9-9.2-20

EE24BTECH11066 - YERRA AKHILESH

Question:
Find the area of the region bounded by the curves $y = x^2 + 2$ $y = x$, $x = 0$ and $x = 3$.
solution: The parameters of the conic are

Equations
$y = x^2 + 2$
$y = x$
$x = 0$
$x = 3$

TABLE 0: Given Equations

Conic	**Parameters**
V	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
u	$\frac{-1}{2}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$
f	0

The equation of a parabola in Matrix form is

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{0.1}$$

The equation of a line in vector form is

$$\mathbf{x} = \mathbf{h} + \kappa \mathbf{m} \tag{0.2}$$

For the given parabola $y = x^2 + 2$, The values of $\mathbf{V}$,$\mathbf{u}$,f are

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{0.3}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix} \tag{0.4}$$

$$f = -2 \tag{0.5}$$

For the given line $x = 0$, The values of $\mathbf{h_1}$, $\mathbf{m_1}$ are

$$\mathbf{h_1} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.6}$$

$$\mathbf{m_1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{0.7}$$

Substituing $x = 0$ line equation in parabola equation gives the values of κ

$$(\mathbf{h} + \kappa\mathbf{m})^\top \mathbf{V} (\mathbf{h} + \kappa\mathbf{m}) + 2\mathbf{u}^\top (\mathbf{h} + \kappa\mathbf{m}) + f = 0 \tag{0.8}$$

$$\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \kappa \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right)^\top \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \kappa \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) + 2 \begin{pmatrix} 0 \\ \frac{-1}{2} \end{pmatrix}^\top \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} + \kappa \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right) + -2 = 0 \tag{0.9}$$

$$\begin{pmatrix} \kappa & \kappa \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \kappa \\ \kappa \end{pmatrix} + 2 \begin{pmatrix} 0 & \frac{-1}{2} \end{pmatrix} \begin{pmatrix} \kappa \\ \kappa \end{pmatrix} = 2 \tag{0.10}$$

$$\begin{pmatrix} \kappa & \kappa \end{pmatrix} \begin{pmatrix} \kappa \\ 0 \end{pmatrix} - (\kappa) = 2 \tag{0.11}$$

$$\kappa^2 - \kappa = 2 \tag{0.12}$$

$$\kappa_1 = 2 \tag{0.13}$$

$$\kappa_2 = -1 \tag{0.14}$$

The intersection points are

$$\mathbf{x_1} = \mathbf{h} + \kappa_1 \mathbf{m} \tag{0.15}$$

$$\mathbf{x_1} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \tag{0.16}$$

By taking k_2 as negative, the point of intersection will be below the x-axis but the given parabola is above the x-axis so we neglect that point of intersection.
similarly for $x = 3$ line intersection points are

$$\mathbf{x_2} = \begin{pmatrix} 3 \\ 11 \end{pmatrix} \tag{0.17}$$

The Area under the curve $y = x^2 + 2$ is given by

$$A_1 = \int_0^3 \left(x^2 + 2\right) dx \tag{0.18}$$

$$A_1 = \left(\frac{x^3}{3} + 2x\right)\Big|_0^3 \tag{0.19}$$

$$A_1 = \left(\frac{3^3}{3} + 2 \cdot 3\right) - (0) \tag{0.20}$$

$$A_1 = (9 + 6) \tag{0.21}$$

$$A_1 = 15 \tag{0.22}$$

The Area under the line $y = x$ is given by

$$A_2 = \int_0^3 (x)\,dx \tag{0.23}$$

$$A_2 = \left(\frac{x^2}{2}\right)\Big|_0^3 \tag{0.24}$$

$$A_2 = \frac{3^2}{2} - 0 \tag{0.25}$$

$$A_2 = \frac{9}{2} \tag{0.26}$$

$$A_2 = 4.5 \tag{0.27}$$

The area of region bounded by the line $x = y$ and the parabola $y = x^2 + 2$ is given by

$$A = A_1 - A_2 \tag{0.28}$$

$$A = 15 - 4.5 \tag{0.29}$$

$$A = 10.5 \tag{0.30}$$

The area of region bounded by the line $x = y$ and the parabola $y = x^2 + 2$ is 10.5.