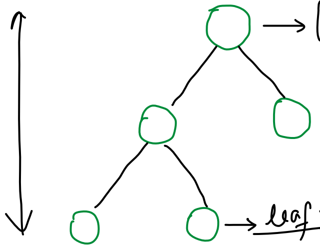
Plan of Attack

30 November 2023 10:42



→ [entropy/gini]
similarity

→ leaf node

Regression

$$\text{Similarity Score} = \frac{\left(\sum \text{Residuals}\right)^2}{N + \lambda} \quad (1)$$

Classification

$$\text{Similarity Score} = \frac{\left(\sum \text{Residuals}\right)^2}{\sum^{N} [P(1 - P)] + \lambda} \quad (2)$$

reg

$$\text{Output value} = \frac{\sum \text{Residuals}}{N + \lambda} \quad (3)$$

$$\text{Output Value} = \frac{\left(\sum \text{Residuals}\right)}{\sum^{N} [P(1 - P)] + \lambda} \quad (4)$$

$\boxed{1}$ intro

$\boxed{2}$ regression

$\boxed{3}$ classification

deep dive → goal

Prerequisite

30 November 2023 10:44

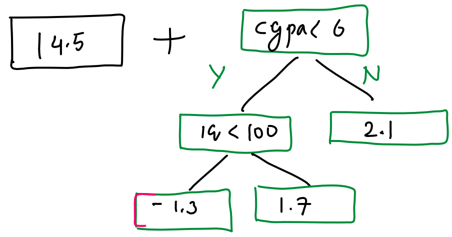
- → Xgboost → Reg
  ↘ Classi
- → Gradient Boosting →
- → Tree (basic)

Boosting as an [Additive Model]

30 November 2023 12:04

Xgboost → 2 model

cgpa	iq	package
6.5	71	13
5	91	10
9.1	120	16



Xgboost

train

o/p leaf weight

{9, 100}

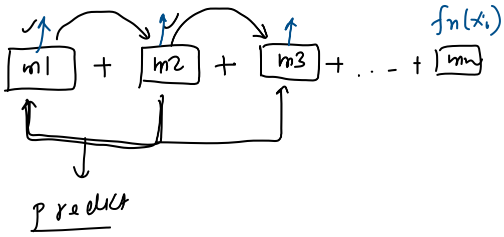
$\hat{y}_i = 14.5 + 2.1$

$= \boxed{16.6}$ lpa

$\boxed{m_1 + m_2}$

$\hat{y}_i = f_1(x_i) + f_2(x_i) + \dots + f_n(x_i)$

$$\boxed{\hat{y}_i = \sum_{i=1}^{n} f_i(x_i)} \rightarrow \text{xgboost}$$



[reg]

$$\boxed{\text{output}} = \boxed{\frac{\text{sum of residuals}}{N + \lambda}}$$

$1 + \lambda$ origin story

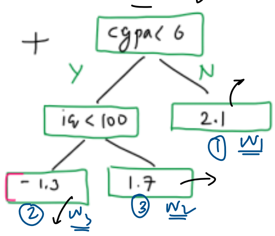
XGBoost Loss Function

30 November 2023 13:19

cgpa	iq	package (y_i)
6.5	71	13
5	91	10
9.1	120	16

5 leaf

$$\left[\underset{m1}{\boxed{14.5}} + \underset{m2}{\text{tree}} \right] \rightarrow \hat{y}_i = m_1 + m_2$$



$f_1(x_i) + \boxed{f_2(x_i)}$ → value

14.5

3 output values

loss function ?

$$\boxed{\hat{y}_i} = \sum_{i=1}^{2} f_i(x_i)$$

$$\left[\text{output} = \frac{\Sigma \text{ sum of residual}}{N + \lambda} \right]$$

number of residuals

→ $\boxed{y_i \quad \hat{y}_i}$ $\boxed{\text{reduce}}$

$y_i \quad \hat{y}_i$ → relation

↳ loss function → minimize

reg mse

$$L(y_i, \hat{y}_i) = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$\boxed{\text{Xgboost}}$ → loss functions

