Deakin University

SIG718 – Real World Analytics

Task End Term Assessment

Submitted by

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Attempt # 1
Date 24th December 2023

Question 1

A garment factory produces shirts and pants for Kmart chain. The contract is such that quality control is done before shipping and all products supplied to Kmart satisfying the quality requirements would be accepted by the chain. The factory employs 20 workers in the cutting department, 50 workers in the sewing department, and 14 workers in the packaging department. The garment factory works 8 productive hours a day (no idle time during these 8 hours). There is a daily demand for at most 180 shirts. The demand for pants is unlimited. Each worker can participate only in one activity- the activity to which they are assigned. The table below gives the time requirements (in minutes) and profit per unit for the two garments.

	Process time in mins per Operation			
Garment/Operation	**Cutting**	**Sewing**	**Packaging**	**Profit per unit ($)**
SHIRTS	40	40	20	10
PANTS	20	100	20	8

a) Explain why a Linear Programming (LP) model would be suitable for this case study.

The case given above, can be formulated into an objective function and it's respective constraints. The management of the factory will be looking to maximize the profits within the production constraints that is highlighted in terms of time, capacity and demand for the two products (shirts, pants). Therefore, to determine the optimal solution of daily production and maximizing profits, the LP model is best suited for this case study. In simple words, there are many variables and constraints **governing** the variables. This can be fixed in a linear programming model with the goal to find the optimal solution.

b) Formulate a LP model to help the factory management to determine the optimal daily production schedule that maximises the profit while satisfying all constraints.

# Workers in Cutting Department	20
# Workers in Sewing Department	50
# Workers in Packaging Department	14

Each employee works 8 hours per day.

Therefore,

Total time for Cutting in Mins/per day	20 (workers) * 8 (hours) * 60 (mins)	9600 mins
Total time for Sewing in Mins/ per day	50 (workers) * 8 (hours) * 60 (mins)	24000 mins
Total time for Packaging in Mins/ per day	14 (workers) * 8 (hours) * 60 (mins)	6720 mins

Let the **number of shirts** produced be : **s**

Let the **number of pants** produced be : **p**

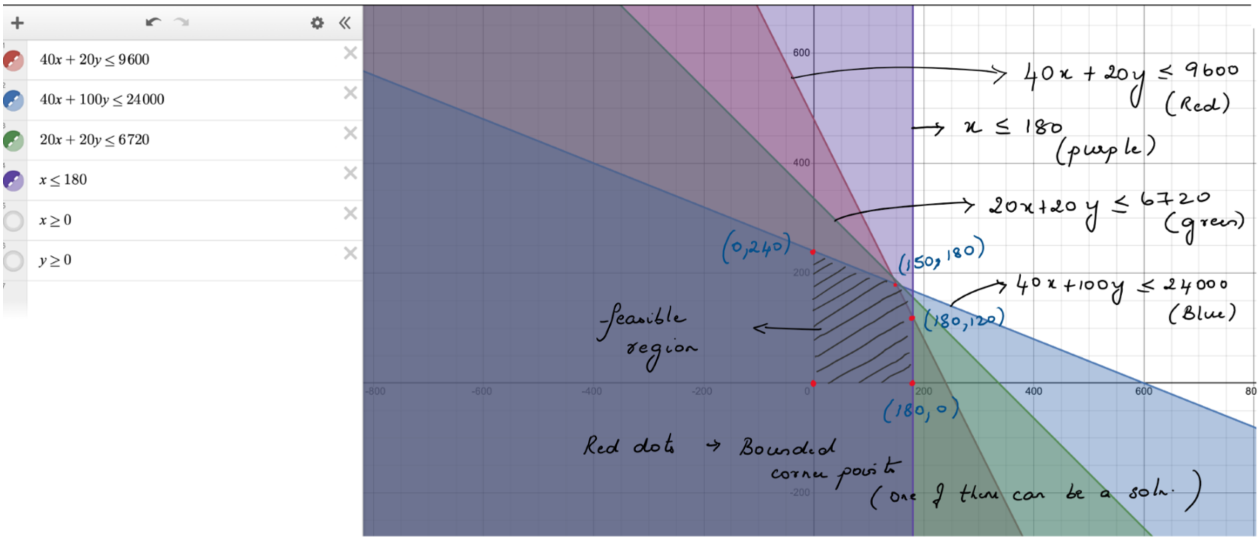
Objective Function : **Max(z) = 10s + 8p**

