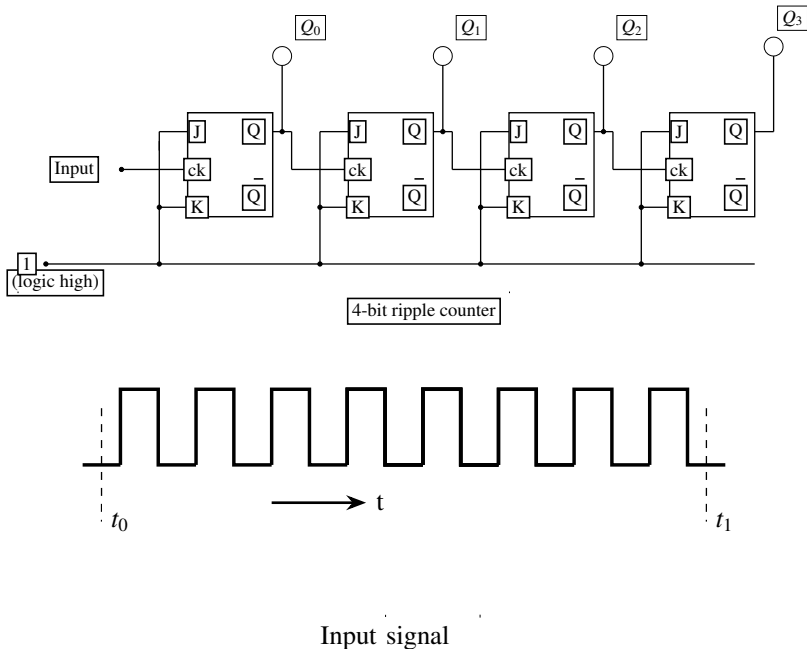


- 40) Consider a 4-bit counter constructed out of four flip-flops. It is formed by connecting the J and K inputs to logic high and feeding the Q output to the clock input of the following flip-flop (see the figure). The input signal to the counter is a series of square pluses and the change of state is triggered by the falling edge. At time $t = t_0$ the outputs are in logic low state ($Q_0 = Q_1 = Q_2 = Q_3 = 0$). Then at $t = t_1$, the logic state of the outputs is

[2020-PH]



- a) $Q_0 = 1, Q_1 = 0, Q_2 = 0, Q_3 = 0$
 b) $Q_0 = 0, Q_1 = 0, Q_2 = 0, Q_3 = 1$
 c) $Q_0 = 1, Q_1 = 0, Q_2 = 1, Q_3 = 0$
 d) $Q_0 = 0, Q_1 = 1, Q_2 = 1, Q_3 = 1$
- 41) Consider the Lagrangian $L = a\left(\frac{dx}{dt}\right)^2 + b\left(\frac{dy}{dt}\right)^2 + cxy$, where a, b and c are constants. If p_x and p_y are the momenta conjugate to the coordinates x and y respectively, then the Hamiltonian is

[2020-PH]

a) $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} - cxy$

b) $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} - cxy$

c) $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} + cxy$

d) $\frac{p_x^2}{a} + \frac{p_y^2}{b} + cxy$

42) Which one of the following matrices does NOT represent a proper rotation in a plane? [2020-PH]

a) $\begin{pmatrix} -\sin \theta & \cos \theta \\ -\cos \theta & -\sin \theta \end{pmatrix}$

b) $\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$

c) $\begin{pmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{pmatrix}$

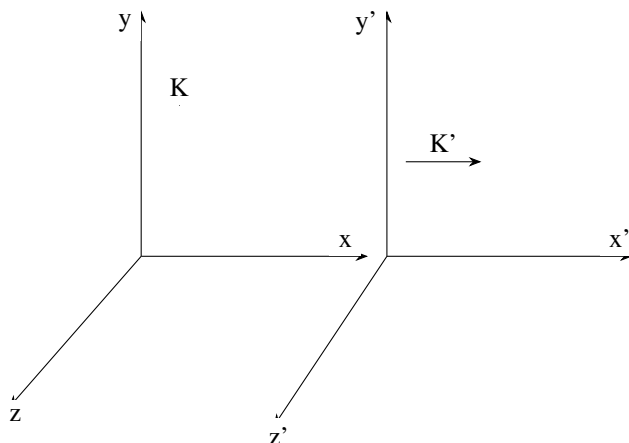
d) $\begin{pmatrix} -\sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{pmatrix}$

