------ CEN 4072/6070 Software Testing & Verification ------

1. a. would not; b. would; c. would not; d. would not; e. would

Eliminating the final term (for which y is not an integer) yields the following initial values: (x=1, y=24), (x=2,y=12), (x=3, y=4), and (x=4, y=1)

- d. i. necessarily true; ii. necessarily true; iii. not necessarily true; iv. necessarily true; v. not necessarily true
- 3. a. true; b. false; c. false; d. true; e. false; f. true; g. false; h. true
- 4. a.  $P \Rightarrow wlp(S,Q)$

- b.  $wlp(while b do S, Q) = [wp(while b do S, Q) V \neg wp(while b do S, true)]$
- c. From the wlp ROI, we need to show:  $P \Rightarrow wlp(S,Q)$ .

wlp(while x<>5 do x:=x-1, y=17) = wp(while x<>5 do x:=x-1, y=17) V 
$$\neg wp(while x<>5 do x:=x-1, true)$$
 =  $(y=17 \land x\geq 5) \lor x<5$  by observation

Clearly,  $(x=3) \Rightarrow [(y=17 \land x \ge 5) \lor x < 5], P \Rightarrow wlp(S,Q)$  as desired.

- 5. b
- 6. function of if then statement:  $(y>0 \rightarrow x,y := x+1,-y) \mid y \le 0 \rightarrow x,y := x,y)$  function of final two assignment statements: (x,y := 3,x-1)

Therefore, the function of the compound program is...

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(x,y := 3,x-1) \circ (y>0 \to x,y := x+1,-y) \mid y \le 0 \to x,y := x,y)
= (x,y := (y>0 \to 3,(x+1)-1) \mid (y \le 0 \to 3,x-1))
= (y>0 \to x,y := 3,(x+1)-1) \mid y \le 0 \to x,y := 3,x-1))
= (y>0 \to x,y := 3,x \mid y \le 0 \to x,y := 3,x-1))
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- 7. a. invalid; b. invalid; c. invalid; d. invalid; e. valid; f. valid; g. valid; h. invalid; i. valid; j. valid
- 8. **Does term(***t*, *T***)?**

We use the Method of Well Founded Sets with measure x to prove T will terminate for any initial value of x in D(t) – i.e., for any  $x \ge 0$ . If x is initially 0, the predicate "x < >0" evaluates to false and T terminates immediately. If x is initially >0:

- i. the value of x decreases by 1 with each execution of the loop body (via x := x-1).
- ii. the value of x is bounded from below since when x becomes equal to 0, the loop must terminate because "x <>0" (i.e., the loop predicate) becomes false.
- iii. the value of x may assume only a finite number of values  $[(x_0, x_0-1, x_0-2, ..., 0)]$  since it decreases by an integral amount (1) with each iteration of the loop body.

Therefore, T terminates for any initial value of  $x \ge 0$  and we conclude that term(t, T) holds.

Does 
$$\neg p \Rightarrow (t = I)$$
?

$$(x=0) \Rightarrow (t = (x,y := 0,y+2(0))$$
  
=  $(x,y := 0,y)$ )  
=  $(x,y := x,y)$ )  
= I

## Does $p \Rightarrow (t = t \circ g)$ ?

As p is  $x \neq 0$ , there are 2 cases to consider: x < 0 and x > 0.

case a:

$$(x<0) \Rightarrow (t = undefined)$$
  
 $(x<0) \Rightarrow (t \circ g = undefined \circ g)$ 

since 
$$((x<0) \circ g(x<0)) = \text{true}$$
  
= undefined)

case b:

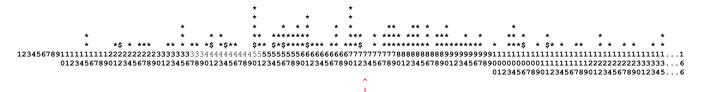
$$(x>0) \Rightarrow (t = (x,y := 0,y+2x))$$
  
 $(x>0) \Rightarrow (t \circ g = (x,y := 0,y+2x) \circ (x,y := x-1,y+2)$ 

since 
$$((x \ge 0) \circ g(x > 0)) = \text{true}$$
  
=  $(x,y := 0,(y+2)+2(x-1))$   
=  $(x,y := 0,(y+2+2x-2))$   
=  $(x,y := 0,y+2x)$ 

Therefore,  $p \Rightarrow (t = t \circ g)$ 

- 9. a.  $y+2x = y_0+2x_0$ 
  - b. t(3,-2) = (0,4)
  - c. No. Intermediate state (5,-7) is inconsistent with q(X) when  $X_0 = (3,-2)$  since y+2x = -7+2(5) = 3 whereas  $y_0+2x_0 = -2+2(3) = 4$ . (This is equivalent to observing that  $t(5,-7) = (0,3) \neq (0,4) = t(3,-2)$ .)
  - d. Yes. Intermediate state (8,-12) is consistent with q(X) when  $X_0 = (3,-2)$  since y+2x = -12+2(8) = 4 and  $y_0+2x_0 = -2+2(3) = 4$ .
  - e. Neither are produced by T. For (5,-7), this is to be expected since these values are not consistent with q(X). (8,-12) is consistent with q(X), but the method by which T computes t(3,-2) does not happen to generate (8,-12) as an intermediate state.
- 10. a. true; b. false; c. true; d. true; e. false
- 11. a. false; b. false; c. true; d. true; e. false
- 12. a. true; b. true; c. false; d. false; e. true

## Histogram of Raw Scores



\* - CEN 6070 students

\$ - CEN 4072 students