Subject to the following Constraints

1) **Cutting Department** Constraint : **40s + 20p <= 9600**
2) **Sewing Department** Constraints: **40s + 100p <= 24000**
3) **Packaging Department Constrain**: **20s + 20p <= 6720**
4) **Shirt Demand** Constraint: **s <= 180**
5) **Non-Negativity** Constraint: **s>= 0 ; p >= 0**

c) Use the graphical method to find the optimal solution. Show the feasible region and the optimal solution on the graph. Annotate all lines on your graph. What is the optimal daily profit for the factory?

Note: you can use graphical solvers available online but make sure that your graph is clear, all variables involved are clearly represented and annotated, and each line is clearly marked and related to the corresponding equation.



Let us take the objective function now

10s + 8p → Maximize

Substituting, the boundary points of the feasible region from the graph.

(180,0)	10(180) + 0	1800
(180,120)	10(180) + 8(120)	2760
(150,180)	10(150) + 8(180)	**2940**
(0,240)	0 + 8(240)	1920

Maximization of the profit happens at (150, 180) where the profit generated daily is 2940.

d) Find the range for the profit ($) per shirt (if any) that can be obtained without affecting the optimal point of part (c).

For finding the profit per shirt we would need to plug in values for "s" using the trial and error method while keeping "p" at the optimal value. All the constraints also needs to be satisfied in this process.

Let Profit per shirt = **r**

Let us take the equation profit per r(150) + 8(180) = 2940

The profit per shirt is within the range of 10 – 16 dollars.

****

Question 2

A factory makes three products called Bloom, Amber, and Leaf, from three materials. containing Cotton, Wool and Nylon. The following table provides details on the sales. price, production cost and purchase cost per ton of products and materials respectively.

(all are in dollars)

	Sales Price	Production Cost		Purchase Price
Bloom	60	5	**Cotton**	40
Amber	55	4	**Wool**	45
Leaf	60	5	**Nylon**	30

The maximal demand (in tons) for each product, the minimum cotton and wool proportion in each product are as follows:

	Demand	Min Cotton Proportion	Min Wool Proportion
Bloom	4200	50%	40%
Amber	3200	60%	40%
Leaf	3500	50%	30%

Hints:
1. Let $x_{ij} \geq 0$ be a decision variable that denotes the number of tons of products j for $j \in \{1 = \text{Bloom}, 2 = \text{Amber}, 3 = \text{Leaf}\}$ to be produced from Materials $i \in \{C=\text{Cotton}, W=\text{Wool}, N=\text{Nylon}\}$.
2. The proportion of a particular type of Material in a particular type of Product can be calculated as:
e.g., the proportion of Cotton in product Bloom is given by:

$$\frac{x_{C1}}{x_{C1} + x_{W1} + x_{N1}}$$

a) Formulate an LP model for the factory that maximises the profit, while satisfying the demand and the cotton and wool proportion constraints.

Let use represent the following

Product Produced	Alias
Bloom	1
Amber	2
Leaf	3

Material Used	Alias
Cotton	C
Wool	W
Nylon	N

Each of the 3 products (Bloom, Amber, Leaf) contains a mix of the 3 materials (Cotton, Wool, Nylon). Therefore the equations of the material mix in each of the products are

Bloom [1] → XC1 + XW1 + XN1
Amber [2] → XC2 + XW2 +XN2
Leaf [3] → XC3 + XW3 + XN3

Let us now formulate the Total Material used in all the 3 products.

