Roll No.:.......................

National Institute of Technology, Delhi

Name of the Examination: B. Tech.

Branch : All Branches

Title of the Course : Linear Algebra & Complex Analysis

Semester : II

Course Code : MAL151

Time: 3 Hours

Maximum Marks: 50

Section A

[All parts of section A are compulsory. Each part is of 01 mark]

Q.1. Attempt the following:

i. If A is 3×7 matrix with Rank of A as 2 then the null space of A is

(A) 1 (B) 2 (C) 7 (D) 5

ii. Find the dimensions of the null space and the column space of the given matrix

$$A=\begin{bmatrix} 1 & -3 & -5 & 3 & 0 \\ -2 & 1 & 3 & -4 & 1 \end{bmatrix}$$

(A) dim Nul A = 4, dim Col A = 1 (B) dim Nul A = 3, dim Col A = 2

(C) dim Nul A = 2, dim Col A = 3 (D) dim Nul A = 3, dim Col A = 3

iii. The value of the integral $\int_0^{1+i}\left(x^2+iy\right)dz$, along the path $y=x^2$ is

(A) $\frac{1}{3}(5i-3)$ (B) $\frac{1}{6}(5i-1)$ (C) $\frac{1}{6}(5i+1)$ (D) $\frac{1}{3}(3i-1)$

iv. The value of the integral $\oint_C z^3 dz$, where C is the unit circle $|z|=1$ is

(A) $2\pi i$ (B) 0 (C) $2(\pi i-1)$ (D) πi

v. If $f(z)=u+iv$ is analytic function and $u=x^2-y^2$ then v is

(A) $2xy+c$ (B) $2x^2y+c$ (C) $2xy^2+c$ (D) $2x^2y^2+c$

vi. Let $A=\begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$. An eigen value of A is 2. The basis for the corresponding eigen space is

(A) $\left\{\begin{bmatrix}1\\2\\0\end{bmatrix},\begin{bmatrix}-3\\0\\1\end{bmatrix}\right\}$ (B) $\left\{\begin{bmatrix}1\\0\\1\end{bmatrix},\begin{bmatrix}-1\\0\\1\end{bmatrix}\right\}$ (C) $\left\{\begin{bmatrix}1\\2\\1\end{bmatrix},\begin{bmatrix}1\\-2\\0\end{bmatrix}\right\}$ (D) $\left\{\begin{bmatrix}1\\0\\2\end{bmatrix},\begin{bmatrix}3\\0\\-2\end{bmatrix}\right\}$

vii. Which one of the following statement is true for $f(z)=z^2$

(A) Differentiable everywhere (B) Nowhere differentiable

(C) Differentiable only at origin (D) Analytic

viii. The function $\frac{\sin z}{z^4}$ has singularity at $z=0$ which is a

(A) pole of order 4 (B) removable singularity (C) pole of order 3 (D) essential singularity

ix. The value of the integral $\oint_C \frac{1}{z+1}dz$, where C is circle $|z|=2$ is

(A) 0 (B) $2\pi i$ (C) $2\pi ie$ (D) $\frac{2\pi i}{e}$

x. The value of the integral $\oint_C \frac{1}{z(z+2)}dz$, where C is circle $|z|=1$ is

(A) 0 (B) $2\pi i$ (C) πi (D) $3\pi i$

Section B

[Attempt any 04 questions of 05 marks each]

Q.2. Define Eigen values and Eigen vectors of a matrix. Obtain the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

Q.3. Construct a matrix A having Eigen values $-1, 2, 2$ and corresponding Eigen vectors (1,2,3), (1,1,0), (0,1,1) respectively.

Q.4. State and prove the necessary and sufficient conditions for the derivative of a function $f(z)$ of a complex variable.

Q.5. Evaluate $\oint_{|z|=2} \frac{e^z}{z} dz$ and hence show that $\int_0^{2\pi} e^{\cos\theta} \cos(\sin\theta)\, d\theta = 2\pi$.

Q.6. Find Laurent Series expansion of $f(z) = \frac{2z-3}{z(z+1)(z-2)}$ in the region $1 < |z+1| < 3$.

Section C

[Attempt any 02 questions of 10 marks each]

Q.7. (A) Use the Gram - Schmidt process to produce an orthogonal basis for W from the basis of W given by {(1,-1,-1,1,1), (2,1,4,-4,2), (5,-4,-3,7,1)}. **(04 Marks)**

(B) Let $A = \begin{bmatrix} -6 & 3 & -27 & -33 \\ 6 & -5 & 25 & 28 \\ 8 & -6 & 34 & 38 \\ 14 & -21 & 49 & 29 \end{bmatrix}$ be a matrix. Construct a matrix N whose columns form a basis for the Nul A (null space of A), and construct a matrix R whose rows form a basis for Row A (row space of A). **(06 Marks)**

Q.8. (A) Show that the Eigen values of the Hermitian matrix are always real. **(04 Marks)**

(B) Given a function $f(z) = \begin{cases} \frac{2xy(x+iy)}{x^2+y^2}, & \text{if } z \neq 0 \\ 0, & \text{if } z = 0 \end{cases}$, show that the CR-Equations are satisfied at origin however the derivative of the function $f(z)$ at origin does not exist. **(06 Marks)**

Q.9. (A) Determine the poles of the function and residue at each pole for $f(z) = \frac{z^2}{(z^2+1)(z^2-4)(z-1)^2}$. **(03 Marks)**

(B) Find the analytic function $f(z) = u + iv$, given that $v = 2xy$. **(03 Marks)**

(C) Evaluate using Cauchy Residue theorem $\oint_{|z+2-i|=6} \frac{dz}{(z-1)^3 (z^2+1)}$. **(04 Marks)**