Lissajous Figures and Capturing One-Time Events on a CRO

Indian Institute of Technology Hyderabad

Lab Assignment : 01

EE1200: Electrical Circuits Lab

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Contents

1 Experiment Objectives

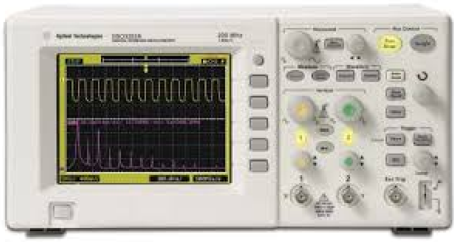
- To Plot at least 6 Lissajous figures on the oscilloscope.
- To Justify the pattern you see on CRO with theory.
- How do you capture one-time event on CRO with an example

2 Materials Used

1. Oscilloscope (CRO): To display the Lissajous figures and observe the patterns.
2. Function Generator: To provide the input signals with adjustable frequencies and amplitudes.
3. Probes: To connect the function generator to the oscilloscope.
4. Data Recorder: To capture the one-time event displayed on the oscilloscope screen.

2.1 ***Oscilloscope (CRO)***

An oscilloscope is an electronic test instrument used to visualize and analyze the waveform of electrical signals. It graphically displays how the voltage of a signal changes over time, allowing for the inspection of signal characteristics such as amplitude, frequency, waveform shape, and noise.



2.1.1 Purpose

Displaying the Lissajous Figures

- The resulting Lissajous figures are intricate patterns that depend on the frequency and phase relationship between the two signals. The oscilloscope plots the waveform on its screen, displaying the figure in real time.
- By adjusting the frequency and phase difference between the two signals, you can observe different Lissajous patterns (ellipses, circles, straight lines, etc.). The oscilloscope helps you see how the shape of the Lissajous figure changes with these variations.
- One of the critical aspects of Lissajous figures is the phase difference between the two input signals. By adjusting the phase difference on the function generator, you can observe how it affects the shape of the figure on the oscilloscope. This helps to understand the theoretical concept of phase in waveforms.

2.1.2 Role in Experiment

- The X-input controls the horizontal deflection (time or an external signal).
- The Y-input controls the vertical deflection (voltage amplitude or an external signal).
- By applying two signals of varying frequency and phase to the X and Y channels, the CRO generates Lissajous patterns.
- The trigger feature allows the display to stabilize or capture one-time events (e.g., a transient waveform).

2.2 *Function Generator*

Generating Sinusoidal Signals
A function generator is an electronic instrument used to create various types of electrical waveforms, typically in the form of sine, square, triangular, and other periodic signals. These waveforms are adjustable in terms of frequency, amplitude, and waveform shape, allowing for precise control over the signal parameters.

2.2.1 Purpose

- The function generator allows you to adjust the frequencies independently, enabling you to observe how different frequency relationships affect the Lissajous figures.
- The function generator controls the amplitude (peak voltage) of the output signals.
- The function generator allows for the adjustment of the phase between the two signals.
- In experiments where one-time or transient waveforms need to be captured, the function generator can generate pulse signals or sudden events.

2.2.2 Role in Experiment

- Generates two signals with adjustable frequencies, amplitudes, and phase differences.
- One signal is sent to the X-input (horizontal), and the other to the Y-input (vertical) of the CRO to create Lissajous figures.
- It ensures precise control over signal characteristics, which directly affect the shapes of the figures.

2.3 *Probes*

2.3.1 *Purpose*

- Connects the function generator outputs to the oscilloscope inputs.

2.3.2 *Role in Experiment*

- Transmits the generated signals to the oscilloscope without distortion.
- Ensures the signals retain their original characteristics (amplitude, phase, and frequency).
- Some probes offer attenuation (e.g., 10x), which is useful for reducing signal amplitude if needed.

Feature	**1x Probe**	**10x Probe**
Attenuation Factor	No attenuation (1:1)	10 times attenuation (10:1)
Signal Voltage Range	Limited to the oscilloscope's input range	Allows measurement of higher voltages without damaging the scope
Sensitivity	More sensitive to small signals	Less sensitive, suitable for larger signals
Loading Effect	Higher capacitive loading on the circuit	Lower capacitive loading on the circuit
Voltage Division	Does not attenuate the input signal	Reduces the signal amplitude by 10x before entering the oscilloscope

2.4 *Data Recorder*

2.4.1 *Purpose*

- Captures and stores the observed waveforms or figures for analysis.

2.4.2 *Role in Experiment*

- Records one-time events or transient waveforms displayed on the oscilloscope.
- Useful for documenting results and comparing them with theoretical predictions.

3 Lissajous figures

Lissajous figures are complex, looped patterns displayed on an oscilloscope screen when two sinusoidal signals are applied to its horizontal (X-axis) and vertical (Y-axis) inputs. These figures are used to analyze the relationship between the two signals, such as their frequency, amplitude, and phase difference.

