

Problem Set 7: Functional Verification

Hints and Notes

1. Given

P1 = while $x > 1$ do $x := x - 1$; $z := z * x$ end_while

P2 = while $x \geq 1$ do $z := z * x$; $x := x - 1$ end_while

P3 = while $x \neq 1$ do $z := z * x$; $x := x - 1$ end_while

$f1 = (x > 1 \rightarrow x, z := 1, zx! \mid x = 1 \rightarrow I)$

$f2 = (x \geq 1 \rightarrow x, z := 1, z(x-1)! \mid x < 1 \rightarrow I)$

$f3 = (x > 1 \rightarrow x, z := 0, zx! \mid x = 1 \rightarrow x, z := 0, z$
 $\mid x < 1 \rightarrow I)$

Determine the correctness relationship between each program and function (C = Complete and Sufficient, S = Sufficient Only, N = Neither).

P1:

```
while  $x > 1$  do  $x := x - 1; z := z * x$  end_while
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$:= 1, z(x-1)!$

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$x < 1 \rightarrow x, z := x, z \quad (I)$

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while $x > 1$ do $x := x - 1; z := z * x$ end_while

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$p1: (x \geq 1 \rightarrow x, z := 1, z(x-1)! \mid \text{true} \rightarrow I)$

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$p1: (x \geq 1 \rightarrow x, z := 1, z(x-1)! \mid \text{true} \rightarrow I)$

or

$(x > 1 \rightarrow x, z := 1, z(x-1)! \mid \text{true} \rightarrow I)$

2. Use the correctness condition for sequencing to prove $f = [P]$ where $f = (x, y := x+2, y(x^2+2x))$ and P is: $y := y*x; x := x+2; y := y*x$

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S1

S2

S3

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S1

S2

S3

Therefore, $P = \mathbf{S1; S2; S3}$. Does $f = \mathbf{s3} \circ \mathbf{s2} \circ \mathbf{s1}$?

4. a. Determine the function of the following program.

```
temp := x
x := y
y := temp
if temp > z then
    y := z
    z := temp
    if x > y then
        temp := x
        x := y
        y := temp
    end_if
end_if
```

4. a. Determine the function of the following program.

```
temp := x
x := y
y := temp
if temp > z then
    y := z
    z := temp
    if x > y then
        temp := x
        x := y
        y := temp
    end_if
end_if
```

The diagram illustrates the scope of variables x , y , and z . A red bracket labeled $S1$ groups the assignments $x := y$ and $y := temp$. A red bracket labeled $S2$ groups the entire `if temp > z then` block, including its nested `if x > y then` block.

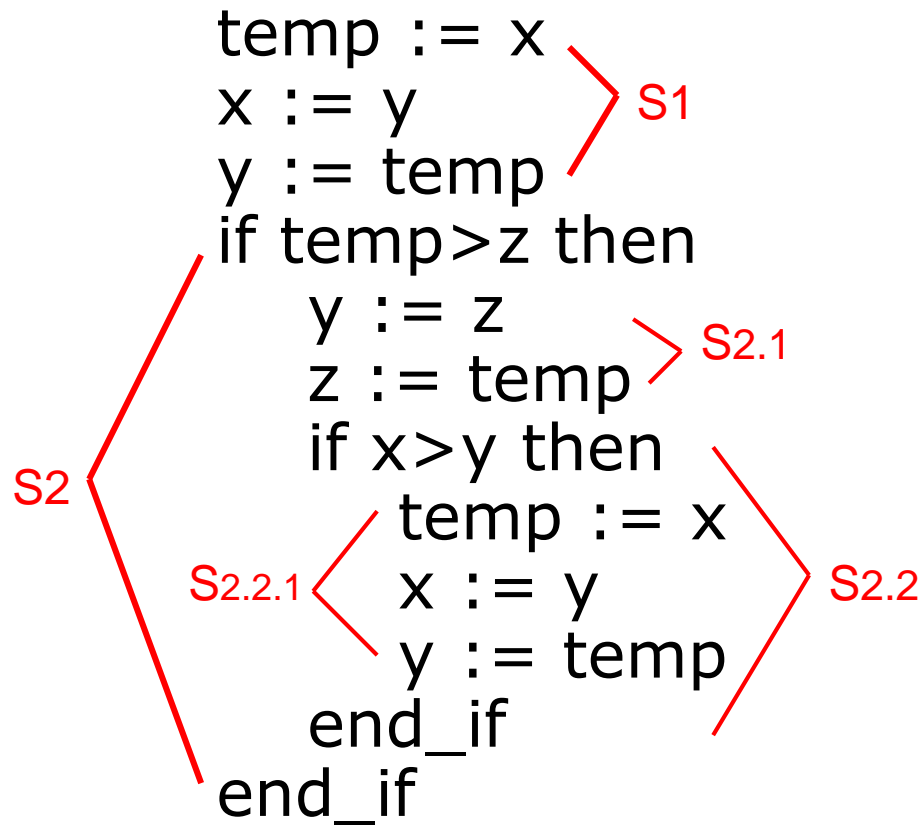
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    z := temp
    if x > y then
        temp := x
        x := y
        y := temp
    end_if
end_if
```

