ROI Analysis and Review of Foundations

1. Interpreting conditional assertions
2. Assertions of weak and strong program correctness are conditionals!
3. Where do ROI’s come from?
4. How to analyze the validity of a hypothesized ROI

conditional assertions

- Interpretation of "A => B": "**<u>if</u>** A is true, **<u>then</u>** B **must** (<u>in every case</u>) be true."
- When is "A => B" <u>false</u> (i.e., "invalid")?

conditional assertions

- Interpretation of "A => B": "**if** A is true, **then** B **must** (in every case) be true."
- When is "A => B" false (i.e., "invalid")?

*If and only if there exists a case for which **A is true** but **B is false**.*

conditional assertions

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- Find a counterexample that PROVES

"x is even => x>10"

is false (i.e., "invalid")?

conditional assertions

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Let x=4. Then “x is even” is true, but “x>10” is false.

conditional assertions (cont'd)

- Interpretation of hypothesized ROI:

$$\frac{A,\ B,\ C}{D}\ ?$$

conditional assertions (cont'd)

- Interpretation of hypothesized ROI:

$$\frac{A,\ B,\ C}{D}\ ?$$

"**<u>if</u>** A and B and C are true, **<u>then</u>** D **must** (<u>in every case</u>) be true."

conditional assertions (cont'd)

- Interpretation of hypothesized ROI:

$$\frac{A,\ B,\ C}{D}\ ?$$

"**if** A and B and C are true, **then** D **must** (in every case) be true."

- When is the ROI false (i.e., "invalid")?

conditional assertions (cont'd)

- Interpretation of hypothesized ROI:

$$\frac{A,\ B,\ C}{D} \ ?$$

"**if** A and B and C are true, **then** D **must** (in every case) be true."

- When is the ROI false (i.e., "invalid")?

*If and only if there exists a case for which **A and B and C are true,** but **D is false**.*

conditional assertions (cont'd)

- Find a counterexample that PROVES

$$\frac{(x=17) \Rightarrow (x \text{ is odd})}{x=17} \; ?$$

is "invalid" (i.e., false)?

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Let x=4. Then "(x=17)=>(x is odd)" is true but "x=17" is false.

conditional assertions (cont'd)

- Find a counterexample that PROVES

$$\frac{(x=17) \Rightarrow (x \text{ is odd})}{x=17} \; ?$$

  is "invalid" (i.e., false)?

  Let x=4. Then "(x=17)=>(x is odd)" is true but "x=17" is false.

- Very important corollary: (A=>B) **≠>** A

A ***really smokin'*** counter-example…

- Consider the following assertion/ROI:

"People who wear red shirts do not smoke."

=

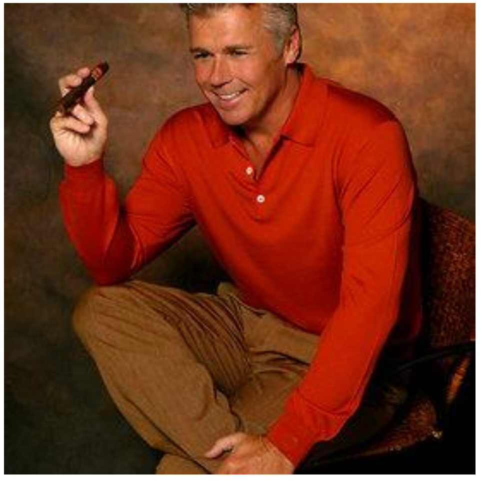
Wears red shirts(X) => Does not smoke(X)

=

$$\frac{\text{Wears red shirts(X)}}{\text{Does not smoke(X)}} \ \textbf{?}$$

A *really smokin'* counter-example… (cont'd)

- Is the assertion/ROI valid (true)?
- No. Proof by counterexample:



- This person satisfies the antecedent, but not the consequent!

weak correctness: {P} S {Q}

- Interpretation of {P} S {Q}: "**<u>if</u>** the input (initial state) satisfies pre-condition P and (**<u>if</u>**) program S executes and terminates, **<u>then</u>** the output (final state) **must** (<u>in every case</u>) satisfy post-condition Q."

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- When is {P} S {Q} false (i.e., "invalid")?

