Bode Plot of Magnitude and Phase Response for Cascaded RC Low-Pass Filters

Indian Institute of Technology
Hyderabad

Lab Assignment : 03

EE1200: Electrical Circuits Lab

Harshil Rathan Y	**EE24BTECH11064**
Y Akhilesh	**EE24BTECH11066**

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1 Experiment Objectives

- To analyze the frequency response of 1-stage, 2-stage, and 3-stage RC low-pass filters by measuring the magnitude and phase response.
- To plot the Bode plots (magnitude and phase) and compare the experimental results with theoretical predictions.
- To Compare Single-Stage and Multi-Stage Filters and analyze how the frequency response changes when multiple RC stages are cascaded.

2 Thoery : Bode Plot

- An RC circuit consists of a resistor (R) and Capacitor (C) in series or parallel configurations. These circuits are fundamental in signal processing, especially for filtering applications.
- The Bode plot is a graphical representation of a system's frequency response, showing how the gain and phase of the output signal change with frequency.

RC Low-Pass Filter

- A 1-stage RC filter consists of a single of a resistor (R) and a capacitor (C) connected in series, with the output taken across the capacitor.
- The transfer function for a 1-stage RC low pass filter is given by

$$H(s) = \frac{1}{1 + sRC}$$

where s = jω

Cascading RC Low-Pass Filter

- When multiple RC sections are cascaded, the overall transfer function becomes the product of individual transfer functions.

For n identical RC stages

$$H_n(S) = (\frac{1}{1 + sRC})^n$$

Transfer function for a 2-Stage RC Low-pass filter

$$H(s) = \frac{1}{(1 + sRC)^2}$$

Transfer function for a 3 Stage RC Low-Pass filter

$$H(s) = \frac{1}{(1 + sRC)^3}$$

- Each additional stage increases the roll-off rate by -20 dB/decade, making the overall filter steeper. The phase response is also affected, introducing additional phase lag.

Cutoff Frequency

It is given by

$$f_c = \frac{1}{2\pi RC}$$

- At this frequency, the output voltage drops to $\frac{1}{\sqrt{2}}$ (about 70.7) of the input voltage, corresponding to a -3 dB gain reduction in the Bode plot. Beyond f_c, the filter attenuates signals at a rate of -20 dB/decade for a single stage.

2.1 *Magnitude Plot*

It is a Gain vs Frequency plot

The Magnitude response in dB is given by

$$|H(j\omega)|_{\mathrm{dB}} = 20\log_{10}\left(\frac{1}{\sqrt{1+(\omega RC)^2}}\right)$$

- At Low frequencies ($\omega \ll \omega_c$) The gain is approximately 0
- At cut-off frequency ($\omega = \omega_c$) The gain drops to -3 dB
- At High Frequencies ($\omega \gg \omega_c$) The gain decreases at a slope of -20 dB/decade

2.2 *Phase Plot*

It is a Phase Shift vs Frequency plot, given by

$$\angle H(j\omega) = -\tan^{-1}(\omega RC)$$

- At Low frequencies ($\omega \ll \omega_c$) The phase shift is approximately $0°$
- At cut-off frequency ($\omega = \omega_c$) The phase shift is approx $-45°$
- At High Frequencies ($\omega \gg \omega_c$) The phase shift approaches $-90°$

3 Components Used

Function Generator

- Supplies and Generates an adjustable periodic waveforms (e.g., sine waves) to evaluate the frequency response.
- Enables variation of signal frequency to observe magnitude and phase response.

Oscilloscope

- Displays the voltage waveform over time, enabling real-time signal analysis.
- Measures amplitude attenuation and phase shift across different filter stages.
- Provides frequency-domain analysis when used in conjunction with Fast Fourier Transform (FFT) functions.

Resistors

- Control the charging and discharging time of capacitors.
- Influence the cutoff frequency .
- Affect the overall attenuation and phase characteristics of the filter.

In this experiment, we used $10k\Omega$ resistors

Capacitors

- Store and release charge, controlling signal attenuation at different frequencies.
- Influence the phase shift of the output signal relative to the input.
- Play a key role in defining the filter's behavior when cascaded.

In this experiment, we used $220\mu F$ capacitors.

Breadboard

- Facilitates quick circuit assembly and reconfiguration.
- Simplifies testing of different cascading configurations.

