

Quiz 5 – Spring 2017 – Solution Notes

1.

	P
f_1	N
f_2	S
f_3	C

$$[P] = (x \geq y \rightarrow x, y := y, x \mid \text{true} \rightarrow I) = (x > y \rightarrow x, y := y, x \mid \text{true} \rightarrow I)$$

2. d. $(xy > 0 \rightarrow x, y := x + xy, 3 \mid xy < 0 \rightarrow x, y := x - xy, 3 \mid \text{true} \rightarrow x, y := x, 3)$

$$[P] = [S3] \circ [S2] \circ [S1] \text{ where}$$

$$[S1] = (x, y := x, xy)$$

$$[S2] = (y > 0 \rightarrow x, y := x + y, y \mid y \leq 0 \rightarrow x, y := x - y, y)$$

$$[S3] = (x, y := x, 3)$$

$$= (x, y := x, 3) \circ (y > 0 \rightarrow x, y := x + y, y \mid y \leq 0 \rightarrow x, y := x - y, y) \circ (x, y := x, xy)$$

$$= (x, y := x, 3) \circ (xy > 0 \rightarrow x, y := x + xy, xy \mid xy \leq 0 \rightarrow x, y := x - xy, xy)$$

$$= (xy > 0 \rightarrow x, y := x + xy, 3 \mid xy \leq 0 \rightarrow x, y := x - xy, 3)$$

3. a. $g = (x, y := x + 1, y - 1)$

b. $p: y \neq 0 \quad \neg p: y = 0$

c. i. measure: y

ii. All values of y in the domain of f , i.e., for $y \geq 0$.

iii. 0

d. Proof that $p \Rightarrow (f = f \circ g)$: As p is $y \neq 0$, there are 2 cases to consider: $y < 0$ and $y > 0$.

case a: $y < 0$

$$(y < 0) \Rightarrow (f = \text{undefined})$$

$$(y < 0) \Rightarrow (f \circ g = \text{undefined} \circ (x, y := x + 1, y - 1))$$

$$\text{since } (y < 0) \circ g(y < 0) = \text{true}$$

$$= \text{undefined})$$

$$\text{Therefore, } (y < 0) \Rightarrow (f = f \circ g)$$

case b: $y > 0$

$$(y > 0) \Rightarrow (f = (x, y := x + y, 0))$$

$$(y > 0) \Rightarrow (f \circ g = (x, y := x + y, 0) \circ (x, y := x + 1, y - 1))$$

$$\text{since } ((y \geq 0) \circ g(y > 0)) = \text{true}$$

$$= (x, y := (x + 1) + (y - 1), 0)$$

$$= (x, y := x + y, 0))$$

$$\text{Therefore, } (y > 0) \Rightarrow (f = f \circ g)$$

$$\text{Therefore, } p \Rightarrow (f = f \circ g)$$