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- When is {P} S {Q} false (i.e., "invalid")?

If <u>and only if</u> there exists a case for which Q will be false when S terminates, given that P held before S executes.

weak correctness: {P} S {Q} (cont'd)

- Example: true or false?

{y=17} if x<>0 do y := 0 {y=0}

weak correctness: {P} S {Q} (cont'd)

- Example: true or false?

{y=17} if x<>0 do y := 0 {y=0}

The assertion is FALSE (invalid) since Q will not hold on termination when x is initially equal to 0 (which is not precluded by the given pre-condition).

strong correctness: {P} S {Q} *strongly*

- Interpretation of {P} S {Q} *strongly:*
  "{P} S {Q} <u>AND</u> P => S terminates"

strong correctness: {P} S {Q} *strongly*

- Interpretation of {P} S {Q} *strongly:*
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- When is "{P} S {Q} *strongly*" false (i.e., "invalid")?

strong correctness: {P} S {Q} *strongly*

- Interpretation of {P} S {Q} *strongly:*
  "{P} S {Q} <u>AND</u> P => S terminates"
- When is "{P} S {Q} *strongly*" false (i.e., "invalid")?

  If <u>and only if</u> either {P} S {Q} is false <u>OR</u> P $\not\Rightarrow$ S terminates

strong correctness: {P} S {Q} *strongly* (cont'd)

- Example: true or false?

{y=17∧|x|=1} while y<>0 do y := y-x {y=0} *strongly*

strong correctness: {P} S {Q} *strongly* (cont'd)

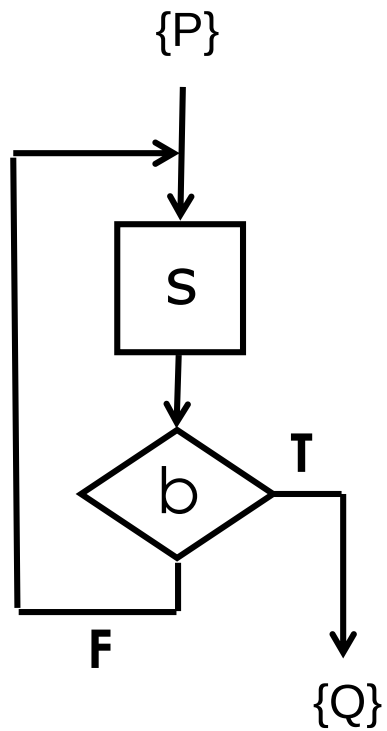
- Example: true or false?

$\{y=17 \wedge |x|=1\}$ while y<>0 do y := y-x $\{y=0\}$ *strongly*

The assertion is FALSE (invalid) since P ≠> S terminates (x may equal -1).

From Problem Set 5: Axiomatic Verification
“Hints and Notes”

4. Prove the following assertion using a suitable Rule of Inference for the Repeat_Until-Loop. Clearly state the Rule of Inference and show all steps. (Hint: Do NOT include P => I as an antecedent in your rule.)



$$\frac{\qquad\qquad\qquad\qquad}{\{P\}\ \text{repeat } s \text{ until } b\ \{Q\}}$$

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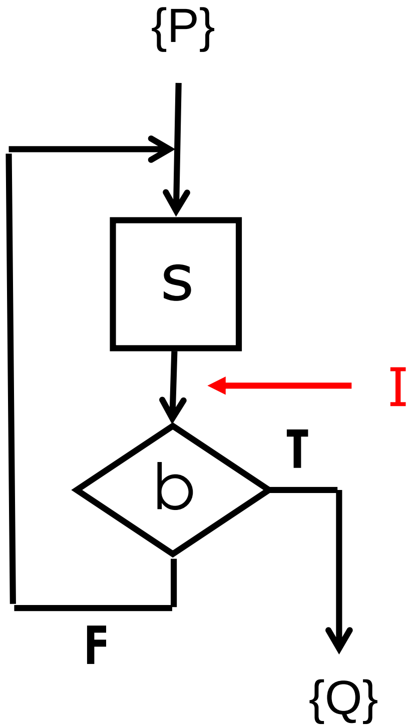