Cotton [C] → XC1 + XC2 + XC3
Wool [W] → XW1 + XW2 + XW3
Nylon [N] → XN1 + XN2 + XN3

Product/Material	C	W	N	Product-Material Mix
1	XC1	XW1	XN1	XC1 + XW1 + XN1
2	XC2	XW2	XN2	XC2 + XW2 + XN2
3	XC3	XW3	XN3	XC3 + XW3 + XN3
Total Material Used	XC1+ XC2 + XC3	XW1 + XW2 +XW3	XN1 + XN2 + XN3	

To formulate the Objective Function let us consider the below notations,

<u>Objective Function</u>

Maximize Profit [z] → Sales – [Production Cost + Material Purchase Cost]

<u>Sales:</u>

Total Sales → 60[Bloom] + 55[Amber] + 60[Leaf]

→ 60 [XC1 + XW1 + XN1] + 55 [XC2 + XW2 +XN2] + 60 [XC3 + XW3 + XN3]

<u>Production Cost of Products:</u>

Product Cost → 5[Bloom] + 4 [Amber] + 5[Leaf]

→ 5 [XC1 + XW1 + XN1] + 4 [XC2 + XW2 +XN2] + 5 [XC3 + XW3 + XN3]

<u>Purchase Cost of Materials:</u>

Purchase Cost → 40 [Cotton] + 45 [Wool] + 30 [Nylon]

→ 40 [XC1 + XC2 + XC3] + 45 [XW1 + XW2 + XW3] + 30 [XN1 + XN2 + XN3]

Therefore, from the above 3 formulated equations the final objective function calculation is:

XC1[60-5-40] + XW1[60-5-45] + XN1[60-5-30] + XC2[55-4-40] + XW2[55-4-45] + XN2[55-4-30] + XC3[60-5-40] + XW3[60-5-45] + XN3[60-5-30]

Max[Z] → 15XC1 + 10XW1 + 25XN1 + 11XC2 + 6XW2 + 21XN2 + 15XC3 + 10XW3 + 25XN3

<u>Constraints:</u>

The above objective function is subjected to the following constraints

1) **Demand Constraints**

   The maximum demand for each of the 3 products (measured in tons)

Product	Constraint
Bloom <= 4200	XC1 + XW1 + XN1 <= 4200
Amber <= 3200	XC2 + XW2 + XN2 <= 3200
Leaf <= 3500	XC3 + XW3 + XN3 <= 3500

2) **Cotton Material Proportion Constraint**

   The minimum cotton proportion constraint

Minimum Cotton Proportion in each of the Products	Constraint Construct	Constraint
50% of Bloom	XC1/(XC1+XW1+ XN1) >=0.5	0.5XC1 – 0.5XW1 – 0.5XN1 >= 0
60% of Amber	XC2/(XC2+XW2+ XN2) >=0.6	0.4XC2 – 0.6XW2 – 0.6XN2 >= 0
50% of Leaf	XC3/(XC3+XW3+ XN3) >=0.5	0.5XC3 – 0.5XW3 – 0.5XN3 >= 0

3) **Wool Material Proportion Constraint**

   The minimum cotton proportion constraint

Minimum Wool Proportion in each of the Products	Constraint Construct	Constraint
40% of Bloom	XW1/(XC1+XW1+ XN1) >=0.4	-0.4XC1 + 0.6XW1 – 0.4XN1 >= 0
40% of Amber	XW2/(XC2+XW2+ XN2) >=0.4	-0.4XC2 + 0.6XW2 – 0.4XN2 >= 0
30% of Leaf	XW3/(XC3+XW3+ XN3) >=0.3	-0.3XC3 + 0.7XW3 – 0.3XN3 >= 0

