Assignment 6

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***Abstract*—This document deals with QR decomposition and Singlular Value Decomposition**

Download all python codes from

https://github.com/surbhi0912/EE5609/

and latex-tikz codes from

https://github.com/surbhi0912/EE5609/

1 Problem

1. Find the QR decomposition of $\mathbf{V} = \begin{pmatrix} 16 & 12 \\ 12 & 9 \end{pmatrix}$

2. Find the vertex of a parabola $(4x + 3y + 15)^2 = 5(3x - 4y)$ using SVD and verify solution using least squares.

2 Solution

2.1 QR decomposition of V

Let the column vectors of $\mathbf{V}$ be α and β:

$$\alpha = \begin{pmatrix} 16 \\ 12 \end{pmatrix} \tag{2.1.1}$$

$$\beta = \begin{pmatrix} 12 \\ 9 \end{pmatrix} \tag{2.1.2}$$

We can express

$$\alpha = k_1 \mathbf{u}_1 \tag{2.1.3}$$

$$\beta = r_1 \mathbf{u}_1 + k_2 \mathbf{u}_2 \tag{2.1.4}$$

where

$$k_1 = \|\alpha\| = \sqrt{16^2 + 12^2} = 20 \tag{2.1.5}$$

$$\mathbf{u}_1 = \frac{\alpha}{k_1} = \frac{1}{20}\begin{pmatrix} 16 \\ 12 \end{pmatrix} = \begin{pmatrix} \frac{4}{5} \\ \frac{3}{5} \end{pmatrix} \tag{2.1.6}$$

$$r_1 = \frac{\mathbf{u}_1^T \beta}{\|\mathbf{u}_1\|^2} = 15 \tag{2.1.7}$$

$$\mathbf{u}_2 = \frac{\beta - r_1 \mathbf{u}_1}{\|\beta - r_1 \mathbf{u}_1\|} = \frac{1}{0}\begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.1.8}$$