Roll No...............

National Institute of Technology Delhi

Name of the Examination: B.Tech.

End Semester Exam (2018-19)

Branch: EEE **Semester-III**

Course Title: Ordinary Differential Equations & Transforms **Course Code: MAL 201**

Max Time: 3 hrs **Total Marks: 50**

Note:

1. In Section A, all questions are **compulsory** with each part carrying 1 mark each.
2. In Section B, attempt any **FOUR** questions with each question carrying 5 marks.
3. In Section C, attempt any **TWO** questions with each question carrying 10 marks.

Section A

1. (a) Which of the following is not the property of convolution?
   i. $f * 1 = f$
   ii. $f * (g_1 + g_2) = f * g_1 + f * g_2$
   iii. $(f * g) * v = f * (g * v)$
   iv. $f * 0 = 0$

   (b) Find the Laplace transform of Bessel function $J_0(x)$.

   (c) Find the regular singular points of the following differential equation:

   $$(x-3)(2x+1)\frac{d^2y}{dx^2} - 2x^2\frac{dy}{dx} + 5xy = 0.$$

   (d) Find the fourier coefficient a_1 in the fourier series representation of function x^2 in the interval $(-l, l)$.

   (e) Find the differential equation whose independent solutions are e^x and xe^x.

   (f) Find the integrating factor of the differential equation

   $$y(xy + 2x^2y^3)dx + x(xy - x^2y^2)dy = 0.$$

   (g) Solve $(D^2+1)^3 y = 0$, where $D = \frac{d}{dx}$

   (h) State conditions that are sufficient for the existence of the Fourier transform.

   (i) What is the generating function for Bessel function $J_n(x)$.

   (j) Prove that the Laplace transform is a linear operation.

[1 × 10 =**10 Marks**]

Section B

2. Obtain the series solution of the equation

$$x(1-x)\frac{d^2y}{dx^2} - (1+3x)\frac{dy}{dx} - y = 0.$$

3. Prove Rodrigue's formula for Legendre polynomials.

4. Evaluate $\mathcal{L}^{-1}\left[\frac{1}{(s^2+1)(s^2+9)}\right]$ and using it evaluate $\mathcal{L}^{-1}\left[\frac{1}{(s^2+1)(s^2+4)(s^2+9)}\right]$.

5. Show that the Fourier integral representation of the function $f(x) = \begin{cases} 0, & \text{when } x \leq 0 \text{ or } x \geq \pi \\ \sin x, & \text{when } 0 \leq x \leq \pi. \end{cases}$ is

$$f(x) = \frac{1}{\pi} \int_0^\infty \frac{\cos u(\pi - x) + \cos ux}{1 - u^2} du.$$

6. Solve the system of differential equations

$$\frac{dx_1}{dt} = 2x_1 - 5x_2$$
$$\frac{dx_2}{dt} = x_1 - 2x_2$$

[5 × 4 =**20 Marks**]

Section C

7. (a) Show that $\frac{1}{\pi} \int_0^\pi \cos(x \cos \phi) d\phi$ satisfies Bessel's equation of order zero. [**5 Marks**]
   (b) Prove that $(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x)$. [**5 Marks**]

8. (a) Using Laplace transform, solve the following initial value problem:

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = 5 \sin t, \quad \text{where } y(0) = y'(0) = 0.$$

[**5 Marks**]

   (b) State and prove the existence theorem for Laplace transforms. [**5 Marks**]

9. (a) Find the Fourier series for $f(x)$, if $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi. \end{cases}$
   Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$. [**5 Marks**]
   (b) Let $f(x)$ be continuous on the x-axis and $f(x) \to 0$ as $|x| \to \infty$. Furthermore, let $f'(x)$ be absolutely integrable on the x-axis. Then prove that

$$\mathcal{F}\{f'(x)\} = \iota w \mathcal{F}\{f(x)\}, \quad \iota \text{ represents iota.}$$

   Also, prove that successive application of the above gives $\mathcal{F}\{f''(x)\} = -w^2 \mathcal{F}\{f(x)\}$.

[**5 Marks**]