Assignment 4

Surbhi Agarwal

Abstract—**This document finds the equation of tangent to the circle which is parallel to another given line.**

Download all python codes from

https://github.com/surbhi0912/EE5609/

and latex-tikz codes from

https://github.com/surbhi0912/EE5609/

1 Problem

Find the equations of the tangents to the circle

$$\mathbf{x}^T\mathbf{x} - \begin{pmatrix} 4 & 3 \end{pmatrix}\mathbf{x} + 5 = 0$$

that are parallel to the line

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 0$$

2 Solution

The given equation of the circle is

$$\mathbf{x}^T\mathbf{x} - \begin{pmatrix} 4 & 3 \end{pmatrix}\mathbf{x} + 5 = 0 \tag{2.0.1}$$

Comparing this to the general second degree equation,

$$\mathbf{V} = \mathbf{I}, \quad \mathbf{u} = \begin{pmatrix} -2 \\ \frac{-3}{2} \end{pmatrix}, \quad f = 5 \tag{2.0.2}$$

Tthe centre $\mathbf{c}$ and radius r are given by

$$\mathbf{c} = -\mathbf{u} = \begin{pmatrix} 2 \\ \frac{3}{2} \end{pmatrix}, r = \sqrt{\mathbf{u}^T\mathbf{u} - f} = \frac{\sqrt{5}}{2} \tag{2.0.3}$$

The equation of the given line is

$$\begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{x} = 0 \tag{2.0.4}$$

Comparing with standard equation of a line $\mathbf{n}^T\mathbf{x} = c$,

$$\mathbf{n}^T = \begin{pmatrix} 1 & 1 \end{pmatrix} \tag{2.0.5}$$

$$\implies \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.0.6}$$

Since the tangent is parallel to this line, it's normal vector is parallel to the normal vector of the given line.

$$\implies \mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.0.7}$$

And its direction vector $\mathbf{m}$ is given by

$$\mathbf{n}^T\mathbf{m} = 0 \tag{2.0.8}$$

$$\implies \begin{pmatrix} 1 & 1 \end{pmatrix}\mathbf{m} = 0 \tag{2.0.9}$$

$$\implies \mathbf{m} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{2.0.10}$$

So, the equation for the tangent can be written as:

$$L : \mathbf{x} = \mathbf{q} + \lambda\mathbf{m} \tag{2.0.11}$$

The point of contact $\mathbf{q}$ is given as

$$\mathbf{q} = \kappa\mathbf{n} - \mathbf{u} \tag{2.0.12}$$

where,

$$\kappa = \pm\sqrt{\frac{\mathbf{u}^T\mathbf{u} - f}{\mathbf{n}^T\mathbf{n}}} \tag{2.0.13}$$

Substituting values from (2.0.2) and (2.0.7) into (2.0.13)

$$\kappa = \pm\frac{\sqrt{5}}{2\sqrt{2}} \tag{2.0.14}$$

Substituting values from (2.0.2), (2.0.7), (2.0.14)

$$\mathbf{q} = \pm\frac{\sqrt{5}}{2\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} -2 \\ \frac{-3}{2} \end{pmatrix} \tag{2.0.15}$$

$$\mathbf{q}_1 = \frac{1}{2}\begin{pmatrix} \sqrt{\frac{5}{2}} + 4 \\ \sqrt{\frac{5}{2}} + 3 \end{pmatrix} \tag{2.0.16}$$

$$\mathbf{q}_2 = \frac{1}{2}\begin{pmatrix} -\sqrt{\frac{5}{2}} + 4 \\ -\sqrt{\frac{5}{2}} + 3 \end{pmatrix} \tag{2.0.17}$$

From this, we get the equation of tangent is

$$(\mathbf{V}\mathbf{q}+\mathbf{u})^T\mathbf{x}+\mathbf{u}^T\mathbf{q}+f=0 \qquad \text{Here } \mathbf{V}=I \tag{2.0.18}$$

Substituting $\mathbf{q}=\kappa\mathbf{n}-\mathbf{u}$

$$(\kappa\mathbf{n}-\mathbf{u}+\mathbf{u})^T\mathbf{x}+\mathbf{u}^T(\kappa\mathbf{n}-\mathbf{u})+f=0 \tag{2.0.19}$$

$$\implies \kappa\mathbf{n}^T\mathbf{x}+\kappa\mathbf{u}^T\mathbf{n}-\mathbf{u}^T\mathbf{u}+f=0 \tag{2.0.20}$$

Substituing values from (2.0.2), (2.0.7) and (2.0.14), we get the equation of the tangents as :

$$\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x}=\frac{7+\sqrt{10}}{2} \tag{2.0.21}$$

and

$$\begin{pmatrix}1 & 1\end{pmatrix}\mathbf{x}=\frac{7-\sqrt{10}}{2} \tag{2.0.22}$$

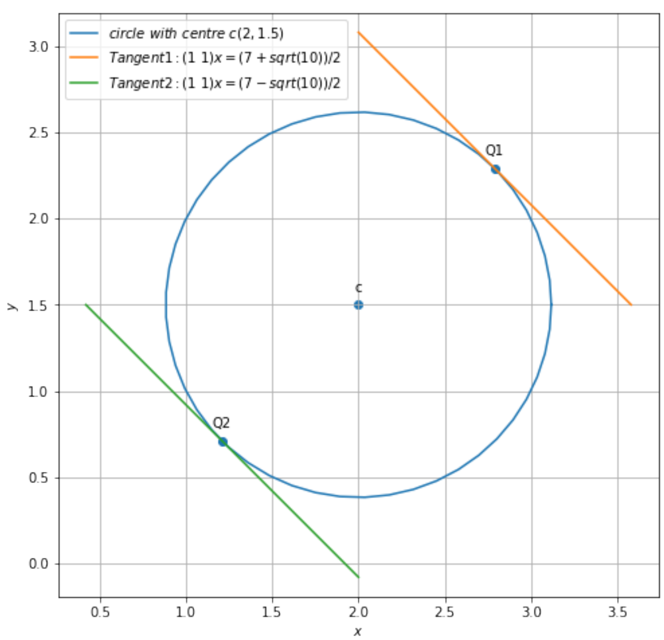

Fig. 1: Tangents to a given circle that are parallel to the line (1 1)x = 0