

# **APPLE STOCK PRICE FORECASTING**



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## Introduction:

**A little bit about the company's stock, its value and information from a stakeholder's point of view:**

### **About Apple:**

- Founded by Steve Jobs, Steve Wozniak, and Ronald Wayne in April 1976.
- Initial purpose was to sell Wozniak's Apple I personal computer.
- Went public in 1980 to instant financial success.
- Launched the first iPhone in 2007.

Our project is on Apple INC. stocks from the years- 2007 to 2018. We chose this dataset for our project to understand underlying trends, seasonality and forecasts of this trillion-dollar worth, American multinational technology firm that designs, develops, and sells consumer electronics, computer software, and online services.

### **Trivia:**

- If you invested in Apple a decade ago, you'd probably be feeling pretty good about it today. According to CNBC calculations, a \$1,000 investment made in early August 2008 would be worth more than \$9,222.50 as of August 2, 2018, or over nine times as much, including price appreciation and excluding dividends.
- There was a stock split of 7:1, that happened on June 9<sup>th</sup>, 2014. The Kaggle dataset that we started with had values that were not adjusted for this split and hence there was a break in the graph. To avoid this we switched to the Quantmod dataset which had already divided historic values by 7 and hence had no breaks.
- The challenge was to download the Quantmod data into excel for data massaging which we did by the following commands:

```
library(quantmod)
getSymbols("AAPL")
library(WriteXLS)
dataFrame <- data.frame(AAPL)
WriteXLS::testPerl()#This needed Perl installation as well!
WriteXLS(dataFrame, ExcelFileName = "aapl.xlsx",row.names = TRUE )
```

## About our dataset:

Date	AAPL.Open	AAPL.High	AAPL.Low	AAPL.Close		
2007-01-03	12.327143	12.368571	11.7	11.971429		
2007-01-04	12.007143	12.278571	11.974286	12.237143		
2007-01-05	12.252857	12.314285	12.057143	12.15		
2007-01-08	12.28	12.361428	12.182858	12.21		
2007-01-09	12.35	13.282857	12.164286	13.224286		
2007-01-10	13.535714	13.971429	13.35	13.857142		
2007-01-11	13.705714	13.825714	13.585714	13.685715		
2007-01-12	13.512857	13.58	13.318571	13.517143		
2007-01-16	13.668571	13.892858	13.635715	13.871428		
2007-01-17	13.937143	13.942857	13.545714	13.564285		
2007-01-18	13.157143	13.158571	12.721429	12.724286		
2007-01-19	12.661428	12.807143	12.588572	12.642858		
2007-01-22	12.734285	12.737143	12.235714	12.398571		
2007-01-23	12.247143	12.501429	12.215714	12.242857		
2007-01-24	12.382857	12.45	12.297143	12.385715		
2007-01-25	12.444285	12.642858	12.29	12.321428		
2007-01-26	12.444285	12.481428	12.141429	12.197143		
2007-01-29	12.328571	12.378572	12.218572	12.277143		
2007-01-30	12.347143	12.355714	12.178572	12.221429		
AAPL.Close_Updated		quantMod AAPL Data		AnnualData_253		

- Our dataset consists of stocks from January 2007 to November, 2018. It had 3001 rows originally.
- To ensure a consist frequency across years we found the year with maximum number of trading days i.e. 253 (2008 which was a leap year) and then strategically added the closing value of previous trading day to the next non-trading day for one or more dates as needed to make all years have 253 records. Original data had mix of 250, 251, 252 and 253. The modified data to match frequency of 253 finally had 3015 records.
- For analysis purpose, we divided the data into two parts:  
**Training data** which consists of 80% of our data and **Test Data** which is 20% of the dataset.

### Code:

#### 80% Train - Fit model

**20% Test; Forecast values and use accuracy function to compute errors, model which will give least error is best model**

```
library(fpp2)
library(readxl)
```

**Here we are Importing Apple dataset in projectData variable:**

```
projectData <- read_excel("C:/Users/user/Desktop/Business Forecasting/BF  
Project/aapl.xlsx",sheet = "AAPL.Close_Updated")
```

**Extracting adjusted close price in appl\_close variable:**

```
appl_close <- projectData$AAPL.Close
```

**Making close price of Apple(appl\_close) into time-series data with frequency 253:**

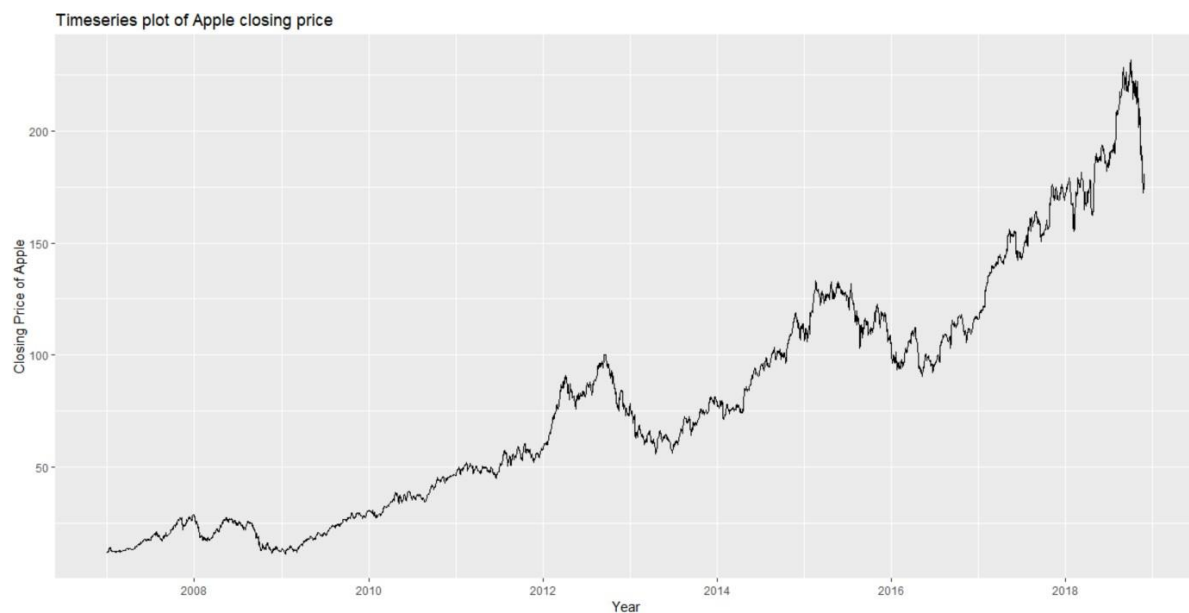
**Frequency is 253 as in one year we have only 253 data entries(records)**

```
appl_close_ts <- ts(data=appl_close, frequency=253, start=c(2007,1))
```

**Plotting close price to observe the behaviour of the data:**

```
autoplot(appl_close_ts) + xlab("Year") + ylab("Closing Price of Apple")+  
ggtitle("Timeseries plot of Apple closing price")
```

**OUTPUT:**



## Data analysis:

To deep dive into the data, we have used the following techniques and methodologies:

1. Autocorrelation
2. STL Decomposition
- Forecasting Models:**
3. Mean Model
4. Naïve Model
5. Seasonal Naïve Model
6. Drift Model
7. Regression Model
8. Holts Linear Model
9. ARIMA Model

### 1. Autocorrelation

Covariance and Correlation measures the extent of a linear relationship between two variables.

Autocorrelation measures the linear relationship between lagged values of a time series.

There are several autocorrelation coefficients, depending on the lag length.

For example, we can measure the relationship between  $y_t$  and

$y_{t-1}$ . Or relationship between  $y_t$  and  $y_{t-2}$ , and so on.

Auto correlation is calculated as follows:

$$r_k = \frac{\sum_{t=k+1}^T (y_t - \bar{y})(y_{t-k} - \bar{y})}{\sum_{t=1}^T (y_t - \bar{y})^2},$$

Where  $r_k$  is the relationship between  $y_t$  and  $y_{t-k}$  and T is length of the series.

Assumption: The observations are equi-spaced.

#### Autocorrelation Function:

The bar in the ACF indicates the autocorrelation, with values between -1 and 1. The first bar indicates how successive values of y relate to each other. The second bar indicates how y values two periods apart relate to each other. The kth bar is almost the same as the sample correlation between  $y_t$  and  $y_{t-k}$ .

Autocorrelation is defined over different lags, we can plot out the ACF. The plot shows that significant autocorrelation exists in this data.

#### Trend:

A trend exists when there is a long-term increase or decrease in the data. It does not have to be linear. Sometimes we will refer to a trend “changing direction” when it might go from an increasing trend to a decreasing trend.

**Seasonal:**

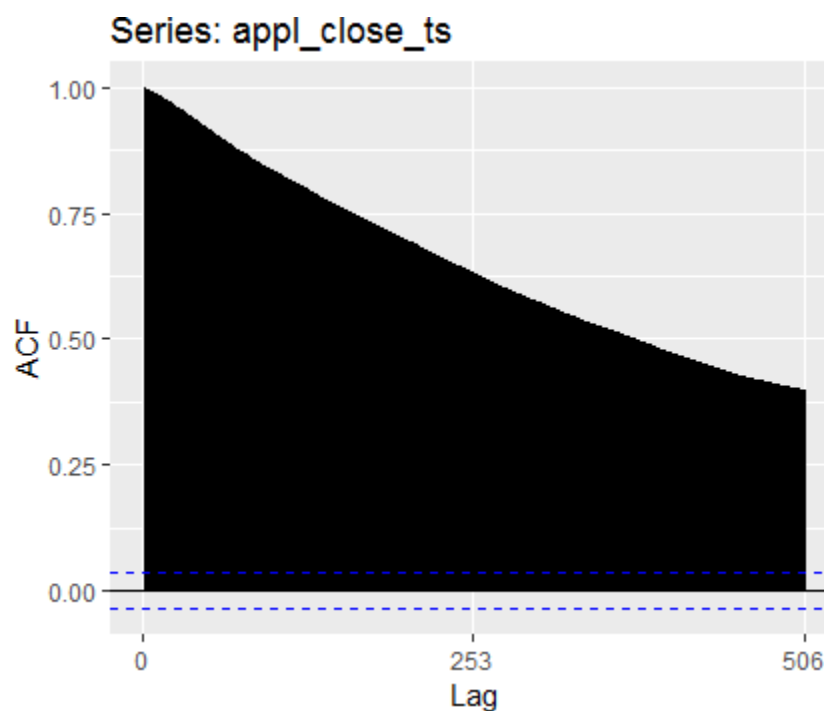
A seasonal pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week). Seasonality is always of a fixed and known period. Hence, seasonal time series are sometimes called periodic time series.

**Cyclic:**

A cyclic pattern exists when data exhibit rises and falls that are not of fixed period. The duration of these fluctuations is usually of at least 2 years.

