

9-9.2-20

EE24BTECH11066 - YERRA AKHILESH

Question:

Find the area of the region bounded by the curves $y = x^2 + 2$, $y = x$, $x = 0$ and $x = 3$.

solution: The parameters of the conic are

Equations
$y = x^2 + 2$
$y = x$
$x = 0$
$x = 3$

TABLE 0: Given Equations

Conic	Parameters
V	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
u	$-\frac{1}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
f	0

$$L : x_i = h + \kappa_i m \quad (0.1)$$

Where,

$$\kappa_i = \frac{1}{m^\top V m} (-m^\top (V h + u) \pm \sqrt{[m^\top (V h + u)]^2 - g(h)(m^\top V m)}) \quad (0.2)$$

For the curves $y = x^2 + 2$ and $y = x$, we find the points of intersection by solving

$$x^2 + 2 = x \quad (0.3)$$

Rearranging the equation:

$$x^2 - x + 2 = 0 \quad (0.4)$$

Since this quadratic has no real roots, we calculate the area between the curves over the interval $x = 0$ to $x = 3$.

The area between the curves is given by:

$$A = \int_0^3 (x - (x^2 + 2)) dx \quad (0.5)$$

Simplifying the integrand:

$$A = \int_0^3 (x - x^2 - 2) dx \quad (0.6)$$

Solving the integral:

$$A = \left[\frac{x^2}{2} - \frac{x^3}{3} - 2x \right]_0^3 \quad (0.7)$$

Evaluating at the bounds:

$$A = \left[\frac{9}{2} - \frac{27}{3} - 6 \right] - [0] \quad (0.8)$$

Simplifying:

$$A = \frac{9}{2} - 9 - 6 = \frac{9}{2} - 15 = \frac{-21}{2} \quad (0.9)$$

Taking the absolute value, the area of the region is:

$$A = \frac{21}{2} \quad (0.10)$$

