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EE24BTECH11066 - YERRA AKHILESH

16) The portion of the line $4x + 5y = 20$ in the first quadrant is trisected by the lines L_1 and L_2 passing through the origin. The tangent of an angle between the lines L_1 and L_2 is :

a) $\frac{8}{5}$ b) $\frac{25}{41}$ c) $\frac{2}{5}$ d) $\frac{30}{41}$

17) Let $\bar{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\bar{b} = 3\left(\hat{i} - \hat{j} + \hat{k}\right)$. Let $\bar{c}$ be the vector such that $\bar{a} \times \bar{c} = \bar{b}$ and $\bar{a} \cdot \bar{c} = 3$. Then $\bar{a} \cdot \left(\left(\bar{c} \times \bar{b}\right) - \bar{b} - \bar{c}\right)$ is equal to :

a) 32 b) 24 c) 20 d) 36

18) If $a = \lim\limits_{x \to 0} \frac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4}$ and $b = \lim\limits_{x \to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$, then the value of ab^3 is :

a) 36 b) 32 c) 25 d) 30

19) Consider the matrix $f(x) = \begin{pmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Given below are two statements :
Statement 1: $f(-x)$ is the inverse of the matrix $f(x)$.
Statement 2: $f(x)f(y) = f(x+y)$.

In the light of the above statements, choose the correct answer from the options given below

a) Statement 1 is false but Statement 2 is true
b) Both Statement 1 and Statement 2 are false
c) Statement 1 is true but Statement 2 is false
d) Both Statement 1 and Statement 2 are true

20) The function $f : N - \{1\} \to N$; defined by $f(n)$ = the highest prime factor of n, is :

a) both one-one and onto
b) one-one only
c) onto only
d) neither one-one nor onto

21) The least positive integral value of α, for which the angle between the vectors $\alpha\hat{i} - 2\hat{j} + 2\hat{k}$ and $\alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k}$ is acute, is _____

22) Let for a differentiable function $f : (0, \infty) \to R, f(x) - f(y) \geq ln\left(\frac{x}{y}\right) + x - y, \forall x, y \in (0, \infty)$. Then $\sum_{n=1}^{20} f'\left(\frac{1}{n^2}\right)$ is equal to _____

23) If the solution of the differential equation $(2x + 3y - 2)\,dx + (4x + 6y - 7)\,dy = 0$, $y(0) = 3$, is $\alpha x + \beta y + 3ln\,|2x + 3y - \gamma| = 6$, then $\alpha + 2\beta + 3\gamma$ is equal to _____

24) Let the area of the region $\left\{(x, y) : x - 2y + 4 \geq 0, x + 2y^2 \geq 0, x + 4y^2 \leq 8, y \geq 0\right\}$ be $\frac{m}{n}$, where m and n are coprime numbers. Then $m + n$ is equal to _____

25) If $8 = 3 + \frac{1}{4}(3 + p) + \frac{1}{4^2}(3 + 2p) + \frac{1}{4^3}(3 + 3p) + \cdots \infty$, then the value of p is _____

26) A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let $a = P(X = 3)$, $b = P(X \geq 3)$ and $c = P(X \geq 6|X > 3)$. Then $\frac{b+c}{a}$ is equal to _____

27) Let the set of all $a \in \mathbb{R}$ such that the equation $\cos 2x + a \sin x = 2a - 7$ has a solution be $[p, q]$ and $r = \tan 9° - \tan 27° - \frac{1}{\cot 63°} + \tan 81°$, then pqr is equal to _____

28) Let $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$, $x \in \mathbb{R}$. Then $f'(10)$ is equal to _____

29) Let $A = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} B_1 & B_2 & B_3 \end{pmatrix}$, where B_1, B_2, B_3 are column matrices, and $AB_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, AB_2 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}, AB_3 = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$. If $\alpha = |B|$ and β is the sum of all the diagonal elements B, then $\alpha^3 + \beta^3$ is equal to _____

30) If α satisfies the equation $x^2 + x + 1 = 0$ and $(1 + \alpha)^7 = A + B\alpha + C\alpha^2, A, B, C \geq 0$, then $5(3A - 2B - C)$ is equal to _____