3.1 *Applications of Lissajous Figures:*

3.1.1 *Phase difference measurement*

- The shape and orientation of the figure indicate the phase difference between the two signals.

3.1.2 Frequency Comparison:

- By observing the pattern, one can determine the frequency ratio between the two signals.

3.1.3 Signal Analysis:

- Used in electronics and physics labs to analyze signal properties.

Lissajous figures are a powerful visual tool for understanding signal relationships and remain widely used in signal analysis and testing environments.

4 Procedure

4.1 Connect the components

- Connect the oscilloscope's X and Y inputs to output channels of the function generator
- Ensure that the triggering is set to Normal to stabilize the waveform
- Set the oscilloscope's time base and voltage scale to appropriate levels based on signal characteristics

5 Lissajous figures : Examples

5.1 Pattern 1

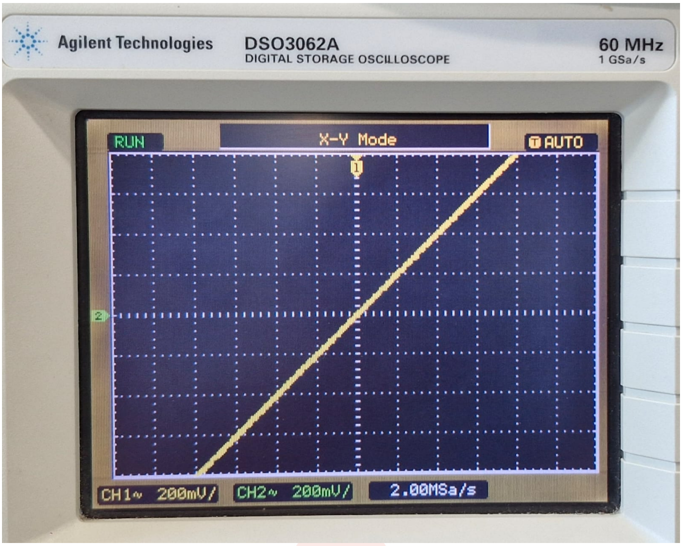
Observation and Analysis: Sinusoidal Signals

Both the signals were sinusoidal and the following conditions were applied and observed:

1) **Frequency Ratio** ($f_x : f_y$): Both sinusoidal signals were set to have the same frequency, i.e.,
$$f_x : f_y = 1 : 1.$$
This ensured the periodicity of the signals was synchronized.
2) **Phase Difference** (ϕ):
We set the phase difference equal to 0, which resulted in a straight line
    - At $\phi = 0°$ or $180°$, the figure collapsed into a straight line.
3) **Amplitudes** (A_x, A_y):
The amplitudes of both signals are set to be the same for the ease of analysis.
    - $A_x = A_y$

Input Signals

Paramters	**Channel - 1**	**Channel - 2**
Frequency	200.00 Hz	200.00 Hz
Phase	0°	0°
High	5.00 V	5.00 V
Low	-5.00 V	-5.00 V



5.1.1 Mathematical Proof

Signals:

$$x(t) = A_x \sin(\omega t), \quad y(t) = A_y \sin(\omega t + \phi)$$

According to the given conditions

- $\phi = 0$
- $A_x = A_y = A$

$$x(t) = A \sin(\omega t), \quad y(t) = A \sin(\omega t)$$

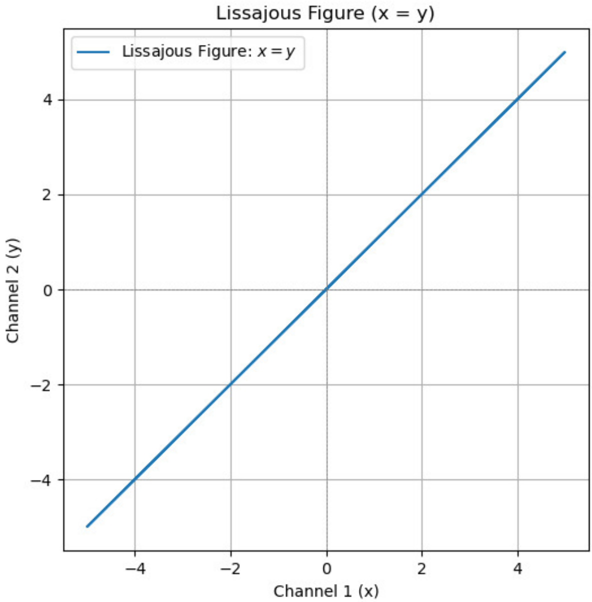
Result
It is clear that

$$x(t) = y(t)$$

The graph on the CRO is $x = y$

5.1.2 Python Plotting

To ensure that the observed reading and the mathematical proofs are correct we run a matlab code and cross check the figure obtained with the one on the CRO



