Roll No.:......................

National Institute of Technology, Delhi

Name of the Examination: B. Tech.
End Semester Examination (November-December, 2019)

Branch	: **EEE**	**Semester**	: **3rd**
Title of the Course	: **Ordinary Differential Equations and Transforms**	**Course Code**	: **MAL 201**

Time: 3 Hours **Maximum Marks: 50**

Note: Read instructions on each section carefully.

Section A [10 Marks]

Note: Questions 1-4 carries 1 mark. Questions 5-7 carries 2 marks.

1. The solution to the integral-differential equation $\frac{dy}{dt}-\frac{1}{2}\int_0^t (t-\tau)^2 y(\tau)d\tau = t, \quad y(0)=1$, is

(A) $\sin t$ (B) $\cos t$ (C) $\cos 2t - \sin 2t$ (D) $\frac{1}{2}(e^t - e^{-t})$ (E) $\frac{1}{2}(e^t + e^{-t})$

2. The Legendre's polynomial $P_5(x) = \lambda(63x^5 - 70x^3 + 15x)$, where λ is equal to

(A) $\frac{1}{2}$ (B) $\frac{1}{5}$ (C) $\frac{1}{8}$ (D) $\frac{1}{10}$ (E) $\frac{1}{3}$

3. The auxiliary equation of $4x^2\frac{d^2y}{dx^2} + y = 0$ is

(A) $4m^2 + 4m + 1 = 0$ (B) $4m^2 + 1 = 0$ (C) $4m^2 - 4m + 1 = 0$ (D) $4m^2 - 4m - 1 = 0$

4. By method of undetermined coefficient , the choice of particular integral for $\frac{d^2y}{dx^2} - 4y = 5e^{-2x}$ is

(A) Ce^{-2x} (B) Cxe^{-2x} (C) Cx^2e^{-2x} (D) Cx^3e^{-2x} (E) Ce^{2x}

5. Determine the constants $\alpha, \beta, y_0, and\, y_0'$ so that $Y(s) = \frac{2s-1}{s^2+s+2}$ is the Laplace transform of the solution to the initial value problem $y'' + \alpha y' + \beta y = 0,\ y(0) = y_0,\ y'(0) = y_0'$.

6. What are the Dirichlet conditions for the expansion of a function as a Fourier series?

7. State and Prove Modulation theorem for complex Fourier transform.

Section B [20 Marks]

Note: Attempt any 4 questions. Each question carries 5 marks.

8. Solve the differential equation by the method of variation of parameters $\frac{d^2y}{dx^2}-y=\frac{2}{1+e^x}$.

9. Solve the differential equation $(2x-1)^2\frac{d^2y}{dx^2}+(2x-1)\frac{dy}{dx}-2y=8x^2-2x+3$.

10. Find the power series solution for $\frac{d^2y}{dx^2}-x\frac{dy}{dx}-y=0$ about the point x=1. Find polynomial solution of at least 4th order.

11. Obtain the Fourier series expansion of $f(x)=x\sin x$ as a cosine series in $(0,\pi)$. Hence show that $\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-......=\frac{\pi-2}{4}$.

12. An inductor of 2 Henrys, a resistor of 16 Ohms and a capacitor of 0.02 Farads are connected in series with an e.m.f. of E volts. At t = 0 the charge on the capacitor and current in the circuit are zero. Find the charge and current at any time t > 0 if E = 100sin3t (volts) using Laplace transform.

Section C [20 Marks]

Note: Attempt any 2 questions. Each question carries 10 marks.

Q.13. Write the Bessel's Differential equation of order n. Describe all solutions of Bessel's differential equation using power series.

Q.14. (A) Find the Fourier sine transform of $e^{-|x|}$. Hence show that $\int_0^\infty \frac{x\sin mx}{1+x^2}dx=\frac{\pi e^{-m}}{2},\ m>0$.

(B) Use appropriate Fourier transform to solve the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, in a square plate of length $L=\pi$ with conditions $u(0,y)=0,\ u(\pi,y)=0$ and $u(x,0)=0,\ u(x,\pi)=x^2$.

Q.15. (A) Use appropriate Fourier transform to solve the diffusion equation $\frac{\partial u}{\partial t}=2\frac{\partial^2 u}{\partial x^2}$, if $u(0,t)=0,\ u(x,0)=e^{-x}\ for\ x>0$ and $u(x,t)$ is bounded where x>0, t>0.

(B) Find the Laplace transform of the function $f(t)=t\int_0^t \frac{e^{-t}\sin t}{t}dt$.