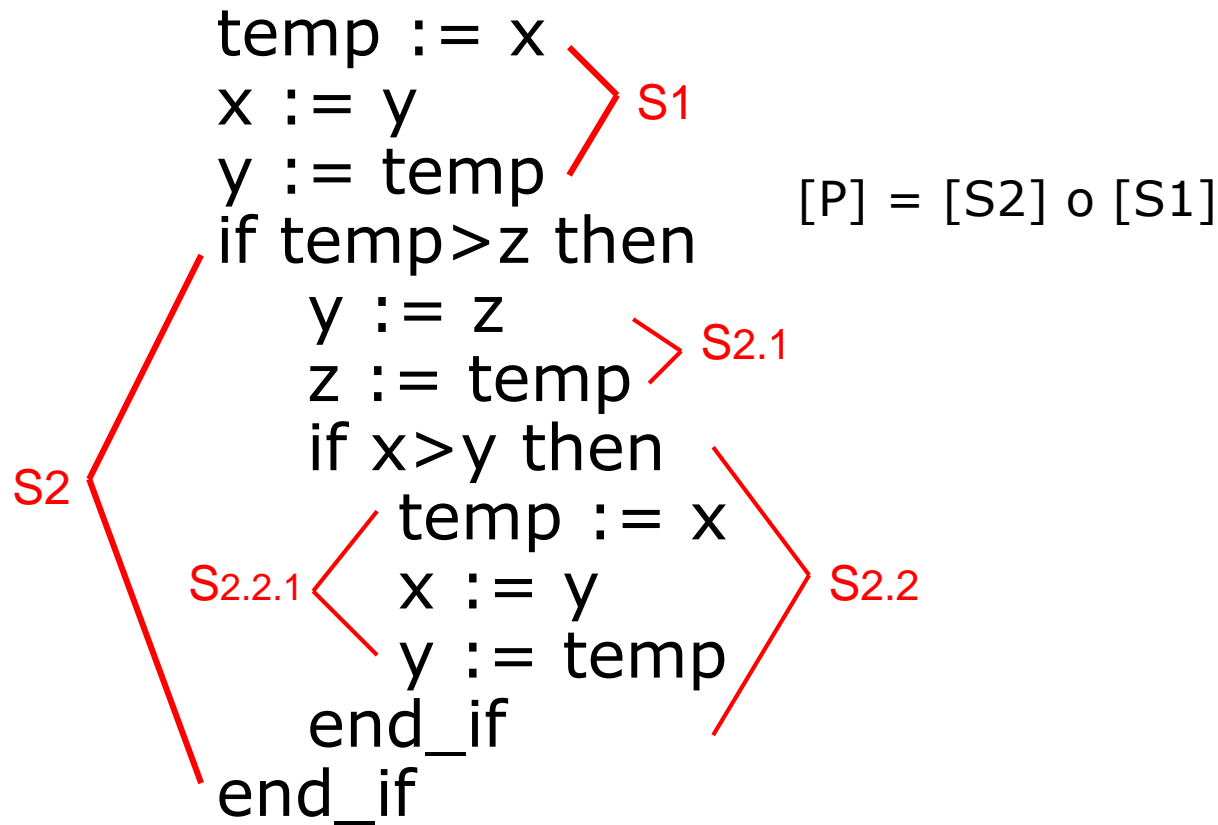
The diagram illustrates the hierarchical structure of the code blocks using red brackets and labels:

