Sample solution for COP 5536/ AD 711R Summer 2003

1. (15)

   (a) (10) Using the aggregate method,
   First, determine an upper bound on the sum of the costs for the first n months.
   The sum of the actual monthly costs for the first n months is

$$\begin{aligned} &= 80 \times \lfloor n/12 \rfloor + 40 \times (\lfloor n/3 \rfloor - \lfloor n/12 \rfloor) + 20 \times (n - \lfloor n/3 \rfloor) \\ &= 40 \times \lfloor n/12 \rfloor + 20 \times \lfloor n/3 \rfloor + 20 \times n \\ &\leq 40 \times n/12 + 20 \times n/3 + 20 \times n \\ &= 20 \cdot (1/6 + 1/3 + 1) \\ &= 20 \cdot (3/2) \\ &= 30 \cdot n \end{aligned}$$

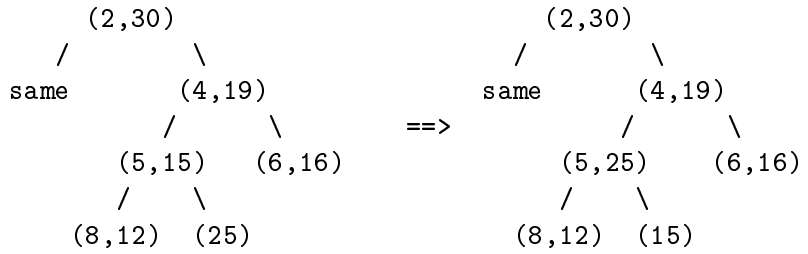
   (b) (5) Using the amortized cost calculated in part(a), fill in the following table.

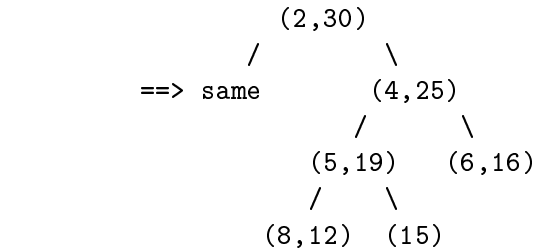
Month	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Actual cost	20	20	40	20	20	40	20	20	40	20	20	80
Amortized cost	30	30	30	30	30	30	30	30	30	30	30	30
Potential()	10	20	10	20	30	20	30	40	30	40	50	0

Table 1: Maintenance Contract

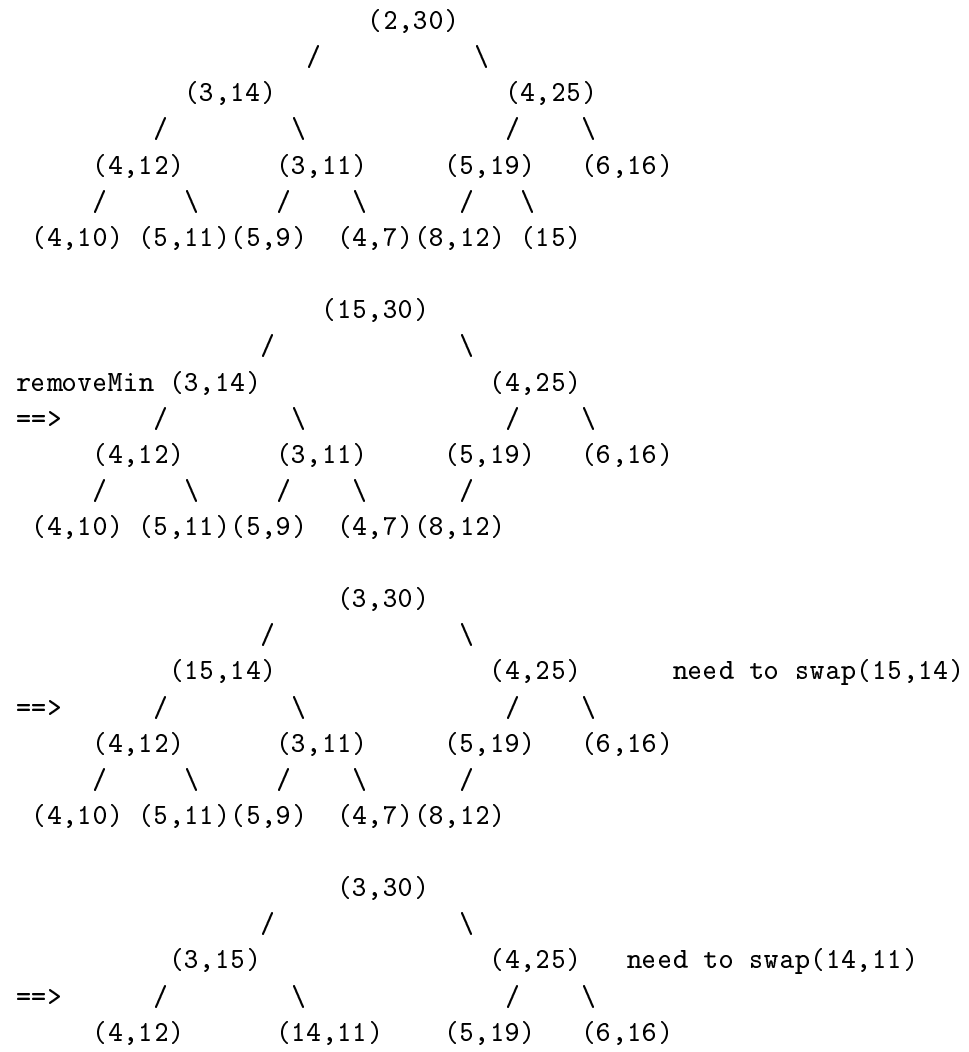
2. (12)

