Spring 2004
make-up exam
Exam02_solution

question 1.

```
                              min
                               |
                               V
   20        3                 1
   |       /   \           /   /   |
   25    7     5       13    15   18
         |           /   |        |
         9          14  26  17
                    |        |
                    19       30
```

(a)
  delete (1) = min

```
              min
               |
               V
   20        3           13        15       18
   |       /   \        /   \       |
   25    7     5      14    26     17
         |           |              |
         9           19             30
```

  joins two min F-heaps with same degree 2 and degree 1

```
          3                 15      18
      /    /  |           /   |
   (F)13   7   5      (F)20   17
   /   |   |           |      |
  14  26  9           25     30
  |
  19
```

(b) DecreaseKey

```
            3                 15      18     14     19
       /     /   |          /   |
(T) 13     7   5      (F) 20   17
        |    |               |      |
        26   9               25     30
```

(2)

  (a)

```
                        10
                     /        \
                   9          12
                 /   \      /     \
               4       6   11       13
             / \     / \
            2    3  5    7
```
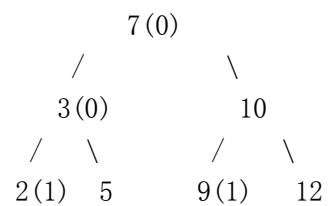
  (b) insert 8 (LR)

```
                        10
                     /        \
                   5          12
                 /   \      /     \
               3       7   11       13
             / \     / \
            2    4  6    9
```

```
                        7(0)
                     /        \
                   5(1)          10
                 /   \       /      \
               3       6   9(1)       12
             /   \         /        /   \
            2      4       8       11    13
```

  insert 1 (LL)

```
                        7(0)
                     /          \
                   3(0)          10
                 /   \        /      \
               2(1)   5     9(1)       12
```

```
              /       / \    /       / \
        1        4   6  8       11  13
```
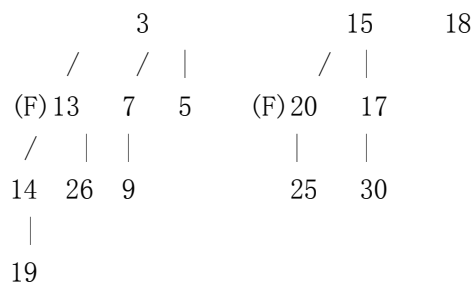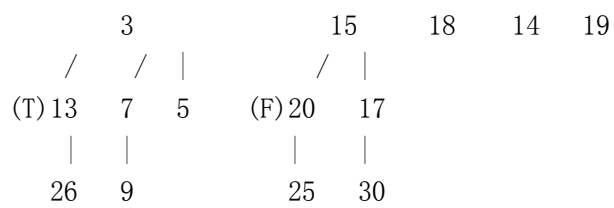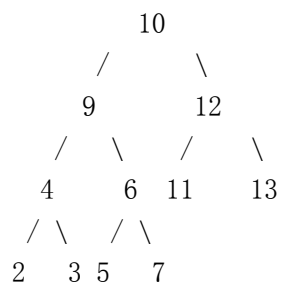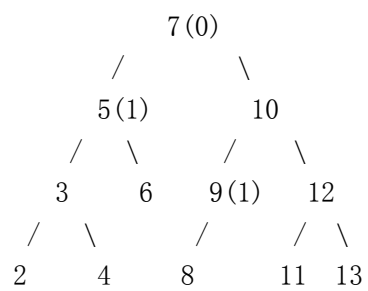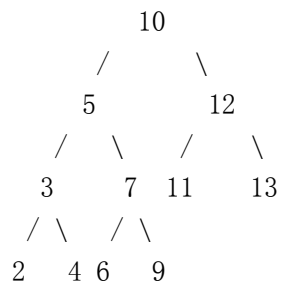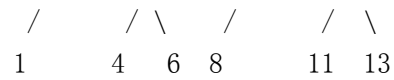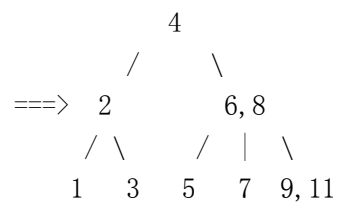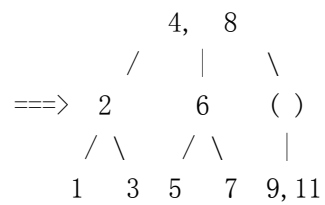
3.
  (a) 2-3 tree with 11 keys

```
                   4,  8
                 /   |   \
              2      6      10
            / \    / \     / \
          1   3  5   7   9   11
```
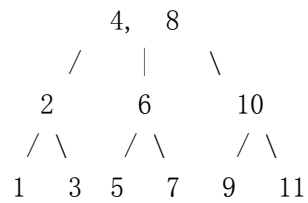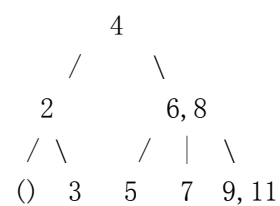
  (b) delete "10"
      First, transform deletion from interior to deletion from a leaf.

```
                   4,  8
                 /   |   \
              2      6      9
            / \    / \     / \
          1   3  5   7  ( )   11
```

```
                   4,  8
                 /   |   \
        ===>  2      6      ( )
            / \    / \      |
          1   3  5   7    9,11
```

```
                   4
                 /     \
        ===>  2         6,8
            / \      /  |  \
          1   3    5   7  9,11
```

