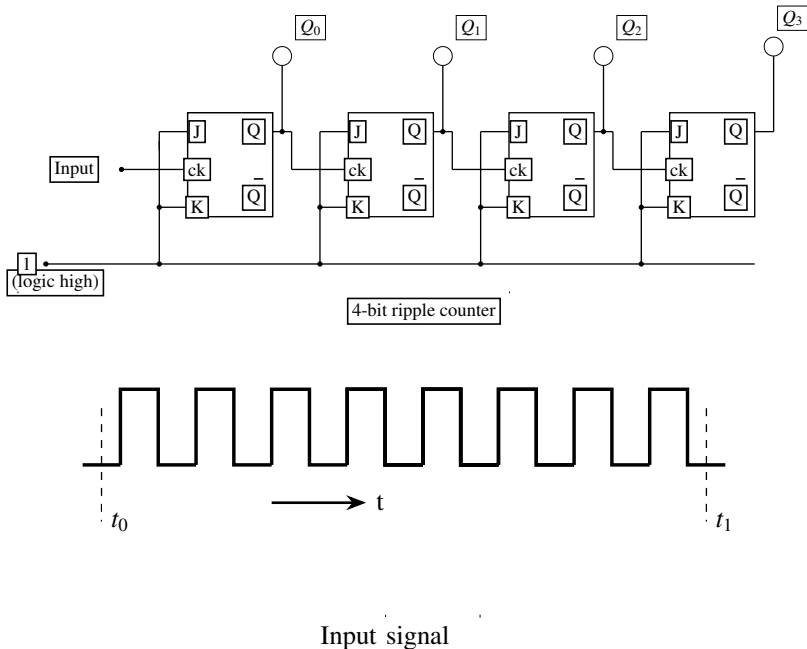


- 40) Consider a 4-bit counter constructed out of four flip-flops. It is formed by connecting the J and K inputs to logic high and feeding the  $Q$  output to the clock input of the following flip-flop (see the figure). The input signal to the counter is a series of square pluses and the change of state is triggered by the falling edge. At time  $t = t_0$  the outputs are in logic low state ( $Q_0 = Q_1 = Q_2 = Q_3 = 0$ ). Then at  $t = t_1$ , the logic state of the outputs is [2020-PH]



- a)  $Q_0 = 1, Q_1 = 0, Q_2 = 0, Q_3 = 0$   
 b)  $Q_0 = 0, Q_1 = 0, Q_2 = 0, Q_3 = 1$   
 c)  $Q_0 = 1, Q_1 = 0, Q_2 = 1, Q_3 = 0$   
 d)  $Q_0 = 0, Q_1 = 1, Q_2 = 1, Q_3 = 1$
- 41) Consider the Lagrangian  $L = a\left(\frac{dx}{dt}\right)^2 + b\left(\frac{dy}{dt}\right)^2 + cxy$ , where  $a, b$  and  $c$  are constants. If  $p_x$  and  $p_y$  are the momenta conjugate to the coordinates  $x$  and  $y$  respectively, then the Hamiltonian is [2020-PH]

a)  $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} - cxy$

b)  $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} - cxy$

c)  $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} + cxy$

d)  $\frac{p_x^2}{a} + \frac{p_y^2}{b} + cxy$

42) Which one of the following matrices does NOT represent a proper rotation in a plane? [2020-PH]

a)  $\begin{pmatrix} -\sin \theta & \cos \theta \\ -\cos \theta & -\sin \theta \end{pmatrix}$

b)  $\begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$

c)  $\begin{pmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{pmatrix}$

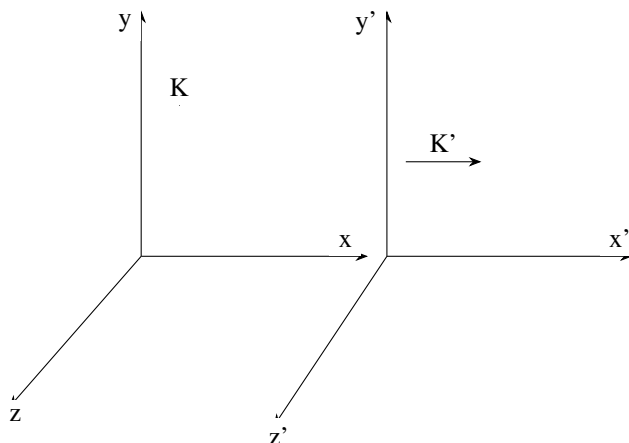
d)  $\begin{pmatrix} -\sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{pmatrix}$