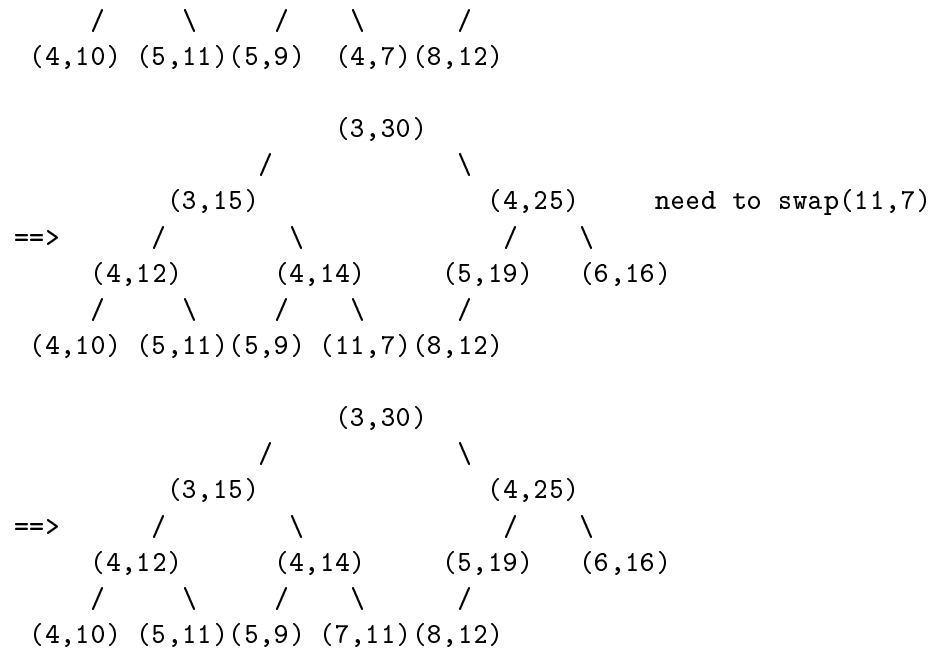
   (a) *Insert*(25):




(2,30)
==> same
(4,25)
(5,19)
(6,16)
(8,12)
(15)

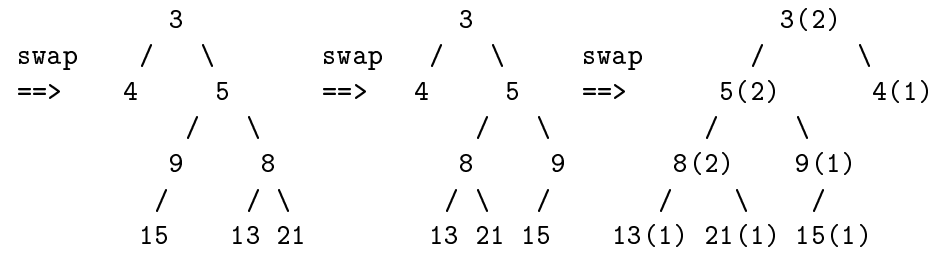
(b) *RemoveMin*()


