

- 16) The function $f : N - \{1\} \rightarrow N$; defined by $f(n) =$ the highest prime factor of n , is :
- both one-one and onto
 - one-one only
 - onto only
 - neither one-one nor onto

- 17) Consider the function.

$$f(x) = \begin{cases} \frac{a(7x-12-x^2)}{b[x^2-7x+12]}, & x < 3, \\ \frac{\sin(x-3)}{2^{x-[x]}}, & x > 3, \\ b, & x = 3. \end{cases}$$

where $[x]$ denotes the greatest integer less than or equal to x . If S denotes the set of all ordered pairs (a, b) such that $f(x)$ is continuous at $x = 3$, then the number of elements in S is :

- 2
 - 4
 - Infinitely many
 - 1
- 18) Four distinct points $(2k, 3k), (1, 0), (0, 1)$ and $(0, 0)$ lie on a circle for k equal to :

- $\frac{3}{13}$
- $\frac{2}{13}$
- $\frac{5}{13}$
- $\frac{1}{13}$

- 19) Let a_1, a_2, \dots, a_{10} be 10 observations such that $\sum_{k=1}^{10} a_k = 50$ and $\sum_{k < j} a_k \cdot a_j = 1100$. Then the standard deviation of a_1, a_2, \dots, a_{10} is equal to :

- $\sqrt{115}$
- 5
- 10
- $\sqrt{5}$

- 20) If (a, b) be the orthocentre of the triangle whose vertices are $(1, 2), (2, 3)$ and $(3, 1)$,

and $I_1 = \int_a^b x \sin(4x - x^2) dx, I_2 = \int_a^b \sin(4x - x^2) dx$, then $36 \frac{I_1}{I_2}$ is equal to :

- 80
- 88
- 66

d) 72

- 21) If $8 = 3 + \frac{1}{4}(3+p) + \frac{1}{4^2}(3+2p) + \frac{1}{4^3}(3+3p) + \cdots \infty$, then the value of p is _____
- 22) If α satisfies the equation $x^2 + x + 1 = 0$ and $(1 + \alpha)^7 = A + B\alpha + C\alpha^2$, $A, B, C \geq 0$, then $5(3A - 2B - C)$ is equal to _____
- 23) Let $A = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$, $B = (B_1 \ B_2 \ B_3)$, where B_1, B_2, B_3 are column matrices, and $AB_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $AB_2 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$, $AB_3 = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$. If $\alpha = |B|$ and β is the sum of all the diagonal elements B , then $\alpha^3 + \beta^3$ is equal to _____
- 24) Let for a differentiable function $f : (0, \infty) \rightarrow \mathbb{R}$, $f(x) - f(y) \geq \ln\left(\frac{x}{y}\right) + x - y$, $\forall x, y \in (0, \infty)$. Then $\sum_{n=1}^{20} f'\left(\frac{1}{n^2}\right)$ is equal to _____
- 25) A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let $a = P(X = 3)$, $b = P(X \geq 3)$ and $c = P(X \geq 6 | X > 3)$. Then $\frac{b+c}{a}$ is equal to _____
- 26) Let $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$, $x \in \mathbb{R}$. Then $f'(10)$ is equal to _____
- 27) If the solution of the differential equation $(2x + 3y - 2)dx + (4x + 6y - 7)dy = 0$, $y(0) = 3$, is $\alpha x + \beta y + 3 \ln|2x + 3y - \gamma| = 6$, then $\alpha + 2\beta + 3\gamma$ is equal to _____
- 28) Let the area of the region $\{(x, y) : x - 2y + 4 \geq 0, x + 2y^2 \geq 0, x + 4y^2 \leq 8, y \geq 0\}$ be $\frac{m}{n}$, where m and n are coprime numbers. Then $m + n$ is equal to _____
- 29) Let the set of all $a \in \mathbb{R}$ such that the equation $\cos 2x + a \sin x = 2a - 7$ has a solution be $[p, q]$ and $r = \tan 9^\circ - \tan 27^\circ - \frac{1}{\cot 63^\circ} + \tan 81^\circ$, then pqr is equal to _____
- 30) The least positive integral value of α , for which the angle between the vectors $\alpha \hat{i} - 2\hat{j} + 2\hat{k}$ and $\alpha \hat{i} + 2\alpha \hat{j} - 2\hat{k}$ is acute, is _____