Aim ↬ To understand and implement the Discrete Time Fourier Series and its inverse in order to analyze and synthesize periodic discrete-time signals using MATLAB.

Software Required ↬ MATLAB

Theory ↬

The Discrete Time Fourier Series (DTFS) is a mathematical tool used to represent a periodic discrete-time signal as a sum of sinusoidal components. These components are complex exponentials with different frequencies, amplitudes, and phases. The DTFS is particularly important in digital signal processing (DSP) because it allows for the analysis and synthesis of periodic signals in the frequency domain.

The Fourier coefficients C_k for the signal x[n] are calculated using the formula:

$$\mathrm{C_k} = \frac{1}{N}\sum_{k=0}^{N-1} x[n]e^{-jk\Omega_0 n}$$

- k is the frequency index, which ranges from 0 to N−1.
- C_k represents the contribution of the frequency component $\frac{2\pi k}{N}$ to the overall signal.
- $\Omega_0 = \frac{2\pi k}{N}$ is the fundamental frequency.

These coefficients capture the signal's frequency content, allowing the signal to be understood and analyzed in terms of its sinusoidal components.

Properties of DTFS ↴

1] Linearity ↪

$$ax_1[n] + bx_2[n] \leftrightarrow aC_k + bD_k$$

2] Time Shifting ↪

$$x[n - n_0] \leftrightarrow C_k e^{-\mathrm{j}\mathrm{k}\Omega_0 n_0}$$

3] Frequency Shifting ↪

$$x[n]e^{jk_0\Omega_0 n} \leftrightarrow C_{k-k_0}$$

4] Time Reversal ↪

$$x[-n] \leftrightarrow C_{-k}$$

5] Periodic Convolution ↪

$$x[n] \circledP h[n] \leftrightarrow C_k . H_k . N$$

6] Multiplication in Time Domain ↪

$$x[n] . h[n] \leftrightarrow C_k * H_k$$

7] Conjugate Symmetry ↪

$$C_k \leftrightarrow C^*{}_{N-k}$$

8] Duality ↪

$$C_n \leftrightarrow \frac{1}{N} x[-k]$$

9] First Difference Theorem ↪

$$x[n] - x[n-1] \leftrightarrow \left(1 - e^{-j\mathrm{k}\Omega_0}\right) . C_k$$

10] Parseval's Power Property ↪

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |C_k|^2$$

Inverse Discrete Time Fourier Series ↴

The Inverse Discrete Time Fourier Series allows us to reconstruct the original periodic discrete-time signal from its Fourier coefficients. This process is crucial for signal synthesis and analysis in the frequency domain.

The Inverse DTFS is given by:

$$x[n] = \sum_{k=0}^{N-1} \mathrm{C_k} e^{jk\Omega_0 n}$$

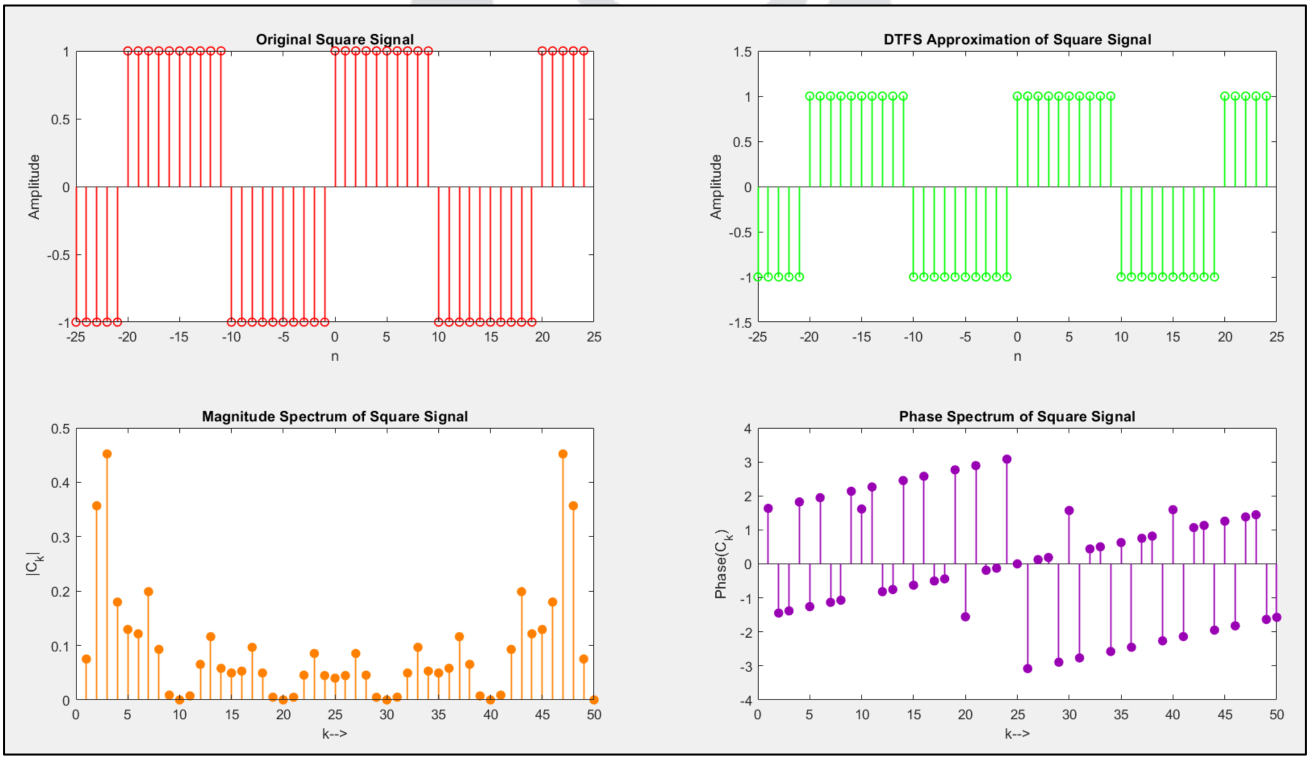
This equation shows how the original signal x[n] can be synthesized by summing all the sinusoidal components represented by the coefficients C_k.