- S1** groups the first three lines: `temp := x`, `x := y`, and `y := temp`.
- S2** groups the entire conditional block starting from `if temp > z then` down to `end_if`.
- S2.1** groups the first two lines of the conditional block: `y := z` and `z := temp`.
- S2.2** groups the nested conditional block: `if x > y then`, `temp := x`, `x := y`, `y := temp`, and `end_if`.

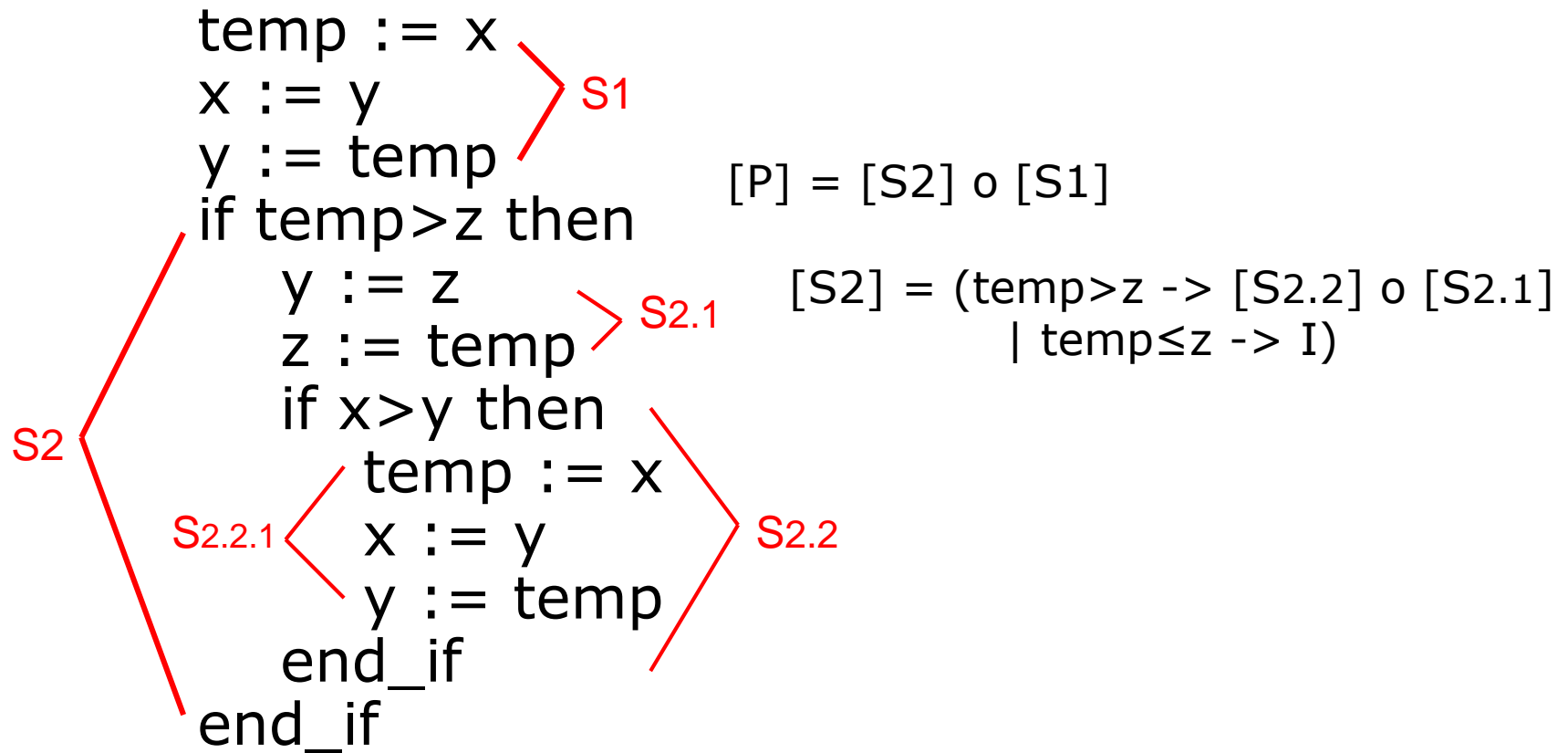
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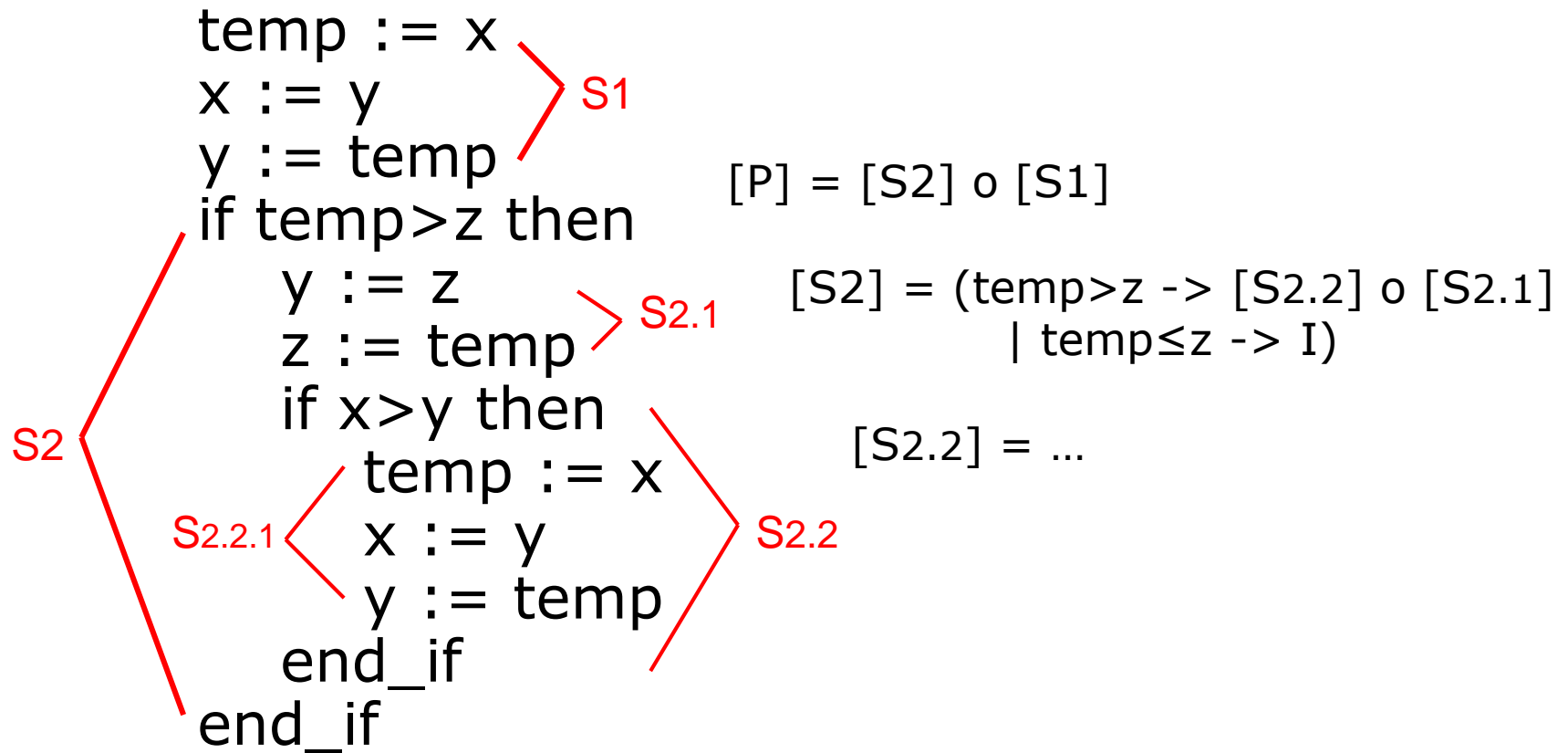
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5. For program A below, hypothesize a function a for $[A]$ and prove $a = [A]$.

```
while  $x < y + b$  do  
   $x := x + 2$   
   $y := y + 1$   
end_while
```

