

Q1 House robbing:-

Sol

Pseudo code :-

function maxpossible rob (array) :-

arraysize = size of array

maxrob = list [i'th element of it shows max amount he can rob without robbing two houses adjacently].

maxrob[1] = array[1]

maxrob[2] = array[2].

for i in 3 to size of array

maxrob[i] = max(maxrob[i-1], maxrob[i-2] + array[i])

return maxrob[size of array]

Proof of correctness:-

at any index i, he has two possible options. i) does not rob the current house, instead, rob the previous house. ii) Rob the current house and house 2 houses back as well. He will choose the strategy which gives him maximum possible amount. we have implemented the same in our algo and hence algo is correct.

Running time complexity:-

Running time complexity of our algo is $O(n)$.

Q3

Sol.

Pseudo Code:-

$dp = []$

$dp[0] = \text{infinite}$ [if we are at 0, we can never take it to 1]

$dp[1] = 0$ [if we are at 1, we are already there].

$dp[2] = 1$ [if we are at 2, we can take it to 1 in one step, $n/2$]

$dp[3] = 1$ [if we are at 3, we can take it to 1 in one step $n = n/2$].

for i in ~~1~~ 4 to n

$dp[i] = dp[i-1] + 1$

~~if~~

if $(i \% 2 == 0)$

$dp[i] = \min(dp[i], 1 + dp[i/2])$

if $(i \% 3 == 0)$

$dp[i] = \min(dp[i], 1 + dp[i/3])$

print $dp[n]$.

Proof of correctness:-

for any number i , we have three possible ways. 1) first take it $i-1$, then take it to 1

ii) if it is divisible by 2, first take it $I/2$, then ~~then~~ to 1.

iii) if it is divisible by 3, first take to $I/3$ then to 1.

we will choose both which take least no steps.

we have implemented the same in our algo and hence our algo is correct.

time complexity:-

time complexity of our algo is $O(n)$.

Q5

Sol.

take input n (no of shops)

take input cost (costs of all three vegetables in n shops).

for i in 2 to n .

$cost[i][0] += \min(cost[i-1][0], cost[i-1][2])$

$cost[i][1] += \text{minimum of } (cost[i-1][0], cost[i-1][2])$

$cost[i][2] += \text{minimum of } (cost[i-1][0], cost[i-1][1])$.

print (minimum value of $cost[n-1]$).

proof of correctness :-

let, we are planning to buy i 'th vegetable from j 'th shop. Then we will have to buy vegetables from previous shop ($j-1$) that are not i 'th vegetable. so we will buy vegetable which costs us least.

$$\text{i.e. } \underset{\substack{| \\ \text{cost}[i][0]}}{\text{cost}[i][0]} = \min(\underset{\substack{| \\ \text{cost}[j-1][1]}}{\text{cost}[j-1][1]}, \underset{\substack{| \\ \text{cost}[j-1][2]}}{\text{cost}[j-1][2]})$$

we have implemented same in our algo and hence our algo is correct

time complexity :-

time complexity of our algo is $O(n)$.

Q4

Sol. function maxprice(price).

n = size of array price

for i in 2 to $n-1$

for j in 1 to $i-1$

price[i] = max(price[i], price[j] + price[i-j]).

n = take input integer

price = take input array of integers

ans = -1

maxprice(price).

for i in 1 to $n-1$:

ans = maximum of (ans, price[i] + price[n-i]).

print ans.

proof of correctness:-

we have to sell the goldbar in its pieces.
So, we need to determine the maximum value of these pieces. The maximum of these pieces could either be the value given itself or we can increase it by further dividing it into subpieces. we have done the same in our algo and hence our algo is correct.

time complexity:-

time complexity of our algo is $O(n^2)$.

Q8

Sol

pseudo code:-
function computeLPSarray(string) :-

n = size of string

lps = array of size n, initialise all elements by 0.

lps[0] = 0.

i = 1

length = 0.

while i < n :-

if string[i] == string[length] :-

length += 1

lps[i] = length

i += 1.

else

if length != 0

length = lps[length-1]

else

lps[i] = 0 ; i += 1.

return lps.

function solution(string):-

revstr = ~~at~~ reverse of original string.

Concat = string + "\$" + revstr

lps = computeLPS Array (Concat).

return (size of string) - (last element of lps array)

String = take input string.

print (solution (string)).

Proof of correctness:-

Suppose we are given a string. ~~then to make~~ and we have to make it palindrome by appending min number of char(s). if we somehow calculate the max size of suffix substring that is palindrome, then our answer will be (size of string) - (size of maximum length suffix substring that is palindrome). we have done the same in our algo and hence this is correct.

time Complexity:-

time complexity of the algo is $O(n)$.

