are the mid-points of sides AB and AC of ΔABC
2.	Area of ΔPQR = Area of ΔPQC	2.	Both are standing on PQ and between PQ // BC
3.	Area of ΔPQC = $\frac{1}{2}$ × Area of ΔAPC	3.	Median PQ bisects ΔAPC
4.	Area of ΔAPC = $\frac{1}{2}$ × Area of ΔABC	4.	Median PC bisects ΔABC
5.	Area of ΔPQR = $\frac{1}{2} \times \frac{1}{2}$ × Area of ΔABC ∴ Area of ΔPQR = $\frac{1}{4}$ × Area of ΔABC.	5.	From (2), (3) and (4)

Proved



9. ***In the figure, PQRS is a parallelogram. Q and S are joined to any point M on the diagonal PR. Prove that area of ΔPQM = area of ΔPSM.***

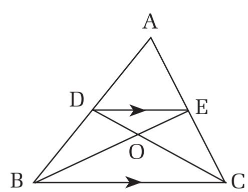
Solution:

Construction: Q and S are joined so that diagonal QS intersects diagonal PR at O.

Proof

	Statements		Reasons
1.	OQ = OS and OP = OR	1.	Diagonals of parallelogram bisect to each other
2.	ΔPOQ = ΔPOS	2.	Median PO bisects ΔPQS
3.	ΔMOQ = ΔMOS	3.	Median MO bisects ΔMQS
4.	ΔPOQ – ΔMOQ = ΔPOS – ΔMOS	4.	Subtracting (3) from (2)
5.	ΔPQM = ΔPSM	5.	Remaining part of whole

Proved



10. ***In the given figure, DE // BC. Prove that:***
i) ΔBOD = ΔCOE (ii) ΔBAE = ΔCAO

Solution:

Given and To Prove: Go through the text.

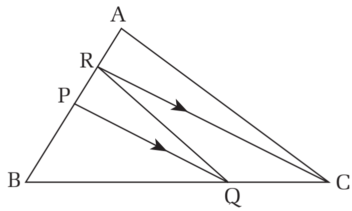
Proof

	Statements		Reasons
1.	ΔBED = ΔCED	1.	Both are standing on DE and between DE // BC
2.	ΔBED – ΔDOE = ΔCED – ΔDOE	2.	Subtracting same ΔDOE from both side of (1)

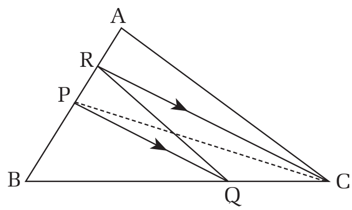
3.	$\Delta BOD = \Delta COE$	3.	Remaining part of whole
4.	$\Delta BED + \Delta ADE = \Delta CED + \Delta ADF$	4.	Adding same ΔADE in (1)
5.	$\Delta BAE = \Delta CAD$	5.	By whole part axiom

Proved

11. In the given figure, P is the mid-point of AB and Q is any point on the side BC. CR meets AB at R and CR // PQ. Prove that : Area of $\Delta BQR = \frac{1}{2}$ area of ΔABC.



Solution:



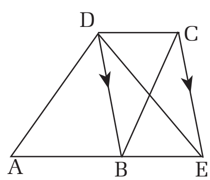
Construction : P and C are joined.

Proof

	Statements		Reasons
1.	$\Delta PQR = \Delta CPQ$	1.	Both are standing on PQ and between RC // PQ
2.	$\Delta PQR + \Delta PBR = \Delta CPQ + \Delta PBR$	2.	Adding same ΔPBR in (1)
3.	$\Delta BQR = \Delta BPC$	3.	By whole part axiom
4.	$\Delta BPC = \frac{1}{2}\Delta ABC$	4.	Median PC bisects ΔABC
5.	$\Delta BQR = \frac{1}{2}\Delta ABC$ $\therefore$ Area of $\Delta BQR = \frac{1}{2}$ Area of ΔABC	5.	From (3) and (4)

Proved

12. The line draw through the vertex C of the quadrilateral ABCD parallel to the diagonal DB meets AB produced at E. Prove that, area of quadrilateral ABCD = area of ΔDAE.



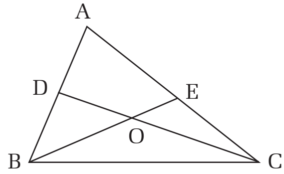
Solution:

Proof

	Statements		Reasons
1.	$\Delta BCD = \Delta BED$	1.	Both are standing on BD and between BD // EC
2.	$\Delta BCD = \Delta ABD = \Delta BED + \Delta ABD$	2.	Adding same ΔABD in (1)
3.	Quad. ABCD = ΔDAE	3.	By whole part axiom

Proved

13. In the given triangle ABC, two medians BE and CD are intersecting at O. Prove that:
area of ΔBOC = area of quad. ADOE



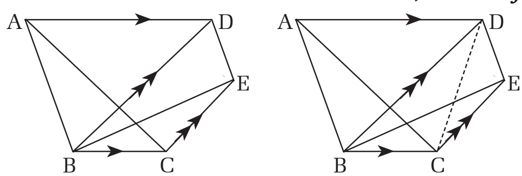
Solution:

Proof

	Statements		Reasons
1.	$\Delta BEC = \frac{1}{2}\Delta ABC$	1.	Median BE bisects ΔABC
2.	$\Delta ADC = \frac{1}{2}\Delta ABC$	2.	Median CD bisects ΔABC
3.	ΔBEC = ΔADC	3.	From (1) and (2)
4.	ΔBEC – ΔEOC = ΔADC – ΔEOC	4.	Subtracting same ΔEDC from both sides of (3)
5.	ΔBOC = Quad. ADOE	5.	Remaining parts of whole

Proved

14. In the adjoining figure, it is given that AD // BC and BD // CE. Prove that, area of ΔABC = area of ΔBOE.



Solution:

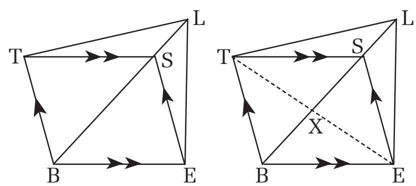
Construction: C and D are joined

Proof

	Statements		Reasons
1.	ΔABC = ΔDBC	1.	Both are standing on BC and between AD // BC
2.	ΔDBC = ΔBDE	2.	Both are standing on BD and between BD // CE
3.	ΔABC = ΔBDE	3.	From (1) and (2)
4.	∴ area of ΔABC = area of ΔBDE	4.	

Proved

15 In the given figure, BEST is a parallelogram. Diagonal BS is produced to point L. Prove that ΔLST and ΔLSE are equal in area



Solution:

Construction: T and E are joined.

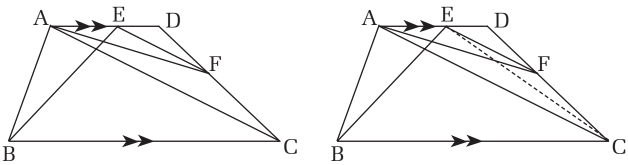
Proof

	Statements		Reasons
1.	TX = XE and SX = XB	1.	Diagonals of ▱ do bisect each other
2.	ΔLTX = ΔLEX	2.	Median LX bisects ΔLTE

3.	$\Delta STX = \Delta SEX$	3.	Median SX bisects ΔSTE
4.	$\Delta LTX - \Delta STX = \Delta LEX - \Delta SEX$	4.	Subtracting (3) from (2)
5.	ΔLST and ΔLSE $\therefore$ ΔLST and ΔLSE are equal in area	5.	Remaining part of whole

Proved

16. *In the given figure, AD // BC. If the area of ΔABE and ΔACF are equal then prove that EF // AC.*



Solution:

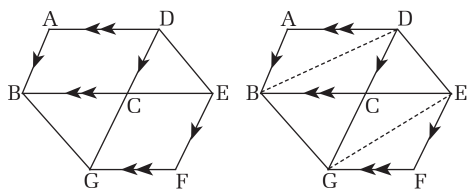
Construction: E and C are joined

Proof

	Statements		Reasons
1.	Area of ΔABE = Area of ΔACE	1.	Both are standing on AE and between AE // BC
2.	Area of ΔABE = Area of ΔACF	2.	Given
3.	Area of ΔACE = Area of ΔACF	3.	From (1) and (2)
4.	EF // AC	4.	From (3), both are standing on same base AC and between EF and AC

Proved

17. *In the figure, AD // BE // GF and AB // DG // EF. If the area of parallelograms ABCD and CEFG are equal, then prove that DE // BG.*



Solution:

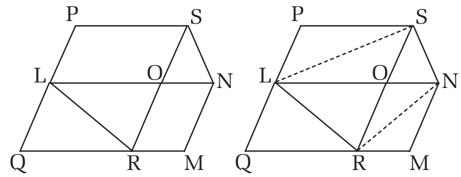
Construction: B, D and G, E are joined

Proof

	Statements		Reasons
1.	area of ▱ ABCD = area of ▱ CEFG	1.	Given
2.	area of $\Delta BCD = \frac{1}{2}$ area of ▱ ABCD and area of $\Delta GCE = \frac{1}{2}$ area of ▱ CEFG.	2.	Diagonals bisects the parallelograms
3.	area of ΔBCD = area of ΔGCE	3.	From (i) and (ii)
4.	area of $(\Delta BCD + \Delta BCG)$ = area of $(\Delta GCE + \Delta BCG)$	4.	Adding ΔBCG on both sides of (iii)
5.	area of ΔDBG = area of ΔEBG	5.	whole part axiom
6.	DE // BG	6.	ΔDBG and ΔEBG with equal areas are standing on the same base BG and between the same lines DE and BG

Proved

18. ***In the adjoining figure, PQRS and LQMN are two parallelogram of equal area. Prove that LR // SN.***



Solution:

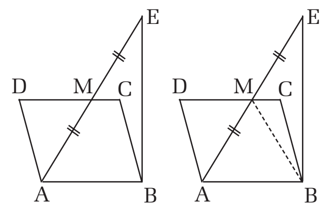
Construction: S, L and N, R are joined

Proof

	Statements		Reasons
1.	▱ PQRS = ▱ LQMN (in area)	1.	Given
2.	ΔSLR = $\frac{1}{2}$ ▱ PQRS (in area)	2.	Both are standing on the same base SR and between PQ // SR
3.	ΔNLR = $\frac{1}{2}$ ▱ LQMN (in area)	3.	Both are standing on the same base and between LN // QM
4.	ΔSLR = ΔNLR (in area)	4.	From (2) and (3)
5.	LR // SN	5.	From (4), both Δs are standing on the same base LR and between SN and LR From (4), both Δ^s are standing on the same base LR and between SN and LR

Proved

19. ***In the given figure, M is the mid-point of AE, then prove that area of ΔABE is equal to the area of parallelogram ABCD.***



Solution:

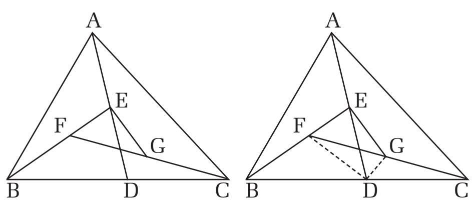
Construction: M and B are joined

Proof

	Statements		Reasons
1.	ΔMAB = $\frac{1}{2}$ ΔABE	1.	Median MR bisects ΔABE
2.	ΔMAB = $\frac{1}{2}$ ▱ ABCD	2.	Both are standing on same base AB and between DC // AB
3.	ΔABE = ▱ ABCD	3.	from (1) and (2)

Proved

20. ***In the given ΔABC; D, E, F and G are mid-point of BC, AD, BE and CF respectively. Prove that ΔABC = 8 ΔEFG***



Solution:

Construction: G, D and FD are joined

Proof

	Statements		Reasons
1.	DG // BF i.e, DG // BE	1.	D and G are mid-point of sides BC and FC in ΔFBC

2.	$\Delta EFG = \Delta EFD$	2.	Both are standing on the same base EF and between EF // GD
3.	$\Delta EFD = \frac{1}{2}\Delta BED$	3.	Median DF bisects ΔBED
4.	$\Delta BED = \frac{1}{2}\Delta ABD$	4.	Median BE bisects ΔABD
5.	$\Delta ABD = \frac{1}{2}\Delta ABC$	5.	Median AD bisects ΔABC
6.	$\Delta EFG = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\Delta ABC$ $\therefore \Delta ABC = 8\ \Delta EFG$	6.	From (2), (3), (4) and (5)

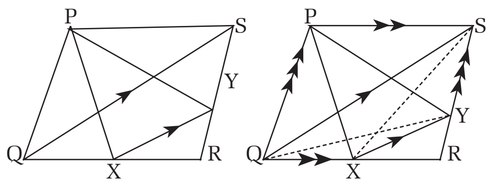
Proved

21. ***In the figure alongside, PQRS is a parallelogram. X and Y are any point on QR and RS respectively such that XY // QS. Prove that area of ΔPQX = area of ΔPSY.***

Solution:

Construction: Q, Y and X, S are joined

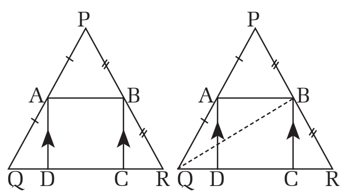
Proof



	Statements		Reasons
1.	Area of ΔPQX = Area of ΔSQX	1.	Both are standing on QX and between PS // QX
2.	Area of ΔSQX = Area of SQY	2.	Both are standing on QS and between QS // XY
3.	Area of ΔSQY = Area of ΔPSY	3.	Both are standing on Sy and between PQ // SR
4.	Area of ΔPQX = Area of ΔPSY	4.	From (1), (2) and (3)

Proved

22. ***In ΔPQR, A and B are the mid-point of the sides PQ and PR respectively. D and C are two points of QR such that AD // BC. Prove that $\square$ ABCD = $\frac{1}{2}\Delta PQR$.***



Solution:

Construction: B and Q are joined

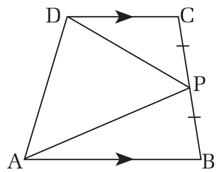
Proof

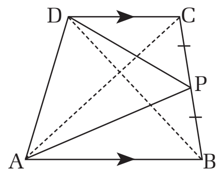
	Statements		Reasons
1.	AB // QR	1.	A and B are mid-point of sides PQ and PR in ΔPQR
2.	ABCD is a parallelogram	2.	Being opposite side parallel

3.	$\Delta ABQ = \frac{1}{2} \square ABCD$	3.	Both are standing on AB and between AB // QR
4.	$\Delta ABQ = \frac{1}{2} \Delta PQB$	4.	Median AB bisects ΔPQB
5.	$\square ABCD = \Delta PQB$	5.	From (3) and (4)
6.	$\Delta PQB = \frac{1}{2} \Delta ABC$	6.	Median QB bisects ΔABC
7.	$\square ABCD = \frac{1}{2} \Delta ABC$	7.	From (5) and (6)

Proved

23. ***In the trapezium ABCD, AB // DC and P is the mid-point of BC.***
Prove that: $\Delta APD = \frac{1}{2}$ trap. ABCD.





Solution:

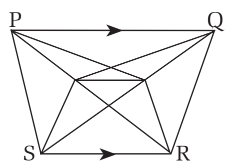
Construction: A, C and B, D are joined

Proof

	Statements		Reasons
1.	$\Delta DAB = \Delta CAB$	1.	Both are standing on AB and between DC // AB
2.	$\Delta CAB = 2\ \Delta APB$	2.	Median AP bisects ΔCAB
3.	$\Delta DAB = 2\ \Delta APB$	3.	From (1) and (2)
4.	$\Delta DBC = 2\ \Delta DPC$	4.	Median DP bisects ΔDBC
5.	$\Delta DAB + \Delta DBC = 2\ \Delta APB + 2\ \Delta DPC$	5.	Adding (3) and (4)
6.	Trap. ABCD $= 2(\Delta APB + \Delta DPC)$ or $\Delta APB + \Delta DPC = \frac{1}{2}$ trap. ABCD	6.	By whole part axiom
7.	$\Delta APD = \frac{1}{2}$ trap. ABCD	7.	Remaining part of Whole

Proved

24. ***In the given trapezium PQRS, PQ // SR. M and N are the mid-point of diagonals PR and QS respectively. Prove that: $\Delta PNR = \Delta QMS$.***



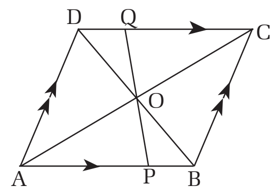
Solution:

Proof

	Statements		Reasons
1.	$\Delta PSR = \Delta QSR$	1.	Both are standing on SR and between PQ // SR
2.	$\Delta MSR = \frac{1}{2} \Delta PSR$ & $\Delta NSR = \frac{1}{2} \Delta QSR$	2.	Medians bisect the triangles
3.	$\Delta MSR = \Delta NSR$	3.	From (1) and (2)

4.	MN // SR	4.	From (3), Δ^S standing on SR and beteen MN and SR
5.	ΔPMN = ΔQMN	5.	Both are standing on MN and between PQ // MN
6.	ΔMNR = ΔMNS	6.	Both are standing on MN and between MN // SR
7.	ΔPMN + ΔMNR = ΔQMN + ΔMNS	7.	Adding (5) and (6)
8.	ΔPMR = ΔQMS	8.	Whole part axiom

Proved



25. In the given figure, ABCD is a parallelogram in which diagonal AC and BD intersect at O. A line segment through O meets at P and DC at Q.
Prove that: area of trap. APQD = $\frac{1}{2}$ area of parm. ABCD.

