APPLICATION OF GAME THEORY IN THREE-PERSON SUPPLY CHAIN ANALYSIS

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BY

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2020

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ABSTRACT

Game theory is generally described as the study of mathematical models concerning how rational individuals or organizations make Decisions in a competitive environment. Due to competition cooperative existing amongst supply chain members, especially in Analyzing the interactions and coordination between supply chain members. This has resulted in a substantial body of works aimed at providing further insights into how game theory can be used to explain the nature of the problems that can arise in basic and complex supply chains. Three-person supply chain game comprising of a supplier, a retailer, and a consumer. This is an unfortunate excluded as a consumers preferences and decisions will directly affect market demand and total supply chain performance. A further complication total arises in a three-person game is the possibility that each player may act individually as a self maximizer or may wish to join in a coalition with one other player to achieve a better outcomes.

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Notations

Decision Variables

S	The shortage controlled by the seller	Units
P	Selling price charged by the buyer to the customer	\$/units
M	Marketing expenditure incurred by the buyer	\$/units
V	The price charged by the seller to the buyer	\$/units
Q	Lot size determined by the buyer	Units

Input Parameters

A_S	Seller's setup cost	\$/setup
A_b	Buyer's ordering cost	\$/order
C_S	Seller's production cost including purchasing cost	\$/units
w	Seller's marginal selling price	\$/units Where $W > 1$
c_1	Buyer's Shortage Cost	\$/units
c_2	Seller's Shortage Cost	\$/units
A	Price elasticity of demand function	$\alpha > 1$
B	Marketing Expenditure elasticity of demand	$0 < \beta < 1, \beta + 1 < \alpha$
u	Scaling constant for production	$u > 1$
ς	Quality level elasticity of demand	$0 < \varsigma < 1$
ξ	Disutility coefficient	$0 \leq \xi \leq 1$
C_3	Consumer's disutility caused by Shortages	\$/units
$V(x)$	Consumer's valuation of product	$X = x$
I_s	Capital cost rate for the seller during the credit period	
D	Market demand rate	Units/cycle
$F(\cdot)$	Cumulative probability distribution function (cdf)	

CHAPTER 1
INTRODUCTION

In recent years, researchers have found that game theory is an essential tool in investigating the behavior of supply chain members and in modeling the complex interactions that often occur between them. In a supply chain management system, the consumer is the terminal point of the chain, behavior and preference will directly affect market demand and total chain performance without a proper and better understanding of consumer's behavior and preference, orderly trading will be disrupted which can harm the proper functioning of the whole supply chain. Examples of this abound in recent history, such as the recent collapse of the several film companies and the shrinking of a media giant company, as the result of consumers shifting from traditional film and newspaper to digital and internal media,
and the loss of traditional market to competitors.

It is therefore crucial to investigate consumer's behavior and preference by including in the supply chain study, and the investigate its performance can be enhanced or improved through interaction with other members in the chain. We propose a three-person supply chain which consists of a single supplier, retailer who will interact with a strategic consumer in several non-coperative games. In each Production cycle, the retailer orders a quantity of product from the supplier and sells it to the consumer. The supplier will determine the wholesale price and allowable shortage. The consumer's decision whether to purchase the product will depend on valuation of the product's quality. In a three-person game, each player may act individually as a self maximizer or some may form coalitions in an attempt to achieve a better outcome.

The best scenario will be obtained in a grand coalition among all the participants in the chain, but this is unrealistic in supply chains with three or more players due to factors such as cost sharing, profit sharing and a variety of strategic considerations. Therefore, we will limit the scope of by investigating a non-cooperative stackelberg leader.

Follows game under two scenarios :-

1 No coalition is formed among the players and each acts non-cooperatively and
2 A selection of two players act cooperatively by forming coalition against the third player.

The choice of a stackelberg game is due to the fact that it reflects many interactions prevailing in the market place where one or more players may have more power than the other players. Hence the strategies of the players will depend on whether they are the leader or follower in the game. under the first scenario, we will let each player takes turn as the leader and the other two players assume the role of the followers. Under the second scenario, we consider two types of coalition among players:

Supplier- Retailer coalition (called SRC)
Retailer-Consumer coalition (called RCC)

In each case, the coalition has more power than the lone player i.e, the coalition is the leader while the lone player the follower. The effectiveness of leadership and coalition formulation to a player's strategy and profit will be compared under these two scenerios.

1.1) Retailer's model :-

We itemize the terms that comprise the retailer's annual profit function after placing an order Q :

PD = Sales revenue

VD = Purchase cost

$A_b \frac{D}{Q}$ = Ordering cost

$C_bDF(R)$ = Lost sale cost due to inspection

iVI = Holding cost;

C_1B = Shortage cost;

Where $I = \frac{[Q(1-u^{-1})-s]^2}{2Q(1-u^{-1})}$ and $B = \frac{s^2}{2Q(1-u^{-1})}$ using the idea developed by Johnson and Montgomery.

The retailer profit function can be expressed as :

Retailer's annual profit = Sales revenue – Purchase cost – Ordering cost – Lost sale cost – Holding cost – Shortage cost.

