Implementation of Sigmoid Regression from Scratch.

This Notebook has two Section:

1. Building Helper Function: In this section we will implement various helper function required for scratch implementation of Sigmoid Regression.
2. Putting Helper Function to Action: In this section we will compile all our helper function to train and test the Sigmoid Regression on Provided Dataset.

Building a Helper Function

Sigmoid Function:

A function $[\sigma: R → R] $ is said to be a sigmoid function, if the function is bounded, differentiable, real function that is defined for all real input values and has a non-negative derivative at each point and exactly on inflection point. The sigmoid fucntion has a characterstic "S" shaped curved also known as sigmoid curve. [--- Wikipedia]

A comon example of a sigmoid function is the logistic function described as below:

$$\sigma(x) = \frac{1}{1+e^{-x}},$$

for $x \in R$.

The next two code blocks construct and plot this function. ___

```
def logistic_function(x):
  """
  Computes the logistic function applied to any value of x.
  Arguments:
    x: scalar or numpy array of any size.
  Returns:
    y: logistic function applied to x.
  """
  import numpy as np
  y = 1/(1 + np.exp(-x))
  return y

import numpy as np

def test_logistic_function():
    """
    Test cases for the logistic_function.
    """
    # Test with scalar input
    x_scalar = 0
    expected_output_scalar = round(1 / (1 + np.exp(0)), 3)  # Expected
output: 0.5
    assert round(logistic_function(x_scalar), 3) ==
expected_output_scalar, "Test failed for scalar input"

    # Test with positive scalar input
    x_pos = 2
    expected_output_pos = round(1 / (1 + np.exp(-2)), 3)  # Expected
output: ~0.881
    assert round(logistic_function(x_pos), 3) == expected_output_pos,
"Test failed for positive scalar input"

    # Test with negative scalar input
    x_neg = -3
    expected_output_neg = round(1 / (1 + np.exp(3)), 3)  # Expected
output: ~0.047
    assert round(logistic_function(x_neg), 3) == expected_output_neg,
"Test failed for negative scalar input"

    # Test with numpy array input
    x_array = np.array([0, 2, -3])
    expected_output_array = np.array([0.5, 0.881, 0.047])  # Adjusted
expected values rounded to 3 decimals
    # Use np.round to round the array element-wise and compare
    assert np.all(np.round(logistic_function(x_array), 3) ==
expected_output_array), "Test failed for numpy array input"

    print("All tests passed!")

# Run the test case
test_logistic_function()
```

```
All tests passed!
```

Explanation of the test cases:

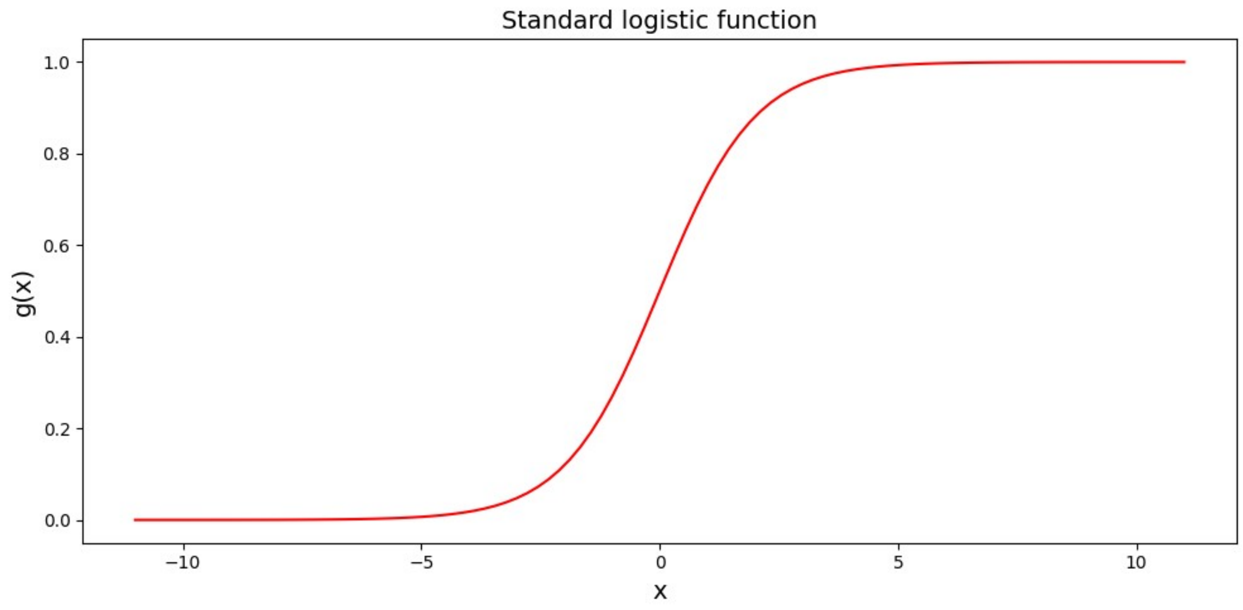
Scalar input test: We check if the function gives the correct output for a scalar input (e.g., 0, 2, -3).

Numpy array test: We verify that the function can handle a numpy array as input and return the correct element-wise logistic function values.

Assertions: np.isclose and np.allclose are used for comparison to avoid issues with floating-point precision errors.

Running this test will check that the function works for both scalar and array inputs and ensures the correctness of the output values.

```python
# Plooting the sigmoid function:
import matplotlib.pyplot as plt
import numpy as np
plt.figure(figsize = (10, 5))
x = np.linspace(-11, 11, 100)
plt.plot(x, logistic_function(x), color = 'red')
plt.xlabel("x", fontsize = 14)
plt.ylabel("g(x)", fontsize = 14)
plt.title("Standard logistic function", fontsize = 14)
plt.tight_layout()
plt.show()
```



Loss Function:

In general, loss function corresponds to observed error value between target value and predicted value for single observation/data points. For Sigmoid Regression and Binary Classification we use log - loss given by:

$$L(y, \hat{y}) = -y \log(\hat{y}) - (1-y) \log(1-\hat{y}),$$

Where:

- y : True target value (taking values 0 or 1)
- $\hat{y}$: Predicted target value (predicted probability of y being 1 and vice versa.)