Solution:

Proof

	Statements		Reasons
1.	In ΔDOQ and ΔBOP i) $\angle$ODQ = $\angle$DBP (A) ii) OD = OB (S) iii) $\angle$DOQ = $\angle$BOP (A)	1.	i) DC // AB, being alternate angles ii) Diagonal AC bisects the diagonal BD in ABCD iii) Vertically opposite angles
2.	ΔDOQ $\cong$ ΔBOP	2.	By A. S. A axiom
3.	Area of ΔDOQ = Area of ΔBOP	3.	Area of congruent triangles are equal
4.	ΔDOQ + Quad. APOD = ΔBOD + Quad. APOD	4.	Same quad. APOD is added on both sides of (3)
5.	Trap. APQD = ΔABD	5.	Whole part axiom
6.	ΔABD = $\frac{1}{2}\square$ ABCD	6.	Diagonal BD bisects ABCD
7.	Trap. APQD = $\frac{1}{2}\square$ ABCD	7.	From (5) and (6)

Proved

Unit 14 Geometry - Construction

Allocated teaching periods	7

Competency

- To construct triangles and quadrilaterals having equal areas and show their interrelationship logically

Learning Outcomes

- To construct triangles and quadrilaterals having equal areas and show their interrelationship.

Level-wise learning objectives

S.N.	LEVELS	OBJECTIVES
1.	Application (A)	- To sketch the rough figure and construct the triangle and quadrilaterals having equal areas and show their interrelationship.

Required Teaching Materials/ Resources

Geometric instruments (ruler, compass, protractor), geo-board, pencils, marker, ICT tool etc

Pre-knowledge: properties of quadrilaterals (square, rectangle, parallelogram, rhombus) and triangles

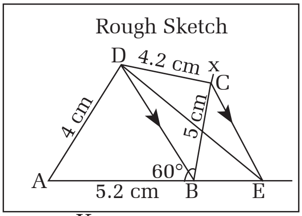
Teaching Activities

1. Recall the area of triangles standing on the same base and between the same parallel lines
2. Discuss about the procedures of construction of given triangle and required triangle having equal area.
3. Explain about the procedures of construction of given triangle and required rectangle having equal area. For this,
   (i) Construct the triangle with the given measurements
   (ii) Through the vertex, draw the perpendicular bisector of the base of triangle
   (iii) Draw the square having the base being half of that of the triangle and within the same parallels
4. Discuss about the procedures of construction of given triangle and required parallelogram having equal area. For this,
   (i) Construct the triangle with the given measurements
   (ii) Through the vertex, draw the perpendicular bisector of the base of triangle
   (iii) Draw the parallelogram with the given angle or side on the base being half of that of the triangle and within the same parallels
5. Recall the area of parallelograms standing on the same base and between the same parallel lines.
6. Discuss about the procedures of construction of given parallelogram and required parallelogram having equal area. For this,
   (i) Construct the parallelogram with the given measurements
   (ii) Produce the upper side and draw the required parallelogram on the base that of the parallelogram within the same parallels
7. Discuss about the procedures of construction of given parallelogram and required triangle having equal area. For this,
   (i) Construct the parallelogram with the given measurements
   (ii) Produce the upper side and draw the required triangle on the base which double of the base of parallelogram within the same parallels
8. Discuss about the procedures of construction of given quadrilateral and required triangle having equal area.

Solution of selected questions from Vedanta Excel in Mathematics

1. ***Construct a quadrilateral ABCD having AB = 5.2 cm, BC = 5 cm, CD = 4.2 cm, AD = 4 cm and ∠ABC = 60°. Construct ΔADE whose area is equal to the quadrilateral ABCD.***

Solution:



1st phase: Construction of quadrilateral ABCD

Steps

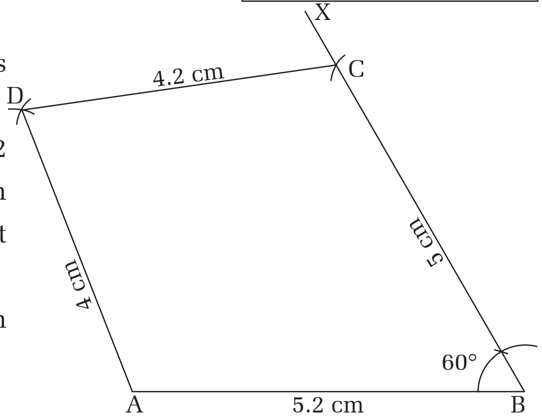
i) Draw AB = 5.2 cm

ii) AT B, draw ∠ABX = 60°

iii) From B, draw an arc with radius BC = 5 cm to cut BX at C.

iv) From C, draw an arc with radius CD=4.2 cm and from A draw another arc with radius AD = 4 cm. These two arc intersect each other at D.

Join C, D and D,A. Thus ABCD is the given quadrilateral.



2nd Phase: Construction of ΔADE.

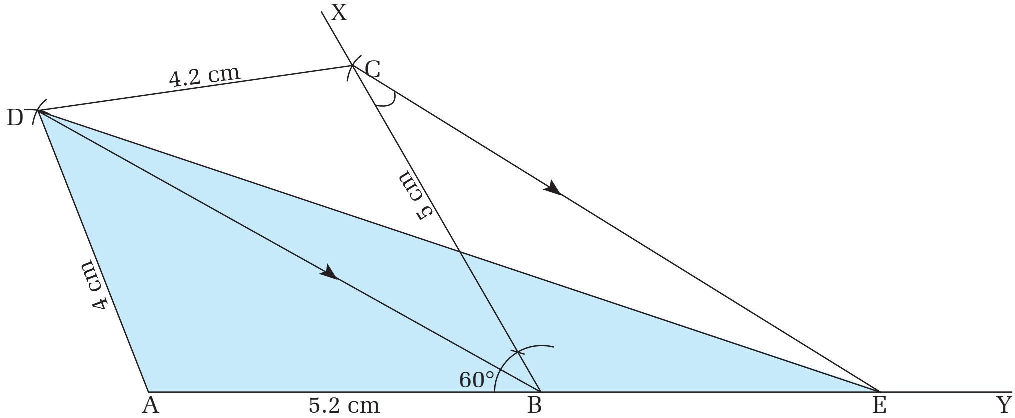
Steps

i) Join D and B.

ii) Measure ∠DBC and copy the angle at C. So that DB // CE through C to meet AB produced at E.

iii) Join D and E and get a triangle ADE.

Hence ΔADE is the required triangle whose area is equal to the area of quadrilateral ABCD.



Checking theoretically: $\Delta ADE = \Delta ABD + \Delta DBE$ [whole part axiom]

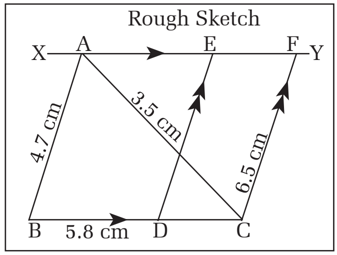
$= \Delta ABD + \Delta CDB$ [$\because \Delta DBE = \Delta CDB$]

$=$ Quad. ABCD [whole part axiom]

$\therefore \Delta ADE =$ Quad. ABCD

2. ***Construct a triangle ABC in which a = 5.8 cm, b = 3.5 cm and c = 4.7 cm. Construct a parallelogram with a side 6.5 cm and equal in area to the triangle.***

Solution:



1st phase: Construction of ΔABC.

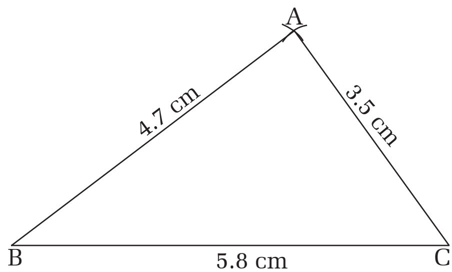
Steps

i) Draw BC = 5.8 cm

ii) From C, draw an arc with radius AC = 3.5 cm and from A draw another arc radius AB = 4.7cm. These two arc intersect at A.

iii) Join A, B and A, C.

Thus, ABC is the given triangle



2nd Phase: Construction of parallelogram CDEF.

Steps

i) Through the vertex A, draw XY // BC.

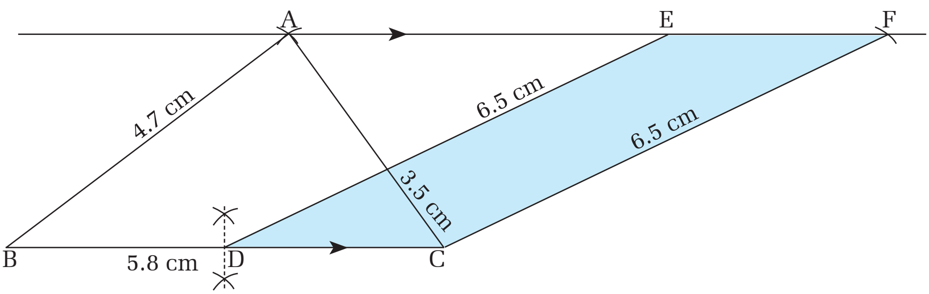
ii) With the help of perpendicular bisector of BC, mark the mid-point D of BC.

iii) Draw DE = 6.5 cm produced to meet XY at E.

iv) From the point E, Draw an arc with radius EF = DC to cut XY at F.

v) Join F and C.

Thus, CDEF is the required parallelogram.



Checking theoretically: ▱ CDEF $= CD \times$ height (h)

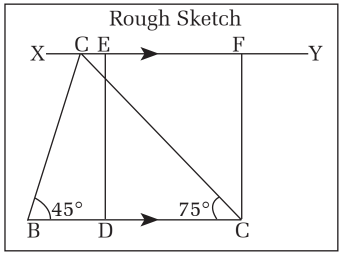
$= \frac{1}{2} BC \times$ height (h) [Δ and ▱ have equal height]

$=$ Area of ΔABC

$\therefore$ ▱ CDEF $= \Delta ABC$

3. ***Construct a triangle ABC in which AB = 5cm, ∠ABC = 75° and ∠ABC = 60°. Then construct a rectangle equal in area to the triangle ABC.***

Solution:



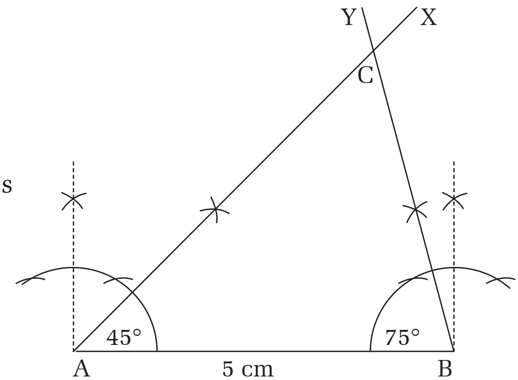
1st phase: Construction of ΔABC.

Steps

i) Find the measure of ∠BAC = 45°.
[∠A + ∠B + ∠C = 180°]

ii) Draw AB = 5cm

iii) At A, draw ∠BAC = 45° and at B.
Draw ∠ABC = 75°. These two lines intersect at C.
Thus, ABC is the given triangle.



2nd phase: Construction of rectangle

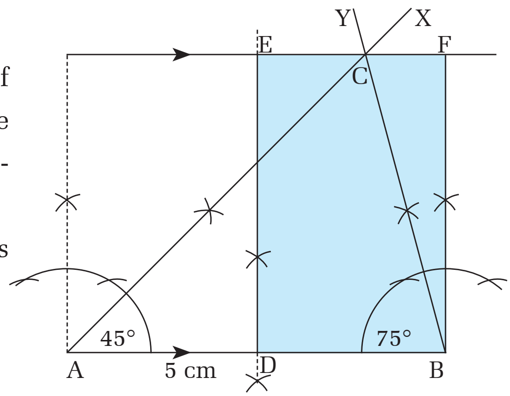
Steps

i) Through the vertex C, draw PQ // AB.

ii) With the help of perpendicular bisector of AB, mark the mid-point of AB. Fix that the perpendicular bisector DE within the parallel lines as a side of rectangle BDEF.

iii) From the point E, draw an arc with radius EF = DB to cut PQ at F.

iv) Join F and B.
Thus, BDEF is the required rectangle.



Checking theoretically: Rectangle BDEF $= BD \times DE$

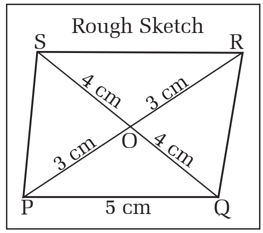
$= \frac{1}{2} AB \times DE \quad [\because BD = \frac{1}{2} AB]$

$= \Delta ABC \quad [\because \Delta ABC = \frac{1}{2} AB \times DE]$

$\therefore$ Rectangle BDEF $= \Delta ABC$

4. ***Construct a parallelogram PQRS in which PQ = 5 cm, diagonal PR = 6 cm and diagonal QS = 8 cm. Construct a ΔPSA whose area is equal to the area of parallelogram.***

Solution:



1^{st} **Phase:** Construction of parallelogram PQRS.

Steps

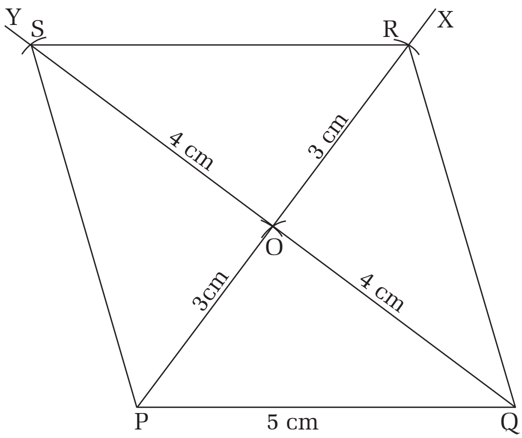
i) Draw PQ = 5 cm

ii) From P, draw an arc with raduus PO = $\frac{1}{2}$ PR = $\frac{1}{2} \times 6$ cm = 3 cm and another arc with radius QO = $\frac{1}{2}$ QS = $\frac{1}{2} \times 8$ cm = 4 cm. These two arc intersects at O.

iii) Join PO and produce to X, take an radius OR = 3 cm and cut OX at R.

iv) Join Q and O and prouce to Y, take an arc with radius OS = 4 cm and cut OY at S.

v) Join P and S, Q and R and R and S. Thus PQRS is the given parallelogram.

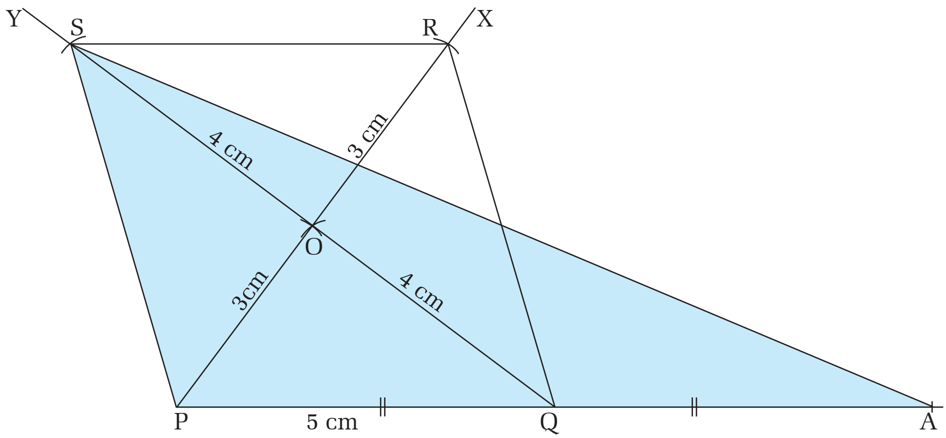


2^{nd} **phase**

Steps

i) Produce PQ to A such that PQ = QA

ii) Join S and A. Now, PSA is the required triangle.



Checking theoretically: $\Delta PSA = \frac{1}{2} PA \times \text{height}$ $[\because \Delta = \frac{1}{2} b \times h]$

$= \frac{1}{2} \times 2\, PQ \times \text{height}$ $[\because PA = 2\, PQ]$

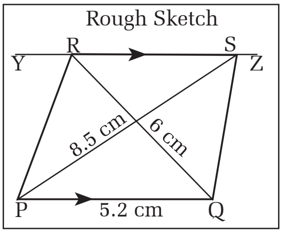
$= PQ \times \text{height}$ $[\because \Delta \text{ and } \square \text{ have same height}]$

$= \square\, PQRS$ $[\because \square = b \times h$

$\therefore \Delta PSA = \square\, PQRS$

5. ***Construct a triangle PQR in which PQ = 5.2 cm, QR = 6 cm and ∠Q = 60°. Construct another triangle SPQ which is equal in area to the ΔPBR and side PS = 8.5 cm.***

Solution:



1st phase: Construction of ΔPQR

Steps

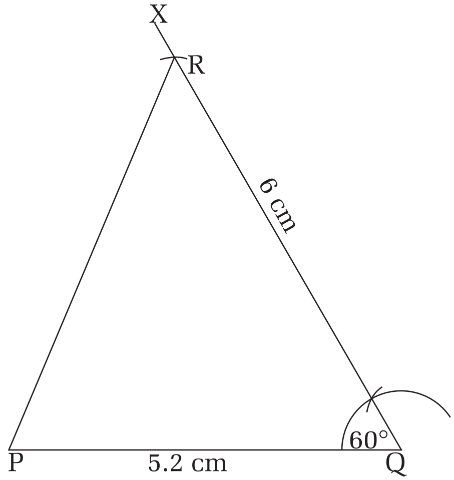
i) Draw PQ = 5.2 cm

ii) At Q, draw ∠PQX = 60°

iii) Take an arc of 6 cm and cut QX at R from Q

iv) Join P and R.

Thus, PQR is the given triangle.