You may assume that the function of the while_do body, G , is:

$$g = (x, y, b := x + 2, y + 1, b)$$

5. What is a ?

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$x = y + b \rightarrow x, y, b := x, y, b$
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$x > y + b \rightarrow x, y, b := x, y, b$

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$$x > y + b \rightarrow x, y, b := x, y, b$$

Therefore, the hypothesized $a =$

$$(x \leq y + b \rightarrow x, y, b := -x + 2y + 2b, -x + 2y + b, b \mid \text{true} \rightarrow I)$$

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or equivalently,

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$$(x \leq y + b \rightarrow x, y, b := -x + 2y + 2b, -x + 2y + b, b \mid \text{true} \rightarrow I)$$

or equivalently,

$$(x < y + b \rightarrow x, y, b := -x + 2y + 2b, -x + 2y + b, b \mid \text{true} \rightarrow I)$$

You can choose the one that is easier to work with!

6. For program C below, hypothesize a function c for $[C]$ and prove $c = [C]$.

```
y := 1
k := 0
repeat
  y := y*2
  k := k+1
until k=n
```

6. What is c ?

```
y := 1
k := 0
repeat
  y := y*2
  k := k+1
until k=n
```

6. What is c ?

c1 { $y := 1$
 $k := 0$

c2 { repeat
 $y := y * 2$
 $k := k + 1$ } **G**
until $k = n$

6. What is c ?

c_1 { $y := 1$
 $k := 0$

c_2 { repeat
 $y := y * 2$ } G
 $k := k + 1$
until $k = n$

$C = C_1; C_2$

6. What is c ?

$c1$ { $y := 1$
 $k := 0$

$c2$ { repeat
 { $y := y*2$
 $k := k+1$ } G
 until $k=n$

$C = C1; C2$

$c1 = [C1] = (y, k := 1, 0)$ by observation

6. What is c ?

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 $k := 0$

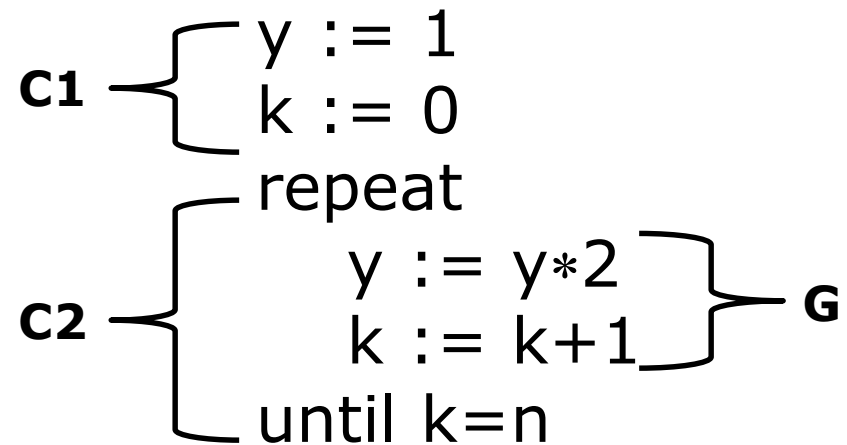
$c2$ { repeat
 $y := y*2$
 $k := k+1$ } G
until $k=n$

