

3. Mathematically derive the average runtime complexity of the non-random pivot version of quicksort.

Ans: → Each partition splits the array into two parts: size k and $n-k-1$.
→ On average, the pivot splits the array into two roughly equal parts, so $k \approx \frac{n}{2}$,

The recurrence relation is:

$$T(n) = T(k) + T(n-k-1) + O(n)$$

For an average split, $k = n/2$:

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

Using the master Theorem for Recurrence Relations:

$$T(n) = 2T\left(\frac{n}{2}\right) + O(n)$$

Here:

$$\rightarrow a = 2$$

$$\rightarrow b = 2$$

$$\rightarrow k = 1$$

Since $a = b^k$:

$$T(n) = O(n \log n)$$

Thus, the average time complexity is: $O(n \log n)$