5.2 Pattern 2

Observation and Analysis: Sinusoidal Signals

Both the signals were taken to be sinusoidal and the following conditions were applied

1) **Frequency Ratio** (f_x : f_y): Both sinusoidal signals were set to have the same frequency, i.e.,
$$f_x : f_y = 1 : 1.$$
This ensured the periodicity of the signals was synchronized.
2) **Phase Difference** (ϕ):
We set the phase difference equal to 90°
   - At $\phi = 90°$, the figure collapsed into a circle.
3) **Amplitudes** (A_x, A_y):
Again, the amplitudes of both signals are set to be the same for the ease of analysis.
   - $A_x = A_y$

Input Signals

Paramters	**Channel - 1**	**Channel - 2**
Frequency	200.00 Hz	200.00 Hz
Phase	90°	0°
High	5.00 V	5.00 V
Low	-5.00 V	-5.00 V

5.2.1 *Mathematical Proof*

Signals:

$$x(t) = A_x \sin(\omega t), \quad y(t) = A_y \sin(\omega t + \phi)$$

According to the given conditions

- $\phi = 90^\circ$
- $A_x = A_y = A$

$$x(t) = A\sin(\omega t), \quad y(t) = A\sin(\omega t + 90)$$

$$y(t) = A\sin(\omega t + 90) = A\cos\omega t$$

The parametric equations now describe a circle

$$\frac{x^2}{A^2} + \frac{y^2}{A^2} = \cos^2(\omega t) + \sin^2(\omega t) = 1$$

Since the Amplitude $A = 5$, the radius of the Lissajous figure is

$$\text{Radius} = 5 = A$$

Result:

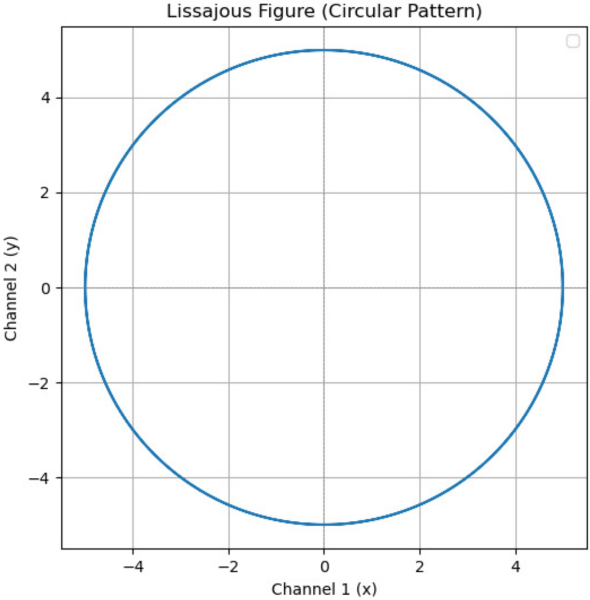
The equation of the cicle is

$$\frac{x^2}{25} + \frac{y^2}{25} = \cos^2(\omega t) + \sin^2(\omega t) = 1$$

The graph on the CRO is $x^2 + y^2 = 25$

5.2.2 *Python Plotting*

To ensure that the observed reading and the mathematical proofs are correct we run a matlab code and cross check the figure obtained with the one on the CRO



5.3 Pattern 3

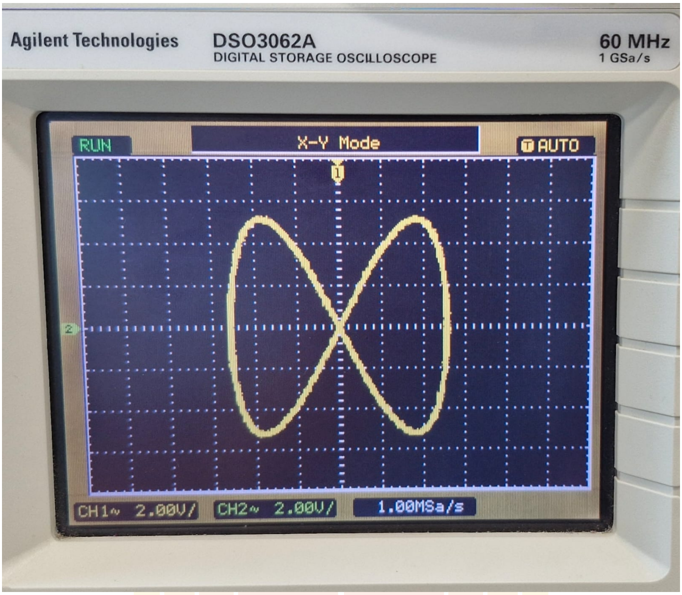
Observation and Analysis: Sinusoidal Signals

Both the signals were taken to be sinusoidal for ease of mathematical proof and the following conditions were applied

