Hands on - 12

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a) Aggregate Method:-

→ In the aggregate method, we analyze the total cost for a sequence of operations and then divide it by the no. of operations to get the average cost (amortized cost) per operation.

1) Inserting an element without resizing: For each insertion where there is enough space in the table, the cost is constant. $O(1)$

2) Resizing: Every time the table doubles in size, the cost is proportional to the no. of elements being copied to the new table. if the table size is k before doubling, copying all k elements takes $O(k)$ time.

Total Cost of n Insertions:-

⇒ The cost of inserting n elements : $O(n)$ for the Insertions.

⇒ The Cost of resizing : The first doubling involves copying 1 element, the next involves copying 2, then 4 and so-on.

⇒ The sum of these doubling operations is;

$$O(1+2+4+8+\dots 2k) \sim O(n)$$

⇒ Amortized Cost per Insertion : $\dfrac{O(n)+O(n)}{n} = O(1)$

so, using aggregate method, the amortized time complexity for inserting n elements is $O(1)$.

ⓑ Accounting Method:-

⇒ The accounting method assigns "Credits" to each operation to account for the costs of future expensive operations.

1) Assigning Credits:

Each insertion will be charged 3 credits

⟹ 2 credits for the insertion itself, which pays constant time $O(1)$ operation.

⟹ 1 credit to help pay for the cost of future resizing operations.

2. Cost of insertion:

⟹ when no resizing happens, the cost is exactly 1 credit for the insertion.

⟹ when a resizing happens, it costs $O(c)$ for copying k elements. but since we have 1 credit saved for each previous insertion, we have enough credits to cover the resizing.

Resizing & Cost:-

⟹ when the table doubles the cost of copying elements doubles as well.

⟹ The total no. of credits that we collect is 3 credits per insertion, for n insertions, resulting in $3n$ credits.

⟹ each resizing is covered by the saved credits. The total no. of resizing operations is proportional to the no. of doublings (about $\log n$ times).

Final Amortized Cost:-

- Inserting n elements costs 1 credit each.
- Total credits collected $3n$
- Cost of each resizing operation is already covered by Saved Credits.

Amortized Cost per Insertion:-

$$= \frac{3n}{n}$$

$$= O(1)$$

∴ $O(1)$ is the amortized time Complexity for inserting n elements using accounting method for a dynamic table that doubles in size.