Roll No.:........................

National Institute of Technology, Delhi

Name of the Examination: B. Tech. (Makeup Examiniation)

Branch : ECE, EEE and CSE **Semester** : I

Title of the Course : Advanced Calculus **Course Code** : MAL101

Time: 3 Hours **Maximum Marks: 50**

Note: This question paper is divided into three sections A, B and C, and each section must be solved with rules given as follows:

Section A

(Contains Ten (10) questions of 01 mark each, and all questions are compulsory)

Q. 1.

(*a*) What is removable discontinuity? Explain with one example.

(*b*) Explain graphically the left hand and right hand derivative of $y = \frac{x}{|x|}$.

(*c*) What is Sandwich theorem? Explain with one example.

(*d*) Show that $\lim_{x\to\infty} \frac{1}{x} = 0$ using $\varepsilon - \delta$ definition.

(e) Find the unit normal vector to the level surface $x^2y + 2xz = 4$ at the point (2, -2, 3).

(*f*) Define Riemann sum of Integral.

(*g*) State the second derivative theorem for the test of local extrema.

(*h*) State the young's Theorem.

(*i*) Define the concavity of $f(x)=\sin x$ in $[0, \pi]$.

(*j*) State the Green's Theorem.

Section B

(Attempt any four questions in this section).

Q. 2. State and prove the Young's theorem.

Q. 3. Sketch the graph of the function

$$y = \frac{x}{x^2+1}$$

Be sure to identify in writing all local max and mins, regions where the function is increasing/decreasing, points of inflection, symmetries, and vertical or horizontal asymptotes (if any of these behaviors occur).

Q. 4. Let V, W be non-zero vectors, and let α be the angel between then. Then

$$\cos\alpha = \frac{V \cdot W}{\|V\|\|W\|}$$

Q. 5. Find the volume under the plan $z = 8x + 6y$ over the region: $R : \{(x, y) : 0 \le x \le 1, 0 \le y \le 2x^2\}$. Also show the graphical representation.

Q. 6. Find the area of the region in the x-y plan bounded by the curve $\rho^2 = a^2 \cos 2\phi$.

Section C
(Attempt any two questions in this section)

Q.7(●). Evaluate $\iint_R e^{\frac{x-y}{x+y}} dA$, where $R : \{(x, y) : x \ge 0, y \ge 0, x + y = 1\}$ by using Jacobian.

Q. 8. Verify the Gauss divergence theorem in the plane for $F = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$, taken over the rectangular parallelepiped $0 \le x \le a, \quad 0 \le y \le b, \quad 0 \le z \le c$.

Q. 9. (a) Prove that series $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \ldots\ldots\ldots$ converges and also find its sum.

Q.9. (b) Show that the function $f(x, y)$ is continuous but not differential at origin.

$$f(x,y) = \begin{cases} x\sin\left(\frac{1}{x}\right) + y\sin\left(\frac{1}{y}\right), & xy \ne 0 \\ x\sin\left(\frac{1}{x}\right), & y = 0,\ x \ne 0 \\ y\sin\left(\frac{1}{y}\right), & x = 0,\ y \ne 0 \\ 0, & x = 0, y = 0 \end{cases}$$