

Assignment 10

Surbhi Agarwal

Download all latex-tikz codes from

<https://github.com/surbhi0912/EE5609/>

Substituting this in (2.0.8)

$$k \sin \beta \cos \theta + k \cos \beta \sin \theta = 1 \quad (2.0.10)$$

$$\implies k \sin(\theta + \beta) = 1 \quad (2.0.11)$$

1 PROBLEM

Given that there are real constants a, b, c, d such that the identity

$$\lambda x^2 + 2xy + y^2 = (ax + by)^2 + (cx + dy)^2 \quad (1.0.1)$$

holds for all $x, y \in \mathbb{R}$. This implies

- 1) $\lambda = -5$
- 2) $\lambda \geq 1$
- 3) $0 < \lambda < 1$
- 4) there is no such $\lambda \in \mathbb{R}$

Substituting (2.0.9) in (2.0.3),

$$\lambda = k^2 \sin^2 \beta + k^2 \cos^2 \beta \quad (2.0.12)$$

$$= k^2(\sin^2 \beta + \cos^2 \beta) \quad (2.0.13)$$

$$= k^2 \quad (2.0.14)$$

Now, since $|\sin(\theta + \beta)| \leq 1$, then from (2.0.11), we get $|k| \geq 1$, hence $k^2 \geq 1$. Using this in (2.0.14),

$$\lambda \geq 1 \quad (2.0.15)$$

So from the given options, option 2) $\lambda \geq 1$ is correct.

2 SOLUTION

Given that

$$\lambda x^2 + 2xy + y^2 = (ax + by)^2 + (cx + dy)^2 \quad (2.0.1)$$

Arranging this in form of a matrix,

$$\begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} \lambda & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} a^2 + c^2 & ab + cd \\ ab + cd & b^2 + d^2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (2.0.2)$$

From this, we get

$$\lambda = a^2 + c^2 \quad (2.0.3)$$

$$ab + cd = 1 \quad (2.0.4)$$

$$b^2 + d^2 = 1 \quad (2.0.5)$$

Let

$$b = \cos \theta, d = \sin \theta \quad \forall \theta \in \mathbb{R} \quad (2.0.6)$$

Substituting this in (2.0.5)

$$\cos^2 \theta + \sin^2 \theta = 1 \quad (2.0.7)$$

Substituting (2.0.6) in (2.0.4)

$$a \cos \theta + c \sin \theta = 1 \quad (2.0.8)$$

$$\text{let } a = k \sin \beta, c = k \cos \beta \quad \forall k, \beta \in \mathbb{R} \quad (2.0.9)$$