$C = C1; C2$

$c1 = [C1] = (y, k := 1, 0)$ by observation

$c2 = [C2] = (k < n \rightarrow y, k := y2^{n-k}, n)$ by hypothesis

6. What is c ?



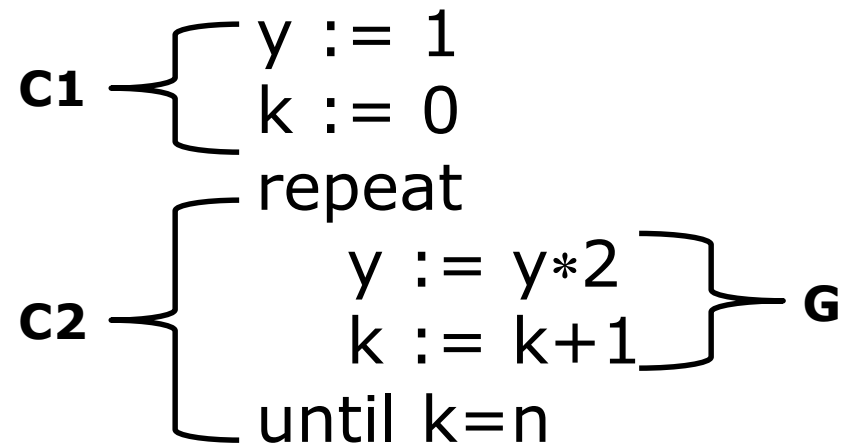
$C = C1; C2$

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$c2 = [C2] = (k < n \rightarrow y, k := y2^{n-k}, n)$ by hypothesis

Therefore, the hypothesized $c = [C] = c2 \circ c1$

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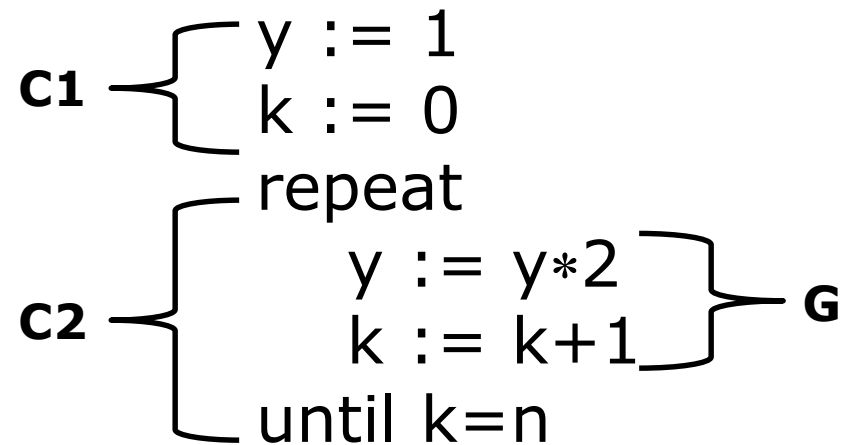
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Therefore, the hypothesized $c = [C] = c2 \circ c1$

$= (k < n \rightarrow y, k := y2^{n-k}, n) \circ (y, k := 1, 0)$

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$c1 = [C1] = (y, k := 1, 0)$ by observation

$c2 = [C2] = (k < n \rightarrow y, k := y2^{n-k}, n)$ by hypothesis

Therefore, the hypothesized $c = [C] = c2 \circ c1$

$= (k < n \rightarrow y, k := y2^{n-k}, n) \circ (y, k := 1, 0)$

$= (n > 0 \rightarrow y, k := 2^n, n)$

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