2nd phase: Construction of ΔSPQ

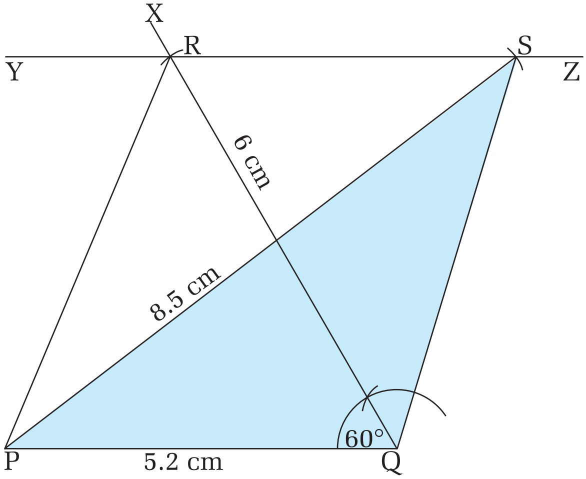
Steps

i) Through the vertex R, draw YZ // PQ.

ii) From the point P, draw an arc with radius PS = 8.5 cm and YZ at S.

iii) Join Q and S.

Thus SPQ is the required triangle.

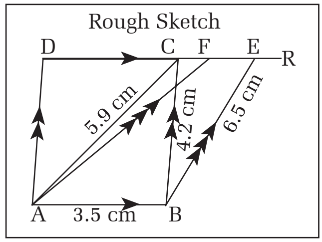


Checking theoretically:

Area of ΔPQR = Area of ΔSPQ [Both are standing on the same base PQ and between YZ // PQ]

6. ***Construct a parallelogram ABCD in which AB = 3.5 cm, AC = 5.9 cm and BC = 4.2 cm. Construct another parallelogram ABEF where BE = 6.5 cm.***

Solution:



1st phase: Construction of ▱ ABCD

Steps

i) Draw AB = 3.5 cm

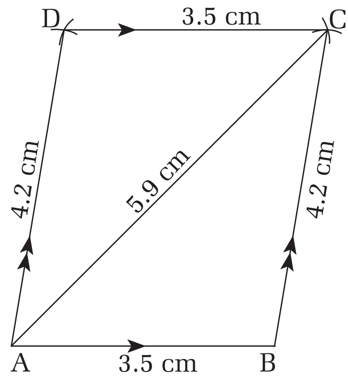
ii) From A, draw an arc with radius AC = 5.9 cm and from B draw another arc with radius BC = 4.2 cm. These two arc intersect each other at C.

iii) Join A, C and B, C.

iv) From C, draw an arc with radius CD = AB = 3.5 cm and from A, draw another arc with radius AD = BC = 4.2 cm. These two arc intersect at D.

v) Join A, D and C, D.

Thus, ABCD is the given parallelogram.



2nd phase: Constuction of ▱ ABEF.

Steps

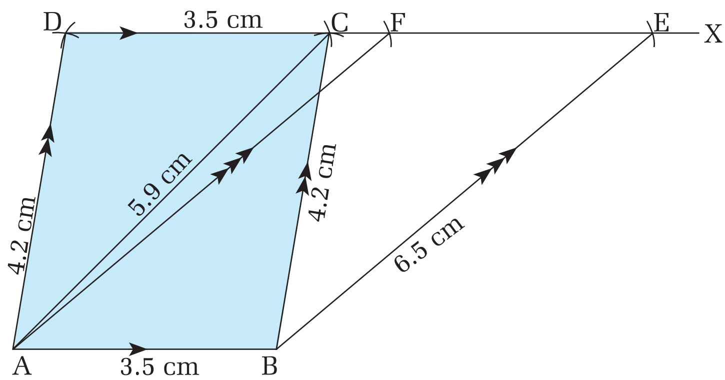
i) Produce DC to X.

ii) From B, draw an arc with radius BE = 6.5 cm to cut DX at E.

iii) From A, draw another arc with radius AF = BE = 6.5 cm to cut DX at F.

iv) Join B, E and A, F

Thus, ABEF is the required parallelogram.



Checking theoretically:

▱ ABCD = ▱ ABEF [Both are standing on the same base AB and between DX // AB]

Unit 15 Geometry - Circle

Allocated teaching periods	15

Competency

- To prove logically and experimentally the properties on arc and angle of circle and use logically their inter-relationship.

Learning Outcomes

- To prove /verify the theorems/properties related to arcs and angle of circle
- To interpret and represent the relationship between properties of arcs and angles of circle and solve the related problems

Level-wise learning objectives

S.N.	LEVELS	OBJECTIVES
1.	Knowledge (K)	- To recall the parts of the circle - To tell the relation between arc and the angle at the centre of circle subtended by it. - To tell the relation between arc and the angle at the circumference of circle subtended by it - To write down the relation between the equal arcs of circle and angles subtended by it. - To tell the relation between equal chords and the corresponding arcs - To write the relation between angles at the centre and circumference of circle standing on the same arc - To tell the angle at the semi-circle - To state the relationship between the inscribed angles subtended by the same arc - To state the relationship between the opposite angles of the cycle quadrilateral - To write down the relationship between the radius and the tangent touching each other at the point of contact
2.	Understanding (U)	- To solve the problems related to the properties of arcs and angles of circle - To solve the problems related to radius and tangent to the circle
3.	Application (A)	- To verify/prove the theorems on properties of arcs and angles of the circle

4.	High Ability (HA)	- To prove theorems logically by drawing required diagrams/ figures

Geo-board, graph board, circular shaped colourful chart-paper, scale, scissors, pencil, , marker, thread, rubber-band ICT tools etc

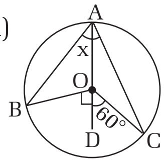
Pre-knowledge: Parts of circle

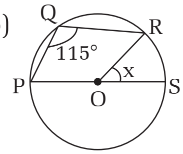
Teaching Activities

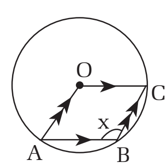
1. Recall the parts of the circle by using figures on chart-paper or using ICT tools
2. Divide the students into groups and encourage them to find the relationship between the arcs and angles subtended by it at the centre and circumference
3. Draw the required figures and experimentally verify that the relationship between equal arcs angles subtended by them at the centre or at the circumference with discussion.
4. Discuss about the relationship between the equal chords and their corresponding arcs
5. Verify experimentally and prove logically the following theorems under discussion
   (i) The angle at the centre of a circle is twice the angle at the circumference standing on the same arc
   (ii) The angle in the semi-s=circle is always a right angle
   (iii) The angles on the same segment of a circle are equal or the angles at the circumference of a circle standing on the same arc are equal
   (iv) The opposite angle s of a cyclic quadrilateral are supplementary
6. Change the name of the figure and ask to prove or verify the theorems
7. Call the students one at a time to prove the theorem on the board.
8. With recalling the properties on arcs and angles of a circle, solve the simple problems from in the textbook in group or individually
9. To develop critical thinking of the students, ask the problems or theorems related to the properties or theorems on circle
10. Show with explanation the theorems
    (i) The tangent to the circle is perpendicular to radius of the circle drawn at the point of contact.
    (ii) The length of two tangents to the circle at the points of contact from the same external point are equal
    (iii) The angles in alternate segments are equal
11. Solve the problems given in the exercise in groups or individually
12. Give the set of problems in group and

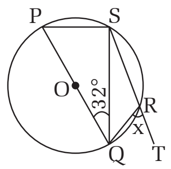
Solution of selected questions from Vedanta Excel in Mathematics

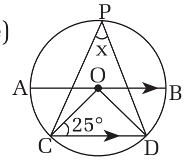
1. ***Find the sizes of unknown angles in the following figures:***

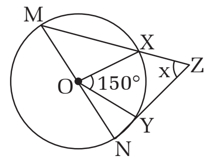
a) 

b) 

c) 

d) 

e) 

f) 

Solution:

a) i) $\angle BOC = \angle BOD + \angle COD = 90° + 60° = 150°$

ii) $\angle BAC = \frac{1}{2} \angle BOC$ [Inscribed angle is half of the angle at central of a circle standing on the same arc BC]

$\therefore x = \frac{1}{2} \times 150 = 75°$

b) $\angle PQR = \frac{1}{2}$ Reflex $\angle POR$ [Inscribed angle is half of the central angle of a circle standing on the same arc PSR]

or, $115° = \frac{1}{2} (\angle POS + \angle ROS)$

or, $230 = 180° + x \quad \therefore 50°$

c) i) $\angle AOC = \angle ABC = x$ [Being opposite angles of parallelogram DABC]

ii) Reflex $\angle AOC = 2\angle ABC = 2x$ [Being central angle twice the inscribed angle standing on the same arc]

iii) Obtuse $\angle AOC$ + Reflex $\angle AOC = 360°$

or, $x + 2x = 360° \quad \therefore x = 120°$

d) i) $\angle PSQ = 90°$ [Angle in semi-circle]

ii) $PQS + \angle PSQ + \angle SPQ = 180°$

or, $32° + 90° + \angle SPQ = 180°$

$\therefore \angle SPQ = 58°$

iii) $\angle QRT = \angle SPQ$ [Exterior angle of cyclic quadrilateral is equal to its opposite interior angle]

$\therefore x = 58°$

e) i) $\angle ABC = \angle BCD$ [AB // CD, alternate angles]

$= 32°$

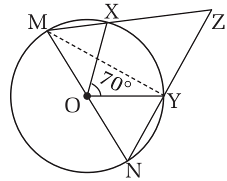
ii) $\angle OCB = \angle OBC = 32°$ [$\because OB = OC$]

iii) $\angle OCD = \angle OCB + \angle BCD = 32° + 32° = 64°$

iv) $\angle ODC = \angle OCD = 64°$ [$\because$ OC = OD]

v) $\angle OCD + \angle ODC + \angle COD = 180°$ or, $64° + 64° + \angle COD = 180°$ $\therefore \angle COD = 50°$

vi) $\angle CPO = \frac{1}{2} \angle COD = \frac{1}{2} \times 52° = 26°$



f) Construct: M and Y are joined

i) $\angle MYN = 90°$ [Angle in semi-circle]

ii) $\angle XMY = \frac{1}{2} \angle XOY = \frac{1}{2} \times 50° = 25°$

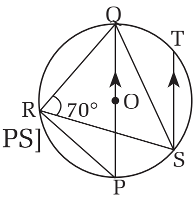
iii) $\angle YMZ + \angle MZY = \angle MYN$ [Exterior angle of Δ is equal to the sum of opposite interior angles]

or, $25° + x = 90°$

$\therefore x = 65°$

2. ***In the figure given alongside, O is the centre of the circle and PQ is a diameter. Chord ST is parallel to the diameter. If $\angle QRS = 70°$, find the measure of $\angle QST$.***

Solution:



Construct: R and P are joined

i) $\angle PRS + \angle SRQ = \angle PRQ = 90°$ [angle in semi-circle]

or, $\angle PRS + 70° = 90°$ $\therefore PRS = 20°$

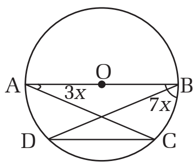
ii) $\angle PQS = \angle PRS = 20°$ [Inscribed angles standing on the same arc PS]

iii) $\angle QST = \angle PQS = 20°$ [PQ // ST, alternate angles]

$\therefore QST = 20°$

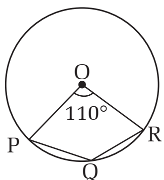
Extra questions

1. ***Find the size of unknown angles in the following figures.***

a) 

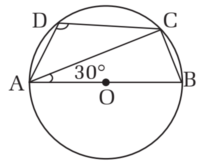
$\angle BDC = ?$

[Ans: 30°]

b) 

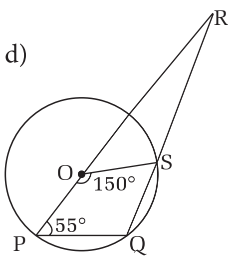
$\angle PQR = ?$

[Ans: 125°]

c) 

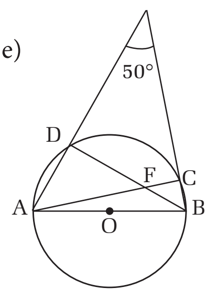
$x = ?$

[Ans: 120°]

d) 

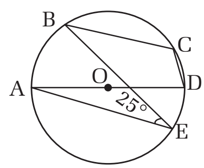
$\angle PRQ = ?$

[Ans: 20°]

e) 

$\angle AFB = ?$

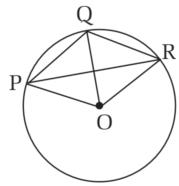
[Ans: 80°]

f) 

$\angle BCD = ?$

[Ans: 115°]

1. ***In the given figure, O is the centre of the circle. Prove that $\angle PQR = 2(\angle PRQ + \angle QPR)$***



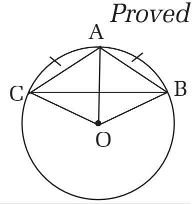
Solution:

Proof

	Statements		Reasons
1.	$\angle POQ = 2\angle PRQ$	1.	Being the central angle twice the inscribed angle standing on the same arc
2.	$\angle QOR = 2\angle QPR$	2.	same as (1)
3.	$\angle POQ + QOR = 2\angle PRQ + 2\angle QPR$	3.	Adding (1) and (2)
4.	$\angle POR = 2(\angle PRQ + QPR)$	4.	From (3) by whole part axiom

Proved

2. ***In the given figure, O is the centre of the circle. If $\overset{\frown}{AC} = \overset{\frown}{AB}$ and BC is an bisector of $\angle ABO$, prove that ΔABD is an equilateral triangle.***



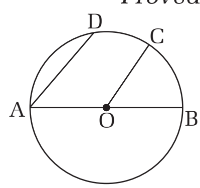
Solution:

Proof

	Statements		Reasons
1.	$\angle AOB = \angle AOC$	1.	Central angles subtended by equal area
2.	$\angle ABC = \frac{1}{2}\angle AOC$	2.	Inscribed angle is half of central angle standing on the same arc
3.	$\angle ABC = \frac{1}{2}\angle AOB$	3.	From (1) and (2)
4.	$\angle ABC = \frac{1}{2}\angle OBA$	4.	BC is an angle bisector of $\angle ABO$
4.	$\angle AOB = \angle OBA$	5.	From (3) and (4)
6.	$\angle OBA = \angle OAC$	6.	$OA = OB$
7.	$\angle AOB = \angle OBA = \angle OAC$	7.	From (5) and (6)
8.	AOB is an equilateral triangle	8.	From (7), being each angle equal

Proved

3. ***In the given figure, O is the centre of the circle, AB the diameter and $\overset{\frown}{BC} = \overset{\frown}{CD}$. Prove that AD // OC.***

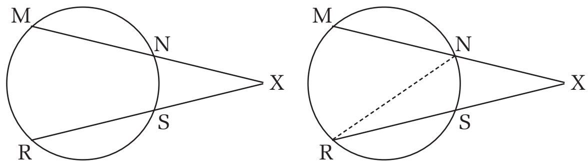


Solution:

Proof

	Statements		Reasons
1.	$\overset{\frown}{BCD} = 2\,\overset{\frown}{BC}$	1.	$\overset{\frown}{BC} = \overset{\frown}{CD}$
2.	$\angle BOC \overset{\circ}{=} \overset{\frown}{BC}$	2.	Relation between central angle and its corresponding arc
3.	$\angle BAD \overset{\circ}{=} \frac{1}{2}\overset{\frown}{BCD}$	3.	Relation between inscribed angle and its corresponding arc
4.	$\angle BAD \overset{\circ}{=} \frac{1}{2} \times 2\overset{\frown}{BC}$	4.	From (1) and (2)
5.	$\angle BOC = \angle BAD$	5.	From (2) and (4)
6.	AD // OC	6.	From (5), alternate angle are equal

4. ***In the figure along side, chords MN and RS of a circle intersect at X. Prove that:*** $\angle MXR \overset{\circ}{=} \frac{1}{2}(\overset{\frown}{MR} - \overset{\frown}{NS})$



Solution:

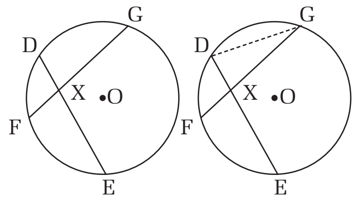
Construction: R and N are joined.

Proof

	Statements		Reasons
1.	$\angle MNR \overset{\circ}{=} \frac{1}{2}\overset{\frown}{MR}$ and $\angle NRX \overset{\circ}{=} \frac{1}{2}\overset{\frown}{NS}$	1.	Relation between inscribed angles and their opposite arcs
2.	$\angle MXR + \angle NRX = \angle MNR$	2.	Exterior angle of a triangle is equal to sum of its opposite interior angles
3.	$\angle MXR + \frac{1}{2}\overset{\frown}{NS} \overset{\circ}{=} \frac{1}{2}\overset{\frown}{MR}$ $\therefore \angle MXR \overset{\circ}{=} \frac{1}{2}(\overset{\frown}{MR} - \overset{\frown}{NS})$	3.	From (1) and (2)

Proved

5. ***In the given circle, chords DE and FG intersect within the circle at X. Prove that:*** $\angle DXF \overset{\circ}{=} \frac{1}{2}(\overset{\frown}{DF} - \overset{\frown}{GE})$



Solution:

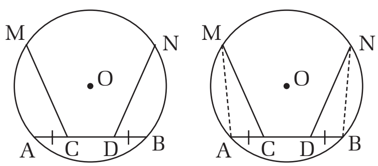
Construction: D and G are joined

Proof

	Statements		Reasons
1.	$\angle DGF \overset{\circ}{=} \frac{1}{2}\overset{\frown}{DF}$ and $\angle EDG \overset{\circ}{=} \frac{1}{2}\overset{\frown}{GE}$	1.	Relation between inscribed angle in opposite site.
2.	$\angle DXF = \angle OGF + \angle EDG$	2.	Exterior angle of a triangle is equal to the sum of opposite interior angles
3.	$\angle DXF \overset{\circ}{=} \overset{\frown}{DF} + \frac{1}{2}\overset{\frown}{GE}$	3.	
4.	$\therefore \angle DXF \overset{\circ}{=} \frac{1}{2}(\overset{\frown}{DF} + \overset{\frown}{GE})$	4.	

