Roll No.:.......................

National Institute of Technology, Delhi

Name of the Examination: B. Tech.

Branch	**: All Branches**	**Semester**	**: II**
Title of the Course	**: Linear Algebra & Complex Analysis**	**Course Code**	**: MAL151**

Time: 3 Hours **Maximum Marks: 50**

Section A

[All parts of section A are compulsory. Each part is of 01 mark]

Q.1. Attempt the following:

i. Let $A=\begin{bmatrix}4 & -1 & 6\\ 2 & 1 & 6\\ 2 & -1 & 8\end{bmatrix}$. An eigen value of A is 2. The basis for the corresponding eigen space is

(A) $\left\{\begin{bmatrix}1\\2\\0\end{bmatrix},\begin{bmatrix}-3\\0\\1\end{bmatrix}\right\}$ (B) $\left\{\begin{bmatrix}1\\0\\1\end{bmatrix},\begin{bmatrix}-1\\0\\1\end{bmatrix}\right\}$ (C) $\left\{\begin{bmatrix}1\\2\\1\end{bmatrix},\begin{bmatrix}1\\-2\\0\end{bmatrix}\right\}$ (D) $\left\{\begin{bmatrix}1\\0\\2\end{bmatrix},\begin{bmatrix}3\\0\\-2\end{bmatrix}\right\}$

ii. If A is 7×9 matrix with 2-dimensional null space, then the Rank of A is

(A) 9 (B) 2 (C) 7 (D) 5

iii. The value of the integral $\int_0^{1+i}\left(x^2+iy\right)dz$, along the path $y=x^2$ is

(A) $\frac{1}{3}(5i-3)$ (B) $\frac{1}{6}(5i-1)$ (C) $\frac{1}{6}(5i+1)$ (D) $\frac{1}{3}(3i-1)$

iv. The value of the integral $\oint_C \log z\,dz$, where C is the unit circle $|z|=1$ is

(A) $2\pi i$ (B) 0 (C) $2(\pi i-1)$ (D) πi

v. The value of p for which the function $f(z)=\frac{1}{2}\log\left(x^2+y^2\right)+i\tan^{-1}\left(\frac{px}{y}\right)$ is analytic is

(A) $p=1$ (B) $p=0$ (C) $p=-1$ (D) $p=2$

vi. If $f(z)=u+iv$ is analytic function and $v=y^2-x^2$ then u is

(A) $2xy+c$ (B) $2x^2y+c$ (C) $2xy^2+c$ (D) $2x^2y^2+c$

vii. Which one of the following statement is true for $f(z)=Im(z)$

(A) Differentiable everywhere (B) Nowhere differentiable
(C) Differentiable only at origin (D) Analytic

viii. The function $\frac{1-e^{2z}}{z^4}$ has singularity at z=0 which is a

(A) pole of order 4 (B) removable singularity (C) pole of order 3 (D) essential singularity

ix. The value of the integral $\oint_C \frac{e^{-z}}{z+1}dz$, where C is circle $|z|=2$ is

(A) 0 (B) $2\pi i$ (C) $2\pi ie$ (D) $\frac{2\pi i}{e}$

x. The value of the integral $\oint_C \frac{\sin\pi z^2+\cos\pi z^2}{(z-1)(z-2)}dz$, where C is circle $|z|=3$ is

(A) 0 (B) $2\pi i$ (C) $4\pi i$ (D) $6\pi i$

Section B

[Attempt any 04 questions of 05 marks each]

Q.2. Let $H_1 = span(v_1, v_2)$ and $H_2 = span(v_3, v_4)$, where $v_1 = \begin{bmatrix} 5 \\ 3 \\ 8 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}, v_4 = \begin{bmatrix} 0 \\ -12 \\ -28 \end{bmatrix}$. Then H_1 and H_2 are subspaces of R^3. Infact H_1 and H_2 are planes in R^3 through the origin and they intersect at a line through origin. Find a non-zero vector that generates that line.

Q.3. Let $B = \{b_1, b_2, \ldots\ldots, b_n\}$ be a basis for the vector space V. Then show that the coordinate mapping $x \mapsto [x]_B$ is one-to-one linear transformation from V onto R^n.

Q.4. State and prove the necessary and sufficient conditions for the derivative of a function $f(z)$ of a complex variable.

Q.5. Determine the analytic function $f(z) = u + iv$, if $u - v = \dfrac{\cos x + \sin x - e^{-y}}{2(\cos x - \cosh y)}$ and $f\left(\dfrac{\pi}{2}\right) = 0$.

Q.6. Find Laurent Series expansion of $f(z) = \dfrac{3z^2 - 5}{z(z+1)(z-2)}$ in the region $1 < |z+1| < 3$.

Section C

[Attempt any 02 questions of 10 marks each]

Q.7. **(A)** If $v_1, v_2, \ldots, v_r$ are eigen vectors that correspond to distinct eigen values $\lambda_1, \lambda_2, \ldots, \lambda_r$ of an $n \times n$ matrix A, then show that the set $\{v_1, v_2, \ldots, v_r\}$ is linearly independent. **(04 Marks)**

(B) Let $A = \begin{bmatrix} -6 & 3 & -27 & -33 & -13 \\ 6 & -5 & 25 & 28 & 14 \\ 8 & -6 & 34 & 38 & 18 \\ 12 & -10 & 50 & 41 & 23 \\ 14 & -21 & 49 & 29 & 33 \end{bmatrix}$ be a matrix. Construct a matrix N whose columns form a basis for the Nul A (null space of A), and construct a matrix R whose rows form a basis for Row A (row space of A). **(06 Marks)**

Q.8. **(A)** If a function $f(z)$ and its conjugate are both analytic in a given domain D. Then prove that the function must be constant on domain D. **(04 Marks)**

(B) Given a function $f(z) = \begin{cases} \dfrac{2xy(x+iy)}{x^2+y^2}, & if\ z \neq 0 \\ 0, & if\ z = 0 \end{cases}$, show that the CR-Equations are satisfied at origin however the derivative of the function $f(z)$ at origin does not exist. **(06 Marks)**

Q.9. **(A)** Determine the poles of the function and residue at each pole for $f(z) = \dfrac{z^2 - 2z}{(z^2+1)(z^2-4)(z-1)^2}$. **(03 Marks)**

(B) Find the analytic function $f(z) = u + iv$, given that $u = a(1 + \cos\theta)$, a is any fixed constant. **(03 Marks)**

(C) Evaluate using Cauchy Residue theorem $\displaystyle\oint_{|z+2-i|=6} \frac{dz}{(z-2)^3(z^2+4)}$. **(04 Marks)**