4 Procedure

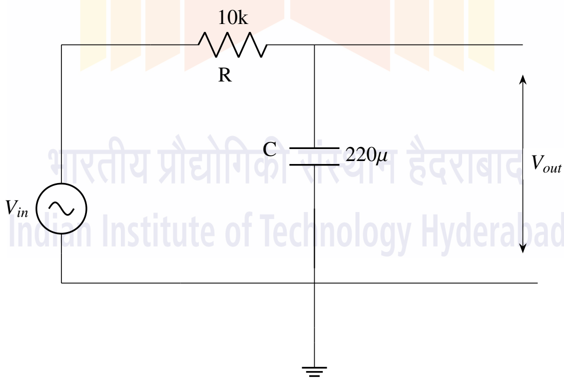
- Choose resistor (R) and capacitor (C) values to achieve a desired cutoff frequency and calculate the cutoff frequency $f_c = \frac{1}{2\pi RC}$
- Connect all the components on the breadboard and build 1-Stage, 2-Stage, 3-Stage RC Low-Pass filters
- Connect the function generator to the input of the filter.
- Connect the oscilloscope to both the input and output of the filter.
- Set the function generator to produce a sinusoidal waveform with amplitude 5V
- Start with the cutoff frequency and gradually increase the frequency in multiples of that cutoff freq.
- At each frequency, measure the output voltage using the oscilloscope.
- Calculate and record the gain for every V_{out} and plot the required bode plot

$$Gain(dB) = 20\log_{10}\left(\frac{V_{out}}{V_{in}}\right)$$

- Use the oscilloscope to measure the phase difference between the input and output signals.
- At each frequency, note the phase shift and plot the required bode plot

5 1-Stage

5.1 Circuit Diagram



5.2 Mathematical Analysis for Bode Plot

5.2.1 Magnitude Plot

The Transfer function is given by

$$H(s) = \frac{V_{out}}{V_{in}} \qquad H(s) = \frac{\frac{1}{j\omega C}}{1 + \frac{1}{j\omega C}}$$

$$H(s) = \frac{1}{1 + j\omega RC}$$

The magnitude of the transfer function:

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}}$$

For our given values of R and C,

$$RC = 2.2$$

Thus magnitude simplifies to:

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (4.84)\omega^2}}$$

The gain in decibels is given by,

$$\text{Gain} = 20\log_{10}|H(j\omega)| = -20\log_{10}\sqrt{1 + 4.84\omega^2}$$

For different values of ω:
- When $\omega \ll a$:

$$\text{Gain} \approx 0$$

- When $\omega = a$:

$$\text{Gain} \approx -20\log_{10}(\sqrt{2}) \approx -10\log_{10} 2 = -3.010299.....$$

- When $\omega \gg a$:

$$\text{Gain} \approx -10\log_{10}(2.2\omega)$$

5.2.2 Phase plot

The transfer function is given by

$$H(j\omega) = \frac{1}{1 + j\omega RC}$$

The phase is given by

$$\angle H(j\omega) = \tan^{-1}\left(\frac{\text{Imaginary part}}{\text{Real part}}\right) = \tan^{-1}(-\omega RC)$$

On substituting $R = 10k\Omega$ and $C = 220\mu F$

$$RC = 2.2$$

$$\angle H(j\omega) = \tan^{-1}(-2.2\omega)$$

- When $\omega \ll a$: Using small angle approximations $\tan^{-1}(x) = x$

$$\angle H(j\omega) \approx -2.2\omega$$

- When $\omega = a$: The phase is approximately $-45°$

$$\angle H(j\omega) = \tan^{-1}(-1) = -45°$$

- When $\omega \gg a$: The phase reaches $-90°$

$$\angle H(j\omega) \approx \tan^{-1}(-\infty) = -90°$$

Where $a = \frac{1}{RC}$, the cutoff value of ω

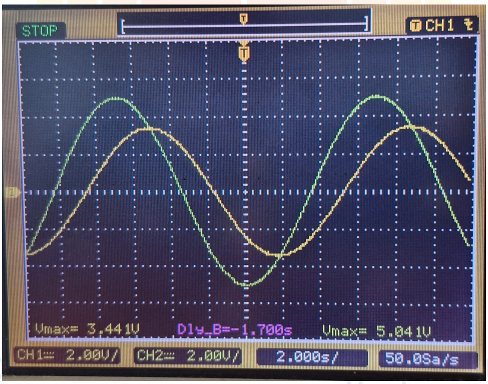
The phase changes from 0° to 90° as ω increases, it reaches -45° at $\omega = \frac{1}{RC}$

$$Phase = \begin{cases} 0°, & \omega < 0.1a \\ \text{Slope} \;\; = -45° dB/dec & \omega \approx a \\ -90°, & \omega > 10a \end{cases}$$

