

Assignment 2

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Abstract—This document finds the foot of the perpendicular from the origin to the given planes

Download all python codes from

<https://github.com/surbhi0912/EE5609/tree/master/Assignment2/codes>

and latex-tikz codes from

<https://github.com/surbhi0912/EE5609/tree/master/Assignment2>

Since \mathbf{P} lies on the given plane, it must satisfy the equation of the plane. Therefore,

$$\mathbf{n}^T \mathbf{P} = c \quad (2.0.5)$$

$$\Rightarrow \mathbf{n}^T (k\mathbf{n}) = c \quad (2.0.6)$$

$$\Rightarrow k\mathbf{n}^T \mathbf{n} = c \quad (2.0.7)$$

3 SOLUTION

1 PROBLEM

For the following planes, find the coordinates of the foot of the perpendicular drawn from the origin

a) $(2 \ 3 \ 4)\mathbf{x} = 12$

b) $(3 \ 4 \ -6)\mathbf{x} = 0$

c) $(1 \ 1 \ 1)\mathbf{x} = 1$

d) $(0 \ 5 \ 0)\mathbf{x} = -8$

2 EXPLANATION

The equation of a plane is given as

$$\mathbf{n}^T \mathbf{x} = c \quad (2.0.1)$$

where \mathbf{n} = normal vector to the plane

Let $\mathbf{O} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ be the origin and $\mathbf{P} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ be the foot of the perpendicular drawn from the origin to the plane.

The position vector from \mathbf{O} to \mathbf{P} is $(\mathbf{P} - \mathbf{O})$

Since $(\mathbf{P} - \mathbf{O})$ is perpendicular to the plane, it is parallel to the normal of the plane \mathbf{n} . So,

$$\mathbf{P} - \mathbf{O} = k\mathbf{n} \quad (2.0.2)$$

$$\mathbf{P} = k\mathbf{n} + \mathbf{O} \quad (2.0.3)$$

where k is any scalar quantity.

Since \mathbf{O} is a null vector,

$$\Rightarrow \mathbf{P} = k\mathbf{n} \quad (2.0.4)$$

a) $(2 \ 3 \ 4)\mathbf{x} = 12$

Comparing with (2.0.1), we get $\mathbf{n}^T = (2 \ 3 \ 4)$ and $c = 12$. Using this in (2.0.7)

$$k(2 \ 3 \ 4) \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} = 12 \quad (3.0.1)$$

$$\Rightarrow 29k = 12 \quad (3.0.2)$$

$$\Rightarrow k = \frac{12}{29} \quad (3.0.3)$$

$$\mathbf{P} = k\mathbf{n} \Rightarrow \mathbf{P} = \frac{12}{29} \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \quad (3.0.4)$$

$$\Rightarrow \mathbf{P} = \begin{pmatrix} \frac{24}{29} \\ \frac{36}{29} \\ \frac{48}{29} \end{pmatrix} \quad (3.0.5)$$

b) $(3 \ 4 \ -6)\mathbf{x} = 0$

Comparing with (2.0.1), we get $\mathbf{n}^T = (3 \ 4 \ -6)$ and $c = 0$. Using this in (2.0.7)

$$k(3 \ 4 \ -6) \begin{pmatrix} 3 \\ 4 \\ -6 \end{pmatrix} = 0 \quad (3.0.6)$$

$$\Rightarrow -11k = 0 \quad (3.0.7)$$

$$\Rightarrow k = 0 \quad (3.0.8)$$

$$\mathbf{P} = k\mathbf{n} \implies \mathbf{P} = 0 \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix} \quad (3.0.9)$$

$$\implies \mathbf{P} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad (3.0.10)$$

This shows that the plane $(3 \ 4 \ -6)\mathbf{x} = 0$ passes through the origin.

$$c) (1 \ 1 \ 1)\mathbf{x} = 1$$

Comparing with (2.0.1), we get $\mathbf{n}^T = (1 \ 1 \ 1)$ and $c = 1$. Using this in (2.0.7)

$$k \begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 1 \quad (3.0.11)$$

$$\implies 3k = 1 \quad (3.0.12)$$

$$\implies k = \frac{1}{3} \quad (3.0.13)$$

$$\mathbf{P} = k\mathbf{n} \implies \mathbf{P} = \frac{1}{3} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \quad (3.0.14)$$

$$\implies \mathbf{P} = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{pmatrix} \quad (3.0.15)$$

$$d) (0 \ 5 \ 0)\mathbf{x} = -8$$

$$\implies (0 \ -5 \ 0)\mathbf{x} = 8$$

Comparing with (2.0.1), we get $\mathbf{n}^T = (0 \ -5 \ 0)$ and $c = 8$. Using this in (2.0.7)

$$k \begin{pmatrix} 0 & -5 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ -5 \\ 0 \end{pmatrix} = 8 \quad (3.0.16)$$

$$\implies 25k = 8 \quad (3.0.17)$$

$$\implies k = \frac{8}{25} \quad (3.0.18)$$

$$\mathbf{P} = k\mathbf{n} \implies \mathbf{P} = \frac{8}{25} \begin{pmatrix} 0 \\ -5 \\ 0 \end{pmatrix} \quad (3.0.19)$$

$$\implies \mathbf{P} = \begin{pmatrix} 0 \\ -8 \\ 0 \end{pmatrix} \quad (3.0.20)$$

