10.4.1.2.4

EE24BTECH11066 - YERRA AKHILESH

Question:

A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Solution :

To solve the problem, let the speed of the train be x km/h. The given conditions are :

-The train travels 480 km at uniform speed x.
-If the speed is reduced by 8 km/h i.e $(x-8)$, the train would take 3 hours more to cover the same distance.

Time taken at speed x is,

$$t_1 = \frac{480}{x} \tag{0.1}$$

Time taken at speed $x-8$ is,

$$t_2 = \frac{480}{x-8} \tag{0.2}$$

using given conditions,

$$t_2 - t_1 = 3 \tag{0.3}$$

$$\frac{480}{x-8} - \frac{480}{x} = 3 \tag{0.4}$$

on simplifying,

$$\frac{480x - 480(x-8)}{x(x-8)} = 3 \tag{0.5}$$

$$480 \cdot 8 = 3x(x-8) \tag{0.6}$$

$$3x^2 - 24x - 3840 = 0 \tag{0.7}$$

$$x^2 - 8x - 1280 = 0 \tag{0.8}$$

We can solve the above equation using fixed point iterations. First we separate x, from the above equation and make an update equation of the below sort.

$$x = g(x) \implies x_{n+1} = g(x_n) \tag{0.9}$$

Applying the above update equation on our equation, we get

$$x_{n+1} = \frac{x_n^2 - 1280}{8} \tag{0.10}$$

Now we take an initial value x_0 and iterate the above update equation. But we realize that the updated values always approach infinity for any initial value.
Thus we will alternatively use **Newton's Method** for solving equations.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \tag{0.11}$$

Where we define $f(x)$ as,

$$f(x) = x^2 - 8x - 1280 \tag{0.12}$$

$$f'(x) = 2x - 8 \tag{0.13}$$

Thus, the new update equation is,

$$x_{n+1} = x_n - \frac{x_n^2 - 8x_n - 1280}{2x_n - 8} \tag{0.14}$$

Taking the initial guess as $x_0 = 39$, we can see that x_n converges with x as,

$$x = 40.014285714 \approx 40 \tag{0.15}$$

Alternatively, we can use the Secant method for solving equations.

$$x_{n+1} = x_n + f(x_n) \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} \tag{0.16}$$

Newton's method is very powerful but has the disadvantage that the derivative may sometimes be a far more difficult expression than $f(x)$ itself and its evaluation therefore it may be more computationally expensive. The secant's method is more computationally cheap as the equation of the derivative is avoided by taking 2 starting points.

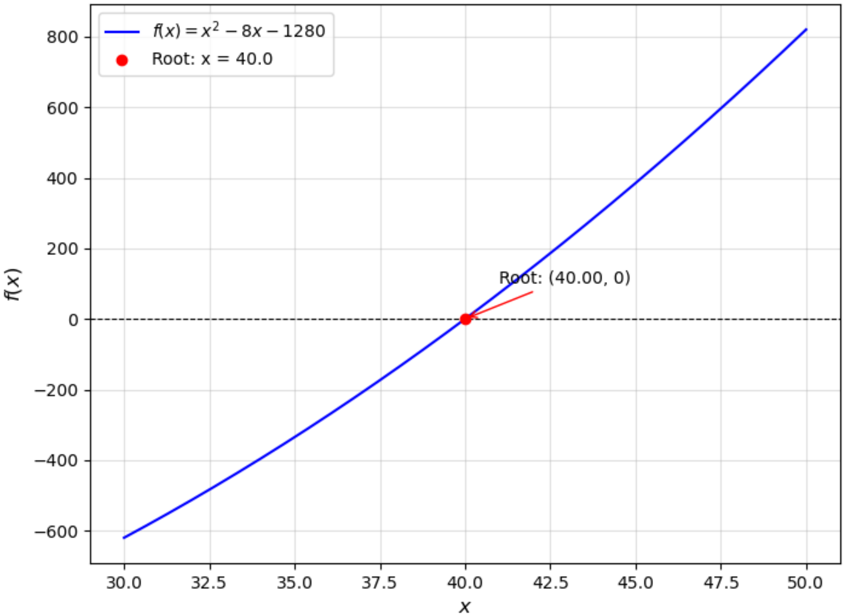

Fig. 0.1: Solution of the given function