Roll No...............

National Institute of Technology Delhi

Name of the Examination: B.Tech.

Make-up Exam, July (2019)

Branch: All Branches | **Semester-I**

Course Title: Advanced Calculus | **Course Code: MAL 101**

Max Time: 3 hrs | **Total Marks: 50**

Note:

1. In Section A, all questions are **compulsory** with each part carrying 1 mark each.
2. In Section B, attempt any **FOUR** questions with each question carrying 5 marks.
3. In Section C, attempt any **TWO** questions with each question carrying 10 marks.

Section A

1. (a) Find $\frac{\partial f}{\partial y}$ if $f(x, y) = y \sin xy$.

   (b) State Euler's mixed derivative Theorem.

   (c) State first part of fundamental theorem of Integral calculus.

   (d) State Gauss Divergence theorem.

   (e) Evaluate limit for the function $f(x) = \begin{cases} 0, & x = 0 \\ \frac{1}{x}, & x \neq 0 \end{cases}$ at the point $x = 0$.

   (f) Give an example of a function which is not differentiable at exactly 4 points.

   (g) What do you mean by *The method of Lagrange's multiplier?*

   (h) Does every discontinuous function is Riemann integrable?

   (i) If $u = x^3 + y^3 + z^3 - 3xyz$, then evaluate $\nabla \times \nabla u$.

   (j) State Fubini's theorem (Strong form).

[$1 \times 10 =$ **10 Marks**]

Section B

2. Find the linearization of the function $f(x, y, z) = \tan^{-1}(xyz)$ at the points $\hat{e}_1$, $\hat{e}_1 + \hat{e}_2$ and $\hat{e}_1 + \hat{e}_2 + \hat{e}_3$, where $\hat{e}_i$ $(i = 1, 2, 3)$ are unit vectors along three co-ordinate axis.

3. Find the length of the curve $y = \left(\frac{x}{2}\right)^{\frac{2}{3}}$ from $x = 0$ to $x = 2$.

4. Prove that the volume of a unit sphere is $\frac{4}{3}\pi$ units by using triple integrals.

5. Find the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of normal to the surface $x \log z - y^2 = -4$ at $(-1, 2, 1)$.

6. Use Green's theorem to find out the counterclockwise circulation and outward flux for the field $\vec{F} = (x + y)\hat{i} - (x^2 + y^2)\hat{j}$ over the triangular curve C bounded by $y = 0$, $x = 1$ and $y = x$.

[$5 \times 4 =$ **20 Marks**]

Section C

7. (a) Find the equations for tangent plane and normal line at the point $(2, -3, 18)$ on the surface $x^2 + y^2 - 2xy - x + 3y - z = -4$. [5 Marks]

   (b) Find Taylor series and the Taylor polynomial generated by $f(x) = e^x$ at $x = 0$. [5 Marks]

8. (a) Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the $x-$axis and the line $y = x - 2$. [5 Marks]

(b) Change the order of integration and hence evaluate $\displaystyle\int_0^a \int_{\sqrt{ax}}^a \frac{y^2\,dxdy}{\sqrt{y^4 - a^2x^2}}$. [5 Marks]

9. (a) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that $div(r^n\vec{r}) = (n+3)r^n$. Hence show that $\frac{\vec{r}}{r^3}$ is solenoidal. [5 Marks]

(b) Verify Stoke's theorem for the field $\vec{F} = y\hat{i} - x\hat{j}$ over the hemisphere $S : x^2 + y^2 + z^2 = 9,\ z \geq 0$ and its bounding circle $C : \vec{r} = x^2 + y^2 = 9,\ z = 0$. [5 Marks]