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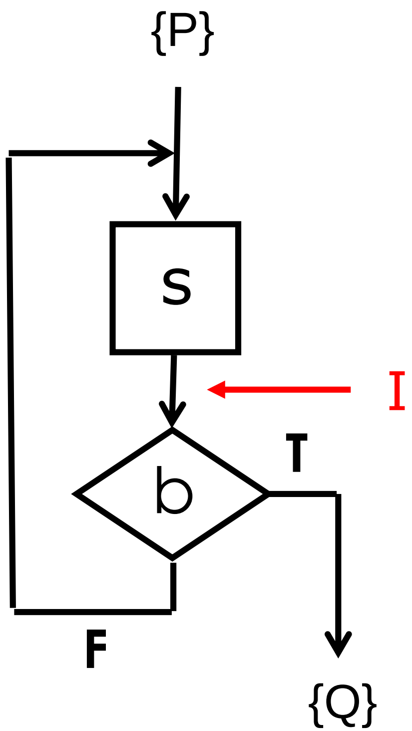

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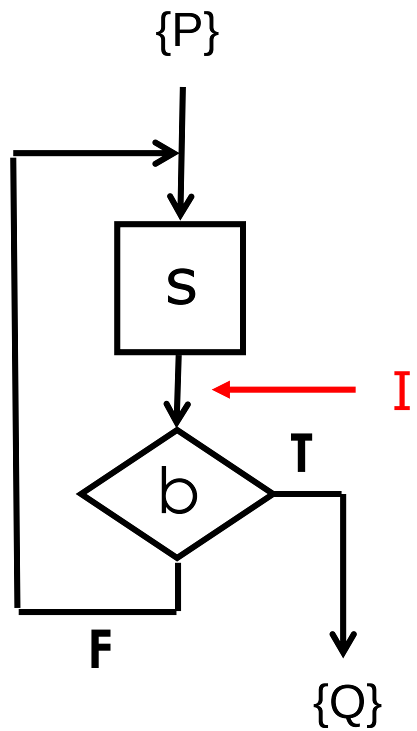

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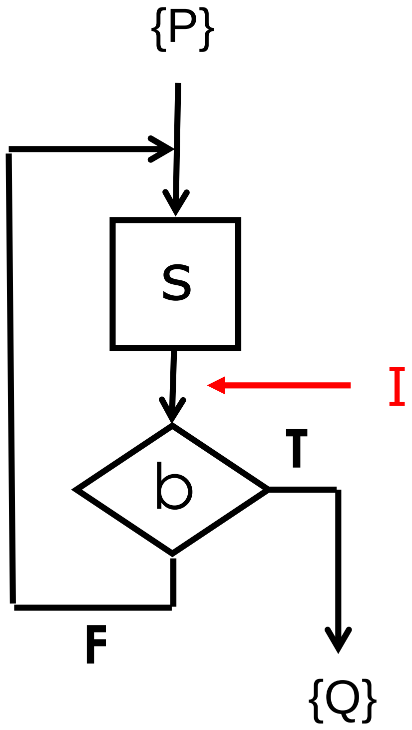

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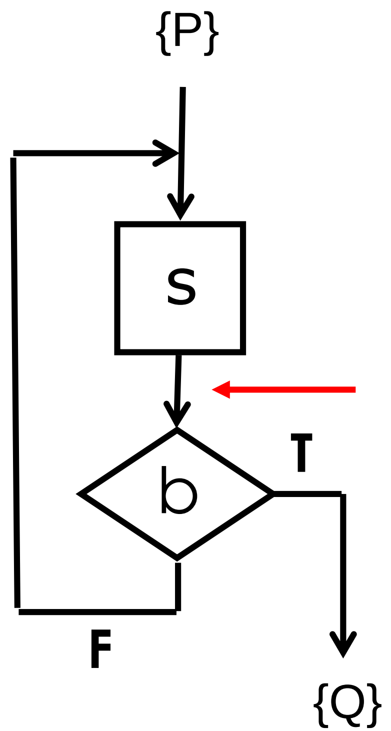

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case 1 (s executes just once): ?

case 2 (s executes just twice): ?

case n>2: ?

