

26 sept. (M) - Lib

Roll No.:.....

## National Institute of Technology Delhi

Name of the Examination: B. Tech.

Branch: ECE, EEE, and CSE

Title of the Course: Advanced Calculus

Time: 2 Hours

Note: Attempt all questions.

Scientific calculators are not permitted.

Semester: 1st

Course Code: MAL 101

Maximum Marks: 25

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Q. 1. Show that for the function

$$f(x, y) = \begin{cases} (x^2 + y^2) \log(x^2 + y^2), & \text{if } (x^2 + y^2) \neq 0, \\ 0, & \text{if } (x^2 + y^2) = 0, \end{cases}$$

$f_{xy}(0, 0) = f_{yx}(0, 0)$ , even though the conditions of Schwarz's theorem are not satisfied.

[3]

Q. 2. Examine the following function for extreme values

$$f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2.$$

[4]

Q. 3. Find the critical points of

$$f(x) = x^{2/3}(x - 2).$$

Identify the interval on which  $f$  is increasing and decreasing. Find the functions local and absolute extrema values.

[3]

Q. 4. Examine the continuity at the origin of the following function

$$f(x, y) = \begin{cases} \frac{xy^9}{x^2 + y^{18}}, & \text{if } (x, y) \neq (0, 0), \\ 0, & \text{if } (x, y) = (0, 0). \end{cases}$$

[3]

Q. 5. State and prove sufficient conditions for differentiability of a function of two variable at a point.

[3]

Q. 6. Trace the curve  $y = \frac{x^2 + 1}{x^2 - 1}$ .

[3]

Q. 7. Expand  $e^x \sin^2 x$  in ascending powers of  $x$  upto  $x^5$ .

[3]

Q. 8. Suppose  $f(x)$  is continuous and differentiable on  $[6, 15]$ . Also suppose that  $f(6) = -2$  and  $f'(x) \leq 10$  for all  $x$  in  $[6, 15]$ . What is the largest possible value for  $f(15)$ ?

[3]