Proved

6. ***In the given figure, O is the centre of circle. C and D are two points on the chord AB. If AC = BD and*** $\overset{\frown}{AM} = \overset{\frown}{BN}$***, prove that*** $\angle ACM = \angle BDN$



Solution:

Construction: M, A and N, B are joined

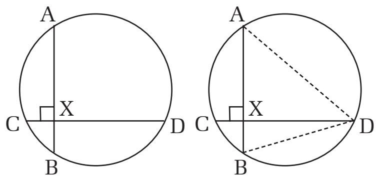
Proof

	Statements		Reasons
1.	$\overset{\frown}{AM} = \overset{\frown}{BN}$	1.	Given
2.	$\overset{\frown}{AM} + \overset{\frown}{MN} = \overset{\frown}{BN} + \overset{\frown}{MN}$	2.	Same $\overset{\frown}{MN}$ is added to both sides of (1)
3.	$\overset{\frown}{AMN} = \overset{\frown}{MNB}$	3.	By whole part axiom
4.	$\angle ABN = \angle MAB$ i.e, $\angle ABD = \angle MAC$	4.	Equal arcs subtend equal inscribed angles

5.	In ΔMAC and ΔNBD (i) $MA = NB$ (S) (ii) $\angle MAC = \angle ABD$ (A) (iii) $AC = BD$ (S)	5.	(i) $\overset{\frown}{AM} = \overset{\frown}{BN}$ (ii) From (iv) (iii) Given
6.	$\Delta MAC \cong \Delta NBD$	6.	By S.A.S axiom
7.	$\angle ACM = \angle BDN$	7.	Corresponding angles of congruent triangles are equal

Proved

7. *In the given figure, two chords AB and CD intersect at right angle at X. Prove that, $\overset{\frown}{AD} - \overset{\frown}{CA} = \overset{\frown}{BD} - \overset{\frown}{BC}$.*



Solution:

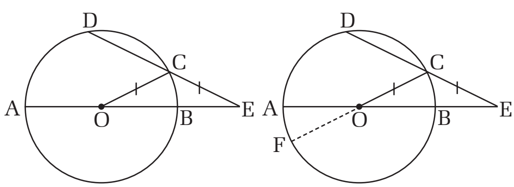
Construction: A and B are joined to D

Proof

	Statements		Reasons
1.	$\angle BAD \overset{\circ}{=} \frac{1}{2}\overset{\frown}{BD}$ and $\angle ADC \overset{\circ}{=} \frac{1}{2}\overset{\frown}{CA}$	1.	Relation between inscribed angles and their opposite arcs
2.	$\angle BAD + \angle ADC = \angle AXC$	2.	Exterior angle of triangle in equal to sum of opposite angles
3.	$\frac{1}{2}\overset{\frown}{BD} + \frac{1}{2}\overset{\frown}{CA} \overset{\circ}{=} 90°$ $\therefore \overset{\frown}{BD} + \overset{\frown}{CA} \overset{\circ}{=} 180°$	3.	From (1) and (2), $\angle AXC = 90°$
4.	$\angle ABD \overset{\circ}{=} \frac{1}{2}\overset{\frown}{AD}$ and $\angle BDC \overset{\circ}{=} \frac{1}{2}\overset{\frown}{BC}$	4.	Same as (1)
5.	$\angle ABD + \angle BDC = \angle AXD$	5.	Same as (2)
6.	$\frac{1}{2}\overset{\frown}{AD} + \frac{1}{2}\overset{\frown}{BC} \overset{\circ}{=} 90°$ $\therefore \overset{\frown}{AD} + \overset{\frown}{BC} \overset{\circ}{=} 180°$	6.	From (4) and (5), $\angle AXD = 90°$
7.	$\overset{\frown}{AD} + \overset{\frown}{BC} = \overset{\frown}{BD} + \overset{\frown}{CA}$ $\therefore \overset{\frown}{AD} - \overset{\frown}{CA} = \overset{\frown}{BD} - \overset{\frown}{BC}$	7.	From (3) and (6)

Proved

8. *In the adjoining figure, O is the centre of a circle in which ΔOCE is an isosceles triangle. Prove that $\overset{\frown}{BC} - \frac{1}{3}\overset{\frown}{AD}$.*



Solution:

Construction: CO is produced to meet the circumference at F.

Proof

	Statements		Reasons
1.	$\angle BOC = \angle AOF$	1.	Vertically opposite angles
2.	$\overset{\frown}{BC} = \overset{\frown}{AF}$	2.	From (i)
3.	$\angle BOC = \angle OEC$	3.	$OC = CE$
4.	$\angle DCF = \frac{1}{2}\overset{\frown}{DAF}$	4.	Relation between inscribed angle and its opposite arc

5.	$\angle BOC + \angle OEC = \angle DCF$	5.	Relation among exterior and opposite interior angles of Δ
6.	$\overset{\frown}{BC} + \overset{\frown}{BC} = \frac{1}{2}\overset{\frown}{DAF}$	6.	From (3), (4) and (5)
7.	$4\overset{\frown}{BC} = \overset{\frown}{AD} + \overset{\frown}{AF} = \overset{\frown}{AD} + \overset{\frown}{BC}$ or, $3\overset{\frown}{BC} = \overset{\frown}{AD}$ $\therefore \overset{\frown}{BC} = \frac{1}{3}\overset{\frown}{AD}$	7.	From (2) and (6)

Proved

9. In the given alongside, AD // BC. Prove that $\angle AYC = \angle BXD$.

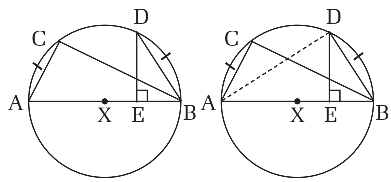
Solution:

Proof

	Statements		Reasons
i.	$\overset{\frown}{AB} = \overset{\frown}{DC}$	i.	AD // BC
ii.	$\overset{\frown}{AB} + \overset{\frown}{BC} = \overset{\frown}{DC} + \overset{\frown}{BC}$	ii.	Same $\overset{\frown}{BC}$ added to both sides of (i)
iii.	$\overset{\frown}{ABC} = \overset{\frown}{BCD}$	iii.	whole part axiom
iv.	$\angle AYC = \angle BXD$	iv.	From (iii), equal arc subtend equal angles at circumference

Proved

10. In the figure, X is the centre of a circle. AB is a diameter of the circle. If DE $\perp$ AB and $\overset{\frown}{AC} = \overset{\frown}{BD}$, prove that, $\angle ABC = \angle BDE$.



Solution:

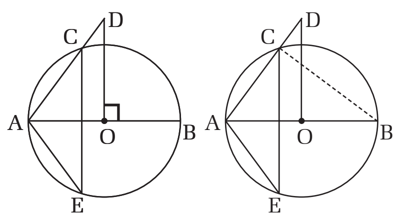
Construction: A and D are joined

Proof

	Statements		Reasons
1.	In ΔABD and ΔBED 1. $\angle ADB = \angle BED = 90°$ 2. $\angle ABD = \angle EBD$ 3. $\angle BAD = \angle BDE$	1.	 1. Given 2. Common angles 3. Remaining angles of Δ^S
2.	$\angle ABC = \angle BAD$	2.	$\overset{\frown}{AC} = \overset{\frown}{BD}$
3.	$\angle ABC = \angle BDE$	3.	From 1 (iii) and (2)

Proved

11. In the given figure, O is the centre of the circle, AB the diameter and DO $\perp$ AB. Prove that $\angle AEC = \angle ODA$.



Solution:

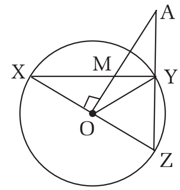
Construction: B and C are joined

Proof

	Statements		Reasons
1.	$\angle ABC = 90°$	1.	Angle in a semi-circle
2.	In ΔABC and ΔAOD (i) $\angle ACB = \angle AOD$ (ii) $\angle CAB = \angle DAO$ (iii) $\angle ABC = \angle ODA$	2.	 (i) Both angles measures 90° (ii) Common angles (iii) Remaining angles of Δ^S
3.	$\angle ABC = \angle AEC$	3.	Inscribed angles standing on $\overset{\frown}{AC}$
4.	$\angle ODA = \angle AEC$ $\therefore \angle AEC = \angle ODA$	4.	From 2(iii) and (3)

Proved

12. ***In the adjoining figure, O is the centre of circle. If AMD ⊥ XOZ, then prove that ∠OAZ = ∠XYO***



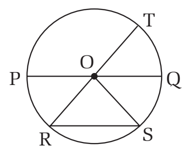
Solution:

Proof

	Statements		Reasons
1.	$\angle XYZ = 90°$	1.	Angle in a semi-circle
2.	In ΔXYZ and ΔOAZ (i) $\angle XYZ = \angle AOZ$ (ii) $\angle XZY = \angle OZA$ (iii) $\angle ZXY = \angle OAZ$	2.	 (i) Each angle measure 90° (ii) Common angles (iii) Remaining angles of Δ^S
3.	$\angle ZXY = \angle XYO$	3.	$OX = OY$
4.	$\angle OAZ = \angle XYO$	4.	From 2 (iii) and (3)

Proved

13. ***In the adjoining figure, POQ and ROT are two diameters of a circle with centre at O. If Q is the mid-point of arc TQS and ∠QOR is obtuse, prove that PQ // RS.***



Solution:

Proof (Process I)

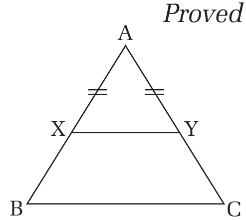
	Statements		Reasons
1.	$\angle TOQ \overset{\circ}{=} \overset{\frown}{TQ}$	1.	Relation between central angle and its opposite arc
2.	$\overset{\frown}{TQS} = 2\,\overset{\frown}{TQ}$	2.	Being Q the mid-point of $\overset{\frown}{TQS}$
3.	$\angle TRS \overset{\circ}{=} \frac{1}{2}\,\overset{\frown}{TQS}$	3.	Relation between inscribed angle and its opposite arc
4.	$\angle TRS \overset{\circ}{=} \frac{1}{2} \times 2\,\overset{\frown}{TQ}$	4.	From (2) and (3)
5.	$\angle TOQ = \angle TRS$	5.	From (1) and (4)
6.	PQ // RS	6.	Being corresponding angles equal

(Process II)

	Statements		Reasons
1.	$\overset{\frown}{PR} = \overset{\frown}{TQ}$	1.	$\angle POR = \angle TOQ$

2.	$\overset{\frown}{TQ} = \overset{\frown}{QS}$	2.	Given
3.	$\overset{\frown}{PR} = \overset{\frown}{QS}$	3.	From (1) and (2)
4.	PQ // RS	4.	From (3)

Proved



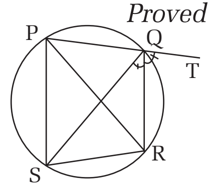
14. ΔABC is an isosceles triangle and XY // BC. If XY cuts AB at X and AC at Y, prove that the four points X, B, C and Y are concyclic.

Solution:

Proof

	Statements		Reasons
1.	∠ABC = ∠ACB	1.	Being base angles of isosceles triangle
2.	∠ABC + ∠BXY = 180°	2.	XY // BC, Co-interior angles
3.	∠ACB + BXY = 180°	3.	From (1) and (2)
4.	XBCY is a cyclic quadrilateral	4.	From (3)
5.	Points X, B, C and Y are concyclic	5.	From (4), being vertices of cyclic quad.

Proved



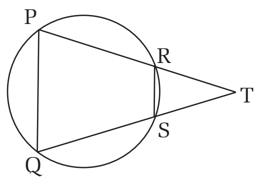
15. In the given figure, QR is the bisector of ∠SQT and PQRS is a cyclic quadrilateral. Prove that ΔPSR is an isosceles triangle.

Solution:

Proof

	Statements		Reasons
1.	∠SQR = ∠RQT	1.	Being QR the bisector of ∠SQT
2.	∠SQR = ∠SPR	2.	Being inscribed angle standing on same arc SR
3.	∠RQT = ∠PSR	3.	Exterior angle of cyclic quadrilateral is equal to its opposite interior angle
4.	∠SPR = ∠PSR	4.	From (1), (2) and (3)
5.	ΔPSR is an isosceles triangle	5.	From (4), base angles are equal

Proved



16. In the adjoining figure, TPQ is an isosceles triangle in which TP = TQ. A circle passing through P and Q cuts TP and TQ at R and S respectively. Proof that PQ // RS

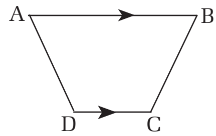
Solution:

Proof

	Statements		Reasons
1.	∠TPQ = ∠TQP	1.	TP = TQ, base angles of isosceles triangle
2.	∠TPQ = ∠SRT	2.	Relation between exterior and opposite interior angles of cyclic quadrilateral PQRS
3.	∠TQP = ∠SRT	3.	From (1) and (2)
4.	PQ // RS	4.	From (3), being corresponding angle equal

Proved

17. *In the given figure, ABCD is a quadrilateral in which AB // DC and $\angle ADC = \angle BCD$. Prove that the point A, B, C and D are concyclic.*



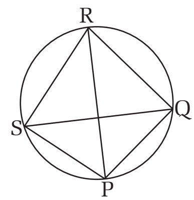
Solution:

Proof

	Statements		Reasons
1.	$\angle ADC = \angle BCD$	1.	Given
2.	$\angle ADC + \angle BAD = 180°$	2.	AB // DC, co-interior angles
3.	$\angle BCD + \angle BAD = 180°$	3.	From (1) and (2)
4.	ABCD is a cyclic quadrilateral	4.	From (3), opposite angles are supplementary
5.	Point A, B, C and D are concyclic	5.	From (4), being vertices of cyclic quad.

Proved

18. *In the adjoining figure, PQRS is a cyclic quadrilateral. If the diagonal PR and QS are equal, prove that QR = PS and PQ // SR.*



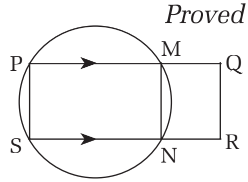
Solution:

Proof

	Statements		Reasons
1.	$\overset{\frown}{PQR} = \angle SPQ$	1.	Arc opposite to equal chord PR and SQ
2.	$\overset{\frown}{QR} = \overset{\frown}{SP}$	2.	same arc PQ is subtracted from both side of (1)
3.	QR = PS	3.	Corresponding chord on equal arcs
4.	PQ // SR	4.	From (2)

Proved

19. *In the figure, PQRS is a parallelogram. The inscribed circle cuts PQ at M and SR at N. Prove that $\angle MNS = \angle PQR$.*



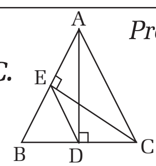
Solution:

Proof

	Statements		Reasons
1.	$\angle SPM + \angle MNS = 180°$	1.	Opposite angles of cyclic quad. PMNS
2.	$\angle SPM + PQR = 180°$	2.	PS // QR, co-interior angles
3.	$\angle MNS = \angle PQR$	3.	From (1) and (2)

Proved

20. *Int the triangle ABC, $AD \perp BC$ and $CE \perp AB$. Prove that $\angle BDE = \angle BAC$.*

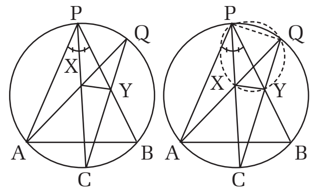


Solution:

Proof

	Statements		Reasons
1.	$\angle AEC = \angle ADC$	1.	Both are right angles
2.	A, E, D and C are concyclic point	2.	From (1), angles on same segment AC towards same side are equal of AC
3.	AEDC is a cyclic quadrilateral	3.	From (2)
4.	$\angle BDE = \angle BAC$	4.	Relation between exterior and opposite interior angles of cyclic quad. AEDC

21. *If PC is bisector of $\angle APB$, then prove that XY // AB.*



Solution:

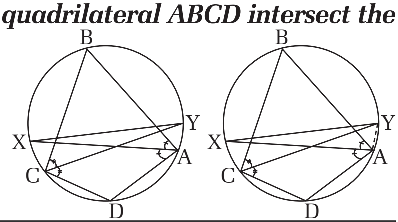
Construction: P and Q are joined

Proof

	Statements		Reasons
1.	$\angle APC = \angle CPB = \angle AQC$	1.	Given and $\angle APC = \angle AQC$
2.	Points P, X, Y and Q are concyclic	2.	As $\angle XPY = \angle XQY$ on same segment XY, from (1)
3.	$\angle PQX = \angle PYX$	3.	Inscribed angles standing on $\overset{\frown}{PX}$
4.	$\angle PQA = \angle PBA$	4.	Inscribed angles standing on $\overset{\frown}{PA}$
5.	$\angle PYX = \angle PBA$	5.	From (3) and (4)
6.	XY // AB	6.	From (5), being corresponding angles equal

Proved

22. *The bisector of opposite angles $\angle A$ and $\angle C$ of a cyclic quadrilateral ABCD intersect the corresponding circle at the point X and Y respectively. Prove that XY is the diameter of the circle.*



Solution:

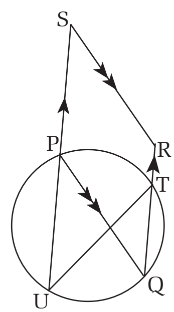
Constructed: A and Y are joined

Proof

	Statements		Reasons
1.	$\angle BCD = 2\angle BCY$ and $\angle BAD = 2\angle BAX$	1.	Given
2.	$\angle BCD + \angle BAD = 180°$	2.	Being opposite angles of cyclic quad. ABCD
3.	$2\angle BCY + 2\angle BAX = 180°$ $\therefore \angle BCY + \angle BAX = 90°$	3.	From (1) and (2)
4.	$\angle BCY = \angle BAY$	4.	Inscribed angles standing on $\overset{\frown}{BY}$
5.	$\angle BAY + \angle BAX = 90°$ $\therefore \angle XAY = 90°$	5.	From (3) and (4), and whole part axiom
6.	XY is diameter	6.	From (5)

Proved

23. *In the given figure, PQRS is a parallelogram. Prove that UTRS is a cyclic quadrilateral.*



Solution:

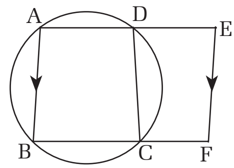
Proof

	Statements		Reasons
1.	$\angle PSR = \angle PQR$	1.	Being opposite angles of ▱ PQRS
2.	$\angle PQR = \angle PUT$	2.	Inscribed angles standing on $\overset{\frown}{PT}$

3.	∠PSR = ∠PUT	3.	From (1) and (2)
4.	∠PUT + ∠UTR = 180°	4.	RT // SU, co-interior angles
5.	∠PSR + ∠UTR = 180°	5.	From (3) and (4)
6.	UTRS is a cyclic quadrilateral	6.	From (5), opposite angles are supplementary.