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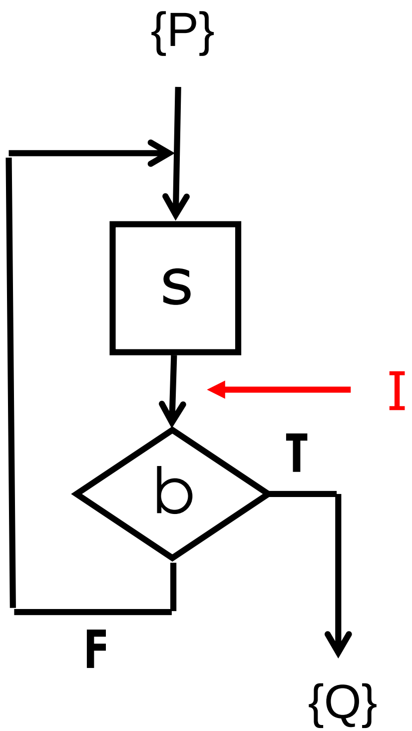

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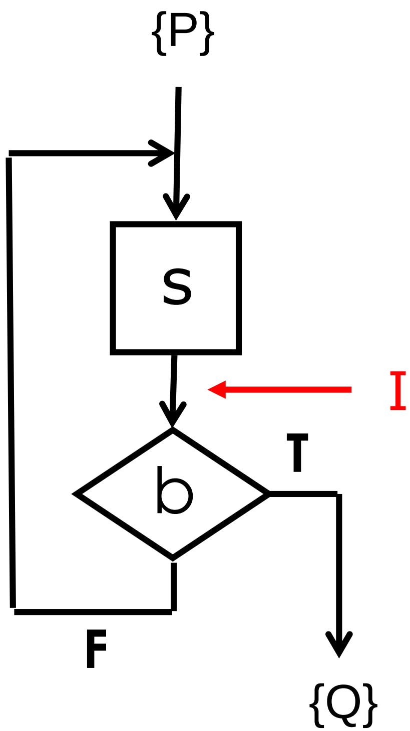

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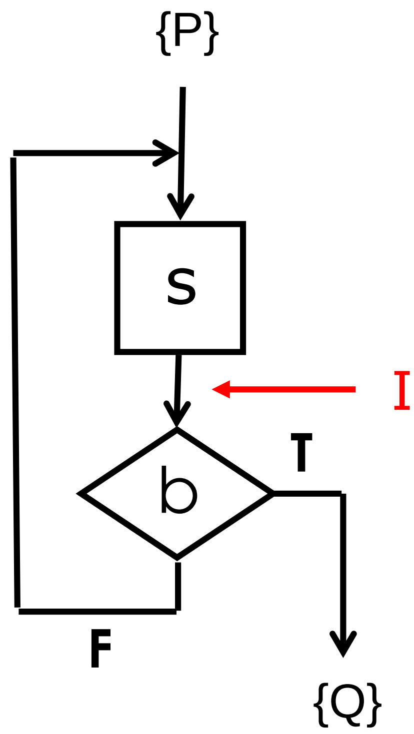

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case 2 (s executes just twice): √

case n>2: √

6. Consider the following HYPOTHESIZED rules of inference for the "while" construct:

a.
$$\frac{P \Rightarrow (\neg b \wedge Q)}{\{P\} \text{ while } b \text{ do } s \; \{Q\}} \, ?$$

b.
$$\frac{\{P \wedge b\} \; s \; \{I\}, \; \{I \wedge b\} \; s \; \{I\}, \; (I \wedge \neg b) \Rightarrow Q}{\{P\} \text{ while } b \text{ do } s \; \{Q\}} \, ?$$

…Clearly indicate whether or not the rule is **valid.** If valid, provide an assertion of the form {P} while b do S {Q} for which it *could* be used. If not valid, prove this by providing a counterexample.

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Is the rule **valid**?

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$$\frac{P \Rightarrow (\neg b \land Q)}{\{P\}\ \text{while } b \text{ do } s\ \{Q\}}\ ?$$

- case 0: Does the ROI antecedent => that Q will hold if s executes 0 times given that P holds initially?

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- case N>0: Does the ROI antecedent => that Q will hold if s executes 1 or more times given that P holds initially?