The basic intution behind the log-loss function is, the loss value should be minimum when our predicted probability values are closer to true target value.

```python
def log_loss(y_true, y_pred):
    """
```

```
  Computes log loss for true target value y ={0 or 1} and predicted
target value y' inbetween {0-1}.
  Arguments:
    y_true (scalar): true target value {0 or 1}.
    y_pred (scalar): predicted taget value {0-1}.
  Returns:
    loss (float): loss/error value
  """
  import numpy as np
  # Ensure y_pred is clipped to avoid log(0)
  y_pred = np.clip(y_pred, 1e-10, 1 - 1e-10)
  loss = -(y_true * np.log(y_pred)) - ((1-y_true) * np.log(1- y_pred))
  return loss
```

Verify the Intution.

```
# Test function:
y_true, y_pred = 0, 0.1
print(f'log loss({y_true}, {y_pred}) ==> {log_loss(y_true, y_pred)}')
print("+++++++++++++--------------------------+++++++++++++++++++++++
+")
y_true, y_pred = 1, 0.9
print(f'log loss({y_true}, {y_pred}) ==> {log_loss(y_true, y_pred)}')
```

```
log loss(0, 0.1) ==> 0.10536051565782628
+++++++++++++--------------------------++++++++++++++++++++++++
log loss(1, 0.9) ==> 0.10536051565782628
```

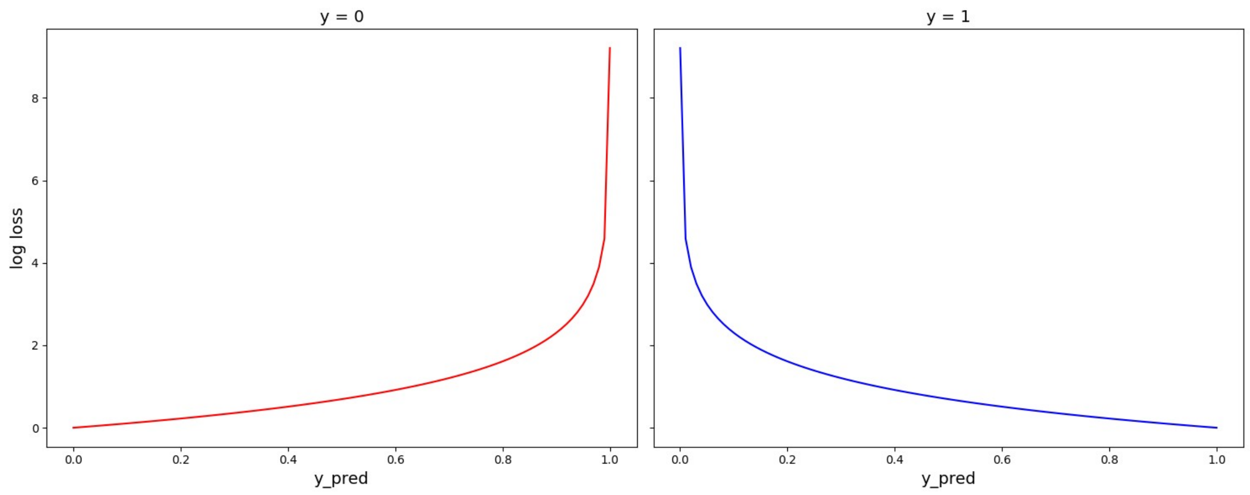
```
# Test function:for
y_true, y_pred = 0, 0.9
print(f'log loss({y_true}, {y_pred}) ==> {log_loss(y_true, y_pred)}')
print("+++++++++++++--------------------------+++++++++++++++++++++++
+")
y_true, y_pred = 1, 0.1
print(f'log loss({y_true}, {y_pred}) ==> {log_loss(y_true, y_pred)}')
```

```
log loss(0, 0.9) ==> 2.302585092994046
+++++++++++++--------------------------++++++++++++++++++++++++
log loss(1, 0.1) ==> 2.3025850929940455
```

Test the Loss Function

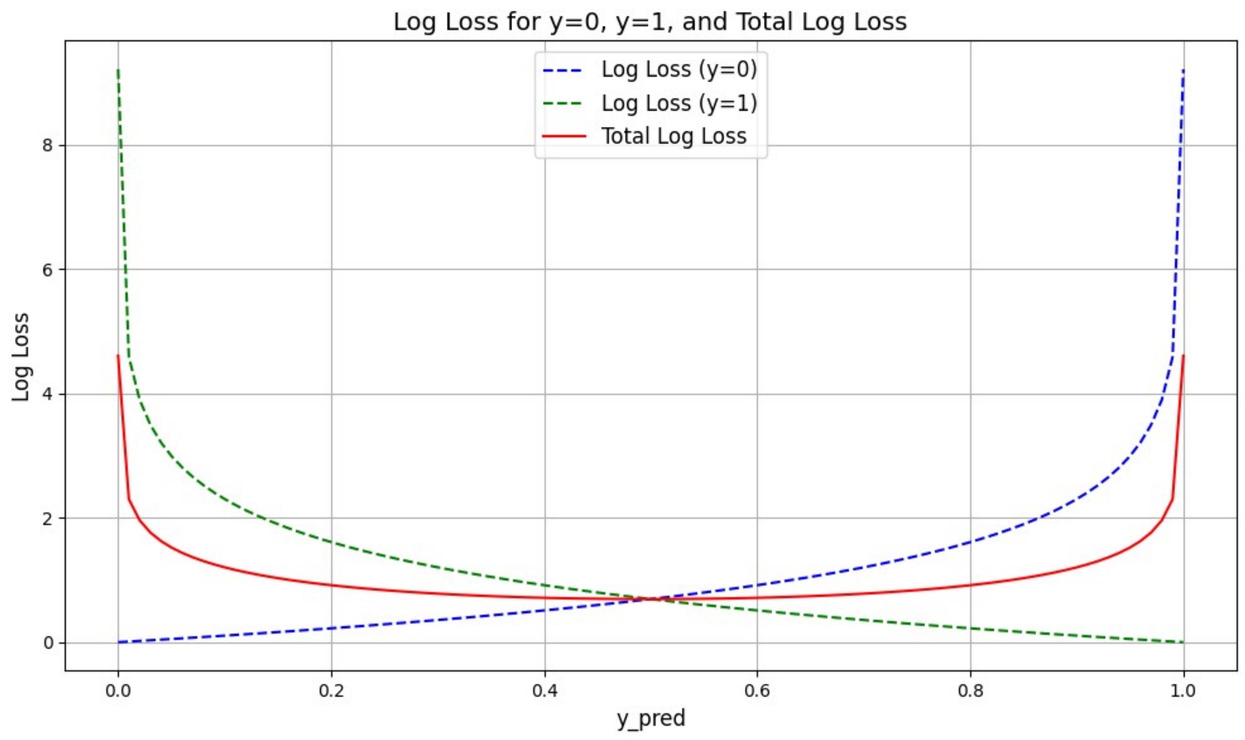
```
def test_log_loss():
    """
    Test cases for the log_loss function.
    """
    import numpy as np

    # Test case 1: Perfect prediction (y_true = 1, y_pred = 1)
    y_true = 1
    y_pred = 1
    expected_loss = 0.0  # Log loss is 0 for perfect prediction
    assert np.isclose(log_loss(y_true, y_pred), expected_loss), "Test
failed for perfect prediction (y_true=1, y_pred=1)"

    # Test case 2: Perfect prediction (y_true = 0, y_pred = 0)
    y_true = 0
    y_pred = 0
    expected_loss = 0.0  # Log loss is 0 for perfect prediction
    assert np.isclose(log_loss(y_true, y_pred), expected_loss), "Test
failed for perfect prediction (y_true=0, y_pred=0)"

    # Test case 3: Incorrect prediction (y_true = 1, y_pred = 0)
    y_true = 1
    y_pred = 0
    try:
        log_loss(y_true, y_pred)  # This should raise an error due to
log(0)
    except ValueError:
        pass  # Test passed if ValueError is raised for log(0)

    # Test case 4: Incorrect prediction (y_true = 0, y_pred = 1)
    y_true = 0
    y_pred = 1
    try:
        log_loss(y_true, y_pred)  # This should raise an error due to
log(0)
    except ValueError:
        pass  # Test passed if ValueError is raised for log(0)

    # Test case 5: Partially correct prediction
    y_true = 1
    y_pred = 0.8
    expected_loss = -(1 * np.log(0.8)) - (0 * np.log(0.2))  # ~0.2231
    assert np.isclose(log_loss(y_true, y_pred), expected_loss,
atol=1e-6), "Test failed for partially correct prediction (y_true=1,
y_pred=0.8)"

    y_true = 0
    y_pred = 0.2
    expected_loss = -(0 * np.log(0.2)) - (1 * np.log(0.8))  # ~0.2231
    assert np.isclose(log_loss(y_true, y_pred), expected_loss,
atol=1e-6), "Test failed for partially correct prediction (y_true=0,
y_pred=0.2)"

    print("All tests passed!")

# Run the test case
test_log_loss()
```

```
All tests passed!

# Plot the loss Function:
import matplotlib.pyplot as plt
fig, ax = plt.subplots(1, 2, figsize = (15, 6), sharex = True, sharey
= True)
y_pred = np.linspace(0.0001, 0.9999, 100)
ax[0].plot(y_pred, log_loss(0, y_pred), color = 'red')
ax[0].set_title("y = 0", fontsize = 14)
ax[0].set_xlabel("y_pred", fontsize = 14)
ax[0].set_ylabel("log loss", fontsize = 14)
ax[1].plot(y_pred, log_loss(1, y_pred), color = 'blue')
ax[1].set_title("y = 1", fontsize = 14)
ax[1].set_xlabel("y_pred", fontsize = 14)
plt.tight_layout()
plt.show()
```



