8.1.3

EE24BTECH11066 - YERRA AKHILESH

Question:
Find the area of the region bounded by $x^2 = 4y, y = 2, y = 4$ and the y-axis in the first quadrant.
Solution:
Theoritical Solution:
Express x in terms of y,

$$x = 2\sqrt{y} \tag{0.1}$$

$$(or) \tag{0.2}$$

$$x = -2\sqrt{y} \tag{0.3}$$

But we should take first equation as we have to find the area in the first quadrant,
Area under the curve is given by,

$$A = \int_2^4 2\sqrt{y}\,dy \tag{0.4}$$

$$A = 2\left(\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right)\Bigg|_2^4 \tag{0.5}$$

$$A = \frac{4}{3}\left(4^{\frac{3}{2}} - 2^{\frac{3}{2}}\right) \tag{0.6}$$

$$A = \frac{4}{3}\left(8 - 2\sqrt{2}\right) \tag{0.7}$$

$$A = 6.895 \tag{0.8}$$

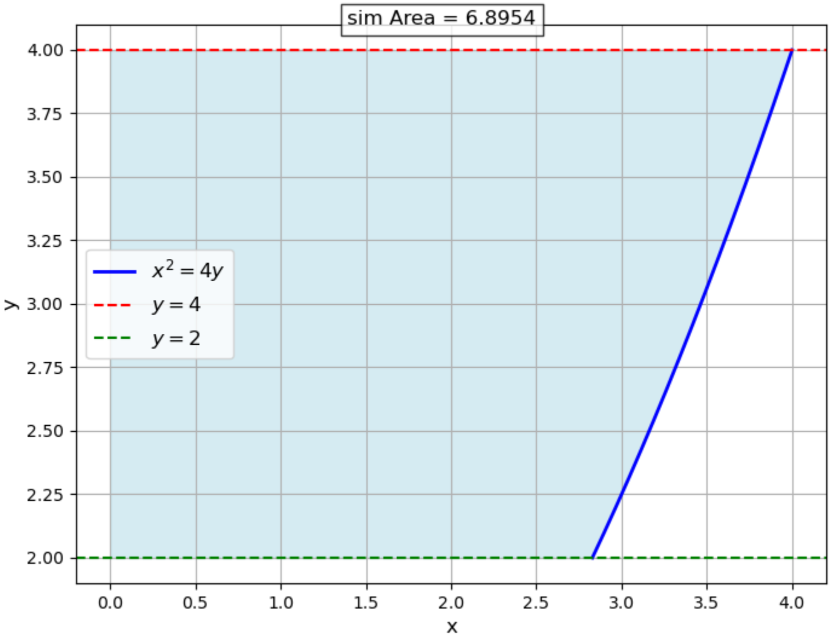
Computational Solution: Trapezoidal method
The difference equation for any general integral is as follows

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right] \tag{0.9}$$

For n=1000,

$$\int_2^4 f(y)\,dy \approx \frac{h}{2}\left[(2\sqrt{y_0}) + 2\sum_{i=1}^{999}(2\sqrt{y_i}) + (2\sqrt{y_{1000}})\right] \tag{0.10}$$

'y' varies from 2 to 4 and x varies accordingly.
The code sums up the required values iteratively for 'n'(say 1000) intervals
By computing it iteratively(computationally) we get area as 6.895 sq. units
Hence, the area we calculated theoretically is verified



sim Area = 6.8954
4.00
3.75
3.50
3.25
3.00
2.75
2.50
2.25
2.00
y
$x^2 = 4y$
$y = 4$
$y = 2$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
x