Welcome!

Introduction to Mathematical Thinking

September 5th, 2018

Suraj Rampure

Why does this course exist?

IMT has been in progress since October 2017!

1. CS 70 is hard! We're here to help. By exposing you to difficult concepts in advance, you'll have time to think about them without the stress of declaring.
2. Math, especially discrete math is fun! We want you to feel the same way.
    - We'll look at cool applications, e.g. message encryption
    - In an ideal world, non-CS/math students could take this course just for fun

Instructor: Suraj Rampure

TAs: Edward Im, Ziv Lotzky, Josh Geng

  

Faculty Advisor

Logistics

Lecture: Wednesdays, 6:30-8:00PM, LeConte 3

Discussion: Mondays, 6:30-8:00PM, LeConte 3

Attendance is **mandatory** to both! Let us know in advance if you can't make it.

Tu 2-3, Th 5-6

- Will also have office hours, probably on Tuesdays and Thursdays (TBA)
- We will use Piazza, you will receive invites today
- Course website is at http://imt-decal.org, where all content is posted (including videos, textbook readings)
- Textbook is at http://book.imt-decal.org
- imt-decal@berkeley.edu is your main point of contact

Grading

sad!

Course is out of 100 points, need 70 to pass.

- **10 homeworks**, graded on effort (5 points each, total of 50)
- **Midterm**, ~~Wednesday, October 17th~~ (20 points)
- **Final**, Wednesday, November 28th (30 points)

Homeworks will be released after lecture on Wednesday, and due on Monday at 6:30PM, at the start of the following week's discussion. In that discussion, we'll go over the problem set that was just due. This way, everyone has had a chance to attempt the problems.

Homeworks will be submitted on Gradescope (you'll receive an invitation tonight). **Homework 1 comes out tonight!**

Syllabus

Topic	Weeks
Set Theory, Functions and Bijections, Number Sets, Propositional Logic	2
Proof Techniques	2
Number Theory	2
Midterm	
Counting (Combinatorics)	2
Polynomials (properties, combinatorics w/ polynomials)	2
Final	

Attendance

Fill out the form at **http://tinyurl.com/imtday1**. You will be dropped from the class if you don't fill this out!

Attendance is especially important this week, as we'll be emailing out enrollment codes tonight. **The deadline to add units without a fee is this Friday, so make sure to enroll on CalCentral ASAP!**

Lecture 1: Set Theory, Functions

http://book.imt-decal.org, Ch. 1.1, 1.2

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1.1: Sets and Set Operations

Definition: Set

In mathematics, a **set** is a well-defined collection of objects.
By well-defined, we mean that for any object x, we can easily determine whether or not x is an element (also referred to as a "member") of the set.

S : set

x : element

$x \in S$

"belongs to"

$x \notin S$

"does not belong to"

We define sets in one of two ways:

- By listing out all of the elements in the set, e.g.
  $A_1 = \{1, 3.14, -15, \pi + e\}$,
  $A_2 = \{..., -3, -2, -1, 0, 1, 2, ...\}$ or
  $A_3 = \{\text{Lebron, Jordan, Kobe}\}$
- By stating a property that determines the set, e.g.
  $A_4 = \{x : x + 3 \text{ is prime}\}$ or $A_5 = \{t : t^2 - 5t + 6 = 0\}$

such that

set-builder notation

$S = \{ t^2 : 0 \leq t \leq 10, t \in \mathbb{Z} \}$

[t**2 for t in range(11)]

Definition: Cardinality

The **cardinality** $|S|$ of a finite set $|S|$ is the number of elements in S.

$S = \{1, 2, 7, 10, 15\}$

$|S| = 5$

$A \supset B$

> **Definition: Subset**
>
> Given some set A, another set B is a **subset** of A if and only if every element in B is also an element of A. Equivalently, we can say A is a **superset** of B.