5.3 *Observations and Calculations*

Let us now verify the values obtained on the oscilloscope with the ones obtained on simulation

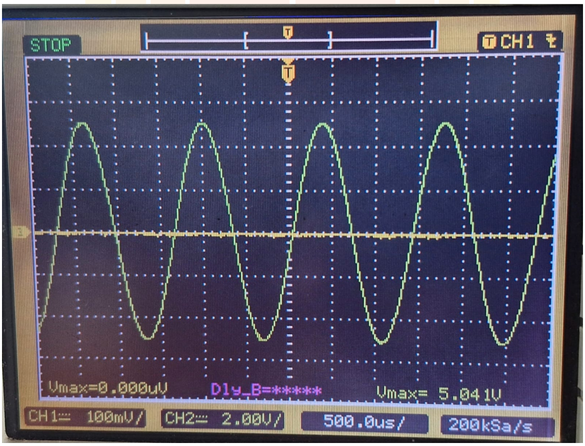
- **Frequency =** $0.0723Hz$



- Gain = -3.316
- Phase = -44.98 ≈ -45
- **Frequency =** $7.23Hz$

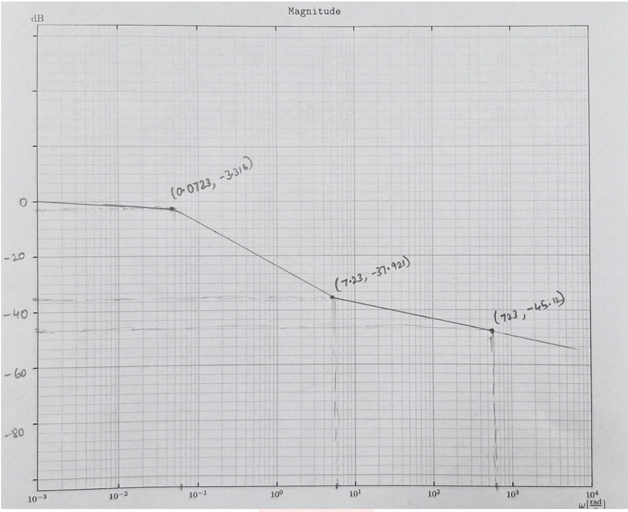
- Gain = -37.921
- Phase = -89.42

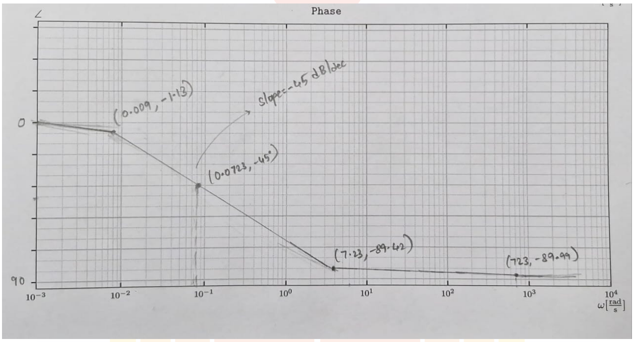
- **Frequency** = $723Hz$



- Gain ≈ -45
- Phase = -89.99 $\approx$ -90

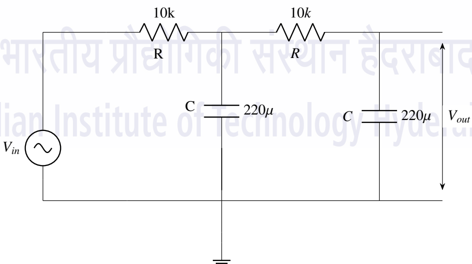
5.4 Plotting Bode Plot





6 2-Stage

6.1 Circuit Diagram



6.2 Mathematical Analysis for Bode Plot

6.2.1 Magnitude Plot

The Transfer function for a two-stage RC circuit is given by:

$$H(j\omega) = \frac{1}{(1 + j\omega RC)^2}$$

The magnitude of the transfer function is:

$$|H(j\omega)| = \frac{1}{1 + (\omega RC)^2}$$

For the given values of R and C:

$$RC = 2.2$$

Thus, the magnitude simplifies to:

$$|H(j\omega)| = \frac{1}{1 + 4.84\omega^2}$$

The gain in decibels is given by:

$$\text{Gain} = 20\log_{10}(|H(j\omega)|) = -20\log_{10}\left(1 + 4.84\omega^2\right)$$

