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Advanced Data Structures (COP 5536 /NTU AD 711R)
Final

CLOSED BOOK
90 Minutes

Name: _____

NOTE:
1. **For all problems, use only the algorithms discussed in class/text.**
2. All answers will be graded on correctness, efficiency, clarity, elegance and other normal criteria that determine quality.
3. The points assigned to each question are provided in parentheses.

1. (6) Assuming that a B-tree of height h is kept on a disk and only one node can be fetched or written at a time, how many disk accesses are needed to *delete* an element in the worst case? Give an explanation of how you arrived at your answer.
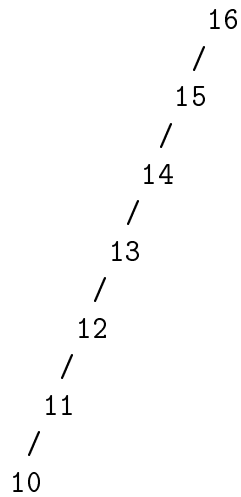
2. (14) splay tree

(a) (8) *Insert* the following keys into an initially empty instance of a splay tree assuming that this is a *bottom-up* splay tree.

$$1, 3, 5, 7, 2, 6, 4$$

Draw the splay tree following each insertion.

(b) (6) Consider the following *top-down* splay tree:

```
        16
       /
      15
     /
    14
   /
  13
 /
12
 /
11
 /
10
```

Perform a *split* operation with respect to the node with the key 10, showing each step.

3. (10) Insert the following keys into an initially empty instance of Patricia:

$$0100, 1000, 1010, 0010, 0101$$

Draw the Patricia instance following each insertion. Then delete the key 0100, and draw the resulting instance. (show each step)

4. (10) You are given a *Bloom filter* that consists of $m = 13$ memory bits and *two* hash functions $f_1()$ and $f_2()$ defined as below:

$$
\begin{aligned}
f_1(k) &= k \bmod m \\
f_2(k) &= (2 \times k) \bmod m
\end{aligned}
$$

, where k is a given key. Assume that all m bits of the *Bloom filter* are initially set to 0.

(a) Show the Bloom filter bits following the insertion of the key 17.

(b) Into the Bloom filter of (a) (i.e., following the insertion of the key 17) insert 19. Show the resulting Bloom filter bits.

(c) For the filter of (b), give a key value that results in a filter error (i.e., the Bloom filter response is *Maybe* even though the key is not in the filter).

5. (10) Describe the *2-dimensional range tree* structure. Derive the formulas for the preprocessing time P, the space required S, and the query time Q.