$y_i \rightarrow \hat{y}_i \rightarrow$ relation

Gradient Boosting –

flexible → loss function

↳ differentiable

$$\left[L(y_i, \hat{y}_i) \right]$$

mse   log loss

differentiable → loss-function → regularization parameter

optimization

$$m_1 + \boxed{m_2}$$

base   dt

$$L = \sum_{i=1}^{n} L(y_i, \hat{y}_i) + \boxed{\Omega(f_2(x_i))}$$

minimize

$$L = \sum_{i=1}^{n} L(y_i, \hat{y}_i)$$

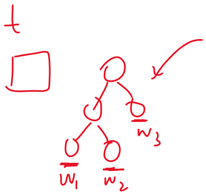
loss

$$\Omega(f) = \gamma T + \frac{1}{2} \lambda \|w\|^2$$

3

regularization   # leaf nodes   regularization

t



$$\frac{1}{2}\lambda(w_1^2 + w_2^2 + w_3^2)$$

The Derivation

30 November 2023 13:57

objective func

$$\mathcal{L} = \underbrace{\sum_{i=1}^{n} L(y_i, \hat{y}_i)}_{\text{loss function}} + \underbrace{\Omega(f_k)}_{\text{regular}}$$

$$\Omega(f_k) = \gamma T + \frac{1}{2}\lambda \|w\|^2$$

$T \rightarrow$ leaf

$\|w\|^2 \rightarrow$ leaf w

boosting → mean □ □ □

↓

xgboost → 1 model □ → objective

Stage I

mean

$\boxed{m1}$

$f_1(x_i)$ → $\mathcal{L}^{\langle 1 \rangle} = \sum_{i=1}^{n} L(y_i, \hat{y}_i)$

f_2 f_1 X

Stage II

mean

$\boxed{m1}$ $\boxed{m2}$ decision

$f_1(x_i)$ $f_2(x_i)$ → $\mathcal{L}^{\langle 2 \rangle} = \sum_{i=1}^{n} L(y_i, f_1(x_i) + f_2(x_i)) + \Omega(f_2(x_i))$

$$\hat{y}_i = f_1(x_i) + f_2(x_i)$$

Stage III

mean

$\boxed{m1}$ $\boxed{m2}$ $\boxed{m3}$

$f_1(x_i)$ $f_2(x_i)$ $f_3(x_i)$ → $\mathcal{L}^{\langle 3 \rangle} = \sum_{i=1}^{n} \left(y_i, f_1(x_i) + f_2(x_i) + f_3(x_i) \right) + \underbrace{\Omega(f_3(x_i))}$

$$\underbrace{f_1(x_i) + f_2(x_i) + \dots f_{t-1}(x_i)} \rightarrow \hat{y}_i^{\langle t-1 \rangle}$$

Stage $\boxed{t}$

mean

$\boxed{m1}$ $\boxed{m2}$ . . . . $\boxed{m_t}$

$f_1(x_i)$ $f_2(x_i)$ $f_t(x_i)$

last decis

$$\rightarrow \mathcal{L}^{\langle t \rangle} = \sum_{i=1}^{n} L(y_i, \underbrace{f_1(x_i) + f_2(x_i) + \dots + f_t(x_i)}) + \Omega(f_t(x_i))$$

$$\mathcal{L}^{\langle t \rangle} = \sum_{i=1}^{n} L(y_i, \hat{y}_i^{\langle t-1 \rangle} + f_t(x_i)) + \Omega(f_t)$$

At any stage t

$$\underbrace{\mathcal{L}^{\langle t \rangle}} = \sum_{i=1}^{n} L(y_i, \hat{y}_i^{\langle t-1 \rangle} + f_t(x_i)) + \Omega(f_t(x_i))$$