43) A uniform magnetic field $\vec{B} = B_0 \hat{y}$ exists in an inertial frame K . A perfect conducting sphere moves with a constant velocity $\vec{v} = v_0 \hat{x}$ with respect to this inertial frame. The rest frame of the sphere is K' (see figure). The electric and magnetic fields in K and K' are related as

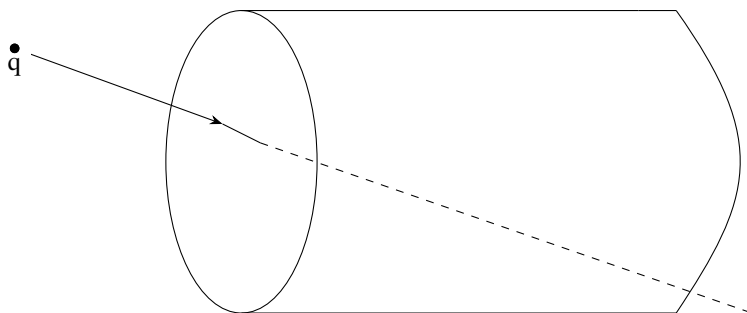
$$\begin{aligned} \vec{E}'_{\parallel} &= \vec{E}_{\parallel} & \vec{E}'_{\perp} &= \gamma \left(\vec{E}_{\perp} + \vec{v} \times \vec{B} \right) \\ \vec{B}'_{\parallel} &= \vec{B}_{\parallel} & \vec{B}'_{\perp} &= \gamma \left(\vec{B}_{\perp} - \frac{\vec{v}}{c^2} \times \vec{E} \right) \end{aligned}$$

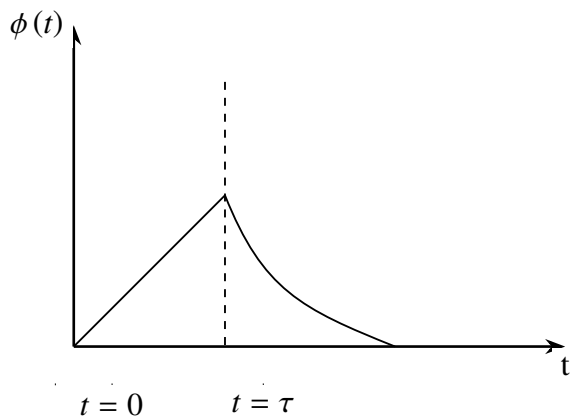
$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}.$$

The induced surface charge density on the sphere (to the lowest order in $\frac{v}{c}$) in the frame K' is [2020-PH]

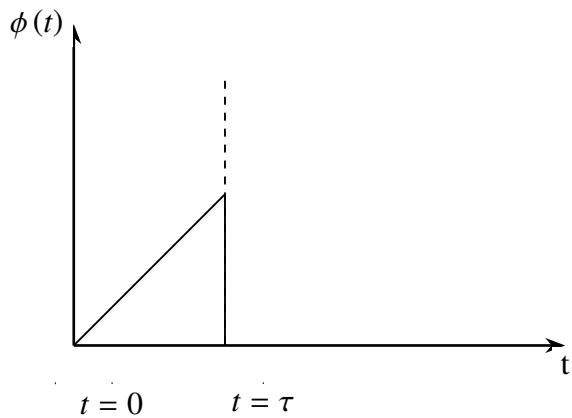


- a) maximum along z'
 - b) maximum along y'
 - c) maximum along x'
 - d) uniform over the sphere
- 44) A charge q moving with uniform speed enters a cylindrical region in free space at $t = 0$ and exits the region at $t = \tau$ (see figure). Which one of the following options best describes the time dependence of the total electric flux $\phi(t)$, through the entire surface of the cylinder?
- [2020-PH]