$B \subseteq A$ "b is a subset of A"

$B \subset A$ "b is a proper subset of A"

→ proper: not equal

$A \supseteq B$

$A \supset B$

$x \geq y$

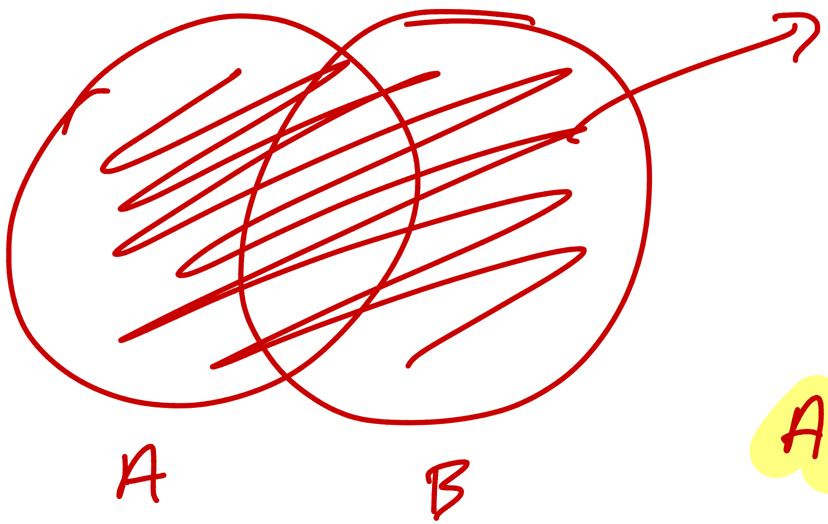
$x > y$

↑ not! greater than / less than signs

Set Operations

Definition: Union

The **union** of two sets A, B is the set of everything contained by either A or B.



$A \cup B$

$A = \{1, 2, 3, 4\}$

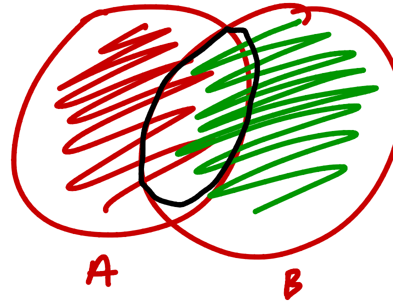
$B = \{3, 4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$A \cup B = \{ x : x \in A \text{ or } x \in B \}$

$\vee$

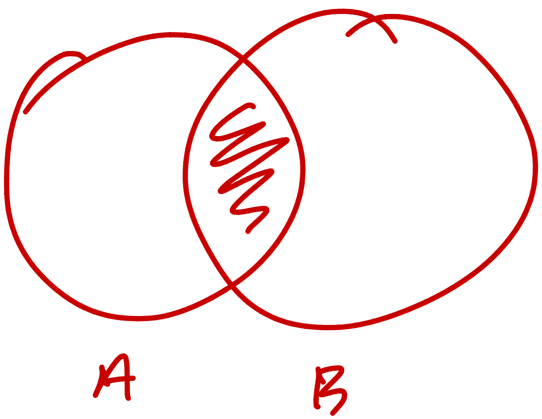
such that



$$|A \cup B| = |A| + |B| - |A \cap B|$$

Principle of Inclusion-Exclusion

> **Definition: Intersection**
>
> The **intersection** of two sets A, B is the set of everything contained in both A and B.



conjunction (means "and")