$f_t(x_i)$

↳ minimize

Problem with Objective Function

30 November 2023 20:40

$$\mathcal{L}^{<t>} = \boxed{\sum_{i=1}^{n} L\left(y_i, \hat{y}_i^{<t-1>} + f_t(x_i)\right)} + \Omega\left(f_t(x_i)\right)$$

minize

[optimization] ~~Yes~~

$\hat{y}_i$ — decis

obj min

approximate

optimize

$$\frac{\partial L}{\partial f_t(x_i)} = 0$$

diffrentiable

loss

Linear reg

inp	out (y_i)
exp x_i	salary

$$\hat{y}_i = m(exp_i) + b$$

$$L = \sum_{i=1}^{n} L(y_i, \hat{y}_i) = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$L = \sum_{i=1}^{n} (y_i - m x_i - b)^2$$

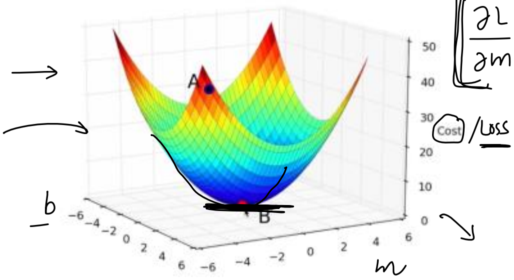
minimum

$$\frac{\partial L}{\partial m} = 0 \qquad \frac{\partial L}{\partial b} = 0$$

$m \Rightarrow$

$(m, b) \rightarrow$ $\underset{min}{b}$ grad desc

$$\frac{\partial L}{\partial m} \quad \frac{\partial L}{\partial b} = 0$$



b

m

Cost / Loss

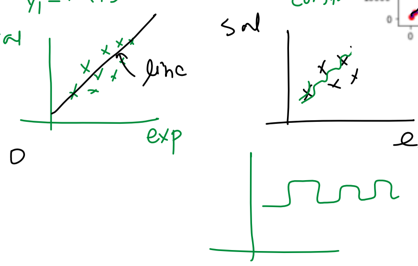
$L(\hat{y}_i)$

$\rightarrow L$ $\quad \mathcal{L}^{<t>}$

$m, b \quad f_t(x_i)$

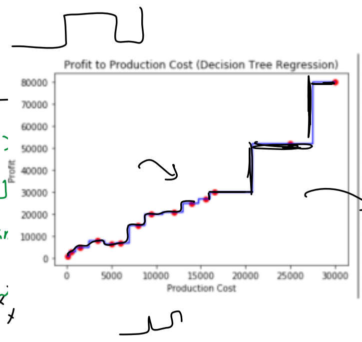
$\hat{y}_i = mx + b$

$\hat{y}_i = \hat{y}_i^{<t-1>}$ — constant

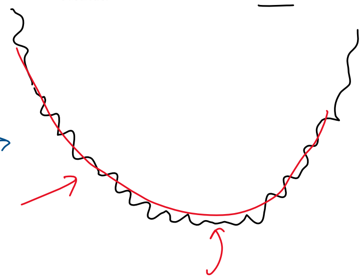


sal — linc — exp

sal — exp



piecewise constant



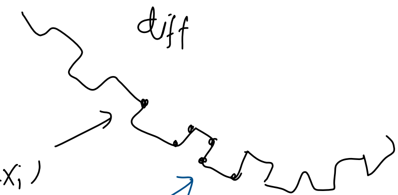
some

often → smooth

→ diffrentable

$$\frac{\partial L}{\partial f_t(x_i)}$$

Taylor series



diff

$f_t(x_i)$

non-diff

The Solution - Taylor Series

01 December 2023 00:02

powerful approx techqi

Objective → usable

↓

Taylor series

complex function → approx → polynomial

↓

e^x ⟶ $3x^2+2$

accurate approx

↑

$3x^2 + 3x^3 + x^4 + x^5 \ldots\ldots$

How

e^x → polynomial

→ derivaties

$\boxed{e^x}$ → e^x

$f(x)$ point → $\boxed{a}$

$$f(x) \simeq f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3$$

$+ \ldots\ldots -$

$f(x) = e^x$ $\boxed{a=0}$

$f(a) = e^a = 1$

$$\frac{f'(a)}{1!}(x-a) = \frac{e^a}{1}(x-a) = e^0(x-0) = x$$

$$\frac{f''(a)}{2!}(x-a)^2 = \frac{e^a}{2!}(x-a)^2 = \frac{e^0(x)^2}{2!} = \frac{x^2}{2}$$

