

Roll No.:.....

# National Institute of Technology, Delhi

**Examination: B. Tech. Mid Semester Examination October 2023 (Autumn Semester)**

Branch : CSE, ECE, EEE

Semester : 1st

Title of the Course : Advanced Calculus

Course Code : MALB 101

Time: One and Half Hours

Maximum Marks: 25

Note : All sections are compulsory.

Course Outcomes: Student will be able to:		Cognitive Levels
C01	Understand the theory and methods of Differential, Integral and Vector Calculus	Understanding Level-II
C02	Apply different methods for solving problems in Differential, Integral and Vector Calculus	Applying Level-III
C03	Analyze sequence and series for its convergence. Analyse function for continuity and differentiability. Analyse curves and surfaces for concavity, inflection points, maxima and minima. Compare different integration techniques for finding area and volume.	Analyzing Level-IV
C04	Evaluate extreme points for function of several variables. Evaluate limits. Evaluate limit of sequences and sum of some convergent series. Evaluate multiple integrals in rectangular, polar, cylindrical, and spherical coordinates.	Evaluating Level-V
C05	Create power series. Formulate problems on maxima and minima. Combine vector differential calculus and vector integral calculus. Construct counter-examples for theorems and arguments. Formulate problems on integral and vector calculus.	Creating Level-VI

Q.No.	Question	Marks	CO	BL
1	It is given that for the function $f(x) = x^3 + bx^2 + ax$ , $x \in [1, 3]$ , the Rolle's theorem holds with $c = 2 + \frac{1}{\sqrt{3}}$ . Evaluate the values of $a$ and $b$ .	3	CO4	L5
2	Compute the interval where function is increasing or decreasing and locate the extreme values for the function $f(x) = (x - 2)^3(x - 3)^2$ .	3	CO2	L3
3	Analyse the ranges of values of $x$ for which the curve $y = x^4 - 6x^3 + 12x^2 + 4x + 10$ is concave up or down. Also, find inflection points (if any).	3	CO3	L4

4	<p>Let a function <math>f(x, y)</math> is defined as</p> $f(x, y) = \begin{cases} (x^2 + y) \sin \left( \frac{1}{x^2 + y^2} \right), & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$ <p>Evaluate <math>f_x(0,0)</math> and <math>f_y(0,0)</math> (if they exist), else prove that it does not exist.</p>	4	CO4	L5
5	<p>Analyse and sketch the curve for <math>f(x) = \frac{x^3 + 1}{x^2}</math> providing all features (including symmetry, increasing/decreasing, maxima/minima, concavity, inflections, asymptotes).</p>	4	CO3	L4
6	<p>Create the linearisation of the functions at <math>x = 0</math></p> <p>(a) <math>f(x) = \frac{1}{1 + \tan x}</math>      (b) <math>f(x) = \sqrt{1 + x} + \sin x - \frac{1}{2}</math></p>	4	CO5	L6
7	<p>Apply <math>\epsilon - \delta</math> definition to prove the limit or show that the limit does not exist (at origin)</p> <p>(a) <math>f(x, y) = \begin{cases} \frac{xy^2}{x^2 + y^2}, &amp; (x, y) \neq (0, 0) \\ 0, &amp; (x, y) = (0, 0) \end{cases}</math></p> <p>(b) <math>f(x, y) = \frac{y}{x^2 - y}</math>, where <math>y \neq x^2</math></p>	4	CO2	L3