4) Non-Negativity Constraints

The below variables under consideration should be >= 0

XC1	XW1	XN1
XC2	XW2	XN2
XC3	XW3	XN3

R- Code to Solve the LP Model

```
##################################################################
# title: "Suraj-Code.R"
# output: R Script
# Student Name: Suraj Mathew Thomas
# Student ID: S223509398
# Subject: SIG 718 - Real World Analytics
# Assessment: End Term Assessment
##################################################################

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## Question 2 b) Solve the model using R/R Studio. Find the optimal profit and optimal values of the decision variables.

##################################################################

library(lpSolveAPI) #Calling the lpSolveAPI library

# Initialising the model with 9  constraints and 9 decision variables
FactoryModel = make.lp(9, 9)


# Specifying that we want to maximize our objective function
lp.control(FactoryModel, sense = "max")


# Setting up our objective
set.objfn(FactoryModel, c(15,10,25,11,6,21,15,10,25))
```

```
# Constraints

# Demand side Constraints
set.row(FactoryModel, 1, c(1,1,1), indices = c(1,2,3)) #Bloom
set.row(FactoryModel, 2, c(1,1,1), indices = c(4,5,6)) #Amber
set.row(FactoryModel, 3, c(1,1,1), indices = c(7,8,9)) #Leaf

# Proportion Constraint of Cotton
set.row(FactoryModel, 4, c(0.5,-0.5,-0.5), indices = c(1,2,3)) #Bloom
set.row(FactoryModel, 5, c(0.4,-0.6,-0.6), indices = c(4,5,6)) #Amber
set.row(FactoryModel, 6, c(0.5,-0.5,-0.5), indices = c(7,8,9)) #Leaf

# Proportion Constraint of Wool
set.row(FactoryModel, 7, c(-0.4,0.6,-0.4), indices = c(1,2,3)) #Bloom
set.row(FactoryModel, 8, c(-0.4,0.6,-0.4), indices = c(4,5,6)) #Amber
set.row(FactoryModel, 9, c(-0.3,0.7,-0.3), indices = c(7,8,9)) #Leaf


set.rhs(FactoryModel, c(4200,3200,3500,0,0,0,0,0,0)) #Setting the RHS of the equation in the order of the constraints given
set.type(FactoryModel, c(1:9),"real") #making sure we are dealing with real numbers
set.bounds(FactoryModel, lower = rep(0,9), upper = rep(Inf,9)) #setting the model bounds

set.constr.type(FactoryModel, c("<=","<=","<=",">=",">=",">=",">=",">=",">=")) #Setting the equation constructors

solve(FactoryModel)

get.objective(FactoryModel) #141850

get.variables(FactoryModel) #2100, 1680,  420, 1920, 1280, 0,  1750, 1050,  700
```

Output:

```
Console   Background Jobs
R 4.3.1 · ~/
>
> # Proportion Constraint of Cotton
> set.row(FactoryModel, 4, c(0.5,-0.5,-0.5), indices = c(1,2,3)) #Bloom
> set.row(FactoryModel, 5, c(0.4,-0.6,-0.6), indices = c(4,5,6)) #Amber
> set.row(FactoryModel, 6, c(0.5,-0.5,-0.5), indices = c(7,8,9)) #Leaf
>
> # Proportion Constraint of Wool
> set.row(FactoryModel, 7, c(-0.4,0.6,-0.4), indices = c(1,2,3)) #Bloom
> set.row(FactoryModel, 8, c(-0.4,0.6,-0.4), indices = c(4,5,6)) #Amber
> set.row(FactoryModel, 9, c(-0.3,0.7,-0.3), indices = c(7,8,9)) #Leaf
> set.rhs(FactoryModel, c(4200,3200,3500,0,0,0,0,0,0))
> set.type(FactoryModel, c(1:9),"real")
> set.bounds(FactoryModel, lower = rep(0,9), upper = rep(Inf,9))
> set.constr.type(FactoryModel, c("<=","<=","<=",">=",">=",">=",">=",">=",">="))
> solve(FactoryModel)
[1] 0
> get.objective(FactoryModel)
[1] 141850
> get.variables(FactoryModel)
[1] 2100 1680  420 1920 1280    0 1750 1050  700
>
```

