9-9.2-20

EE24BTECH11066 - YERRA AKHILESH

Question:
Find the area of the region bounded by the curves $y = x^2 + 2$, $y = x$, $x = 0$ and $x = 3$.
solution: The parameters of the conic are

Equations
$y = x^2 + 2$
$y = x$
$x = 0$
$x = 3$

TABLE 0: Given Equations

Conic	**Parameters**
V	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
u	$\frac{-1}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
f	0

$$L : x_i = h + \kappa_i m \tag{0.1}$$

Where,

$$\kappa_i = \frac{1}{m^\top V m}(-m^\top(Vh + u) \pm \sqrt{[m^\top(Vh + u)]^2 - g(h)(m^\top V m)} \tag{0.2}$$

For the curve $y = x$, the parameters are

$$h_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, m_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{0.3}$$

Substituting from the above in (0.2)

$$\kappa_i = 1, -1 \tag{0.4}$$

yielding the points of intersection

$$a_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, a_1 = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \tag{0.5}$$

Similarly, for the curve $y = x^2 + 2$ (0.2)

$$h_1 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}, m_1 = \begin{pmatrix} 1 \\ x^2 \end{pmatrix} \tag{0.6}$$

yielding

$$\kappa_i = 1, -1 \tag{0.7}$$

from which, the points of intersection are

$$a_2 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}, a_3 = \begin{pmatrix} 3 \\ 11 \end{pmatrix} \tag{0.8}$$

Thus, the area of the region between the curves $y = x^2 + 2$ and $y = x$ from $x = 0$ to $x = 3$ is given by

$$\int_0^3 \left| x - \left(x^2 + 2 \right) \right| \, dx \tag{0.9}$$

Evaluating the integral:

$$A = \int_0^3 \left| x - \left(x^2 + 2 \right) \right| \, dx \tag{0.10}$$

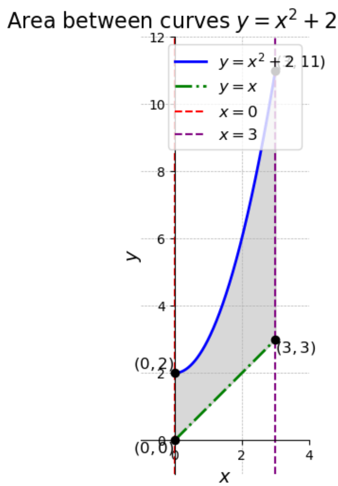
Evaluating the expressions step by step:

$$A = \left[\frac{x^2}{2} - \frac{x^3}{3} - 2x \right]_0^3 \tag{0.11}$$

Simplifying:

$$A = \left[\frac{9}{2} - \frac{27}{3} - 6 \right] = \frac{9}{2} - 9 - 6 = \frac{21}{2} \tag{0.12}$$

Thus, the area enclosed is $\frac{21}{2}$.


Area between curves $y = x^2 + 2$ and $y = x$
$y = x^2 + 2$
$y = x$
$x = 0$
$x = 3$
(3, 11)
(3, 3)
(0, 2)
(0, 0)
x
y