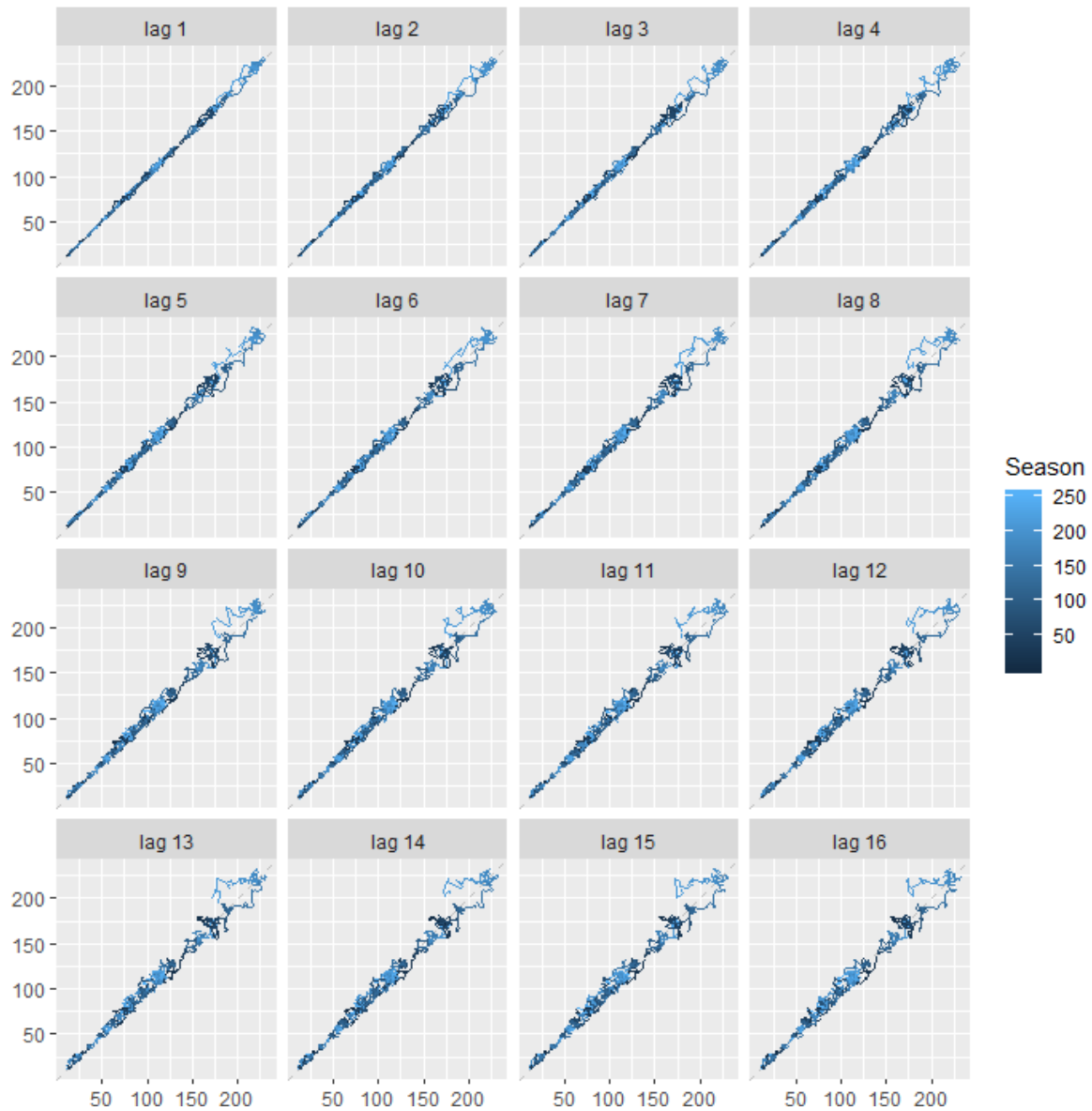
**Code for Autocorrelation:****Autocorrelation to find the trend and seasonality:**

```
ggAcf(appl_close_ts)#Constant decrease in ACF is due to trend.
```

**OUTPUT:**

**Lag plots:**  
`gglagplot(appl_close_ts)`

**OUTPUT:**



NOTE: As seen above there is no seasonality in evident from the first several lags.



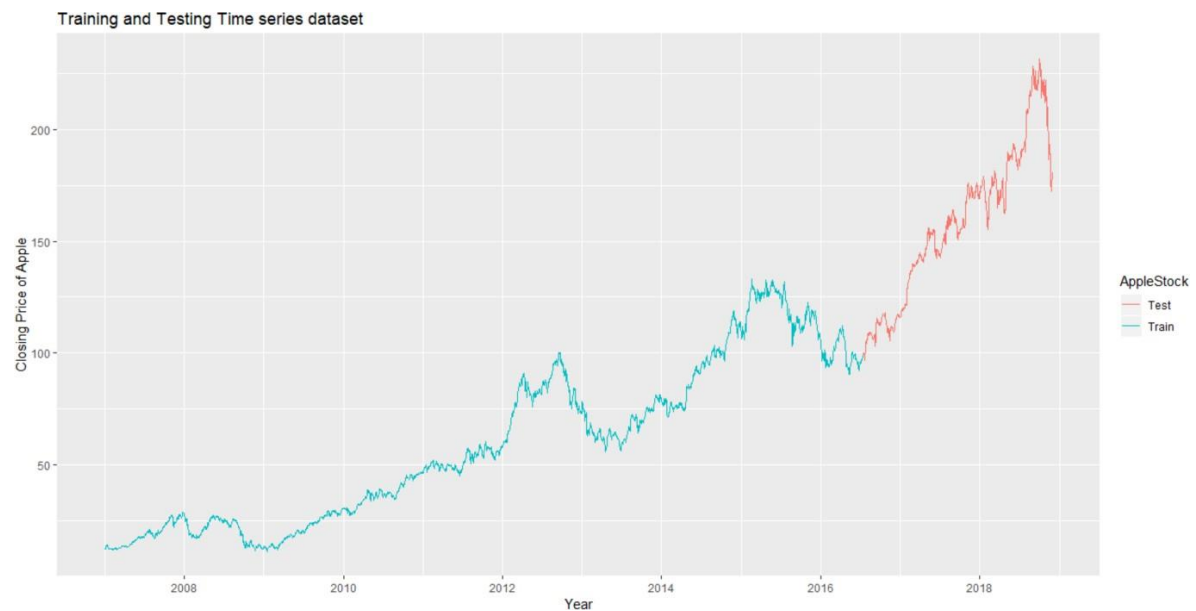
**Splitting data in Training and Testing (80:20) set for forecasting:**  
**3015 total records, 2412 training and 603 is test set so we will use h=603**

```
appl_training <- window(appl_close_ts, start = c(2007,1), end=c(2016,135))  
appl_testing <- window(appl_close_ts, start= c(2016,136), end=c(2018,232))
```

**Plotting Training and testing set:**

```
autoplot(appl_training,series="Train") +  
autolayer(appl_testing,series="Test") +  
xlab("Year") + ylab("Closing Price of Apple") +  
ggtitle("Training and Testing Time series dataset") +  
guides(colour=guide_legend(title="AppleStock"))
```

**OUTPUT:**



## 2. STL Decomposition

- We are decomposing apple close price of the stocks to analyze Trend- Cycle and Seasonality.
- We have used the additive model in this case as the additive model is useful when the seasonal variation is relatively constant over time.
- Additive model

Additive:  $y_t = \text{Seasonal} + \text{Trend} + \text{Random}$

$$\square\square = \square\square + \square\square + \square\square$$

Decomposition procedures are used in time series to describe the trend and seasonal factors in a time series. More extensive decompositions might also include long-run cycles, holiday effects, day of week effects and so on. Here, we'll only consider trend and seasonal decompositions.

### Code for STL Model:

**Decomposing apple close price of stock to analyze Trend- Cycle and Seasonality using STL:**

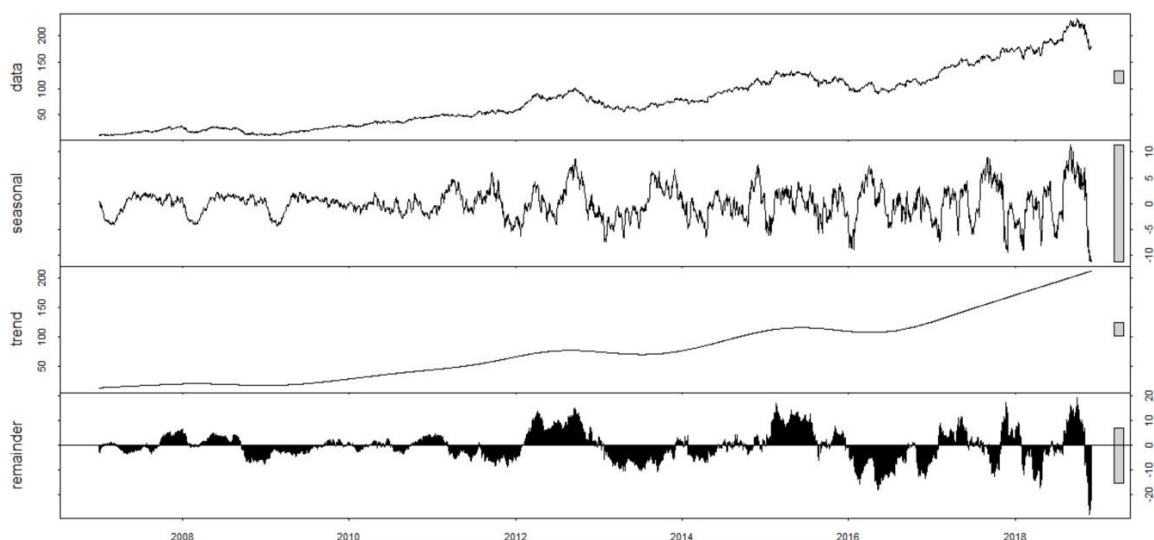
**s.window is the number of consecutive years to be used in estimating each value in the seasonal component. The user must specify s.window as there is no default. Setting it to be infinite is equivalent to forcing the seasonal component to be periodic (i.e., identical across years).**

```
appl_stl<-stl(appl_close_ts, s.window = 5)
```