For different values of ω:
- When $\omega \ll a$:

$$\text{Gain} \approx 0$$

- When $\omega = a$ (cutoff frequency):

$$\text{Gain} \approx -20\log_{10} 2 \approx -6.0205\,\text{dB}$$

- When $\omega \gg a$ (high frequency):

$$\text{Gain} \approx -20 \times 2\log_{10}(2.2\omega) \approx -40\log(2.2\omega)$$

6.2.2 Phase plot

For a two-stage RC circuit, the transfer function is given by

$$H(j\omega) = \frac{1}{(1 + j\omega RC)^2}$$

The phase response is

$$\angle H(j\omega) = \tan^{-1}(-\omega RC) + \tan^{-1}(-\omega RC)$$

$$\angle H(j\omega) = 2\tan^{-1}(-\omega RC)$$

On substituting $R = 10k\Omega$ and $C = 220\mu F$

$$RC = 2.2$$

$$\angle H(j\omega) = 2\tan^{-1}(-2.2\omega)$$

- When $\omega \ll a$: Using small angle approximations $\tan^{-1}(x) = x$

$$\angle H(j\omega) \approx 2(-2.2\omega) \approx -4.4\omega$$

- When $\omega = a$: The phase is approximately $-90°$

$$\angle H(j\omega) = 2\tan^{-1}(-1) = 2(-45°) = 2(-45°) = -90°$$

- When $\omega \gg a$: The phase reaches $-180°$

$$\angle H(j\omega) \approx 2\tan^{-1}(-\infty) = 2(-90°) = -180°$$

Where $a = \frac{1}{RC}$, the cutoff value of ω

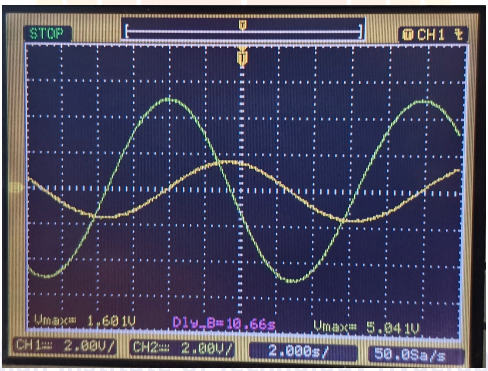
Thus, for a two-stage RC filter, the phase shifts from $0°$ to $-180°$, reaching $-90°$ at $\omega = \frac{1}{RC}$ with a steeper slope of $-90°$/decade compared to the $-45°$/decade slope of a single-stage RC filter.

$$\text{Phase} = \begin{cases} 0°, & \omega < 0.1a \\ \text{Slope} = -90° \text{ dB/dec}, & \omega \approx a \\ -180°, & \omega > 10a \end{cases}$$

6.3 *Observations and Calculations*

Let us now verify the values obtained on the oscilloscope with the ones obtained on simulation