1) **Frequency Ratio** ($f_x : f_y$):
Both the signals are sinusoidal and the frequencies were set to be different in the ratio, i.e.,
$$f_x : f_y = 1 : 2$$
This ensured that the signals maintained a consistent relationship over the time.
2) **Phase Difference** (ϕ):
We set the phase difference equal to 0°
   - At $\phi = 0°$, the figure collapsed into a bowtie .
3) **Amplitudes** (A_x, A_y):
Again, the amplitudes of both signals are set to be the same for the ease of analysis.
   - $A_x = A_y$

Input Signals

Paramters	**Channel - 1**	**Channel - 2**
Frequency	400.00 Hz	800.00 Hz
Phase	0°	0°
High	5.00 V	5.00 V
Low	-5.00 V	-5.00 V



5.3.1 Mathematical Proof

Signals:

$$x(t) = A_x \sin(\omega_1 t + \phi_1), \quad y(t) = A_y \sin(\omega_2 t + \phi_2)$$

According to the given conditions

- $\phi_1 = \phi_2 = 0°$
- $A_x = A_y = 5$
- $\omega_1 = 2\pi f_1 = 2\pi(400)$
- $\omega_2 = 2\pi f_2 = 2\pi(800)$

On substituting the values

$$x(t) = 5\sin(2\pi(400)t), \quad y(t) = 5\sin(2\pi(800)t)$$

Eliminating t to Find the Lissajous Curve
Using the trigonometric identity:

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

we derive the relationship between x and y.

Rewrite $x(t)$

$$x(t) = 5\sin(2\pi(400)t)$$

Rewrite $y(t)$

$$y(t) = 5\sin(2\pi(800)t) = 5\sin(2\cdot 2\pi(400)t)$$

Let:

$$\theta = 2\pi(400)t$$

Then:

$$x = 5\sin(\theta)$$

$$y = 5\sin(2\theta)$$

From the trigonometric identity:

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

we can write:

$$y = 5\cdot 2\sin(\theta)\cos(\theta) = 10\sin(\theta)\cos(\theta)$$

Substitute $\sin(\theta) = \frac{x}{5}$:

$$y = 10\cdot\frac{x}{5}\cdot\cos(\theta) = 2x\cos(\theta)$$

To eliminate $\cos(\theta)$, use:

$$\cos^2(\theta) = 1 - \sin^2(\theta) = 1 - \left(\frac{x}{5}\right)^2$$

$$\cos(\theta) = \pm\sqrt{1 - \left(\frac{x}{5}\right)^2}$$

Result:

Substitute back:

$$y = 2x\cdot\sqrt{1 - \left(\frac{x}{5}\right)^2}$$

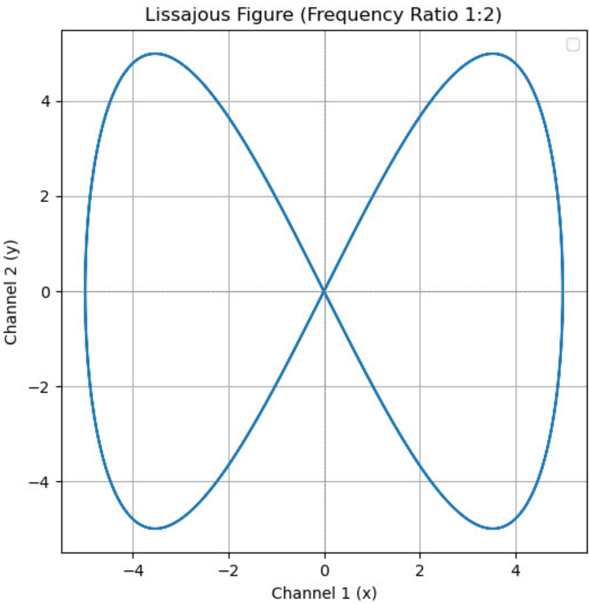
The graph on the CRO is

$$y = 2x\cdot\sqrt{1 - \left(\frac{x}{5}\right)^2}$$

This can be further verified using python to plot the curve and verifying it with the figure obtained on the CRO

5.3.2 Python Plotting

To ensure that the observed readings and the mathematical proofs are correct we run a matlab code and cross check the figure obtained with the one on the CRO



5.4 Pattern 4

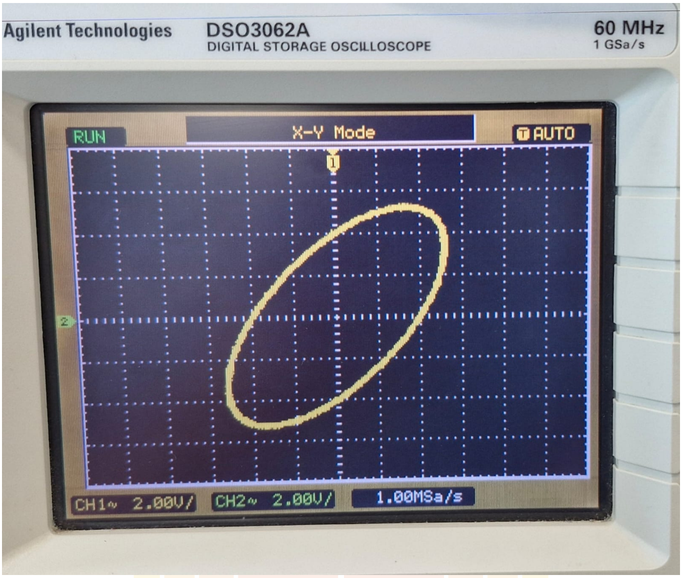
Observation and Analysis: Sinusoidal Signals

