january 27th shift1 2024

EE24BTECH11066 - YERRA AKHILESH

16) The function $f : N - \{1\} \rightarrow N$; defined by $f(n) =$ the highest prime factor of n, is : [27th January shift1,2024]
    a) both one-one and onto
    b) one-one only
    c) onto only
    d) neither one-one nor onto
17) Consider the function.
$$f(x) = \begin{cases} \frac{a(7x-12-x^2)}{b|x^2-7x+12|}, & x < 3, \\ \frac{\sin(x-3)}{2^{x-[x]}}, & x > 3, \\ b, & x = 3. \end{cases}$$
where $[x]$ denotes the greatest integer less than or equal to x. If S denotes the set of all ordered pairs (a, b) such that $f(x)$ is continuous at $x = 3$, then the number of elements in S is : [27th January shift1,2024]
    a) 2
    b) 4
    c) Infinitely many
    d) 1
18) Four distinct points $(2k, 3k), (1, 0), (0, 1)$ and $(0, 0)$ lie on a circle for k equal to : [27th January shift1,2024]
    a) $\frac{3}{13}$
    b) $\frac{2}{13}$
    c) $\frac{5}{13}$
    d) $\frac{1}{13}$
19) Let $a_1, a_2, \cdots a_{10}$ be 10 observations such that $\sum_{k=1}^{10} a_k = 50$ and $\sum_{\forall k<j} a_k \cdot a_j = 1100$. Then the standard deviation of $a_1, a_2, \cdots a_{10}$ is equal to : [27th January shift1,2024]
    a) $\sqrt{115}$
    b) 5
    c) 10
    d) $\sqrt{5}$
20) If (a, b) be the orthocentre of the triangle whose vertices are $(1, 2), (2, 3)$ and $(3, 1)$, and $I_1 = \int_a^b x \sin\left(4x - x^2\right) \mathrm{dx}, I_2 = \int_a^b \sin\left(4x - x^2\right) \mathrm{dx}$, then $36\frac{I_1}{I_2}$ is equal to : [27th January shift1,2024]

a) 80
b) 88
c) 66
d) 72

21) If $8 = 3 + \frac{1}{4}(3 + p) + \frac{1}{4^2}(3 + 2p) + \frac{1}{4^3}(3 + 3p) + \cdots \infty$, then the value of p is ______ [27th January shift1,2024]

22) If α satisfies the equation $x^2 + x + 1 = 0$ and $(1 + \alpha)^7 = A + B\alpha + C\alpha^2, A, B, C \geq 0$, then $5(3A - 2B - C)$ is equal to ______ [27th January shift1,2024]

23) Let $A = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}, B = \begin{pmatrix} B_1 & B_2 & B_3 \end{pmatrix}$, where B_1, B_2, B_3 are column matrices, and $AB_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, AB_2 = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}, AB_3 = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$. If $\alpha = |B|$ and β is the sum of all the diagonal elements B, then $\alpha^3 + \beta^3$ is equal to ______ [27th January shift1,2024]

24) Let for a differentiable function $f : (0, \infty) \to R, f(x) - f(y) \geq ln\left(\frac{x}{y}\right) + x - y, \forall x, y \in (0, \infty)$. Then $\sum_{n=1}^{20} f'\left(\frac{1}{n^2}\right)$ is equal to ______ [27th January shift1,2024]

25) A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let $a = P(X = 3), b = P(X \geq 3)$ and $c = P(X \geq 6 | X > 3)$. Then $\frac{b+c}{a}$ is equal to ______ [27th January shift1,2024]

26) Let $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3), x \in \mathbb{R}$. Then $f'(10)$ is equal to ______ [27th January shift1,2024]

27) If the solution of the differential equation $(2x + 3y - 2)\,\mathrm{dx} + (4x + 6y - 7)\,\mathrm{dy} = 0$, $y(0) = 3$, is $\alpha x + \beta y + 3ln|2x + 3y - \gamma| = 6$, then $\alpha + 2\beta + 3\gamma$ is equal to ______ [27th January shift1,2024]

28) Let the area of the region $\left\{(x, y) : x - 2y + 4 \geq 0, x + 2y^2 \geq 0, x + 4y^2 \leq 8, y \geq 0\right\}$ be $\frac{m}{n}$, where m and n are coprime numbers. Then $m + n$ is equal to ______ [27th January shift1,2024]

29) Let the set of all $a \in \mathbb{R}$ such that the equation $\cos 2x + a \sin x = 2a - 7$ has a solution be $[p, q]$ and $r = \tan 9^\circ - \tan 27^\circ - \frac{1}{\cot 63^\circ} + \tan 81^\circ$, then pqr is equal to ______ [27th January shift1,2024]

30) The least positive integral value of α, for which the angle between the vectors $\alpha\hat{i} - 2\hat{j} + 2\hat{k}$ and $\alpha\hat{i} + 2\alpha\hat{j} - 2\hat{k}$ is acute, is ______ [27th January shift1,2024]