National Institute of Technology, Delhi

Name of Examination: B.Tech.(Makeup Examination)

Branch	**: CSE/ECE/EEE**	**Semester: 2nd**
Title of the Course	**: Linear Algebra and Complex Analysis**	**Course Code: MAL 151**
Time: 3Hours		**Maximum Marks: 50**

Note: This question paper is divided into three sections **A, B, C**. Section **A** contains One questions having Ten parts of 01 mark and all parts are Compulsory. Section **B** contains Five questions of 05 marks and any four questions are to be attempted. Section **C** contains Three questions of 10 marks and any two questions are to be attempted.

Section A

Q1 Answer the following:

(i) If λ is an eigen value of a nonsingular matrix A, then show that $\frac{1}{\lambda}$ is a eigen value of A^{-1}.

(ii) Is the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ diagonalizable? (Justify your answer)

(iii) The vectors $u_1 = (1, 1, 1, 1)$ and $u_2 = (1, -3, 4 - 2)$ are orthogonal. Express the vector (1,3,5,7) as a linear combination of u_1 and u_2.

(iv) Find a non-zero vector that is orthogonal to $u_1 = (1, 2, 1, 2)$ and $u_2 = (1, 2, 3, 4)$ in $\mathbb{R}^3$.

(v) The value of $f'(z) = \frac{d}{dz} f(z)$ in polar form is:

(A) $\frac{-i\,e^{-i\theta}}{r}(u_r + i\,v_r)$ (B) $\frac{i\,e^{-i\theta}}{r}(u_r + i\,v_r)$ (C) $\frac{e^{-i\theta}}{r}(u_r + i\,v_r)$ (D) None of these.

(vi) If $f(z)$ is analytic and uniformly bounded in every domain, then

(A) $f(z) = 0$. (C) $f(z)$ is constant.
(B) $f(z)$ is discontinuous. (D) None of these.

(vii) The residues of $f(z) = \frac{\cot(\pi z)}{z^4}$ at $z = 0$ is

(A) 0. (B) $-\pi^2/45$. (C) -1. (D) $-\pi^3/45$.

(viii) Which of the following has the removable singularity

(A) $\sin(\frac{1}{z})$. (B) $\frac{\sin(z)-z}{Z^3}$. (C) $\frac{\cos z}{z^2}$. (D) $\frac{e^z-1}{z^2}$.

(ix) The value of $\int_0^1 ze^{2z}\,dz$ will be equal to

(A) e. (B) $\frac{e^2+1}{4}$. (C) $\frac{e^2-1}{4}$. (D) None of these.

(x) If C is a circle $|z - a| = r$, then $\int_C \frac{dz}{(z-a)^n} = 2\pi i$, when

(A) $n = 1$. (B) $n \neq 1$. (C) $n = 0$. (D) None of these.

Section B

Q2. (a.) Find the value of integral

$$\int_0^{i+1} (x - y + i\,x^2)\,dz$$

(I) along the straight line from $z = 0$ to $z = 1 + i$.
(II) along the real axis from $z = 0$ to $z = 1$ and along a line parallel to imaginary axis from $z = 1$ to $z = 1 + i$.

(b.) Expand $\frac{1}{z(z^2-3z+2)}$ for the region $1 < |z| < 2$. (3+2 marks)

Q3. Prove that, in polar form, the Cauchy-Riemann equations for an analytic function $f(z) = u(x, y) + i\,v(x, y)$ can be written as

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta},$$

where $z = x + iy = re^{i\theta}$.

Q4. If $f(z)$ is an analytic function of z and if $f'(z)$ is continuous at each point within and on a simple closed contour C, then prove that

$$\int_C f(z)\,dz = 0.$$

Q5. Find an analytic function $f(z) = u + i\,v$ in term of z such that $Re\{f'(z)\} = 3x^2 - 4y - 3y^2$ and $f(1 + i) = 0$.

Q6. Suppose that $\{v_1, v_2, \cdots, v_n\}$ are non-zero eigenvectors of a matrix A belonging to distinct eigen values $\{\lambda_1, \lambda_2, \cdots, \lambda_n\}$. Then $\{v_1, v_2, \cdots, v_n\}$ are linear independent.

Section C

Q7. (a.) Find the Taylor's Laurent's Series of the following function with center $z = 1$.

$$f(z) = \frac{2z^2 + 9z + 5}{z^3 + z^2 - 8z - 12}$$

with and give the region of convergence of the series.
(b.) Prove or disprove that the function $f(z) = |z|^2$ is analytic at origin. (7+3 marks)

Q8. (a.) Use calculus of Residue, evaluate

$$\int_{-\infty}^{+\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9}\,dx.$$

(b.) Evaluate

$$\int_C \frac{z}{z^2 + 4z + 3}\,dz,$$

where C is the ellipse $(x - 2)^2 + 4y^2 = 4$, positively oriented. (7+3 marks)

Q9. Find a basis and the dimension of Row space, Column Space and Null space of the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{bmatrix}.$$