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8.1 The Per-Unit System

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In many engineering situations, it is useful to scale or normalize quantities. This is commonly done in power system analysis, and the standard method used is referred to as the per-unit system. Historically, this was done to simplify numerical calculations that were made by hand. Although this advantage has been eliminated by using the computer, other advantages remain:

- Device parameters tend to fall into a relatively narrow range, making erroneous values conspicuous.
- The method is defined in order to eliminate ideal transformers as circuit components.
- The voltage throughout the power system is normally close to unity.

Some disadvantages are that component equivalent circuits are somewhat more abstract. Sometimes phase shifts that are clearly present in the unscaled circuit are eliminated in the per-unit circuit.

It is necessary for power system engineers to become familiar with the system because of its wide industrial acceptance and use and also to take advantage of its analytical simplifications. This discussion is limited to traditional AC analysis, with voltages and currents represented as complex phasor values. Per-unit is sometimes extended to transient analysis and may include quantities other than voltage, power, current, and impedance.

The basic per-unit scaling equation is

$$\textbf{Per-unit value} = \frac{\textbf{actual value}}{\textbf{base value}}. \tag{8.1}$$

The base value always has the same units as the actual value, forcing the per-unit value to be dimensionless. Also, the base value is always a real number, whereas the actual value may be complex. Representing a complex value in polar form, the angle of the per-unit value is the same as that of the actual value.

Consider complex power

$$\mathbf{S} = \mathbf{V}\mathbf{I}^* \tag{8.2}$$

or

$$S\angle\theta = V\angle\alpha \; I\angle -\beta$$

where $\mathbf{V}$ = phasor voltage, in volts; $\mathbf{I}$ = phasor current, in amperes.

Suppose we arbitrarily pick a value S_{base}, a real number with the units of volt-amperes. Dividing through by S_{base},

$$\frac{S\angle\theta}{S_{base}} = \frac{V\angle\alpha \; I\angle -\beta}{S_{base}}.$$

We further define

$$V_{base}\, I_{base} = S_{base}. \tag{8.3}$$

Either V_{base} or I_{base} may be selected arbitrarily, but not both. Substituting Eq. (8.3) into Eq. (8.2), we obtain

$$\frac{S\angle\theta}{S_{base}} = \frac{V\angle\alpha\left(I\angle -\beta\right)}{V_{base}\, I_{base}}$$

$$S_{pu}\angle\theta = \left(\frac{V\angle\alpha}{V_{base}}\right)\left(\frac{I\angle -\beta}{I_{base}}\right)$$

$$\mathbf{S}_{pu} = V_{pu}\angle\alpha\left(I_{pu}\angle -\beta\right)$$

$$\mathbf{S}_{pu} = \mathbf{V}_{pu}\, \mathbf{I}_{pu}{}^* \tag{8.4}$$

The subscript *pu* indicates per-unit values. Note that the form of Eq. (8.4) is identical to Eq. (8.2). This was not inevitable, but resulted from our decision to relate V_{base} I_{base} and S_{base} through Eq. (8.3). If we select Z_{base} by

$$Z_{base} = \frac{V_{base}}{I_{base}} = \frac{V_{base}^2}{S_{base}}. \tag{8.5}$$

Convert Ohm's law:

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \tag{8.6}$$

into per-unit by dividing by Z_{base}.

$$\frac{\mathbf{Z}}{Z_{base}} = \frac{\mathbf{V}/\mathbf{I}}{Z_{base}}$$

$$\mathbf{Z}_{pu} = \frac{\mathbf{V}/V_{base}}{\mathbf{I}/I_{base}} = \frac{\mathbf{V}_{pu}}{\mathbf{I}_{pu}}.$$

Observe that

$$\mathbf{Z}_{pu} = \frac{\mathbf{Z}}{Z_{base}} = \frac{R + jX}{Z_{base}} = \left(\frac{R}{Z_{base}}\right) + j\left(\frac{X}{Z_{base}}\right)$$

$$Z_{pu} = R_{pu} + jX_{pu} \tag{8.7}$$

Thus, separate bases for R and X are not necessary:

$$Z_{base} = R_{base} = X_{base}$$

By the same logic,

$$S_{base} = P_{base} = Q_{base}$$

Example 1:

(a) Solve for **Z**, **I**, and **S** at Port ab in Fig. 8.1a.
(b) Repeat (a) in per-unit on bases of V_{base} = 100 V and S_{base} = 1000 V. Draw the corresponding per-unit circuit.