Q2

Hard problem

Sol

function findMinAmount (arr) :-

n = size of arr

ans = 0.

ansarr = [] (empty array).

incLeft = [].

incLeft[1] = 1.

for i in 2 to n:

if arr[i] > arr[i-1] :

incLeft[i] = incLeft[i-1] + 1.

else :

incLeft[i] = 1.

incRight = arr[1] (array of n element, all equal to 1).

i = n - 1

while (i > 0) :-

if (arr[i] > arr[i+1])

incRight[i] = incRight[i+1] + 1

i = i - 1

ans = 0

for i in 1 to n.

ans += max(incLeft[i], incRight[i]).

return ans

n = take input integer (size of array).

array = take input array of integers

print (findMinAmount(array)).

Proof of correctness:-

for any integer n , ~~the~~ let p be the number it is continuously greater than from left and let q be the number it is continuously greater than from right. So, the minimum no. of awards we have to give to integer n is $\max(p, q) + 1$ for example

(1, 3, 5, 4, 3, 2, 1).

So consider 5, it is greater than continuously 2 elements from its left and continuously 4 elements from its right. So we have to give it at least $\max(2, 4) + 1 = 5$ awards

time complexity:-

time complexity of the problem is $O(n)$.

Q6

Pseudo code

Sol

function

finalMaxAmount(r , years, investment, f_1 , f_2 , initialAmount, scheme, amount) :-

for I in ~~range~~ 1 to investment :-

amount[0].append(initialAmount * $r(I)(0)$).

scheme[0].append(-1).

for J in 2 to years :-

amount[J].append(-1).

scheme[J].append(-1).

for I -prev in ~~range~~ 1 to investment

if (I -prev = 1)

temp = (amount[J-1](1) - f_1) * $r(I)(J)$

if (temp > amount[J](I))

scheme[J](I) = I -prev

amount[J](I) = temp.

else

temp = (amount[J-1](I -prev) - f_2) * $r(I)(J)$

if (temp > amount[J](I))

scheme[J](I) = I -prev

amount[J](I) = temp.

return maximum of (amount (years-1)).

years = take input integer

investment = take input integer

r = take input double dimensional array

$f_1 =$ take input integer
 $f_2 =$ take input integer

Scheme = empty array.

initial_amount = take input integer

amount = [] (empty array).

ans = final_maxAmount(r, years, investment, f_1, f_2 , initial_amount)

final_invest_company = index of max value of amount(years).

SchemeAns = [] (empty array).

Y = years

while (final_invest_company \neq -1)

~~final~~ SchemeAns.append (final_invest_company).

final_invest_company = Scheme(Y) (final_invest_company)

Y -= 1.

print(years, ans)

print(scheme).

Proof of correctness :- for any particular year, for any investment, we can compute its maximum value, ~~by~~ by assuming it previously was investing in its scheme (i from 0 to no of investments) and then taking maximum value of it. In this way, we can compute the maximum value of these investments in last year and maximum of which will give us the

answer. we are doing the same in our algo and hence our algo is correct.

time complexity :-

time complexity of our algo

is $n \times n = O(n^2)$.

where n = no. of investments.

Q7

Sol. Pseudo Code :-

function lcs(string1, string2, i, j, ans) :-

if (string1[i] == string2[j])

ans += string1[i]

~~if (string1~~

if (i == size of string1 or j == size of string2)

return ans

else

return lcs(string1, string2, i+1, j+1, ans)

ans1 = "" [empty string]

ans2 = "" [empty string].

if (i != size of string1)

ans1 = lcs(string1, string2, i+1, j, ans1)

if (j != size of string2)

ans2 = lcs(string1, string2, i, j+1, ans2)

if ((size of ans) < (size of ans1))

ans = ans1

if ((size of ans) < (size of ans2))

ans = ans2

return ans

String 1 = take input string

String 2 = take input string

ans = empty string

ans = lcs(string1, string2, 0, 0, ans).

print(ans).

proof of correctness :-

we can compute longest common sub string recursively. let's assume i 'th char of s_1 and j 'th char of s_2 matches. Then we can add this char to till computed LCS and recursively compute remaining portion of LCS. if i 'th char of s_1 and j 'th char of s_2 does not match, we store till computed LCS somewhere and compute LCS of $(s_1[i+1] \dots)$ and $s_2[j+1 \dots]$ and $s_2[0 \dots j]$ and $(s_1[i] \dots)$ and maximum size of any of these will be our answer we are doing the same in our algo and hence our algo is correct.

time complexity :-

is $O(n * m)$

time complexity of the algo

where
 n = size of string 1
 m = size of string 2