Applications of Derivatives

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1) For $x \in \left(0, \frac{5\pi}{2}\right)$, define $f(x) = \int\limits_0^x \sqrt{t} \sin t \, dt$. Then f has [2011]

   a) local minimum at π and 2π

   b) local minimum at π and local maximum at 2π
   c) local minimum at π and local maximum at 2π
   d) local maximum at π and 2π

2) A spherical balloon is filled with 4500π cubic meters of helium gas. If a leak in the balloon causes the gas to escape at the rate of 72π cubic meters per minute, then the rate(in meters per minute) at which the radius of the balloon decreases 49 minutes after the leakage began is : [2012]

   a) $\frac{9}{7}$

   b) $\frac{7}{9}$

   c) $\frac{2}{9}$

   d) $\frac{9}{2}$

3) Let $a, b \in \mathbb{R}$ be such that the function f given by $f(x) = ln\,|x| + bx^2 + ax$, $x \neq 0$ has extreme values at $x = -1$ and $x = 2$
   Statement-1 : f has local maximum at $x = -1$ and at $x = 2$.
   Statement-2 : $a = \frac{1}{2}$ and $b = \frac{-1}{4}$ [2012]

   a) Statement-1 is false, Statement-2 is true.

   b) Statement-1 is true, Statement-2 is true;statement-2 is a correct explanation for Statement-1.

   c) Statement-1 is true, statement-2 is true;statement-2 is not a correct explanation for Statement-1
   .
   d) Statement-1 is true, statement-2 is false.

4) A line is drawn through the point$(1, 2)$ to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ is: [2012]

   a) $\frac{-1}{4}$

   b) -4

   c) -2

   d) $\frac{-1}{2}$

5) The intercepts on $x-$axis made by tangents to the curve, $y = \int\limits_0^x |t| \, dt, x \in R$, which are parallel to the line $y = 2x$, are equal to : [JEE M 2013]

   a) ± 1

   b) ± 2

   c) ± 3

   d) ± 4

6) If f and g are differentiable functions in $[0, 1]$ satisfying $f(0) = 2 = g(1)$,$g(0)$ and $f(1) = 6$, then for some $c \in [0, 1]$ [JEE M 2014]
   a) $f'(c) = g'(c)$

   b) $f'(c) = 2g'(c)$

   c) $2f'(c) = g'(c)$

d) $2f'(c) = 3g'(c)$

7) Let $f(x)$ be a polynomial of degree four having extreme values at $x = 1$ and $x = 2$. If $\lim_{x\to 0} \left(1 + \frac{f(x)}{x^2}\right) = 3$, then $f(2)$ is equal to : [JEE M 2015]

a) 0

b) 4

c) -8

d) -4

8) Consider : $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right)$, $x \in \left(0, \frac{\pi}{2}\right)$. A normal to $y = f(x)$ at $x = \frac{\pi}{6}$ also passes through the point : [JEE M 2016]

a) $\left(\frac{\pi}{6}, 0\right)$

b) $\left(\frac{\pi}{4}, 0\right)$

c) $(0, 0)$

d) $\left(\frac{2\pi}{3}\right)$

9) A wire of length 2 units is cut into two parts which are bent respectively to form a square of side=x units and a circle of radius=r units. If the sum of the areas of the square and the circle so formed is minimum, then: [JEE M 2016]

a) $x = 2r$

b) $2x = r$

c) $2x = (\pi + 4r)$

d) $(4 - \pi)x = \pi r$

10) The function $f : R \to \left(\frac{-1}{2}, \frac{1}{2}\right)$ defined as $f(x) = \frac{x}{1+x^2}$, is : [JEE M 2016]

a) neither injective nor surjective

b) invertible

c) injective but not surjective

d) surjective but not injective

11) The normal to the curve $y(x-2)(x-3) = x+6$ at the point where the curve intersects the y-axis passes through the point : [JEE M 2017]

a) $\left(\frac{1}{2}, \frac{1}{3}\right)$

b) $\left(\frac{-1}{2}, \frac{-1}{2}\right)$

c) $\left(\frac{1}{2}, \frac{1}{2}\right)$

d) $\left(\frac{1}{2}, \frac{-1}{3}\right)$

12) Twenty meters of wire is available for fencing off a flower-bed in the form of a circular sector. Then the maximum area($insq.m$) of the flower-bed, is: [JEE M 2017]

a) 30

b) 12.5

c) 10

d) 25

13) The eccentricity of an ellipse whose centre is at the origin is $\frac{1}{2}$. If one of its directices is $x = -4$, then the equation of the normal to it at $\left(1, \frac{3}{2}\right)$ is : [JEE M 2017]

a) $x + 2y = 4$

b) $2y - x = 2$

c) $4x - 2y = 1$

d) $4x + 2y = 7$

14) Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local

minimum value of $h(x)$ is: [JEE M 2018]

a) -3

b) $-2\sqrt{2}$

c) $2\sqrt{2}$

d) 3

15) If the curves $y^2 = 6x, 9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is: [JEE M 2018]

a) $\frac{7}{2}$

b) 4

c) $\frac{9}{2}$

d) 6