National Institute of Technology, Delhi

Name of Examination: B.Tech.

Branch	**: CSE/ECE/EEE**	**Semester: 2nd**
Title of the Course	**: Linear Algebra and Complex Analysis**	**Course Code: MAL 151**
Time: 3Hours		**Maximum Marks: 50**

Note: This question paper is divided into three sections **A, B, C**. Section **A** contains One questions having Ten parts of 01 mark and all parts are Compulsory. Section **B** contains Five questions of 05 marks and any four questions are to be attempted. Section **C** contains Three questions of 10 marks and any two questions are to be attempted.

Section A

Q1 Answer the following:

(i) Prove that 0 must be an eigen value of a singular matrix.

(ii) Is the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ diagonalizable? (Justify your answer)

(iii) The vectors $u_1 = (1, 2, 1)$, $u_2 = (2, 1, -4)$ and $u_3 = (3, -2, 1)$ are orthogonal. Express the vector (8,3,-5) as a linear combination of u_1, u_2, u_3.

(iv) Find a non-zero vector that is orthogonal to $u_1 = (1, 2, 1)$ and $u_2 = (2, 5, 4)$ in $\mathbb{R}^3$.

(v) Let $T\colon \mathbb{R}^n \to \mathbb{R}^m$ be an onto linear mapping and the matrix A be such that $T(x) = Ax, \quad x \in \mathbb{R}^n$. The rank of A is

(A) n (B) m (C) $m - n$ (D) $n - m$

(vi) The function $f(z) = \frac{xy}{x^2+y^2}$ when $z \neq 0$ and $f(0) = 0$ is

(A) continuous at $z = 0$.
(B) discontinuous at $z = 0$.
(C) constant.
(D) not predictable.

(vii) The sum of the residues of $f(z) = \frac{1}{z^2+1}$ is

(A) 0. (B) 1. (C) -1. (D) None of these.

(viii) Which of the following has the removable singularity

(A) $\sin(\frac{1}{z})$. (B) $\frac{\sin^2 z}{Z^3}$. (C) $\frac{\cos z - 1}{z^2}$. (D) e^z.

(ix) Laurent series expansion of $f(z) = \frac{1}{(z-1)(z-2)(z-3)}$ is not possible for

(A) $1 < |z| < 2$. (B) $1 < |z| < 3$. (C) $|z| > 3$. (D) All of them.

(x) The value of $f(2)$ where $f(a) = \int_C \frac{2z^2-z-2}{z-a}\,dz$ and C is circle $|z| = 2.5$, positively oriented, is

(A) $8\pi i$. (B) $4\pi i$. (C) 4. (D) 0.

Section B

Q11. (a.) Show that an analytic function with constant modulus in a domain is constant.
(b.) Suppose that $u(x,y), v(x,y)$ satisfy the Laplace equation. Is it always true that the function $f(z) = u + i\,v$ is analytic? Also justify your answer. (3+2 marks)

Q12. Derive the Necessary condition for a function $f(z) = u(x,y) + i\,v(x,y)$ to be analytic.

Q13. Let $f(z)$ be analytic everywhere inside and on a simple closed contour C, taken in positive sense. If z_0 is any point interior to C, then prove that

$$f(z_0) = \frac{1}{2\pi i}\int_C \frac{f(z)}{(z-z_0)}\,dz$$

Q14. Let

$$u(x,y) = x^2 - y^2 + \frac{x^2}{x^2+y^2}.$$

Construct the analytic function $f(z) = u + i\,v$ in term of z if possible.

Q15. (a.) Find the dimension and a basis of the subspace $W = \{(a,b,c) : a+b+c = 0\}$ of $\mathbb{R}^3$.
(b.) If a set $\{u_1, u_2, \cdots, u_n\}$ is orthogonal, then prove that the set $\{u_1, u_2, \cdots, u_n\}$ is linear independent. (2+3 marks)

Section C

Q16. Find all Taylor's and Laurent's Series of

$$f(z) = \frac{2z-3}{z^3-6z^2+9z-4}$$

with center $z = 2$ and give the region of convergence of each series.

Q17. (a.) Use calculus of Residue, evaluate

$$\int_0^{2\pi} \frac{1}{(5-3\sin\theta)^2}\,d\theta.$$

(b.) Evaluate

$$\int_C \frac{1}{z^2+4}\,dz,$$

where C is the circle $|z - i| = 2$, oriented clockwise. (7+3 marks)

Q18. Let $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$. Find all the eigen values of the matrix A. Is A diagonalizable? If yes, find the matrix P such that $P^{-1}AP$ is diagonal.