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EE24BTECH11066 - YERRA AKHILESH

1) Consider $\mathbb{R}^2$ with the usual topology. Let $S = \left\{(x, y) \in \mathbb{R}^2 : x \text{ is an integer}\right\}$. Then S is
   a) open but NOT closed
   b) both open and closed
   c) neither open nor closed
   d) closed but NOT open
2) Suppose $X = \{\alpha, \beta, \delta\}$. Let
$$\mathscr{T}_1 = \{\phi, X, \{\alpha\}, \{\alpha, \beta\}\} \text{ and} \mathscr{T}_2 = \{\phi, X, \{\alpha\}, \{\beta, \delta\}\}.$$
   Then
   a) both $\mathscr{T}_1 \cap \mathscr{T}_2$ and $\mathscr{T}_1 \cup \mathscr{T}_2$ are topologies
   b) neither $\mathscr{T}_1 \cap \mathscr{T}_2$ nor $\mathscr{T}_1 \cup \mathscr{T}_2$ is a topology
   c) $\mathscr{T}_1 \cup \mathscr{T}_2$ is a topology but $\mathscr{T}_1 \cap \mathscr{T}_2$ is NOT a topology
   d) $\mathscr{T}_1 \cap \mathscr{T}_2$ is a topology but $\mathscr{T}_1 \cap \mathscr{T}_2$ is NOT a topology
3) For a positive integer n, let $f_n : \mathbb{R} \to \mathbb{R}$ be defined by
$$f_n(x) = \begin{cases} \frac{1}{4n+5}, & \text{if } 0 \le x \le n, \\ 0, & otherwise. \end{cases}$$
   Then $\{f_n(x)\}$ converges to zero
   a) uniformly but NOT in L^1 norm
   b) uniformly and also in L^1 norm
   c) pointwise but NOT uniformly
   d) in L^1 norm but NOT pointwise
4) Let P_1 and P_2 be two projection operators on a vector space. Then
   a) $P_1 + P_2$ is a projection if $P_1P_2 = P_2P_1 = 0$
   b) $P_1 - P_2$ is a projection if $P_1P_2 = P_2P_1 = 0$
   c) $P_1 + P_2$ is a projection
   d) $P_1 - P_2$ is a projection
5) Consider the system of linear equations
$$x + y + z = 3$$
$$x - y - z = 4$$
$$x - 5y + kz = 6$$
   Then the value of k for which this system has an infinite number of solutions is

a) $k = -5$ b) $k = 0$ c) $k = 1$ d) $k = 3$

6) Let

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ x & y & z \end{pmatrix}$$

and let $V = \left\{(x, y, z) \in \mathbb{R}^3 : det\,(A) = 0\right\}$. Then the dimension of V equals

a) 0 b) 1 c) 2 d) 3

7) Let $S = \{0\} \cup \left\{\frac{1}{4n+7} : n = 1, 2, \ldots\right\}$. Then the number of analytic functions which vanish only on S is

a) infinite b) 0 c) 1 d) 2

8) It is given that $\sum_{n=0}^{\infty} a_n z^n$ converges at $z = 3 + i4$. Then the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n z^n$ is

a) ≤ 5 b) ≥ 5 c) < 5 d) > 5

9) The value of α for which $G = \{\alpha, 1, 3, 9, 19, 27\}$ is a cyclic group under multiplication modulo 56 is

a) 5 b) 15 c) 25 d) 35

10) Consider $\mathbb{Z}_{24}$ as the additive group modulo 24. Then the number of elements of order 8 in the group $\mathbb{Z}_{24}$ is

a) 1 b) 2 c) 3 d) 4

11) Define $f : \mathbb{R}^2 \to \mathbb{R}$ by

$$f(x, y) = \begin{cases} 1, & \text{if } xy = 0, \\ 2, & \textit{otherwise}. \end{cases}$$

If $S = \{(x, y) : \text{f is continuous at the point}\,(x, y)\}$, then

a) S is open c) $S = \phi$
b) S is connected d) S is closed

12) Consider the linear programming problem,

$$Max.z = c_1 x_1 + c_2 x_2, c_1, c_2 > 0, \text{ subject to}$$

$$\begin{aligned} x_1 + x_2 &\leq 3 \\ 2x_1 + 3x_2 &\leq 4 \\ x_1, x_2 &\geq 0. \end{aligned}$$

Then,

a) the primal has an optimal solution but the dual does NOT have an optimal solution
b) both the primal and the dual have optimal solutions
c) the dual has an optimal solution but the primal does NOT have an optimal solution
d) neither the primal nor the dual have optimal solutions

13) Let $f(x) = x^{10} + x - 1, x \in \mathbb{R}$ and let $x_k = k, k = 0, 1, 2, \ldots, 10$. Then the value of the divided difference $f[x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_10]$ is

a) -1 b) 0 c) 1 d) 10

14) Let X and Y be jointly distributed random variables having the joint probability density function

$$f(x, y) = \begin{cases} \frac{1}{\pi}, & \text{if } x^2 + y^2 \leq 1, \\ 0, & otherwise. \end{cases}$$

Then $P(Y > max(X, -X)) =$

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) $\frac{1}{6}$

15) Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed chi-square random variables, each having 4 degrees of freedom. Define $S_n = \sum_{i=1}^{n} X_i^2, n = 1, 2, \ldots$. If $\frac{S_n}{n} \xrightarrow{P} \mu$, as $n \to \infty$, then $\mu =$

a) 8 b) 16 c) 24 d) 32

16) Let $\{E_n : n = 1, 2, \ldots\}$ be decreasing sequence of Lebesgue measurable sets on $\mathbb{R}$ and let F be a Lebesgue measurable set on $\mathbb{R}$ such that $E_1 \cap F = \mu$. Suppose that F has Lebesgue measure 2 and the Lebesgue measure of E_n equals $\frac{2n+2}{3n+1}, n = 1, 2, \ldots$. Then the Lebesgue measure of the set $\left(\cap_{n=1}^{\infty} E_n\right) \cup F$ equals

a) $\frac{5}{3}$ b) 2 c) $\frac{7}{3}$ d) $\frac{8}{3}$

17) The extremum for the variational problem

$$\int_0^{\frac{\pi}{8}} \left((y')^2 + 2yy' - 16y^2\right) \mathrm{dx}, y(0) = 0, y\left(\frac{\pi}{8}\right) = 1,$$

occurs for the curve

a) $y = \sin(4x)$

b) $y = \sqrt{2}\sin(2x)$

c) $y = 1 - \cos(4x)$

d) $y = \frac{1-\cos(8x)}{2}$