Roll No.:........................

National Institute of Technology, Delhi

Examination: B. Tech. End Semester Examination December 2023 (Autumn Semester)

Branch	**: CSE, ECE, EEE**	**Semester**	**: 1st**
Title of the Course	**: Advanced Calculus**	**Course Code**	**: MALB 101**

Time: Three Hours **Maximum Marks: 50**

Note : All sections are compulsory.

Course Outcomes: Student will be able to:		**Cognitive Levels**
CO1	Understand the theory and methods of Differential, Integral and Vector Calculus	**Understanding Level-II**
CO2	Apply different methods for solving problems in Differential, Integral and Vector Calculus	**Applying Level-III**
CO3	Analyze sequence and series for its convergence. Analyse function for continuity and differentiability. Analyse curves and surfaces for concavity, inflection points, maxima and minima. Compare different integration techniques for finding area and volume.	**Analyzing Level-IV**
CO4	Evaluate extreme points for function of several variables. Evaluate limits. Evaluate limit of sequences and sum of some convergent series. Evaluate multiple integrals in rectangular, polar, cylindrical, and spherical coordinates.	**Evaluating Level-V**
CO5	Create power series. Formulate problems on maxima and minima. Combine vector differential calculus and vector integral calculus. Construct counter-examples for theorems and arguments. Formulate problems on integral and vector calculus.	**Creating Level-VI**

Q.No.	Question	Marks	CO	BL
1	Suppose you are designing a cylindrical storage tank with hemispherical ends for storing liquid petroleum gas for a company. The company wants the tank to hold 8000 m^3 of gas and wants you to use smallest amount of material possible to built the tank. Formulate the minimisation problem. What radius and height of the tank do you recommend for the cylindrical portion of the tank?	5	CO5	L6
2	Define second derivative test for local extreme values. Find the absolute maximum and minimum of the function $f(x, y) = x^2 - xy + y^2 + 1$ on the closed triangular plate in the first quadrant bounded by the lines $x = 0$, $y = 4$, $y = x$.	5	CO4	L5
3	(i) State and prove the first fundamental theorem of integral calculus. (ii) Analyse the convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n (x+2)^n}{n}$. Also, give the interval and radius of convergence.	5	CO3	L4

4	State alternating series test for convergence. Discuss the absolute and conditional convergence for the following: (i) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n!)^2}{(2n)!}$ (ii) $\sum_{n=1}^{\infty} (-1)^n \frac{tan^{-1}n}{n^2+1}$	5	CO2	L3
5	Consider the region in the first quadrant bounded from above by a parabola $y = x^2$ and below by x-axis and on the right by line x=2. Find the volume of the solid generated by revolving the region (i) about the x-axis, (ii) about the y-axis.	5	CO3	L4
6	Sketch the region of integration, reverse the order of integration and hence evaluate the integral $\int_0^8 \int_{\sqrt[3]{x}}^2 \frac{1}{y^4+1} dydx$	5	CO5	L6
7	Convert the integral $\int_{-1}^1 \int_0^{\sqrt{1-y^2}} \int_0^x (x^2+y^2) dzdxdy$ into an equivalent integral in cylindrical coordinates and then evaluate the integral.	5	CO4	L5
8	Define Green's theorem in tangential and normal form. Apply Green's theorem to calculate the circulation and outward flux created by the field $\vec{F} = (y^2 - x^2)\vec{i} + (x^2 + y^2)\vec{j}$ on the curve $C:$ the triangle bounded by $y = 0, x = 3, y = x$	5	CO2	L3
9	Use the surface integral in Stokes theorem to calculate the flux of the curl of the vector field F across the surface S in the direction of the outward unit normal n, when $\vec{F} = x^2y\vec{i} + 2y^3z\vec{j} + 3z\vec{k}$ and $S: \vec{r}(r,\theta) = rcos\theta\vec{i} + rsin\theta\vec{j} + r\vec{k}, 0 \le r \le 1, 0 \le \theta \le \pi$	5	CO5	L6
10	Define conservative field. Analyze if $\vec{F} = (e^x cosy + yz)\vec{i} + (xz - e^x siny)\vec{j} + (xy + z)\vec{k}$ is conservative over its natural domain and find potential function for it.	5	CO3	L4