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The rule could be employed, for example, to prove:

{x=17} while x<0 do x := 0 {x>0}

6. Consider the following HYPOTHESIZED rules of inference for the "while" construct:

b. $$\frac{\{P \wedge b\}\ s\ \{I\},\ \{I \wedge b\}\ s\ \{I\},\ (I \wedge \neg b) \Rightarrow Q}{\{P\}\ \text{while } b \text{ do } s\ \{Q\}}\ ?$$

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Is the rule **valid**?

case 0: Do the ROI antecedents => that Q will hold if b is initially false and the loop terminates with s not executing, given that P holds initially?

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Is the rule **<u>valid</u>**?

case 0: Do the ROI antecedents => that Q will hold if b is initially false and the loop terminates with <u>s not executing</u>, given that P holds initially?

NO, since neither of the first two antecedents implies that I will hold when the loop terminates for this case. (Therefore, the third antecedent cannot be used to deduce that Q will hold.)

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<u>Question</u>: How can this be *proven* using a *counterexample?*

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Question: How can this be *proven* using a *counterexample?*

Answer: (1) Identify a specific, concrete program of the form while b do s together with pre- and post-conditions such that **{P} while b do s {Q} does NOT hold**. (2) Identify an invariant I such that **all three antecedents of the rule DO hold**. This would prove that the rule is not valid for the same reason that x=4 serves as a counterexample proving the rule: **[“x is even” ⇒ x>10]** is not valid.

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Proof:

```
{y≠17}                I: y=17
   while x>0 do
      y := 17
      x := x-1
   end_while
{y=17}
```

The three antecedents hold for the invariant y=17 but the consequent does not since the initial value of x may be ≤ 0 initially, in which case Q would not hold on termination.

More examples:

7. Consider the following HYPOTHESIZED rule of inference for the “repeat until" construct:

$$\frac{\{\text{true}\}\ S\ \{I\}\ \text{strongly},\ (I \wedge b) \Rightarrow Q}{\{P\}\ \text{repeat}\ S\ \text{until}\ b\ \{Q\}\ \text{strongly}}\ ?$$

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– case 1: Do the ROI antecedents => that Q will hold if S executes just once given that P holds initially?

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– case 1: Do the ROI antecedents => that Q will hold if S executes just once given that P holds initially? YES

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- case 1: Do the ROI antecedents => that Q will hold if S executes just once given that P holds initially? YES
- case 2: …that Q will hold if S executes exactly twice…?

More examples:

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– case 2: …that Q will hold if S executes exactly twice…? YES
– case N: …that Q hold if S executes exactly N times…?

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- case 2: …that Q will hold if S executes exactly twice…? YES
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- Do the antecedents => that (<u>repeat</u> S <u>until b</u>) will **terminate** given that P holds initially?

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- case 1: Do the ROI antecedents => that Q will hold if S executes just once given that P holds initially? YES
- case 2: …that Q will hold if S executes exactly twice…? YES
- case N: …that Q hold if S executes exactly N times…? YES

- Do the antecedents => that (<u>repeat</u> S <u>until b</u>) will **terminate** given that P holds initially? **NO, so the ROI is invalid.**

More examples:

8. Consider the following HYPOTHESIZED rule of inference for the “repeat until" construct:

$$\frac{P \Rightarrow (b \wedge Q)}{\{P\}\ \text{repeat S until b}\ \{Q\}}\ ?$$

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– case 1: Does the ROI antecedent => that Q will hold if S executes just once given that P holds initially?

NO, since P and (b∧Q) holding initially do NOT imply that either P or (b∧Q) will hold after S executes.

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$$\frac{P \Rightarrow (b \wedge Q)}{\{P\}\ \text{repeat } S \text{ until } b\ \{Q\}} \ ?$$

– case 1: Does the ROI antecedent => that Q will hold if S executes just once given that P holds initially?

NO, since P and (b∧Q) holding initially do NOT imply that either P or (b∧Q) will hold after S executes.

Exercise: PROVE the ROI is invalid using a suitable *counterexample.*

ROI Analysis and Review of Foundations

1. Interpreting conditional assertions
2. Assertions of weak and strong program correctness are conditionals!
3. Where do ROI’s come from?
4. How to analyze the validity of a hypothesized ROI