plot for total loss

```
# Generate predicted probabilities
y_pred = np.linspace(0.0001, 0.9999, 100)

# Compute log losses for y = 0 and y = 1
log_loss_0 = log_loss(0, y_pred)
log_loss_1 = log_loss(1, y_pred)

# Compute total log loss as a weighted sum (assuming equal weights
here for simplicity)
total_log_loss = 0.5 * log_loss_0 + 0.5 * log_loss_1

# Plot the results
plt.figure(figsize=(10, 6))
plt.plot(y_pred, log_loss_0, label="Log Loss (y=0)", color="blue",
linestyle="--")
plt.plot(y_pred, log_loss_1, label="Log Loss (y=1)", color="green",
linestyle="--")
plt.plot(y_pred, total_log_loss, label="Total Log Loss", color="red")
plt.title("Log Loss for y=0, y=1, and Total Log Loss", fontsize=14)
plt.xlabel("y_pred", fontsize=12)
plt.ylabel("Log Loss", fontsize=12)
plt.legend(fontsize=12)
plt.grid(True)
plt.tight_layout()
plt.show()
```



Cost Function - Binary Classification:

As described above, we determine the cost function as an average of loss function value calculated for each observation/datapoints.

Let $y=[y_1, \ldots, y_n]$ be the true target values{(0 or 1)}

and $\hat{y}=[\hat{y}_1, \ldots, \hat{y}_n]$ be the corresponding predicted target values in between $\{[0 \le \hat{y} \le 1]\}$,

Then the cost function be:

$$Cost(y, \hat{y}) = \frac{1}{n}\sum_{i=1}^{n} L(y_i, \hat{y}_i).$$

```
# Cost function - using vectorization
def cost_function(y_true, y_pred):
    """
    Computes log loss for inputs true value (0 or 1) and predicted
value (between 0 and 1)
    Args:
      y_true    (array_like, shape (m,)): array of true values (0 or
1)
      y_pred (array_like, shape (m,)): array of predicted values
(probability of y_pred being 1)
    Returns:
      cost (float): nonnegative cost corresponding to y_true and
y_pred
    """
    assert len(y_true) == len(y_pred), "Length of true values and
length of predicted values do not match"
    n = len(y_true)
    loss_vec = np.array([log_loss(y_true[i], y_pred[i]) for i in
range(n)])
    cost = np.dot(loss_vec, np.ones(n)) / n
    return cost

y_true, y_pred = np.array([0, 1, 0]), np.array([0.4, 0.6, 0.25])
print(f"cost_function({y_true}, {y_pred}) = {cost_function(y_true,
y_pred)}")
```

```
cost_function([0 1 0], [0.4  0.6  0.25]) = 0.43644443999458743
```

```
import numpy as np

# Test function for the cost_function
def test_cost_function():
    # Test case 1: Simple example with known expected cost
    y_true = np.array([1, 0, 1])
    y_pred = np.array([0.9, 0.1, 0.8])

    # Expected output: Manually calculate cost for these values
    # log_loss(y_true, y_pred) for each example
    expected_cost = (-(1 * np.log(0.9)) - (1 - 1) * np.log(1 - 0.9) +
                     -(0 * np.log(0.1)) - (1 - 0) * np.log(1 - 0.1) +
                     -(1 * np.log(0.8)) - (1 - 1) * np.log(1 - 0.8)) /
3

    # Call the cost_function to get the result
    result = cost_function(y_true, y_pred)

    # Assert that the result is close to the expected cost with a
tolerance of 1e-6
    assert np.isclose(result, expected_cost, atol=1e-6), f"Test
failed: {result} != {expected_cost}"

    print("Test passed for simple case!")
```

```
# Run the test case
test_cost_function()

Test passed for simple case!
```

Extending the cost function for logistic regression to be used with model parameters.

Function we are estimating:

$$\hat{y} = \sigma\left(x \cdot w^T + b\right) = \frac{1}{1 + e^{-\left(x \cdot w^T + b\right)}}.$$

Where:

- w parameters (Coefficinet of feature variable) also known as weights. -b parameters (intercept of the function) also known as bias.

Assume;

- $X_{n \times d} \in R$: Feature Matirix of $n \times d$ independent variables;
- Y_n: Vector of n dependent variables;
- $\hat{Y}_n$: Vector of n predicted variables; and are represented as follows:

$$ \mathbf{X_{n\times d}} = \begin{pmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,d} \newline x_{2,1} & x_{2,2} & \cdots & x_{2,d} \newline \vdots & \vdots & \ddots & \vdots \newline x_{n,1} & x_{n,2} & \cdots & x_{n,d} \end{pmatrix},\;\;\;\; \mathbf{Y_n} = \begin{pmatrix} y_1 \newline y_2 \newline \vdots \newline y_n \end{pmatrix},\;\;\;\; \mathbf{\hat{Y}_n} = \begin{pmatrix} \sigma\left(\mathbf{x_1} \cdot \mathbf{{w^T}} + b_1\right) \newline \sigma\left(\mathbf{x_2} \cdot \mathbf{w^T} + b_2\right) \newline \vdots \newline \sigma\left(\mathbf{x_n} \cdot \mathbf{w^T} + b_n\right) \end{pmatrix}. \tag{2} $$