Proved

24. ***In the given figure, AB and EF are parallel to each other. Prove that CDEF is a cyclic quadrilateral.***



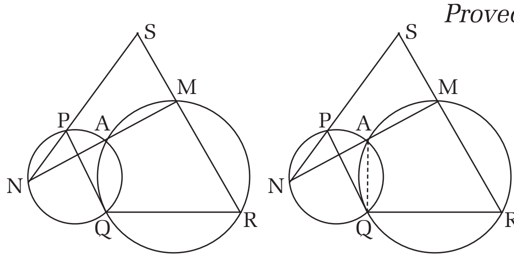
Solution:

Proof

	Statements		Reasons
1.	∠BAD = ∠DCF	1.	Being exterior angle of cyclic quadrilateral equal to its opposite interior angles
2.	∠BAD + ∠AEF = 180°	2.	AB // EF, co-interior angles
3.	∠DCF + ∠AEF = 180°	3.	From (1) and (2)
4.	CDEF is a cyclic quadrilateral	4.	From (3), being opposite angles supplementary.

Proved

25. ***In the figure, NPS, MAN and RMS are straight lines. Prove that PQRS is a cyclic quadrilateral.***



Solution:

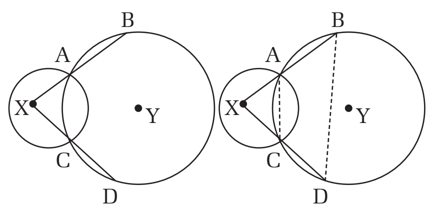
Construction: A and Q are joined

Proof

	Statements		Reasons
1.	∠NPQ = ∠NAQ	1.	Inscribed angles standing on $\overset{\frown}{NQ}$
2.	∠NAQ = ∠QRS	2.	Relation between exterior and interior angles of cyclic quadrilateral
3.	∠NPQ = ∠QRS	3.	From (1) and (2)
4.	PQRS is a cyclic quadrilateral	4.	From (3), being exterior and its opposite interior angles of quad. PQRS equal.

Proved

26. ***In the adjoining figure, X and Y are the centres of the circles which intersect at A and C. XA and XC are produced to meet the other circle at B and D. Prove that AB = CD.***



Solution:

Construction: A, C and B, D are joined.

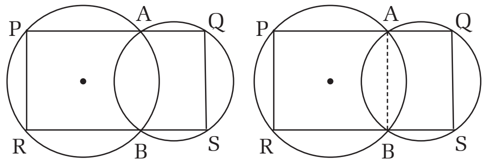
Proof

	Statements		Reasons
1.	∠XAC = ∠XCA	1.	XA = XC
2.	∠XAC = ∠XCD	2.	Relation between exterior and opposite interior angle of cyclic quadrilateral
3.	∠XCA = ∠XBD	3.	Same as (2)

4.	XB = XD	4.	From (3)
5.	AB = CD	5.	Subtracting XA = XC from (4)

Proved

27. ***In the adjoining figure, two circles intersect at A and B. The straight line PAQ meets the circles at P and Q and the straight line RBS meets the circles at R and S. Prove that PR // QS.***



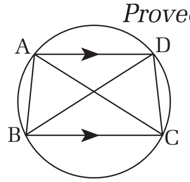
Solution:

Proof

	Statements		Reasons
1.	$\angle APR = \angle ABS$	1.	Exterior and opposite interior angles of cyclic quadrilateral are equal
2.	$\angle ABS + \angle AQS = 180°$	2.	Opposite angles of cyclic quad. are supplementary
3.	$\angle APR + \angle AQS = 180°$	3.	From (1) and (2)
4.	PR // QS	4.	From (3), being the sum of co-interior angles equal to 180°

Proved

28. ***Prove that a cyclic trapezium is always isosceles and its diagonals are equal.***



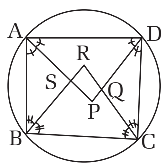
Solution:

Proof

	Statements		Reasons
1.	$\overset{\frown}{AB} = \overset{\frown}{DC}$	1.	AB // BC
2.	AB = CD	2.	Corresponding chords on equal arcs
3.	ABCD is an isosceles trapezium	3.	From (2) and given
4.	$\overset{\frown}{ABC} = \overset{\frown}{BCD}$	4.	Adding same $\overset{\frown}{BC}$ on both sides of (1)
5.	AC = BD	5.	Corresponding chords on equal arcs

Proved

29. ***Prove that the quadrilateral formed by angle bisectors of cyclic quadrilateral in also cyclic.***



Solution:

Proof

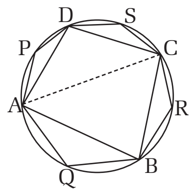
	Statements		Reasons
1.	$\angle RBC + \angle BCR + \angle BRC = 180°$	1.	Sum of angles of triangle
2.	$\frac{1}{2}\angle ABC + \frac{1}{2}\angle BCD + \angle BRC = 180°$	2.	$\angle RBC = \frac{1}{2}\angle ABC$, $\angle BCR = \frac{1}{2}\angle BCD$
3.	$\angle PAD + \angle ADP + \angle APD = 180°$	3.	Sum of angles of triangle
4.	$\frac{1}{2}\angle BAD + \frac{1}{2}\angle ADC + \angle APD = 180°$	4.	$\angle PAD = \frac{1}{2}\angle BAD$, $\angle ADP = \frac{1}{2}\angle ADC$

5.	$\frac{1}{2}\angle ABC + \frac{1}{2}\angle BCD + \angle BRC + \frac{1}{2}\angle BAD + \frac{1}{2}\angle ADC + \angle APD = 360°$ or, $\frac{1}{2}(\angle ABC + \angle BCD + \angle BAD + \angle ADC) + \angle BRC + \angle APD = 360°$	5.	Adding (2) and (4)
6.	$\frac{1}{2} \times 360° + \angle BRC + \angle APD = 360$ $\therefore BRC = \angle APD = 180°$	6.	Sum of angles of quad. = 360°
7.	PQRS is a cyclic quadrilateral	7.	From (6)

Proved

30. Prove that the sum of the angles in the four segments exterior to a cyclic quadrilateral is equal to six right angles.

Solution:



Given: ABCD is a cyclic quadrilateral. $\angle P$, $\angle Q$, $\angle R$ and $\angle S$ are the angles in the four segments exterior to the cyclic quadrilateral ABCD.

To Prove: $\angle P + \angle Q + \angle R + \angle S = 6$ right angles

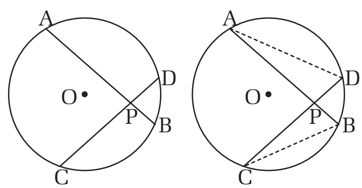
Construction: A and C are joined

Proof

	Statements		Reasons
1.	$\angle P + \angle ACD = 180°$, $\angle Q + \angle ACB = 180°$, $\angle R + \angle BAC = 180°$, $\angle S + \angle CAD = 180°$,	1.	Sum of opposite angles of cyclic quadrilaterals
2.	$\angle P + \angle Q + \angle R + \angle S + \angle ACD + \angle ACB + \angle BAC + \angle CAD = 720°$	2.	From (1), adding sidewise
3.	$\angle P + \angle Q + \angle R + \angle S + \angle BCD + \angle BAD = 720°$	3.	From (2), by whole part axiom
4.	$\angle P + \angle Q + \angle R + \angle S + 180 = 720°$ or, $\angle P + \angle Q + \angle R + \angle S = 540°$ $= 6 \times 90° = 6$ right angles	4.	$\angle BCD + \angle BAD = 180°$

Proved

31. In the given figure, AB and CD are two chords of a circle which are inter secting each other at P such that AP = CP. Prove that: AB = CD.



Solution:

Construction: A, D and B, C are joined.

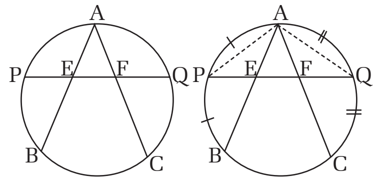
Proof

	Statements		Reasons
1.	In ΔAPD and ΔCPB; (i) $\angle ADC = \angle CBA$ (ii) $\angle BAD = \angle BCD$ (iii) AP = CP	1.	 (i) Inscribed angles on $\overset{\frown}{AC}$ (ii) Inscribed angles on $\overset{\frown}{BD}$ (iii) Given
2.	$\Delta APD \cong \Delta CPB$	2.	BY A.A.S axiom

3.	PD = PB	3.	Corresponding sides of congruent triangles
4.	AP + PB = CP + PD	4.	Adding 1 (iii) and (3)
5.	AB = CD	5.	Whole part axiom

Proved

32. *In the given figure, P and Q are mid-point of $\overset{\frown}{AB}$ and $\overset{\frown}{AC}$ respectively. Prove that AE = AF.*



Solution:

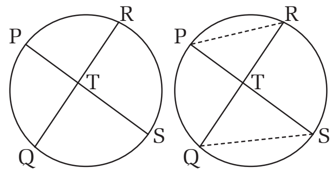
Construction: P and Q are joined to A.

Proof

	Statements		Reasons
1.	$\angle BAP = \angle AQP$	1.	$\overset{\frown}{PB} = \overset{\frown}{AP}$
2.	$\angle APQ = \angle CAQ$	2.	$\overset{\frown}{AQ} = \overset{\frown}{QC}$
3.	$\angle BAP + \angle APQ = \angle AQP + \angle CAQ$	3.	Adding (1) and (2)
4.	$\angle AEF = \angle AFE$	4.	From (3) exterior angle of ΔAPE and ΔAQF
5.	AE = AF	5.	From (4)

Proved

33. *In the adjoining figure, $\overset{\frown}{PQ} = \overset{\frown}{RS}$. Prove that QT = ST and PT = RT.*



Solution:

Construction: Q, S and P, R are joined.

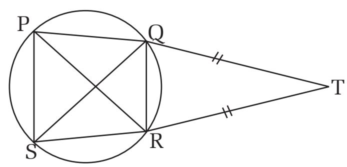
Proof

	Statements		Reasons
1.	PR // QS	1.	$\overset{\frown}{PQ} = \overset{\frown}{RS}$
2.	$\angle RPS = \angle PSQ$	2.	PR // QS, alternate angles
3.	$\angle PRQ = \angle PSQ$	3.	Inscribed angle on $\overset{\frown}{PQ}$
4.	$\angle RPS = \angle PRQ$	4.	From (2) and (3)
5.	PT = RT	5.	From (4)
6.	$\angle PSQ = \angle RPS$	6.	PR // QS, alternate angles
7.	$\angle RPS = \angle RQS$	7.	Inscribed angles on $\overset{\frown}{RS}$
8.	$\angle PSQ = \angle RQS$	8.	From (6) and (7)
9.	QT = ST	9.	From (8)

Proved

34. *In the adjoining figure, PQRS is a cyclic quadrilateral PQ and SR are produced to meet at T. If QT = RT, prove that*

i) PS // QR   ii) PR = QS.



Solution:

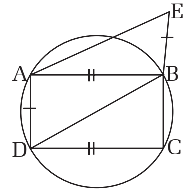
Proof

	Statements		Reasons
1.	$\angle TQR = \angle TRQ$	1.	QT = RT

2.	$\angle TPS = \angle TRQ$	2.	Exterior and opposite interior angles of cyclic quadrilateral
3.	$\angle TQR = \angle TPS$	3.	From (1) and (2)
4.	PS // QR	4.	From (iii), corresponding angles are equal
5.	$\overset{\frown}{SR} = \overset{\frown}{PQ}$	5.	PS // QR
6.	$\overset{\frown}{PSR} = \overset{\frown}{QPS}$	6.	Same arc PS is added to each side of (5)
7.	PR = QS	7.	Corresponding chords on equal arcs

Proved

35. ***In the adjoining figure, AB = CD and BEd = AD. Prove that ADBE is a parallelogram.***



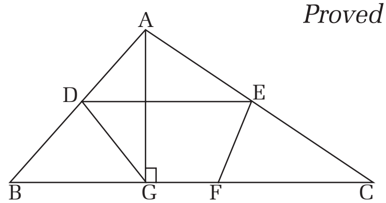
Solution:

Proof

	Statements		Reasons
1.	$\overset{\frown}{AB} = \overset{\frown}{DC}$	1.	AB = DC
2.	AD // BC $\quad\therefore$ AD // CE	2.	From (1)
3.	AD = BE	3.	Given
4.	AE = DB and AE // DB	4.	From (2) and (3)
5.	ADBE is a parallelogram	5.	From (2), (3) and (4)

Proved

36. ***In the figure, if D, E and F are the mid-point of sides AB, AC and BC respectively and AG ⊥ BC. Prove that DEFG is a cyclic quadrilateral.***



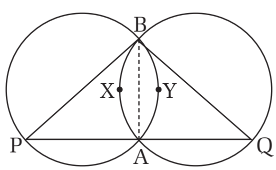
Solution:

Proof

	Statements		Reasons
1.	DE // BC and EF // AB	1.	Being D, E and F the mid-point of sides AB, AC and BC in ΔABC.
2.	DEFB is a parallelogram	2.	From (1)
3.	$\angle DEF = \angle FBD$	3.	Being opposite angles of ▱ DEFB
4.	AD = BD = GD	4.	mid-point of hypotenuse being equidistance from each vertex in a right angled triangle
5.	$\angle DBG = \angle BGD$	5.	From (4); BD = GD
6.	$\angle DBG = \angle DEF$	6.	From (3) and (5)
7.	DEFG is a cyclic quadrilateral	7.	From (6), being exterior angle equal to opposite interior angle

Proved

37. ***Two equal circle intersect each other at A and B. If a straight line PQ is drawn through A to touch the circumference of one circle at P and the other at Q, prove that BP = BQ.***



Solution:

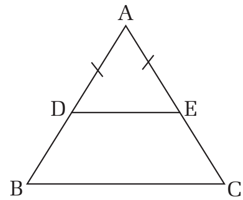
Construction: A and B are joined.

