Roll No. ____________

National Institute of Technology Delhi

Name of the Examination: B.Tech.

Mid-Semester Examination (March, 2019)

Branch : All Branches

Course Title : Linear Algebra & Complex Analysis

Semester : II

Course Code : MAL 151

Time: 2 hrs

Total Marks: 25

Note: All questions are compulsory

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1. Each question is of one mark. [05 Marks]
   (a) Let A be an idempotent matrix ($A^2 = A$). Show that if λ is an eigenvalue of A, then $\lambda = 0$ or $\lambda = 1$.
   (b) Let $B = P^{-1}AP$. If v is an eigenvector of A corresponding to the eigenvalue λ, find the eigenvector of B corresponding to λ.
   (c) Let V be a vector space and $u, v \in V$. Then,
   $$\|u+v\| = \|u-v\| \iff u.v = 0.$$
   (d) Find all the eigenvalues of $A = \begin{bmatrix} 1001 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1001 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1001 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1001 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1001 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1001 \end{bmatrix}$.
   (e) Let $A = \begin{bmatrix} -8 & -2 & -9 \\ 6 & 4 & 8 \\ 4 & 0 & 4 \end{bmatrix}$ and $w = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$. Determine if w is in Col A. Is w in Nul A ?

2. Diagonalize the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ and hence, compute A^5. [05 Marks]

3. Find the solution of the system of linear equations: [05 Marks]
$$\begin{aligned} x + 2y + 3z + w &= 4 \\ 2x + 3y + z - w &= 4 \\ 3x + 4y - z + 2w &= 5 \\ 2x + 3y + z + 4w &= 5 \end{aligned}$$

4. Look at the collection $\{x_1, x_2, x_3, x_4\}$ of four linearly independent vectors in $\mathbb{R}^5$:
$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}.$$
   Let $W = \text{Span}\{x_1, x_2, x_3, x_4\}$. Construct an orthogonal basis for the subspace W of $\mathbb{R}^5$. [05 Marks]

5. Suppose u, v and w are vectors in an arbitrary vector space V over a field $\mathbb{F}$. Show that $\{u, v, w\}$ is linearly independent if and only if $\{u+v, v+w, w+u\}$ is linearly independent. [05 Marks]