

- 27) Consider a three-dimensional crystal of  $N$  inert gas atoms. The total energy is given by  $U(R) = 2N \epsilon \left[ p \left( \frac{\sigma}{R} \right)^{12} - q \left( \frac{\sigma}{R} \right)^6 \right]$ , where  $p = 12.13$ ,  $q = 14.45$ , and  $R$  is the nearest neighbour distance between two atoms. The two constants,  $\epsilon$  and  $R$ , have the dimensions of energy and length, respectively. The equilibrium separation between two nearest neighbour atoms in units of  $\sigma$  (rounded off to two decimal places) is \_\_\_\_\_ [2019-PH]

- 28) The energy-wavevector ( $E - k$ ) dispersion relation for a particle in two dimensions is  $E = Ck$ , where  $C$  is a constant. If its density of states  $D(E)$  is proportional to  $E^p$  then the value of  $p$  is \_\_\_\_\_ [2019-PH]

- 29) A circular loop made of a thin wire has radius 2 cm and resistance  $2\Omega$ . It is placed perpendicular to a uniform magnetic field of magnitude  $|\vec{B}_0| = 0.01$  Tesla. At time  $t = 0$  the field starts decaying as  $\vec{B} = \vec{B}_0 e^{-\frac{t}{t_0}}$ , where  $t_0 = 1$  s. The total charge that passes through a cross section of the wire during the decay is  $Q$ . The value of  $Q$  in  $\mu C$  (rounded off to two decimal places) is \_\_\_\_\_ [2019-PH]

- 30) The electric field of an electromagnetic wave in vacuum is given by

$$\vec{E} = E_0 \cos(3y + 4z - 1.5 \times 10^9 t) \hat{x}.$$

The wave is reflected from the  $z = 0$  surface. If the pressure exerted on the surface is  $\alpha \epsilon_0 E_0^2$ , the value of  $\alpha$  (rounded off to one decimal place) is \_\_\_\_\_ [2019-PH]

- 31) The Hamiltonian for a quantum harmonic oscillator of mass  $m$  in three dimensions is

$$H = \frac{p^2}{2m} + \frac{1}{2} m \omega^2 r^2$$

where  $\omega$  is the angular frequency. The expectation value of  $r^2$  in the first excited state of the oscillator in units of  $\frac{\hbar}{m\omega}$  (rounded off to one decimal place)

[2019-PH]

- 32) The Hamiltonian for a particle of mass  $m$  is  $H = \frac{p^2}{2m} + kqt$  where  $q$  and  $p$  are the generalized coordinate and momentum, respectively,  $t$  is time and  $k$  is a constant. For the initial condition,  $q = 0$  and  $p = 0$  at  $t = 0$ ,  $q(t) \propto t^\alpha$ . The value of  $\alpha$  is \_\_\_\_\_ [2019-PH]

- 33) At temperature  $T$  Kelvin ( $K$ ), the value of the Fermi function at an energy  $0.5eV$  above the Fermi energy is 0.01. Then  $T$ , to the nearest integer, is \_\_\_\_\_  
 $(k_B = 8.62 \times 10^{-5} \frac{eV}{K})$  [2019-PH]
- 34) Let  $|\psi_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |\psi_2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$  represent two possible states of a two-level quantum system. The state obtained by the incoherent superposition of  $|\psi_1\rangle$  and  $|\psi_2\rangle$  is given by a density matrix that is defined as  $\rho \equiv c_1|\psi_1\rangle\langle\psi_1| + c_2|\psi_2\rangle\langle\psi_2|$ . If  $c_1 = 0.4$  and  $c_2 = 0.6$ , the matrix element  $\rho_{22}$  (rounded off to one decimal place) is \_\_\_\_\_ [2019-PH]
- 35) A conventional type-I superconductor has a critical temperature of  $4.7K$  at zero magnetic field and a critical magnetic field of  $0.3$  Tesla at  $0$  K. The critical field in Tesla at  $2$  K (rounded off to three decimal places) is \_\_\_\_\_ [2019-PH]
- 36) Consider the following Boolean expression:

$$(\overline{A} + \overline{B}) [\overline{A(B+C)}] + A(\overline{B} + \overline{C})$$

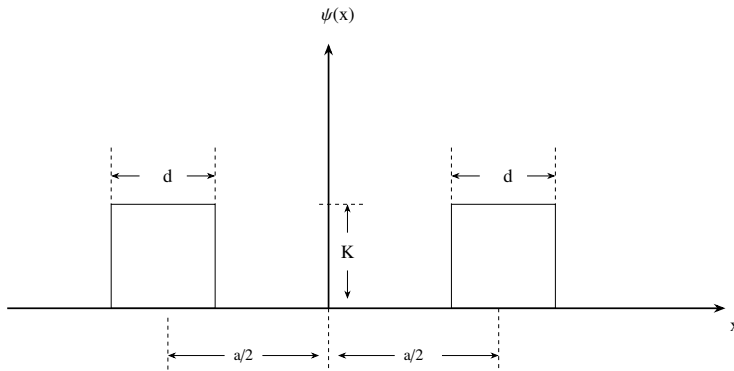
It can be represented by a single three-input logic gate. Identify the gate. [2019-PH]

- a) AND                      b) OR                      c) XOR                      d) NAND

- 37) The value of the integral  $\int_{-\infty}^{\infty} \frac{\cos(kx)}{x^2+a^2} dx$ , where  $k > 0$  and  $a > 0$ , is \_\_\_\_\_ [2019-PH]

- a)  $\frac{\pi}{a} e^{-ka}$                       b)  $\frac{2\pi}{a} e^{-ka}$                       c)  $\frac{\pi}{2a} e^{-ka}$                       d)  $\frac{3\pi}{2a} e^{-ka}$

- 38) The wave function  $\psi(x)$  of a particle is as shown below



Here  $K$  is a constant, and  $a > d$ . The position uncertainty  $\langle \Delta x \rangle$  of the particle is

a)  $\sqrt{\frac{a^2+3d^2}{12}}$

b)  $\sqrt{\frac{3a^2+d^2}{12}}$

c)  $\sqrt{\frac{d^2}{6}}$

d)  $\sqrt{\frac{d^2}{24}}$

- 39) A solid cylinder of radius  $R$  has total charge  $Q$  distributed uniformly over its volume. It is rotating about its axis with angular speed  $\omega$ . The magnitude of the total magnetic moment of the cylinder is [2019-PH]

a)  $QR^2\omega$

b)  $\frac{1}{2}QR^2\omega$

c)  $\frac{1}{4}QR^2\omega$

d)  $\frac{1}{8}QR^2\omega$