Both the signals were taken to be sinusoidal and the following conditions were applied

1) **Frequency Ratio** (f_x : f_y): Both sinusoidal signals were set to have the same frequency, i.e.,
$$f_x : f_y = 1 : 1.$$
This ensured the periodicity of the signals was synchronized.
2) **Phase Difference** (ϕ): We set the phase difference to any value between 0° and 90° and observed different ellipses
   - For $0° < \phi < 90°$, the resulting figure formed an ellipse. The orientation and shape of the ellipse depended on the phase difference, with the axes determined by the relative amplitudes and frequencies of the signals.
   - The phase is set to 50° in this condition
3) **Amplitudes** (A_x, A_y): Again, the amplitudes of both signals are set to be the same for the ease of analysis.
   - $A_x = A_y = A$

Input Signals

Paramters	Channel - 1	Channel - 2
Frequency	200.00 Hz	200.00 Hz
Phase	50°	0°
High	5.00 V	5.00 V
Low	-5.00 V	-5.00 V



5.4.1 Mathematical Proof

Signals:

$$x(t) = A_x \sin(\omega t + \phi_1), \quad y(t) = A_y \sin(\omega t + \phi_2)$$

According to the given conditions

- $\phi_1 = 50°$
- $\phi_2 = 0°$
- $A_x = A_y = 5$
- $\omega = 2\pi(200)$

The parametric equations for the signals are:

$$x(t) = 5\sin(2\pi(200)t + 50°), \quad y(t) = 5\sin(2\pi(200)t)$$

Expanding $x(t)$ using the angle addition formula:

$$\begin{aligned} x(t) &= 5\left[\sin(2\pi(200)t)\cos(50°) + \cos(2\pi(200)t)\sin(50°)\right] \\ &= 5\sin(2\pi(200)t)\cos(50°) + 5\cos(2\pi(200)t)\sin(50°) \end{aligned}$$

$y(t)$ remains:

$$y(t) = 5\sin(2\pi(200)t)$$

To describe the parametric relationship, substitute $\sin(2\pi(200)t) = S$ and $\cos(2\pi(200)t) = C$:

$$x(t) = 5S\cos(50^\circ) + 5C\sin(50^\circ)$$
$$y(t) = 5S$$

Eliminating t:

$$\frac{y(t)}{5} = S$$
$$x(t) = 5S\cos(50^\circ) + 5\sqrt{1-S^2}\sin(50^\circ)$$

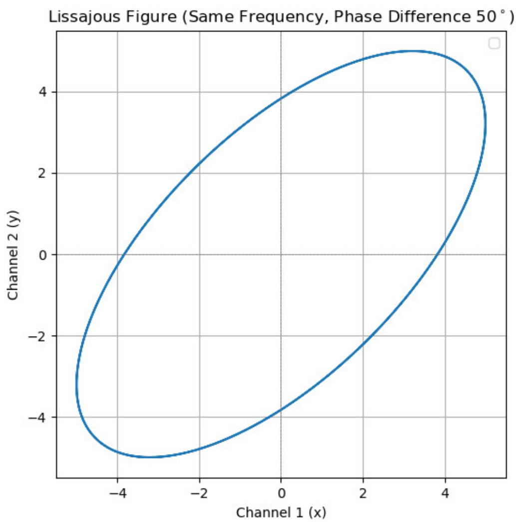
Squaring and adding the equations, we obtain the general equation of the ellipse:

$$\left(\frac{x}{5\cos(50^\circ)}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$$

Result
The parametric equations describe an ellipse with axes determined by $5\cos(50^\circ)$ and 5. The graph on the CRO is a tilted ellipse.

