Roll No.:......................

National Institute of Technology, Delhi

Name of the Examination: B. Tech.

Branch	: EEE	Semester	: 3rd
Title of the Course	: Ordinary Differential Equations and Transforms	Course Code	: MAL 201

Time: 3 Hours

Maximum Marks: 50

Section A

[All parts of section A are compulsory. Each part is of 01 mark]

Q.1. Attempt each of the following:

i. The solution of the differential equation $\frac{dy}{dx}+\frac{y}{x}=0$ is

(A) $x^2+y^2=c$ (B) $\frac{x^2}{2}+\frac{y^2}{2}=c$ (C) $xy=c$ (D) $\frac{x}{y}=c$

ii. The particular solution of a Second order differential equation will contain

(A) Two constants (B) One constant (C) No constant (D) Three constant

iii. The differential equation $2\frac{dy}{dx}+x^2y=2x+3,\ \ y(0)=5$ is

(A) Linear (B) Nonlinear (C) Linear with fixed constants (D) Undeterminable

iv. A differential equation is considered to be ordinary if it has

(A) One independent variable (B) One dependent variable (C) Two independent variable (D) None of these

v. Integrating factor of $dy=e^{x-y}\left(e^x-e^y\right)dx$ is

(A) 1 (B) s^2 (C) s (D) s^{-2}

vi. The value of $\int_0^{\infty} te^{-3t}J_0(4t)dt$ is

(A) $\frac{3}{125}$ (B) $\frac{2}{125}$ (C) $\frac{3}{25}$ (D) $\frac{2}{25}$

vii. The inverse Laplace transform of $\frac{e^{-2s}}{s-3}$ is

(A) $e^{3(t-2)}u(t-3)$ (B) $e^{2(t-3)}u(t-2)$ (C) $e^{3(t-2)}u(t-2)$ (D) $e^{2(t-3)}u(t-3)$

viii. At $x=0$, the Fourier series of $f(x)=\begin{cases}0, & -\pi<x<0\\ \pi-x, & 0<x<\pi\end{cases}$ converges to

(A) π (B) 0 (C) $\frac{\pi}{2}$ (D) none of these

ix. If $f(x)=\begin{cases}1, & 0<x<\frac{1}{2}\\ 0, & \frac{1}{2}<x<1\end{cases}$ then b_1 in the half-range sine series is equal to

(A) $\frac{1}{\pi}$ (B) $\frac{2}{\pi}$ (C) $\frac{3}{\pi}$ (D) $\frac{4}{\pi}$

x. The Laplace transform of f(t), t>0 is

(A) $\int_0^{\infty} e^{-st}f(t)dt$ (B) $\int_0^{t} e^{-st}f(t)dt$ (C) $\int_{-\infty}^{\infty} e^{-st}f(t)dt$ (D) $\int_{-\infty}^{0} e^{-st}f(t)dt$

Section B

[Attempt any 04 questions of 05 marks each]

Q.2. Solve the differential equation $(D^2-4D+4)y=5e^{2x}\sin 2x$.

Q.3. Solve the differential equation $x^4D^3y+2x^3D^2y-x^2Dy+xy=\frac{1}{x}$.

Q.4. Determine the half range Fourier cosine series for $f(x)=x, \quad 0<x<\pi$.

Q.5. Find the Fourier transform of $f(x)=\frac{1}{\sqrt{x}}$.

Q.6. Define Laplace transform and inverse Laplace transform of the function. Find the inverse Laplace transform of $F(s)=\frac{s-2}{s^2+6}+\frac{s}{s^2-1}$.

Section C

[Attempt any 02 questions of 10 marks each (06 marks + 04 marks)]

Q.7. (A) Find the Fourier series for the function $f(x)$ if $f(x)$ is defined in $-\pi<x<\pi$ as

$$f(x)=\begin{cases}0, & -\pi<x<0\\ \sin x, & 0<x<\pi\end{cases}$$

(B) Find the Fourier sine transform of $g(x)=\begin{cases}x+1, & 0<x<1\\ 3-x, & 1<x<3\\ 0, & x>3\end{cases}$.

Q.8. (A) Find a series solution around x=0 for the following differential equation $\frac{d^2y}{dx^2}-xy=0$.

(B) Use a convolution integral to find the inverse transform of the following transform

$$H(s)=\frac{1}{(s^2+a^2)^2}.$$

Q.9. (A) Solve the following system of ordinary differential equation by Laplace transformation

$x'=3x-3y+2, \quad y'=-6x-t$ where $x(0)=1, y(0)=-1$.

(B) Find the Laplace transform of the function $f(t)=\begin{cases}t, & if\ t<6\\ -8+(t-6)^2 & if\ t\ge 6\end{cases}$.