The Figures for the planes are shown in order: b b

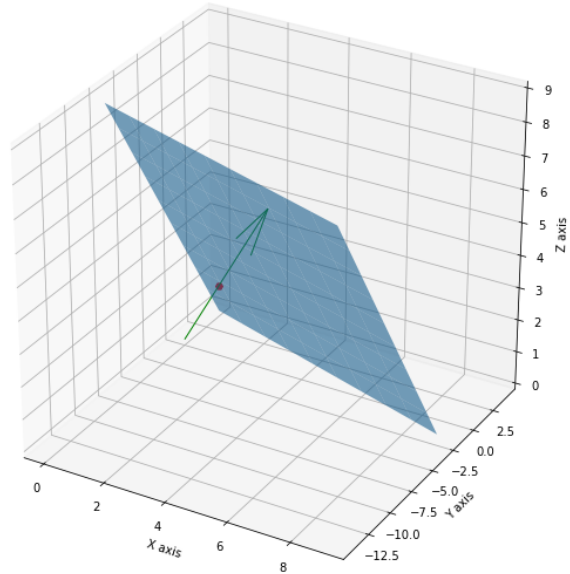


Fig. a: Perpendicular drawn from origin to the plane $(2 \ 3 \ 4)\mathbf{x} = 12$

b b

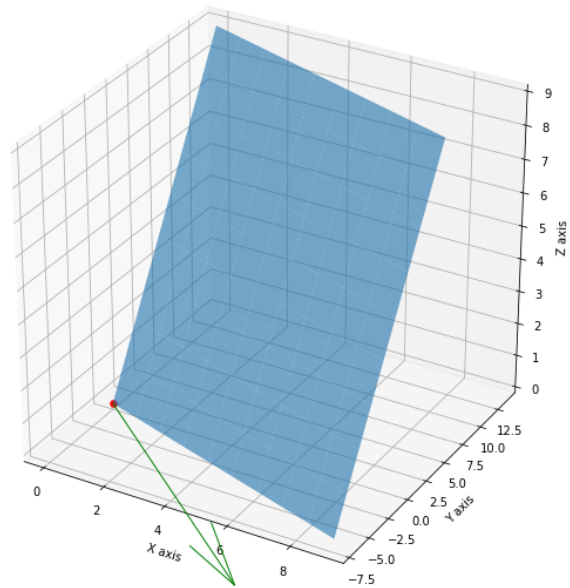


Fig. b: Perpendicular drawn from origin to the plane $(3 \ 4 \ -6)\mathbf{x} = 0$

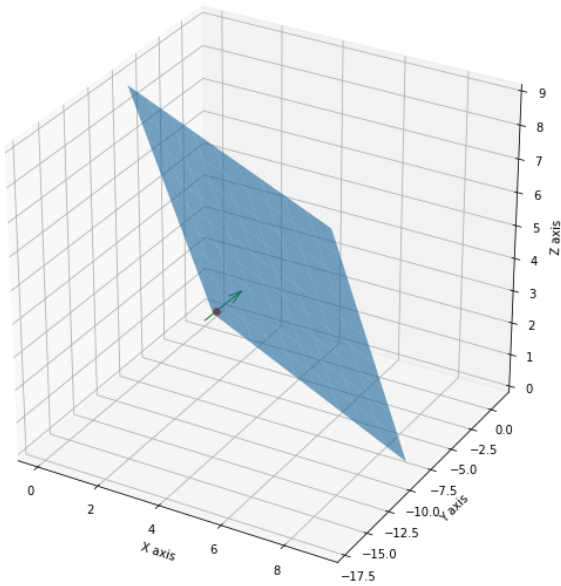


Fig. b: Perpendicular drawn from origin to the plane $\begin{pmatrix} 1 & 1 & 1 \end{pmatrix} \mathbf{x} = 1$

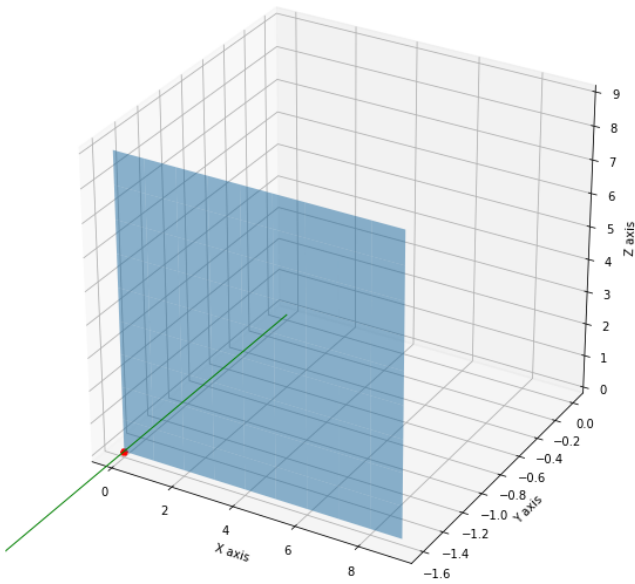


Fig. b: Perpendicular drawn from origin to the plane $\begin{pmatrix} 0 & 5 & 0 \end{pmatrix} \mathbf{x} = -8$