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National Institute of Technology, Delhi

Name of the Examination: B. Tech.

Branch : CSE, ECE, EEE **Semester** : 2nd

Title of the Course : Linear Algebra and Complex Analysis **Course Code** : MAL 151

Time: 1 Hour 30 Mins **Maximum Marks: 25**

Note : All questions are compulsory

Q.1. Let **A** be an **n×n** matrix. Its only eigenvalues are **1, 2, 3, 4, 5** possibly with multiplicities. What is the dimension of the Null space of the matrix $\mathbf{A+I_n}$, where $\mathbf{I_n}$ is the **n×n** identity matrix? Explain your answer. **(02 Marks)**

Q.2. Find all 3x3 matrices which are in reduced row echelon form and have rank 2. **(02 Marks)**

Q.3. Is it possible to have a linear transformation $T: R^2 \to R^2$ for which $T(2,2)=(8,-6)$ and $T(5,5)=(3,-2)$? Explain your answer. **(02 Marks)**

Q.4. Let $A=\begin{bmatrix}1 & 0 & 2\\ 1 & -2 & 0\\ 0 & 0 & -3\end{bmatrix}$ and I be the 3x3 identity matrix. If $6A^{-1}=aA^2+bA+cI$ for $a,b,c \in R$ then find the value of a,b,c. **(02 Marks)**

Q.5. Let $\vec{\mathbf{u}}=\begin{bmatrix}1\\ 1\\ 0\end{bmatrix}$ and $T: R^3 \to R^3$ be the linear transformation such that $T(\vec{\mathbf{x}})=proj_{\vec{\mathbf{u}}}\vec{\mathbf{x}}$. Find the matrix of transformation A s.t. $T(\vec{x})=A\vec{x}$. **(02 Marks)**

Q.6. Let W be the subspace of R^4 generated by the vectors $(1,-2,5,-3)$, $(2,3,1,-4)$ and $(3,8,-3,-5)$. Extend the Basis of W to a Basis of the whole space R^4. **(03 Marks)**

Q.7. Consider $\vec{\mathbf{u}}=\begin{bmatrix}0\\ 1\\ -2\\ 5\end{bmatrix}$ in R^4. Find a basis for the orthogonal complement $\vec{\mathbf{u}}^{\perp}$ of $\vec{\mathbf{u}}$. **(03 Marks)**

Q.8. Discuss the solutions of the system of equations in unknowns x_1, x_2, x_3:

$$\begin{aligned} ax_1 + bx_2 + 2x_3 &= 1 \\ ax_1 + (2b-1)x_2 + 3x_3 &= 1 \\ ax_1 + bx_2 + (b+3)x_3 &= 2b-1 \end{aligned}$$

(03 Marks)

Q.9. Let V be the vector space over C of all polynomials P(x) in a variable x of degree at most 3. Let $T: V \to V$ be the linear transformation given by differentiation with respect to x, i.e. $T[P(x)]=\dfrac{dP(x)}{dx}$. Let A be the matrix of T with respect to some basis of V.

(i) Check whether A is diagonalizable or not.

(ii) Find the null space of $A-I$ and eigen values of $(A+I)^2-I$. **(03 Marks)**

Q.10. Let $v_1=\begin{bmatrix}\frac{2}{3} & \frac{2}{3} & \frac{1}{3}\end{bmatrix}^T$ be a vector in R^3. Use the Gram - Schmidt process to produce an orthonormal basis for R^3 containing the vector v_1. **(03 Marks)**