Interpretation:

- The optimal profit that can be realised subject to satisfying all the constraints is **$141,850**
- The optimal values of Cotton, Wool and Nylon to be used are given below

Product/Material	C	W	N
1	XC1 = 2100	XW1 = 1680	XN1 = 420
2	XC2 = 1920	XW2 = 1280	XN2 = 0
3	XC3 = 1750	XW3 = 1050	XN3 = 700

****

Question 3

Two construction companies, Giant and Sky, bid for the right to build in a field. The possible bids are $ 10 Million, $ 20 Million, $ 30 Million, $ 35 Million and $ 40 Million.

The winner is the company with the higher bid.
The two companies decide that in the case of a tie (equal bids), Giant is the winner and will get the field.

Giant has ordered a survey and, based on the report from the survey, concludes that getting the field for more than $ 35 Million is as bad as not getting it (assume loss), except in case of a tie (assume win). Sky is not aware of this survey.

(a) State reasons why/how this game can be described as a two-players-zero-sum game

1) There are only two players/companies involved (GIANT & SKY)
2) There are 5 Strategies that each company uses wherein there can be ONLY ONE WINNER.
3) in this case the gains of construction company Giant (Player 1) is equal to the losses of construction company Sky (Player 2). This is true because this a bidding war where only one contractor gets the deal to build in a field. The total payoff will be zero.
4) There are also constraints given as to when a player will be considered a winner.

Therefore, considering the above 4 reasons, we can clearly state that this is a Two-Player, Zero-Sum game.

(b) Considering all possible combinations of bids, formulate the payoff matrix for the game.

Let us consider Winning a Game = 1
Let us consider Losing a Game = -1

<u>Conditions</u>

1) Winner is the player with the highest bid.
2) If there is an equal bid from both players (tie), then GIANT [Player 1] wins.
3) If the bid is > $35, and GIANT wins, it is not profitable for them, except when where is a tie.

<u>Pay-Off Matrix [From GIANTS perspective]</u>

		SKY [PLAYER 2]					
GIANT [PLAYER 1]	**Bids in Millions**	**$10**	**$20**	**$30**	**$35**	**$40**	Row Min (MaxMin)
	$10	1	-1	-1	-1	-1	-1
	$20	1	1	-1	-1	-1	-1
	$30	1	1	1	-1	-1	-1
	$35	1	1	1	1	-1	-1
	$40	-1	-1	-1	-1	1	-1
Column Max (Min Max)		1	1	1	1	1	L = -1 U = 1

Here we given that since GIANT has done a market research where anything above 35 is not going to be profitable for them, we are formulating the Payoff matrix likewise where even though GIANT has bid 40 and SKY has bid less, we are conceding a defeat or No-Deal for GIANT except when it is a tie bid where GIANT takes the Deal.

Since L < U , a pure strategy will not result in an equilibrium. The players must use mixed strategies. The game value will be between -1 and 1.

	SKY Wins and it is also profitable for them
	SKY Wins but it might not be profitable
	GIANT Wins and it is also profitable for them [SKY Losses]
	SKY Wins as GIANT pulls out because, the research they did says the deal is not going to be profitable

(c) Explain what a saddle point is. Verify: does the game have a saddle point?

A saddle point, in game theory, is a point in the strategy matrix / playoff matrix where the smallest value in a row is also the largest value in the corresponding column. It is at this point the player would want to hold on to the strategy as any change would result in an unfavourable outcome. There is no incentive to change the strategy. It is the best response for one player regardless of the other player's strategy. Therefore, neither player can improve their payoffs by changing their strategy. In this case, **the matrix does not have a saddle point L < U. The game value will be between L and U.**

The game does not have a saddle point. The lower bound is -1 and the upper bound is +1. Therefore, since they are not identical, mixed strategies needs to be applied.