5.4.2 Python Plotting

To ensure that the observed readings and the mathematical proofs are correct we run a matlab code and cross check the figure obtained with the one on the CRO



5.5 Pattern 5

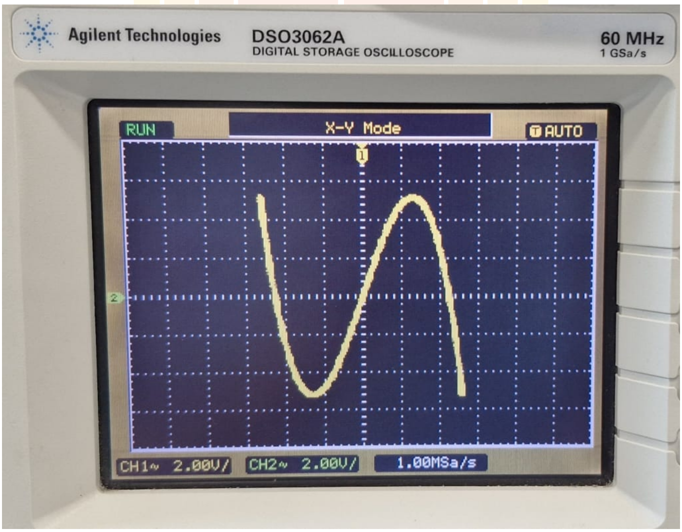
Observation and Analysis: Sinusoidal Signals

Both the signals were taken to be sinusoidal and the following conditions were applied:

1) **Frequency Ratio** ($f_x : f_y$): The sinusoidal signals were set to have a frequency ratio of:
$$f_x : f_y = 1 : 3.$$
This ensured the periodicity of the signals was in a harmonic relationship.
2) **Phase Difference** (ϕ):
We set the phase difference to 0° for both signals, i.e.,
   - $\phi_1 = 0°$
   - $\phi_2 = 0°$
3) **Amplitudes** (A_x, A_y):
The amplitudes of both signals were kept the same for simplicity of analysis:
   - $A_x = A_y = 5$.

Input Signals

Parameters	**Channel - 1**	**Channel - 2**
Frequency	200.00 Hz	600.00 Hz
Phase	0°	0°
High	5.00 V	5.00 V
Low	-5.00 V	-5.00 V



5.5.1 Mathematical Proof

Signals:

The two sinusoidal signals can be expressed as:

$$x(t) = A_x \sin(\omega_1 t + \phi_1), \quad y(t) = A_y \sin(\omega_2 t + \phi_2)$$

Where:

- $\phi_1 = 0^\circ$
- $\phi_2 = 0^\circ$
- $A_x = A_y = 5$ V
- $\omega_1 = 2\pi f_x = 2\pi(200) = 400\pi$ rad/s
- $\omega_2 = 2\pi f_y = 2\pi(600) = 1200\pi$ rad/s

Thus, the signals become:

$$x(t) = 5\sin(400\pi t), \quad y(t) = 5\sin(1200\pi t)$$

Parametric Equations:

The parametric equations for the two signals are:

$$x(t) = 5\sin(400\pi t)$$
$$y(t) = 5\sin(1200\pi t)$$

Lissajous Figure:

For a frequency ratio $f_x : f_y = 1 : 3$, the resulting Lissajous figure will have three loops along the y-axis and one loop along the x-axis. The general parametric form of a Lissajous curve with these frequency ratios is:

$$\frac{x}{A_x} = \sin(\omega_1 t), \quad \frac{y}{A_y} = \sin(\omega_2 t)$$

Substituting $\omega_1 = 400\pi$ and $\omega_2 = 1200\pi$:

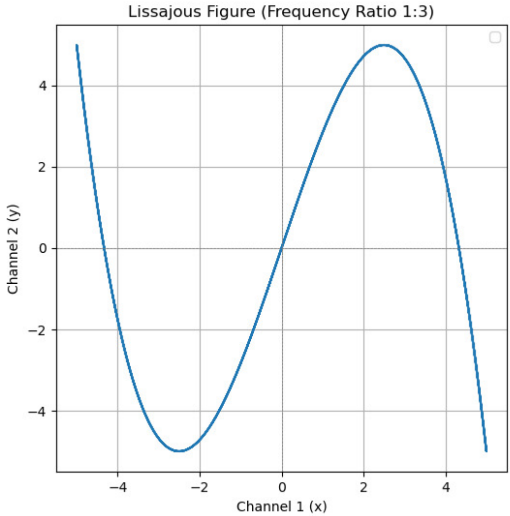
$$\frac{x}{5} = \sin(400\pi t), \quad \frac{y}{5} = \sin(1200\pi t)$$

These parametric equations describe the Lissajous figure, which in this case has a harmonic relationship of $f_x : f_y = 1 : 3$.

In conclusion, the graph formed on the CRO is a Lissajous figure with the expected shape, demonstrating the harmonic relationship between the two sinusoidal signals with the given frequency ratio.

