

Roll No.:.....

National Institute of Technology, Delhi

Name of the Examination: B. Tech.

Branch : EEE
Title of the Course : Ordinary Differential
Equations and Transforms

Semester : 3rd
Course Code : MAL 201

Time: 3 Hours

Maximum Marks: 50

Section A

[All parts of section A are compulsory. Each part is of 01 mark]

Q.1. Attempt each of the following:

- i. The solution of the differential equation $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$ is
(A) $\sin^{-1} x - \sin^{-1} y = c$ (B) $\sin^{-1} x + \sin^{-1} y = c$ (C) $\sin^{-1} x = c \sin^{-1} y$ (D) $\sin^{-1} x \cdot \sin^{-1} y = c$
- ii. If $J_n(x)$ defines the Bessel's function of first kind of order n , then which of the following is true for $n > 2$
(A) $\frac{d}{dx}[x^n J_n(x)] = x^{n-1} J_n(x)$ (B) $\frac{d}{dx}[x^n J_n(x)] = x J_{n-1}(x)$
(C) $\frac{d}{dx}[x^n J_n(x)] = x^n J_{n-1}(x)$ (D) $\frac{d}{dx}[x^n J_n(x)] = x J_n(x)$
- iii. The finite Fourier sine transform of $f(x) = x$, $0 \leq x \leq 1$ is
(A) $\frac{1}{n}$ (B) $\frac{(-1)^n}{n}$ (C) $\frac{1 - \cos n\pi}{n}$ (D) 0
- iv. If the Fourier cosine transform of $f(x)$ is $\sqrt{\frac{2}{\pi}} \frac{\omega}{1 + \omega^2}$ then the Fourier transform of $f(ax)$ is
(A) $\sqrt{\frac{2}{\pi}} \frac{a\omega}{a^2 + \omega^2}$ (B) $\sqrt{\frac{2}{\pi}} \frac{\omega}{a^2 + \omega^2}$ (C) $\sqrt{\frac{2}{\pi}} \frac{a}{a^2 + \omega^2}$ (D) $\sqrt{\frac{2}{\pi}} \frac{a + \omega}{a^2 + \omega^2}$
- v. If $\delta(t)$ denotes a unit impulse function, then the Laplace transform of $\frac{d^2 \delta(t)}{dt^2}$ will be
(A) 1 (B) s^2 (C) s (D) s^{-2}
- vi. The value of $\int_0^\infty t e^{-3t} J_0(4t) dt$ is
(A) $\frac{3}{125}$ (B) $\frac{2}{125}$ (C) $\frac{3}{25}$ (D) $\frac{2}{25}$
- vii. The inverse Laplace transform of $\frac{e^{-2s}}{s-3}$ is
(A) $e^{3(t-2)} u(t-3)$ (B) $e^{2(t-3)} u(t-2)$ (C) $e^{3(t-2)} u(t-2)$ (D) $e^{2(t-3)} u(t-3)$
- viii. At $x = 0$, the Fourier series of $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \pi - x, & 0 < x < \pi \end{cases}$ converges to
(A) π (B) 0 (C) $\frac{\pi}{2}$ (D) none of these
- ix. If $f(x) = \begin{cases} 1, & 0 < x < \frac{1}{2} \\ 0, & \frac{1}{2} < x < 1 \end{cases}$ then b_1 in the half-range sine series is equal to

(A) $\frac{1}{\pi}$

(B) $\frac{2}{\pi}$

(C) $\frac{3}{\pi}$

(D) $\frac{4}{\pi}$

- x. The Fourier transform of exponential signal $e^{j\omega_0 t}$ is
(A) a constant (B) a rectangular gate (C) an impulse (D) a series of impulses

Section B

[Attempt any 04 questions of 05 marks each]

Q.2. Solve the differential equation $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$.

Q.3. Solve the differential equation $(x+1)^4 D^3 y + 2(x+1)^3 D^2 y - (x+1)^2 Dy + (x+1)y = \frac{1}{1+x}$.

Q.4. Determine the half range Fourier cosine series for $f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$.

Q.5. Find the Fourier transform of $f(x) = \frac{1}{\sqrt{x}}$.

Q.6. Define Laplace transform and inverse Laplace transform of the function. Find the inverse Laplace transform of $F(s) = \ln\left(\frac{s+1}{s+5}\right)$.

Section C

[Attempt any 02 questions of 10 marks each (06 marks + 04 marks)]

Q.7. (A) Find the Fourier series for the function $f(x)$ if $f(x)$ is defined in $-\pi < x < \pi$ as

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ \sin x, & 0 < x < \pi \end{cases}$$

Hence deduce that $\frac{1}{4}(\pi - 2) = \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \dots$

(B) Find the Fourier sine transform of $g(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0, & x > 2 \end{cases}$.

Q.8. (A) Find the general solution of $2x(x+1)D^2 y - (1-x)Dy + y = 0$ in series about $x = 0$.

(B) Show that $J_n(x) = \frac{x}{2n} [J_{n-1}(x) + J_{n+1}(x)]$ and express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$.

Q.9. (A) Solve the following system of ordinary differential equation by Laplace transformation

$$x - y'' + y = e^{-t} - 1, \quad x' + y' - y = -3e^{-t} + t \quad \text{where } x(0) = 0, y(0) = 1, y'(0) = -2.$$

(B) Find the Laplace transform of periodic function $f(t) = |\sin \omega t|$ with period $\frac{\pi}{\omega}$.