This can be expressed mathematically as

$$\Pi_r(P,Q) = PD - VD - A_b\frac{D}{Q} - C_bDF(R) - iVI - C_1B$$

$$=PD - VD - A_b\frac{D}{Q} - C_bDF(R) - iV\frac{[Q(1-u^{-1})-S]^2}{2Q(1-u^{-1})} - \frac{C_1S^2}{2Q(1-u^{-1})}$$

1.2 Supplier's model :-

After receiving an order to supply the retailer. The supplier charged a unit selling price of V and allowed a possible shortage size of S. The supplier's annual profit function consists of the following terms :

VD = Sales revenue

C_sD = Production cost

$A_s\frac{D}{Q}$ = Setup cost

C_r DF(r) = Rework cost

iC_sI = Holding cost

C_2B = Shortage cost

Supplier's profit = Sales revenue – Production cost – Setup cost – Rework cost – Holding cost – Shortage cost.

Mathematically, this is expressed as

$$\Pi_s(V,S) = VD - C_SD - A_S\frac{D}{Q} - C_rDF(R) - iC_SI - C_2B$$

$$= VD - C_s D - A_s \frac{D}{Q} - C_r DF(R) - iC_S \frac{[Q(1-u^{-1})-S]^2}{2Q(1-u^{-1})} - \frac{C_2 S^2}{2Q(1-u^{-1})} \quad (2)$$

The first order condition with respect to S yields

$$S^* = \frac{i(1-u^{-1})C_S Q}{iC_S + C_2} \quad (3)$$

And since $\frac{d^2 \Pi_S}{ds^2} = -\frac{ic_S + C_2}{(1-u^{-1})Q} < 0$, S^* given by (3) is the optimal solution for the supplier.

Substituting S^* into (2) gives the supplier's corresponding profit as a function of V:

$$\Pi_S(V) = VD - C_S D - A_S \frac{D}{Q} - C_r DF(R) - iC_S I_S - C_2 B_S \quad (4)$$

Where $I_S = \frac{C_2^2 Q(1-u^{-1})}{2(C_S i + C_2)^2}$ and $B_S = \frac{C_2^2 i^2 Q(1-u^{-1})}{2(C_S i + C_2)^2}$ are the positive and negative inventory respectively corresponding to S^*.

since (4) is linear in V, the minimum selling price charged by the supplier is V_0

When $\Pi_S(V) = 0$, $i.e.$, $V_0 = C_S + \frac{A_S}{Q} + C_r F(R) + iC_S I_S D^{-1} + C_2 B_S D^{-1}$

Therefore, supplier can obtain the optimal selling price from

$$V^* = WV_0 = W(C_S + \frac{A_S}{Q} + C_r F(R) + iC_S I_S D^{-1} + C_2 B_S D^{-1}) \quad (5)$$

For some W>1, where W is the supplier's marginal selling price. Thus, the supplier's optimal decision variables are given by V^* and S^*.

1.3 Consumer's model :-

The consumer's purchasing decision is arrived at after determining the product quality X at the point of purchase. This decision is based on judgment of the product's overall excellence or superiority. Although the consumer is aware of the selling price of the product, its true quality is usually not reveal. The decision whether or not to purchase will be based on comparing X against a fixed quality level R determined by the consumer; If the actual quality X >R, the consumer will purchase the product ;otherwise the product will not be purchased.

Suppose on each production cycle T, the retailers quantity Q of the product from the supplier and sells it to the consumer. Then, based on the quality $X_{i,}$ $i = 1 \ldots Q$ of the i^th product, we define

$$I_i = \begin{cases} 1, & if\ X_i \geq R \\ 0, & if\ X_i < R \end{cases}$$

The quantity which is bought by the consumer is $q = \sum_i^Q I_i$ with expected value

$$E(Q) = Q(1 - F(R))$$

And the expected quantity which is rejected by the consumer is therefore $Q - E(Q) = QF(R)$

The consumer's annual utility is given by :

Consumer's annual utility = Consumer valuation – Purchase cost – Disutility due to unaccepted product – Shortage cost.

Or expressed mathematically as :-

$$\Pi_c(R) = \frac{D}{Q}\{[E(V(X)) - P]E(q) - E(S_0)[Q - E(q)] - C_3 S\}$$

$$= \mathrm{D}\left\{E(V(X)) - P - \frac{C_3 S}{Q} - [E(V(X)) - P + E(S_0)]F(R)\right\}$$

(6)

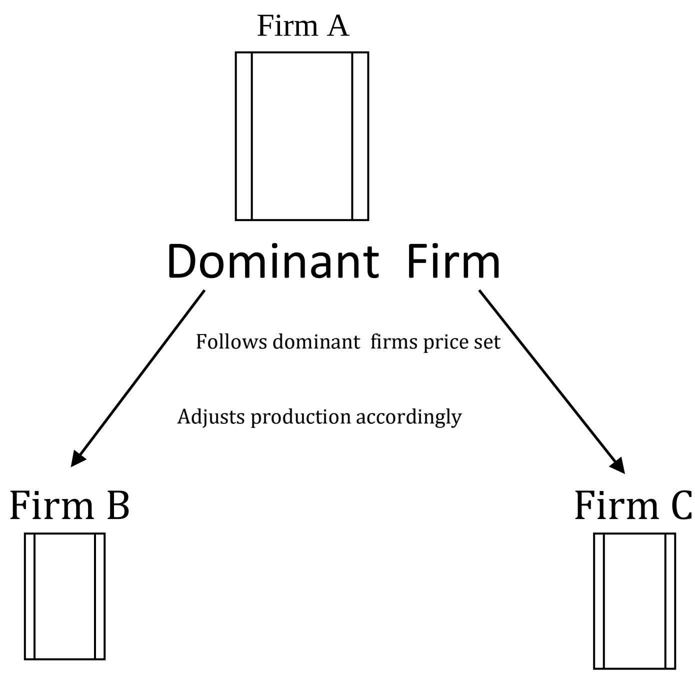
We note that

$$\frac{d\Pi_c}{dR} = \frac{\varsigma}{R}\Pi_c(R) - D[E(V(X)) - P + E(S_0)]f(R)$$

