Roll No...............

National Institute of Technology Delhi

Name of the Examination: B.Tech.

End Semester Exam (2018-19)

Branch: All Branches

Semester-I

Course Title: Advanced Calculus

Course Code: MAL 101

Max Time: 3 hrs

Total Marks: 50

Note:

1. In Section A, all questions are **compulsory** with each part carrying 1 mark each.
2. In Section B, attempt any **FOUR** questions with each question carrying 5 marks.
3. In Section C, attempt any **TWO** questions with each question carrying 10 marks.

Section A

1. (a) Is it possible for a function to have partial derivatives at point without being continuous there? Justify your answer.

   (b) Find the slope of the curve $z = x^2 + y^2$, $x = 1$ using partial derivatives.

   (c) State second part of fundamental theorem of Integral calculus.

   (d) State Stoke's theorem.

   (e) Evaluate right hand limit for the function $f(x) = \begin{cases} 0, & x \leq 0 \\ \sin\frac{1}{x}, & x > 0 \end{cases}$ at the point $x = 0$.

   (f) Find derivative for the function $f(x) = |x|$ at the point $x = 0$.

   (g) What do you mean by the term *Lagrange's Multiplier?*

   (h) Give an example of a function which is not Riemann integrable.

   (i) Evaluate $\int_{x=-\pi/2}^{\pi/2} \int_{y=-33}^{44} y^{4/11}(x^7 + \sin^3 x)\, dydx$.

   (j) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then evaluate ∇r^5.

[1 × 10 =**10 Marks**]

Section B

2. Define the term *linearization* of a function $f(x, y)$ at a point and calculate the same for the function $e^x \cos y$ at the point $(0, \frac{\pi}{2})$.

3. Find the area of the surface generated by revolving the curve $y = x^3$, $0 \leq x \leq \frac{1}{2}$ about the $x-$axis.

4. By changing the order of integration of $\displaystyle\int_0^\infty \int_0^\infty e^{-xy} \overset{\sin px}{\wedge}\, dxdy$, show that $\displaystyle\int_0^\infty \frac{\sin px}{x} dx = \frac{\pi}{2}$.

5. Find the values of a and b such that the surfaces $ax^2 - byz = (a+2)x$ and $4x^2y + z^3 = 4$ cut each other orthogonally at $(1, -1, 2)$.

6. Use divergence theorem to find out the outward flux of $\vec{F} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ across the region cut from solid cylinder $x^2 + y^2 \leq 4$ between the planes $z = 0$ and $z = 1$.

[5 × 4 =**20 Marks**]

Section C

7. (a) Find the direction in which the function $f(x, y, z) = \log(x^2 + y^2 - 1) + y + 6z$ increase and decrease most rapidly at the point $(1, 1, 0)$. Further, find its derivative in these directions.

[5 Marks]

(b) Find Maclaurin series for the function $f(x) = \cos x$ and hence derive it for the function $g(x) = \cos(\frac{x^{\frac{3}{2}}}{\sqrt{2}})$. [5 Marks]

8. (a) Find the area of the *triangular* region in the first quadrant bounded on the left by the $y-$axis and on the right by the curves $y = \sin x$ and $y = \cos x$. [5 Marks]

(b) Evaluate the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, by using triple integrals.

[5 Marks]

9. (a) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then show that $r^{\alpha}\vec{r}$ is an irrotational vector for any value of α, but is solenoidal if $\alpha = -3$. [5 Marks]

(b) Verify Green's theorem for the field $\vec{F} = -x^2y\hat{i} + xy^2\hat{j}$ over the disk $R : x^2 + y^2 \leq a^2$ and its bounding circle $C : \vec{r} = (a\cos t)\hat{i} + (a\sin t)\hat{j}$, $0 \leq t \leq 2\pi$. [5 Marks]