$$\frac{f'''(a)}{3!}(x-a)^3 = \frac{e^a}{3!}(x-a)^3 = \frac{1}{6}x^3 = \frac{x^3}{6}$$

$$e^x \simeq \boxed{1+x} + \frac{x^2}{2} + \frac{x^3}{6} + \ldots$$

Applying Taylor Series

01 December 2023 00:51

$$f(x) \simeq f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 \ldots\ldots$$

$$\frac{\partial f}{dx} =$$

$f(x) = x^2$

$f'(x)\ \frac{\partial f}{\partial x} = 2x$

$f'(a) = 2a$

$$\rightarrow \mathcal{L}^{\langle t \rangle} = \left[\sum_{i=1}^{n} L\left(y_i, \hat{y}_i^{\langle t-1 \rangle} + f_t(x_i)\right)\right] + \Omega(f_t(x_i)) \rightarrow 2^{nd}\ \text{degree}$$

$$x \rightarrow \underbrace{\hat{y}_i^{\langle t-1 \rangle} + f_t(x_i)} \qquad a \rightarrow \hat{y}_i^{\langle t-1 \rangle}$$

$$f(a) = \sum_{i=1}^{n} L(y_i, \hat{y}_i^{\langle t-1 \rangle})$$

gradient g_i

$$f'(a)(x-a) = \sum_{i=1}^{n} \boxed{\frac{\partial L(y_i, \hat{y}_i^{\langle t-1 \rangle})}{\partial \hat{y}_i^{\langle t-1 \rangle}}} f_t(x_i)$$

hessian h_i

$$\frac{f''(a)}{2}(x-a)^2 = \frac{1}{2}\sum_{i=1}^{n} \boxed{\frac{\partial^2 L(y_i, \hat{y}_i^{\langle t-1 \rangle})}{\partial \hat{y}_i^{\langle t-1 \rangle 2}}} f_t^2(x_i)$$

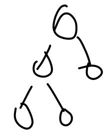
$$\mathcal{L}^{\langle t \rangle} = \sum_{i=1}^{n} L(y_i, \hat{y}_i^{\langle t-1 \rangle}) + g_i f_t(x_i) + \frac{1}{2} h_i f_t^2(x_i) + \Omega(f_t(x_i))$$

Simplification

01 December 2023 01:29

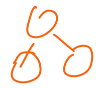
$$\mathcal{L}^{\langle t\rangle} \simeq \sum_{i=1}^{n}\Big[\underbrace{L(y_i,\hat{y}_i^{\langle t-1\rangle})}_{\to 0} + \underbrace{g_i f_t(x_i)} + \underbrace{\tfrac{1}{2}h_i f_t^2(x_i)}\Big] + \underbrace{\Omega(f_t(x_i))}$$

$$\mathcal{L}^{\langle t\rangle} = \sum_{i=1}^{n}\Big[g_i f_t(x_i) + \tfrac{1}{2}h_i f_t^2(x_i)\Big] + \boxed{\Omega(f_t(x_i))} \to \text{expand}$$



T leaf nodes

$$\mathcal{L}^{\langle t\rangle} = \underbrace{\sum_{i=1}^{n}\Big[g_i f_t(x_i) + \tfrac{1}{2}h_i f_t^2(x_i)\Big]}_{i=1\to n \ \#\text{rows}} + \underbrace{\gamma T} + \underbrace{\tfrac{1}{2}\lambda\sum_{j=1}^{T} w_j^2}_{j=1\to T,\ T\to \#\text{leafnodes}}$$



T=2

j=1,2

$w_j \to f_t(x_i)$

$\sum_{i=1}^{n} \to \sum_{j=1}^{T}\sum_{i\in I_j}$

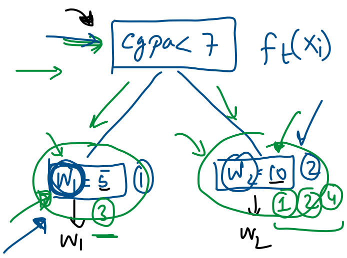
$$\mathcal{L}^{\langle t\rangle} = \sum_{j=1}^{T}\Big[\sum_{i\in I_j} g_i w_j + \tfrac{1}{2}\sum_{i\in I_j} h_i w_j^2\Big] + \gamma T + \tfrac{1}{2}\lambda\sum_{j=1}^{T} w_j^2$$

