National Institute of Technology, Delhi

Name of the examination: B. Tech.

Course: Linear Algebra and Complex Analysis

Branch: EEE/ECE/CSE, II-Semester — Time Duration : **3 Hours**

Course Code : MAL-151 — Total Marks : 50

Name of the Student :____________ — Roll No.:____________

NOTE: This question paper contains eight (8) questions. All questions are compulsory.

Question 1: Answer all the following questions: [5 × 2]-marks

(*A*) What is the value of $\lim_{z\to 0} \frac{z}{\bar{z}}$?

(*i*) 1 (*ii*) −1 (*iii*) 0 (*iv*) does not exists.

(*B*) What is the value of integral $\int_C \frac{z}{z^2+9}dz$, where C is a unit circle $|z| = 1$ in positive direction?

(*i*) $2\pi i$ (*ii*) $-2\pi i$ (*iii*) 0 (*iv*) 1.

(*C*) What is the residue of the function $f(z) = ze^{\frac{1}{z}}$ at $z = 0$?

(*i*) 2! (*ii*) πi (*iii*) $\frac{1}{2!}$ (*iv*) $\frac{1}{6}$.

(*D*) Which of the following is an example of linear transformation $T : R^2 \to R^2$?

(*i*) $T(x, y) = (2x + y, x + |y|)$ (*ii*) $T(x, y) = (x + y, y + 1)$

(*iii*) $T(x, y) = (x - 2y, y)$ (*iv*) $T(x, y) = (5, -1)$.

(*E*) What is the dimension of $\mathbb{C}$ as a vector space over R, where $\mathbb{C}$ is the set of complex numbers?

(*i*) 1 (*ii*) 2 (*iii*) 0 (*iv*) does not exists.

Question 2: Answer all the following questions: [5 × 2]-marks

(*i*) Show that the vectors $(1, 2, 3)$, $(3, -2, 1)$ and $(1, -6, -5)$ are linearly dependent in R^3.

(*ii*) Find the length of orthogonal projection of the vector $y = (7, 6)$ onto $u = (4, 2)$.

(*iii*) Determine where $f'(z)$ exists and find its value, where $f(z) = x^2 + iy^2$.

(*iv*) Evaluate the contour integral $\int_{c_1} \frac{1}{z}dz$, where c_1 is $z = e^{i\theta}$, $0 \le \theta \le \pi$.

(*v*) Find the order of the pole and the corresponding residue for the function $f(z) = \frac{z+4}{z^2+1}$ at $z = i$.

Question 3: Diagonalize the matrix, if possible $\begin{bmatrix} -1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$. 5-marks

Question 4: Find an orthogonal basis for the column space of the matrix 5-marks

$$\begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$$

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Question 5: Find the integrals along the counterclockwise circles [2 + 3]-marks

(i) $\displaystyle\int_{|z|=2021} \frac{e^{3z}}{3z-i} dz$

(ii) $\displaystyle\int_{|z|=2022} \frac{\cos z}{z^6} dz.$

Question 6: Find the Laurent series of the function $f(z) = \dfrac{-1}{(z-1)(z-2)}$ in the domain $1 < |z| < 2$. 5-marks

Question 7: Find an analytic function $f(z) = u(x, y) + iv(x, y)$ corresponding to the real part $u(x, y) = x^3 - 3xy^2$. 5-marks

Question 8: Find the singularities of the function $f(z) = \frac{ze^{\pi z}}{z^4-16} + ze^{\pi z}$ inside c and calculate corresponding residues, where c is the ellipse $9x^2 + y^2 = 9$ in positive direction. Then, evaluate the integration $\displaystyle\int_c f(z)dz$. [1 + 2 + 2]-marks