

# 9.5.14

EE24BTECH11066 - YERRA AKHILESH

## Question:

For the following differential equation, find the particular solution satisfying the given condition:

$$\frac{dy}{dx} - \frac{y}{x} + \csc\left(\frac{y}{x}\right) = 0; y = 0 \text{ when } x = 1$$

**Solution:** Given Differential equation:

$$\frac{dy}{dx} - \frac{y}{x} + \csc\left(\frac{y}{x}\right) = 0 \quad (0.1)$$

let  $t$  be a variable,

$$\frac{y}{x} = t \quad (0.2)$$

$$y = xt \quad (0.3)$$

Differentiate on both sides with  $x$ ,

$$\frac{dy}{dx} = t + x \frac{dt}{dx} \quad (0.4)$$

Substitute the above  $\frac{dy}{dx}$  in given differential equation,

$$t + x \frac{dt}{dx} - t + \csc(t) = 0 \quad (0.5)$$

$$\frac{dt}{dx} = -\frac{\csc(t)}{x} \quad (0.6)$$

$$-\sin t \, dt = \frac{dx}{x} \quad (0.7)$$

Integrating on both sides,

$$\int -\sin t \, dt = \int \frac{1}{x} \, dx \quad (0.8)$$

$$\cos t = \log |cx| \quad (0.9)$$

where  $c$  be integrating constant and it can be obtained by substituting initial values ( $y = 0, x = 1$ )

On substituting,

$$c = e \quad (0.10)$$

Therefore final solution for  $y(x)$  by substituting  $t = \frac{y}{x}$ ,

$$\cos \frac{y}{x} = (1 + \log x) \quad (0.11)$$

$$y(x) = x \cdot \cos^{-1}(1 + \log x) \quad (0.12)$$

Now let us verify this computationally From definition of  $\frac{dy}{dx}$ ,

$$y_{n+1} = y_n + \frac{dy}{dx} \cdot h \quad (0.13)$$

(where  $h$  is small number tending to zero) From the differential equation given,

$$\frac{dy}{dx} = \frac{y}{x} - \csc \frac{y}{x} \quad (0.14)$$

By substituting 0.14 in 0.13,

$$y_{n+1} = y_n + \left( \frac{y_n}{x_n} - \csc \frac{y_n}{x_n} \right) \cdot h \quad (0.15)$$

By taking  $x_1 = 0.2$  and  $y_1 = 0.4361$  and  $h = 0.0032$  going till  $x = 1$  by iterating through the loop and finding  $y_2, y_3, \dots, y_{500}$  and plotting the graph, we can verify if the function we got by solving the differential equation mathematically.