Here:

$$x_i = x_{i,1} \cdot w_{i,1}^T + \ldots + x_{i,d} \cdot w_{i,d}^T + b_i$$

Now rewrite cost funtion defined in (0) for model parameters as:

$$L(w,b) := C(y, \hat{y} \vee X, w, b) = \frac{1}{n}\sum_{i=1}^{n} L\left(y_i, \frac{1}{1 + e^{-\left(x_i \cdot w + b\right)}}\right)$$

$$¿ \frac{1}{n}\sum_{i=1}^{n}\left[-y_i \log\left(\frac{1}{1 + e^{-\left(x_i \cdot w + b\right)}}\right) - \left(1 - y_i\right)\log\left(1 - \frac{1}{1 + e^{-\left(x_i \cdot w + b\right)}}\right)\right].$$

```
# Function to compute cost function in terms of model parameters -
using vectorization
def costfunction_logreg(X, y, w, b):
    """
    Computes the cost function, given data and model parameters.
    Args:
      X (ndarray, shape (m,n)): data on features, m observations with
n features.
      y (array_like, shape (m,)): array of true values of target (0 or
1).
      w (array_like, shape (n,)): weight parameters of the model.
      b (float): bias parameter of the model.
    Returns:
      cost (float): nonnegative cost corresponding to y and y_pred.
    """
    m, n = X.shape
    assert len(y) == m, "Number of feature observations and number of
target observations do not match."
    assert len(w) == n, "Number of features and number of weight
parameters do not match."

    # Compute z using np.dot
    z = np.dot(X, w) + b  # Matrix-vector multiplication and adding
bias

    # Compute predictions using logistic function (sigmoid)
    y_pred = logistic_function(z)

    # Compute the cost using the cost function
    cost = cost_function(y, y_pred)

    return cost
X, y, w, b = np.array([[10, 20], [-10, 10]]), np.array([1, 0]),
np.array([0.5, 1.5]), 1
print(f"cost for logistic regression(X = {X}, y = {y}, w = {w}, b =
{b}) = {costfunction_logreg(X, y, w, b)}")
```

```
cost for logistic regression(X = [[ 10  20]
 [-10  10]], y = [1 0], w = [0.5 1.5], b = 1) = 5.500008350834906
```

```
def compute_gradient(X, y, w, b):
    """
    Computes gradients of the cost function with respect to model
parameters.
    Args:
      X (ndarray, shape (n,d)): Input data, n observations with d
features
      y (array_like, shape (n,)): True labels (0 or 1)
      w (array_like, shape (d,)): Weight parameters of the model
      b (float): Bias parameter of the model
    Returns:
      grad_w (array_like, shape (d,)): Gradients of the cost function
with respect to the weight parameters
```

```
        grad_b (float): Gradient of the cost function with respect to
the bias parameter
    """
    n, d = X.shape  # X has shape (n, d)
    assert len(y) == n, f"Expected y to have {n} elements, but got
{len(y)}"
    assert len(w) == d, f"Expected w to have {d} elements, but got
{len(w)}"

    # Compute predictions using logistic function (sigmoid)
    y_pred = logistic_function(np.dot(X, w) + b)  # Compute z = X * w
+ b

    # Compute gradients
    grad_w = np.dot(X.T, (y_pred - y)) / n  # Gradient w.r.t weights,
shape (d,)
    grad_b = np.dot(np.ones(n), (y_pred - y)) / n  # Gradient w.r.t
bias, scalar

    return grad_w, grad_b
```

Simple Test for Gradient Computations:

```
# Simple test case
X = np.array([[10, 20], [-10, 10]])  # shape (2, 2)
y = np.array([1, 0])  # shape (2,)
w = np.array([0.5, 1.5])  # shape (2,)
b = 1  # scalar

# Assertion tests
try:
    grad_w, grad_b = compute_gradient(X, y, w, b)
    print("Gradients computed successfully.")
    print(f"grad_w: {grad_w}")
    print(f"grad_b: {grad_b}")
except AssertionError as e:
    print(f"Assertion error: {e}")
```

```
Gradients computed successfully.
grad_w: [-4.99991649  4.99991649]
grad_b: 0.4999916492890759
```