**STL decomposed components plot:**

```
plot(appl_stl)
```

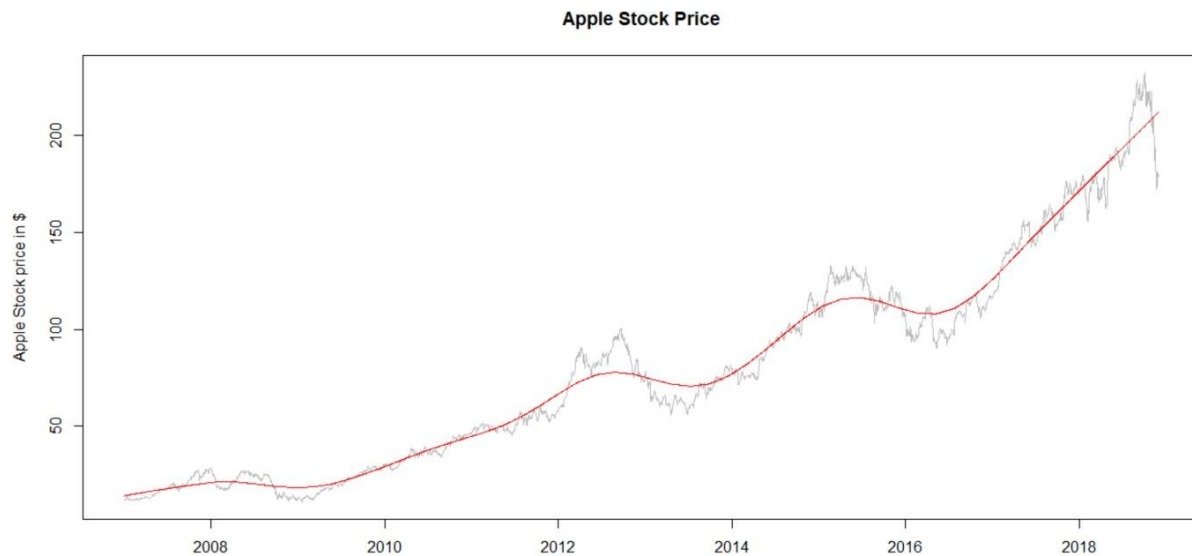
### OUTPUT:



### Data plotted with Trend component:

```
plot(appl_close_ts, col="gray",  
     main="Apple Stock Price",  
     ylab="Apple Stock Price in $", xlab="Time")  
lines(appl_stl$time.series[, "trend"], col="red", ylab="Trend")
```

### OUTPUT:



### Seasonal adjusted plot:

```
seasadj(appl_stl)
```

### OUTPUT:

Time Series:

Start = c(2007, 1)

End = c(2018, 232)

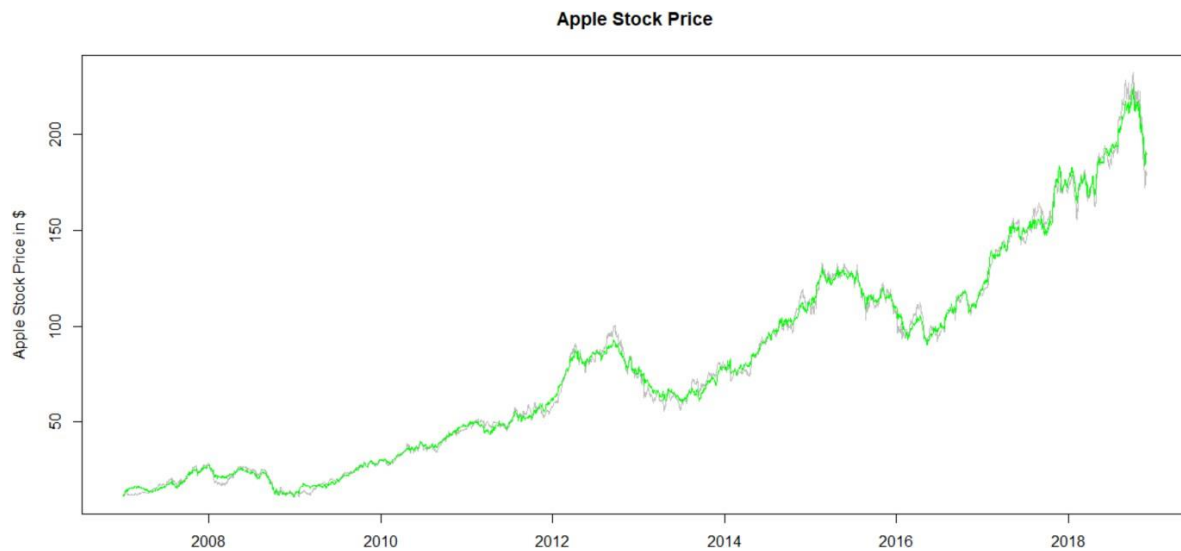
Frequency = 253

```
[1] 13.46080 14.00527 14.66148 14.70105 14.35892 15.27825 15.92082 16.04933  
15.68166 16.73136 16.25500  
[12] 15.48673 15.44751 14.86352 15.05120 16.06401 16.93740 16.62235 16.55765  
17.05807 16.37420 16.07586  
[23] 16.65233 16.83053 16.20228 16.36999 16.30102 15.32146 14.98928 14.18091  
13.79171 13.70429 13.86069  
[34] 14.13232 14.69109 14.78618 14.77876 14.23611 13.67845 13.04997 13.41594  
13.63577 13.31843 13.70172  
[45] 14.50298 14.21896 14.07125 13.82866 14.01929 13.67230 13.55709 13.54164  
13.22786 13.65861 13.85959  
[56] 14.19117 14.55917 14.07408 14.13888 14.25045 13.77179 14.30643 14.05315  
13.52458 13.33182 14.21723  
[67] 14.28527 13.78272 13.81064 13.63415 13.78945 13.80178 14.33279 14.43393  
14.52101 15.16500 15.14578  
[78] 15.39406 15.00174 15.04787 15.80222 15.59214 15.69464 14.61712 14.77858  
14.23301 14.27756 14.76878  
[89] 14.42155 14.68063 14.86220 15.24509 15.37488 15.86979 15.47636 16.02139  
15.86903 15.79537 15.78338
```

```
[100] 15.46777 14.70901 15.19751 15.24460 15.61768 15.73738 15.46351 15.71286  
15.93292 16.40786 17.10174  
[ reached getOption("max.print") – omitted several entries]
```

```
plot(appl_close_ts, col="grey",  
     main="Apple Stock Price",  
     ylab="AAPL Sotck price in $", xlab="Time")  
lines(seasadj(appl_stl),col="green",ylab="Seasonally adjusted")
```

## OUTPUT:



### 3. Mean Model

The forecasts of all future values are equal to the mean of the historical data.

If we let the historical data be denoted by  $\square_1, \dots, \square_T$ , then we can write the forecasts as

$$\hat{y}_{T+h|T} = \bar{y} = (y_1 + \dots + y_T)/T.$$

`meanf(y, h)`

# y contains the time series

# h is the forecast horizons

#### Code for Mean Model:

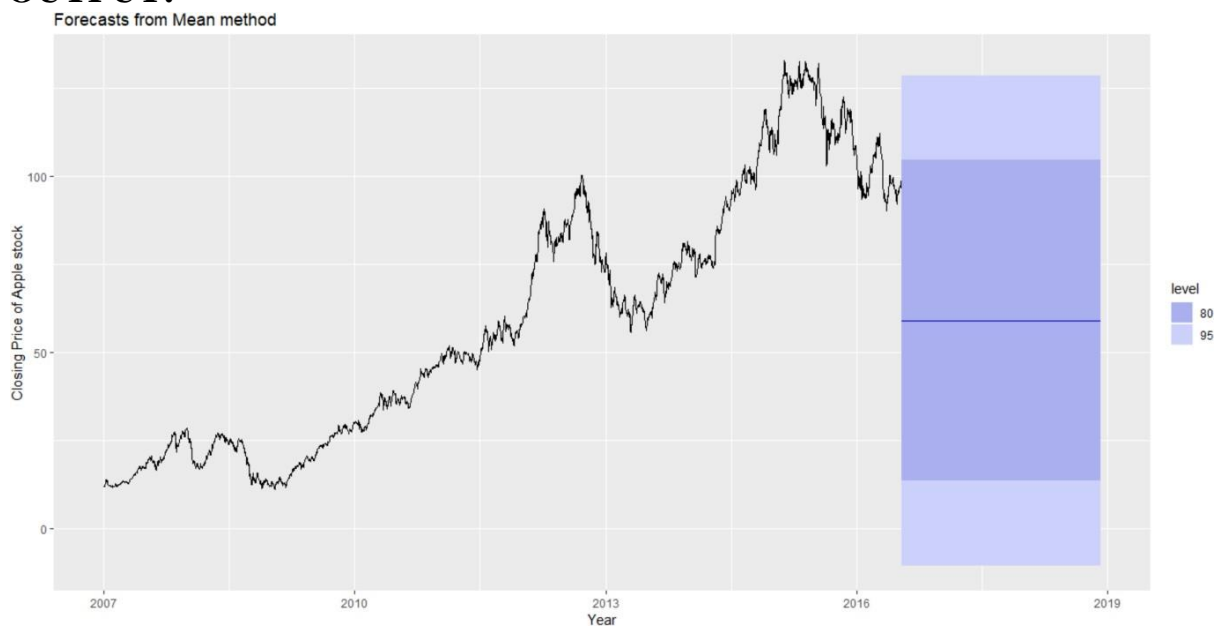
#### Train Mean model of Closing Price:

```
appl_mean <- meanf(appl_training,h=603)
```

#### Forecast of Apple Closing Price using Mean method:

```
autoplot(appl_mean) +  
  xlab("Year") + ylab("Closing Price of Apple stock") +  
  ggtitle("Forecasts from Mean method")
```

#### OUTPUT:



#### Check Residuals:

```
checkresiduals(appl_mean)
```

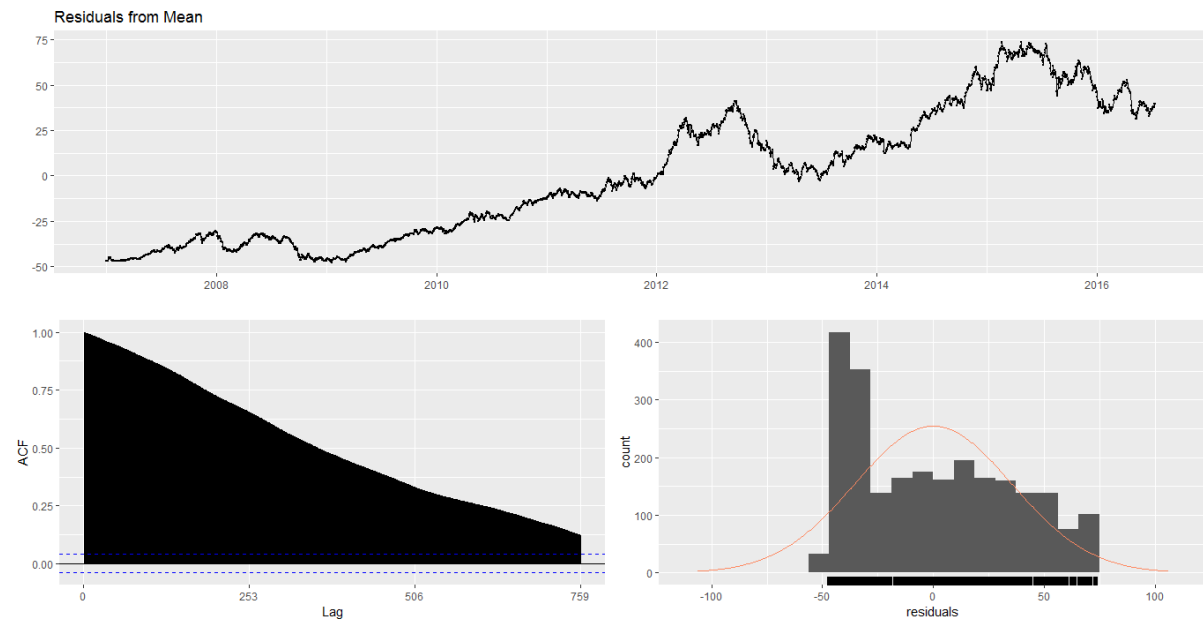
#### OUTPUT:

Ljung-Box test

data: Residuals from Mean

$Q^* = 615100$ ,  $df = 481.4$ ,  $p\text{-value} < 2.2e-16$

Model df: 1. Total lags used: 482.4



NOTE: As seen above the residuals are not randomly distributed. There is a clear trend in the residuals and no normal distribution in histogram. Therefore, this is not a good model for our dataset.

