

9-9.2-20

EE24BTECH11066 - YERRA AKHILESH

Question:

Find the area of the region bounded by the curves $y = x^2 + 2$, $y = x$, $x = 0$ and $x = 3$.

solution: The parameters of the conic are

Equations
$y = x^2 + 2$
$y = x$
$x = 0$
$x = 3$

TABLE 0: Given Equations

Conic	Parameters
V	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
u	$\frac{-1}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
f	0

The equation of a parabola in Matrix form is

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (0.1)$$

The equation of a line in vector form is

$$\mathbf{x} = \mathbf{h} + \kappa \mathbf{m} \quad (0.2)$$

For the given parabola $y = x^2 + 2$, The values of $\mathbf{V}, \mathbf{u}, f$ are

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (0.3)$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix} \quad (0.4)$$

$$f = -2 \quad (0.5)$$

For the given line $x = 0$, The values of \mathbf{h}_1 , \mathbf{m}_1 are

$$\mathbf{h}_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (0.6)$$

$$\mathbf{m}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (0.7)$$

Substituting $x = 0$ line equation in parabola equation gives the values of κ

$$(\mathbf{h} + \kappa \mathbf{m})^\top \mathbf{V} (\mathbf{h} + \kappa \mathbf{m}) + 2\mathbf{u}^\top (\mathbf{h} + \kappa \mathbf{m}) + f = 0 \quad (0.8)$$

$$\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \kappa \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right)^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \kappa \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) + 2 \begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix}^\top \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \kappa \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) + -2 = 0 \quad (0.9)$$

$$\begin{pmatrix} \kappa & \kappa \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \kappa \\ \kappa \end{pmatrix} + 2 \begin{pmatrix} 0 & \frac{-1}{2} \end{pmatrix} \begin{pmatrix} \kappa \\ \kappa \end{pmatrix} = 2 \quad (0.10)$$

$$\begin{pmatrix} \kappa & \kappa \end{pmatrix} \begin{pmatrix} \kappa \\ 0 \end{pmatrix} - (\kappa) = 2 \quad (0.11)$$

$$\kappa^2 - \kappa = 2 \quad (0.12)$$

$$\kappa_1 = 2 \quad (0.13)$$

$$\kappa_2 = -1 \quad (0.14)$$

The intersection points are

$$\mathbf{x}_1 = \mathbf{h} + \kappa_1 \mathbf{m} \quad (0.15)$$

$$\mathbf{x}_1 = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \quad (0.16)$$

as κ_2 is negative so we neglect that point of intersection.

similarly for $x = 3$ line intersection points are

$$\mathbf{x}_2 = \begin{pmatrix} 3 \\ 11 \end{pmatrix} \quad (0.17)$$

The Area under the curve $y = x^2 + 2$ is given by

$$A_1 = \int_0^3 (x^2 + 2) dx \quad (0.18)$$

$$A_1 = \left(\frac{x^3}{3} + 2x \right) \Big|_0^3 \quad (0.19)$$

$$A_1 = \left(\frac{3^3}{3} + 2 \cdot 3 \right) - (0) \quad (0.20)$$

$$A_1 = (9 + 6) \quad (0.21)$$

$$A_1 = 15 \quad (0.22)$$

The Area under the line $y = x$ is given by

$$A_2 = \int_0^3 (x) dx \quad (0.23)$$

$$A_2 = \left(\frac{x^2}{2} \right) \Big|_0^3 \quad (0.24)$$

$$A_2 = \frac{3^2}{2} - 0 \quad (0.25)$$

$$A_2 = \frac{9}{2} \quad (0.26)$$

$$A_2 = 4.5 \quad (0.27)$$

The area of region bounded by the line $x = y$ and the parabola $y = x^2 + 2$ is given by

$$A = A_1 - A_2 \quad (0.28)$$

$$A = 15 - 4.5 \quad (0.29)$$

$$A = 10.5 \quad (0.30)$$

The area of region bounded by the line $x = y$ and the parabola $y = x^2 + 2$ is 10.5.

Area between curves $y = x^2 + 2$ and $y = x$