Implementation of Gradient Descent for Sigmoid Regresssion.

```
def gradient_descent(X, y, w, b, alpha, n_iter, show_cost=False,
show_params=True):
    """
    Implements batch gradient descent to optimize logistic regression
parameters.
```

```
    Args:
      X (ndarray, shape (n,d)): Data on features, n observations with
d features
      y (array_like, shape (n,)): True values of target (0 or 1)
      w (array_like, shape (d,)): Initial weight parameters
      b (float): Initial bias parameter
      alpha (float): Learning rate
      n_iter (int): Number of iterations
      show_cost (bool): If True, displays cost every 100 iterations
      show_params (bool): If True, displays parameters every 100
iterations

    Returns:
      w (array_like, shape (d,)): Optimized weight parameters
      b (float): Optimized bias parameter
      cost_history (list): List of cost values over iterations
      params_history (list): List of parameters (w, b) over iterations
    """
    n, d = X.shape
    assert len(y) == n, "Number of observations in X and y do not
match"
    assert len(w) == d, "Number of features in X and w do not match"

    cost_history = []
    params_history = []

    for i in range(n_iter):
        # Compute gradients
        grad_w, grad_b = compute_gradient(X, y, w, b)

        # Update weights and bias
        w -= alpha * grad_w
        b -= alpha * grad_b

        # Compute cost
        cost = costfunction_logreg(X, y, w, b)

        # Store cost and parameters
        cost_history.append(cost)
        params_history.append((w.copy(), b))

        # Optionally print cost and parameters
        if show_cost and (i % 100 == 0 or i == n_iter - 1):
            print(f"Iteration {i}: Cost = {cost:.6f}")
        if show_params and (i % 100 == 0 or i == n_iter - 1):
            print(f"Iteration {i}: w = {w}, b = {b:.6f}")

    return w, b, cost_history, params_history
```

```
# Test the gradient_descent function with sample data
X = np.array([[0.1, 0.2], [-0.1, 0.1]])  # Shape (2, 2)
y = np.array([1, 0])                      # Shape (2,)
w = np.zeros(X.shape[1])                  # Shape (2,) - same as number
of features
b = 0.0                                   # Scalar
alpha = 0.1                               # Learning rate
n_iter = 100000                            # Number of iterations

# Perform gradient descent
w_out, b_out, cost_history, params_history = gradient_descent(X, y, w,
b, alpha, n_iter, show_cost=True, show_params=False)

# Print final parameters and cost
print("\nFinal parameters:")
print(f"w: {w_out}, b: {b_out}")
print(f"Final cost: {cost_history[-1]:.6f}")
```

```
Iteration 0: Cost = 0.692835
Iteration 100: Cost = 0.662662
Iteration 200: Cost = 0.634332
Iteration 300: Cost = 0.607704
Iteration 400: Cost = 0.582671
Iteration 500: Cost = 0.559128
Iteration 600: Cost = 0.536977
Iteration 700: Cost = 0.516126
Iteration 800: Cost = 0.496487
Iteration 900: Cost = 0.477978
Iteration 1000: Cost = 0.460524
Iteration 1100: Cost = 0.444052
Iteration 1200: Cost = 0.428497
Iteration 1300: Cost = 0.413797
Iteration 1400: Cost = 0.399895
Iteration 1500: Cost = 0.386736
Iteration 1600: Cost = 0.374272
Iteration 1700: Cost = 0.362457
Iteration 1800: Cost = 0.351248
Iteration 1900: Cost = 0.340607
Iteration 2000: Cost = 0.330495
Iteration 2100: Cost = 0.320880
Iteration 2200: Cost = 0.311730
Iteration 2300: Cost = 0.303016
Iteration 2400: Cost = 0.294710
Iteration 2500: Cost = 0.286789
Iteration 2600: Cost = 0.279228
Iteration 2700: Cost = 0.272007
Iteration 2800: Cost = 0.265104
Iteration 2900: Cost = 0.258502
Iteration 3000: Cost = 0.252182
Iteration 3100: Cost = 0.246129
Iteration 3200: Cost = 0.240328
Iteration 3300: Cost = 0.234764
Iteration 3400: Cost = 0.229425
Iteration 3500: Cost = 0.224299
Iteration 3600: Cost = 0.219373
Iteration 3700: Cost = 0.214637
Iteration 3800: Cost = 0.210081
Iteration 3900: Cost = 0.205697
Iteration 4000: Cost = 0.201474
Iteration 4100: Cost = 0.197406
Iteration 4200: Cost = 0.193484
Iteration 4300: Cost = 0.189700
Iteration 4400: Cost = 0.186049
Iteration 4500: Cost = 0.182524
Iteration 4600: Cost = 0.179119
Iteration 4700: Cost = 0.175828
Iteration 4800: Cost = 0.172646
Iteration 4900: Cost = 0.169568
Iteration 5000: Cost = 0.166589
Iteration 5100: Cost = 0.163705
Iteration 5200: Cost = 0.160912
Iteration 5300: Cost = 0.158205
Iteration 5400: Cost = 0.155581
Iteration 5500: Cost = 0.153036
Iteration 5600: Cost = 0.150568
Iteration 5700: Cost = 0.148172
Iteration 5800: Cost = 0.145846
Iteration 5900: Cost = 0.143586
Iteration 6000: Cost = 0.141392
Iteration 6100: Cost = 0.139258
Iteration 6200: Cost = 0.137184
Iteration 6300: Cost = 0.135167
Iteration 6400: Cost = 0.133205
Iteration 6500: Cost = 0.131295
Iteration 6600: Cost = 0.129436
Iteration 6700: Cost = 0.127625
Iteration 6800: Cost = 0.125862
Iteration 6900: Cost = 0.124143
Iteration 7000: Cost = 0.122469
Iteration 7100: Cost = 0.120836
Iteration 7200: Cost = 0.119243
Iteration 7300: Cost = 0.117690
Iteration 7400: Cost = 0.116175
Iteration 7500: Cost = 0.114695
Iteration 7600: Cost = 0.113251
Iteration 7700: Cost = 0.111841
Iteration 7800: Cost = 0.110464
Iteration 7900: Cost = 0.109118
Iteration 8000: Cost = 0.107804
Iteration 8100: Cost = 0.106518
Iteration 8200: Cost = 0.105262
Iteration 8300: Cost = 0.104033
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Iteration 69100: Cost = 0.012036
Iteration 69200: Cost = 0.012018
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Iteration 70000: Cost = 0.011878
Iteration 70100: Cost = 0.011861
Iteration 70200: Cost = 0.011843
Iteration 70300: Cost = 0.011826
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Iteration 71000: Cost = 0.011707
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Iteration 71200: Cost = 0.011673
Iteration 71300: Cost = 0.011656
Iteration 71400: Cost = 0.011640
Iteration 71500: Cost = 0.011623
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Iteration 72100: Cost = 0.011524
Iteration 72200: Cost = 0.011508
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Iteration 72500: Cost = 0.011459
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Iteration 73700: Cost = 0.011269
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Iteration 74000: Cost = 0.011222
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Iteration 74200: Cost = 0.011191
Iteration 74300: Cost = 0.011176
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Iteration 75000: Cost = 0.011069
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Iteration 75500: Cost = 0.010994
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Iteration 78200: Cost = 0.010607
Iteration 78300: Cost = 0.010593
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Iteration 78700: Cost = 0.010538
Iteration 78800: Cost = 0.010524
Iteration 78900: Cost = 0.010510
Iteration 79000: Cost = 0.010497
Iteration 79100: Cost = 0.010483
Iteration 79200: Cost = 0.010470
Iteration 79300: Cost = 0.010456
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Iteration 96800: Cost = 0.008532
Iteration 96900: Cost = 0.008523
Iteration 97000: Cost = 0.008514
Iteration 97100: Cost = 0.008505
Iteration 97200: Cost = 0.008496
Iteration 97300: Cost = 0.008487
Iteration 97400: Cost = 0.008478
Iteration 97500: Cost = 0.008469
Iteration 97600: Cost = 0.008460
Iteration 97700: Cost = 0.008452
Iteration 97800: Cost = 0.008443
Iteration 97900: Cost = 0.008434
Iteration 98000: Cost = 0.008425
Iteration 98100: Cost = 0.008416
Iteration 98200: Cost = 0.008408
Iteration 98300: Cost = 0.008399
Iteration 98400: Cost = 0.008390
Iteration 98500: Cost = 0.008382
Iteration 98600: Cost = 0.008373
Iteration 98700: Cost = 0.008364
Iteration 98800: Cost = 0.008356
Iteration 98900: Cost = 0.008347
Iteration 99000: Cost = 0.008339
Iteration 99100: Cost = 0.008330
Iteration 99200: Cost = 0.008321
Iteration 99300: Cost = 0.008313
Iteration 99400: Cost = 0.008304
Iteration 99500: Cost = 0.008296
Iteration 99600: Cost = 0.008287
Iteration 99700: Cost = 0.008279
Iteration 99800: Cost = 0.008270
Iteration 99900: Cost = 0.008262
Iteration 99999: Cost = 0.008254

Final parameters:
w: [38.51304248 18.83386869], b: -2.8176836626325836
Final cost: 0.008254
```

```
# Simple assertion test for gradient_descent
def test_gradient_descent():
    X = np.array([[0.1, 0.2], [-0.1, 0.1]])  # Shape (2, 2)
    y = np.array([1, 0])                     # Shape (2,)
    w = np.zeros(X.shape[1])                 # Shape (2,)
    b = 0.0                                  # Scalar
    alpha = 0.1                              # Learning rate
    n_iter = 100                             # Number of iterations
    # Run gradient descent
    w_out, b_out, cost_history, _ = gradient_descent(X, y, w, b,
alpha, n_iter, show_cost=False, show_params=False)

    # Assertions
    assert len(cost_history) == n_iter, "Cost history length does not
match the number of iterations"
    assert w_out.shape == w.shape, "Shape of output weights does not
match the initial weights"
    assert isinstance(b_out, float), "Bias output is not a float"
    assert cost_history[-1] < cost_history[0], "Cost did not decrease
over iterations"

    print("All tests passed!")

# Run the test
test_gradient_descent()
```

