

Quiz 3 – Spring 2017 – Solution Notes

$$1. a. \frac{\{P\} S \{I\}, \quad \{I \wedge \neg b\} S \{I\}, \quad (I \wedge b) \Rightarrow Q}{\{P\} \text{ repeat } S \text{ until } b \{Q\}}$$

$$b. iii. (Found \wedge Key = List[Index]) \vee (\neg Found \wedge \forall (1 \leq i \leq Index) \bullet Key \neq List[i])$$

c. i. Proof that "Initialization" holds:

INITIALIZATION: Does  $\{P\} S \{I\}$ ?

$$\begin{aligned} & \{N \geq 1 \wedge Index = 0\} \\ & \quad \mathbf{Index := Index + 1} \\ & \{N \geq 1 \wedge Index = 1\} \\ & \quad \mathbf{Found := (Key = List[Index])} \\ & \{N \geq 1 \wedge Index = 1\} \wedge [(Found \wedge Key = List[1]) \vee (\neg Found \wedge Key \neq List[1])] \\ = & \{N \geq 1 \wedge Index = 1\} \wedge [(Found \wedge Key = List[Index]) \vee (\neg Found \wedge Key \neq List[Index])] \\ => & \{[(Found \wedge Key = List[Index]) \vee (\neg Found \wedge \forall (1 \leq i \leq Index) \bullet Key \neq List[i])]\} \\ = & \{I\} \end{aligned}$$

So,  $\{P\} S \{I\}$ .

ii. Proof that "Preservation" holds:

PRESERVATION: Does  $\{I \wedge \neg b\} S \{I\}$ ?

$$\begin{aligned} & \{[(Found \wedge Key = List[Index]) \vee (\neg Found \wedge \forall (1 \leq i \leq Index) \bullet Key \neq List[i])] \\ & \quad \wedge \neg(Found \vee Index = N)\} \\ = & \{\neg Found \wedge \forall (1 \leq i \leq Index) \bullet Key \neq List[i] \wedge Index \neq N\} \\ & \quad \mathbf{Index := Index + 1} \\ & \{\neg Found \wedge \forall (1 \leq i \leq Index - 1) \bullet Key \neq List[i] \wedge Index - 1 \neq N\} \\ & \quad \mathbf{Found := (Key = List[Index])} \\ & \{[(Found \wedge Key = List[Index]) \vee (\neg Found \wedge \forall (1 \leq i \leq Index) \bullet Key \neq List[i])] \\ & \quad \wedge Index - 1 \neq N\} \\ = & \{I \wedge Index - 1 \neq N\} => I \end{aligned}$$

So,  $\{I \wedge \neg b\} S \{I\}$ .

2. a. would not (S may change the value of z.)
- b. would not (S may change the value of z to an odd number)
- c. would not (S may change the value of z.)
- d. would not (y could be 2 and z could be 1 on termination)
- e. would (if  $z = -5$  and  $y \neq -4$  initially, Q will not hold when S terminates)

3. a. invalid (the antecedent does not guarantee that  $(b \wedge Q)$  will hold AFTER S executes)  
b. invalid (the antecedents do not guarantee that I, P, or Q will hold on termination)  
c. valid (the antecedent guarantees that Q will hold IF the loop terminates)  
d. invalid (the antecedents guarantee that S will terminate, but do NOT guarantee that the LOOP will terminate)  
e. valid (the antecedent guarantees that the loop will terminate in state Q after one iteration)