100

$$\mathcal{L}^{\langle t\rangle} = \sum_{i=1}^{n}\Big[g_i f_t(x_i) + \tfrac{1}{2}h_i f_t^2(x_i)\Big] + \gamma T + \tfrac{1}{2}\lambda\sum_{j=1}^{T} w_j^2$$

100

$$\mathcal{L}^{\langle t\rangle} = \sum_{j=1}^{T}\Big[\sum_{i\in I_j} g_i w_j + \tfrac{1}{2}\sum_{i\in I_j} h_i w_j^2\Big] + \gamma T + \tfrac{1}{2}\lambda\sum_{j=1}^{T} w_j^2$$



cgpa (x_i)	package (y_i)
1 → 7.1	4.5
2 → 8.2	6.9
3 → 6.4	5.1
4 → 9.1	10

$\sum_{i=1}^{n} \to \sum_{j=1}^{T}\sum_{i\in I_j}$

$i\in I_j \to$ instance set of leaf j

2 times

$f_t(x_i) \to w_j$

4 terms

$f_t(x_3)$

decision

$$\to \Big(g_1 f_t(x_1) + \tfrac{1}{2}h_1 f_t^2(x_1)\Big) + \Big(g_2 f_t(x_2) + \tfrac{1}{2}h_2 f_t^2(x_2)\Big) + \dots + g_4 f_t(x_4) + \tfrac{1}{2}h_4 f_t^2(x_4)$$

j=1, j=2

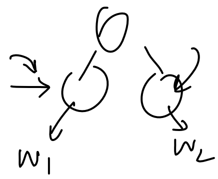
$$\to g_3 f_t(x_3) + \tfrac{1}{2}h_3 f_t^2(x_3) + g_1 f_t(x_1) + \tfrac{1}{2}h_1 f_t^2(x_1) + g_2 f_t(x_2) + \tfrac{1}{2}h_2 f_t^2(x_2) + g_4 f_t(x_4) + \tfrac{1}{2}h_4 f_t^2(x_4)$$

$g_3 w_1 + \tfrac{1}{2}h_3 w_1^2$

$g_1 w_2 + \tfrac{1}{2}h_1 w_2^2$

$$\mathcal{L}^{\langle t\rangle} = \sum_{j=1}^{T}\Big[\sum_{i\in I_j} g_i w_j + \tfrac{1}{2}\sum_{i\in I_j} h_i w_j^2\Big] + \gamma T + \tfrac{1}{2}\lambda\sum_{j=1}^{T} w_j^2$$

$$\mathcal{L}^{\langle t\rangle} = \sum_{j=1}^{T}\Big[\sum_{i\in I_j} g_i w_j + \tfrac{1}{2}\underbrace{\sum_{i\in I_j} h_i w_j^2 + \tfrac{1}{2}\lambda w_j^2}\Big] + \gamma T$$



$$\mathcal{L}^{\langle t\rangle} = \sum_{j=1}^{T}\Big[\sum g_i w_j + \tfrac{1}{2}\Big(\sum_{i\in I_j} h_i + \lambda\Big) w_j^2\Big] + \gamma T$$

$$\mathcal{L}^{\langle t\rangle} = \sum_{j=1}^{T} \left[\underline{\sum_{i\in I_j} g_i w_j} + \frac{1}{2} \underline{\left(\sum_{i\in I_j} h_i + \lambda\right)} w_j^2 \right] + \gamma T$$

w_1 w_2

$\frac{\partial \mathcal{L}}{\partial w_j}$

$$\frac{\partial \mathcal{L}^{\langle t\rangle}}{\partial w_j} = \sum_{i\in I_j} g_i + \left(\sum_{i\in I_j} h_i + \lambda\right) w_j = 0$$