- **Frequency** = $0.0723Hz$



- Gain = -9.967
- Phase = -89.96

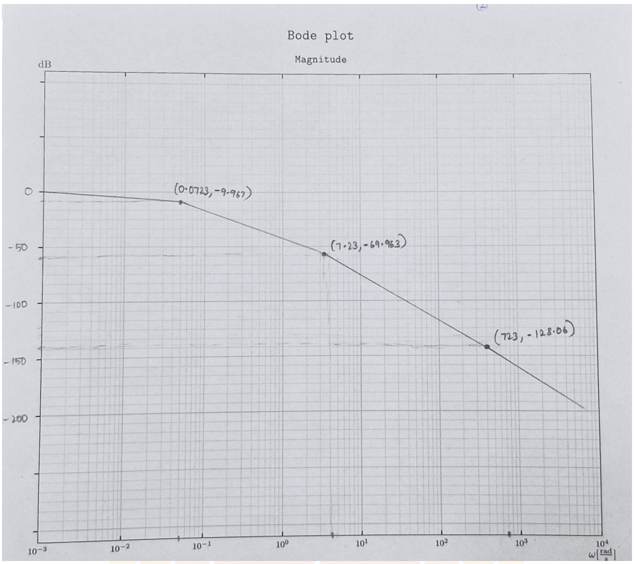
- **Frequency** = $7.23Hz$

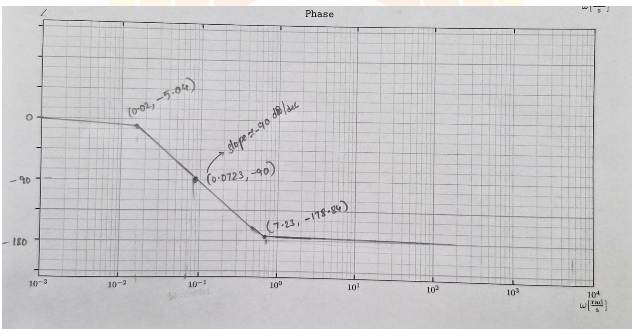
- Gain = -69.963
- Phase = -178.84

- **Frequency =** $723Hz$

- Gain ≈ -128.062
- Phase = -178.98

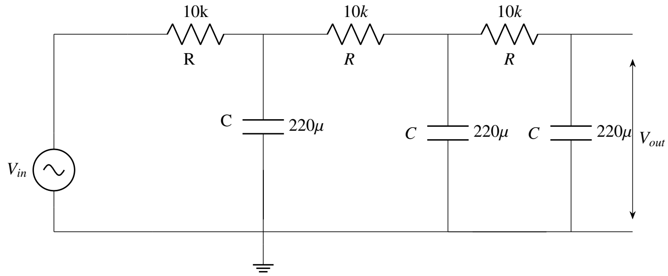
6.4 Plotting Bode Plot





7 3-Stage

7.1 Circuit Diagram



7.2 Mathematical Analysis for Bode Plot

7.2.1 Magnitude Plot for Three-Stage RC Circuit

The Transfer function for a three-stage RC circuit is given by:

$$H(j\omega) = \frac{1}{(1 + j\omega RC)^3}$$

The magnitude of the transfer function is:

$$|H(j\omega)| = \frac{1}{(1 + (\omega RC)^2)^{3/2}}$$

For the given values of R and C:

$$RC = 2.2$$

Thus, the magnitude simplifies to:

$$|H(j\omega)| = \frac{1}{(1 + 4.84\omega^2)^{3/2}}$$

The gain in decibels is given by:

$$\text{Gain} = 20\log_{10}(|H(j\omega)|) = -30\log_{10}\left(1 + 4.84\omega^2\right)$$

For different values of ω:
- When $\omega \ll a$ (low frequency):

$$\text{Gain} \approx 0$$

In this case, the gain is very close to 0 dB because the frequency is much lower than the cutoff frequency.
- When $\omega = a$ (cutoff frequency):

$$\text{Gain} \approx -30\log_{10} 2 \approx -30 \times 0.3010 = -9.03\,\text{dB}$$

- When $\omega \gg a$ (high frequency):

$$\text{Gain} \approx -60\log_{10}(2.2\omega)$$

This reflects the roll-off of the third-order filter with a slope of −60 dB/decade.

7.2.2 *Phase plot*

For a three-stage RC circuit, the transfer function is given by

$$H(j\omega) = \frac{1}{(1 + j\omega RC)^3}$$

The phase response is

$$\angle H(j\omega) = \tan^{-1}(-\omega RC) + \tan^{-1}(-\omega RC) + \tan^{-1}(-\omega RC)$$

$$\angle H(j\omega) = 3\tan^{-1}(-\omega RC)$$

On substituting $R = 10k\Omega$ and $C = 220\mu F$

$$RC = 2.2$$

$$\angle H(j\omega) = 3\tan^{-1}(-2.2\omega)$$

- When $\omega \ll a$: Using small angle approximations $\tan^{-1}(x) = x$

$$\angle H(j\omega) \approx 3(-2.2\omega) \approx -6.6\omega$$

- When $\omega = a$: The phase is approximately $-135°$

$$\angle H(j\omega) = 3\tan^{-1}(-1) = 3(-45°) = -135°$$

- When $\omega \gg a$: The phase reaches $-180°$

$$\angle H(j\omega) \approx 3\tan^{-1}(-\infty) = 3(-90°) = -270°$$

Where $a = \frac{1}{RC}$, the cutoff value of ω

Thus, for a three-stage RC filter, the phase shifts from 0° to −270°, reaching −135° at $\omega = \frac{1}{RC}$ with a steeper slope of −135°/decade compared to the −90°/decade slope of a two-stage RC filter.