### Compute forecast accuracy measures of mean method:

`accuracy(appl_mean, appl_testing)`

### OUTPUT:

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	-1.108569e-15	35.40613	30.85103	-62.64105	93.14712	1.639193	0.9988489	NA
Test set	9.931478e+01	104.83518	99.31478	60.88951	60.88951	5.276846	0.9946694	44.24462

## 4. Naïve Model

For naïve forecasts, we simply set all forecasts to be the value of the last observation. This model is optimal for efficient stock markets as closing price for the stocks is the starting price for the next day.

Forecasts:  $\hat{y}_{n+h|n} = y_n$

### Code for Naïve Model:

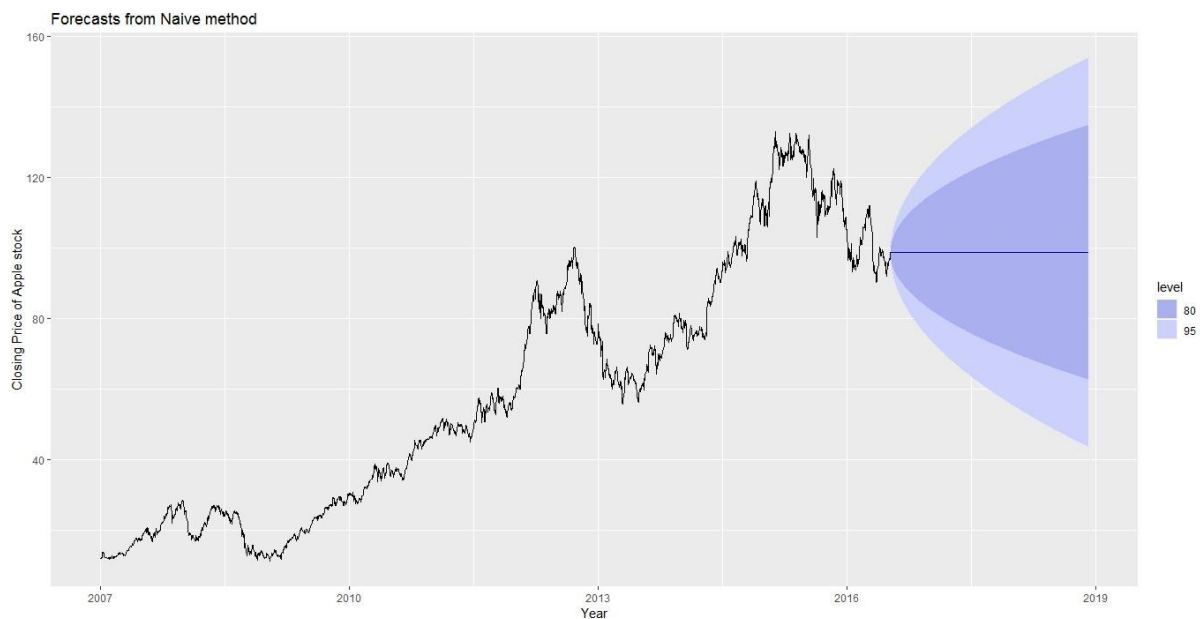
#### Naive model on training data of Closing Price:

```
appl_naive <- rwf(appl_training,h=603)
```

#### Forecast of Apple Closing Price using Naive method:

```
autoplot(appl_naive) +  
  xlab("Year") + ylab("Closing Price of Apple stock") +  
  ggtitle("Forecasts from Naive method")
```

### OUTPUT:



### Check Residuals:

```
checkresiduals(appl_naive)
```

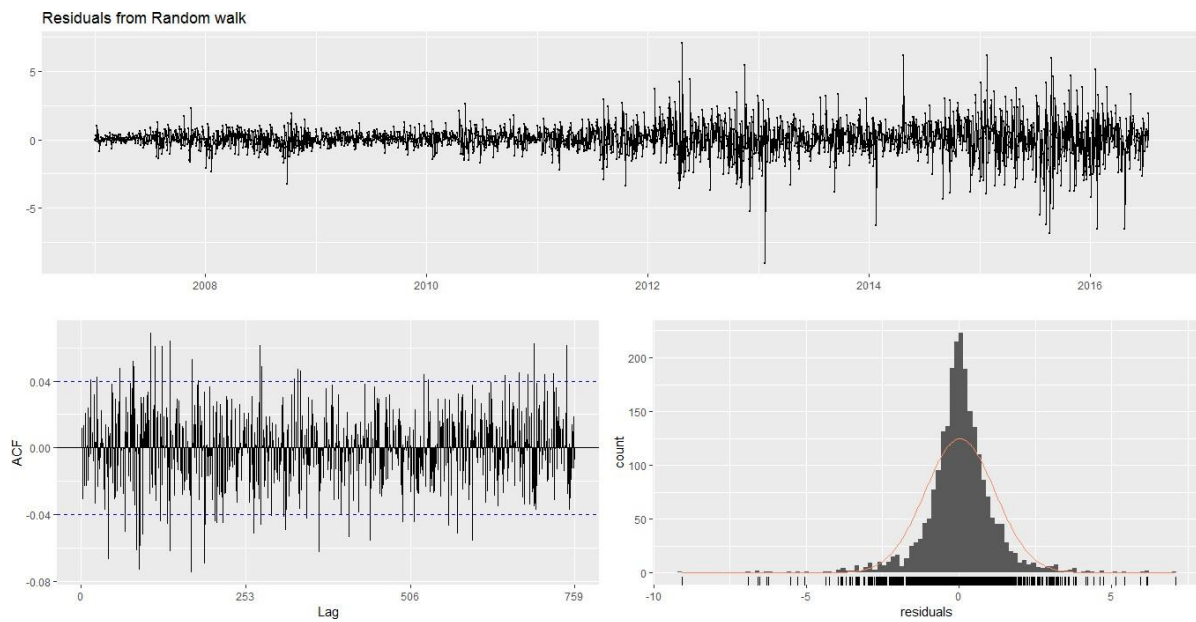
### OUTPUT:

Ljung-Box test

data: Residuals from Random walk

$Q^* = 701.68$ ,  $df = 482.4$ ,  $p\text{-value} = 2.371e-10$

Model df: 0. Total lags used: 482.4



## Computing forecast accuracy measures of Naive method:

`accuracy(appl_naive, appl_testing)`

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	0.03600521	1.145799	0.7558108	0.0652478	1.47613	0.04015814	0.01164376	NA
Test set	59.63733107	68.436841	59.6495035	34.6333753	34.64592	3.16932919	0.99466939	27.03928



## 5. Seasonal Naive Model

We set each forecast to be equal to the last observed value from the same season of the year (e.g., the same month of the previous year).

Formally, the forecast for time  $T+h$  is written as

$$\hat{y}_{T+h|T} = y_{T+h-m(k+1)}$$

Where  $m$  = the seasonal period and  $k$  is the integer part of  $(h-1)/m$  (i.e., the number of complete years in the forecast period prior to time  $T+h$ )

### Code for Seasonal Naive Model:

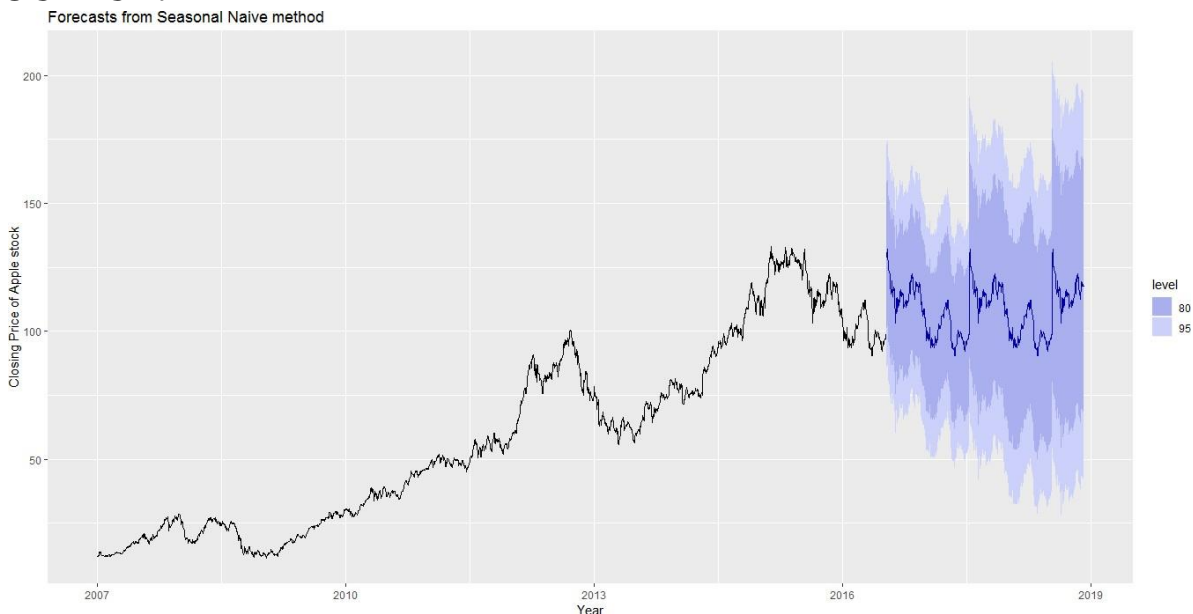
#### Seasonal Naive model of Closing Price on training data:

```
appl_snaive <- snaive(appl_training,h=603)
```