$$\boxed{w_j = -\frac{\sum_{i\in I_j} g_i}{\sum_{i\in I_j} h_i + \lambda}}$$

$g_i / h_i \rightarrow$ formula

the

output

Output Value for Regression

01 December 2023 13:45

$$w_j = \frac{\boxed{-\sum_{i \in I_j} g_i}}{\sum_{i \in I_j} h_i + \lambda}$$

gradient

hessian

$$g = \frac{\partial L(y_i, \hat{y}_i^{<t-1>})}{\partial \hat{y}_i^{<t-1>}} \qquad h = \frac{\partial^2 L(y_i, \hat{y}_i^{<t-1>})}{\partial \hat{y}_i^{<t-1>2}}$$

Loss → reg, classi

Reg → mse → $\boxed{L = \frac{1}{2}\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}$

$$g_i = \frac{\partial L}{\partial \hat{y}_i} = -\sum_{i=1}^{n}(y_i - \hat{y}_i) = \sum_{i=1}^{n}(\hat{y}_i - y_i)$$

prev step

$$= \frac{\sum_{i \in I_j}(y_i - \hat{y}_i^{<t-1>})}{\sum_{i \in I_j} 1 + \lambda} \rightarrow \frac{\text{sum of residual}}{\text{\# of residuals} + \lambda}$$

residual for every point

cgpa	iq	package (y_i)	pred ($\hat{y}_i^{<t-1>}$)	residual
8	80	8	8.5	8-8.5
9	90	9	8.5	9-8.5

$$\frac{\partial}{\partial \hat{y}_i}\frac{\partial L}{\partial \hat{y}_i} = \frac{\partial}{\partial \hat{y}_i}(\hat{y}_i - y_i) = 1$$

$$w_j = \frac{\text{sum of residual}}{\text{\# residuals} + \lambda}$$

Output Value for Classification

01 December 2023 13:45

$$w_j = \frac{-\sum_{i \in I_j} g_i}{\sum_{i \in I_j} h_i + \lambda}$$

$$g = \frac{\partial L(y_i, \hat{y}_i^{<t-1>})}{\partial \hat{y}_i^{<t-1>}}$$

$$h = \frac{\partial^2 L(y_i, \hat{y}_i^{<t-1>})}{\partial \hat{y}_i^{<t-1>2}}$$

loss - log loss → gradient boosting

$$\left[w_j = \frac{\text{sum of residual}}{\sum_{i \in I_j} p_i(1-p_i) + \lambda} \right]$$

p_i → pred prob of prev timestep

Derivation of Similarity score

01 December 2023 13:45

$$\boxed{\mathcal{L}^{\langle t\rangle}} = \sum_{j=1}^{T}\left[\left(\sum_{i\in I_j} g_i\right) w_j + \frac{1}{2}\left(\sum_{i\in I_j} h_i + \lambda\right) w_j^2\right] + \gamma T$$

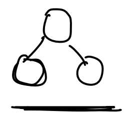
$$w_j = \frac{-\sum_{i\in I_j} g_i}{\sum_{i\in I_j} h_i + \lambda}$$

$$\mathcal{L}^{\langle t\rangle}(q) = \sum_{j=1}^{T}\left[\left(\sum_{i\in I_j} g_i\right)\frac{\left(-\sum_{i\in I_j} g_i\right)}{\sum_{i\in I_j} h_i+\lambda} + \frac{1}{2}\cancel{\left(\sum_{i\in I_j} h_i+\lambda\right)}\frac{\left(\sum_{i\in I_j} g_i\right)^2}{\left(\sum_{i\in I_j} h_i+\lambda\right)^{\cancel{2}}}\right] + \gamma T$$

$$\mathcal{L}^{\langle t\rangle}(q) = \sum_{j=1}^{T}\left[-\frac{\left(\sum_{i\in I_j} g_i\right)^2}{\sum_{i\in I_j} h_i+\lambda} + \frac{1}{2}\frac{\left(\sum_{i\in I_j} g_i\right)^2}{\sum_{i\in I_j} h_i+\lambda}\right] + \gamma T$$

$$\text{Similarity score} = \sum_{j=1}^{T}\left[-\frac{1}{2}\frac{\left(\sum_{i\in I_j} g_i\right)^2}{\sum_{i\in I_j} h_i+\lambda}\right] + \gamma T$$

iter t



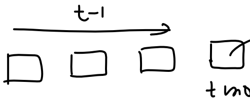
$$\mathcal{L}^{\langle t\rangle}(q) = -\frac{1}{2}\sum_{j=1}^{T}\frac{\left(\sum_{i\in I_j} g_i\right)^2}{\sum_{i\in I_j} h_i+\lambda} + \gamma T$$