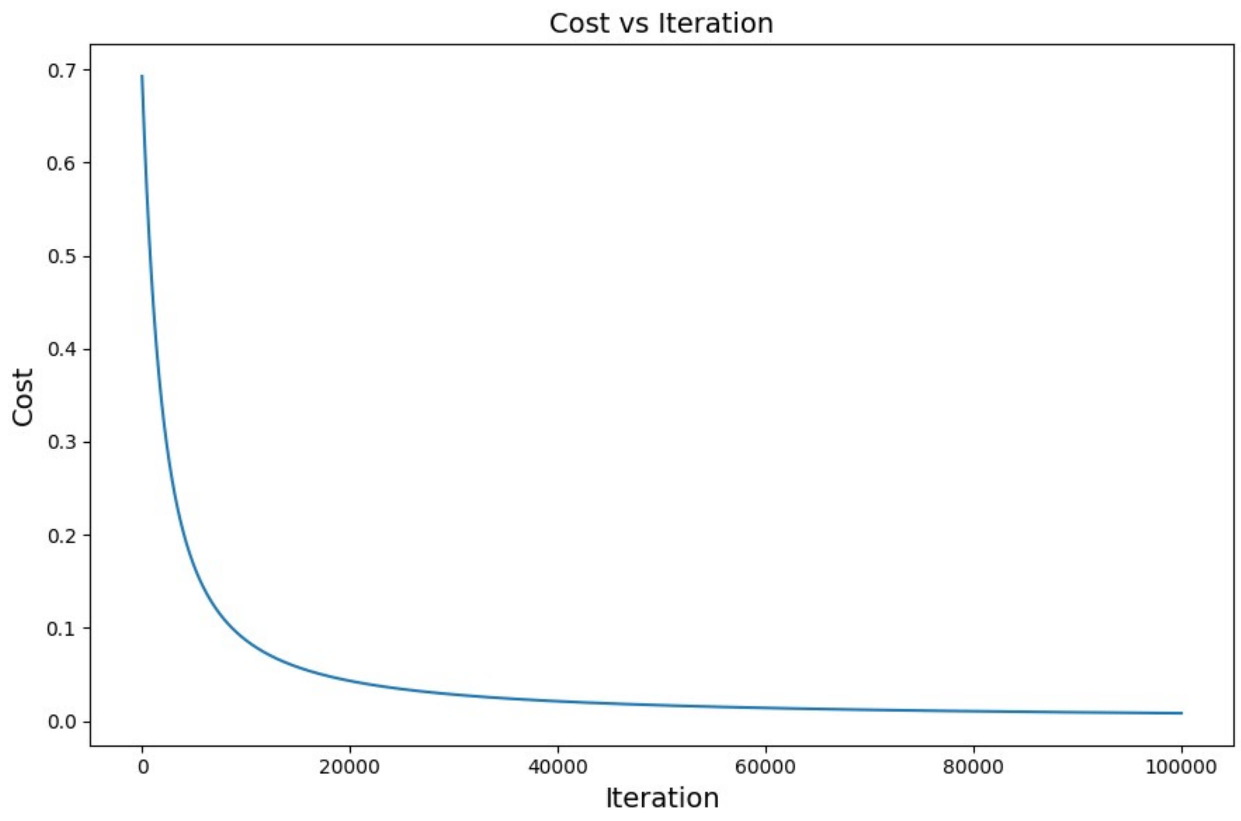
```
All tests passed!
```

Visualizing Convergence of Cost During Gradient Descent:

This plot tracks how the cost decreases over iterations, providing insight into the convergence of the gradient descent algorithm.

It ensures the optimization is progressing as expected, helps detect issues like divergence, and aids in tuning hyperparameters effectively.

```
# Plotting cost over iteration
plt.figure(figsize = (9, 6))
plt.plot(cost_history)
plt.xlabel("Iteration", fontsize = 14)
plt.ylabel("Cost", fontsize = 14)
plt.title("Cost vs Iteration", fontsize = 14)
plt.tight_layout()
plt.show()
```



Decision Function

In Decsion function we perform following:

- Prediction:
  - Utilizing our trained weights and biases we first find the probabilities values y_probab for the x_test. Then, Y_probabiliy value is transformed to discrete class value using Decision boubdry. ___
- Decision Boundry:
  - We first calculate the y_prediction as probabilities value, which is then converted to discrete class by using a threshold value. For instance, we take the threshold to be 0.5,then we classify the observation to class 1 and to class 0 otherwise.

```
import numpy as np

def prediction(X, w, b, threshold=0.5):
    """
    Predicts binary outcomes for given input features based on
logistic regression parameters.

    Arguments:
      X (ndarray, shape (n,d)): Array of test independent variables
(features) with n samples and d features.
```

```
      w (ndarray, shape (d,)): Array of weights learned via gradient
descent.
      b (float): Bias learned via gradient descent.
      threshold (float, optional): Classification threshold for
predicting class labels. Default is 0.5.

    Returns:
      y_pred (ndarray, shape (n,)): Array of predicted dependent
variable (binary class labels: 0 or 1).
    """
    # Compute the predicted probabilities using the logistic function
    y_test_prob = logistic_function(np.dot(X, w) + b)

    # Classify based on the threshold
    y_pred = (y_test_prob >= threshold).astype(int)

    return y_pred

def test_prediction():
    X_test = np.array([[0.5, 1.0], [1.5, -0.5], [-0.5, -1.0]])  #
Shape (3, 2)
    w_test = np.array([1.0, -1.0])                              #
Shape (2,)
    b_test = 0.0                                                #
Scalar bias
    threshold = 0.5                                             #
Default threshold

    # Updated expected output
    expected_output = np.array([0, 1, 1])

    # Call the prediction function
    y_pred = prediction(X_test, w_test, b_test, threshold)

    # Assert that the output matches the expected output
    assert np.array_equal(y_pred, expected_output), f"Expected
{expected_output}, but got {y_pred}"

    print("Test passed!")

test_prediction()

Test passed!
```

Evaluation of the Classifier.

```
# Evaluation Function: Computes confusion matrix, precision, recall,
and F1-score
def evaluate_classification(y_true, y_pred):
    TP = np.sum((y_true == 1) & (y_pred == 1))  # True Positives
    TN = np.sum((y_true == 0) & (y_pred == 0))  # True Negatives
    FP = np.sum((y_true == 0) & (y_pred == 1))  # False Positives
    FN = np.sum((y_true == 1) & (y_pred == 0))  # False Negatives

    # Confusion matrix
    confusion_matrix = np.array([[TN, FP],
                                  [FN, TP]])

    # Precision, recall, and F1-score with safe handling of zero
division
    precision = TP / (TP + FP) if (TP + FP) > 0 else 0.0
    recall = TP / (TP + FN) if (TP + FN) > 0 else 0.0
    f1_score = (2 * precision * recall) / (precision + recall) if
(precision + recall) > 0 else 0.0

    # Ensure metrics are floats
    precision = float(precision)
    recall = float(recall)
    f1_score = float(f1_score)

    # Return metrics as a dictionary
    return confusion_matrix, precision, recall, f1_score
```

Putting Helper Function to Action.