#### Forecast of Apple Closing Price using Seasonal Naive method:

```
autoplot(appl_snaive) +  
  xlab("Year") + ylab("Closing Price of Apple stock") +  
  ggtitle("Forecasts from Seasonal Naive method")
```

### OUTPUT:



### Checking Residuals:

```
checkresiduals(appl_snaive)
```

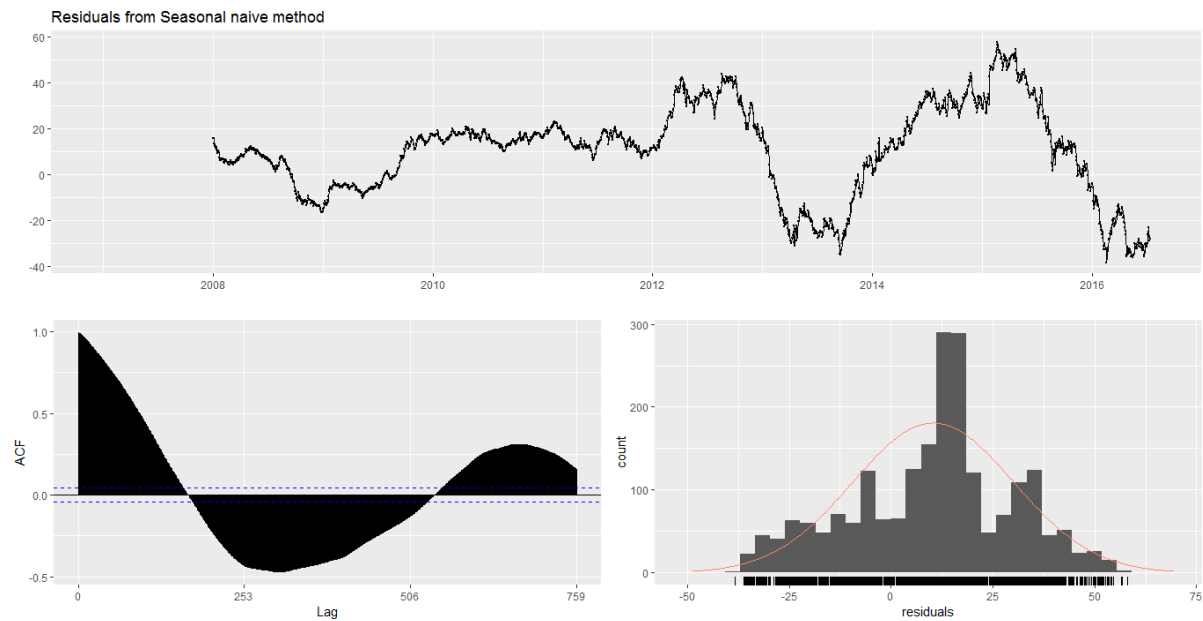
### OUTPUT:

Ljung-Box test

data: Residuals from Seasonal naive method

$Q^* = 249980$ ,  $df = 482.4$ ,  $p\text{-value} < 2.2e-16$

Model  $df$ : 0. Total lags used: 482.4



NOTE: As seen above the residuals are not randomly distributed. There is a clear trend in the residuals and no normal distribution in histogram. Therefore, this is not a good model for our dataset.

**Computing forecast accuracy measures of Seasonal Naive method:**  
`accuracy(appl_snaive, appl_testing)`

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	10.35607	22.25247	18.82086	13.18451	32.88206	1.000000	0.9959151	NA
Test set	50.29479	61.45184	52.62289	28.26428	30.47388	2.795987	0.9907513	24.33266

## 6. Drift Model

A variation on the naïve method is to allow the forecasts to increase or decrease over time, where the amount of change over time (called the **drift**) is set to be the average change seen in the historical data. Thus, the forecast for time  $T+h$  is given by

$$\hat{y}_{T+h|T} = y_T + \frac{h}{T-1} \sum_{t=2}^T (y_t - y_{t-1}) = y_T + h \left( \frac{y_T - y_1}{T-1} \right).$$

This is equivalent to drawing a line between the first and last observations, and extrapolating it into the future.

`rwf(y,h,drift=TRUE)`

### Code for Drift Model:

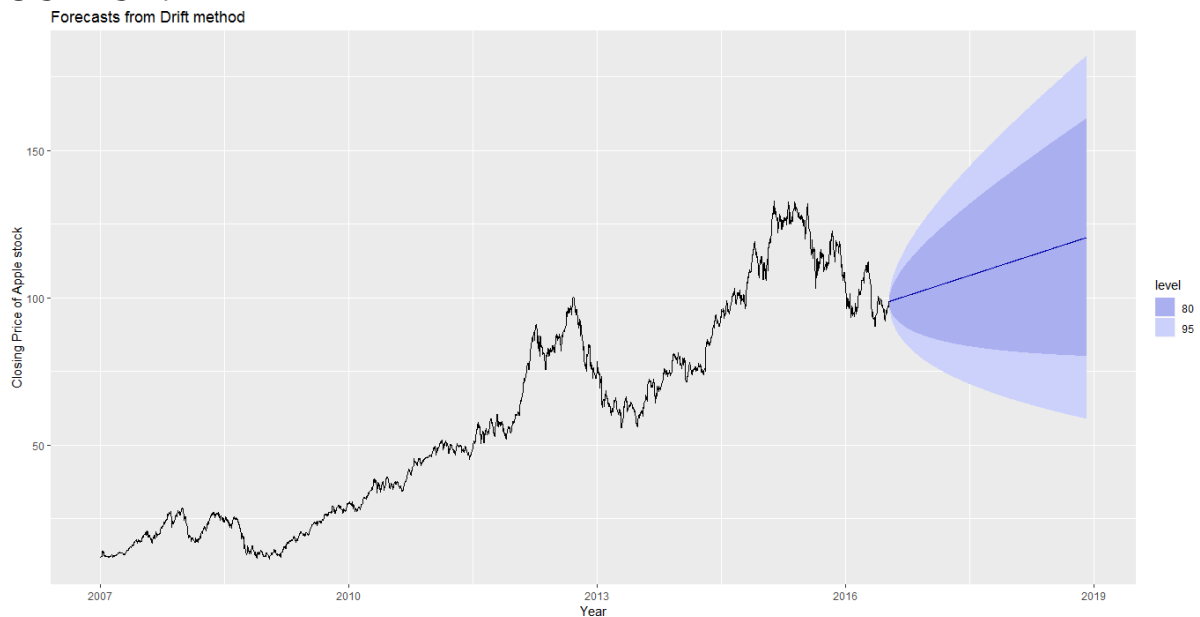
#### Train Drift model of Closing Price:

```
appl_drift <- rwf(appl_training,h=603,drift=TRUE)
```

#### Forecast of Apple Closing Price using Drift method:

```
autoplot(appl_drift) +  
  xlab("Year") + ylab("Closing Price of Apple stock") +  
  ggtitle("Forecasts from Drift method")
```

### OUTPUT:



## Check Residuals:

```
checkresiduals(appl_drift)
```

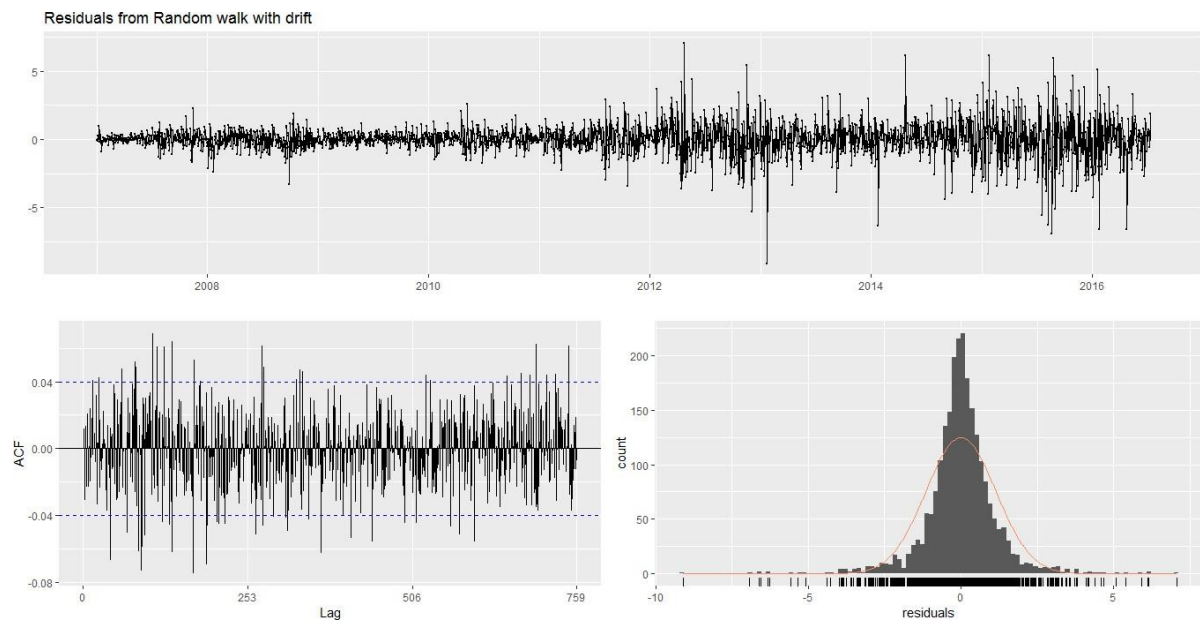
## OUTPUT:

Ljung-Box test

data: Residuals from Random walk with drift

$Q^* = 701.68$ ,  $df = 481.4$ ,  $p\text{-value} = 1.954e-10$

Model df: 1. Total lags used: 482.4



## Computing forecast accuracy measures of Drift method:

```
accuracy(appl_drift, appl_testing)
```

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	9.826522e-15	1.145233	0.7549649	-0.03374931	1.474513	0.0401132	0.01164376	NA
Test set	4.876376e+01	56.040182	48.7780786	28.31563590	28.330391	2.5917028	0.99372814	22.11823

## 7. Regression Model

### Simple Linear Regression:

In the simplest case, the regression model allows for a linear relationship between the forecast variable  $y$  and a single predictor variable  $x$ :

$$y_t = \beta_0 + \beta_1 x_t + \varepsilon_t.$$

The coefficients  $\beta_0$  and  $\beta_1$  denote the intercept and the slope of the line respectively.

The intercept  $\beta_0$  represents the predicted value of  $y$  when  $x=0$

The slope  $\beta_1$  represents the average predicted change in  $y$  resulting from a one unit increase in  $x$ .