Hence the first order condition with respect to R yield

$$F(R) + \frac{R}{\varsigma}f(R) = \frac{E(V(X)) - P\frac{C_3 S}{Q}}{E(V(X)) - P + E(S_0)} \qquad (7)$$

1.4 Stackelberg Model :-



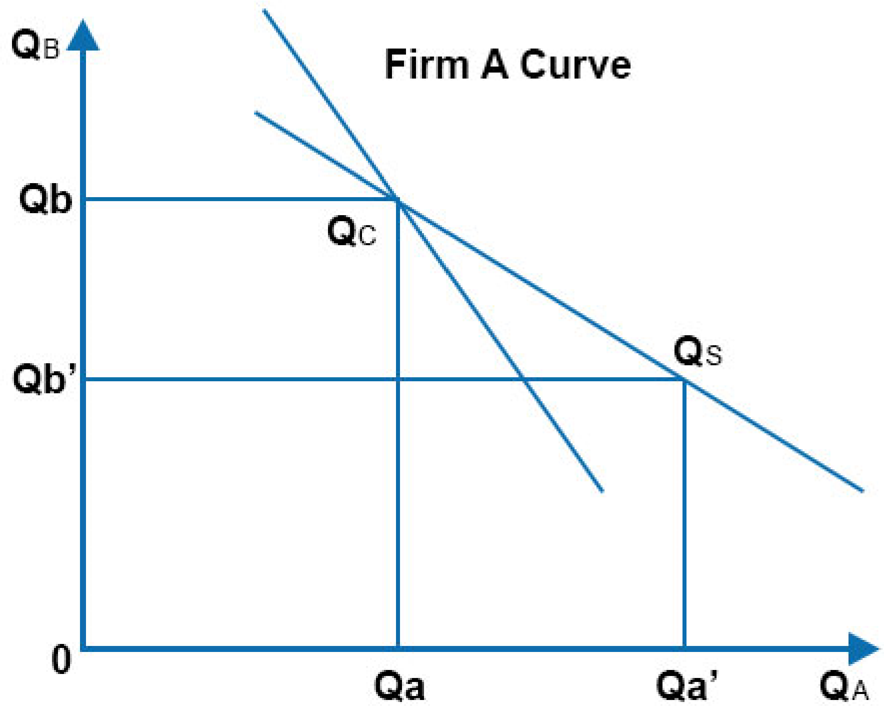
Stackelberg model is a leadership model that allows the firm dominant in the market to set its price first and subsequently, the follower firms optimize their production and price. It was formulated by Heinrich Von Stackelberg in 1934.

In simple words, let us assume a market with three players – A, B, and C. If A is the dominant force, then it will set the price of the product first up. Firms B and C will follow the price set and will accordingly adjust their production basis supply and demand

Understanding The Stackelberg Model graphically :-

An important genesis of this model is that one of the Stackelberg leaders produces more output than it would have produced under the Cournot equilibrium. Similarly, the follower in the Stackelberg model produces less output than that in the Cournot model. In order to demonstrate this, look at the graphical representation below:

Assuming the x-axis represents the production of firm A and y-axis for the production of firm B.The quantities Qc and Qs indicate a point of equilibrium for Cournot and Stackelberg conditions respectively.



If firm A assumes itself as the Stackelberg leader and B as the follower, it will produce Qa' quantity. In consequence, firm B follows with Qb' which is the best it can maximize up to. Notice that Qs is the Stackelberg equilibrium point where the firm A produces more than what it could produce at Qc which is the Courton equilibrium point.

Similarly, when firm B follows after firm A has taken output decision, firm B produces much less than what it could have had it been a Courton game.

1.5 Assumptions :-

1. We assume that the quality X_i of products from a lot size Q, i.e. $X_i, i = 1,2, \ldots Q$, are independent and identically distributed with cdf $F(\cdot)$ and pdf $f(\cdot) = F'(\cdot) > 0$ over the domain [0,1]. This is common knowledge to all players .

2. The demand function should capture the consumer's preferences and choices. It is usually determined by many factors such as price, quality, advertisement and warranty. Among these, price P and quality R are the most important factors that will influence the consumer's decision to purchase the product. Therefore, in order to explore consumer's influence to the demand function, we propose a demand function, which depends on price P and quality R, based on the Cobb-Douglas function given by $D = kP^{-\alpha}R^{\varsigma}$, Where k is a constant and α,ς are the price and quality level elasticity respectively. The Cobb-Dougles function is widely used in economics and supply chain (c.f. Lee and Kim [92], Goyal and Gunasekaran [56], Yu et al.[169], Esmaeili et al.[42]).

3. The hazard function $\frac{f(R)}{1-F(R)} > \frac{\varsigma}{R}$ which, due to the fact that $\frac{\varsigma}{R} = \frac{1}{D}\frac{dD}{dR}$, imposes the condition that hazard rate of the quality R is greater than the rate of change in demand with respect to R and is inversely proportional to the demand.

