Roll No.:.........................

National Institute of Technology, Delhi

Name of the Examination: B. Tech.

Branch	: ECE, EEE and CSE	Semester	: I
Title of the Course	: Advanced Calculus	Course Code	: MAL101

Time: 3 Hours | Maximum Marks: 50

Note: This question paper is divided into three sections A, B and C, and each section must be solved with rules given as follows:
Section A: Contains Ten (10) questions of 01 mark each, and all questions are compulsory.
Section B: Contains Five (05) questions of 5 marks each, and any four (04) are to be attempted.
Section C: Contains Three (03) questions of ten (10) marks each, and any two (02) are to be attempted.
Each part of the questions is of equal marks. Use of Scientific Calculator is not permitted.

Section A

Q. 1.

(*a*) Explain, on which interval $g(x)$ is continuous?

$$g(x)=\begin{cases}2x-4, & \text{if } x>2\\ 1, & \text{if } x=2\\ 2-x, & \text{if } x<2.\end{cases}$$

(*b*) Find two positive numbers whose product is 100 and whose sum is minimum.

(*c*) Given an example of functions f and g, both continuous at $x = 0$, for which the composite $f \circ g$ is discontinuous at $x=0$.

(*d*) Show that $a+1/a < b+1/b$, whenever $1<a<b$.

(e) Find the unit normal vector to the level surface $x^2y + 2xz = 4$ at the point (2, -2, 3).

(*f*) Prove that $curl\ \vec{r} = 0$.

(*g*) Let $a_n = \begin{cases} n/2^n, & \text{if } n \text{ is prime number,}\\ 1/2^n, & \text{otherwise,}\end{cases}$ does $\sum a_n$ converge? Give reason for your answer.

(*h*) Show that vector $V = (\sin y + z)i + (x\cos y - z)j + (x-y)k$ is irrotational.

(*i*) A function which is derivable at a point is necessarily continuous at that point.

(*j*) State the Green's theorem.

Section B

Q. 2. Explain, which of the following series converges or diverges?

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{(2n)!}{(n!)^2}$ (b) $\displaystyle\sum_{n=1}^{\infty}\frac{4^n(n!)^2}{(2n)!}$

(b) **Q. 3.** Calculate the area between the curve $y^2(a+x)=(a-x)^3$ and its asymptotes. Also draw the shaded region.

Q. 4. Show that $div(A\times B)=(B.\nabla)A-B\ div\ A-(A.\nabla)B+A\ div\ B$.

Q. 5. Evaluate $\iint_S F\cdot n\,dS$, where $F=yi+2xj-zk$ and S is the surface of the plane $2x+y=6$ in the first octant cut off by the plane $z=4$.

Q. 6. If $\lim_{n\to\infty} a_n = A \neq 0$. Prove that $|a_n| > \frac{1}{2}|A|$, for all N.

Section C

Q.7(a). Evaluate $\iiint_V \left[\frac{1-x-y-z}{xyz}\right]^{\frac{1}{2}} dxdydz$ by using Jacobian over the tetrahedron $x=0,\ y=0,\ z=0$, and $x+y+z=1$.

Q. 7 (b). Determine the area bounded by curve $xy=2,\ 4y=x^2$, and $y=4$ by using double integral.

Q. 8 . Verify the Gauss divergence theorem in the plane for $F=(x^2-yz)i+(y^2-zx)j+(z^2-xy)k$, taken over the rectangular parallelepiped $0\le x\le a,\ \ 0\le y\le b,\ \ 0\le z\le c$.

Q. 9. (a) Prove that series $\frac{1}{1.3}+\frac{1}{3.5}+\frac{1}{5.7}+\dots\dots\dots$ converges and also find its sum.

Q.9. (b) Show that the function $f(x,y)$ is continuous but not differential at origin.

$$f(x,y)=\begin{cases} x\sin\left(\frac{1}{x}\right)+y\sin\left(\frac{1}{y}\right), & xy\neq 0 \\ x\sin\left(\frac{1}{x}\right), & y=0,\ x\neq 0 \\ y\sin\left(\frac{1}{y}\right), & x=0,\ y\neq 0 \\ 0, & x=0, y=0 \end{cases}$$