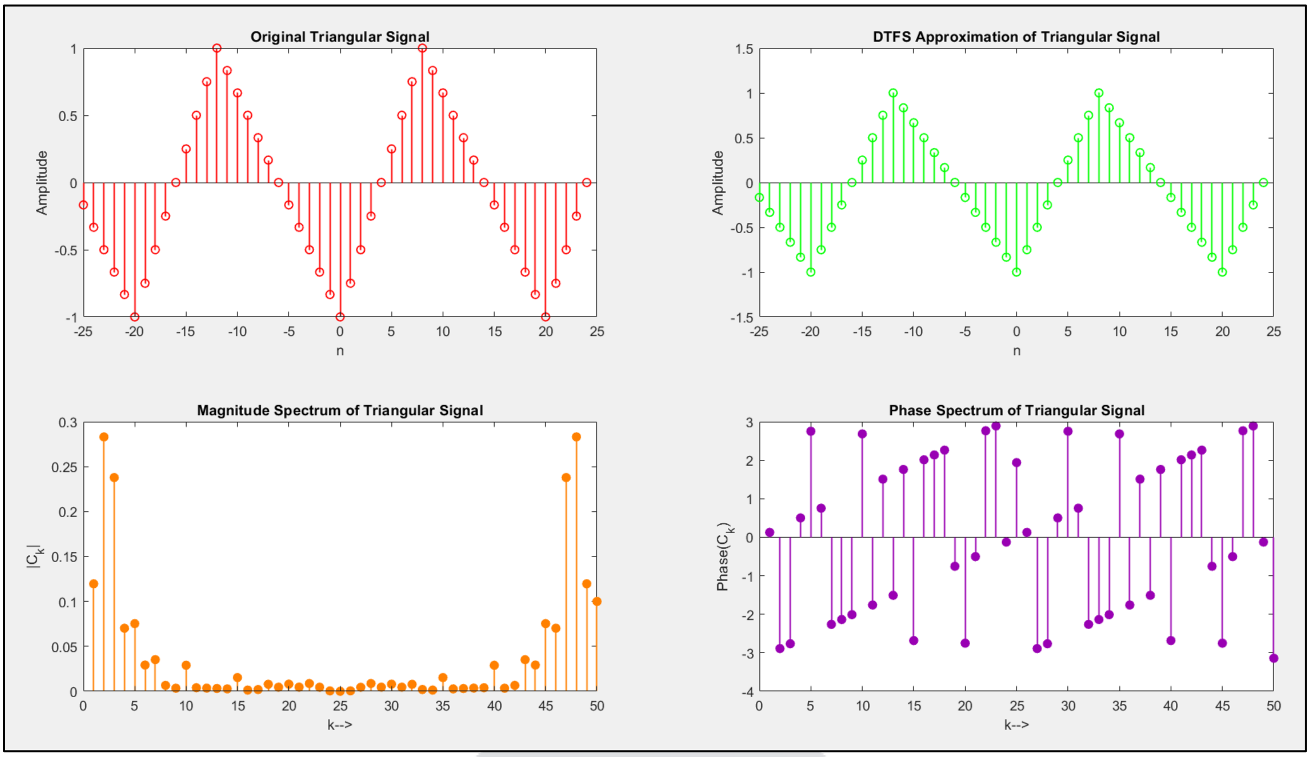
Code ↬

```
%Discrete Time Fourier Series
N = 50;
n = -25:24;

signals = {
    @(n) square(2*pi*n/20), 'Square Signal';
    @(n) sawtooth(2*pi*n/20, 0.4), 'Triangular Signal';
};

for k = 1:size(signals, 1)
    f = signals{k, 1};
    signalName = signals{k, 2};

    f_n = f(n);
    f_approx = zeros(size(n));

    C_k = zeros(1, N);

    for m = 1:N
        C_k(m) = (1/N) * sum(f_n .* exp(-1i * 2 * pi * m * n / length(n)));
    end

    for m = 1:N
        f_approx = f_approx + real(C_k(m) * exp(1i * 2 * pi * m * n / length(n)));
    end

    magnitude = abs(C_k);
    phase = angle(C_k);

    figure;

    subplot(2,2,1);
    stem(n, f_n, 'r', 'LineWidth', 1);
    xlabel('n');
    ylabel('Amplitude');
    title(['Original ', signalName]);

    subplot(2,2,2);
    stem(n, f_approx, 'g', 'LineWidth', 1);
```

```
    xlabel('n');
    ylabel('Amplitude');
    title(['DTFS Approximation of ', signalName]);

    subplot(2,2,3);
    stem(1:N, magnitude, 'filled', 'Color', [1, 0.5, 0], 'LineWidth', 1);
    xlabel('k-->');
    ylabel('|C_k|');
    title(['Magnitude Spectrum of ', signalName]);

    subplot(2,2,4);
    stem(1:N, phase, 'filled', 'Color', [0.6, 0, 0.7], 'LineWidth', 1);
    xlabel('k-->');
    ylabel('Phase(C_k)');
    title(['Phase Spectrum of ', signalName]);
end
```

Output ↬





Result ↬

The Discrete Time Fourier Series and its inverse were implemented in MATLAB to analyze and synthesize periodic discrete-time signals. The DTFS was applied to a sample discrete-time signal, and the resulting reconstruction closely matched the original signal.

Conclusion ↬

The experiment demonstrated the effectiveness of the Discrete Time Fourier Series in representing periodic discrete-time signals using complex exponential components. This method is crucial in digital signal processing for analyzing and understanding the frequency components of discrete-time signals.

Precautions ↬

- Ensure the correct implementation of summation and exponentials to avoid phase errors.
- Verify the signal period N to calculate the DTFS coefficients correctly.
- Consider the impact of numerical precision in MATLAB when working with complex exponentials.