(d) Construct a linear programming model for Company Sky in this game.

Let us consider the following

X1 → Let it be the probability of SKY placing a bid of $10 Million [Strategy 1]
X2 → Let it be the probability of SKY placing a bid of $20 Million [Strategy 2]
X3 → Let it be the probability of SKY placing a bid of $30 Million [Strategy 3]
X4 → Let it be the probability of SKY placing a bid of $35 Million [Strategy 4]
X5 → Let it be the probability of SKY placing a bid of $40 Million [Strategy 5]

Let the game value for SKY be → V

Since we are building a model for SKY, the game value is something that SKY would try to maximize and GIANT would try to minimize.

Each player has 5 strategies

Pay-Off Matrix [From SKY's perspective]

	GIANT [PLAYER 2]						
SKY [PLAYER 1]	**Bids in Millions**	**$10**	**$20**	**$30**	**$35**	**$40**	Row Min (MaxMin)
	$10	-1	-1	-1	-1	1	-1
	$20	1	-1	-1	-1	1	-1
	$30	1	1	-1	-1	1	-1
	$35	1	1	1	-1	1	-1
	$40	1	1	1	1	-1	-1
Column Max (Min Max)		1	1	1	1	1	L = -1 U = 1

Since the market research that GIANT did, states that if they invest more than $35 it is as good as losing the deal because it might not turn profitable, except when the bids are equal (in the case of ($40 vs $ 40) where GIANT wins. Therefore considering this knowledge GIANT will never go for the $40 bid or even if they do they will concede defeat and pull out.

Expected Payoffs

IF GIANT CHOOSES	EXPECTED PAYOFF FOR SKY
Strategy 1	-X1 + X2 + X3 + X4 + X5
Strategy 2	-X1 – X2 + X3 + X4 + X5
Strategy 3	-X1 - X2 – X3 + X4 + X5
Strategy 4	-X1 - X2 – X3 - X4 + X5
Strategy 5	X1 + X2 + X3 + X4 - X5

Objective Function

Player 1 in this case SKY is looking to Maximize his Payoffs. Therefore,

Max (z) = V , where V is the game value for SKY

Constraints

X1 + X2 + X3 + X4 + X5 = 1 [The sum of probabilities should be equal to 1]

V – (-X1 + X2 + X3 + X4 + X5) <= 0
V – (-X1 – X2 + X3 + X4 + X5) <= 0
V – (-X1 - X2 – X3 + X4 + X5) <= 0
V – (-X1 - X2 – X3 - X4 + X5) <= 0
V – (X1 + X2 + X3 + X4 - X5) <= 0

Xi >= 0 (needs to be positive proportions), where i = 1,2,3,4,5

(e) Produce an appropriate code to solve the linear programming model in part (d).

```
library(lpSolveAPI)

SkyModel <- make.lp(0,6) #For now declaring only the objective function variables (5 inputs and 1 output = 6)

lp.control(SkyModel, sense ="max")

#Setting the objective function

set.objfn(SkyModel, c(0,0,0,0,0,1))

#Setting the Constraints

add.constraint(SkyModel, c(1,-1,-1,-1,-1,1),"<=", 0) # GIANT Chooses Strategy 1 this constraint is the expected payoff for SKY

add.constraint(SkyModel, c(1,1,-1,-1,-1,1),"<=", 0) # GIANT Chooses Strategy 2 this constraint is the expected payoff for SKY

add.constraint(SkyModel, c(1,1,1,-1,-1,1),"<=", 0) # GIANT Chooses Strategy 3 this constraint is the expected payoff for SKY

add.constraint(SkyModel, c(1,1,1,1,-1,1),"<=", 0) #GIANT Chooses Strategy 4 this constraint is the expected payoff for SKY

add.constraint(SkyModel, c(-1,-1,-1,-1,1,1),"<=", 0) #GIANT Chooses Strategy 5 this constraint is the expected payoff for SKY

add.constraint(SkyModel, c(1,1,1,1,1,0), "=", 1) #Sum of probabilites constraints, always equal to 1
```