$$A \cap B = \{x \mid x \in A \wedge x \in B\}$$

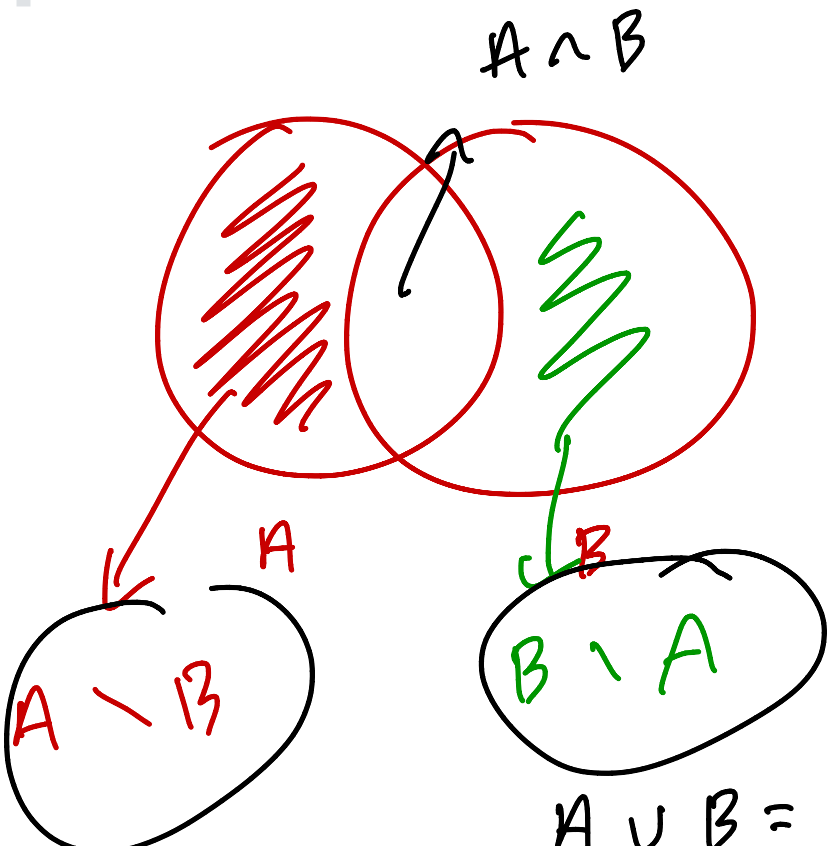
such that

Definition: Difference

The **difference** of two sets A, B is the set of everything contained in A but not contained in B.

$A - B$

$A \setminus B$



$A - B = \{x : x \in A$ and $x \notin B\}$

$A \cup B = (A - B) \cup (B - A) \cup (A \cap B)$

> **Definition: Cartesian Product**
>
> The **Cartesian product** of two sets A, B is the set of all possible combinations of one element in A and one element in B.

$A = \{1, 2, 3\}$

$B = \{x, y, z\}$

$A \times B \neq B \times A$

$A \times B = \{(1, x), (1, y), (1, z), (2, x) \ldots (3, z)\}$

$B \times A = \{(x, 1), (x, 2), (x, 3), \ldots (z, 3)\}$

$= \{(z, 3), (y, 1), (x, 2), \ldots\}$

Universes and Complements

$A \subset U$

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{2, 4, 5, 9\} \to A^c = \{1, 3, 6, 7, 8, 10\}$$

$A^c = \{x : x \notin A\}$

$= U - A$

$U^c = \{x : x \notin U\} = \emptyset$

Definition: De Morgan's Laws

$$\underline{(A \cup B)}^c = \underline{A^c \cap B^c}$$

$$(A \cap B)^c = A^c \cup B^c$$

$U = \{1, 2, \ldots 10\}$

$A = \{1, 2, 5\}$

$B = \{2, 5, 6, 7\}$

1.2: Functions

Definition: Relation, Domain, Codomain

not the only relation!

A relation with *domain* $\underline{A}$ and *codomain* $\underline{B}$ is a subset of $A \times B$.

$A = \{1, 2, 3\}$

$B = \{4, 5, 6\}$

$r: \{\underbrace{(1,4), (2,5), (2,6)}\}$

example of a relation

→ not a function:

→ input 2 has multiple outputs

→ no output for input 3

Definition: Function

We say f is a **function** with *domain* A and *codomain* B if $f \subset A \times B$ such that for every element in A, there is exactly one element in B. Symbolically, we represent this as $f : A \to B$.

$f: \{(1, \_), (2, \_), (3, \_)\}$

functions $\subset$ relations

e.g. $A = \{"\text{Hi}", "\text{Hey}"\}$, $B = \{5, 10\}$
$A \times B = \{("\text{Hi}", 5), ("\text{Hi}", 10), ("\text{Hey}", 5), ("\text{Hey}", 10)\}$

Are the following functions?

