Roll No.:......................

National Institute of Technology, Delhi

Name of the Examination: B. Tech.

Branch	: EEE	Semester	: 3rd
Title of the Course	: Ordinary Differential Equations and Transforms	Course Code	: MAL 201

Time: 3 Hours

Maximum Marks: 50

Section A

[All parts of section A are compulsory. Each part is of 01 mark]

Q.1. Attempt each of the following:

i. The solution of the differential equation $\frac{dy}{dx}+\sqrt{\frac{1-y^2}{1-x^2}}=0$ is

(A) $\sin^{-1}x-\sin^{-1}y=c$ (B) $\sin^{-1}x+\sin^{-1}y=c$ (C) $\sin^{-1}x=c\sin^{-1}y$ (D) $\sin^{-1}x\cdot\sin^{-1}y=c$

ii. If $J_n(x)$ defines the Bessel's function of first kind of order n, then which of the following is true for n>2

(A) $\frac{d}{dx}\left[x^nJ_n(x)\right]=x^{n-1}J_n(x)$ (B) $\frac{d}{dx}\left[x^nJ_n(x)\right]=xJ_{n-1}(x)$

(C) $\frac{d}{dx}\left[x^nJ_n(x)\right]=x^nJ_{n-1}(x)$ (D) $\frac{d}{dx}\left[x^nJ_n(x)\right]=xJ_n(x)$

iii. The finite Fourier sine transform of $f(x)=x,\ 0\le x\le 1$ is

(A) $\frac{1}{n}$ (B) $\frac{(-1)^n}{n}$ (C) $\frac{1-\cos n\pi}{n}$ (D) 0

iv. If the Fourier cosine transform of $f(x)$ is $\sqrt{\frac{2}{\pi}}\frac{\omega}{1+\omega^2}$ then the Fourier transform of $f(ax)$ is

(A) $\sqrt{\frac{2}{\pi}}\frac{a\omega}{a^2+\omega^2}$ (B) $\sqrt{\frac{2}{\pi}}\frac{\omega}{a^2+\omega^2}$ (C) $\sqrt{\frac{2}{\pi}}\frac{a}{a^2+\omega^2}$ (D) $\sqrt{\frac{2}{\pi}}\frac{a+\omega}{a^2+\omega^2}$

v. If $\delta(t)$ denotes a unit impulse function, then the Laplace transform of $\frac{d^2\delta(t)}{dt^2}$ will be

(A) 1 (B) s^2 (C) s (D) s^{-2}

vi. The value of $\int_0^\infty te^{-3t}J_0(4t)$ is

(A) $\frac{3}{125}$ (B) $\frac{2}{125}$ (C) $\frac{3}{25}$ (D) $\frac{2}{25}$

vii. The inverse Laplace transform of $\frac{e^{-2s}}{s-3}$ is

(A) $e^{3(t-2)}u(t-3)$ (B) $e^{2(t-3)}u(t-2)$ (C) $e^{3(t-2)}u(t-2)$ (D) $e^{2(t-3)}u(t-3)$

viii. At $x=0$, the Fourier series of $f(x)=\begin{cases}0, & -\pi<x<0\\ \pi-x, & 0<x<\pi\end{cases}$ converges to

(A) π (B) 0 (C) $\frac{\pi}{2}$ (D) none of these

ix. If $f(x)=\begin{cases}1, & 0<x<\frac{1}{2}\\ 0, & \frac{1}{2}<x<1\end{cases}$ then b_1 in the half-range sine series is equal to

(A) $\frac{1}{\pi}$ (B) $\frac{2}{\pi}$ (C) $\frac{3}{\pi}$ (D) $\frac{4}{\pi}$

x. The Fourier transform of exponential signal $e^{j\omega_0 t}$ is
(A) a constant (B) a rectangular gate (C) an impulse (D) a series of impulses

Section B

[Attempt any 04 questions of 05 marks each]

Q.2. Solve the differential equation $\left(D^2-4D+4\right)y=8x^2e^{2x}\sin 2x$.

Q.3. Solve the differential equation $(x+1)^4 D^3y+2(x+1)^3 D^2y-(x+1)^2 Dy+(x+1)y=\frac{1}{1+x}$.

Q.4. Determine the half range Fourier cosine series for $f(x)=\begin{cases} x, & 0<x<\frac{\pi}{2} \\ \pi-x, & \frac{\pi}{2}<x<\pi \end{cases}$.

Q.5. Find the Fourier transform of $f(x)=\frac{1}{\sqrt{x}}$.

Q.6. Define Laplace transform and inverse Laplace transform of the function. Find the inverse Laplace transform of $F(s)=\ln\left(\frac{s+1}{s+5}\right)$.

Section C

[Attempt any 02 questions of 10 marks each (06 marks + 04 marks)]

Q.7. (A) Find the Fourier series for the function $f(x)$ if $f(x)$ is defined in $-\pi<x<\pi$ as

$$f(x)=\begin{cases} 0, & -\pi<x<0 \\ \sin x, & 0<x<\pi \end{cases}$$

Hence deduce that $\frac{1}{4}(\pi-2)=\frac{1}{1.3}-\frac{1}{3.5}+\frac{1}{5.7}-\dots\dots\dots$

(B) Find the Fourier sine transform of $g(x)=\begin{cases} x, & 0<x<1 \\ 2-x, & 1<x<2 \\ 0, & x>2 \end{cases}$.

Q.8. (A) Find the general solution of $2x(x+1)D^2y-(1-x)Dy+y=0$ in series about $x=0$.

(B) Show that $J_n(x)=\frac{x}{2n}\left[J_{n-1}(x)+J_{n+1}(x)\right]$ and express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$.

Q.9. (A) Solve the following system of ordinary differential equation by Laplace transformation

$x-y''+y=e^{-t}-1,\quad x'+y'-y=-3e^{-t}+t$ where $x(0)=0, y(0)=1, y'(0)=-2$.

(B) Find the Laplace transform of periodic function $f(t)=|\sin\omega t|$ with period $\frac{\pi}{\omega}$.