Transient Response Analysis of an LC Circuit

Lab Assignment : 04

EE1200: Electrical Circuits Lab

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1 Experiment Objectives

- To study the transient response of an LC circuit and observe its oscillatory nature.
- To determine the natural frequency (f_n) of the circuit using theoretical and experimental methods.
- To compare experimental results with theoretical predictions and understand discrepancies.

2 Theory

An LC circuit, consisting of an inductor (L) and a capacitor (C), exhibits oscillatory behavior due to energy exchange between the magnetic field of the inductor and the electric field of the capacitor. The transient response of an LC circuit provides insight into its natural frequency (f_n) and damping characteristics, which are crucial in designing resonant circuits, filters, and oscillators.

$$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0 \tag{0.1}$$

where q is the charge on the capacitor. The oscillation frequency is given by:

$$f_n = \frac{1}{2\pi\sqrt{LC}} \tag{0.2}$$

When resistance is present, damping occurs, and the damping ratio (ξ) is calculated as:

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}} \tag{0.3}$$

3 Components Used

Oscilloscope

- A key instrument to measure voltage variations and observe the transient response of the LC circuit.
- Used for capturing for capturing transient signals.

Inductor

- A passive component that stores energy in its magnetic field when current flows through it.
- Used in the LC circuit to create oscillations when paired with a capacitor.

Capacitor

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- A passive component that stores electrical energy in its electric field.
- When charged and connected to an inductor, it forms a resonant circuit.

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Breadboard and Connecting Wires

- Facilitates the assembly and reconfiguration of the LC circuit.
- Provides an efficient setup for testing transient responses under different conditions.

Power Source

- Used to initially charge the capacitor before disconnecting it to observe the transient response.
- Ensures stable and consistent power delivery for accurate measurements.

Multimeter

- To check the voltage between the capacitor during charging
- Can also verify circuit connections and measure voltage manually

4 Procedure

Preparing the Capacitor

- Connect the 4.7 nF capacitor to a 5 V DC power supply.
- Allow it to fully charge, then carefully disconnect it to retain its charge.

Setting Up the LC Circuit

- Take the charged capacitor and connect it in parallel with the inductor.
- Ensure that the connections are firm and that there is minimal resistance in the wiring.

Observing the Oscillations

- Use an oscilloscope to measure the voltage across the capacitor.
- Observe how the voltage oscillates over time, forming a damped or undamped sinusoidal waveform depending on resistance.

Theoretical Analysis

- Calculate the expected oscillation frequency using $f_n = \frac{1}{2\pi\sqrt{LC}}$.
- If a resistor is present, determine the damping ratio (ξ) and classify the system as underdamped, critically damped, or overdamped.

Comparing Experimental and Theoretical Values

- Extract frequency measurements from the oscilloscope data.
- Compare the observed values with the calculated theoretical values.

Introducing Resistance (Optional)

- Insert a small resistor in series to observe its effect on damping.
- Measure how quickly the oscillations decay and record the results.

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5 Theoretical calculation

Taken values

- $L = 2.2mH$
- $C = 4.7nF$

5.1 Resonant Frequency

Resonant Frequency

$$f_n = \frac{1}{2\pi\sqrt{LC}}$$

On substituting we get

$$f_n = \frac{1}{2\pi\sqrt{10.34 \times 10^{-12}}}$$

$$f_n = \frac{1}{2\pi \times 3.217 \times 10^{-6}}$$

$$f_n = \frac{3.11 \times 10^5}{2\pi}$$

$$f_n = 49.5kHz$$

Thus, the theoretical resonant frequency is approximately 49.5 kHz.

