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EE24BTECH11066 - YERRA AKHILESH

1) The dimension of the vector space $V = \left\{A = \left(a_{ij}\right)_{n\times n} : a_{ij} \in \mathbb{C}, a_{ij} = -a_{ji}\right\}$ over the field $\mathbb{R}$ is

   a) $n^2$ b) $n^2 - 1$ c) $n^2 - n$ d) $\frac{n^2}{2}$

2) The minimal polynomial associated with the matrix $\begin{pmatrix} 0 & 0 & 3 \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{pmatrix}$ is

   a) $x^3 - x^2 - 2x - 3$
   b) $x^3 - x^2 + 2x - 3$
   c) $x^3 - x^2 - 3x - 3$
   d) $x^3 - x^2 + 3x - 3$

3) For the function $f(z) = \sin\left(\frac{1}{\cos\left(\frac{1}{z}\right)}\right)$, the point $z = 0$ is

   a) a removable singularity
   b) a pole
   c) an essential singularity
   d) a non-isolated singularity

4) Let $f(z) = \sum_{n=0}^{15} z^n$ for $z \in \mathbb{C}$. If $C : |z - i| = 2$ then $\oint_C \frac{f(z)}{(z-i)^1 5} =$

   a) $2\pi i\,(1 + 15i)$ b) $2\pi i\,(1 - 15i)$ c) $4\pi i\,(1 + 15i)$ d) $2\pi i$

5) For what values of $\alpha$ and $\beta$, the quadrature formula $\int\limits_{-1}^{1} f(x)\,\mathrm{dx} \approx \alpha f(-1) + f(\beta)$ is exact for all polynomials of degree $\leq 1$?

   a) $\alpha = 1, \beta = 1$ b) $\alpha = -1, \beta = 1$ c) $\alpha = 1, \beta = -1$ d) $\alpha = -1, \beta = -1$

6) Let $f : [0, 4] \to \mathbb{R}$ be a three times continuously differential function. Then the value of $f[1, 2, 3, 4]$ is

   a) $\frac{f''(\xi)}{3}$ for some $\xi \in (0, 4)$
   b) $\frac{f''(\xi)}{6}$ for some $\xi \in (0, 4)$
   c) $\frac{f'''(\xi)}{3}$ for some $\xi \in (0, 4)$
   d) $\frac{f'''(\xi)}{6}$ for some $\xi \in (0, 4)$

7) Which one of the following is TRUE ?
   a) Every linear programming problem has a feasible solution.
   b) If a linear programming problem has an optimal solution then it is unique.
   c) The union of two convex sets is necessarily convex.
   d) Extreme points of the disk $x^2 + y^2 \leq 1$ are the points on the circle $x^2 + y^2 = 1$.

8) The dual of the linear programming problem:

$$\text{Minimize } \mathbf{c}^{\mathrm{T}}\mathbf{x} \text{ subject to } A\mathbf{x} \geq b \text{ and } \mathbf{x} \geq 0 \text{ is}$$

a) Maximize $\mathbf{b}^{\mathrm{T}}\mathbf{w}$ subject to $A^{\mathrm{T}}\mathbf{w} \geq c$ and $\mathbf{w} \geq 0$
b) Maximize $\mathbf{b}^{\mathrm{T}}\mathbf{w}$ subject to $A^{\mathrm{T}}\mathbf{w} \leq c$ and $\mathbf{w} \geq 0$
c) Maximize $\mathbf{b}^{\mathrm{T}}\mathbf{w}$ subject to $A^{\mathrm{T}}\mathbf{w} \leq c$ and $\mathbf{w}$ is unrestricted
d) Maximize $\mathbf{b}^{\mathrm{T}}\mathbf{w}$ subject to $A^{\mathrm{T}}\mathbf{w} \geq c$ and $\mathbf{w}$ is unrestricted

9) The resolvent kernel for the integral equation $u(x) = F(x) + \int_{log2}^{x} e^{(t-x)} u(t)\, \mathrm{dt}$ is

a) $\cos(x-t)$ b) 1 c) $e^{(t-x)}$ d) $e^{2(t-x)}$

10) Consider the metrics $d_2(f,g) = \left(\int_a^b |f(t)-g(t)|^2\, \mathrm{dt}\right)^{\frac{1}{2}}$ and $d_\infty(f,g) = \sup_{t\in[a,b]} |f(t)-g(t)|$ on the space $X = C[a,b]$ of all real valued continuous functions on $[a,b]$. Then which of the following is TRUE ?

a) Both $(X, d_2)$ and $(X, d_\infty)$ are complete.
b) $(X, d_2)$ is complete but $(X, d_\infty)$ is NOT complete.
c) $(X, d_\infty)$ is complete but $(X, d_2)$ is NOT complete.
d) Both $(X, d_2)$ and $(X, d_\infty)$ are NOT complete.

11) A function $f : \mathbb{R} \to \mathbb{R}$ need NOT be Lebesgue measurable if

a) $f$ is monotone
b) $\{x \in \mathbb{R} : f(x) \geq \alpha\}$ is measurable for each $\alpha \in \mathbb{Q}$
c) $\{x \in \mathbb{R} : f(x) = \alpha\}$ is measurable for each $\alpha \in \mathbb{R}$
d) For each open set $G$ in $\mathbb{R}$, $f^{-1}(G)$ is measurable

12) Let $\{e_n\}_{n=1}^{\infty}$ be an orthonormal sequence in a Hilbert space $H$ and let $x(\neq 0) \in H$. Then

a) $\lim\limits_{x\to\infty} \langle x, e_n \rangle$ does not exist
b) $\lim\limits_{x\to\infty} \langle x, e_n \rangle = ||x||$
c) $\lim\limits_{x\to\infty} \langle x, e_n \rangle = 1$
d) $\lim\limits_{x\to\infty} \langle x, e_n \rangle = 0$