-------------------------- CEN 6070 Software Testing & Verification --------------------------

Exam 2 – Summer 2015 – Solution Notes

1. a. would not
   b. would
   c. would not
   d. would not
   e. would
   f. would not
   g. would not
   h. would

2. a. false
   b. false
   c. false
   d. true
   e. false
   f. true
   g. true
   h. true
   i. false

3. a. $P \Rightarrow wlp(K,Q)$

   b. H_0: $(t=0 \land z=yx)$
      H_1: $(t=1 \land z=y(x-1))$
      H_2: $(t=2 \land z=y(x-2))$
      H_k: $t=k \land z=y(x-k))$
      Closed form expression of wlp: $(t\geq 0 \land z=y(x-t)) \lor t<0 \equiv z=y(x-t) \lor t<0$

   c. Clearly, $(|t|=3 \land x=5 \land z=4 \land y=2) \Rightarrow wlp(K,z=yx)$ since t can be either -3, satisfying the second disjunct, or 3, satisfying the first disjunct. Therefore, the instantiated antecedent of the ROI from part (a), holds, and therefore $\{|t|=3 \land x=5 \land z=4 \land y=2\}$ K $\{z=yx\}$ holds. Since, by observation, both x and y are invariant w.r.t. K, we therefore conclude that $z=y'x'$ = (2)(5) = 10 if K happens to terminate.

4. f

5.

	P1	*P2*
f1	N	N
f2	N	S

6. a. invalid
   b. valid
   c. valid
   d. invalid
   e. valid
   f. invalid
   g. valid
   h. valid
   i. invalid
   j. invalid

7. INITIALIZATION: Does $P \Rightarrow I$?

   P: $\{n \geq -17 \wedge t=1 \wedge k=0\} \Rightarrow t=1 \wedge 2^k=1 \Rightarrow t=2^k$

   Therefore $P \Rightarrow I$. √

   PRESERVATION: Does $\{I \wedge b\}$ s $\{I\}$?

   $$I \wedge b: \{ t=2^k \wedge k \neq n \}$$
   $$t := 2*t$$
   $$\{ t=2^{k+1} \wedge k \neq n \}$$
   $$k := k+1$$
   $$\{ t=2^{(k-1)+1} \wedge k-1 \neq 0 \}$$
   $$\Leftrightarrow$$
   $$\{ t=2^k \wedge k-1 \neq 0 \} \Rightarrow I \ \surd$$

   FINALIZATION: Does $(I \wedge \neg b) \Rightarrow Q$?

   $(I \wedge \neg b)$: $(t=2^k \wedge k=n) \Rightarrow t=2^n = Q$ √

8. **Does term*(f, H)?***

   We use the Method of Well Founded Sets with measure k to prove *H* will terminate for any initial values of k and n in D(t) – i.e., for any $k \leq n$. If k is initially equal to n, the predicate "k<>n" evaluates to false and *H* terminates immediately. If k is initially less than n:

   i. the value of k increases by 1 with each execution of the loop body (via k := k+1).

   ii. the value of k is bounded from above when k is initially less than or equal to n since when k becomes equal to n (which is constant), the loop must terminate **because** "$k \neq n$" (i.e., the loop predicate) becomes false.

   iii. the value of k may assume only a finite number of values [(k_0<n, k_0+1, k_0+2, …,n)] since it increases by an integral amount (1) with each iteration of the loop body.

Therefore, *H* terminates for any initial value of $k \le n$ and we conclude that term*(f, H) holds.*

Does $\neg p \Rightarrow (f = I)$?

$(k=n) \Rightarrow (f = (t,k := t2^{\mathbf{0}},k)$

$= \mathbf{I})$

Therefore, $\neg p \Rightarrow (f = I)$.

Does $p \Rightarrow (f = f \circ g)$?

There are 2 cases to consider: $k<n$ and $k>n$.

case a:

$(k<n) \Rightarrow (f = \mathbf{(t,k := t2^{n-k},n)})$

$(k<n) \Rightarrow (f \circ g = (t,k := t2^{n-k},n) \circ$

$(t,k := 2t,k+1)$

since $((k \le n) \circ g(k<n)) = \text{true}$

$= (t,k := (2t)2^{n-(k+1)},n)$

$= (t,k := t2^{n-k-1+1},n)$

$= \mathbf{(t,k := t2^{n-k},n)})$

case b:

$(k>n) \Rightarrow (f = \textbf{undefined})$

$(k>n) \Rightarrow (f \circ g = \text{undefined} \circ g$

since $((k \le n) \circ g(k>n)) = \text{false}$

$= \textbf{undefined})$

Therefore, $p \Rightarrow (f = f \circ g)$.

9. a. i. $t(X_3) = X_n = (0, z_0+5|y_0|)$
   ii. $t(X_n) = X_n = (0, z_0+5|y_0|)$
   iii. $q(X)$: $z+5|y| = z_0+5|y_0|$ or $z = z_0+5(|y_0|-|y|)$

   b. One cannot deduce unique values of y_0 and z_0 from this information, but one CAN deduce that $z_0+5|y_0|= -1$
   c. iii
   d. Yes, q(1,14) is consistent with $X_0 = (y_0, z_0) = (4,-1)$. Equivalently, $t(1,14) = t(4,-1) = (0,19)$. Therefore, a while loop computing $t(4,-1)$ could also generate intermediate state (1,14).

10. a. true
    b. false
    c. true
    d. true
    e. false

11. a. false
    b. false
    c. false
    d. false
    e. true

12. program function: $\mathbf{(x>-1 \rightarrow x,y := x,x+1 \mid x \le -1 \rightarrow x,y := -x-1,-x)}$

Histogram of Scaled Scores

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                                                                     *
                                                                     *
                                           *               *     **  *    *    *    *
         1111111111222222222233333333334444444444555555555566666666667777777777888888888899999999990
1234567890123456789012345678901234567890123456789012345678901234567890123456789012345678901234567890
                                                                    ^
                                                                    |
                                                                   mean
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