(f) Solve the game for Sky using the linear programming model and the code you constructed in parts (d) and (e). Interpret your solution.

```
# Setting the Boundaries

set.bounds(SkyModel, lower = c(0,0,0,0,0,-Inf))

# Defining Labels for the LP model
# Row Names

RowNames <- c("Row1","Row2", "Row3", "Row4", "Row5", "Row6")

#Column Names

ColNames <- c("X1","X2","X3","X4","X5","V")

#Adding the Row and Columns Names to the model

dimnames(SkyModel) <- list(RowNames,ColNames)

SkyModel

solve(SkyModel) #The game has arrived at an optimal solution

get.objective(SkyModel) #0

get.variables(SkyModel) # 0.5 0.0 0.0 0.0 0.5 0.0
```

```
> SkyModel
Model name:
            X1    X2    X3    X4    X5     V
Maximize     0     0     0     0     0     1
Row1         1    -1    -1    -1    -1     1  <=  0
Row2         1     1    -1    -1    -1     1  <=  0
Row3         1     1     1    -1    -1     1  <=  0
Row4         1     1     1     1    -1     1  <=  0
Row5        -1    -1    -1    -1     1     1  <=  0
Row6         1     1     1     1     1     0   =  1
Kind       Std   Std   Std   Std   Std   Std
Type      Real  Real  Real  Real  Real  Real
Upper      Inf   Inf   Inf   Inf   Inf   Inf
Lower        0     0     0     0     0  -Inf
> solve(SkyModel)
[1] 0
> get.objective(SkyModel)
[1] 0
> get.objective(SkyModel)
[1] 0
> get.variables(SkyModel)
[1] 0.5 0.0 0.0 0.0 0.5 0.0
```

The game value that is output through this model is 0 (objective). This is between the defined Lower and Upper values as defined in the payoff matrix (-1,1)

The probability of choosing Strategy 1 for SKY ($10M) → 50%
The probability of choosing Strategy 2 for SKY ($20M) → 0%
The probability of choosing Strategy 3 for SKY ($30M) → 0%
The probability of choosing Strategy 4 for SKY ($35M) → 0%
The probability of choosing Strategy 5 for SKY ($40M) → 50%

Therefore SKY's strategies should be $10M or $40M

The interpretation is that when we solve the problem for SKY using the LP Model in R studio, we get the objective value as 0. This means that the bidding is a tie game. Therefore, in the event of a tie, the constraints state that the winner of the game/bid is GIANT. Therefore, the conclusion is that SKY has lost the bidding war. Therefore we can state that we are 50% confident that GIANT will be the winner in case of a tie when they bid for values $10 Million and $40 Million. SKY losses the bidding war with this outcome.

References

Videos

1) CTC - circle (21 March 2018) 'Game theory [Operations research] – Part 2 – Saddle point- 10 solved examples', YouTube, accessed 22nd December 2023, [Link]
2) William Spaniel (17 January 2012) 'Game Theory 101: What is Nash Equilibrium ? (Spotlight Game)', YouTube, accessed 22nd December 2023, [Link]

Course Material

1) Deakin Masterclass Live (20 December 2023) 'Week 6 Saddle Points and Pure Strategies, Real World Analytics Content SIG718, Deakin University
2) Deakin Masterclass Live (20 December 2023) 'Week 6 Mixed Strategies, Real World Analytics Content SIG718, Deakin University
3) Deakin Masterclass Live (21 December 2023) 'Week 6 Solving 2 – Person Zero – Sum Games using LP, Real World Analytics Content SIG718, Deakin University