Proof

	Statements		Reasons
1.	$\overset{\frown}{AXB} = \overset{\frown}{AYB}$	1.	Corresponding arcs of chord AB in equal circles
2.	$\angle PQB = \angle QPB$	2.	Equal arcs subtend equal angles at the circumference
3.	BP = BQ	3.	From (2)

Proved

38. ***D and E are any two points on equal sides AB and AC of an equilateral triangle ABC such that AD = AE. Prove that B, C, D and E are concyclic.***



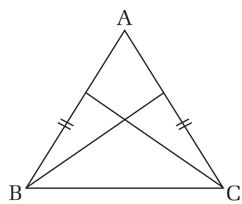
Solution:

Proof

	Statements		Reasons
1.	D and E are mid-point of AB and AC	1.	AB = AC and AD = AE
2.	DE // BC	2.	From (i)
3.	$\angle ADE = \angle AED$	3.	AD = AE
4.	$\angle ABC = \angle ACB$	4.	AB = AC
5.	$\angle ADE = \angle ABC$	5.	DE // BC, corresponding angles
6.	$\angle AED = \angle ABC$	6.	From (3), (4) and (5)
7.	Point B, C, D and E are concyclic	7.	From (6), BCED is a cyclic quadrilateral

Proved

39. ***In an isosceles triangle PQR, PQ = PR. If the bisectors of ∠Q and ∠R meet PR at S and PQ at T. Prove that the points Q, R, S, T are concyclic.***



Solution:

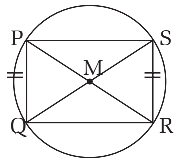
Proof

	Statements		Reasons
1.	In ΔTQR and ΔSQR (i) $\angle TQR = \angle SRQ$ (ii) QR = QR (iii) $\angle SQR = \angle TRQ$	1.	 (i) Base angles of isosceles triangle (ii) Common side (iii) SQ and TR, the bisectors of equal angles $\angle PQR$ and $\angle PRQ$
2.	$\Delta TQR \cong \Delta SQR$	2.	By A.S.A. axiom
3.	$\angle QTR = \angle QSR$	3.	Corresponding angles of congruent triangles
4.	Point Q, R, S and T are concyclic	4.	From (3), angles on the same segment QR

Proved

40. Point P, Q, R and S are concyclic such that arc PQ = arc SR. If the chords PR and QS are intersecting at a point M, prove that:

i) area of ΔPQM = area of ΔSMR ii) chord PR = chord QS



Solution:

Proof (Method I)

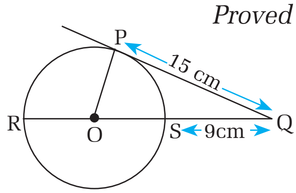
	Statements		Reasons
1.	In ΔPQM and ΔSMR (i) $\angle QPM = \angle MSR$ (ii) PQ = SR (iii) $\angle PQM = \angle SRM$	1.	(i) Inscribed angles on $\overset{\frown}{QR}$ (ii) Corresponding chords on equal arc (iii) Inscribed angles on $\overset{\frown}{RS}$
2.	$\Delta PQM \cong SMR$	2.	By A.S.A. axiom
3.	Area of ΔPQM = area of ΔSMR	3.	Congruent triangle are equal in area
4.	PM = SM	4.	Corresponding sides of congruent triangles
5.	QM = MR	5.	Same as (4)
6.	PM + MR = SM + QM $\therefore$ PR = QS	6.	Adding (4) and (5)

(Method II)

	Statements		Reasons
1.	PS // QR	1.	$\overset{\frown}{PQ} = \overset{\frown}{SR}$
2.	$\Delta PQR = \Delta SQR$	2.	Both are standing on QR and between PS // QR
3.	$\Delta PQM = \Delta SMR$	3.	Subtracting ΔQMR from both sides of (2)
4.	$\overset{\frown}{PQR} = \overset{\frown}{QRS}$	4.	Adding $\overset{\frown}{QR}$ on both side of $\overset{\frown}{PQ} = \overset{\frown}{SR}$
5.	PR = QS	5.	From (4)

Proved

Tangents and Secant



1. In the given figure, O is the centre of the circle. PQ is a tangent and P the point of contact. RS is produce to the point Q. If PQ = 15 cm and SQ = 9 cm, find the value of OS.

Solution:

i) OP $\perp$ PQ [Being radius perpendicular to the tangent at point of contact]

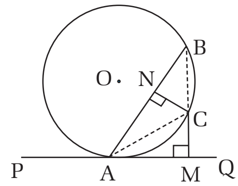
ii) $OP^2 + PQ^2 = OQ^2$ [By pythagoras theorem]

or, $OS^2 + (15\text{ cm})^2 = (OS + 9\text{ cm})^2$ [$\because OP = OS = r$]

or $\cancel{OS^2} + 225\text{ cm}^2 = \cancel{OS^2} + 2 \times OS \times 9\text{ cm} + 81\text{ cm}^2$

$\therefore OS = 8\text{ cm}$

2. ***In the adjoining figure, PQ is tangent to the circle at A. C is the mid-point of arc AB. CM ⊥ PQ and CN ⊥ AB. Prove that CM = CN.***



Solution:

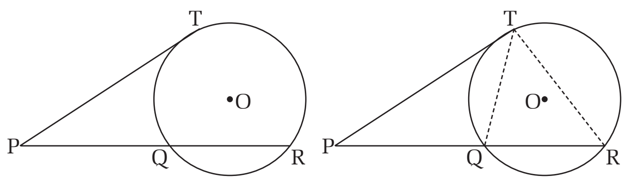
Construction: A and B are joined to C.

Proof

	Statements		Reasons
1.	In ΔAMC and ΔBNC i. ∠AMC = ∠CNB ii. ∠MAC = ∠CBN iii. AC = BC	1.	 i. Both are right angles ii. Angles in alternate segment iii. Corresponding chords on equal arcs
2.	ΔAMC ≅ ΔBNC	2.	By A.A.S. axiom
3.	CM = CN	3.	Corresponding sides of congruent triangles

Proved

3. ***In the adjoining figure, PQR is the secant and PT is the tangent to the circle with centre O. Prove that $PT^2 = PQ.QR$.***



Solution:

Construction: Q and R are joinde to T.

Proof

	Statements		Reasons
1.	In ΔPTR and ΔPQT i. ∠TPR = ∠TPQ ii. ∠PRT = ∠PTQ iii. ∠PTR = ∠PQT	1.	 i. Common angle ii. Angles in alternate segment iii. Remaining angles
2.	ΔPTR ~ ΔTPQ	2.	By A.A.A. axiom
3.	$\frac{PT}{PQ} = \frac{PR}{PT}$ $\therefore PT^2 = PR \times PQ$	3.	Corresponding sides of similar triangles are proportional

Proved

Unit 16 Trigonometry

Allocated teaching periods	10

Competency

- To solve the problems on area, height and distance using trigonometric ratios.

Learning Outcomes

- To find the area of triangle and quadrilateral by using trigonometric ratios
- To solve the simple problems on height and distance using trigonometric ratios

Level-wise learning objectives

S.N.	LEVELS	OBJECTIVES
1.	Knowledge (K)	- To define trigonometry - To tell the six trigonometric ratios - To tell the formula to calculate the area of triangle
2.	Understanding (U)	- To find the area o triangle and quadrilateral - To solve the problems on finding the length of side or angle of triangle or quadrilateral
3.	Application (A)	- To solve the problems on finding the area of quadrilateral formed by finding the triangles - To solve the simple problem based on height and distance
4.	High Ability (HA)	- To prepare a report on the application of trigonometry

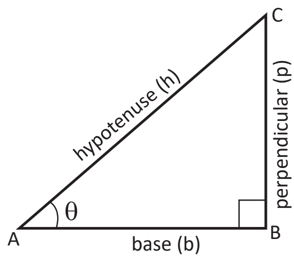
Required Teaching Materials/ Resources

Colourful chart-paper, models of right angled triangles, geometric instruments, colourful markers, hypsometer, clinometers, chart paper with trigonometric ratios of standard angles up to 90^0, ICT tool etc

Pre-knowledge: right angled triangles, Pythagorean triplets and name of parts its sides

Teaching Activities

1. Recall the following six trigonometric ratios from a right angled triangle



(i) $\sin\theta = \frac{p}{h}$ (ii) $\cos\theta = \frac{b}{h}$ (iii) $\tan\theta = \frac{p}{b}$

(iv) $\text{cosec}\theta = \frac{h}{p}$ (v) $\sec\theta = \frac{h}{b}$ (vi) $\cot\theta = \frac{b}{p}$

2. With discussion, explain the trigonometric ratios of some standard angles up to 90^0 Give the project work to tabulate the trigonometric ratios and paste on the math-corner

Trigonometric ratios	Angles				
	0°	**30°**	**45°**	**60°**	**90°**
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

3. With real life examples, discuss about the angles of elevation and depreciation
4. Use clinometers or hypsometer or theodolite to measure the angle of elevation or angle of depression
5. Ask the following questions during the class
   (i) What do you mean by horizontal line and line of sight?
   (ii) What is angle of elevation?
   (iii) Define angle of depression.
   (iv) Are the angle of elevation and angle of depression equal? Why?
6. Explain the importance of trigonometry on solving the real life problems
7. Take an example of height and distance and draw the figure with discussion
8. Make the groups of students and let solve the problems on height and distance by using trigonometric ratios
9. Recall the formulae of finding the area of triangle
   (i) Area of triangle $= \frac{1}{2}$ base $\times$ height
   (ii) Area of triangle = where semi-perimeter (s) $= \frac{a + b + c}{2}$
10. Make the 3 gro$\sqrt{s (s - a) (s - b) (s - c)}$ups of students and give the following work
   Group-A: Find the height of the triangle ABC using trigonometric ratio when sides AB = c, BC = a and $\angle$ABC = q are given.
   Group-B: Find the height of the triangle ABC using trigonometric ratio when sides BC = a, CA = b and $\angle$ACB = q are given.
   Group-C: Find the height of the triangle ABC using trigonometric ratio when sides AB = c, CA = b and $\angle$BAC = q are given.
11. Then discuss about the area of triangle as half of the product of length of any two sides and the sine of angle between them
   1. Similarly discuss about the area of parallelogram using trigonometric ratio.
   2. Solve some problems based on the topic and make the groups of the students and give them the work.

Solution of selected questions from Vedanta Excel in Mathematics

A. High and Distwance

1. ***The circumference of a circular meadow is 220 m and a pillar is situated at the centre of the meadow. If the angles of elevation of the top of the pillar from a point at the circumference of the meadow is found to be 30°, find the height of the pillar.***

Solution:

Let OA be the radius of the circular meadow, OB be the height of the pillar above the ground level and ∠BAO be the angle of elevation of the top of the pillar from a point A at the circumference.

Then,

$\angle BAO\ (\theta) = 30°$, Circumference (C) = 220 m, BD = ?

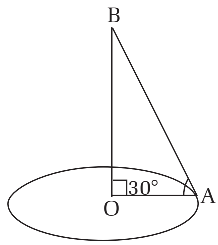
Now, Circumference (C) = $2\pi r$

or, $220\text{ m} = 2 \times \frac{22}{7} \times r$

$\therefore r = OA = 35\text{ m}$

Again,

From right angled triangle on B,



$\tan 30° = \frac{AB\ (p)}{BC\ (b)}$

or, $\frac{1}{\sqrt{3}} = \frac{BO}{35m}$

$\therefore BO = \frac{35}{\sqrt{3}} = 20.2\text{ m}$

Hence, the height of pillar (BO) is 20.2 m

2. ***On a windy day, a girl of height 1.1 m was flying her kite. When the length of string of the kite was 33 metres, it makes an angle of 30° with horizon. At what height was the kite above the ground?***

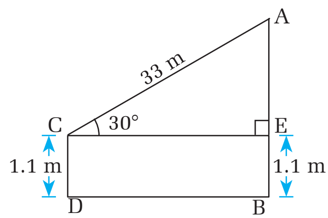
Solution:

Let AB and CD be the height of kite from ground and girl respectively. AC be the length of string of the kite and ∠ACE be the angle made by string of kite with horizon.

Then,

AC = 33 m, CD = EB = 1.1 m, ∠ACE = 30°, AB = ?

Now, from right angled ACE,



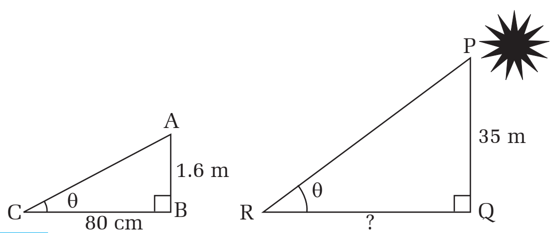
$\sin 30° = \frac{AE\ (p)}{AC\ (h)} = \frac{AE}{33m}$

or, $\frac{1}{2} = \frac{AE}{33m}$ $\therefore AE = 16.5\text{ m}$

Hence the height of kite above the ground AB = AE + EB

= 16.5 m + 1.1 m = 17.6 m

3. ***A man 1.6 m tall and the length of his shadow is 80 cm. Find the length of shadow of the building 35 m at the same time of a day.***



Solution:

Let AB be the height of a man and BC be the length of his shadow PQ be the height of the building and QR be the length of its shadow. $\angle ACB = \angle PRQ$ be the altitude of the sun at the same time of a day.

Then,

$AB = 1.6$ m, $BC = 80$ cm $= 0.8$ m, $PQ = 35$ m, $\angle ACB = \angle PRQ = \theta$ (say), $QR = ?$

Now,

From right angled triangle ABC, $\tan\theta = \dfrac{AB\ (p)}{BC\ (b)} = \dfrac{1.6m}{0.8m} = 2$ .... (i)

From right angled triangle PQR, $\tan\theta = \dfrac{PQ\ (p)}{QR\ (b)} = \dfrac{35m}{QR}$

or $2 = \dfrac{35m}{QR}$ [From (i)]

$\therefore QR = 17.5$ cm

Hence, the length of shadow of building is 17.5 cm.

4. ***A tree 14 m tall high is broken by the wind so that its top touches the ground and makes an angle of 60° with the ground. Find the length of the broken part of the tree.***

Solution:

Let AB be the height of the tree before it was broken, CD be the broken part of the tree and BD be the distance between the foot of the tree and the point on the ground at which the top of the tree touched.

Then,

$AB = 14$ m, The angle of elevation, $\angle BDC = 60°$

Let, $AC = CD = x$ m then $BC = (14 - x)$m

Now,

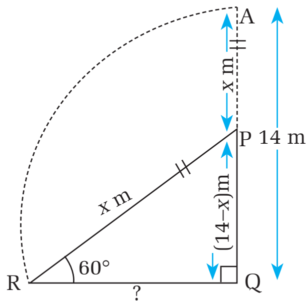
From rt.$\angle$ed ΔBCD, $\sin 60° = \dfrac{CD\ (p)}{CD\ (b)}$

or, $\dfrac{\sqrt{3}}{2} = \dfrac{14 - x}{x}$

or, $\sqrt{3}x = 28 - 2x$

or, $1.732x + 2x = 28$ $\therefore x = 7.5$

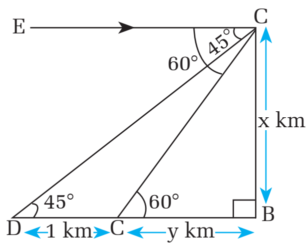
So, the length of broken part of the tree was 7.5 m.



5. ***From an aeroplane flying vertically over a straight road, the angles of depression of two consecutive kilometers stones on the same sides are 45° and 60°. At what height is the aeroplane from the ground ?***

Solution:

Let AB be the height of the aeroplanes from the ground, C and D be the stones and $\angle ADB$ and $\angle ACB$ be the angles of depression of the stones D adn C from the aeroplane.



Then,

$CD = 1$ km, $\angle ADB = 45°$, $\angle ACB = 60°$,

Let $AB = x$ m and $BC = y$ m

Now,

From rt.$\angle$ ed ΔABC, $\tan 60° = \dfrac{AB}{BC} = \dfrac{x\ km}{y\ km}$

or, $\sqrt{3} = \dfrac{x}{y}$

$\therefore y = \dfrac{x}{\sqrt{3}}$ .... (i)

Again,

From rt.$\angle$ ed Δ ABD, $\tan 45° = \dfrac{AB}{BD}$

or, $1 = \dfrac{x}{y+1}$ or, $y + 1 = x$

or, $\dfrac{x}{\sqrt{3}} + 1 = x$ [From (i)]

or, $x + \sqrt{3} = \sqrt{3}x$

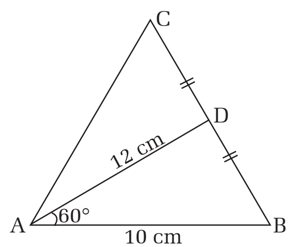
or, $\sqrt{3} = 0.732x$ $\therefore$ x = 2.3660225 km

Hence, the aeroplane is at the height of 2366.025 m from the ground level.

Extra Questions

1. A girl, 1.2 m tall, is flying a kite. When the length of the string of the kite is 180 m, it makes an angle of 300 with the horizontal line. At what height is the kite from the ground? [Ans: 91.2 m]
2. A temple 3 m tall is 90 m away from a tower 33 m high. Find the angel of elevation of the top of the tower from the top of the temple. [Ans: 300]
3. A straight tree of 15 m height is broken by the wind so that its top touches the ground and makes an angle of 300 with the ground. Find the length of broken part of the tree. [Ans: 10 m]

2. Area of triangle



1. ***In the given figure, AD is a median of ΔABC. If ∠BAD = 60°, AD = 12 cm and AB = 10 cm, find the area of ΔABC.***

Solution:

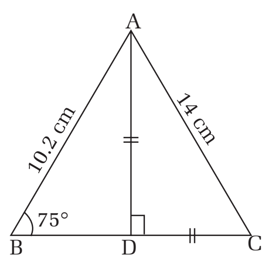
Here, In ΔABC, AB = 10 cm, AD = 12 cm, $\angle$BAD = 60° and BD = CD

Now,

Area of $\Delta ABD = \frac{1}{2} \times AB \times AD \times \sin 60°$

$= \frac{1}{2} \times 10 \text{ cm} \times 12 \text{ cm} \times \frac{\sqrt{3}}{2} = 30\sqrt{3} \text{ cm}^2$

Area of ΔABC = 2 × Area of ΔABD [Median AD bisects ΔABC] $= 2 \times 30\sqrt{3}\text{ cm}^2 = 60\sqrt{3}\text{ cm}^2$



2. ***In the given figure AB = 10.2 cm, AC = 14 cm, AD ⊥ BC and AD = DC. Calculate the area of ΔABC.***

Solution:

Here, AB = 10.2 cm, AC = 14 cm, AD $\perp$ BC and AD = DC

In ΔABD, $\angle$BAD = 180° – (75° + 90°) = 15°

In ΔADC, $\angle$DAC + $\angle$ACD + $\angle$ADC = 180°

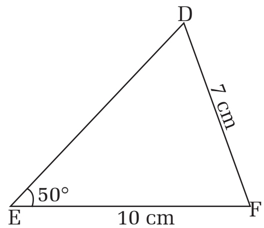
or, 2$\angle$DAC = 90° $\therefore$ $\angle$DAC = 45°

$\therefore$ $\angle$BAC = $\angle$BAD + $\angle$DAC = 15° + 45° = 60°

Now, area of $\Delta ABC = \frac{1}{2} AB \times AC \times \sin\angle BAC = \frac{1}{\cancel{2}} \times \overset{5.1}{\cancel{10.2}} \text{ cm} \times 14 \text{ cm} \times \sin 60°$

$= 5.1 \text{ cm} \times \overset{7}{\cancel{14}} \text{ cm} \times \frac{\sqrt{3}}{\cancel{2}} = 61.83 \text{ cm}^2$

3. ***In the given triangle DEF, DF = 7 cm, EF = 10 cm, area of $\Delta DEF = \frac{35}{2}\sqrt{3}cm^2$ and $\angle DEF = 50°$, find the value of $\angle EDF$.***



Solution:

Here, In ΔDEF, DF = 7 cm, EF = 10 cm area of $\Delta DEF = \frac{35}{2}\sqrt{3}cm^2$ and $\angle DEF = 50°$ $\angle EDF$?