(2,30)
(3,14)
(4,25)
(4,12)
(3,11)
(5,19)
(6,16)
(4,10)
(5,11)
(5,9)
(4,7)
(8,12)
(15)
(15,30)
removeMin
==>
(3,14)
(4,25)
(4,12)
(3,11)
(5,19)
(6,16)
(4,10)
(5,11)
(5,9)
(4,7)
(8,12)
(3,30)
(15,14)
(4,25)
need to swap(15,14)
==>
(4,12)
(3,11)
(5,19)
(6,16)
(4,10)
(5,11)
(5,9)
(4,7)
(8,12)
(3,30)
(3,15)
(4,25)
need to swap(14,11)
==>
(4,12)
(14,11)
(5,19)
(6,16)

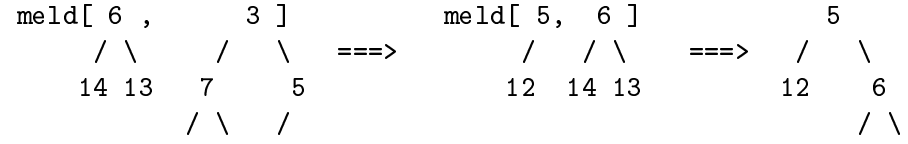


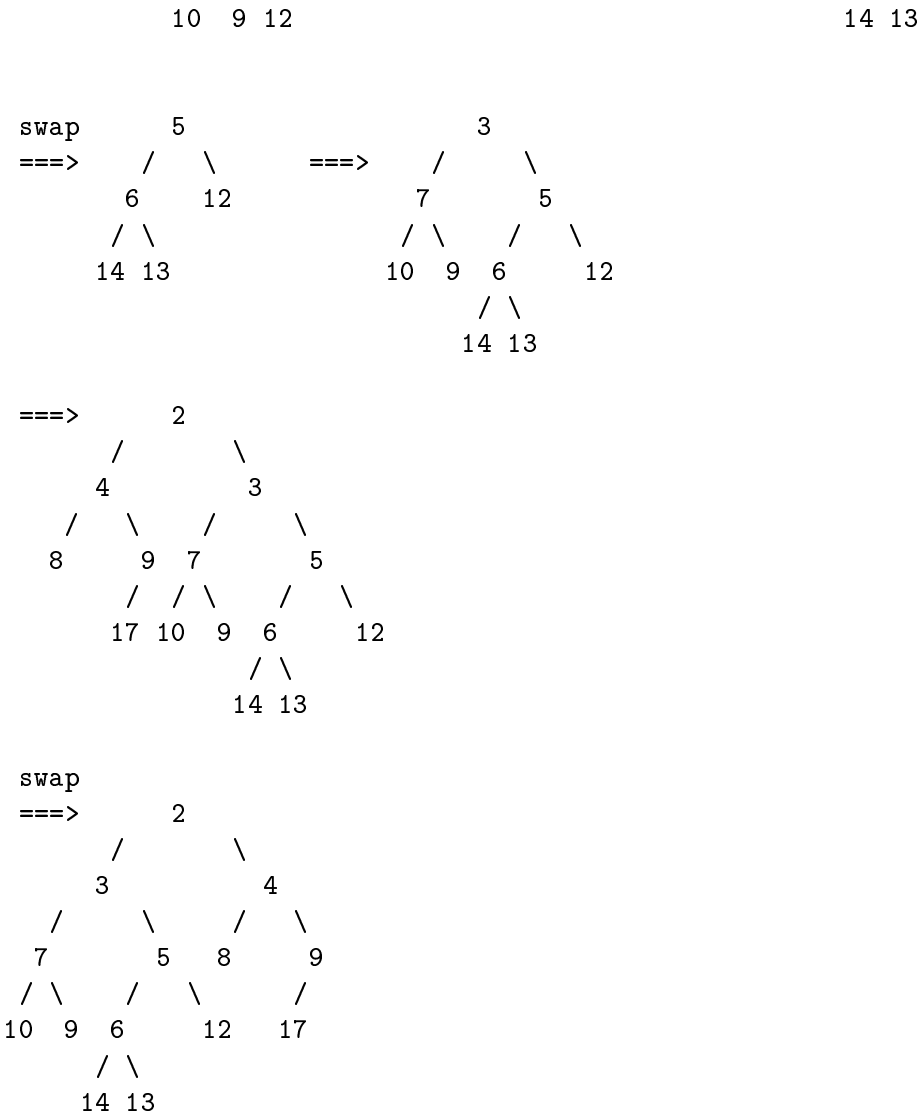
3. (13)

   (a) (5)



   (b) (8) Meld right subtree with smaller root and all of the other tree.