Solution:

(a) $\mathbf{Z}_{\mathbf{ab}} = 8 + j12 - j6 = 8 + j6 = 10\ \angle 36.9°\ \Omega$

$$\mathbf{I} = \frac{\mathbf{V}_{ab}}{\mathbf{Z}_{ab}} = \frac{100\ \angle 0°}{10\ \angle 36.9°} = 10\ \angle -36.9° \quad \text{amperes}$$

$$\mathbf{S} = \mathbf{V}\ \mathbf{I}^* = \left(100\ \angle 0°\right)\left(10\ \angle -36.9°\right)^*$$

$$= 1000\ \angle 36.9° = 800 + j600 \quad \text{VA}$$

$P = 800$ W $\qquad Q = 600$ var

(b) On bases V_{base} and S_{base} = 1000 VA:

$$Z_{base} = \frac{V_{base}^2}{S_{base}} = \frac{\left(100\right)^2}{1000} = 10\ \Omega$$

$$I_{base} = \frac{S_{base}}{V_{base}} = \frac{1000}{100} = 10\ \text{A}$$

$$\mathbf{V}_{pu} = \frac{100\ \angle 0°}{100} = 1\ \angle 0°\ \ \text{pu}$$

$$\mathbf{Z}_{pu} = \frac{8 + j12 - j6}{10} = 0.8 + j0.6\ \ \text{pu}$$

$$= 1.0\ \angle 36.9°\ \ \text{pu}$$

$$\mathbf{I}_{pu} = \frac{\mathbf{V}_{pu}}{\mathbf{Z}_{pu}} = \frac{1\angle 0°}{1\angle 36.9°} = 1\angle -36.9°\ \ \text{pu}$$

$$\mathbf{S}_{pu} = \mathbf{V}_{pu}\ \mathbf{I}_{pu}{}^{*} = \left(1\angle 0°\right)\left(1\angle -36.9°\right)^{*} = 1\angle 36.9°\ \ \text{pu}$$

$$= 0.8 + j0.6\ \ \text{pu}$$

Converting results in (b) to SI units:

$$\mathbf{I} = \left(\mathbf{I}_{pu}\right)I_{base} = \left(1\angle -36.9°\right)\left(10\right) = 10\angle -36.9°\ \ \text{A}$$

$$\mathbf{Z} = \left(\mathbf{Z}_{pu}\right)Z_{base} = \left(0.8 + j0.6\right)\left(10\right) = 8 + j6\ \ \Omega$$

$$\mathbf{S} = \left(\mathbf{S}_{pu}\right)S_{base} = \left(0.8 + j0.6\right)\left(1000\right) = 800 + j600\ \ \text{W, var}$$

The results of (a) and (b) are identical.

For power system applications, base values for S_{base} and V_{base} are arbitrarily selected. Actually, in practice, values are selected that force results into certain ranges. Thus, for V_{base}, a value is chosen such that the normal system operating voltage is close to unity. Popular power bases used are 1, 10, 100, and 1000 MVA, depending on system size.

Impact on Transformers

To understand the impact of pu scaling on transformer, consider the three-winding ideal device (see Fig. 8.2).

