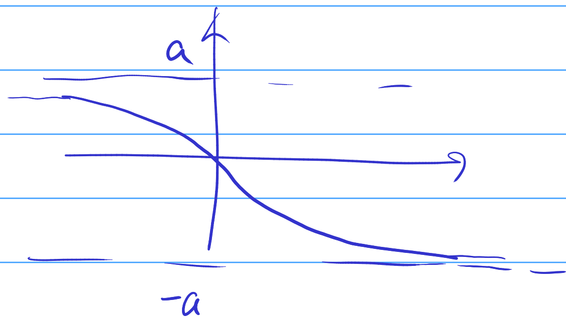
① $\begin{cases} \sum\limits_{1}^{n} x_i^2 \to extr \\ \sum\limits_{1}^{n} x_i^4 \le 1 \end{cases}$ $\quad \underline{\text{Th B-cp}}$

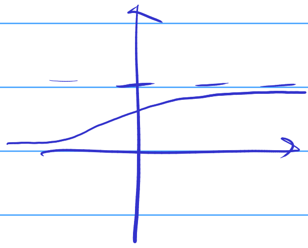
$0 \in absmin$ ①

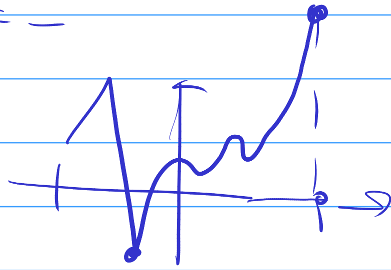
$A_n: x_i = \pm \frac{1}{\sqrt[4]{n}}, i=\overline{1,n}$ $\quad A_n \in absmax$ ①

$A_k: x_i = \pm \frac{1}{\sqrt[4]{k}}, i=\overline{1,k}, 1 \le k < n$

A_k – стац. т.







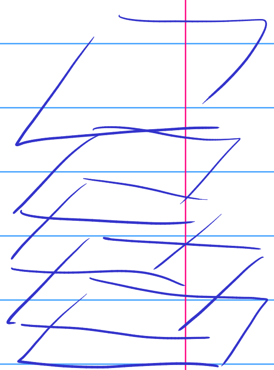
Ⅰ $\geqslant$ Ⅱ

$\sqrt[n]{\underbrace{A \cdot A \cdot A}_{n}} = A = A$

$\sqrt[n]{\prod\limits_{1}^{n} x_i} \le \frac{\sum\limits_{1}^{n} x_i}{n}, \quad x_i \geqslant 0$

$\parallel$

A



$\begin{cases} \sqrt[n]{\prod\limits_{1}^{n} x_i} \to max \\ \sum\limits_{i=1}^{n} x_i = nA \\ x_i \geqslant 0 \end{cases}$

$x_1^{\frac{1}{n}} \cdot x_2^{\frac{1}{n}} \dots x_n^{\frac{1}{n}}$

$x_1^{\frac{1}{n}-1} < 0$

$\prod x_i = y$

$f(y) = y \qquad \sqrt[n]{y_1} > \sqrt[n]{y_2}$, если $y_1 > y_2$

$$(*)\quad \begin{cases} \prod\limits_1^n x_i \to \max \\ \lambda_1 \to \sum\limits_1^n x_i = nA \\ x_i \geq 0,\ i=\overline{1,n} \end{cases} \qquad (\text{БОО } A>0)$$

$\lambda_2 \quad -x_1 \leq 0$

$\lambda_3 \quad -x_2 \leq 0$

$\dots$

$\lambda_{n+1} \quad -x_n \leq 0$

(M) — комп. $\Rightarrow$ дост. max и min

$$\mathcal{L}(x,\lambda) = \lambda_0 \prod_1^n x_i + \lambda_1 \left(\sum_1^n x_i - nA\right) - \sum_{i=1}^n \lambda_{i+1} x_i$$

$$\begin{cases} \mathcal{L}'_{x_i} = \lambda_0 \prod\limits_{j\neq i} x_j + \lambda_1 - \lambda_{i+1} = 0, \quad i=\overline{1,n} \\ \sum\limits_1^n x_i = nA \\ \lambda_{i+1} x_i = 0, \quad i = \overline{1,n} \\ x_i \geq 0 \ \forall i \end{cases}$$

$$\begin{cases} \sum x_i = nA \\ x_i \geq 0 \end{cases} (M) \qquad (-k, k, nA, 0, \dots, 0)$$

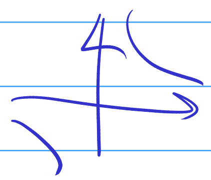
M замкн. $\Leftrightarrow \forall x_n \in M: x_n \to x \in \mathbb{R}^n \Rightarrow x \in M$

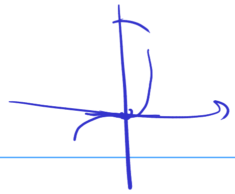
M – огр $\Leftrightarrow \exists r, x: M \subset B(x, r)$

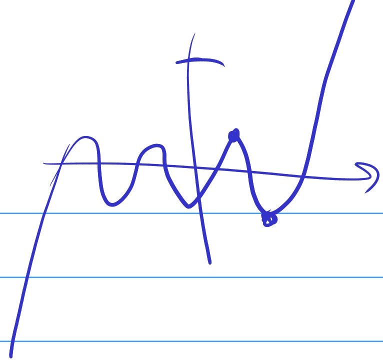
$\underline{k > 1}$ $\quad \left(\frac{A}{k}, A - \frac{A}{k}, 2A, \dots, A\right), \quad \underline{x_i > 0}$

