----------------------- CEN 4072/6070 Software Testing & Verification ---------------------

Exam 2 – Spring 2014 – Solution Notes

1. a. would not; b. would; c. would not; d. would not; e. would

2. a. H_1=wp(S, b∧Q)

=wp(y := y∗x; x := x-1, x=0 ∧ y=24) = **(x=1 ∧ y=24)**

H_2=wp(S, ¬b∧H_1)
=wp(y := y∗x; x := x-1, x=1 ∧ y=24) = **(x=2 ∧ y=12)**

H_3=wp(S, ¬b∧H_2)
=wp(y := y∗x; x := x-1, x=2 ∧ y=12) = **(x=3 ∧ y=4)**

H_4=wp(S, ¬b∧H_3)
=wp(y := y∗x; x := x-1, x=3 ∧ y=4) = **(x=4 ∧ y=1)**

Therefore, the initial (integer) values are: **(x=1, y=24), (x=2, y=12), (x=3, y=4),** and **(x=4, y=1)**

b. H_0=(¬b∧Q)
= **(x=0 ∧ y=24)**

H_1=b∧wp(S, H_0)
=x≠0∧wp(y := y∗x; x := x-1, x=0 ∧ y=24) = **(x=1 ∧ y=24)**

H_2=b∧wp(S, H_1)
=x≠0∧wp(y := y∗x; x := x-1, x=1 ∧ y=24) = **(x=2 ∧ y=12)**

H_3=b∧wp(S, H_2)
=x≠0∧wp(y := y∗x; x := x-1, x=2 ∧ y=12) = **(x=3 ∧ y=4)**

H_4=b∧wp(S, H_3)
=x≠0∧wp(y := y∗x; x := x-1, x=3 ∧ y=4) = **(x=4 ∧ y=1)**

Therefore, the initial (integer) values are: **(x=0, y=24), (x=1, y=24), (x=2, y=12), (x=3, y=4),** and **(x=4, y=1)**

c. wp[y := y∗x; x := x-1, (x=0 ∧ y=24) V (x=1 ∧ y=24) V (x=2 ∧ y=12) V
(x=3 ∧ y=4) V (x=4 ∧ y=1)]
= (x=1 ∧ y=24) V (x=2 ∧ y=12) V (x=3 ∧ y=4) V (x=4 ∧ y=1) V (x=5 ∧ 5y=1)

Eliminating the final term (for which y is not an integer) yields the following initial values: **(x=1, y=24), (x=2,y=12), (x=3, y=4),** and **(x=4, y=1)**

d. i. necessarily true; ii. necessarily true; iii. not necessarily true; iv. necessarily true; v. not necessarily true

3. a. true; b. false; c. false; d. true; e. false; f. true; g. false; h. true

4. a.
$$\frac{P \Rightarrow wlp(S,Q)}{\{P\}\ S\ \{Q\}}$$

b. wlp(while b do S, Q) = [wp(while b do S, Q) V ¬wp(while b do S, true)]

c. From the wlp ROI, we need to show: $P \Rightarrow wlp(S,Q)$.

wlp(while x<>5 do x:=x-1, y=17) = wp(while x<>5 do x:=x-1, y=17) V ¬wp(while x<>5 do x:=x-1, true)
= $(y=17 \wedge x\geq 5) \vee x<5$ by observation

Clearly, $(x=3) \Rightarrow [(y=17 \wedge x\geq 5) \vee x<5]$, $P \Rightarrow wlp(S,Q)$ as desired.

5. b

6. function of if then statement: $(y>0 \rightarrow x,y := x+1,-y) \mid y\leq 0 \rightarrow x,y := x,y)$
function of final two assignment statements: $(x,y := 3,x-1)$

Therefore, the function of the compound program is...

$(x,y := 3,x-1)$ o $(y>0 \rightarrow x,y := x+1,-y) \mid y\leq 0 \rightarrow x,y := x,y)$
$= (x,y := (y>0 \rightarrow 3,(x+1)-1) \mid (y\leq 0 \rightarrow 3,x-1))$
$= (y>0 \rightarrow x,y := 3,(x+1)-1) \mid y\leq 0 \rightarrow x,y := 3,x-1))$
$= (y>0 \rightarrow x,y := 3,x \mid y\leq 0 \rightarrow x,y := 3,x-1))$

7. a. invalid; b. invalid; c. invalid; d. invalid; e. valid; f. valid; g. valid; h. invalid; i. valid; j. valid

8. **Does term*(t, T)?***

We use the Method of Well Founded Sets with measure x to prove *T* will terminate for any initial value of x in D*(t)* – i.e., for any x≥0. If x is initially 0, the predicate "x<>0" evaluates to false and *T* terminates immediately. If x is initially >0:

i. the value of x decreases by 1 with each execution of the loop body (via x := x-1).

ii. the value of x is bounded from below since when x becomes equal to 0, the loop must terminate because "x<>0" (i.e., the loop predicate) becomes false.

iii. the value of x may assume only a finite number of values $[(x_0, x_0-1, x_0-2, \ldots, 0)]$ since it decreases by an integral amount (1) with each iteration of the loop body.

Therefore, *T* terminates for any initial value of x≥0 and we conclude that term*(t, T) holds.*

Does $\neg p \Rightarrow (t = I)$?

$(x=0) \Rightarrow ($ t = (x,y := 0,y+2(0))
= (x,y := 0,y))
= (x,y := x,y))
= I

Does $p \Rightarrow (t = t$ o g) ?

As p is $x \neq 0$, there are 2 cases to consider: $x<0$ and $x>0$.

case a:

$(x<0) \Rightarrow ($ t = **undefined**)

$(x<0) \Rightarrow ($ t o g = undefined o g

since ((x<0) o g(x<0)) = true

= **undefined**)

case b:

$(x>0) \Rightarrow ($ t = **(x,y := 0,y+2x)**)

$(x>0) \Rightarrow ($ t o g = (x,y := 0,y+2x) o
(x,y := x-1,y+2)

since ((x≥0) o g(x>0)) = true

= (x,y := 0,(y+2)+2(x-1))
= (x,y := 0,(y+2+2x-2))
= **(x,y := 0,y+2x)**)

Therefore, $p \Rightarrow (t = t$ o $g)$

9. a. $y+2x = y_0+2x_0$
   b. $t(3,-2) = (0,4)$
   c. No. Intermediate state (5,-7) is inconsistent with q(X) when $X_0 = (3,-2)$ since $y+2x = -7+2(5) = 3$ whereas $y_0+2x_0 = -2+2(3) = 4$. (This is equivalent to observing that $t(5,-7) = (0,3) \neq (0,4) = t(3,-2)$.)
   d. Yes. Intermediate state (8,-12) is consistent with q(X) when $X_0 = (3,-2)$ since $y+2x = -12+2(8) = 4$ and $y_0+2x_0 = -2+2(3) = 4$.
   e. Neither are produced by T. For (5,-7), this is to be expected since these values are not consistent with q(X). (8,-12) is consistent with q(X), but the method by which T computes $t(3,-2)$ does not happen to generate (8,-12) as an intermediate state.

10. a. true; b. false; c. true; d. true; e. false

11. a. false; b. false; c. true; d. true; e. false

12. a. true; b. true; c. false; d. false; e. true

Histogram of Raw Scores

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              *                   *     * *      **  ********     *  ***  * **** *** * ***   *  *     ** *    *          *            *
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         012345678901234567890123456789012345678901234567890123456789012345678901234567890123456789000000000011111111112222222222333333...6
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* - CEN 6070 students
$ - CEN 4072 students