$f = \{("\text{Hi}", 10), ("\text{Hey}", 5)\}$ ✓

$g = \{("\text{Hi}", 5), ("\text{Hey}", 5)\}$ ✓

$h = \{("\text{Hi}", 10), ("\text{Hi}", 5)\}$

→ every input has exactly one output!

domain = {'hi', 'hey'}

CD = {5, 10}

$f = \{(\text{"Hi"}, 10), (\text{"Hey"}, 5)\}$ → CD = range

$g = \{(\text{"Hi"}, 5), (\text{"Hey"}, 5)\}$ → CD ≠ range

{5, 10} ≠ {5}

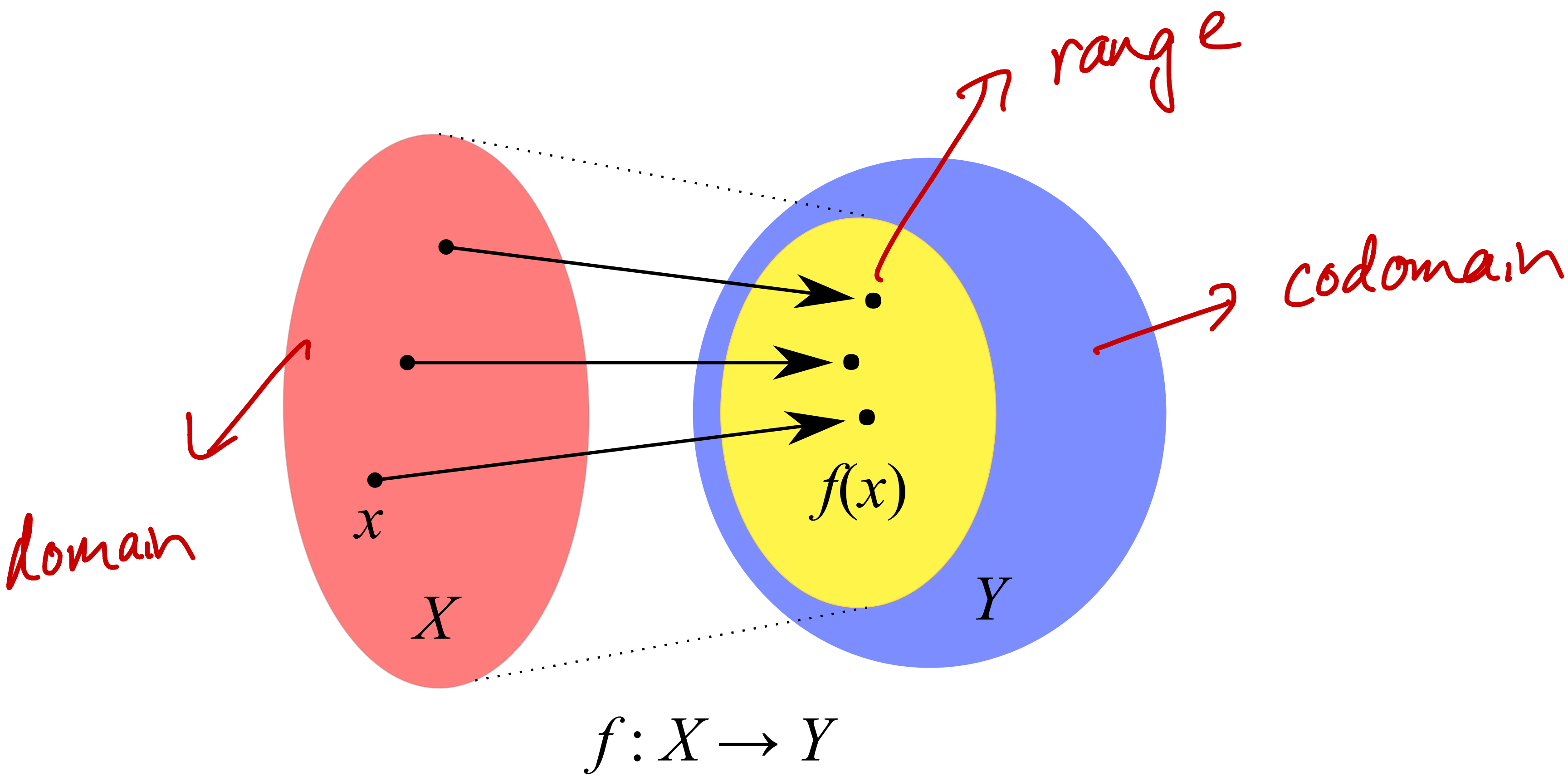
A is the **domain**, as it represents all possible inputs.

