8.1.3

EE24BTECH11066 - YERRA AKHILESH

Question:
Find the area of the region bounded by $x^2 = 4y, y = 2, y = 4$ and the y-axis in the first quadrant.
Solution:
Theoritical Solution:
Express x in terms of y,

$$x = 2\sqrt{y} \tag{0.1}$$
$$(or) \tag{0.2}$$
$$x = -2\sqrt{y} \tag{0.3}$$

But we should take first equation as we have to find the area in the first quadrant,
Area under the curve is given by,

$$A = \int_2^4 2\sqrt{y}\,dy \tag{0.4}$$
$$A = 2\left(\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right)\Bigg|_2^4 \tag{0.5}$$
$$A = \frac{4}{3}\left(4^{\frac{3}{2}} - 2^{\frac{3}{2}}\right) \tag{0.6}$$
$$A = \frac{4}{3}\left(8 - 2\sqrt{2}\right) \tag{0.7}$$
$$A = 6.895 \tag{0.8}$$

Computational Solution: Taking trapezoid-shaped strips of small area and adding them all up. Say we have to find the area of y_x from $x = x_0$ to $x = x_n$, discretize the points on the x axis $x_0, x_1, x_2, \dots, x_n$ such that they are equally spaced with the step size h.
Sum of all trapezoidal areas is given by,

$$A = \frac{1}{2}h\left(y\left(x_1\right) + y\left(x_0\right)\right) + \frac{1}{2}h\left(y\left(x_2\right) + y\left(x_1\right)\right) + \dots + \frac{1}{2}h\left(y\left(x_n\right) + y\left(x_{n-1}\right)\right) \tag{0.9}$$
$$= h\left[\frac{1}{2}\left(y\left(x_0\right) + y\left(x_n\right)\right) + y\left(x_1\right) + \dots + y\left(x_{n-1}\right)\right] \tag{0.10}$$

Let $A\left(x_n\right)$ be the area enclosed by the curve $y\left(x\right)$ from $x = x_0$ to $x = x_n$, $\left(x_0, x_1, \dots x_n\right)$ be equidistant points with step-size h.

$$A\left(x_n + h\right) = A\left(x_n\right) + \frac{1}{2}h\left(y\left(x_n + h\right) + y\left(x_n\right)\right) \tag{0.11}$$

We can repeat this till we get the required area.
Discretizing the steps, making $A(x_n) = A_n, y(x_n) = y_n$ we get,

$$A_{n+1} = A_n + \frac{1}{2}h(y_{n+1} + y_n) \tag{0.12}$$

We can write y_{n+1} in terms of y_n using the first principle of derivative. $y_{n+1} = y_n + hy'_n$

$$A_{n+1} = A_n + \frac{1}{2}h((y_n + hy'_n) + y_n) \tag{0.13}$$

$$A_{n+1} = A_n + \frac{1}{2}h(2y_n + hy'_n) \tag{0.14}$$

$$A_{n+1} = A_n + hy_n + \frac{1}{2}h^2y'_n \tag{0.15}$$

$$x_{n+1} = x_n + h \tag{0.16}$$

In the given question, $y_n = \frac{x_n^2}{4}$ and $y'_n = \frac{x_n}{2}$.
The general difference equation will be given by

$$A_{n+1} = A_n + hy_n + \frac{1}{2}h^2y'_n \tag{0.17}$$

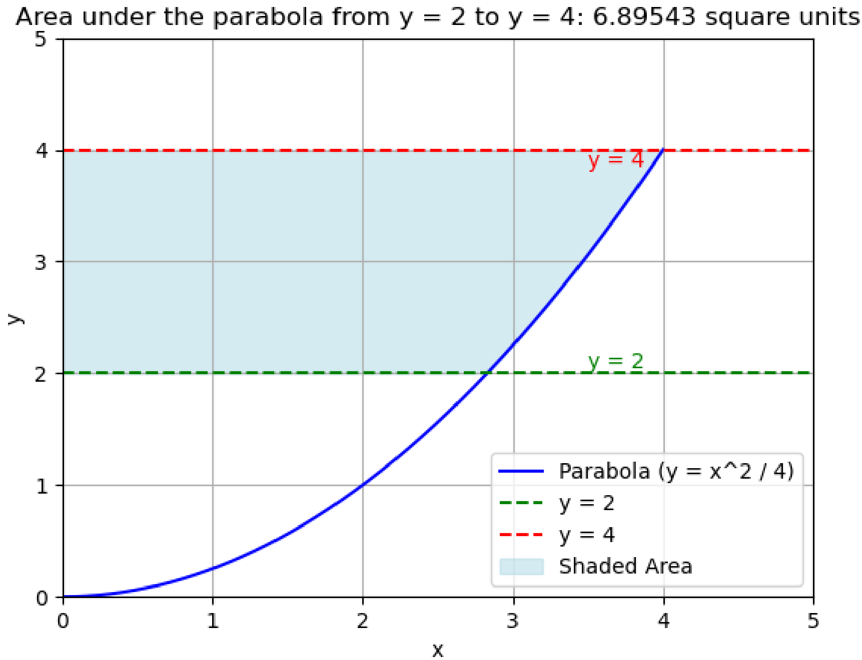
$$A_{n+1} = A_n + h\left(\frac{x_n^2}{4}\right) + \frac{1}{2}h^2\left(\frac{x_n}{2}\right) \tag{0.18}$$

$$x_{n+1} = x_n + h \tag{0.19}$$

Iterating till we reach $x_n = 4$ will return the required area.

Area obtained computationally: 6.895 sq. units.

Area obtained theoretically: $\frac{4}{3}\left(8 - 2\sqrt{2}\right) = 6.895$ sq.units


Area under the parabola from y = 2 to y = 4: 6.89543 square units
y = 4
y = 2
Parabola (y = x^2 / 4)
y = 2
y = 4
Shaded Area
x
y
0
1
2
3
4
5