Question 1.

a. Aggregate method. There are $\lfloor\sqrt{n}\rfloor$ operations that each cost $2\sqrt{k}$, and the other $n - \sqrt{\lfloor n \rfloor}$ operations each cost 1. This yields a total cost of $2 * (1 + 2 + 3 + \ldots + \sqrt{\lfloor n \rfloor}) + (n - \sqrt{\lfloor n \rfloor}) * 1 = 2 * ((\sqrt{\lfloor n \rfloor})* (\sqrt{\lfloor n \rfloor} + 1) / 2) + (n - \sqrt{\lfloor n \rfloor}) \leq n + \sqrt{\lfloor n \rfloor} + n - \sqrt{\lfloor n \rfloor} = 2n$. So the amortized cost is $2n / n = 2$.

b. Accounting method. Charge each operation \$2. When k is not a perfect square, use \$1 to pay for the operation and use the extra \$1 for credit. When k is a perfect square, the preceding $(k - 1) - (\sqrt{k} - 1)^2 = 2\sqrt{k} - 2$ operations have each paid a credit of \$1, which together with the \$2 for the current operation yields exactly enough to pay for its $2\sqrt{k}$ actual cost.

c. Potential method. Define Φ = the number of operations since the most recent perfect square. That is, let $\Phi_\kappa = k - (\sqrt{\lfloor k \rfloor})^2$. When k is not a perfect square, the amortized cost is $c' = c + \Phi_k - \Phi_{k-1} = 1 + (k - (\sqrt{\lfloor k \rfloor})^2) - ((k - 1) - (\sqrt{\lfloor k \rfloor})^2) = 2$. When k is a perfect square, the amortized cost is $c' = c + \Phi_k - \Phi_{k-1} = 2\sqrt{k} + (k - (\sqrt{k})^2) - ((k - 1) - (\sqrt{k} - 1)^2) = 2\sqrt{k} + 0 - (2\sqrt{k} - 2) = 2$.