B is the **codomain**, as it represents all possible outputs; as a result, we say that $f(A) = B$.

The **image**, or **range**, is the set of all actual outputs of a function.

Which of the above functions have a

In the following image, the red area represents the domain, the blue the codomain, and the yellow the range.



range $\subseteq$ codomain

A, B sets

x number

In order to define a function f, we need to specify the domain, codomain, and how to **map** elements of the domain to the codomain. We can do this in two ways:

- List out the pairs (as we did for the previous examples)
- Specify a rule that describes how to get from input to corresponding output

no tail on arrow

$f : \underbrace{A \to B}_{\text{domain, codomain}}$

Function definitions don't necessarily need to be algebraic!

$$f(x) = \text{the } x^{th} \text{ smallest prime number}$$

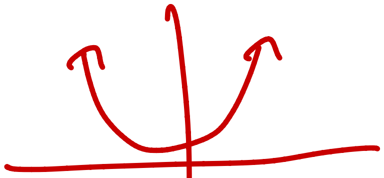
$f(x) = x^2$

$f : x \mapsto x^2$

"x maps to x^2"

Injections, Surjections and Bijections

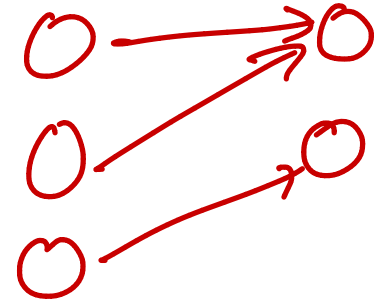
all types of functions



x^2 not injective!

$-a, a \mapsto a^2$, but $a \neq -a$

not an injection!



no 2 inputs can have same output!

Definition: Injection (One-to-one)

We say a function $f : A \to B$ is injective, or **one-to-one**, if no two elements in the input have the same output.

$A = \{1, 2, 3\}$

$B = \{4, 5, 6\}$

$f: \cancel{\{(1,4), (2,4), (3,5)\}}$

not injective!

$$f(x_1) = f(x_2) \implies x_1 = x_2$$

"implies"

Are the following functions injections?

Your answer may change depending on the domain and codomain!

$f(x) = e^x$

$g : x \mapsto \sqrt{x}$

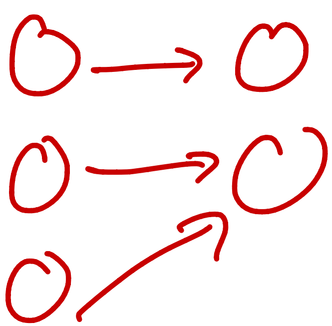
$h(x) = x^2$

Sample Problem

Prove that the *composition* of two injective functions is also injective.

codomain = range

> **Definition: Surjection (Onto)**
>
> We say a function $f : A \rightarrow B$ is surjective, or **onto**, if every element in B is mapped to by an element in A, i.e. when the codomain and range are the same set (that is, all possible outputs are actually seen as outputs).



surjection : yes!

injection : no!

What domain and codomain make $f : x \mapsto x^2$ a surjection? Not a surjection?

> **Definition: Bijection**
>
> A function f is bijective if and only if it is both injective and surjective. That is, a function is a bijection if and only if no two elements in the domain map to the same element in the range, and every element in the codomain has something mapping to it from the domain.

Summary of Types of Relations