5.5.2 Python Plotting

To ensure that the observed readings and the mathematical proofs are correct we run a matlab code and cross check the figure obtained with the one on the CRO



5.6 Pattern 6

Observation and Analysis: Sinusoidal Signals

Both the signals were taken to be sinusoidal, and the following conditions were applied:

1) **Frequency Ratio** ($f_x : f_y$): The sinusoidal signals were set to have a frequency ratio of:

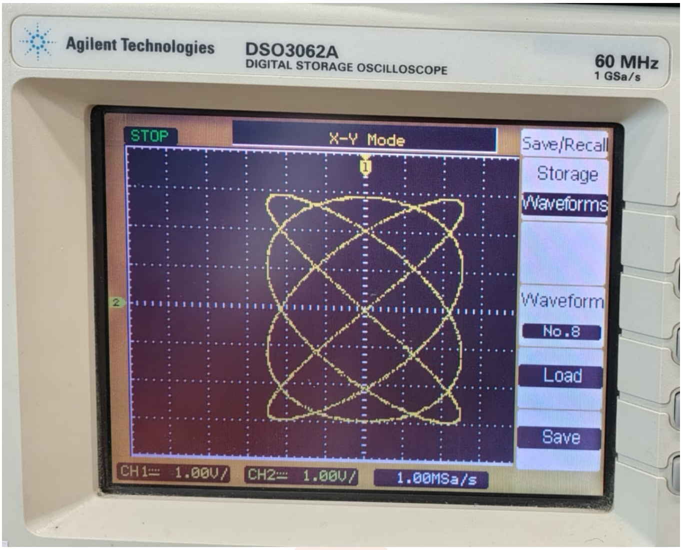
$$f_x : f_y = 4 : 3.$$

This ensured the periodicity of the signals was in a harmonic relationship.

2) **Phase Difference** (ϕ):
We set the phase difference to 85°, i.e.,
   - $\phi_1 = 0°$
   - $\phi_2 = 85°$
3) **Amplitudes** (A_x, A_y):
The amplitudes of both signals were kept the same for simplicity of analysis:
   - $A_x = A_y = 2.5$.

Input Signals

Parameters	**Channel - 1**	**Channel - 2**
Frequency	4000 Hz	3000 Hz
Phase	0°	85°
High	2.50 V	2.50 V
Low	-2.50 V	-2.50 V



5.6.1 Mathematical Proof

Signals:

The two sinusoidal signals can be expressed as:

$$x(t) = A_x \sin(\omega_1 t + \phi_1), \quad y(t) = A_y \sin(\omega_2 t + \phi_2)$$

Where:

- $\phi_1 = 0^\circ$
- $\phi_2 = 85^\circ$
- $A_x = A_y = 2.5$ V
- $\omega_1 = 2\pi f_x = 2\pi(4000) = 8000\pi$ rad/s
- $\omega_2 = 2\pi f_y = 2\pi(3000) = 6000\pi$ rad/s

Thus, the signals become:

$$x(t) = 2.5 \sin(8000\pi t), \quad y(t) = 2.5 \sin(6000\pi t + 85^\circ)$$

Parametric Equations:

The parametric equations for the two signals are:

$$x(t) = 2.5 \sin(8000\pi t)$$
$$y(t) = 2.5 \sin(6000\pi t + 85^\circ)$$

Lissajous Figure:

For a frequency ratio $f_x : f_y = 4 : 3$, the resulting Lissajous figure will have four loops along the x-axis and three loops along the y-axis. The general parametric form of a Lissajous curve with these frequency ratios is:

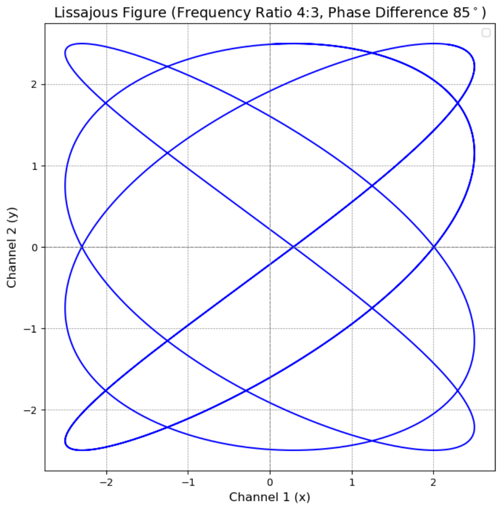
$$\frac{x}{A_x} = \sin(\omega_1 t), \quad \frac{y}{A_y} = \sin(\omega_2 t + \phi_2)$$

Substituting $\omega_1 = 8000\pi$, $\omega_2 = 6000\pi$, and $\phi_2 = 85°$:

$$\frac{x}{2.5} = \sin(8000\pi t), \quad \frac{y}{2.5} = \sin(6000\pi t + 85°)$$

These parametric equations describe the Lissajous figure, which in this case has a harmonic relationship of $f_x : f_y = 4 : 3$.

In conclusion, the graph formed on the CRO is a Lissajous figure with the expected shape, demonstrating the harmonic relationship between the two sinusoidal signals with the given frequency ratio.



6 How to capture an one time event on a CRO

6.1 Introduction

A one-time event refers to a signal or phenomenon that occurs only once or happens sporadically, rather than repeating periodically. In the context of a CRO (Cathode Ray Oscilloscope), it is an electrical signal that does not have a repetitive pattern or waveform.