43) A uniform magnetic field  $\vec{B} = B_0 \hat{y}$  exists in an inertial frame  $K$ . A perfect conducting sphere moves with a constant velocity  $\vec{v} = v_0 \hat{x}$  with respect to this inertial frame. The rest frame of the sphere is  $K'$  (see figure). The electric and magnetic fields in  $K$  and  $K'$  are related as

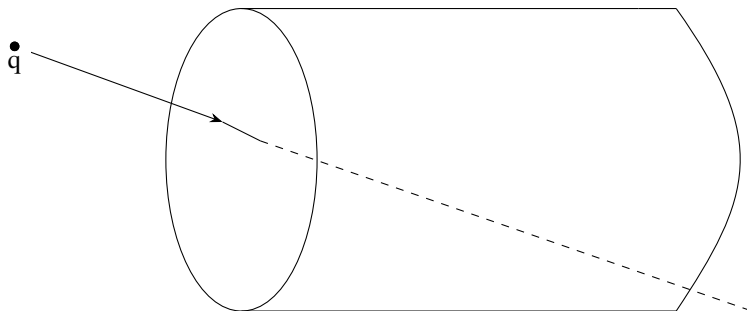
$$\begin{aligned} \vec{E}'_{\parallel} &= \vec{E}_{\parallel} & \vec{E}'_{\perp} &= \gamma \left( \vec{E}_{\perp} + \vec{v} \times \vec{B} \right) \\ \vec{B}'_{\parallel} &= \vec{B}_{\parallel} & \vec{B}'_{\perp} &= \gamma \left( \vec{B}_{\perp} - \frac{\vec{v}}{c^2} \times \vec{E} \right) \end{aligned}$$

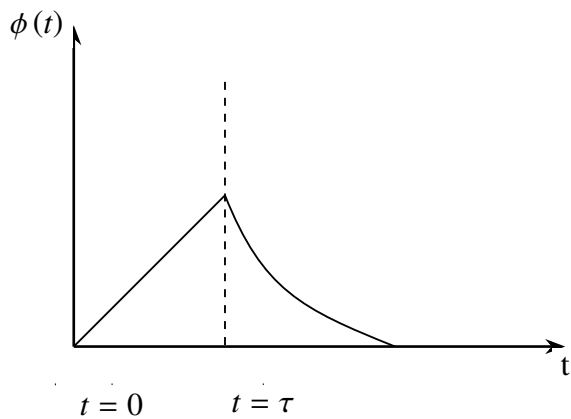
$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}.$$

The induced surface charge density on the sphere (to the lowest order in  $\frac{v}{c}$ ) in the frame  $K'$  is [2020-PH]

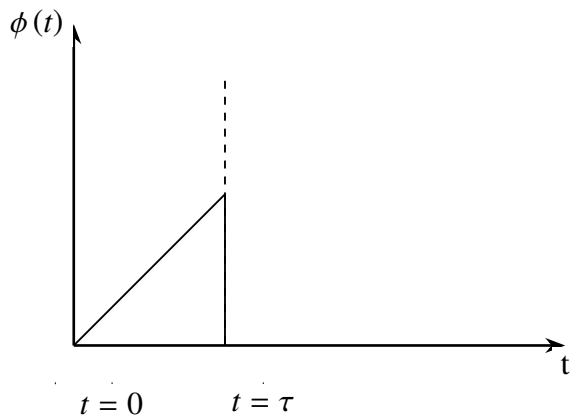


- a) maximum along  $z'$
  - b) maximum along  $y'$
  - c) maximum along  $x'$
  - d) uniform over the sphere
- 44) A charge  $q$  moving with uniform speed enters a cylindrical region in free space at  $t = 0$  and exits the region at  $t = \tau$  (see figure). Which one of the following options best describes the time dependence of the total electric flux  $\phi(t)$ , through the entire surface of the cylinder?
- [2020-PH]

