Roll No.:.......................

National Institute of Technology, Delhi

Name of the Examination: B. Tech. (Mid-term Examination) Sept. 2017

Branch	**: ECE, EEE and CSE**	**Semester**	**: Ist**
Title of the Course	**: Advanced Calculus**	**Course Code**	**: MAL101**

Time: 2 Hours **Maximum Marks: 25**

Note: All questions are compulsory.

Q.1. **(*a*)** What is removable discontinuity? Explain with one example.

(*b*) Explain graphically the left hand and right hand derivative of $y = \dfrac{x}{|x|}$.

(*c*) Give an example of a function of two variable whose partial derivatives f_x and f_y at a point exist even when function is not continuous at that point.

(*d*) State the second derivative theorem for the test of local extrema.

(e) Define the concavity of $f(x) = \sin x$ in $[0, \pi]$.

[1+1+1+1+1]

Q. 2. Show that the function $f(x, y)$ is continuous but not differential at origin.

$$f(x,y) = \begin{cases} x\sin\left(\frac{1}{x}\right) + y\sin\left(\frac{1}{y}\right), & xy \neq 0 \\ x\sin\left(\frac{1}{x}\right), & y = 0,\ x \neq 0 \\ y\sin\left(\frac{1}{y}\right), & x = 0,\ y \neq 0 \\ 0, & x = 0, y = 0 \end{cases}$$

[4]

Q.3. Prove that $\lim_{x\to 2} f(x) = 4$ *if* $f(x) = \begin{cases} x^2, & x \neq 2 \\ 0\ , & x = 2 \end{cases}$ using $\varepsilon - \delta$ definition. Find the δ algebraically.

[3]

Q.4. State and Prove the Lagrange mean value theorem. [1+3]

Q.5. Find the dimension of the closed rectangular box with maximum volume that can be inscribed in a unit sphere. Find the maximum volume. [3]

Q.6. Check the extrema of the function $f(x, y) = 2x^4 - 3x^2y + y^2$. [3]

Q.7. Sketch the graph of the function $f(x, y) = (x+1)^2 / (1 + x^2)$. [3]