4. A consumer's valuation of a product is often highly personal and intrinsic (c.f.Zeithaml [170]) and reflects her overall assessment of the utility of the product and depends on perception of what is received and what should be given back in return. Thus, we assume that consumer's valuation function V(X) depends on the perceived quality X. Referring to Tirole [150], the valuation function $V(X)$ should increases with decreasing rate with respect to X and has a finite upper bound, that is

$$\frac{dV(x)}{dx} > 0, \frac{d^2V(x)}{dx^2} < 0, and \lim_{x\to\infty} V(x) = \tilde{V}$$

The valuation increases with increasing quality level, but at a decreasing rate and never reaching the upper threshold value $\tilde{V}$.

5. This assumptions concerns the adverse effect on the consumer's satisfaction due to products being rejected after inspection at the point of purchase. These rejected products will incur a disuility cost, denote by S_0 , which we will assume to be proportional to $V(X)$ according to $S_0 = \boldsymbol{\xi V(X)}$ $\boldsymbol{where\ 0 \le \xi \le 1}$. According to this condition, the higher is the consumer's valuation of the product, the higher disutility will be incurred if the product is rejected. Notice that the bounds on ξ indicate that the disutility cost is positive and will not exceed the total valuation of the product.

6. We will assume that $\frac{E(V(X))+E(S_0)}{2} > P$, i.e. the average of the expected valuation and disutility is greater than the price the consumer has to pay for the product. Note that this assumption is equivalent to $E(V(X)) - P > P - E(S_0)$, which emphasizes its reasonableness, since the condition states that the profit to the consumer based on the valuation of the product is greater that her loss less the disutility.

7. To ensure that the seller has enough margin to refund the buyer, we let C_b, retailer's unit lost sale due to unacceptable product, be less than the difference between seller's minimum selling price and unit production and setup cost, i.e, $C_b < V_0 - C_S - \frac{A_s}{Q}$ where V_0 is seller's minimum selling price.

8. The production rate r is related to the demand rate d by $r = ud$ where $u > 1$

CHAPTER-2

LITERATURE REVIEW

Over the last few decades, much work has been done towards a better understanding on how to improve the output and benefits to members of different supply chains. Goyal first proposed an integrated inventory model based on the total cost of the producer and purchaser, obtaining The optimum production quantity and order cycle in the process. Lu considered the problems of an Integrated inventory model with a single producer and multiple purchasers and presented a heuristic method for obtaining the model's optimal solution. Goyal and Nebebe proposed an optimum production and transportation policy with a minimum cost to the producers and the purchasers.Chen and Liu proposed an optimal consignment policy considering a fixed fee and a per-unitCommission. Their model produces a higher profit for the manufacturer than the traditional production System and provides a means for the retailer to obtain a larger supply chain profit. Darwish proposed the optimum process mean setting for a single-vendor, single-buyer supply chain where in addition to the optimum process mean, the shipment quantity of product and number of shipments that is required were also determined from model. Chen and Huang addressed the problem how a retailer could hedge the risk of demand uncertainity by purchasing their products from options and online spot markets. Some recent supply chain models which integrated quality and production with inventory. and Supply chain management can also be found in Jeang , Darwish and Odah , Darwish and Goyal , Sana ,Darwish and Abdulmalek , Roy et al.

We introduce a three-person game model concerning the quality level of product from consumer's viewpoint. However, the quality of the product can be determined by The manufacturer who will consider all the relevant costs which are incurred during the production process, so that he can achieve an optimal outcome.

A properly functioning supply chain system is important for industry in order to obtain a mutually beneficial situation for both manufacturer and buyer (or retailer) who comprise the main players of the system . The manufacturer's objective function would include, among other things, sales revenue, manufacturing and management cost, and the buyer's objective function would consider the order Quantity, holding cost, loss of goodwill cost due to the quality of the product supplier by the manufacturer. Since their objectives would often conflict, it is therefore essential to study and solve the Problem of how to get a good trade-off between the objectives of the two participants in a supply chain.

Chapter 3

Aim and objectives :-

The goal of this thesis is to provide a comprehensive study of a supply chain incorporating the supplier, retailer with a strategic consumer as a three-person game and investigation between them and examining the possibility of forming different coalitions among the three players .

CHAPTER 4

Stackelberg games with a single leader and no coalition

In this chapter, we will extend the stackelberg game concept to three players, i.e. the supplier, retailer, and consumer , in a competitive market. One player will be the leader and the remaining players the followers but they are not allowed to cooperate and form a coalition against the leader.

We will let each player takes turn as the leader, and depending on who takes on the leadership role, we will have the Supplier-Stackelberg , the Retailer-Stackelberg and the consumer-Stackelberg game respectively. We will focus here on the welfare of the consumer with respect to interaction with both supplier and retailer. Numerical examples will be provided to compare and contrast between the three --games and to demonstrate the effect that a player's leadership position has on optimal strategies and resulting proof.

4.1 Supplier-Stackelberg game

In this game, the supplier takes on the role of leader, the retailer and consumer are the followers. The supplier makes the first move and select $V\ and\ S$; their best responses to the supplier's first move.

Finally, the supplier's final decision is obtained by selecting the optimal strategy among the best responses by the consumer and retailer.