Now,

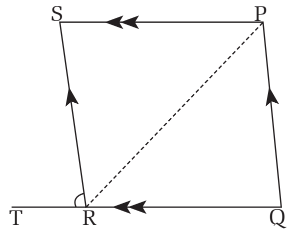
$$\text{Area of } \Delta DEF = \frac{1}{2} EF \times DF \times \sin F$$

or, $\frac{35}{2}\sqrt{3}cm^2 = \frac{1}{2} \times 10 \text{ cm} \times 7 \text{ cm} \times \sin F$

or, $\sin F = \frac{\sqrt{3}}{2}$ $\therefore \angle F = 60°$

$\therefore \angle EDF = 180° - (\angle DEF + \angle EFD) = 180° - (50° + 60°) = 70°$

4. ***In the given figure, PQRS is a parallelogram in which QR is produced to the point T. If the area of the parallelogram is $27\sqrt{3}$ sq.cm, the sides QP = 6 cm and PS = 9 cm, find the measurement of $\angle SRT$.***



Solution:

Here, Area of ▱ PQRS = $27\sqrt{3}cm^2$, QP = 6 cm, PS = QR = 9 cm, $\angle SRT$ = ?

Now,

Area of ▱ PQRS = 2 × area of ΔPQR

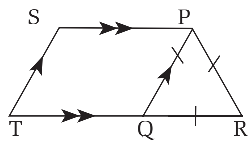
or, $27\sqrt{3}cm^2 = 2 \times \frac{1}{2} \times PQ \times QR \times \sin Q$

or, $27\sqrt{3}cm^2 = 6 \text{ cm} \times 9 \text{ cm} \times \sin Q$

or, $\sin Q = \frac{\sqrt{3}}{2}$ $\therefore \angle PQR = 60°$

Again, $\angle SRT = \angle PQR = 60°$ [PQ // SR, corresponding angles]

5. ***In the alongside, PQTS is a parallelogram and PQR is an equilateral triangle. If TQ = 3 ST and the area of trapezium PSTR is $175\sqrt{3}$ sq.cm, find the length of ST.***



Solution:

Here, $\angle PQR = \angle STQ = 60°$, TQ = 3ST, area of trap. PSTR = $175\sqrt{3}cm^2$, ST = ?

Now,

Area of trap. PSTR = area of STQP + area of ΔPQR

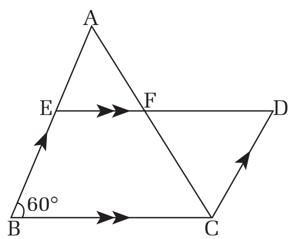
or, $175\sqrt{3}cm^2 = ST \times TQ \times \sin T + \frac{1}{2} \times PQ \times QR \times \sin Q$

or, $175\sqrt{3}cm^2 = ST \times 3\ ST \times \sin 60° + \frac{1}{2} \times ST \times ST \times \sin 60°$ [$\therefore$ST = PQ = QR]

or, $175\sqrt{3}cm^2 = 3 \times \frac{\sqrt{3}}{2} ST^2 + \frac{1}{2} \times \frac{\sqrt{3}}{2} ST^2$

or, $ST^2 = 100 \text{ cm}^2$ $\therefore$ ST = 10 cm

6. ***In the given figure, BCDE is a parallelogram and E is the mid-point of AB. If AE = 3 cm, EF = 4 cm and $\angle B = 60°$, find the area of parallelogram and ΔABC.***



Solution:

Here, AE = BE = 3 cm, EF = 4 cm, $\angle B = 60°$.

AF = CF [$\therefore$In ΔABC, AE = BE and EF // BC]

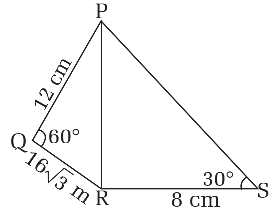
$\therefore BC = 2 \quad = 2 \times 4$ cm $= 8$ cm

Now,

Area of ▱ BCDE $= BE \times BC \times \sin B = 3 \text{ cm} \times 8 \text{ cm} \times \sin 60° = 24\text{cm}^2 \times \frac{\sqrt{3}}{2} = 12\sqrt{3}\text{cm}^2$

Again, Area of $\Delta ABC = \frac{1}{2} \times AB \times BC \times \sin B = \frac{1}{2} \times 6 \text{ cm} \times 8 \text{ cm} \times \sin 60°$

$= 24 \text{ cm2} \times \frac{\sqrt{3}}{2} = 12\sqrt{3}\text{cm}^2$



7. In the figure, PQ = 12 cm, QR = $16\sqrt{3}$cm , RS = 8 cm, ∠PQR = 60° and ∠PSR = 30°. If ΔPQR = 4ΔPSR, find the length of PS.

Solution:

Here, PQ = 12 cm, QR = $16\sqrt{3}$cm, RS = 8 cm, ∠PSR = 60° and ∠PSR = 30°

Now, area of $\Delta PQR = \frac{1}{2} PQ \times QR \times \sin 60° = \frac{1}{2} \times 12 \text{ cm} \times 16\sqrt{3}\text{cm} \times \frac{\sqrt{3}}{2} = 144 \text{ cm}^2$

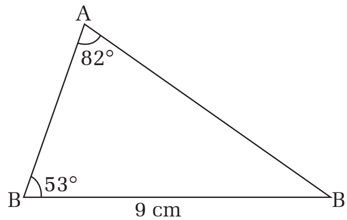
Also, Area of $\Delta PRS = \frac{1}{2} \times PS \times RS \times \sin 30° = \frac{1}{2} \times PS \times 8 \text{ cm} \times \frac{1}{2} = 2$ PS cm

According to question, ΔPQR = 4ΔPSR

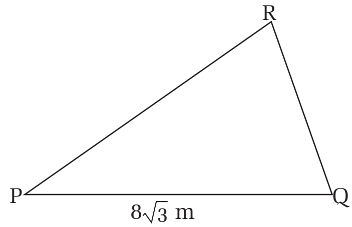
or, $144 \text{ cm}^2 = 4 \times 2 \text{ PS cm}^2 \quad \therefore PS = 18$

Hence PS = 18 cm

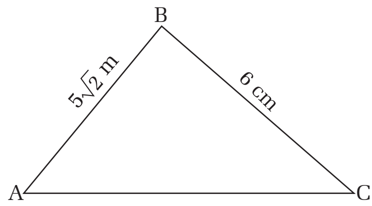
Extra Questions



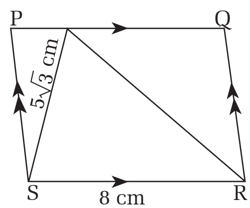
1. In the given ΔABC, ∠BAC = 82°, ∠ABC = 53°, BC = 9 cm and area of ΔABC = 27 cm², find the length of AC.



2. In the given figure alongside, ∠P : ∠Q : ∠R = 3 : 4 : 5, PQ = $8\sqrt{3}$ cm and area of ΔPQR = 42 sq.cm, find length of QR. [Ans: 10 cm]



3. In the given figure alongside, if a = 6 cm, c=$5\sqrt{2}$ cm and the area of ΔABC is 15 cm², find the measure of ∠ABC. [Ans: 45°]



4. In the adjoining figure, MNOP is a parallelogram in which MN = $7\sqrt{2}$ cm, ∠NOP = 45° and area of MNOP = 70 cm², find the length of ON. [Ans: 10 cm]

Unit 17 Statistices

Allocated teaching periods	9

Competency

- Solution of the problems for showing the different social and economic conditions by collecting, arranging, presenting and analyzing the data

Learning Outcomes

- To calculate the mean, median and quartiles from group data
- To analyze and conclude by using central values

Level-wise learning objectives

S.N.	LEVELS	OBJECTIVES
1.	Knowledge (K)	- To define central tendencies - To tell the formula of finding the mean, median and quartiles of continuous data - To identify the meanings of symbols used in the formula of find the median and quartiles
2.	Understanding (U)	- To solved the simple problems based on mean, median and quartile - To find the median and quartile classes from ogive
3.	Application (A)	- To transform the raw data in grouped data and find the mean - To calculate the median and quartiles of continuous data
4.	High Ability (HA)	- To collect the real (primary and secondary) data and analyze the data using the appropriate statistical measure and analyze the data

Required Teaching Materials/ Resources

Colourful chart-paper, colourful markers, chart paper with required formulae, graph-board, graph paper, highlighter etc

Pre-knowledge: Frequency distribution table, mean, median and quartiles of individual and discrete data etc

Teaching Activities

1. Recall definition of the statistics as a branch of mathematics in which facts and data are collected, sorted, displayed and analyzed.
2. Discuss about the use of statistics in decision making and predicting future plans and policies
3. With appropriate example, discuss about the measure of central tendency
(i) The single value which represents the characteristics of entire data.
(ii) Average of the given data is the measure of central tendency.
(iii) There are three types of central tendencies: Mean, Median and Mode
4. Ask about the mean of ungrouped data
5. Discuss about the mean of grouped data with examples and list the following methods and formulae

(i) Direct method: Mean $(\overline{x}) = \frac{\Sigma x}{n}$ (ii) Short cut method : Mean $(\overline{x}) = A + \frac{\Sigma fd}{N}$
where A = assumed mean and d = x – A

(iii) Step-deviation method: mean $(\overline{X}) = A + \frac{\Sigma fd'}{N} \times c$

where A = assumed mean and $d' = \frac{m - A}{h}$

6. Transform the raw data into grouped data and ask to calculate the mean
7. Make the groups of students and give different problems related to mean
8. Make a discussion on the median of grouped data focusing on the following facts
   (i) The median is the positional value below and above which 50% of the items lie.
   (ii) Position of median $= \left(\frac{N}{2}\right)^{th}$ class
   (iii) Median $= \mathbf{L} + \dfrac{\frac{\mathbf{N}}{\mathbf{2}} - \mathbf{c.f.}}{\mathbf{f}} \times \mathbf{c}$ where L = lower limit, c.f. = cumulative frequency of pre-median class, f = frequency median class and i = size of class
9. Solve the problems based on median in groups
10. Discuss about the quartiles by showing the following points on the chart-paper or board
    (i) Quartiles are the positional values which divide the given arranged data into four equal parts.

    25% Q_1 25% Q_2 25% Q_3 25%

    (ii) A distribution is divided into four equal parts by three positional values. So, there are three quartiles.
11. Explain about the first quartile (Q_{1}) of grouped/continuous data
    (i) The first or lower quartile (Q1) is a positional value below which 25% of the items lie and above which 75% items lie. (ii) Position of $Q_1 = \left(\frac{N}{4}\right)^{th}$ class
    (iii) $Q_1 = \mathbf{L} + \dfrac{\frac{\mathbf{N}}{\mathbf{2}} - \mathbf{c.f.}}{\mathbf{f}} \times \mathbf{c}$ where L = lower limit, c.f. = cumulative frequency of pre- Q1 class, f = frequency of Q1 class and i = size of class
12. Solve the problems related to lower quartile in groups
13. Explain about the third quartile (Q3) of grouped/continuous data
    (i) The third or upper quartile (Q_3) is a positional value below which 75% of the items lie and above which 25% items lie. (ii) Position of $Q_3 = \left(\frac{\mathbf{3N}}{\mathbf{4}}\right)^{th}$ class
    (iii) $Q_3 = \mathbf{L} + \dfrac{\frac{\mathbf{3N}}{\mathbf{4}} - \mathbf{c.f.}}{\mathbf{f}} \times \mathbf{c}$ where L = lower limit, c.f. = cumulative frequency of pre-Q3 class, f = frequency of Q3 class and i = size of class
14. Solve the problems related to lower quartile in groups
15. Recall about the less and more than cumulative frequency curves (ogives) and discuss upon the median and the quartile classes from the curves

Solution of selected questions from Vedanta Excel in Mathematics

1. ***The marks obtained by 20 students in an examination of mathematics are given below.***

15,	12,	23,	35,	46,	57,	18,	13,	39,	51,
32,	43,	25,	59,	18,	38,	45,	40,	32,	33

a. Make a frequency table of class interval of 10.

b. Calculate the mean by the following methods

i) Direct method ii) Deviation method (short-cut method)

iii) Step deviation method

Solution:

Here, lowest mark = 15, Highest mark = 59

$\therefore$ Class intervals are 10 - 20, 20 - 30, ..., 50 - 60

a)

Marks	Tally Marks	No.of Students (f)
10 - 20	~~||||~~	5
20 - 30	||	2
30 -40	~~||||~~ |	6
40 - 50	||||	4
50 - 60	|||	3

b) i) Calculating of mean by direct method

Marks	Mid - value (m)	No.of Students (f)	fm
10 - 20	15	5	75
20 - 30	25	2	50
30 -40	35	6	210
40 - 50	45	4	180
50 - 60	55	3	165
		N = 20	Σfm = 680

Now, mean ($\overline{x}$) $= \frac{\Sigma fm}{N} = \frac{680}{20} = 34$

ii) Calculating of mean by derivation (short-cut) method

Let assumed mean (A) be 35.

Marks	Mid - value (m)	No.of Students (f)	d=m – 35	fd
10 - 20	15	5	– 20	– 100
20 - 30	25	2	– 10	– 20
30 -40	35	6	0	0
40 - 50	45	4	10	40
50 - 60	55	3	20	60
		N = 20		Σfd = – 20

Now, mean ($\overline{x}$) $= A + \frac{\Sigma fd}{N} = 35 + \left(-\frac{20}{20}\right) = 35 - 1 = 34$

iii) Calculation of mean by step- deviation method.
Let assumed mean (A) be 35
Class size (h) = 10

Marks	Mid - value (m)	No.of Students (f)	d=m – 35	$d' = \frac{d}{10}$	fd'
10 - 20	15	5	– 20	– 2	– 10
20 - 30	25	2	– 10	– 1	– 2
30 -40	35	6	0	0	0
40 - 50	45	4	10	1	4
50 - 60	55	3	20	2	6
		N = 20			$\sum fd' = -2$

Now, mean $(\overline{x}) = A + \frac{\Sigma fd'}{N} \times h = 35 + \left(\frac{-2}{20}\right) \times 10 = 35 - 1 = 34$

2. ***The median of the following data is 50. If N = 100, find the value of x and y.***

Class	0 - 20	20 - 40	40 - 60	60 - 80	80 - 100
f	14	x	26	y	16

Solution:

Here

Class	f	cf
0 - 20	14	14
20 - 40	x	14 + x
40 - 60	26	40 + x
60 - 80	y	40 + x + y
80 -100	16	56 + x + y
	N = 56 + x + y	

Now, N = 56 + x + y or, 100 = 56 + x + y $\therefore$ x = 44 – y .... (i)
Since, median = 50. So, median class = 40 – 60 where,
L = 40, cf = 14 + x, f = 20, i = 20

We have, Median $= L + \left(\frac{\frac{N}{2} - cf}{f}\right) \times i$

or, $50 = 40 + \left(\frac{\frac{100}{2} - (14+x)}{\cancel{26}\ 13}\right) \times \overset{10}{\cancel{20}}$

or, $\cancel{10} = \left(\frac{50 - 14 - x}{13}\right) \times \cancel{10}$

or, 13 = 36 – x
$\therefore$ x = 23
Putting the value of x in equation (i), we get
23 = 44 – y $\therefore$ y = 21
Hence, required value of x is 23 and y is 21.

3. ***Find the first or lower (Q_1) from the data given below.***

Marks Obtained	0 - 20	0 - 40	0 - 60	0 - 80	0 - 100
No. of students	21	44	66	78	88

Solution:

Cumulative frequency(c.*f*) table

Marks Obtained	No. of Students (*f*)	c.*f*
0 - 20	21	21
20 - 40	23	44
40 - 60	22	66
60 - 80	12	78
80 -100	10	88
	N = 88	

Now, the position of lower quartic (Q_1) $= \left(\frac{N}{4}\right)^{th}$ class $= \left(\frac{88}{4}\right)^{th}$ class $= 22^{nd}$ class

In c.*f*. column, the c.*f*. first greater than 22 is 44 and its corresponding class is 20 – 40.

Here, $L = 20$, $\frac{N}{4} = 22$, $c.f. = 21$, $f = 23$ and $i = 20$

$\therefore$ Lower quartic (Q_1) $= L + \left(\frac{\frac{N}{4} - cf}{f}\right) \times i = 20 + \left(\frac{22-21}{23}\right) \times 20 \quad = 20.87$

4. ***In the first quartic of the data given below is 30.625, find the value of x.***

C.I	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
f	8	x	5	4	3

Solution:

Cumulative frequency table

C.I	*f*	c.*f*
20 - 30	8	8
30 - 40	x	8 + x
40 - 50	5	13 + x
50 - 60	4	17 + x
60 - 70	3	20 + x
	N = 20 + x	

Since, the first quartic (Q_1) class = 30.625, it lies is the class interval 30 - 40.
So, Q_1 class = 30 - 40.

