COP 5536 / AD 711R
Advanced Data Structures

Exam 1 (Feb. 14, 2001)
CLOSED BOOK
50 Minutes

Note.   **All answers will be graded on correctness, efficiency, clarity, elegance and other normal criteria that determine quality. The points assigned to each question are provided in parentheses.**

1. (10) Consider three stack operations, **push(x, S)**, **pop(S)** and **multi-pop(k, S)**: *push(x,s)* puts the element *x* at the top of stack *S*, *pop(S)* removes the top element from the stack *S* and *multi-pop(k, S)* removes the top *k* elements from the stack *S*. A *push(x, S)* and a *pop(S)* operation take *O(1)* time each and a *multi-pop(k, S)* operation takes *O(k)* time (you may assume that $|S| <= k$). The stack is initially empty.

   Show that the amortized complexity of each of *push(x, S)*, *pop(S)* and *multi-pop(k, S)* is *O(1)*.

2. (10) Start with an empty *min binomial heap*,
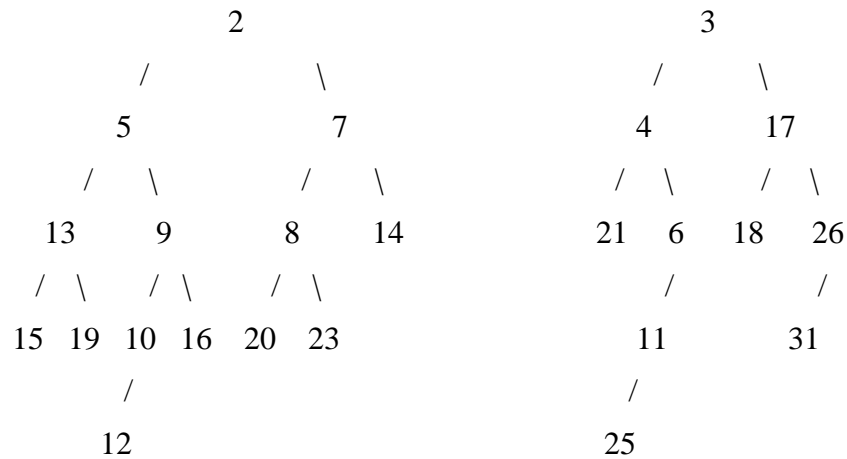
   (a) Insert the following elements into the min binomial heap:

   $$15, 45, 3, 19, 17, 26, 31, 21, 1, 4, 2$$

   Draw the resulting structure.

   (b) Perform a **DeleteMin** operation and show the resulting *min binomial heap*. Also show the intermediate steps in arriving at the result.

3. (10) *Meld* the following *min height-biased leftist trees* using the algorithm in the text. Describe each step.

```
                2                                       3
              /     \                                 /     \
            5         7                             4         17
          /   \     /   \                         /   \     /   \
        13     9   8     14                      21    6   18    26
       /  \   / \   / \                              /             /
     15  19 10  16 20  23                          11            31
         /                                          /
        12                                         25
```

4. (10) Consider 16 bins that have following remaining capacities.

   2, 6, 5, 4, 8, 3, 5, 7, 3, 4, 6, 9, 1, 7, 5, 3

   (a) Construct a max winner tree of the above 16 bins. Consider the bins from left to right.

   (b) Allocate a bin for an item of size 7 using the First Fit strategy. Describe how you found this bin (you must do this using a strategy that takes $O(\log n)$ time, where n is the number of bins). Draw the max winner tree following the allocation.

5. (10) You are given 9 runs that have 50, 100, 250, 300, 350, 400, 450, 500 , 600 equal-length records. It takes **10** seconds to **read** and **20** seconds to **write** one block from/to disk. It takes **10** seconds of **CPU time** to merge one block of records.

   Assume that all input, output, and CPU processing is sequential (I.e. input, output, and CPU processing is not overlapped or done in parallel). The block size is 50 records.

   (a) Give an optimal *4-way* merging scheme to merge the 9 runs into 1.

   (b) What is the total time taken by the optimal scheme?