### Multiple Linear Regression:

When there are two or more predictor variables, the model is called a **multiple regression model**. The general form of a multiple regression model is

$$y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \dots + \beta_k x_{k,t} + \varepsilon_t,$$

where  $y$  is the variable to be forecast and  $x_1, \dots, x_k$  are the  $k$  predictor variables. Each of the predictor variables must be numerical. The coefficients  $\beta_1, \dots, \beta_k$  measure the effect of each predictor after taking into account the effects of all the other predictors in the model. Thus, the coefficients measure the *marginal effects* of the predictor variables.

### Code for Regression Model:

#### Train regression model on Apple closing price:

#### Running regression model with Trend and season as the predictor variables:

```
appl_reg <- tslm(appl_training ~ trend + season)
```

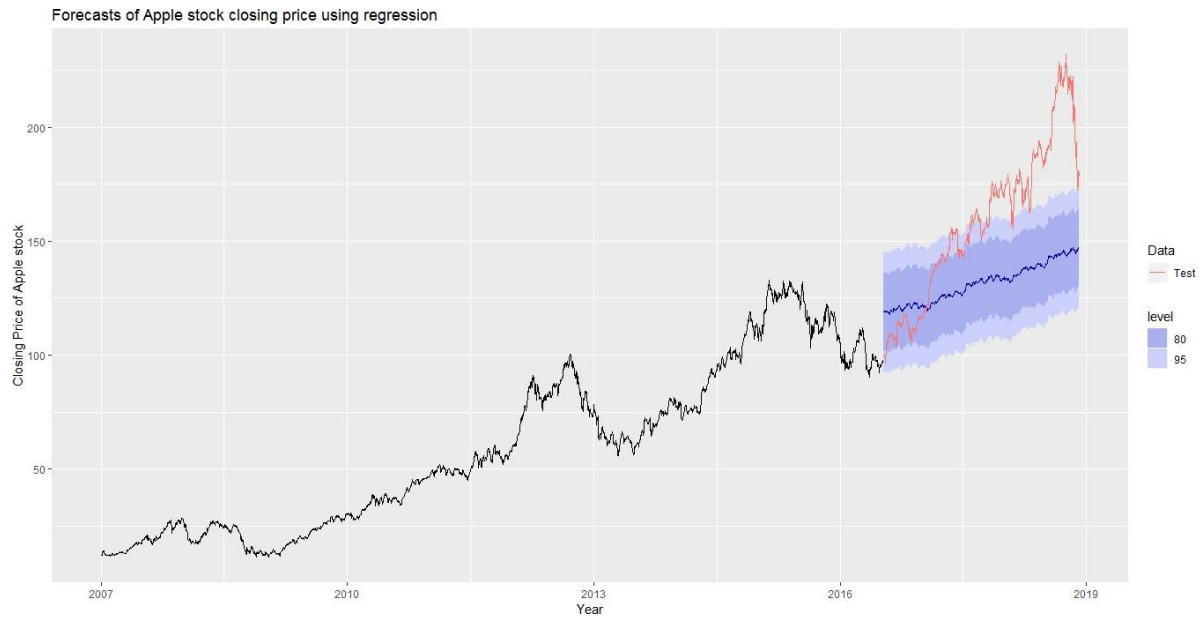
#### Forecast of Apple Closing Price using Regression Model:

```
appl_reg_forecast <- forecast(appl_reg, h= 603)
```

#### Plot of forecast to test data for regression:

```
autoplot(appl_reg_forecast) +  
autolayer(appl_testing, series = "Test") +  
  xlab("Year") + ylab("Closing Price of Apple stock") +  
  ggtitle("Forecasts of Apple stock closing price using regression") +  
  guides(colour = guide_legend(title = "Data"))
```

## OUTPUT:



## Check Residuals:

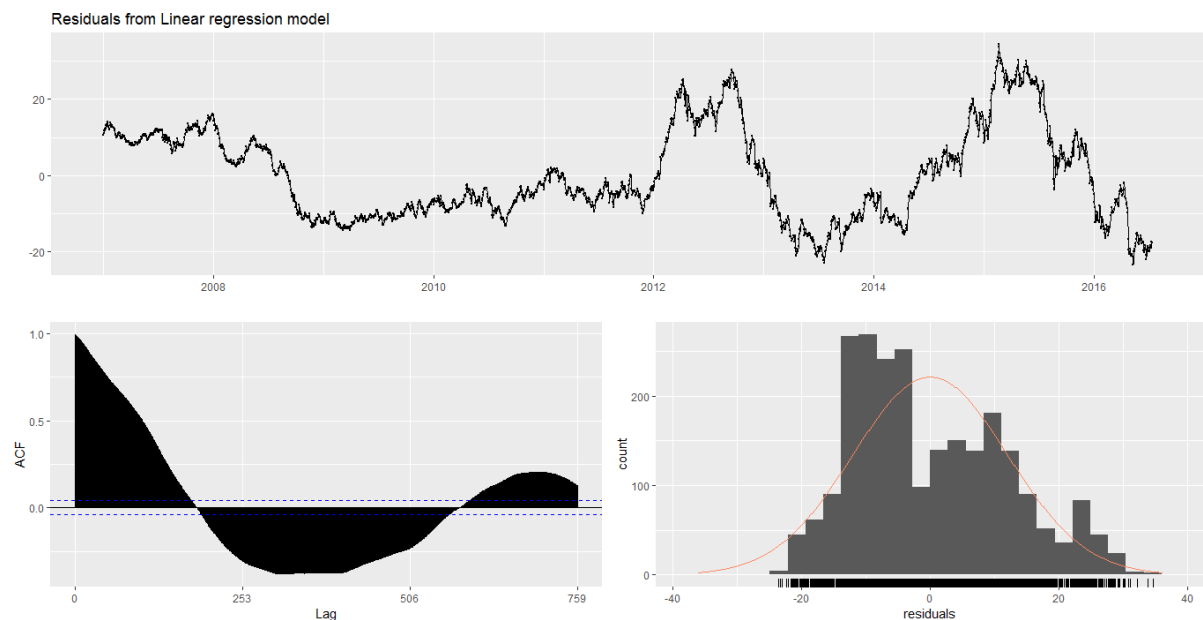
`checkresiduals(appl_reg)`

## OUTPUT:

Breusch-Godfrey test for serial correlation of order up to 482

data: Residuals from Linear regression model

LM test = 2397.2, df = 482, p-value < 2.2e-16



NOTE: As seen above the residuals are not randomly distributed. There is a clear trend in the residuals and no normal distribution in histogram. Therefore, this is not a good model for our dataset.

## Computing forecast accuracy measures of Regression method:

accuracy(appl\_reg\_forecast, appl\_testing)

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	-5.613427e-16	12.01551	10.18912	-2.975144	26.32305	0.5413738	0.9952434	NA
Test set	2.701277e+01	37.24508	30.99776	14.202686	17.86990	1.6469895	0.9928585	14.30352

## 8. Holt Linear Model

Holt Linear also known as double exponential smoothing computes a trend equation through the data using a special weighting function that places the greatest emphasis on the most recent time periods. The forecasting equation changes from period to period.

The forecasting algorithm makes use of the following formulas:

$$\begin{aligned}F_t &= a_t + b_t \\a_t &= X_t + (1 - \alpha) a_{t-1} \\b_t &= b_{t-1} + \alpha (X_t - F_{t-1})\end{aligned}$$

The smoothing constant,  $\alpha$ , dictates the amount of smoothing that takes place. It ranges from zero to one. The forecast at time period T for the value at time period T+k is  $a_T + b_T k$ .

We are using this method to observe the trend of the closing price.

### Code for Holt Linear Model:

#### Forecast of Apple Closing Price using Holt's linear trend method:

```
appl_holt <- holt(appl_training, h= 603)
```

#### Smoothing Parameters:

```
appl_holt[["model"]]
```

#### OUTPUT:

Holt's method

Call:

```
holt(y = appl_training, h = 603)
```

Smoothing parameters:

alpha = 0.9999

beta = 1e-04

Initial states:

l = 13.3848

b = 0.0316

sigma: 1.1464

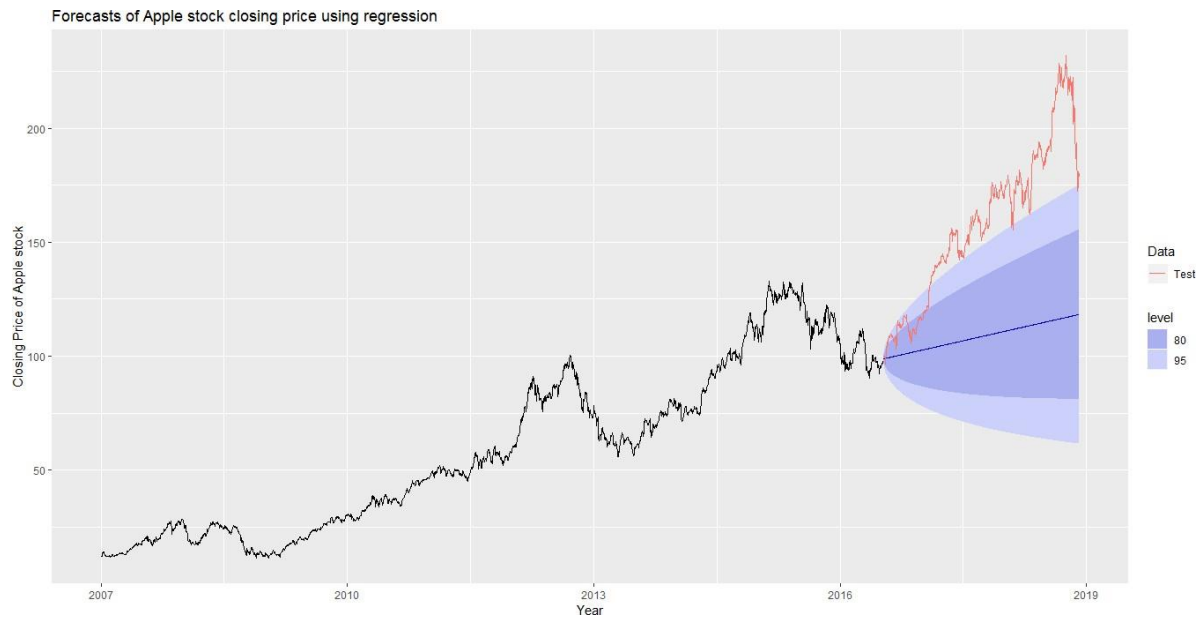
AIC	AICc	BIC
19450.20	19450.23	19479.15

#### Plot of forecast to test data for Holt's linear:

```
autoplot(appl_holt, series = "Forecast") +  
autolayer(appl_testing, series = "Test") +  
  xlab("Year") + ylab("Closing Price of Apple stock") +  
  ggtitle("Forecasts of Apple stock closing price using regression") +  
  guides(colour = guide_legend(title = "Data"))
```



