National Institute of Technology, Delhi

Name of Examination: B.Tech.

Branch **: CSE/ECE/EEE** **Semester: 2nd**
Title of the Course **: Linear Algebra and Complex Analysis** **Course Code: MAL 151**
Time: 2Hours **Maximum Marks: 25**

Q1. (a.) Let A be a matrix of order 16×22. What is maximum possible dimensional of Column space of A if rows of A are not linear independent? (1 mark)
(b.) How many pivot columns must a 4×6 matrix have if its columns span $\mathbb{R}^4$?(1 mark)

Q2. Let T be a one-one linear transformation from a vector space V to a vector sapce W. If a set $\{v_1, v_2, \ldots, v_n\}$ is linear independent, then show that the set $\{T(v_1), T(v_2), \ldots, T(v_n)\}$ is linear independent. (2 marks)

Q3. Prove that the charcteristic root of a Hemitian matrix are real. (2 marks)

Q4. Let $T\colon \mathbb{R}^4 \to \mathbb{R}^3$ be the linear mapping defined by

$$T(x, y, z, t) = (x - y + z + t, \quad x + 2z + t, \quad x + y + 3z - 3t).$$

Detemine whether the map T is onto or not. (2 marks)

Q5. Find the dimension and a basis of the solution space of the system (3 marks)

$$\begin{aligned} x + 2y + 2z - s + 3t &= 0 \\ x + 2y + 3z + s + t &= 0 \\ 3x + 6y + 8z + s + 5t &= 0. \end{aligned}$$

Q6. Use the elementary row operations to find the inverse of the matrix (3 marks)

$$A = \begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{bmatrix}.$$

Q7. Applying Gram-Schmidt Process, convert the basis $\{(1,1,1), (1,1,0), (1,0,1)\}$ of $\mathbb{R}^3$ into orthonormal basis. (3 marks)

Q8. Let $H = \text{Span}\{v_1, v_2\}$ and $K = \text{Span}\{v_3, v_4\}$, where

$$v_1 = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 0 \\ -3 \\ -7 \end{bmatrix}.$$

Clearly H and K are subspaces of $\mathbb{R}^3$. In fact, H and K are planes in $\mathbb{R}^3$ through the origin, and they intersect in a line through origin. Find a non zero vector w that generates that line (i.e., find a vector w that spans the subspace $H \bigcap K$). (4 marks)

Q9 Suppose that A is a 3×3 matrix with eigen values $\lambda_1 = -1$, $\lambda_2 = 0$ and $\lambda_3 = 1$, and corresponding eigenvectors

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

a) Find the matrix A. (3 marks)

b) Compute the matrix A^{20} . (1 marks).