6.2 Apparatus used

- Cathode Ray oscilloscope (CRO)
- Function generator
- Oscilloscope probes

6.3 Oscilloscope Features

To capture one-time events effectively, oscilloscopes have several features specifically designed for such tasks.

Trigger System

The trigger system ensures the oscilloscope captures the signal at the right moment.

- Edge Triggering: Triggers on a rising or falling edge of the signal.
- Slope Triggering: Triggers based on the signal's slope or rate of change.
- Pulse Width Triggering: Captures signals with a specific width.
- Trigger Level: Adjust the voltage level at which the oscilloscope will trigger.

6.3.1 Normal Sweep Mode

Periodic Waveforms

Normal sweep mode is particularly useful for observing signals that repeat periodically, such as:

- Sine waves
- Square waves
- Clock pulses

This mode ensures that the waveform aligns with the trigger point, producing a stable and clear display suitable for analysis.

One-Time Events

In combination with the *single-shot mode*, normal sweep mode is ideal for capturing one-time events. This ensures the oscilloscope records the desired signal only when the trigger condition is met, making it highly effective for capturing:

- Transient signals
- Glitches
- Switching events

The combination of these modes provides a precise and efficient way to analyze unique, non-repeating phenomena in a circuit.

How to set up Sweep mode

- On the oscilloscope, locate the sweep mode settings and choose "Normal."
- Set the trigger source (e.g., CH1, CH2, external).
- Choose the trigger type (e.g., edge, pulse, slope) and level to capture the desired signal.
- Adjust the time/div for horizontal scaling.
- Connect the input signal to the appropriate channel. If the trigger condition is met, the oscilloscope will display a stable waveform.

6.3.2 Comparison: Normal vs. Auto Sweep Mode

- Accurate Analysis: Useful for detailed analysis of signals with specific trigger requirements.
- Eliminates Noise: Prevents irrelevant data from being displayed when no valid trigger is present.

Feature	Normal Mode	Auto Mode
Sweep Activation	Trigger-dependent	Continuous sweeps without a trigger.
Display	Blank if no trigger occurs	Unsynchronized waveforms shown.
Use Case	For synchronized signals	For general observation.

6.3.3 Burst mode

Burst mode is a specialized feature on oscilloscopes that allows you to capture and analyze brief, high-frequency signals or ”bursts” of activity within a longer period of inactivity.

- Burst mode generates a fixed number of cycles of a waveform (e.g., sine, square, triangle) and stops until triggered again.
- This allows you to focus on a specific event without the oscilloscope continuously sweeping across the time axis.
- Burst mode helps conserve memory since only the relevant portions of the signal are recorded, unlike continuous sweep modes, which would store unnecessary data during periods of inactivity.

6.3.4 How to Set Up Burst Mode on an Oscilloscope

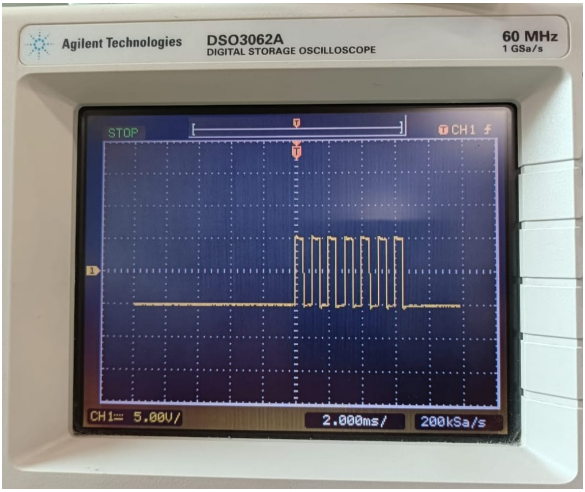
- On the oscilloscope, select Burst Mode from the sweep mode options. This typically involves switching from normal or continuous sweep mode to burst mode.
- Set the oscilloscope's trigger to Burst Trigger
- Set the duration of the burst window. The oscilloscope will capture the signal only during this window.

6.4 Procedure / Experiment

- Set the oscilloscope's sweep mode to Normal to ensure it only displays waveforms when triggered.
- Choose the appropriate channel for the input signal to be analyzed.
- Configure the trigger type (e.g., edge, pulse) that best suits the expected signal.
- Adjust the trigger voltage level to match the point where the one-time event is anticipated.
- Enable the burst mode feature on the function generator.
- Specify the number of cycles for the burst using the burst count setting.
- Set the function generator's trigger mode to Manual.
- Arm the oscilloscope for capture by pressing the Single button.
- Use the function generator's trigger button to manually initiate the one-time event when ready.
- Once the event occurs and satisfies the trigger condition, the oscilloscope will record and display the waveform.

6.5 Result

Here is the captured waveform,



for the required input cycles,