5.2 Voltage across capacitor

The voltage across the capacitor is given by

$$V_c(t) = V_0 e^{-\alpha t}\cos(2\pi f_d t)$$

Assuming practical conditions there is some resistance so we consider damping condition theoretically

- $V_0 = 5$
- $\alpha = \frac{R}{2L}$
- $f_d = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - (\frac{R}{2L})^2}$

R is the resistance of the combined setup including all the components and wires which is unknown

5.3 Finding Natural Frequency

In a damped oscillatory system, the natural frequency (f_n) is related to the damped frequency (f_d) and the damping ratio (ζ). The relationship is given by:

$$f_n = \frac{f_d}{\sqrt{1-\zeta^2}}$$

1. Measurement of Damped Frequency

The damped frequency is determined by measuring the time period of oscillations using an oscilloscope:

$$f_d = \frac{1}{T_d}$$

where T_d is the time period of the damped oscillations.

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2. Determination of the Damping Ratio

The damping ratio (ζ) is calculated using the logarithmic decrement (δ), which is given by:

$$\delta = \ln\left(\frac{V_n}{V_{n+1}}\right)$$

where V_n and V_{n+1} are the peak amplitudes of two successive oscillations. The damping ratio is then computed as:

$$\zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$$

3. Calculation of Natural Frequency

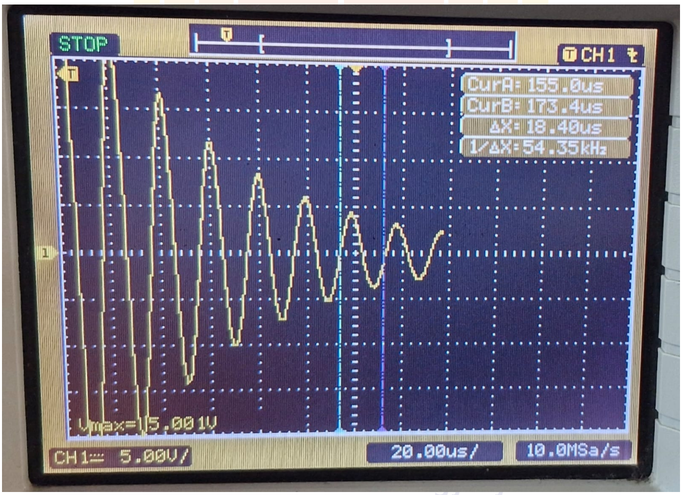
Using the calculated values of f_d and ζ, the natural frequency (f_n) is obtained from:

$$f_n = \frac{f_d}{\sqrt{1 - \zeta^2}}$$

For lightly damped systems (ζ is small), the natural frequency is approximately equal to the damped frequency:

$$f_n \approx f_d$$

6 Observation



The above are the readings obtained on the CRO for the given values of L and C

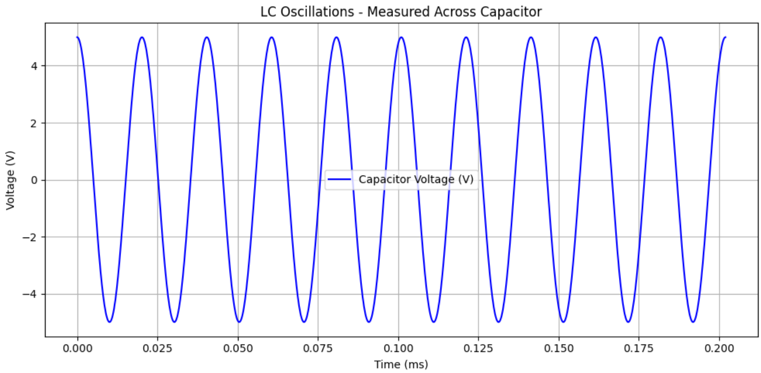
- $V_{max} = 5.001V$
- $T = 18.40\mu s$

- $f = 54.35kHz$

The value of the frequency obtained theoretically is 49.5kHz
The value of frequency obtained experimentally is 54.35kHz
Which matches approximately
Note that the error is due to the resistance between the wires, connecting probes and various other practical factors

7 Verification using Python

Assuming ideal conditions without resistance the LC oscillations would look like



Ideal case is not possible practically as there are resistances for the connecting wires and probes.
We measure the damped oscillations assuming a resistance of 50Ω
The damping factor for 50Ω would be 0.0365