Compiling and Training A sigmoid regression on Dataset.

Some Basic Data operations, Loading, Analysis and Cleaning:

```
from google.colab import drive
drive.mount('/content/drive')
```

```
Drive already mounted at /content/drive; to attempt to forcibly
remount, call drive.mount("/content/drive", force_remount=True).
```

Necessary Imports and Loading of the Dataset:

```
# Load dataset
import numpy as np
import pandas as pd
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import StandardScaler
import matplotlib.pyplot as plt
url =
"https://raw.githubusercontent.com/jbrownlee/Datasets/master/pima-
indians-diabetes.data.csv"
```

```
columns = ['Pregnancies', 'Glucose', 'BloodPressure', 'SkinThickness',
'Insulin', 'BMI', 'DiabetesPedigreeFunction', 'Age', 'Outcome']
data_pima_diabetes = pd.read_csv(url, names=columns)
```

Some Basic Data Cleaning:

```
# Data cleaning
columns_to_clean = ['Glucose', 'BloodPressure', 'SkinThickness',
'Insulin', 'BMI']
data_pima_diabetes[columns_to_clean] =
data_pima_diabetes[columns_to_clean].replace(0, np.nan)
data_pima_diabetes.fillna(data_pima_diabetes.median(), inplace=True)

data_pima_diabetes.info()

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 101 entries, 0 to 100
Data columns (total 4 columns):
 #   Column      Non-Null Count  Dtype
---  ------      --------------  -----
 0   HouseAge    101 non-null    object
 1   HouseFloor  101 non-null    object
 2   HouseArea   101 non-null    object
 3   HousePrice  101 non-null    object
dtypes: object(4)
memory usage: 3.3+ KB
```

Summary Statistics:

```
data_pima_diabetes.describe()
```

```
{"summary":"{\n  \"name\": \"data_pima_diabetes\",\n  \"rows\": 4,\n
\"fields\": [\n    {\n      \"column\": \"HouseAge\",\n
\"properties\": {\n        \"dtype\": \"string\",\n
\"num_unique_values\": 4,\n        \"samples\": [\n          62,\n
\"5\",\n          \"101\"\n        ],\n        \"semantic_type\":
\"\",\n        \"description\": \"\"\n      }\n    },\n    {\n
\"column\": \"HouseFloor\",\n      \"properties\": {\n
\"dtype\": \"string\",\n        \"num_unique_values\": 4,\n
\"samples\": [\n          6,\n          \"27\",\n          \"101\"\n
],\n        \"semantic_type\": \"\",\n        \"description\": \"\"\n
}\n    },\n    {\n      \"column\": \"HouseArea\",\n
\"properties\": {\n        \"dtype\": \"string\",\n
\"num_unique_values\": 3,\n        \"samples\": [\n          \"101\",\
n          \"HouseArea\",\n          \"1\"\n        ],\n
\"semantic_type\": \"\",\n        \"description\": \"\"\n      }\
n    },\n    {\n      \"column\": \"HousePrice\",\n
\"properties\": {\n        \"dtype\": \"string\",\n
\"num_unique_values\": 3,\n        \"samples\": [\n          \"101\",\
n          \"HousePrice\",\n          \"1\"\n        ],\n
\"semantic_type\": \"\",\n        \"description\": \"\"\n      }\
n    }\n  ]\n}","type":"dataframe"}
```

Train Test Split and Standard Scaling of the Dataset:

Train - Test Split:

```
# Train-test split
X = data_pima_diabetes.drop(columns=['Outcome']).values
y = data_pima_diabetes['Outcome'].values
X_train, X_test, y_train, y_test = train_test_split(X, y,
test_size=0.2, random_state=42, stratify=y)
```

Standarize the Input Dataset:

```
# Standardize features
scaler = StandardScaler()
X_train_scaled = scaler.fit_transform(X_train)
X_test_scaled = scaler.transform(X_test)
```

Initialize the parameters and Hyper-parameters:

```
# Initialize parameters
w = np.zeros(X_train_scaled.shape[1])
b = 0.0
alpha = 0.1
n_iter = 1000
```

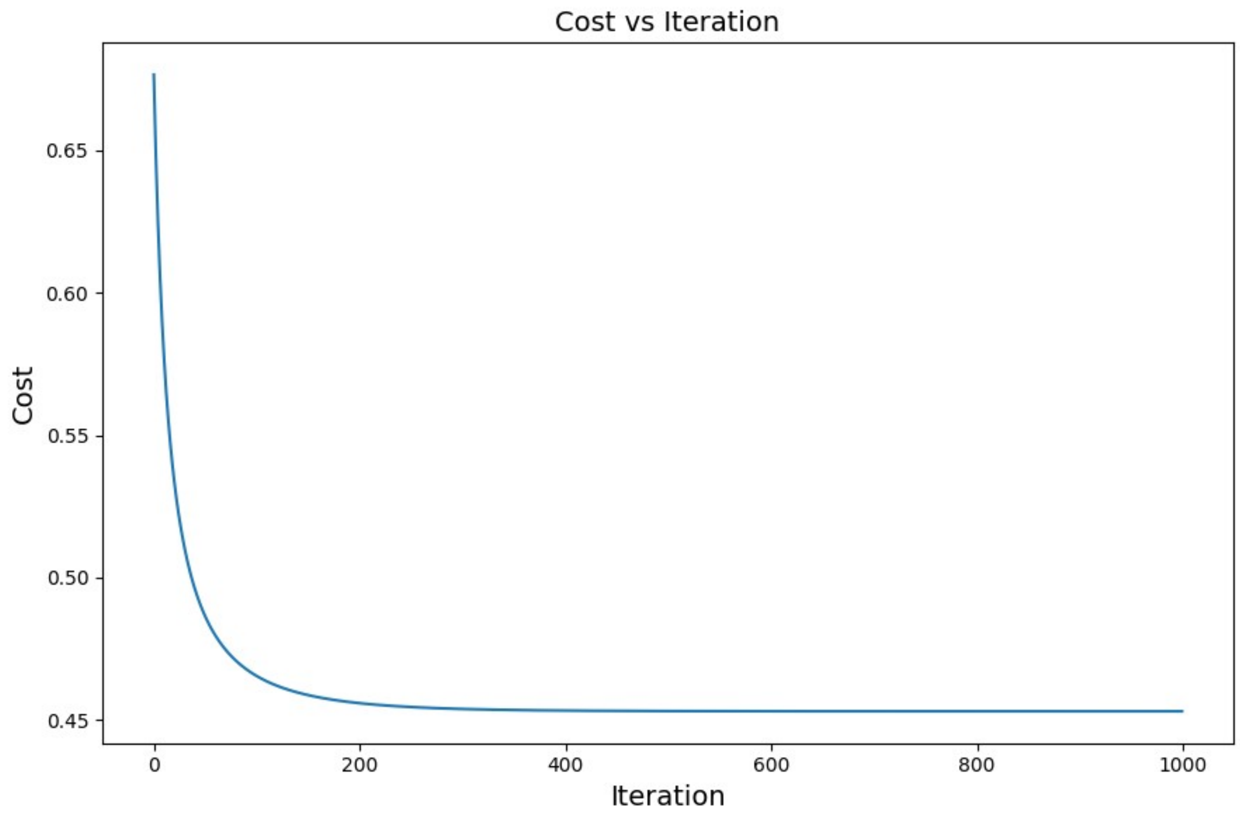
Start the Training:

```
# Train model
print("\nTraining Logistic Regression Model:")
w, b, cost_history,params_history = gradient_descent(X_train_scaled,
y_train, w, b, alpha, n_iter, show_cost=True, show_params=False)
```

```
Training Logistic Regression Model:
Iteration 0: Cost = 0.676575
Iteration 100: Cost = 0.465441
Iteration 200: Cost = 0.455913
Iteration 300: Cost = 0.453874
Iteration 400: Cost = 0.453316
Iteration 500: Cost = 0.453148
Iteration 600: Cost = 0.453096
Iteration 700: Cost = 0.453079
Iteration 800: Cost = 0.453074
Iteration 900: Cost = 0.453072
Iteration 999: Cost = 0.453071
```

