Roll No.:......................

National Institute of Technology, Delhi

Name of the Examination: B. Tech.

End Semester Examination (November-December, 2019)

Branch	: CSE, ECE, EEE	Semester	: 1st
Title of the Course	: Advanced Calculus	Course Code	: MAL 101

Time: 3 Hours | Maximum Marks: 50

Note: Read instructions on each section carefully.

Section A [10 Marks]

Note: Questions 1-4 carries 1 mark. Questions 5-7 carries 2 mark.

Q.1. A line is drawn through the point (1, 2) forming a right triangle with the positive x- and y-axes. The slope of the line forming the triangle of least area is

(A). -1 (B). -2 (C). -4 (D). $-1/2$ (E). -3

Q.2. If the graph of $f(x) = x^3 + ax^2 + bx - 4$ has a point of inflection at (1, –6), what is the value of b?

(A). -3 (B). 0 (C). 1 (D). 3 (E). Information insufficient

Q.3. The value of double integral $\int_0^4 \int_{\sqrt{x}}^2 \cos(y^3)\, dy dx$ is

(A) $\frac{1}{3}(1-\cos 8)$ (B) $\frac{1}{3}\sin 8$ (C) $\frac{1}{3}\cos 8$ (D) $3(1-\cos 8)$ (E) $3\sin 8$ (F) $3\cos 8$

Q.4. The value of the integral $\oint_C \left[(y^3 + x^2)dx + (3y^2x + x)dy\right]$ when C is the boundary of positively oriented triangle with vertices (0,0), (0,4), and (2,2) is

(A) -2 (B) 2 (C) -4 (D) 0 (E) 4

Q.5. What do you mean by a vector field to be conservative? Is the field $F(x, y) = x\hat{i}$ conservative?

Q.6. Check the convergence of the series $\sum_{n=1}^{\infty} \frac{n^3 x^n}{3^n}$.

Q.7. Evaluate the integral $\int_0^1 \int_0^1 x \max(x, y)\, dy dx$ where max(x, y) is a maximum function.

Section B [20 Marks]

Note: Attempt any 4 questions. Each question carries 5 marks.

Q.8. A company has been asked to design a storage tank for a liquid petroleum gas. The specifications require a cylindrical tank with hemispherical ends, and the tank is to hold 8000 m^3 of gas. It is also desired to use the smallest amount of material possible in building the tank. What radius and height do you recommend for the cylindrical portion of the tank?

Q.9. State and prove ratio test for the convergence of a series. Check the convergence of series with terms $a_n = \dfrac{4^n n! n!}{(2n)!}$.

Q.10. Find the volume of the solid generated by revolving the region which is bounded by the parabola $y = x^2$ and the line $y = 1$ about (i) the line $y = 1$ and (ii) the line $y = -1$.

Q.11. Find the work done by the force $\vec{\mathbf{F}} = (x^2 - y^3)\vec{\mathbf{i}} + (x+y)\vec{\mathbf{j}}$ in moving a particle along closed path C containing the curves $x + y = 0$, $x^2 + y^2 = 16$, and $y = x$ in the first and fourth quadrants.

Q.12. Find the flux of the vector field $\vec{\mathbf{F}} = -x\vec{\mathbf{i}} - y\vec{\mathbf{j}} + z^2\vec{\mathbf{k}}$ outward (normal away from the z-axis) through the portion of the cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 2$.

Section C [20 Marks]

Note: Attempt any 2 questions. Each question carries 10 marks.

Q.13. **(A)** Find the interval of convergence of series $\sum_{n=1}^{\infty} \dfrac{(-1)^n 3^{2n} (x-2)^n}{3n}$.

(B) State and prove integral test for convergence of a series.

Q.14. **(A)** Convert the triple integral $\int_{-1}^{1} \int_{0}^{\sqrt{1-y^2}} \int_{0}^{x} (x^2 + y^2)\, dz dx dy$ to cylindrical coordinates and evaluate.

(B) Evaluate the double integral $\int_{1}^{2} \int_{1/y}^{y} \sqrt{\dfrac{y}{x}} e^{\sqrt{xy}}\, dx dy$ by appropriate substitution.

Q.15. **(A)** Find the circulation using Stokes theorem for the vector field $\vec{\mathbf{F}} = x^2 y\vec{\mathbf{i}} + 2y^3 z\vec{\mathbf{j}} + 3z\vec{\mathbf{k}}$ across the surface S having position vector $\vec{\mathbf{r}}(r,\theta) = r\cos\theta\vec{\mathbf{i}} + r\sin\theta\vec{\mathbf{j}} + r\vec{\mathbf{k}},\ 0 \le r \le 1,\ 0 \le \theta \le 2\pi$.

(B) Find the flux of field using divergence theorem, where $\vec{\mathbf{F}} = \sqrt{x^2 + y^2 + z^2}\left(x\vec{\mathbf{i}} + y\vec{\mathbf{j}} + z\vec{\mathbf{k}}\right)$ across the boundary of the region $1 \le x^2 + y^2 + z^2 \le 2$.