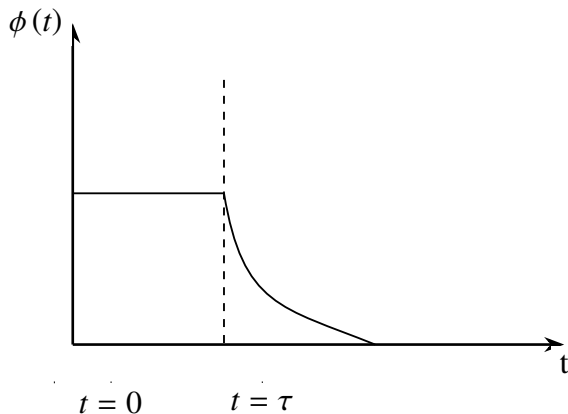




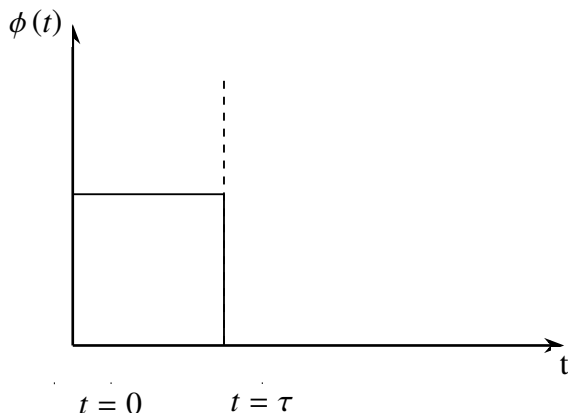
a)



b)



c)



d)

- 45) Consider a one-dimensional non-magnetic crystal with one atom per unit cell. Assume that the valence electrons (i) do not interact with each other and (ii) interact weakly with the ions. If  $n$  is the number of valence electrons per unit cell, then at 0 K, [2020-PH]

- a) the crystal is metallic for any value of  $n$
- b) the crystal is non-metallic for any value of  $n$
- c) the crystal is metallic for even values of  $n$
- d) the crystal is metallic for odd values of  $n$

- 46) According to the Fermi gas model of the nucleus, the nucleons move in a spherical volume of radius  $R (= R_0 A^{\frac{1}{3}}$ , where  $A$  is the mass number and  $R_0$  is an empirical constant with the dimension

The Fermi energy of the nucleus  $E_F$  is proportional to

[2020-PH]

a)  $R_0^2$

b)  $\frac{1}{R_0}$

c)  $\frac{1}{R_0^2}$

d)  $\frac{1}{R_0^3}$

47) Consider a two dimensional crystal with 3 atoms in the basis. The number of allowed optical branches ( $n$ ) and acoustic branches ( $m$ ) due to the lattice vibrations are [2020-PH]

a)  $(n, m) = (2, 4)$

b)  $(n, m) = (3, 3)$

c)  $(n, m) = (4, 2)$

d)  $(n, m) = (1, 5)$

48) The internal energy  $U$  of a system is given by  $U(S, V) = \lambda V^{\frac{2}{3}} S^2$ , where  $\lambda$  is a constant of appropriate dimensions;  $V$  and  $S$  denote the volume and entropy, respectively. Which one of the following gives the correct equation of state of the system? [2020-PH]

a)  $\frac{PV^{\frac{1}{3}}}{T^2} = \text{constant}$

b)  $\frac{PV}{T^{\frac{2}{3}}} = \text{constant}$

c)  $\frac{P}{V^{\frac{1}{3}}T} = \text{constant}$

d)  $\frac{PV^{\frac{2}{3}}}{T} = \text{constant}$

49) The potential energy of a particle of mass  $m$  is given by  $U(x) = a \sin\left(k^2 x - \frac{\pi}{2}\right)$ ,  $a > 0, k^2 > 0$ .

The angular frequency of small oscillations of the particle about  $x = 0$  is [2020-PH]

a)  $k^2 \sqrt{\frac{2a}{m}}$

b)  $k^2 \sqrt{\frac{a}{m}}$

c)  $k^2 \sqrt{\frac{a}{2m}}$

d)  $2k^2 \sqrt{\frac{a}{m}}$

- 50) The radial wave function of a particle in a central potential is given by  $R(r) = A \frac{r}{a} \exp\left(-\frac{r}{2a}\right)$ , where  $A$  is the normalization constant and  $a$  is positive constant of suitable dimensions. If  $\gamma a$  is the most probable distance of the particle from the force center, the value of  $\gamma$  is \_\_\_\_\_. [2020-PH]
- 51) A free particle of mass  $M$  is located in a three-dimensional cubic potential well with impenetrable walls. The degeneracy of the fifth excited state of the particle is \_\_\_\_\_. [2020-PH]
- 52) Consider the circuit given in the figure. Let the forward voltage drop across each diode be 0.7 V. The current  $I$  (in mA) through the resistor is \_\_\_\_\_. [2020-PH]