Observe the Cost History:

```
# Plot cost history
plt.figure(figsize=(9, 6))
plt.plot(cost_history)
plt.xlabel("Iteration", fontsize=14)
plt.ylabel("Cost", fontsize=14)
plt.title("Cost vs Iteration", fontsize=14)
plt.tight_layout()
plt.show()
```



Did the model overfitt?

```
# Test model
y_train_pred = prediction(X_train_scaled, w, b)
y_test_pred = prediction(X_test_scaled, w, b)

 # Evaluate train and test performance
train_cost = costfunction_logreg(X_train_scaled, y_train, w, b)
test_cost = costfunction_logreg(X_test_scaled, y_test, w, b)
print(f"\nTrain Loss (Cost): {train_cost:.4f}")
print(f"Test Loss (Cost): {test_cost:.4f}")
```

```
Train Loss (Cost): 0.4531
Test Loss (Cost): 0.5146
```

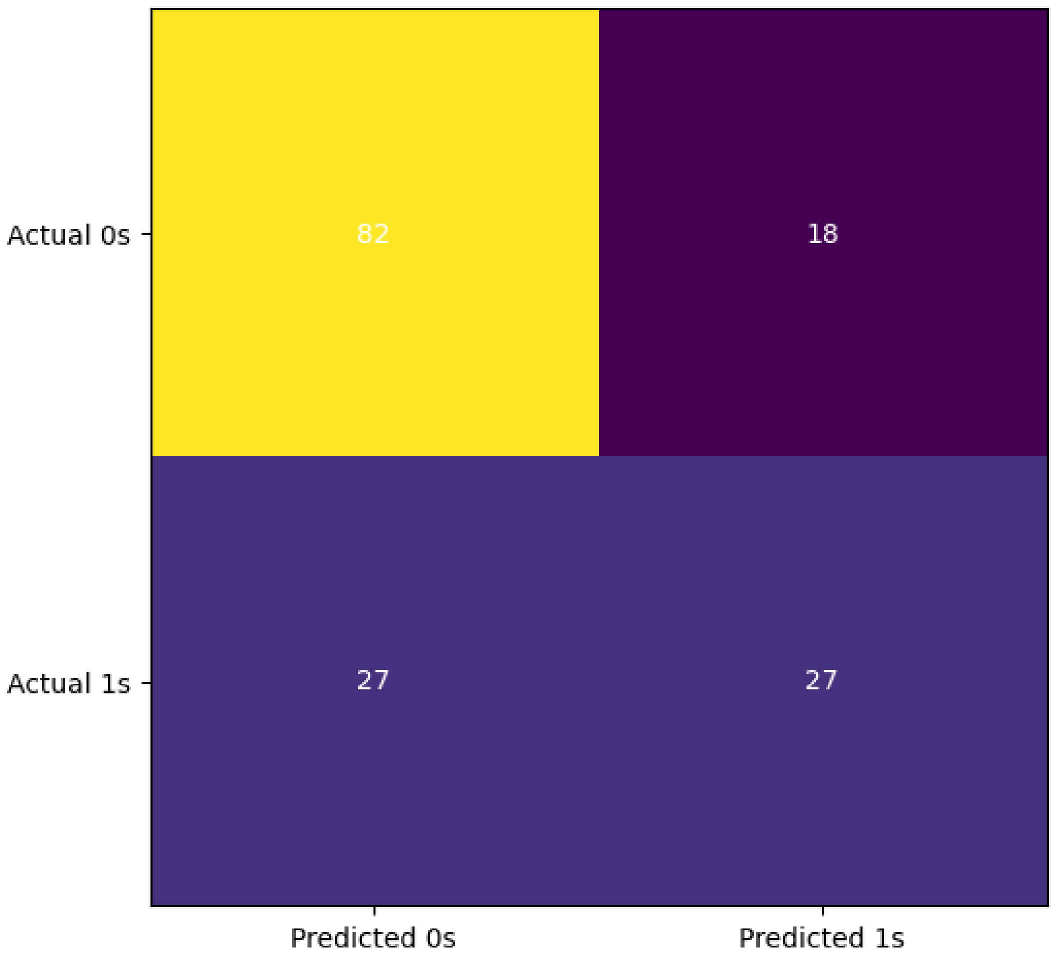
How well my model did?

```
# Accuracy on test data
test_accuracy = np.mean(y_test_pred == y_test) * 100
print(f"\nTest Accuracy: {test_accuracy:.2f}%")

# Evaluation
confusion_matrix, precision, recall, f1_score =
evaluate_classification(y_test, y_test_pred)
print(f"\nConfusion Matrix:\n{confusion_matrix}")
print(f"Precision: {precision:.2f}")
print(f"Recall: {recall:.2f}")
print(f"F1-Score: {f1_score:.2f}")
```

```
Test Accuracy: 70.78%

Confusion Matrix:
[[82 18]
 [27 27]]
Precision: 0.60
Recall: 0.50
F1-Score: 0.55
```

Visualize the Confusion Matrix - Optional.

```
# Visualizing Confusion Matrix
fig, ax = plt.subplots(figsize=(6, 6))
ax.imshow(confusion_matrix)
ax.grid(False)
ax.xaxis.set(ticks=(0, 1), ticklabels=('Predicted 0s', 'Predicted
1s'))
ax.yaxis.set(ticks=(0, 1), ticklabels=('Actual 0s', 'Actual 1s'))
ax.set_ylim(1.5, -0.5)
for i in range(2):
    for j in range(2):
        ax.text(j, i, confusion_matrix[i, j], ha='center',
va='center', color='white')
plt.show()
```


Actual 0s
Actual 1s
Predicted 0s
Predicted 1s
82
18
27
27