Roll No.:.....

National Institute of Technology, Delhi

Name of the Examination: B. Tech.

Branch	: EEE, ECE,CSE	Semester	: 3rd
Title of the Course	: Signals and Systems	Course Code	: ECB 204

Time: 3 Hours | Maximum Marks: 50

Note : 1. This question paper has 3 sections: A, B and C. All the sections are compulsory. **Section A** carries only one question (Q1) having 10 parts of 01 mark each and all the parts are compulsory. **Section B** contains five questions (Q2 to Q6) of 5 marks each and any four are to be answered. **Section C** contains three questions (Q7 to Q9) of 10 marks each and any two are to be answered.

Section A

Q1. a) Specify the value of the real parameter σ which ensure that the following integral converges:

$$\int_{-5}^{5} e^{-5t}\, e^{-(\sigma+j\omega)t}\, dt$$

b) Evaluate the laplace transform of the following signal. Also, specify its ROC.

$$x(t) = e^{-5t}u(t-1)$$

c) Determine the constraint on $r = |z|$ for the following sum to converge:

$$\sum_{n=-1}^{\infty} \left(\frac{1}{2}\right)^{n+1} z^{-n}$$

d) Consider the signal:

$$x[n] = \begin{cases} \left(\frac{1}{3}\right)^n \cos\left(\frac{\pi}{4}n\right) & n \le 0 \\ 0, & n > 0 \end{cases}$$

Determine the poles and ROC for X(z).

e) A real valued signal x(t) is known to be uniquely determined by its samples when the sampling frequency is $\omega_s = 10{,}000\pi$. For what value of ω is X(jω) guaranteed to be zero?

f) Let x(t) be a signal with the Nyquist rate ω_0. Determine the Nyquist rate for $x^2(t)$.

g) Find discrete-time Fourier transform of the discrete-time signal $x[n] = \left(\frac{1}{3}\right)^n u[n-1]$.

h) Find continuous-time Fourier transform of $x(t) = e^{-2|t-2|}$.

i) Find the discrete-time Fourier series coefficients a_k for the sequence $x[n] = n^2,\ -2 \le n \le 1$. The sequence $x[n]$ is periodic with period N = 4.

j) Find the continuous-time Fourier series coefficients a_k of the signal $x(t) = e^{-|t|}$, $-\pi \leq n \leq \pi$. The signal $x(t)$ is periodic with period T = 2π.

Section B

Q2. Let $x(t) = u(t-3) - u(t-5)$ and $h(t) = e^{-3t}u(t)$.

a) Compute $y(t) = x(t) * h(t)$.

b) Compute $g(t) = \frac{d}{dt}[x(t)] * h(t)$.

c) How is g(t) related to y(t).

Q3. Let $x[n]$ be a real and odd periodic signal with period $N = 7$ and Fourier coefficients a_k. Given that $a_{15} = j$, $a_{16} = 2j$ and $a_{17} = 3j$. Determine the values of a_0, a_{-1}, a_{-2} and a_{-3}.

Q4. Compute the Fourier transform of the following signal:

$$\sum_{k=0}^{\infty} \alpha^k \delta(t - kT), \quad |\alpha|<1$$

Q5. An RLC circuit whose capacitor voltage and inductor current are initially zero constitutes an LTI system. Consider the series RLC circuit. Let the voltage across the voltage source be x(t) and let the voltage measured across the capacitor be the output signal y(t). Show that this system is stable if the values of R, L and C are all positive.

Q6. Consider the following system function for stable LTI system. Without using the inverse Z transform, determine whether or not the corresponding system is causal.

$$\frac{z-1}{z^2 + \frac{1}{2}z - \frac{3}{16}}$$

Section C

Q7. Suppose g(t) = x(t)cos(t) and the Fourier transform of g(t) is

$$G(j\omega) = \begin{cases} 1, & |\omega| \leq 2 \\ 0, & otherwise \end{cases}$$

a) Determine x(t).

b) Find the Fourier transform $X_1(j\omega)$ of $x_1(t)$ such that

$$g(t) = x_1(t)cos\left(\frac{2}{3}t\right)$$

Q8. A signal x(t) having the frequency domain representation X(jω) is sampled in every T_s seconds and is denoted by x(nT_s). Find the Fourier transform of the sampled signal in terms of X(jω).

Q9. a) Show that if x(t) is an even function then X(s) = X(-s).

b) Show that if x(t) is an odd function then X(s) = -X(-s).

c) Determine if the pole zero plot shown in Fig. 1 correspond to an even function. If yes then indicate the required ROC.

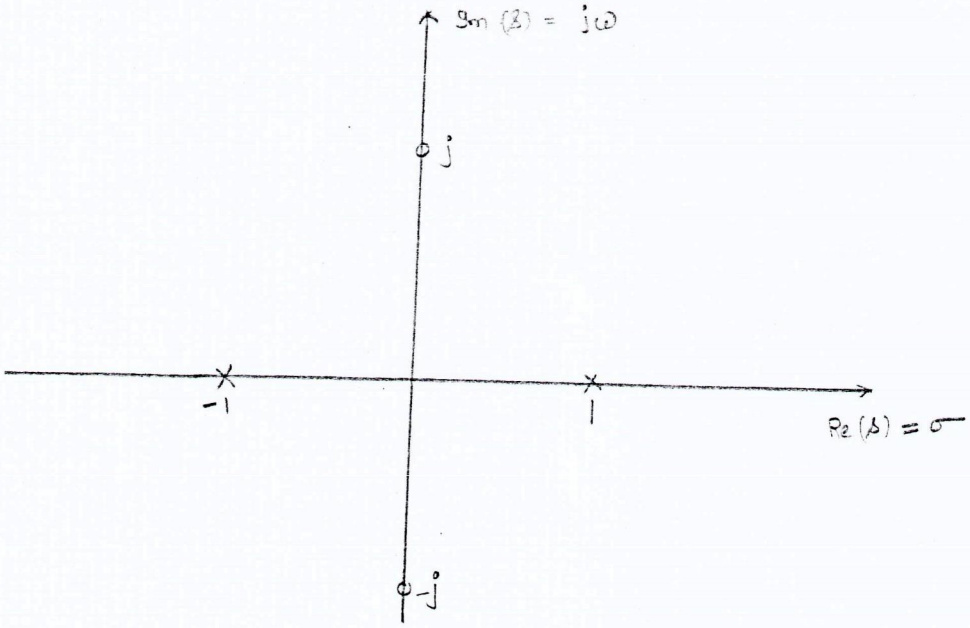

FIG. 1