

ASDS 5303 Project 2
Due by: Friday Oct 4, 2024

While discussions are allowed, please ensure that you work on your project independently.
In the final submission include:

- Your code
- Your Output
- *Justification where necessary*

Q1: use sample function to generate a random 10x10 matrix A with integers in $[-10, 10]$ and compute the determinant of A.

Q2: Verify whether A is invertible. Justify.

Q3:

- a). If your A is NOT invertible., then regenerate A for this question. Otherwise, continue to 3b).
- b). Verify the determinant of inverse of A equals to 1 over the determinant of A.

Q4:

- a). Compute the null space of A.
- b). How many solutions do you expect $Ax=b$ to have and why? b can be any 10-dimensional real vectors.

Q5:

- a). For the null space obtained in 4a) Provide the vector(s) that spans the null space. Define the spanning set.
- b). What is the different between a vector that span a vector space and the spanning set

Q6:

- a). Generate a random 3 x 5 matrix A with integers in $[-10, 10]$ and a solution vector b with real numbers in $[-5, 5]$.
- b). Obtain the vector space including all the solutions of the system of equations $Ax=b$.

Q7:

- a). Generate a random 4 x 8 matrix, find the rank of A, and the rank of transpose of A. Compare these two ranks and show your result.
- b). Find the rank of $\text{null}(A)$, and compare the $\text{rank}(A) + \text{null}(A)$ with the number of columns of A.

Q8:

- a). Generate two random vectors v and u with the size defined by yourself. Generate a matrix from these two vectors. (Hint: use $v * u^T$)
- b). Find the rank of this matrix. Find the ranks of vector v and u, respectively. Compare these three ranks with each other and give your conclusion.

Q9:

- a). Generate a square matrix A with real numbers, of a size defined by yourself. Find an orthogonal basis of image of A and an orthogonal basis of null of A.
- b). Find the orthonormal Bases of image of A and null of A, respectively.

Q10:

- a). Generate a real random square matrix A with the size defined by yourself. Find all the eigenvalues and corresponding eigenvectors of A.
- b). Check if the eigenvectors from different eigenvalues are orthogonal to each other. If so, please state your conclusion.

Bonus Question:

The following questions will be graded as bonus points.

- a). Generate a random square, and invertible matrix A with the size defined by yourself.
- b). Design the Gram-Schmit Algorithm in R and find the orthonormal basis of image of A .
- c). Generate a real random square matrix A with a size greater than 1000.
- d). Try if you can get all the eigenvalues of A and all the eigenvectors of A . How many eigenvalues and eigenvectors can you get from R?

Challenge Yourself:

The following questions will not be graded but are provided as means to challenge yourself. Material for these questions may not have been covered in class so completing this section is optional

- a). For matrix A , defined in Bonus Question a). Find the QR decomposition of A . Write down the Q and R matrices, respectively.
- b). For matrix A , defined in Bonus Question c). can you just get the largest eigenvalue of A and its eigenvector.
- c). For matrix A , defined in Bonus Question c). can you just get the smallest eigenvalue of A and its eigenvector