Therefore, the optimal solutions $V^*\ and\ S^*$ to this game are obytained by solving the following nonlinear optimization problem :b

$$\max \Pi_S(V,S) = VD - C_S D - A_S \frac{D}{Q} - C_r DF(R) - iC_S \frac{[Q(1-u^{-1})-S]^2}{2Q(1-u^{-1})} - \frac{C_2 S^2}{2Q(1-u^{-1})} \tag{8}$$

Subject to $$P = \frac{\alpha[V + A_b Q^{-1} + C_b F(R)}{\alpha - 1} \tag{9}$$

$$iV(1-u^{-1})^2Q^2 = 2A_b(1-u^{-1})D + (iV + C_1)S^2 \qquad (10)$$

$$F(R) + \frac{R}{\varsigma}f(R) = \frac{E\big(V(X)\big)-P-C_3SQ^{-1}}{E\big(V(X)\big)-P+E(S_0)} \qquad (11)$$

In the following, we denote $T(R) = F(R) + \frac{R}{\varsigma}f(R)$ to simplify the notation ,

where T(R) is the right hand side of equation (11) , which implies that $0 < T(R) < 1$, Since $E\big(V(x)\big) - P - C_3SQ^{-1} < E\big(V(X)\big) - P + E(S_0)$. Solving equation (9) and (10) simultaneously give :-

$$P = \frac{\alpha}{\alpha-1}\{V + C_bF(R) + A_b\frac{(\alpha-1)[E\big(V(X)\big)\big(1-T(R)\big)-E(S_0)T(R)-\alpha[1-T(R)][V+C_bF(R)]}{\alpha A_b[1-T(R)]+(\alpha-1)C_3S}\} \qquad (12)$$

$$Q = \frac{\alpha A_b[1-T(R)]+(\alpha-1)C_3S}{(\alpha-1)\big[E\big(V(X)\big)\big(1-T(R)\big)-E(S_0)T(R)\big]-\alpha[1-T(R)][V+C_bF(R)]} \qquad (13)$$

Using equation (10) in combination with equation (12) and (13) will give us R once f(·) is known.

Thus , the optimization problem reduces to a non-constrained nonlinear optimization problem which can be used to solve for V^* and S^* numerically.

Since $0 < T(R) < 1$ and $\alpha > 1$ equation (12) (13) also provide us with the following observation:

OBSERVATION : when supplier is the leader, the retailer's selling price is increasing in V, and the order quantity Q is increasing in S . Thus, consumer's utility is dependent on P, Q, V and S.

4.2 Retailer-Stackelberg Game :-

In this game, the retailer is the leader, and the supplier and consumer assume the role of followers. The game sequence proceeds in a similar manner outlined in 4.1 but with the leadership role assumed by the retailer.

Thus, the retailer would seek to determine most profitable selling price and order quantity, based on the supplier and consumer's best responses, so that profit would be maximized.

Therefore, the retailer's optimization problem can be expressed as :

$$\max\ \Pi_r(P,Q) = PD - VD - A_b\frac{D}{Q} - C_bDF(R) - iV\frac{[Q(1-u^{-1})-S]^2}{2Q(1-u^{-1})} - \frac{C_1S^2}{2Q(1-u^{-1})} \qquad (14)$$

Subject to
$$S = \frac{i(1-u^{-1})C_SQ}{iC_S+C_2} \qquad (15)$$

$$V = W(C_S + \frac{A_S}{Q} + C_rF(R) + iC_SI_SD^{-1} + C_2B_SD^{-1}) \qquad (16)$$

$$F(R) + \frac{R}{\varsigma}f(R) = \frac{E\big(V(X)\big)-P-C_3SQ^{-1}}{E\big(V(X)\big)-P+E(S_0)} \qquad (17)$$

Substituting equation (15) and (16) gives

$$F(R) + \frac{R}{\varsigma}f(R) = \frac{(iC_S+C_2)\big(E\big(V(X)\big)-P\big)-iC_3C_S(1-u^{-1})}{(iC_S+C_2)(E\big(V(X)\big)-P+E(S_0))} \qquad (18)$$

And then equation (15) and (18) into (14) to yield an objective function without the constraints.

An application of the first order condition with respect to P and Q give

$$P = \frac{\alpha}{\alpha-1}\left[V + \frac{A_b}{Q} + C_bF(R)\right] - \frac{\alpha}{\alpha-1}\left[V - W\left(C_S + \frac{A_S}{Q} + C_rF(R)\right)\right](1 + iI_SD^{-1})$$

$$-\frac{P[WC_r\big(1+iI_SD^{-1}\big)+C_b}{\alpha-1}\frac{F(R)+\frac{R}{\varsigma}f(R)-1}{E\big(V(X)\big)-P+E(S_0)} \qquad (19)$$

$$Q = \frac{(A_b+WA_S)D+iWA_SI_S}{(iVI_S+C_1B_S)+W(iC_SI_S+C_2B_S)(1+iI_SD^{-1})} \tag{20}$$

The retailer's optimal decision variables $P^* \ and\ Q^* \ can\ be$ obtained by solving equation

(19) and (20) simultaneously.

NOTE :- In a three-person game where there is no coalition among the players. The consumer's utility is greater in the Retailer-Stackelberg game than in the Supplier-Stackelberg game.

