SIG718 Real World Analytics
Assessment Task 3, 2023

Total Marks = 100, Weighting - 30%

Your final submission should consist of:

1. ``name-report.pdf'': A pdf file (created in any word processor) containing the solutions of the questions, appropriate graphs, tables and references. The file should be labelled with your name. This file should consist of up to 8 pages, 9 pages with title page).
2. ``name-code.R'': Two codes combined in one R file, labelled with yourname.R, with codes for the LP models for Question 2 and Question 3.

- Your assignment will not be assessed if we cannot reproduce your results with your R codes, R codes are missing or not working.
- Your assignment will not be assessed if your report is missing from the submission.
- Reference style: Harvard

1. A garment factory produces shirts and pants for Kmart chain. The contract is such that quality control is done before shipping and all products supplied to Kmart satisfying the quality requirements would be accepted by the chain. The factory employs 20 workers in the cutting department, 50 workers in the sewing department, and 14 workers in the packaging department. The garment factory works 8 productive hours a day (no idle time during these 8 hours). There is a daily demand for at most 180 shirts. The demand for pants is unlimited. Each worker can participate only in one activity- the activity to which they are assigned. The table below gives the time requirements (in minutes) and profit per unit for the two garments.

	Amount (minutes) per operation			
	Cutting	Sewing	Packaging	Profit per unit($)
Shirts	40	40	20	10
Pants	20	100	20	8

a) Explain why a Linear Programming (LP) model would be suitable for this case study.

[5 marks]

b) Formulate a LP model to help the factory management to determine the optimal daily production schedule that maximises the profit while satisfying all constraints.

[10 marks]

c) Use the graphical method to find the optimal solution. Show the feasible region and the optimal solution on the graph. Annotate all lines on your graph. What is the optimal daily profit for the factory?

[10 marks]

Note: you can use graphical solvers available online but make sure that your graph is clear, all variables involved are clearly represented and annotated, and each line is clearly marked and related to the corresponding equation.

d) Find the range for the profit ($) per shirt (if any) that can be obtained without affecting the optimal point of part (c).

[5 marks]

2. A factory makes three products called Bloom, Amber, and Leaf, from three materials containing Cotton, Wool and Nylon. The following table provides details on the sales price, production cost and purchase cost per ton of products and materials respectively.

	Sales price	Production cost		Purchase price
Bloom	\$60	\$5	Cotton	\$40
Amber	\$55	\$4	Wool	\$45
Leaf	\$60	\$5	Nylon	\$30

The maximal demand (in tons) for each product, the minimum cotton and wool proportion in each product are as follows:

	Demand	min Cotton proportion	min Wool proportion
Bloom	4200	50%	40%
Amber	3200	60%	40%
Leaf	3500	50%	30%

a) Formulate an LP model for the factory that maximises the profit, while satisfying the demand and the cotton and wool proportion constraints.

[20 Marks]

b) Solve the model using R/R Studio. Find the optimal profit and optimal values of the decision variables.

[20 Marks]

Hints:

1. Let $x_{ij} \geq 0$ be a decision variable that denotes the number of tons of products j for $j \in \{1 = Bloom, 2 = Amber, 3 = Leaf\}$ to be produced from Materials $i \in \{C$=Cotton, W=Wool, N=Nylon$\}$.

2. The proportion of a particular type of Material in a particular type of Product can be calculated as:
e.g., the proportion of Cotton in product Bloom is given by: $\dfrac{x_{C1}}{x_{C1} + x_{W1} + x_{N1}}$.

3. Two construction companies, Giant and Sky, bid for the right to build in a field. The possible bids are $ 10 Million, $ 20 Million, $ 30 Million, $ 35 Million and $ 40 Million. The winner is the company with the higher bid.

   The two companies decide that in the case of a tie (equal bids), Giant is the winner and will get the field.

   Giant has ordered a survey and, based on the report from the survey, concludes that getting the field for more than $ 35 Million is as bad as not getting it (assume loss), except in case of a tie (assume win). Sky is not aware of this survey.

   (a) State reasons why/how this game can be described as a two-players-zero-sum game

   [5 Marks]

   (b) Considering all possible combinations of bids, formulate the payoff matrix for the game.

   [5 Marks]

   (c) Explain what is a saddle point. Verify: does the game have a saddle point?

   [5 Marks]

   (d) Construct a linear programming model for Company Sky in this game.

   [5 Marks]

   (e) Produce an appropriate code to solve the linear programming model in part (d).

   [5 Marks]

   (f) Solve the game for Sky using the linear programming model and the code you constructed in parts (d) and (e). Interpret your solution.

   [5 Marks]