

NOTE1: This is the solution set for exam 2.

NOTE2: To keep you up to date on class scores, the homework and exam averages follow (for local students). For NTU/FEEDS students, the HWs and exams come in at different times from different students, hence your statistics cannot be meaningful until all work has been completed.

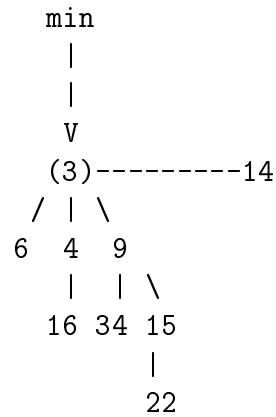
For Local Students Only

| | | AVG | STD |
|------|--|-----|------|
| HW 1 | | 32 | 7.4 |
| | | | |
| HW 2 | | 39 | 10.5 |
| | | | |
| EX 1 | | 37 | 4.6 |
| | | | |
| EX 2 | | 37 | 7.8 |
| | | | |

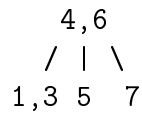
1. (10) Consider a min binomial heap with the following elements:

3, 6, 4, 2, 16, 22, 15, 34, 9, 14.

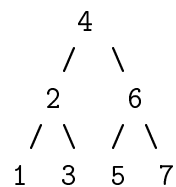
Each element defines a min tree of the binomial heap. Perform a DeleteMin operation and show the resulting min trees.



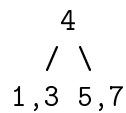
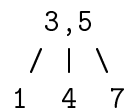
2. (10) Follow the 2-3 tree algorithms discussed in the class/text.



- (a) (5) Draw the resulting 2-3 tree after inserting 2.

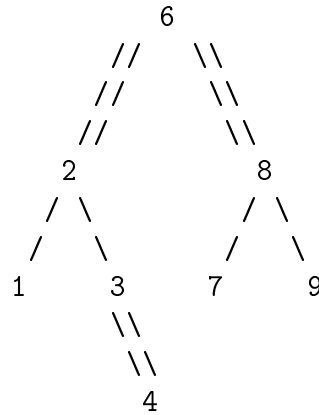


- (b) (5) Draw all possible trees resulting from the operation delete 6 (seperate from part a, ie. on the original tree not the tree resulting from part a).



3. (10) Use the bottom-up (2-pass) style algorithms of Red-Black Trees for this question.

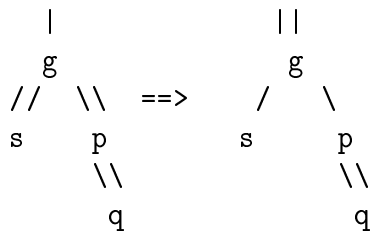
- (a) (5) Insert the keys 9, 8, 7, 2, 6, 1, 3 and 4 into an initially empty red-black tree in the order shown, showing the resulting tree following each insertion.



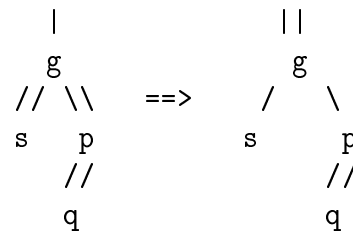
- (b) (5) Draw the figures for RRR and RLr color changes and RRb, RLb rotations for the insert operation in a red-black tree.

You can find the LL mirror images in our book, pages 596-597.

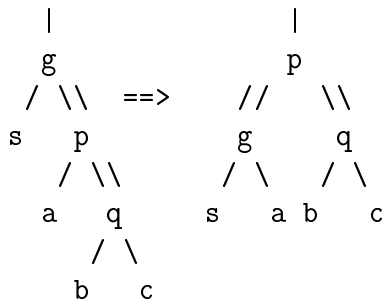
o RRR



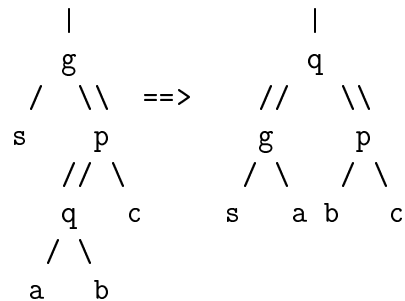
o RLr



o RRb



o RLb



4. (10)

- (a) (5) A single 2-3-4 tree can be represented by many red-black trees. How many red-black trees represent a 2-3-4 tree with x 2-nodes, y 3-nodes, and z 4-nodes (where x , y and z are variables/numbers, an example is $x=1$, $y=2$, and $z=4$)?

Answer: 2^y

- (b) (5) What is the number of failure/external nodes in this red-black tree?

Answer: $= 1 + x + 2y + 3z$

5. (10) Using the optimal binary search tree algorithm discussed in class, compute r_{ij} , and c_{ij} , $0 \leq i < j \leq 3$ for the identifier set $(a_1, a_2, a_3) = (\text{do}, \text{for}, \text{while})$ with $p_1 = 3/10$, $p_2 = 1/5$, $p_3 = 5/20$, $q_0 = 1/20$, $q_1 = 1/20$, $q_2 = 1/10$, $q_3 = 1/20$. Using the r_{ij} 's, construct the optimal binary search tree.

```
( a1, a2, a3 ) = ( do, for, while )
multiply by 20 to make simple
p1=3/10=6, p2=4, p3=5
q0=1/20=1, q1=1/20=1, q2=1/10=2, q3=1/20=1
```

```
w00 = q0, w11 = q1, w22 = q2, w33 = q3
c00 = c11 = c22 = c33 = 0
r00 = r11 = r22 = r33 = 0
```

```
w01 = q0 + q1 + p1 = 1 + 1 + 6 = 8
c01 = w01 + min{ c00 + c11 } = 8
r01 = 1
```

```
w12 = q1 + q2 + p2 = 1 + 2 + 4 = 7
c12 = w12 + min{ c11 + c22 } = 7
r12 = 2
```

```
w23 = q2 + q3 + p3 = 2 + 1 + 5 = 8
c23 = w23 + min{ c22 + c33 } = 8
r23 = 3
```

```
w02 = q0+q1+q2+p1+p2 = 1 + 1 + 2 + 6 + 4 = 14
c02 = w02 + min{ c00+c12, c01+c22 } = 14 + min{7,8} = 21
r02 = 1
```

```
w13 = q1+q2+q3+p2+p3 = 1 + 2 + 1 + 4 + 5 = 13
c13 = w13 + min{ c11+c23, c12+c33 } = 13 + min{8,7} = 20
r13 = 3
```

```
w03 = q0+q1+q2+q3+p1+p2+p3 = 20
```

$c_{03} = w_{03} + \min\{c_{00}+c_{13}, c_{01}+c_{23}, c_{02}+c_{33}\} = 20 + \min\{21, 16, 20\} = 36$
 $r_{03} = 2$

OBST: $r_{03}=2$ for
 / \
 $r_{01}=1$ $r_{23}=3$ \implies / \
 do while