4.3 Consumer-Stackelberg game :-

In this game, consumer has the role of leader , and the supplier and retailer assume the role of followers. The consumer's decision will be based on inspection of products at the point of purchase

To determine the quality level which is accepted. Proceeding as in the previous games, the consumer's

Optimization problem based on the follower's best responses is :-

$$\max \Pi_c(R) = D\{E\big(V(X)\big) - P - \frac{C_3S}{Q} - \big[E\big(V(X)\big) - P + E(S_0)\big]F(R)\} \tag{21}$$

Subject to
$$P = \frac{\alpha[V+A_bQ^{-1}+C_bF(R)]}{\alpha-1} \tag{22}$$

$$S = \frac{i(1-u^{-1})C_SQ}{iC_S+C_2} \tag{23}$$

$$V = W(C_S + \frac{A_S}{Q} + C_rF(R) + iC_SI_SD^{-1} + C_2B_SD^{-1}) \tag{24}$$

$$iV(1-u^{-1})^2Q^2 = 2A_b(1-u^{-1})D + (iV + C_1)S^2 \tag{25}$$

Substituting equation (24) into equation (22) and (25) , will eliminate V from those equations. The resulting equations can be solved simultaneously to obtain P,Q and S.

Substituting these into (21) will produce a nonlinear optimization problem with no constraints which can be solved numerically for the optimal decision variable R^*

CHAPTER-5

Stackelberg games with coalitions

In a three-person game, two players may form a coalition at the expense of the third player to gain an edge in the market. Coalition between supplier and retailer is quite common in supply chain management, and it is usually achieved by implementing some incentive schemes such as discount policy, credit option and buyback policy, but may seem less so between retailer and consumer. However, cooperation between consumer and retailer is quite common;

for example,

a coalition between the retailer and consumer could manifest itself in sharing of information, negotiation for suitable prices and suitable warranty term.

In this chapter we will investigate coalitional Stackelberg game under two scenarios:-

(i)The retailer and consumer form a coalition and has more power than the supplier (RCC)

(ii) The supplier and retailer form a coalition and has more power than the consumer (SRC).

In other word, the coalition is the leader and the remaining single player the follower of the Stackelberg game.

A justification for the above coalitions is seen in the phenomena which often occur in practice Where the consumer may negotiate the warranty and return term with the retailer against the supplier, and the retailer and supplier may coordinate their inventory planning and selling price against the consumer.

5.1 RCC-Stackelberg game :-

In the RCC-Stackelberg game, retailer and consumer cooperate with each other against the supplier. The coalition jointly acts as leader and the supplier as follower.

The coalition first select a strategy and the supplier responds with his best strategy. The coalition then replies by selecting the cooperative pareto optimum solution among the best responses offered by supplier through carrying out the joint optimization of the weighted sum of their profit functions:

$$Z_{rc} = T\Pi_r(P,Q) + (1-T)\Pi_C(R)$$

Where $0 < \mathcal{T} < 1$. **Thus, the Rcc's problem becomes:**

$$\begin{aligned} max \quad Z_{rc} = \mathcal{T}\Big[&PD - VD - A_b\frac{D}{Q} - C_bDF(R) - iV\frac{[Q(1-u^{-1})-S]^2}{2Q(1-u^{-1})} - \frac{c_1S^2}{2Q(1-u^{-1})}\Big] \\ &+(1-\mathcal{T})D\Big\{E\big(V(X)\big) - P - \frac{C_3S}{Q} - \big[E\big(V(X)\big) - P + E(S_0)\big]F(R)\Big\} \end{aligned} \tag{26}$$

Subject to

$$S = \frac{i(1-u^{-1})C_SQ}{iC_S+C_2}$$

$$V = W(C_S + \frac{A_S}{Q} + C_rF(R) + iC_SI_SD^{-1} + C_2B_SD^{-1}) \tag{27}$$

Substituting equations (26) and (27) into equation (25), reduces the problem to an unconstrained nonlinear optimization problem.

It would be interesting here to compare the results between the RCC-Stackelberg game and Consumer-Stackelberg game, where the consumer holds a leadership position, to see whether gains any advantage by forming a coalition with the retailer.

To further explore the effect of coalition game to consumer's welfare compare those games without coalition, we obtain the following equation by using first order condition with respect to P as:

$$P = \frac{\alpha}{\alpha-1}\left[V + \frac{A_b}{Q} + C_bF(R)\right] - \frac{\alpha}{\alpha-1}\left[V - W\left(C_S + \frac{A_S}{Q} + C_rF(R)\right)\right](1 + iI_SD^{-1})$$

$$-\frac{1-\mathcal{T}}{\mathcal{T}(\alpha-1)}\left[\alpha \Pi_C(R) D^{-1}+\big(1-F(R)\big)P\right]$$

Notice that in both Consumer-Stackelberg game and RCC-Stackelberg game, consumer has the same position, i.e., in the leadership position, and the supplier is the follower.

5.2 SRC-Stackelberg game :-

In SRC-Stackelberg game, supplier and retailer form a coalition against consumer. The game flow is similar to the RCC game of the previous section. We will investigate the conflict between SRC and

Consumer, and let SRC acts as a leader. The SRC designed based on concept of Pareto efficient solution as weighted sum of supplier and retailer's profit function, so that the supplier and retailer work together

To determine their decision variables. SRC will make the first move, and maximize their Pareto profit function based on consumer's best strategy. The SRC's problem become :

$$\max \quad Z_{Sr} = \lambda \Pi_S(V,S) + (1-\lambda)\Pi_b(P,Q) \qquad (28)$$

Subject to $$F(R)+\frac{R}{\varsigma}f(R)=\frac{E\big(V(X)\big)-P-C_3SQ^{-1}}{E\big(V(X)\big)-P+E(S_0)} \qquad (29)$$