## OUTPUT:



## Check Residuals:

`checkresiduals(appl_holt)`

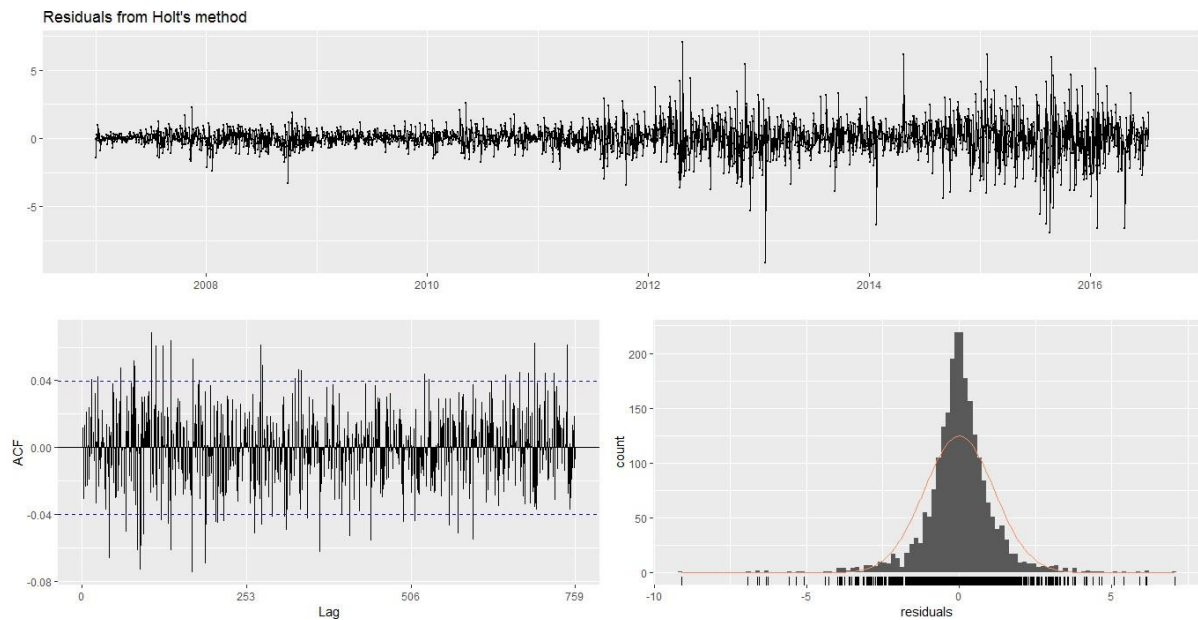
## OUTPUT:

Ljung-Box test

data: Residuals from Holt's method

$Q^* = 701.59$ ,  $df = 478.4$ ,  $p\text{-value} = 1.105e-10$

Model df: 4. Total lags used: 482.4



**Computing forecast accuracy measures of Holt's linear:**  
accuracy (appl\_holt, appl\_testing)

**OUTPUT:**

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	0.003056777	1.145438	0.7553488	-0.02641243	1.478918	0.0401336	0.01175102	NA
Test set	49.870360582	57.298729	49.8844639	28.95859209	28.973123	2.6504879	0.99385790	22.61763

## 9. ARIMA

ARIMA models provide another approach to time series forecasting. Exponential smoothing and ARIMA models are the two most widely used approaches to time series forecasting and provide complementary approaches to the problem. While exponential smoothing models are based on a description of the trend and seasonality in the data, ARIMA models aim to describe the autocorrelations in the data.

### Auto Regressive (AR) Model:

In a multiple regression model, we forecast the variable of interest using a linear combination of predictors. In an autoregression model, we forecast the variable of interest using a linear combination of *past values of the variable*. The term *autoregression* indicates that it is a regression of the variable against itself.

Thus, an autoregressive model of order  $p$  can be written as

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \varepsilon_t,$$

where  $\varepsilon_t$  is white noise. This is like a multiple regression but with *lagged values* of  $y_t$  as predictors. We refer to this as an **AR(p) model**, an autoregressive model of order  $p$ .

Autoregressive models are remarkably flexible at handling a wide range of different time series patterns.

### Moving Averages (MA) Model:

Rather than using past values of the forecast variable in a regression, a moving average model uses past forecast errors in a regression-like model.

$$y_t = c + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \dots + \theta_q \varepsilon_{t-q},$$

where  $\varepsilon_t$  is white noise. We refer to this as an **MA(q) model**, a moving average model of order  $q$ . Of course, we do not *observe* the values of  $\varepsilon_t$ , so it is not really a regression in the usual sense.

Notice that each value of  $y_t$  can be thought of as a weighted moving average of the past few forecast errors.

A moving average model is used for forecasting future values, while moving average smoothing is used for estimating the trend-cycle of past values.

### Code for ARIMA Model:

#### Using Auto.Arima to fit best ARIMA model (finding p,d,q)

```
fit <- auto.arima(appl_training)
fit
```

### OUTPUT:

```
Series: appl_training
ARIMA(0,1,0) with drift
```

```
Coefficients:
```

```
drift
0.0360
s.e. 0.0233
```

```
sigma^2 estimated as 1.312: log likelihood=-3748.01
AIC=7500.02 AICc=7500.03 BIC=7511.
```

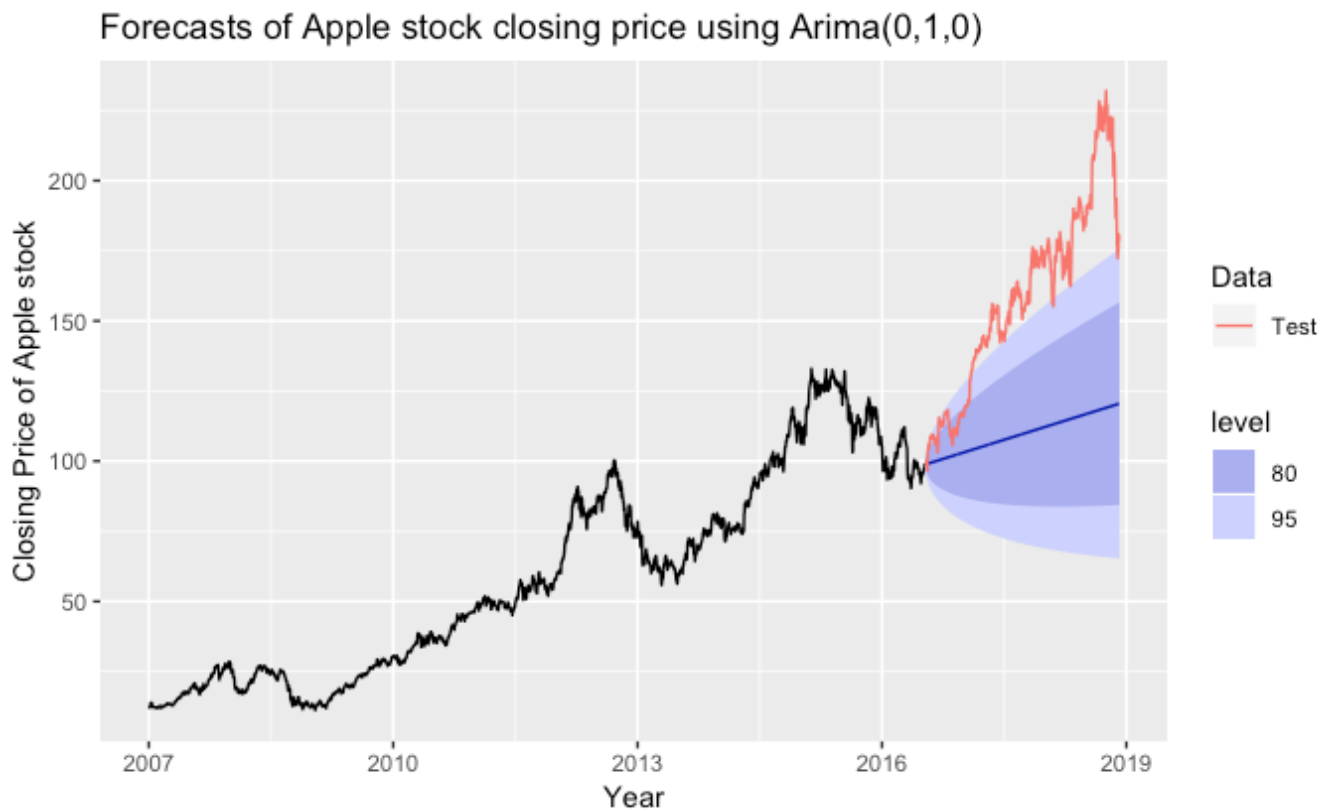
### Fitting ARIMA(0,1,0) for apple training data:

```
fit2 <- Arima(appl_training,order=c(0,1,0),include.drift=TRUE)
appl_arima <- forecast(fit2, h=603)
```

### Plot of forecast to test data for Arima:

```
autoplot(appl_arima, series = "Forecast") +  
  autolayer(appl_testing, series = "Test") +  
  xlab("Year") + ylab("Closing Price of Apple stock") +  
  ggtitle("Forecasts of Apple stock closing price using Arima(0,1,0)") +  
  guides(colour = guide_legend(title = "Data"))
```

