1. Pdf = $\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$

$$L = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_i-\mu)^2}{2\sigma^2}}$$

$$L = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^n e^{-\frac{1}{2}\sum_{i=1}^{n}\left(\frac{x_i-\mu}{\sigma}\right)^2}$$

Take ln

$$\ln(L) = n \ln\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) + \ln\left(e^{-\frac{1}{2\sigma^2}(x_i-\mu)^2}\right)$$

$$= n\left(\ln(1) - \ln(\sqrt{2\pi\sigma^2})\right) - \frac{1}{2\sigma^2}\sum(x_i-\mu)^2$$

$$= -n \ln(\sqrt{2\pi\sigma^2}) - \frac{1}{2\sigma^2}\sum(x_i-\mu)^2$$

$$\ln(L) = -\frac{n}{2}\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum(x_i-\mu)^2$$

for μ

$$\frac{\partial \ln(L)}{\partial \mu} = 0 \Rightarrow \frac{-1}{2\sigma^2}\sum 2(x_i-\mu)(-1)$$

$$= \frac{1}{\sigma^2}\sum(x_i-\mu) = 0$$

$$\sum_{i=1}^{n} (x_i - \mu) = 0$$

$$\Sigma(x_i) - n\mu = 0$$

$$\Sigma(x_i) = n\mu$$

$$\mu = \frac{\Sigma(x_i)}{n} = \bar{x} = \text{Sample mean.}$$

for σ^2

$$\frac{\partial \ln(L)}{\partial \sigma^2} = 0$$

$$\Rightarrow \cancel{-\frac{n}{2} \times \frac{1}{2\pi\sigma^2} \times 2\pi \times \sigma}$$

$$= \frac{-n}{2} \times \frac{1}{2\pi\sigma^2} \times 2\pi + \frac{1}{2\sigma^4} \Sigma(x_i - \mu)^2$$

$$\Rightarrow \frac{-n}{2\sigma^2} + \frac{1}{2\sigma^4} \Sigma(x_i - \mu)^2 = 0$$

$$\Rightarrow -n\sigma^2 + \Sigma(x_i - \mu)^2 = 0 \times 2\sigma^4$$

$$\Rightarrow \Sigma(x_i - \mu)^2 = n\sigma^2$$

$$\Rightarrow \sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}$$

9. pdf $= m_{C_{x_i}} \theta^{x_i} (1-\theta)^{m-x_i}$

$$L = \prod_{i=1}^{n} m_{C_{x_i}} \theta^{x_i} (1-\theta)^{m-x_i}$$

$$\ln(L) = \sum_{i=1}^{n} \left(\ln(m_{C_{x_i}}) + x_i \ln\theta + (m-x_i)\ln(1-\theta) \right)$$

$$\frac{\partial \ln(L)}{\partial \theta} = \sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{(m-x_i)}{1-\theta} \right) = 0$$

$$\sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{(m-x_i)}{1-\theta} \right) = 0$$

$$\frac{\Sigma x_i}{\theta} - \frac{n.m - \Sigma x_i}{1-\theta} = 0$$

$$\frac{\Sigma x_i}{\theta} = \frac{n.m - \Sigma x_i}{1-\theta}.$$

$$\Sigma x_i - \cancel{\theta \Sigma x_i} = n.m.\theta - \cancel{\Sigma x_i \theta}.$$

$$\theta = \frac{\Sigma x_i}{n.m}$$

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