Assignment 2

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***Abstract*—This document finds the foot of the perpendicular from the origin to the given planes**

Download latex-tikz codes from

https://github.com/surbhi0912/EE5609/tree/master/Assignment2

1 Problem

For the following planes, find the coordinates of the foot of the perpendicular drawn from the origin

a) $\begin{pmatrix} 2 & 3 & 4 \end{pmatrix} \mathbf{x} = 12$

b) $\begin{pmatrix} 3 & 4 & -6 \end{pmatrix} \mathbf{x} = 0$

c) $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \mathbf{x} = 1$

d) $\begin{pmatrix} 0 & 5 & 0 \end{pmatrix} \mathbf{x} = -8$

2 Explanation

The equation of a plane is given as

$$\mathbf{n}^T \mathbf{x} = c \tag{2.0.1}$$

where $\mathbf{n}$ = normal vector to the plane

Let $\mathbf{O} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ be the origin and $\mathbf{P} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ be the foot of the perpendicular drawn from the origin to the plane.

The position vector from $\mathbf{O}$ to $\mathbf{P}$ is $(\mathbf{P} - \mathbf{O})$

Since $(\mathbf{P} - \mathbf{O})$ is perpendicular to the plane, it is parallel to the normal of the plane $\mathbf{n}$. So,

$$\mathbf{P} - \mathbf{O} = k\mathbf{n} \tag{2.0.2}$$

$$\mathbf{P} = k\mathbf{n} + \mathbf{O} \tag{2.0.3}$$

where k is any scalar quantity.

Since $\mathbf{O}$ is a null vector,

$$\implies \mathbf{P} = k\mathbf{n} \tag{2.0.4}$$

Since $\mathbf{P}$ lies on the given plane, it must satisfy the equation of the plane. Therefore,

$$\mathbf{n}^T \mathbf{P} = c \tag{2.0.5}$$

$$\implies \mathbf{n}^T (k\mathbf{n}) = c \tag{2.0.6}$$

$$\implies k\mathbf{n}^T \mathbf{n} = c \tag{2.0.7}$$

3 Solution

a) $\begin{pmatrix} 2 & 3 & 4 \end{pmatrix} \mathbf{x} = 12$

Comparing with (2.0.1), we get $\mathbf{n}^T = \begin{pmatrix} 2 & 3 & 4 \end{pmatrix}$ and $c = 12$. Using this in (2.0.7)

$$k \begin{pmatrix} 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = 12 \tag{3.0.1}$$

$$\implies 29k = 12 \tag{3.0.2}$$

$$\implies k = \frac{12}{29} \tag{3.0.3}$$

$$\mathbf{P} = k\mathbf{n} \implies \mathbf{P} = \frac{12}{29} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \tag{3.0.4}$$

$$\implies \mathbf{P} = \begin{pmatrix} \frac{24}{29} \\ \frac{36}{29} \\ \frac{48}{29} \end{pmatrix} \tag{3.0.5}$$

b) $\begin{pmatrix} 3 & 4 & -6 \end{pmatrix} \mathbf{x} = 0$

Comparing with (2.0.1), we get $\mathbf{n}^T = \begin{pmatrix} 3 & 4 & -6 \end{pmatrix}$ and $c = 0$. Using this in (2.0.7)

$$k \begin{pmatrix} 3 & 4 & -6 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \\ -6 \end{pmatrix} = 0 \tag{3.0.6}$$

$$\implies -11k = 0 \tag{3.0.7}$$

$$\implies k = 0 \tag{3.0.8}$$

$$\mathbf{P} = k\mathbf{n} \implies \mathbf{P} = 0 \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \tag{3.0.9}$$

$$\implies \mathbf{P} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{3.0.10}$$

This shows that the plane $\begin{pmatrix} 3 & 4 & -6 \end{pmatrix} \mathbf{x} = 0$ passes through the origin.

c) $\begin{pmatrix}1 & 1 & 1\end{pmatrix}\mathbf{x} = 1$

Comparing with (2.0.1), we get $\mathbf{n}^T = \begin{pmatrix}1 & 1 & 1\end{pmatrix}$ and $c = 1$. Using this in (2.0.7)

$$k\begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} = 1 \tag{3.0.11}$$

$$\implies 3k = 1 \tag{3.0.12}$$

$$\implies k = \frac{1}{3} \tag{3.0.13}$$

$$\mathbf{P} = k\mathbf{n} \implies \mathbf{P} = \frac{1}{3}\begin{pmatrix}1\\1\\1\end{pmatrix} \tag{3.0.14}$$

$$\implies \mathbf{P} = \begin{pmatrix}\frac{1}{3}\\\frac{1}{3}\\\frac{1}{3}\end{pmatrix} \tag{3.0.15}$$

d) $\begin{pmatrix}0 & 5 & 0\end{pmatrix}\mathbf{x} = -8$

Comparing with (2.0.1), we get $\mathbf{n}^T = \begin{pmatrix}0 & 5 & 0\end{pmatrix}$ and $c = -8$. Using this in (2.0.7)

$$k\begin{pmatrix}0 & 5 & 0\end{pmatrix}\begin{pmatrix}0\\5\\0\end{pmatrix} = -8 \tag{3.0.16}$$

$$\implies 25k = -8 \tag{3.0.17}$$

$$\implies k = \frac{-8}{25} \tag{3.0.18}$$

$$\mathbf{P} = k\mathbf{n} \implies \mathbf{P} = \frac{-8}{25}\begin{pmatrix}0\\5\\0\end{pmatrix} \tag{3.0.19}$$

$$\implies \mathbf{P} = \begin{pmatrix}0\\\frac{-8}{5}\\0\end{pmatrix} \tag{3.0.20}$$