4. (10)

   (a) (4) *Insert* does not need pairwise combine.

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7 - 12 - 4 - 23 - 3 - 24 - 13 - 20 - 19 - 5
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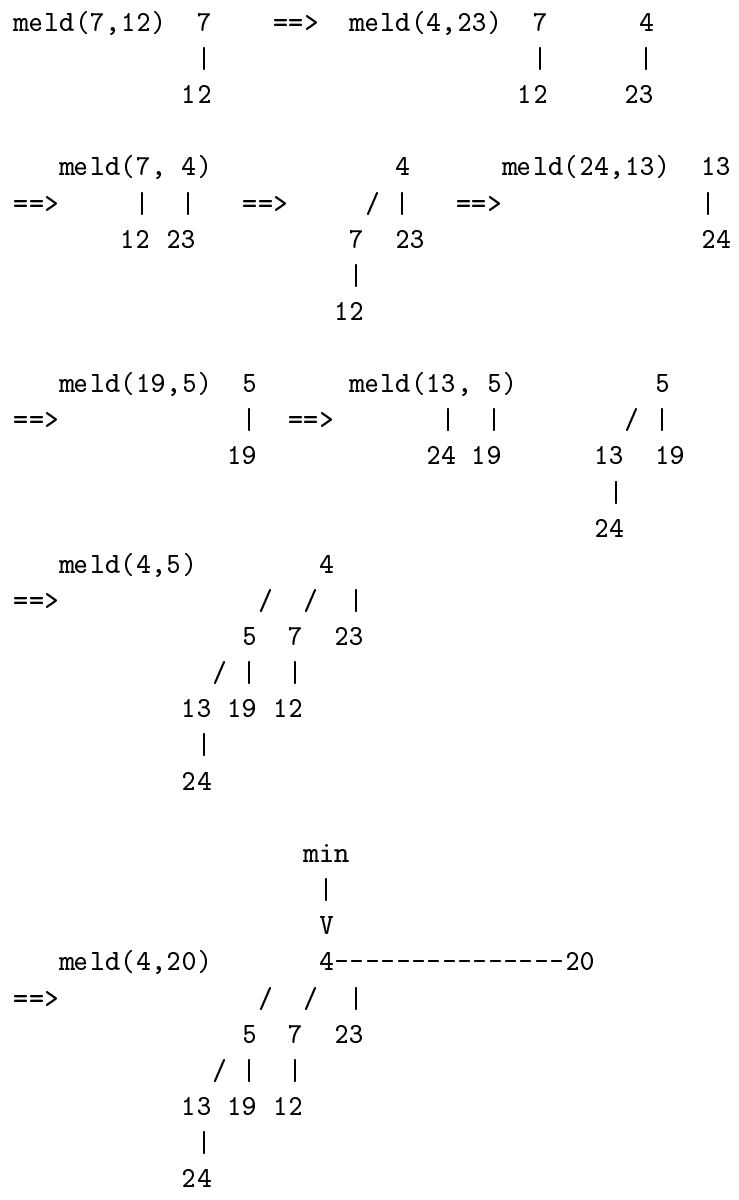
(b) (6)



5. (10) amortized analysis

◯ actual cost
- the actual cost of odd operation = 1 (corresponds to a leaf node)
- the actual cost of even operation = the number of nodes connected by the edges labelled 0 without going throu any edge labelled 1.

→ So, we can mark all the corresponding nodes in a binary tree.

the number of nodes marked by the sequence of operations ¡ 2n-1 (since we discard the leaf node 0)

• the total number of nodes in a binary tree = 2n-1, where n is the number of leaf nodes.

So, we can assign amortized cost 2 to each operation.

• Clearly, P(n) - P(0) ¿ 0, (note that we throw the leaf node 0 away, and assume that P(0) = 0) since the to cost of the sequence of operations can't exceeds the total number of nodes in the binary tree