Substituting equation (29) into (28) , reduce the problem as a non-constrained non-linear optimization problem. The first order condition with respect to V,S,P and Q, yield the following equations:

$$\lambda = \frac{2DQ(1-u^{-1})+i[Q(1-u^{-1})-S]^2}{4DQ(1-u^{-1})+i[Q(1-u^{-1})-S]^2} \qquad (30)$$

Note that $0<\lambda<1$, as it desired.

$$P=\frac{\alpha}{\alpha-1}\left[V+\frac{A_b}{Q}+C_bF(R)\right]$$

$$-\frac{\alpha}{\alpha-1}\left[V-C_S-\frac{A_S}{Q}-C_rF(R)\right](1+iID^{-1})$$

$$-\frac{P[C_r(1+iID^{-1})+C_b]}{\alpha-1}\frac{F(R)+\frac{R}{\varsigma}f(R)-1}{E\big(V(X)\big)-P+E(S_0)} \quad (31)$$

$$Q=\frac{2D[\lambda A_S+(1-\lambda)A_b-\frac{C_3S[\lambda C_r+(1-\lambda)C_b}{E(V(X))-P-E(S_0)}]}{(1-u^{-1})[\lambda iC_S+(1-\lambda)iV]}$$

$$+\frac{[\lambda(iC_S+C_2)+(1-\lambda)(iV+C_1)]S^2}{(1-u^{-1})[\lambda iC_S+(1-\lambda)iV]} \quad (32)$$

$$S=\frac{(1-u^{-1})[Q[\lambda iC_s+(1-\lambda)iV]+\frac{C_3D[\lambda C_r+(1-\lambda)C_b]}{E\big(V(X)\big)-P+E(S_0)}]}{\lambda(iC_S+C_2)+(1-\lambda)(iV+C_1)} \quad (33)$$

Once pdf f(x) is given, solve the above equations simultaneously and combining this with the equation obtained by the first order condition with respect to λ, we can obtain the optimal strategy for supplier-retailer coalition. It is worthwhile to investigate what is the effect of Supplier-retailer coalition to consumer's Welfare. By comparing equation (31) with equation (19) which obtained from Retailer-Stackelberg game, since both α>W, W>1 .We have $\alpha[V-(C_S+\frac{A_S}{Q}+C_rF(R))]>\alpha[V-W(C_S+\frac{A_S}{Q}+C_rF(R))]$, Thus, result $P^{src}<P^{rs},\ which\ implies\ Q^{src}>Q^{rs}$. Further, compare equation (33) with equation (15) ,we see that S^{src} is not only depend on supplier's cost $C_s, C_r\ and\ C_2$, but also depend on retailer's unit shortage cost C_1 and lost sale cost C_b, and consumer's cost C_3, thus, it is not observe

That whether S^{src} will exceed S^{rs} or not. However, we notice that $C_r\ and\ C_b$ are sufficient small, so that the term which involving $C_r\ and\ C_b$ in equation (33) will not make much contribution to S^{src}, Thus, we would expect that if retailer's unit shortage cost C_2 i.e.,$C_1>\frac{V}{C_s}C_2,\ then\ S^{src}<S^{rs}$.

CHAPTER-6

Numerical examples and discussion

We provide the following examples to demonstrate SRC-Stackelberg game and RCC-Stackelberg game. We will use the same assumption regarding to the actual quality X, consumer's valuation V(X) and disutility S_0. These numerical examples will illustrate the interaction between players when two of the players form a coalition against the third player. The results will reveal players advantage/disadvantage in the games regarding to their leadership position.

6.1)Example 1 :- This example deals with SRC-Stackelberg game. Following the solution procedure, we obtain supplier and retailer's optimal pareto efficient profit $Z_{sr} = 175.87$ Supplier's strategy is $V = 2.94$, $S = 13.51$ with corresponding retailer's strategy $P = 4.53, Q = 1215.78$, and the weight parameter λ= 0.5073. This yields consumer's quality level R= 0.4374 with utility $\Pi_c = 743.35$

Example 2 :- In RCC-Stackelberg game, according to the solution procedure results the optimal Pareto efficient profit $Z_{rc} = 435.58$. The retailer's optimal strategy is $P = 4.53$, $Q = 1696.82$, Consumer's quality level $R = 0.7702$ and the corresponding weight parameter $\mathcal{T}$= 0.8209. This yields the supplier's optimal strategy $V = 2.13$, $S = 20.12$, and corresponding profit $\Pi_s = 92.15$

Numerical results show that consumer's utility in SRC-Stackelberg game is larger than in Retailer-Stackelberg game, and the utility in RCC-Stackelberg game is larger than in Consumer-Stackelberg game. We also observed that consumer's utility in RCC-Stackelberg game is less than in Retailer-Stackelberg game. Thus, from consumer's perspective, in the coalition game, is more favorite the SRC-Stackelberg game rather than RCC-Stackelberg game. This perhaps when supplier and retailer working together to determine their decisions, the inventory will be well organized, i.e, The shortage size is quite lower, and the retailer's selling price also is lower than when retailer working with consumer. Therefore, results consumer receiving higher utility even it is the follower in SRC-Stackelberg.