$\downarrow$

$(0, A, 2A, \dots, A)$







$$\left\{\begin{array}{l}\sum x_i = nA \\ \underbrace{x_i \geq 0}\end{array}\right. (M) \qquad \left\{\begin{array}{l} A_k \in M \\ A_k \to A \in \mathbb{R}^n \end{array}\right.$$

$A \in M$?

$$\begin{array}{l} \quad\ \ 0 \quad\ \ 0 \qquad\qquad 0 \\ \quad\ \ \wedge\!\!| \quad\ \ \wedge\!\!| \qquad\qquad \wedge\!\!| \\ A_k = (a_1^k, \ a_2^k, \ \dots, \ a_n^k) \\ \quad\ \ \downarrow_{k\to\infty} \ \downarrow \qquad\qquad \downarrow \\ \quad\ \ \underbrace{(a_1, \ \ a_2, \ \ \dots, \ \ a_n) \\ \quad\ \ \ \vee\!\!| \quad\ \ \vee\!\!| \qquad\qquad \vee\!\!| \\ \quad\ \ \ 0 \quad\ \ \ 0 \qquad\qquad 0} \end{array}$$

$$\begin{array}{c} \sum a_i^k = nA \\ {\scriptstyle k\to\infty}\downarrow \qquad \downarrow \\ \underbrace{\sum a_i = nA} \end{array}$$

$\prod x_i$

$$\left\{\begin{array}{l} \mathcal{L}'_{x_i} = \lambda_0 \prod\limits_{j\neq i} x_j + \lambda_1 - \lambda_{i+1} = 0, \quad i=\overline{1,n} \\ \sum x_i = nA \\ \lambda_{i+1} x_i = 0, \quad i = \overline{1,n} \\ x_i \geq 0, \quad i=\overline{1,n} \end{array}\right.$$

$\boxed{(2n+1)\text{-ур.}}$

a) $\lambda_0 = 0$

$\lambda_1 = \lambda_{i+1} \ \forall i=\overline{1,n} \Rightarrow \lambda_i = \lambda_j$

$\lambda = 0$?!

б) $\lambda_0 \neq 0 \qquad \lambda_0 = 1$

$\boxed{n\text{-ур-й!!!}} \to \left\{\begin{array}{l} \prod\limits_{j\neq i} x_j + \lambda_1 - \lambda_{i+1} = 0 \\ \sum x_i = nA \\ \lambda_{i+1} x_i = 0, \quad x_i \geq 0 \end{array}\right. \quad (*)$

$$\prod_{i=1}^{n} x_i \to extr$$

$$x_i \geq 0$$

Если в реш. (*) есть хоть 1 $x_i = 0 \Rightarrow \prod x_i = 0 = \varphi_{\min}$ на M,

т.к. $\varphi_0(x) \geq 0 \quad \forall x \in M$

Ост-сь р-ть случай:

$\forall i \quad x_i \neq 0 \Rightarrow \lambda_{i+1} = 0 \quad \forall i$

$$\times x_i \; \Big| \quad \begin{cases} \prod_{j \neq i} x_j + \lambda_1 = 0 \quad \forall i \\ \sum_i^n x_i = nA \\ x_i > 0 \end{cases}$$

n

$$\begin{cases} \prod_1 x_j + \lambda_1 x_i = 0 \quad \forall i = \overline{1,n} \\ \sum x_i = nA \\ x_i > 0 \end{cases}$$

$\lambda_1 x_i = \lambda_1 x_j \quad \forall i,j$

$\lambda_1 \neq 0$, т.к. $\prod x_j > 0$

$\Downarrow$

$x_i = x_j = A$

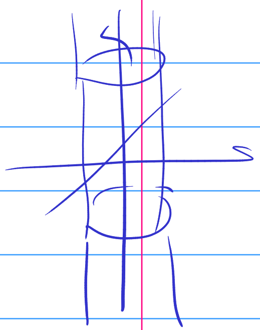
|

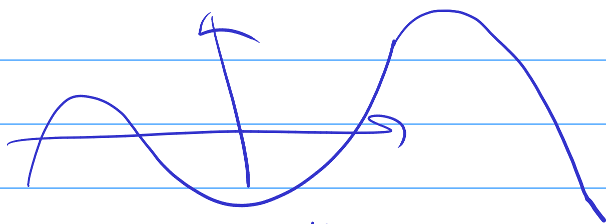
т. max по Th В-са

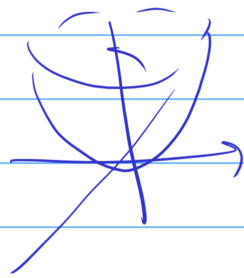
$$-\frac{\prod x_j + \lambda_1 x_1 = 0}{\prod x_j + \lambda_1 x_2 = 0}$$

По th В-са max и min в (*) дост-ся

Min дост-ся в тех т., кот. явл. реш. сист. (*) и у кот. хоть одна нулев. к-та.







$\Downarrow$

$(A, \dots, A) \in$ glob max