$$\text{Phase} = \begin{cases} 0°, & \omega < 0.1a \\ \text{Slope} = -135° \text{ dB/dec}, & \omega \approx a \\ -270°, & \omega > 10a \end{cases}$$

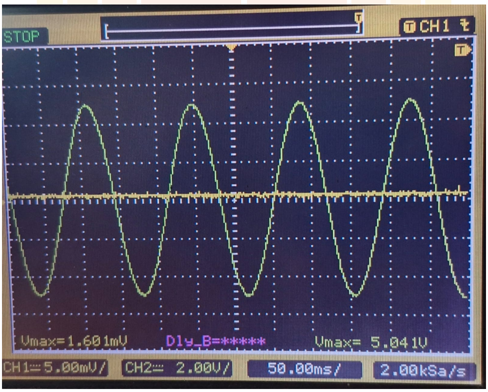
7.3 *Observations and Calculations*

Let us now verify the values obtained on the oscilloscope with the ones obtained on simulation

- **Frequency** = $0.0723Hz$

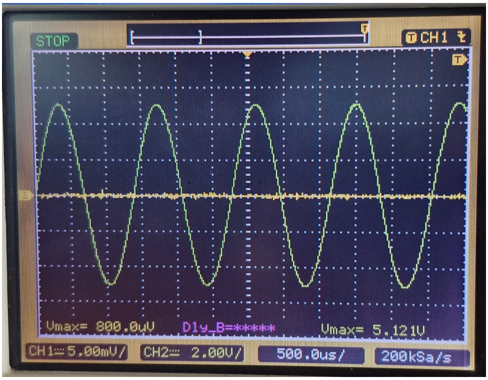
- Gain = -15.98
- Phase = -134.94

- **Frequency** = $7.23Hz$



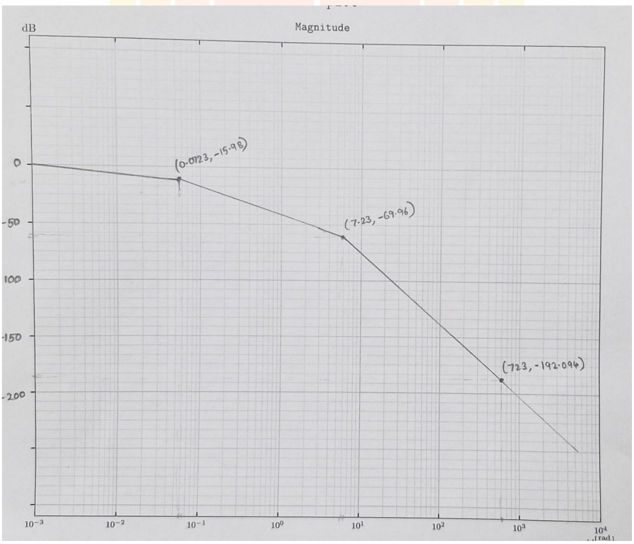
- Gain = -69.96
- Phase = -268.26

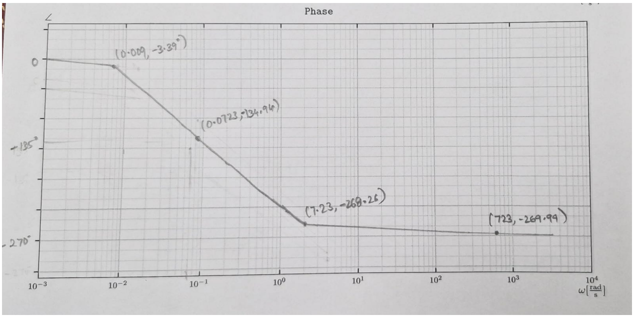
- **Frequency** = $723Hz$



- Gain = -192.094
- Phase = -269.97

7.4 Plotting Bode Plot





Phase
0
-135°
-270°
(0.009, -3.39°)
(0.0723, -134.96)
(7.23, -268.26)
(723, -269.99)
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
10^{4}
$\omega[\frac{rad}{s}]$