Roll No.:.......................

National Institute of Technology, Delhi

Name of the Examination: B. Tech.

Branch	: All	**Semester**	: 3rd
Title of the Course	: Linear Algebra and Complex Analysis	**Course Code**	: MAL 151

Time: 3 Hours **Maximum Marks: 50**

Section A

[All parts of section A are compulsory. Each part is of 01 mark]

Q.1. Attempt each of the following:

i. Which of the following sets of vectors are bases for R^2

(A) {(0, 1), (1, 1)} (B) {((1, 1), (2, 2)} (C) {((1, 1), (0, 0)} (D) none of these

ii. The sum of the residues of $f(z)=\dfrac{1}{(z^2+1)^3}$ is

(A) 0 (B) 1 (C) -1 (D) z

iii. If $f(z)=\dfrac{\sin z}{z}$, then z=1 is

(A) its removable singularity (B) its isolated singularity (C) its essential singularity (D) none of these

iv. Which of the following functions have an essential singularity at z=0

(A) $\dfrac{1}{z}$ (B) $\dfrac{1}{z(z-1)}$ (C) $e^{1/z}$ (D) $\dfrac{\sin z}{z}$

v. The region of validity of Taylor's series about z=0 of function e^z is

(A) $|z|=0$ (B) $|z|<1$ (C) $|z|>1$ (D) $|z|<\infty$

vi. Find all linear maps L: $R^3 \rightarrow R^3$ whose kernel is exactly the plane $\{(x_1, x_2, x_3)\in R^3 \mid x_1+2x_2-x_3=0\}$.

vii. If A is an invertible 3 × 3 matrix and λ is an eigen value of A, then show that 1/λ is an eigen value of A^{-1}.

viii. For which real numbers x do the vectors: (x, 1, 1, 1), (1, x, 1, 1), (1, 1, x, 1), (1, 1, 1, x) not form a basis of R^4.

ix. What are the standard basis of R^3.

x. Give example of a function which is differentiable at origin but not analytic at origin.

Section B

[Attempt any 04 questions of 05 marks each]

Q.2. . Find a basis for the solutions to the following system of linear equations:

$$x_1 + 2x_2 - x_3 + x_4 = 0$$
$$-x_1 - 2x_2 + 3x_3 + 5x_4 = 0$$
$$-x_1 - 2x_2 - x_3 - 7x_4 = 0$$

Q.3. Find the eigenvalues and eigenvectors of the following matrix:

$$A = \begin{bmatrix} 4 & -4 & 2 \\ 2 & -2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Q.4. Compute the dimension of the intersection of the following two planes in R^3

$x + 2y - z = 0,$

$3x - 3y + z = 0.$

Q.5. State and prove Cauchy integral theorem.

Q.6. Find Laurent Series expansion of $f(z) = \dfrac{z}{(z-1)(z-2)}$ in the region $0 \le |z-1| \le 1$.

Section C

[Attempt any 02 questions of 10 marks each]

Q.7. Consider the system of equations

$x + y - z = a$

$x - y + 2z = b.$

a) Find the general solution of the homogeneous equation.

b) A particular solution of the inhomogeneous equations when a = 1 and b = 2 is x = 1, y = 1, z = 1. Find the most general solution of the inhomogeneous equations.

c) Find some particular solution of the inhomogeneous equations when a = −1 and b = −2.

d) Find some particular solution of the inhomogeneous equations when a = 3 and b = 6. **(10 Marks)**

Q.8. (A) By integrating around a unit circle, evaluate

$$\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4\cos\theta} d\theta$$ **(05 Marks)**

(B) Let A be a square matrix. If the eigenvectors $v_1, \ldots, v_k$ have distinct eigenvalues, show that these vectors are linearly independent. **(05 Marks)**

Q.9. (A) Determine the poles of the function and residue at each pole for $f(z) = \dfrac{e^z}{z^3(z-2)}$. **(03 Marks)**

(B) Evaluate $\oint_{|z|=2} \dfrac{e^z}{z} dz$ and hence show that $\int_0^{2\pi} e^{\cos\theta} \cos(\sin\theta) d\theta = 2\pi$. **(03 Marks)**

(C) Evaluate using Cauchy Residue theorem $\oint_{|z|=2} \dfrac{z^2+2}{z^2(z-1)} dz$ **(04 Marks)**