  (c) delete "1"

```
                   4
                 /     \
              2         6,8
            / \      /  |  \
          ()  3    5   7  9,11
```

```
                  4
               /     \
    ===>   ( )       6,8
               |     / | \
             2,3   5  7  9,11


                  6
               /     \
    ===>    4          8
           / \        / \
         2,3  5     7  9,11
```

(4)
 (a)
```
      10     I(5)      10      LLb         8        I(1)         8
      //     ===>      //      ===>      // \\      ===>       // \\
     8                8                 5    10                5    10
                      //                                       //
                      5                                        1


     LLr      8          I(3)        8          LRb          8
    ===>    / \          ===>      /   \        ===>       /   \
          5    10                5     10                3     10
          //                     //                     // \\
          1                      1                      1    5
                                  \\
                                   3


     I(7)      8          RRr          8
    ===>     /   \        ===>       //  \
           3    10                 3    10
           // \\                   /    \
          1    5                  1      5
             \\                        \\
              7                         7
```
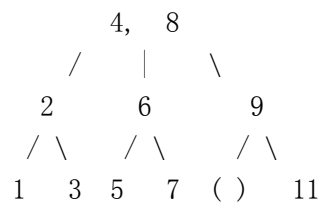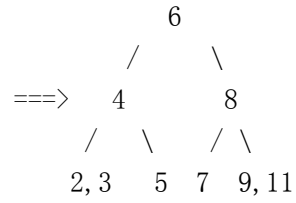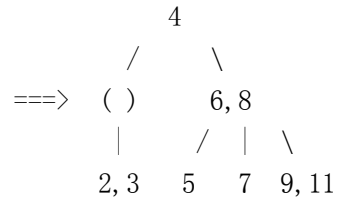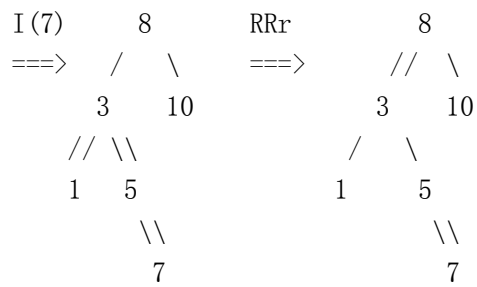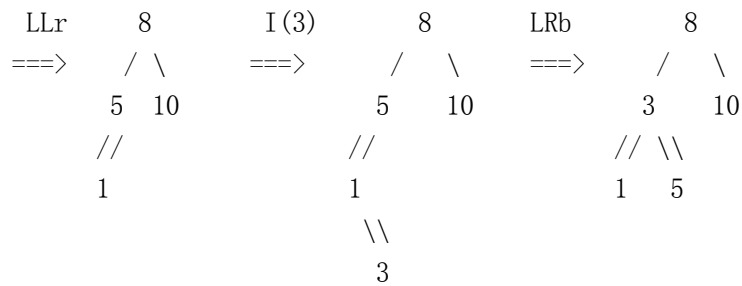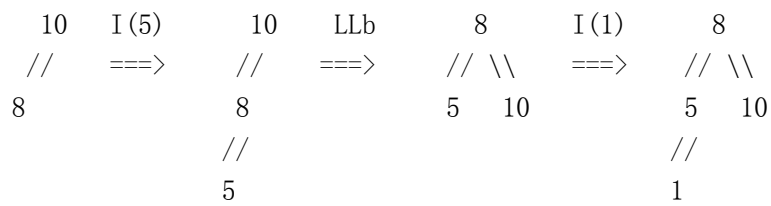
(b) Step 1. Follow the left child pointers from the root of B to
          first node x whose rank equals rank(S) = 1.

```
   6 <--- x
  //
  5
```

Step 2. Combine S, 4, and subtree rooted at 6.

```
        4
      /   \
     3     6
    //    //
    2     5
```

Step 3. Combine the result of Step 2 to node "7" through a red pointer.

```
        11
       //    \
      7      13
    //    \
    4      9
   / \   // \\
  3   6  8   10
 // //
 2   5
```

Step 4. Imbalance (LLb --> rotate)

```
        7
      //      \\
     4          11
    / \        /  \
   3   6      9   13
  // //     // \\
  2   5     8   10
```