Here, $L = 30$, $\frac{N}{4} = \frac{20 + x}{4}$, $c.f. = 8$, $f = x$ and $c = 10$

Again, $(Q_1) = L + \left(\frac{\frac{N}{4} - cf}{f}\right) \times c$

or, $30.625 = 30 + \left(\frac{\frac{20 + x}{4} - 8}{x}\right) \times 10$

or, $0.625 = \left(\frac{20 + x - 32}{4x}\right) \times 10$

or, $2.5x = 10x - 120$

or, $7.5x = 120 \quad \therefore x = 16$

So, the value of x is 16.

5. ***In the data given below, If the third quartic (Q_3) is 60, find the value of p.***

Marks Obtained	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
f	3	5	p	5	p	p	3

Solution:

Cumulative frequency table

Marks Obtained	No. of students (f)	c.f
10 - 20	3	3
20 - 30	5	8
30 -40	p	8 + p
40 - 50	5	13 + p
50 - 60	p	13 + 2p
60 - 70	p	13 + 3p
70 - 80	3	16 + 3p
	N = 16 + 3p	

Since, the third quartic (Q_3) = 60, it lies is the class interval of 60 - 70.

So, Q_3 class = 60 - 70.

Here, L = 60, $\frac{3N}{4} = \frac{3(16 + 3p}{4}$, c.$f$. = 13 + 2p, f = p and c = 10

Now, $(Q_3) = L + \left(\frac{\frac{3N}{4} - cf}{f}\right) \times c$

or, $60 = 60 + \left(\frac{3\left(\frac{16 + 3p}{4}\right) - (13 + 2p)}{p}\right) \times 10$

or, $0 = \left(\frac{48 + 9p - 52 - 8p}{4p}\right) \times 10$

or, $0 = (p - 4) \times 10$

$\therefore p = 4$

Hence, required value of p is 4.

Extra Questions

1. Find the median from the data given below. [Ans:33.5]

Marks obtained	0-10	0-20	0-30	0-40	0-50
No. of students	4	12	24	44	62

2. If the median of the following data is 26 years, find the value of m. [Ans:30]

Age in years	0-10	10-20	20-30	30-40	40-50	50-60
No. of people	5	12	m	10	8	5

3. Calculate the first quartile from the data given below if it lies in class (8 – 16). [Ans:10 cm]

Height in cm	0-8	8-16	16-24	24-32	32-40	40-48
No. of plants	7	p	5	6	8	2

Unit 17 Probability

Allocated teaching periods	11

Competency

- Study of probability and to solve the daily life problems through mathematical structures
- To solve the problems and uses of probability on daily life activities

Learning Outcomes

- To identify the probability and solve the probability based on random sampling and empirical probability.
- To find the probability by using the addition and multiplication laws of mutually exclusive events.
- To solve the problem related to probability of dependent and independent events.

Level-wise learning objectives

S.N.	LEVELS	OBJECTIVES
1.	Knowledge (K)	- To define random experiment, sample space, event, mutually exclusive events, dependent/ independent events etc. - To tell the formula of an event E. - To tell the probability of certain/uncertain event - To state the addition laws of mutually exclusive events. - To state the multiplication laws of independent events.
2.	Understanding (U)	- To find the probability of an event in an experiment. - To find the probability of mutually exclusive events by using addition laws - To find the probability of independent events by using multiplication laws - To show the probabilities in tree-diagram

Required Teaching Materials/ Resources

Coin, cubical dice, number cards, playing cards, spinner, charts with definitions of related terminologies, formulae chart,

Teaching learning strategies

Pre-knowledge: Meaning of outcomes, possibility, probability etc.

Teaching Activities

1. Take a coin and ask the following questions:

(i) How many faces does it have?
(ii) How many faces can be drawn when a coin in tossed at once?
(iii) What are the possible outcomes in tossing a coin at once?
(iv) Is it certain to draw head?

2. Roll die and ask the following questions:
(i) How many faces does it have?
(ii) What are the possible outcomes when a die rolled once?
3. Similarly, take a pack of playing cards, number cards and discuss upon the outcomes, sample space, favourable outcomes, events, probability etc.
4. Give and ask some more real life problems like probability of rising sun due east, probability of raining, probability of getting 7 on a die etc.
5. Explain about the formula of probability of an event E, $P(E) = \frac{n(E)}{n(S)}$
6. From a die, ask the probabilities of getting each natural number, odd numbers, even numbers, prime numbers, square numbers, cube numbers etc.
7. Present different types of cards from a deck of 52 playing cards: red/ black, spade/ club, heart/diamond, ace, 2, 3, …, J, Q, K etc. and ask their probability.
   Note: The probability of impossible event $P(E) = 0$ and the probability of certain event $P(C) = 1$.
8. Divide the students into five groups and do the following activities:
   Group-A: Give a coin and tell them to find P (H) and P (T)
   Group-B: Give a die and tell them to find P (1), P (2), P (3), P (4), P (5) and P (6)
   Group-C: Give a spinner with three sectors of red, blue and black colour and tell them to find P (R) and P (B) and P (W).
   Group D: Give a pack of 52 playing cards and tell them to find P (Red card), P (King), P (Spade) andP (Face card).
   Group E: Give a pack of bag of containing 5 white, 3 red and 4 green marbles of same shape and size and tell them to find P (G), P (W) and P (R).
   Then tell the students to present their answers in classroom.
9. Discuss on probability scale.
10. Call two students in front of the classroom, give a coin and tell one student to toss it 20 times and another student to fill the following table.

Outcome	Tally bar	Frequency
Head	…..	…..
Tail	….	…..

   Then ask the students to find P (H) and P (T) and explain with examples of empirical probability.
11. Ask the students whether head and tail occur simultaneously or not. Similarly, give or ask some real life examples which cannot happen at the same time. Then discuss about mutually exclusive events.

Note:

(i) The events which cannot happen at the same time are called mutually exclusive events. In the other word, if the occurrence of an event excludes the occurrence of other event/s are called mutually exclusive events.

(ii) If A and B are two mutually exclusive events then $n(A \cap B) = 0$ and, $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$ and $P(\overline{A \cup B}) = 1 - P(A \cup B)$

(iii) If A, B and C are two mutually exclusive events then $P(A \text{ or } B \text{ or } C) = P(A \cup B \cup C) = P(A) + P(B) + P(C)$

12. Divide the students into five groups and provide 1/1 problems based on addition law of mutually exclusive events.
13. Give some examples of independent events such as occurrence of head does not affect the occurrence of 5 when a coin is tossed and die is rolled at once. Discuss on independent events.

Note:

(i) Two events are said to be independent if the occurrence of an event affects the occurrence of another event.

(ii) If A and B are be independent events then P (A and B) = P (AÇB) = P (A) ´ P(B)

(iii) If A, B and C are three independent events then P (AÇBÇC) = P (A)´ P (B)´ P (C)

14. Demonstrate the probability of depend and independent events in tree diagram and encourage the students to solve the related problems.

Solution of selected questions from Vedanta Excel in Mathematics

1. ***From a well-suffled pack of 52 cards, a card is drawn at random. What is the probability of getting neither a black jack or a red queen.***

Solution:

Let, J and Q denote the events of getting a black jack and queen respectively.

Then, n(S) = 52, n(J) = 2, n(Q) = 4

Now,

Probability of getting either black jack or queen $= P(J \text{ or } Q) = P(J\cup Q)$

$= P(J) + P(Q)$

$= \dfrac{n(J)}{n(S)} + \dfrac{n(Q)}{n(S)} = \dfrac{2}{52} + \dfrac{2}{52} = \dfrac{1}{13}$

Probability of getting neither black jack not red queen $= P(\overline{J \cup Q}) = 1 - \dfrac{1}{13} = \dfrac{12}{13}$

2. ***Two unbiased dice are rolled. Find the probability that the sum of the number on the two facer is either divisible by 5 or divisible by 6.***

Solution:

Let D_5 and D_6 denote the event of getting the sum of number be divisible by 5 and 6 respectively.

Then,

$$S = \left\{\begin{matrix} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6), \end{matrix}\right\}$$

$\therefore n(S) = 36$

$D_5 = \{(1,4), (2,3), (3,2), (4,1), (4,6), (5,5), (6,4)\}$ $\therefore n(D_5) = 7$

$D_6 = \{(1,5), (2,4), (3,3), (4,2), (5,1), (6,6)\}$ $\therefore n(D_6) = 6$

Now $p(D_5 \text{ or } D_6) = p(D_5) + p(D_6) = \dfrac{7}{36} + \dfrac{6}{36} = \dfrac{13}{36}$

Hence, the probability that the sum of number on the two faces is either divisible by 5 or divisible by 6 is $\dfrac{13}{36}$

3. ***A number card is drawn randomly from the set of numbered cards, numbered from 6 to 39. Find the probability that the card may be a prime number or cubed number.***

Solution:

Let P and C denote the event of getting a prime and cubed numbered cards respectively.

Then, S = {6, 7, 8, ..., 39}

P = {7, 11, 13, 17, 19, 23, 29, 31,37} $\therefore n(P) = 9$

C ={8, 27} $\therefore n(C) = 2$

Now, probability of getting prime or cubed numbered card

$P(P \text{ or } C) = P(P) + P(C) = \dfrac{9}{34} + \dfrac{2}{34} = \dfrac{11}{34}$

So, the probability that the card is a prime number or cubed number is $\dfrac{11}{34}$

4. ***A natural number is chosen at random from the first 30 natural numbers. What is the probability that the number chosen is either multiple of 3 or a multiple of 4 ?***

Solution:

Let M_3 and M_4 denote the events of getting a multiple of 3 and multiple of 4 respectively.

Then, $S = \{1, 2, 3, \ldots\ldots, 30\}$ $\therefore n(S) = (30 - 1) + 1 = 30$

$M_3 = \{3, 6, 9, 12, 15, 18, 21, 24, 27, 30\}$ $\therefore n(M_3) = 10$

$M_4 = \{4, 8, 12, 16, 20, 24, 28\}$ $\therefore n(M_4) = 7$

$n(M_3 \cap M_4) = \{12, 24\}$ $\therefore n(M_3 \cap M_4) = 2$

Now, $P(M_3 \text{ or } M_4) = P(M_3) + P(M_4) - P(M_3 \cap M_4)$

$$= \frac{n(M_3)}{n(S)} + \frac{n(M_4)}{n(S)} - \frac{n(M_3 \cap M_4)}{n(S)}$$

$$= \frac{10}{30} + \frac{7}{30} - \frac{2}{30}$$

$$= \frac{15}{30} = \frac{1}{2}$$

So, the probability that the number chosen is either multiple of 3 or a multiple of 4 is $\frac{1}{2}$.

5. ***If a card is drawn at random from a pack of 52 cards and at the same time a marble is drawn at random from a bag containing 2 red marbles and 3 blue marbles. Find the probability of getting a blue marble and a king.***

Solution:

Let K, B and R denote the events of getting king card, blue and red marbles respectively.

Then, For drawing a card: $n(S_1) = 52$, $n(K) = 4$ $\therefore P(K) = \frac{n(K)}{n(S)} = \frac{4}{52} = \frac{1}{13}$

For drawing a marble: $n(S_2) = 2 + 3 = 5$, $n(B) = 3$ $\therefore P(B) = \frac{n(B)}{n(S)} = \frac{3}{5}$

Since the events are independent

$\therefore P(K \text{ and } B) = P(K) \times P(B) = \frac{1}{13} \times \frac{3}{5} = \frac{3}{65}$

Hence the required probability is $\frac{3}{65}$.

6. ***A bag contains 5 black, 7 blue and 4 yellow balls. A ball is drawn at random and it is replaced, then another ball is drawn. Find the probability that:***
   - ***i) the first is blue and the second is black***
   - ***ii) both of them are yellow***
   - ***iii) both of them are of the same colour***
   - ***iv) the first is black and the second is yellow***
   - ***v) both of them are not black***

Solution:

Let B_k, B_e and Y denote the event of getting a black, blue and yellow balls respectively.

Then, $n(B_k) = 5$, $n(B_e) = 7$ and $n(Y) = 4$

$\therefore$ Total number of balls n(S) = 5 + 7 + 4 = 16

Since the ball which is draw at first is replaced to draw another ball, it is an independent event.

Now,

i) $P(B_e \text{ and } B_k) = P(B_e \cap B_k) = P_1(B_e) \times P_2(B_k)$

$= \dfrac{n(B_e)}{n(S)} \times \dfrac{n(B_K)}{n(S)} = \dfrac{7}{16} \times \dfrac{5}{16} = \dfrac{35}{256}$

ii) P(Both are yellow) $= P(YY) = P(Y \cap Y)$

$= P_1(Y) \times P_2(Y) = \dfrac{n(Y)}{n(S)} \times \dfrac{n(Y)}{n(S)}$

$= \dfrac{4}{16} \times \dfrac{4}{16} = \dfrac{1}{16}$

iii) P(Both are of same colour) $= P(B_e\, B_e \text{ or } B_k\, B_k \text{ or } YY)$

$= P(B_e\, B_e) + P(B_k\, B_k) + P(YY)$

$= \dfrac{7}{16} \times \dfrac{7}{16} + \dfrac{5}{16} \times \dfrac{5}{16} + \dfrac{4}{16} \times \dfrac{4}{16}$

$= \dfrac{49 + 25 + 16}{256} = \dfrac{\cancel{90}\,45}{\cancel{256}\,128} = \dfrac{45}{128}$

iv) $P(B_k \text{ and } Y) = P(B_k) \times P(Y) = \dfrac{5}{16} \times \dfrac{4}{16} = \dfrac{5}{64}$

v) $P(\overline{B_k \cup B_k}) = P_1(\bar{B}_k) \times P_2(\bar{B}_k)$

$= [1 - P(B_k)]\,[1 - P(B_k)]$

$= \left(1 - \dfrac{5}{16}\right)\left(1 - \dfrac{5}{16}\right) = \dfrac{12}{256}$

7. ***A can solve 90% of the problems given in the exercise of probability and B can solve 70%. What is the probability that at least one of them will solve a problem selected at random from the exercise ?***

Solution:

Here, $p(A) = \dfrac{90}{100} = \dfrac{9}{10}$ and $p(B) = \dfrac{70}{100} = \dfrac{7}{10}$

Since, the event of solving problem is independent.

So, $p(A \text{ and } B) = p(A \cap B) = p(A) \times p(B) = \dfrac{9}{10} \times \dfrac{7}{10} = \dfrac{63}{100}$

Also, the problem may be solved by both the students A and B.

So, the events are non - mutually exclusive.

Now, p(solving problem by at least one of them) $= p(A \cup B) = p(A) + p(B) - p(A \cap B)$

$= \dfrac{7}{10} + \dfrac{7}{10} - \dfrac{63}{100} = \dfrac{63}{100} = 0.97.$

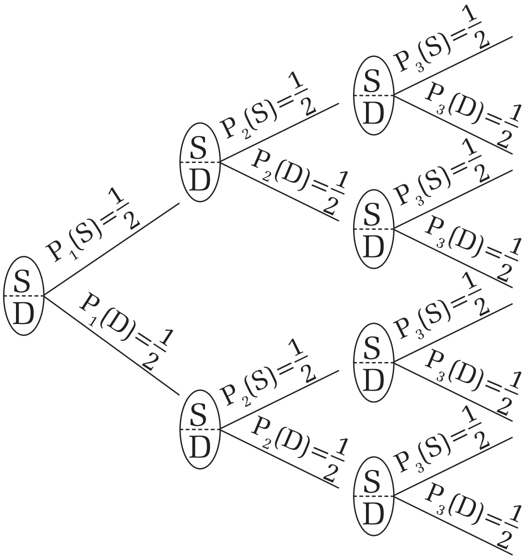
8. ***Three children were born in a family. By drawing a tree diagram, find the following probabilities.***

i) at least two daughters ***ii) all of them are boys*** ***iii) at least a boy***

Solution:

Let S and D denote the events of having son and daughter respectively.

Drawing a probability tree diagram:



SSS→P(SSS) $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

SSD→P(SSD) $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

SDS→P(SDS) $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

SDD→P(SDD) $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

DSS→P(DSS) $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

DSD→(DSD) $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

DDS→P(DDS) $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

DDD→P(DDD) $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Now,

i) P(at least two daughter) = P(SDD) + P(DSD) + P(DDS) + P(DDD) $= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$

ii) P(all are boys) = P(SSS) $= \frac{1}{8}$

iii) P(at least a boys) = P(SSS) + P(SSD) + P(SDS) + P(SDD) + P(DSS) + P(DSD) + P(DDS)

$$= \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{7}{8}$$

Extra questions

1. A box contains the lottery tickets numbered from 3 to 32. If a ticket is drawn at random, what is the probability of the ticket bearing square or cube number? [Ans: $\frac{1}{5}$]
2. An ace of diamond is lost from a deck of 52 playing cards and a card is drawn at random. What is the probability of getting black faced card or ace? [Ans: $\frac{3}{17}$]
3. A dice is rolled and a coin is tossed at the same time, find the probability of occurring even number on the dice and head on the coin. [Ans: $\frac{1}{4}$]
4. A card is draw from a well-shuffled pack of 52 cards and at the same time a marble is drawn at random from a bag containing 2 green and 3 red marbles of same shape and size. Find the probability of getting a spade and green marble. [Ans: $\frac{1}{10}$]
5. A bag contains 1 yellow, 1 black and 1 green balls of same shape and size. Two balls are drawn at randomly one after another without replacement; show the probabilities of all possible outcomes in a tree-diagram.
6. A coin is tossed thrice successively. Show the probabilities of all the possible outcomes in a tree diagram and find the probability of getting all heads. [Ans: $\frac{1}{8}$]
7. Three children were born in the interval of five years. Find the probability of having the at least one son by drawing a tree-diagram. [Ans: $\frac{7}{8}$]