We also observed that supplier's profit is better in coalition games rather than in those games without coalitions, and the best profit which the supplier will get is from SRC-Stackelberg game, which is the game that the retailer will get the worst payoff. This indicate that the supplier will prefer to form a coalition with retailer to improve profit. In opposite of the supplier, the retailer is more likely to form a coalition with consumer rather than with supplier, since in RCC-Stackelberg game, it will gain more than in SRC-Stackelberg Game. However, as we indicated before, the consumer is more preferable to a cooperation between supplier and retailer. Thus, in this three-person game, the relationship between players are conflict with their preferences of interest. However among these complex relationship, we observed that the retailer plays a crucial role in this three-person game, it Can cooperate with either the supplier or the consumer. This may provide a opportunity for the retailer to coordinate the supply chain, so that an efficient outcome will bee achieved among the players.

Table 1: Sensitivity analysis of Non-cooperative Stackelberg model with respect to α

	SS game			RS game			CS game		
α	Π_s	Π_b	Π_c	Π_s	Π_b	Π_c	Π_s	Π_b	
1.7	70.39	191.17	64.57	42.71	304.78	453.52	36.05	282.25	360.[illegible]
1.8	63.70	154.77	81.59	41.05	254.46	455.49	34.43	235.27	360.[illegible]
1.9	57.09	127.93	97.56	39.14	213.56	448.23	32.54	197.06	351.[illegible]
2.0	50.91	107.02	110.16	37.07	179.95	434.56	30.29	162.85	334.[illegible]
2.1	45.27	90.20	119.15	34.92	152.08	416.55	28.38	139.66	316.4[illegible]
2.3	35.59	65.0	128.07	30.60	109.24	373.13	24.13	99.74	269.[illegible]

6.2 Sensitivity analysis

In this section, we carry out sensitivity analysis on the parameters α and ς. These two parameters directly affect the demand function and hence each player's profit. We compute each player's profit for all non-cooperative and cooperative game discussed in chapter 4 and chapter 5 By varying α (for a fixed ς) and ς (for a fixed α) . All the other parameters are kept the same as in previous example. The results are displayed in Table 1 and Table 2 for non-cooperative games and in table 3 and table 4 for coalition games. From the results in table 1 and table 2, regardless of the values of α and ς, both supplier and retailer have the leadership advantage insofar as profit is concerned , whereas

Table 6.2.1: Sensitivity analysis of Non-cooperative Stackelberg model with respect to α

	SS game			RS game			CS game		
ς	Π_s	Π_b	Π_c	Π_s	Π_b	Π_c	Π_s	Π_b	Π_c
0.4	88.17	229.77	104.98	52.95	371.02	761.84	45.08	341.03	632.34
0.5	82.18	216.44	90.15	49.92	348.45	653.16	41.82	319.61	535.73
0.6	77.45	206.14	79.31	47.63	330.75	587.76	39.39	303.76	462.70
0.7	73.61	197.91	70.96	44.50	316.55	505.66	37.52	291.68	405.81
0.8	70.39	191.17	64.57	42.71	304.78	453.52	36.05	282.25	360.39
0.9	67.66	185.53	59.31	41.22	294.86	410.78	34.86	274.74	323.41

Table 6.2.2:-Sensitivity analysis of Coalition Stackelberg model with respect to α

	SRC game			RCC game		
A	Π_s	Π_b	Π_c	Π_s	Π_b	Π_c
1.7	175.87	175.87	743.35	92.15	435.58	435.58
1.8	150.36	150.36	754.94	83.15	370.86	370.86
1.9	129.31	129.31	753.94	75.14	315.97	315.97
2.0	111.74	111.74	743.63	67.99	269.34	269.34
2.1	96.93	96.93	726.60	61.56	229.67	229.67
2.3	73.57	73.57	679.52	50.53	166.99	166.99

Table 6.2.3:- Sensitivity analysis of Coalition Stackelberg model with respect to ς

	SRC game			RCC game		
ς	Π_s	Π_b	Π_c	Π_s	Π_b	Π_c
0.4	215.12	215.12	1258.16	85.19	493.43	493.43
0.5	201.71	201.71	1076.36	87.44	476.68	476.68
0.6	191.25	191.25	938.32	89.29	461.67	461.67
0.7	182.82	182.82	830.18	90.83	448.06	448.06
0.8	175.87	175.87	473.35	92.15	435.58	435.58
0.9	170.03	170.03	672.22	93.29	424.06	424.06

Chapter 7

Conclusion :-

We investigate a three-person supply chain game comprising a supplier and a retailer interacting with a strategic consumer. We also considered the interaction between the participants of the supply chain using the concept of a stackelberg game, with and without formulation of a coalition. Consequently, several game models were presented and numerical examples for these games provided. Without coalition, both analytical results and numerical examples indicated that the supplier and retailer have the leadership advantage whereas the consumer is better off when the retailer is the leader. In games with coalition, both supplier and consumer would prefer game where there is coalition between supplier and retailer, but the retailer will prefer to cooperate with the consumer. This shows that the retailer holds the key in these three-person games, since both supplier's profit and consumer's utility will rely on retailer's preference whether to work alone or enter into a coalition with the other players.

A sensitivity analysis was also performed on the parameters of the demand function to compare the profits between all the games discussed when these parameters were varied.

Future works :-

An important extension based on the three-person games would be to investigate whether some incentive schemes by either one or more players might be brought to bear on the system to improve channel performance and thereby achieve better outcomes for all players. Furthermore, a statistical analysis of the copula method applied to supply chains is lacking in this thesis. It would be interesting to perform an empirical case study to explore the correlation between player's types and to identify the copula functions that best fit various type distribution using real data.

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