6.5.2.2

EE24BTECH11066 - YERRA AKHILESH

Question:
Find the Maximum and Minimum values of the function

$$y(x) = -|x+1| + 3$$

Solution :
Threotical Solution :

As $y(x)$ is modular function, The vertex of the function is at the point where,

$$|x+1| = 0 \tag{0.1}$$
$$x = -1 \tag{0.2}$$

Substitute $x = -1$ in the function, gives :

$$y(-1) = 3 \tag{0.3}$$

Hence, the vertex is at $(-1, 3)$, which is the **maximum** point and it can be explained by ,

$$y = \begin{cases} -(x+1) + 3 & \text{if } x \geq -1, \\ (x+1) + 3 & \text{if } x < -1 \end{cases}$$

The derivative of y is:

$$\frac{dy}{dx} = \begin{cases} -1 & \text{if } x > -1, \\ 1 & \text{if } x < -1. \end{cases}$$

At $x = -1$, the derivative does not exist because of the abrupt change in slope. However, we can observe the behavior of the function:
- For $x < -1$, the derivative $\frac{dy}{dx} = 1 > 0$, indicating that the function is increasing.
- For $x > -1$, the derivative $\frac{dy}{dx} = -1 < 0$, indicating that the function is decreasing.

Therefore, **maximum** value of $y(x) = 3$ at $x = -1$

Now,
As $x \to \infty$ or $x \to -\infty$, the absolute value $|x+1| \to \infty$, and the negative of this term dominates. Thus:

$$y \to -\infty \tag{0.4}$$

This means the function decreases without bound, so there **is no minimum value.**

Computational solution :

Maximum value of the function can be done by **Gradient Ascent** method:

- **Choose a starting point** - x_0 away from $x = -1$.
- **Update the position iteratively**:

$$x_{n+1} = x_n + \eta \cdot \frac{dy}{dx}$$

  Here, $\eta = 0.01$, η is learning rate.
- **Behaviour in each region**:
  for $x > -1$: $\frac{dy}{dx} = -1$,

$$x_{n+1} = x_n - \eta \tag{0.5}$$

  This causes x_n to decrease toward $x = -1$.
  for $x < -1$: $\frac{dy}{dx} = 1$,

$$x_{n+1} = x_n + \eta \tag{0.6}$$

  This causes x_n to increase toward $x = -1$.
  At $x = -1$:
  the gradient changes direction abruptly, and the iteration stops because the function value is maximized at this point.

Minimum value of the function can be done by **Gradient decent** method:

Here,

$$x_{n+1} = x_n - \eta \cdot \frac{dy}{dx} \tag{0.7}$$

Similarly finding behaviour in each region:
for $x > -1$:

$$x_{n+1} = x_n + \eta \tag{0.8}$$

This causes x_n to increase indefinitely, moving away from $x = -1$.
for $x < -1$:

$$x_{n+1} = x_n - \eta \tag{0.9}$$

This causes x_n to decrease indefinitely, moving away from $x = -1$.

The function decreases without bound as $x \to \infty$ or $x \to -\infty$, so **gradient descent** will not converge to a minimum. The iteration will continue indefinitely. so, **No Minimum exists**

Computational results :

-Absolute Maximum

$$x \approx -1, \; y(x) \approx 3 \tag{0.10}$$

-No Absolute Minimum