### OUTPUT:



## Check Residuals:

```
checkresiduals(appl_arima)
```

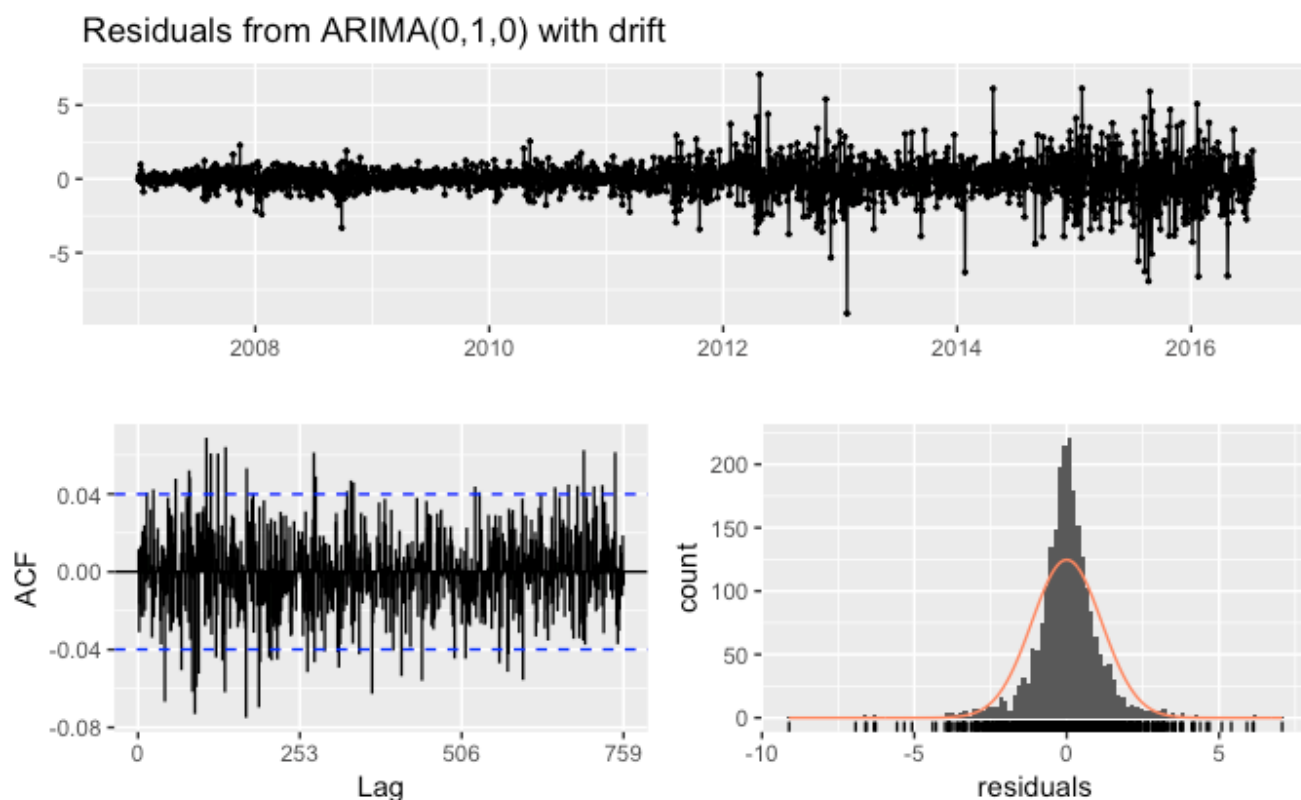
## OUTPUT:

Ljung-Box test

data: Residuals from ARIMA(0,1,0) with drift

$Q^* = 701.94$ ,  $df = 481.4$ ,  $p\text{-value} = 1.875e-10$

Model df: 1. Total lags used: 482.4



## Compute forecast accuracy measures of ARIMA(0,1,0) model with drift:

```
accuracy (appl_arima, appl_testing)
```

## OUTPUT:

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
Training set	4.948349e-06	1.144996	0.7546568	-0.03369398	1.473943	0.04009683	0.01164362	NA
Test set	4.876376e+01	56.040182	48.7780786	28.31563590	28.330391	2.59170285	0.99372814	22.11823

> |

## CONCLUSION:

### Which Model to choose?

Mean, Seasonal Naïve, Regression are rejected based on Residuals check.

For the remaining models we compared the RMSE of training data as well as test data set as depicted below:

#### For Training Set

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
<b>Mean</b>	-1.108569e-15	35.40613	30.85103	-62.64105	93.14712	1.639193	0.9988489	NA
<b>Naïve</b>	0.03600521	1.145799	0.7558108	0.0652478	1.47613	0.04015814	0.01164376	NA
<b>Seasonal Naïve</b>	10.35607	22.25247	18.82086	13.18451	32.88206	1.000000	0.9959151	NA
<b>Drift</b>	9.826522e-15	1.145233	0.7549649	-0.03374931	1.474513	0.0401132	0.01164376	NA
<b>Regression</b>	-5.613427e-16	12.01551	10.18912	-2.975144	26.32305	0.5413738	0.9952434	NA
<b>Holt Linear</b>	0.00305677	1.145438	0.7553488	-0.02641243	1.478918	0.0401336	0.01175102	NA
<b>ARIMA</b>	4.948349e-06	1.144996	0.7546568	-0.03369398	1.473943	0.04009683	0.01164362	NA

#### For Test Set

	ME	RMSE	MAE	MPE	MAPE	MASE	ACF1	Theil's U
<b>Mean</b>	9.931478e+01	104.83518	99.31478	60.88951	60.88951	5.276846	0.9946694	44.24462
<b>Naïve</b>	59.63733107	68.436841	59.6495035	34.6333753	34.64592	3.16932919	0.99466939	27.03928
<b>Seasonal Naïve</b>	50.29479	61.4518	52.62289	28.26428	30.47388	2.795987	0.9907513	24.33266
<b>Drift</b>	4.876376e+01	56.040182	48.7780786	28.31563590	28.330391	2.5917028	0.99372814	22.11823
<b>Regression</b>	2.701277e+01	37.2450	30.99776	14.202686	17.86990	1.6469895	0.9928585	14.30352
<b>Holt Linear</b>	49.870360582	57.298729	49.8844639	28.95859209	28.973123	2.6504879	0.99385790	22.61763
<b>ARIMA</b>	4.876376e+01	56.040182	48.7780786	28.31563590	28.330391	2.59170285	0.99372814	22.11823

After comparing **RMSE and complexity** of **Naïve, Drift, Holt Linear and ARIMA**, we've arrived on a conclusion that Drift Model is the most suitable one among all the other models which we have implemented as forecasting using Drift model is simpler to implement. Also, the ARIMA model suggested using an automated function `auto.arima` may not be optimal, so

we will predict future values using Drift Model.

## Predicting Future Apple Stock Price:

**As we compared all the models, we found out that drift method is the best with good accuracy on test data:**

**Now we will forecast future values:**

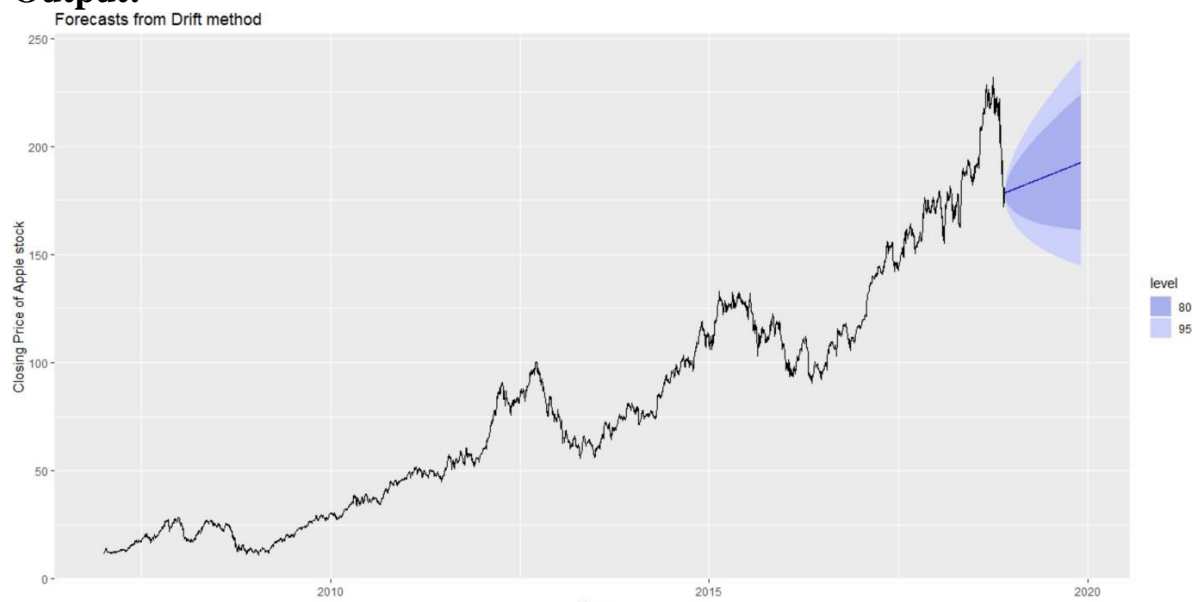
**For that we will train model on the whole dataset:**

```
future_drift <- rwf(appl_close_ts,h= 253, drift = TRUE)
```

**Plotting Predictted Values:**

```
autoplot(future_drift) +  
  xlab("Year") + ylab("Closing Price of Apple stock") +  
  ggtitle("Forecasts from Drift method")
```

**Output:**



## Forecast values:

forecast(future\_drift)

## Output:

```
> forecast(future_drift)
```

	Point	Forecast	Lo 80	Hi 80	Lo 95	Hi 95
2018.917		178.6353	176.7435	180.5271	175.7421	181.5285
2018.921		178.6906	176.0147	181.3664	174.5983	182.7829
2018.925		178.7458	175.4681	182.0236	173.7330	183.7587
2018.929		178.8011	175.0157	182.5865	173.0118	184.5904
2018.933		178.8564	174.6235	183.0893	172.3827	185.3301
2018.937		178.9117	174.2740	183.5494	171.8189	186.0044
2018.941		178.9669	173.9568	183.9771	171.3046	186.6293
2018.945		179.0222	173.6653	184.3792	170.8295	187.2150
2018.949		179.0775	173.3947	184.7603	170.3864	187.7687
2018.953		179.1328	173.1416	185.1240	169.9700	188.2956
2018.957		179.1881	172.9034	185.4728	169.5765	188.7997
2018.960		179.2433	172.6781	185.8086	169.2027	189.2840
2018.964		179.2986	172.4642	186.1331	168.8463	189.7510
2018.968		179.3539	172.2603	186.4475	168.5052	190.2026
2018.972		179.4092	172.0654	186.7530	168.1778	190.6405
2018.976		179.4645	171.8786	187.0503	167.8629	191.0661
2018.980		179.5197	171.6991	187.3404	167.5591	191.4804
2018.984		179.5750	171.5263	187.6237	167.2656	191.8844
2018.988		179.6303	171.3597	187.9009	166.9815	192.2791
2018.992		179.6856	171.1987	188.1724	166.7060	192.6651
2018.996		179.7408	171.0430	188.4387	166.4386	193.0431
2019.000		179.7961	170.8921	188.7002	166.1786	193.4137
2019.004		179.8514	170.7458	188.9570	165.9255	193.7773
2019.008		179.9067	170.6037	189.2097	165.6789	194.1344
2019.012		179.9620	170.4655	189.4584	165.4384	194.4855



## CHALLENGES:

- The data that we fetched from Quant Mod did not have uniform number of records for each year. Hence, we had to plug in values using the naïve model to make each year have equal number of records.
- Selection of model for prediction of apple stock was another challenge as Drift and Arima forecasting models showed identical accuracies.
- ARIMA model achieved through auto.arima function had a similar accuracy and a low AICc as compared to Holt model.
- auto.arima function may not be optimal as it is automated and we get the model as (0,1,0) with drift. Thus, when  $c=0$  and  $d=1$ , the long-term forecasts will go to a non-zero constant. We chose DRIFT model to be the best fit for our data set since rwf with Drift is equivalent by definition to Arima(0,1,0).

## REFERENCES:

**[1] Kaggle:**

<https://www.kaggle.com/dgawlik/nyse>

**[2] Forecasting: Principles and Practice by Rob J Hyndman and George Athanasopoulos:**

<https://otexts.org/fpp2/>

**[3] Quamtmod:**

<https://www.quantmod.com/>