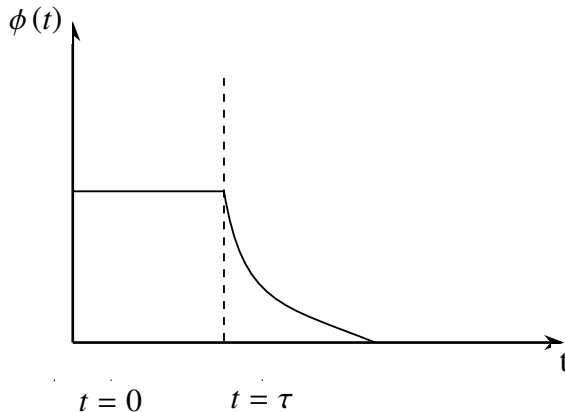




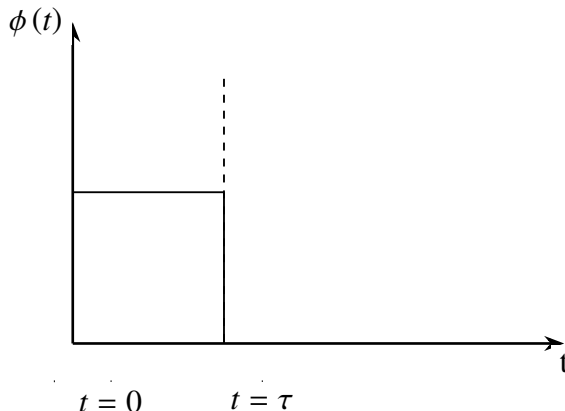
a)



b)



c)



d)

- 45) Consider a one-dimensional non-magnetic crystal with one atom per unit cell. Assume that the valence electrons (i) do not interact with each other and (ii) interact weakly with the ions. If n is the number of valence electrons per unit cell, then at 0 K, [2020-PH]

- the crystal is metallic for any value of n
 - the crystal is non-metallic for any value of n
 - the crystal is metallic for even values of n
 - the crystal is metallic for odd values of n
- 46) According to the Fermi gas model of the nucleus, the nucleons move in a spherical volume of radius $R = R_0 A^{\frac{1}{3}}$, where A is the mass number and R_0 is an empirical constant with the dimensions of length. The Fermi energy of the nucleus E_F is

proportional to

[2020-PH]

a) R_0^2

b) $\frac{1}{R_0}$

c) $\frac{1}{R_0^2}$

d) $\frac{1}{R_0^3}$

- 47) Consider a two dimensional crystal with 3 atoms in the basis. The number of allowed optical branches (n) and acoustic branches (m) due to the lattice vibrations are [2020-PH]

a) $(n, m) = (2, 4)$

b) $(n, m) = (3, 3)$

c) $(n, m) = (4, 2)$

d) $(n, m) = (1, 5)$

- 48) The internal energy U of a system is given by $U(S, V) = \lambda V^{\frac{2}{3}} S^2$, where λ is a constant of appropriate dimensions; V and S denote the volume and entropy, respectively. Which one of the following gives the correct equation of state of the system? [2020-PH]

a) $\frac{PV^{\frac{1}{3}}}{T^2} = \text{constant}$

b) $\frac{PV}{T^{\frac{2}{3}}} = \text{constant}$

c) $\frac{P}{V^{\frac{1}{3}} T} = \text{constant}$

d) $\frac{PV^{\frac{2}{3}}}{T} = \text{constant}$

- 49) The potential energy of a particle of mass m is given by $U(x) = a \sin\left(k^2 x - \frac{\pi}{2}\right)$, $a > 0, k^2 > 0$.

The angular frequency of small oscillations of the particle about $x = 0$ is [2020-PH]

a) $k^2 \sqrt{\frac{2a}{m}}$

b) $k^2 \sqrt{\frac{a}{m}}$

c) $k^2 \sqrt{\frac{a}{2m}}$

d) $2k^2 \sqrt{\frac{a}{m}}$

- 50) The radial wave function of a particle in a central potential is given by $R(r) = A \frac{r}{a} \exp\left(-\frac{r}{2a}\right)$, where A is the normalization constant and a is positive constant of suitable dimensions. If γa is the most probable distance of the particle from the force center, the value of γ is _____. [2020-PH]
- 51) A free particle of mass M is located in a three-dimensional cubic potential well with impenetrable walls. The degeneracy of the fifth excited state of the particle is _____. [2020-PH]
- 52) Consider the circuit given in the figure. Let the forward voltage drop across each diode be 0.7 V. The current I (in mA) through the resistor is _____. [2020-PH]