2 ← T

gini / entropy

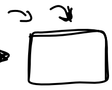
1

t-1

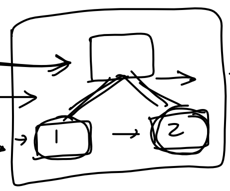


decision tree

t model



leafnode



→ 2 leaf nodes

$$\mathcal{L}^{\langle t\rangle}(q_p) = -\frac{1}{2}\frac{\left(\sum_{i\in I_p} g_i\right)^2}{\sum_{i\in I_p} h_i+\lambda} + \gamma$$

re

$$\boxed{\mathcal{L}^{\langle t\rangle}(q_c)} = -\frac{1}{2}\left[\frac{\left(\sum_{i\in I_L} g_i\right)^2}{\sum_{i\in I_L} h_i+\lambda} + \frac{\left(\sum_{i\in I_R} g_i\right)^2}{\sum_{i\in I_R} h_i+\lambda}\right] + 2\gamma$$

$$\mathcal{L}^{\langle t\rangle}(q_p) \overset{?}{-} \mathcal{L}^{\langle t\rangle}(q_c) \qquad -\gamma$$

$$-\frac{1}{2}\frac{\left(\sum_{i\in I_p} g_i\right)^2}{\ldots} \;-\; \left(-\frac{1}{2}\left[\frac{\left(\sum_{i\in I_L} g_i\right)^2}{\ldots} + \frac{\left(\sum_{i\in I_R} g_i\right)^2}{\ldots}\right] \pm 2\gamma\right)$$

$$-\frac{1}{2}\frac{\left(\sum_{i\in I_P} g_i\right)^2}{\sum_{i\in I_P} h_i+\lambda} + \gamma - \left(-\frac{1}{2}\left[\frac{\left(\sum_{i\in I_L} g_i\right)^2}{\sum_{i\in I_L} h_i+\lambda} + \frac{\left(\sum_{i\in I_R} g_i\right)^2}{\sum_{i\in I_R} h_i+\lambda}\right] \pm 2\gamma\right)$$

$$\left[-\frac{1}{2}\frac{\left(\sum_{i\in I_P} g_i\right)^2}{\sum_{i\in I_P} h_i+\lambda} + \frac{1}{2}\frac{\left(\sum_{i\in I_L} g_i\right)^2}{\sum_{i\in I_L} h_i+\lambda} + \frac{1}{2}\frac{\left(\sum_{i\in I_R} g_i\right)^2}{\sum_{i\in I_R} h_i+\lambda}\right] - \gamma$$

$$\underline{\frac{1}{2}}\left[\frac{\left(\sum_{i\in I_L} g_i\right)^2}{\sum_{i\in I_L} h_i+\lambda} + \frac{\left(\sum_{i\in I_R} g_i\right)^2}{\sum_{i\in I_R} h_i+\lambda} - \frac{\left(\sum_{i\in I_R} g_i\right)^2}{\sum_{i\in I_P} h_i+\lambda}\right] - \underline{\gamma}$$

differ

Final Calculation of Similarity

02 December 2023 01:27

$$\text{Similarity score} = \sum_{j=1}^{T} \left[-\frac{1}{2} \frac{\left(\sum_{i \in I_j} g_i\right)^2}{\sum_{i \in I_j} h_i + \lambda} \right] + \gamma T$$

reg

$g \rightarrow$ sum of residuals

$h \rightarrow \sum 1 \rightarrow N$

Class

$g \rightarrow$ residual

$h \rightarrow p_i(1-p_i)$