Roll No.....

# National Institute of Technology Delhi

**Name of the Examination: B.Tech.**

**Mid Sem Exam (2018-19)**

**Branch: All Branches** **Semester-I**

**Course Title: Advanced Calculus** **Course Code: MAL 101**

**Max Time: 2 hrs** **Total Marks: 25**

**Note: All questions are compulsory**

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1. Prove that $\lim_{x\to 2} f(x) = 4$ if

$$f(x) = \begin{cases} x^2, & x \neq 2 \\ 1, & x = 2. \end{cases}$$

[4 Marks]

2. (a) Let $f(x)$ and $g(x)$ be two functions where $f(x)$ is continuous at $x = 0$, and $g(x) = xf(x)$ for all $x \in \mathbb{R}$. Is $g(x)$ differentiable at $x = 0$? [2 Marks]

   (b) Let $f(x)$ be a function which is continuous on $[\alpha, \beta]$, and differentiable on $(\alpha, \beta)$. Further, let $f(\alpha) = \alpha$ and $f(\beta) = \beta$. Does there exist points $x_1$ and $x_2$ in $(\alpha, \beta)$ such that $\alpha < x_1 < \frac{\alpha+\beta}{2} < x_2 < \beta$ and $f'(x_1) + f'(x_2) = 2$?

   [3 Marks]

3. Let $f(x,y) = \begin{cases} \frac{x\sqrt{|y|}}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$

   (a) Does $\lim_{(x,y)\to(0,0)}$ exist?

   (b) Is $f(x,y)$ continuous at $(0,0)$?

   (c) Is $f(x,y)$ differentiable at $(0,0)$? [4 Marks]

4. Determine the global maximum and global minimum values of

$$f(x,y) = (x^2 - 4x)\cos y$$

over

$$D = \{(x,y) \in \mathbb{R}^2 | 1 \leq x \leq 3, \frac{-\pi}{4} \leq y \leq \frac{\pi}{4}\}.$$

[4 Marks]

5. Sketch the graph of the function $f(x) = \frac{x^2+5x+1}{x^2}$. [4 Marks]

6. Maximize the function $f(x,y,z) = x^2 + 2y - z^2$ subject to the constraints $2x - y = 0$ and $y + z = 0$. [4 Marks]