Quiz 4 – Spring 2017 – Solution Notes

1. a. K (see Problem Set 6 solution notes and slide 42 in (instructor) lecture notes 20 for a
       similar problem)
   b. F (weakest Q-adequate I is post-condition of loop body when wp holds initially)
   c. M (by observation, the loop will not terminate when y≤0)
   d. D (similar to b)
   e. P (loop body will execute twice with result y=0 and x=7)
   f. N (loop will not terminate)
   g. H (if y'>0 the loop will terminate with (y=0 ∧ x=5+y'), otherwise it will not terminate)

2. a. would not (given observation/fact does not disprove assertion since S may not terminate
       when z≤-5)
   b. would (first conjunct of observation/fact implies S will terminate for z=-4,-3,-2, and -1;
       second conjunct implies Q will not hold whenever S terminates)
   c. would not (y≠z does not contradict given post-condition in assertion)

3. a. would (compare with 2a – now S must terminate since the assertion is one of strong
       correctness)
   b. would (given observation/fact implies that S will not terminate with y≠z when z≤-5,
       contradicting assertion of strong correctness)
   c. would not (does not contradict given assertion)

4. a. invalid (antecedent does NOT guarantee that P ⇒ S will terminate, as required for
       consequent)
   b. invalid (consider the counterexample: P is (x=5), S is x:=x+1, and Q is (x=6); here the
       antecedent holds, but the consequent does not)
   c. valid (if the antecedent holds, then P also implies wlp(S, Q), which in turn implies the
       consequent)

5. wlp(if b then S, Q) ≡ (b ∧ wlp(S, Q)) ∨ (¬b ∧ Q)

   wlp(if A>B then Z:=A, Z=Max(A,B))
       = (A>B ∧ wlp(Z:=A, Z=Max(A,B)) ∨ (A≤B ∧ Z=Max(A,B))
       = (A>B ∧ A=Max(A,B)) ∨ (A≤B ∧ Z=Max(A,B))
       = (A>B ∧ A≥B) ∨ (A≤B ∧ Z=Max(A,B))
       = (A>B) ∨ (A≤B ∧ Z=B)
       = (A>B) ∨ (Z=B)

HISTOGRAM OF SCORES

Range	Count
1-5	1
6-10	6
11-15	10
16-20	23
21-25	14
26-30	8
31-35	1