---------------------------- CEN 4072/6070 Software Testing & Verification --------------------------
Exam 2 – Spring 2017 – Solution Notes

1. f. the correctness conditions for f = [*repeat g until p*]

2. vi. (Found ∧ Key=List[Index]) V (¬Found ∧ ∀ (1≤i≤Index) • Key≠List[i])

3. a. ii. Suppose P is x=5, S is (x := x+1; y := 17), b is x=0, Q is (y=17 Λ x=0), and I is y=17.
   b. i. {y=17} while (x>0 V y<>17) do x := x-1 {x≤0}
   c. iv. Suppose P is x=0, S is (x := x-17), b is x<=0, and Q is x≥0.

4. a. M. y>0 ∧ x=17-y
   b. C. y≥0 ∧ x=17-y
   c. R. (x=17-y) V (y≤0)
   d. F. (x=17-y) V (y<0)
   e. O. x=7 ∧ y=0
   f. P. "undefined"
   g. H. [y'>0 ⇒ (y=0 ∧ x=5+y')] ∧ (y'≤0 ⇒ "undefined")

5. (1) term*(f,K)*, (2) *p* ⇒ (*v*=*v*o*g*), (3) ¬*p* ⇒ (*v*=*t*), and (4) *f*=*v*o*h*

6. f. (none of the above)

   [P] = (x,y := x,xy) o (y>0 -> x,y := x+y,y | y≤0 -> x,y := x-y,y) o (x,y := x,3)
   = (x,y := x,xy) o (3>0 -> x,y := x+3,3 | 3≤0 -> x,y := x-3,3)
   = (x,y := x,xy) o (x,y := x+3,3)
   = (x,y := x+3,3(x+3))
   = (x,y := x+3,3x+9)

7. Suppose the initial (input) value of z is -5, and the initial input value of y is any value other than -4. Then P is satisfied, S terminates, and Q will be false since z+1 would equal -4 but y would not.

8. a. *g*=(x,y := x-1,2y)
   b. *p*: x=0
   c. i. measure: x
      ii. All (integer) values of the measure in the domain of *f*, i.e., for every x>0.
      iii. 0
   d. Proof that ¬*(p*o*g)* ⇒ (*f*=*f*o*g*):

   Two proof approaches are illustrated:

   Approach I: determine all cases associated with ¬*(p*o*g)* and then determine case-specific expressions for *f* and *f*o*g* separately for comparison

   ¬[(x=0)o(x,y := x-1,2y)] ⇒ x_0≠1
   Thus, there are two cases to consider: x_0<1 and x_0>1

   <u>case a</u>: x_0<1
   (x<1) ⇒ (*f* = ***undefined*** **)** by defn of *f* for x<1
   (x<1) ⇒ (*f*o*g* = *undefined* o (x,y := x-1,2y)

since ((x>0) $\circ$ g(x<1)) = false
= ***undefined***)

Therefore, (x<1) $\Rightarrow$ (f=$f\circ g$)

<u>case b</u>: x_0>1
(x>1) $\Rightarrow$ (f = **(x,y := 0, y2^x)**) by defn of f for x>1
(x>1) $\Rightarrow$ ($f\circ g$ = (x,y := 0, y2^x) $\circ$ (x,y := x-1,2y)
since ((x>0) $\circ$ g(x>1)) = true
= (x,y := 0, (2y)2^{x-1})
= **(x,y := 0, y2^x)**)

Therefore, (x>1) $\Rightarrow$ (f=$f\circ g$)

Therefore, $\neg$(*p*$\circ$*g*) $\Rightarrow$ (f=$f\circ g$)

Approach II: determine $f\circ g$ **in general** and then compare to f for each case associated with $\neg$(*p*$\circ$*g*)

(1) $f\circ g$ = (x>0 -> x,y := 0, y2^x) $\circ$ (x,y := x-1,2y) (by defn. of f and g)
= (x-1>0 -> x,y := 0, (2y)2^{x-1})
= (x>1 -> x,y := 0, y2^x))

(2) $\neg$(*p*$\circ$*g*) = $\neg$[(x=0)$\circ$(x,y := x-1,2y)] = x-1≠0 = x-1≠0
Therefore, x<1 or x>1 (two cases to consider)

(3) Does (x>1) $\Rightarrow$ (f=$f\circ g$)?
(x>1) $\Rightarrow$ (f = **(x,y := 0, y2^x)**) by defn of f for x>1
(x>1) $\Rightarrow$ ($f\circ g$ = **(x,y := 0, y2^x)**) by defn of $f\circ g$ for x>1
Therefore, (x>1) $\Rightarrow$ (f=$f\circ g$)

(4) Does (x<1) $\Rightarrow$ (f=$f\circ g$)?
(x<1) $\Rightarrow$ (f = **undefined**) by defn of f for x<1
(x<1) $\Rightarrow$ ($f\circ g$ = **undefined**) by defn of $f\circ g$ for x<1
Therefore, (x<1) $\Rightarrow$ (f=$f\circ g$)

Therefore, $\neg$(*p*$\circ$*g*) $\Rightarrow$ (f=$f\circ g$)

9. a. vi. $y=y_0+2(x_0-x) \wedge x\geq 0$
b. X_0 = (3,3), X_1 = (2,5), X_2 = (1,7), X_3 = X_n = (0,9)
c.

$p(X_0)$: true	$g(X_0)$: (2,5)	$f(X_2)$: (0,9)	$q(X_2)$: true
$p(X_2)$: true	$g(X_2)$: (0,9)	$f(X_n)$: (0,9)	q(2,7): false
$p(X_n)$: false	$g(X_n)$: (-1,11)	$f(g(X_n))$: undef	$q(g(X_n))$: false/undef

d. (4,3)
e. i. no
ii. no

10. a. false
b. false
c. true
d. false
e. true

11. a. false
    b. true
    c. true
    d. false
    e. true