For sinusoidal steady-state performance:

$$\mathbf{V}_1 = \frac{N_1}{N_2}\mathbf{V}_2 \tag{8.8a}$$

$$\mathbf{V}_2 = \frac{N_2}{N_3}\mathbf{V}_3 \tag{8.8b}$$

$$\mathbf{V}_3 = \frac{N_3}{N_1}\mathbf{V}_1 \tag{8.8c}$$

and

$$N_1\mathbf{I}_1 + N_2\mathbf{I}_2 + N_3\mathbf{I}_3 = 0 \tag{8.9}$$

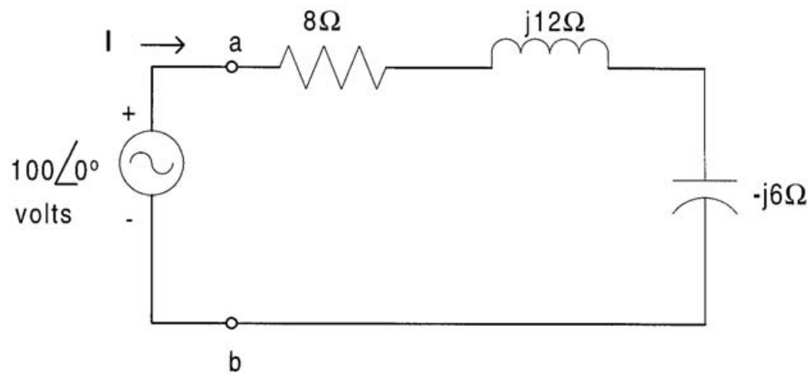


FIGURE 8.1a Circuit with elements in SI units.

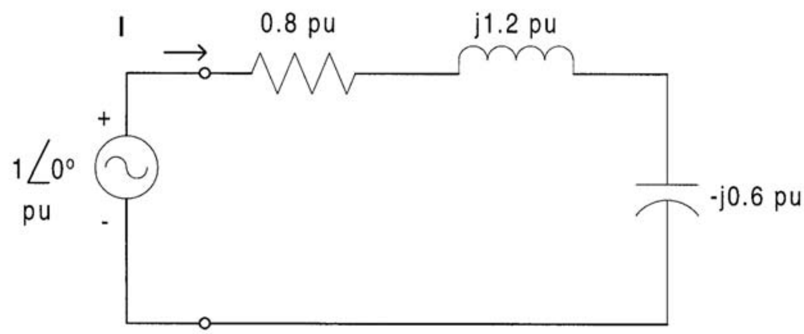


FIGURE 8.1b Circuit with elements in per-unit.

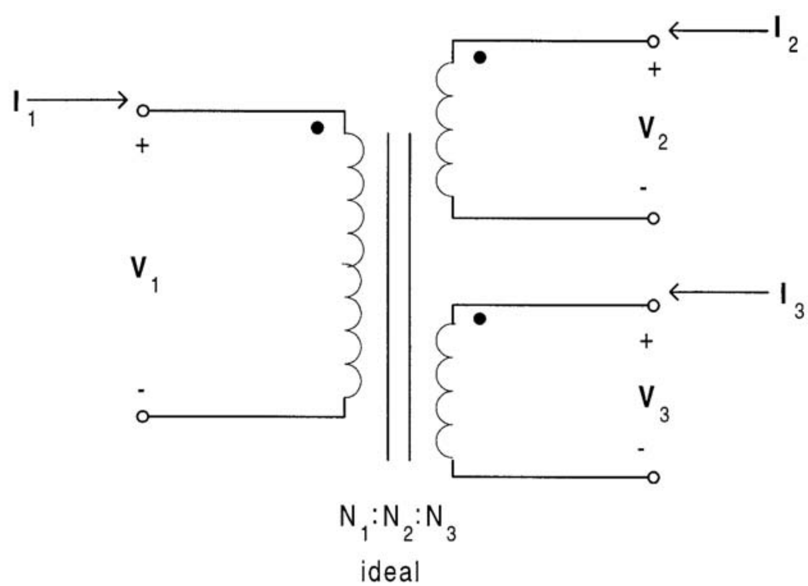


FIGURE 8.2 The three-winding ideal transformer.

Consider the total input complex power **S**.

$$
\begin{aligned}
\mathbf{S} &= \mathbf{V}_1\mathbf{I}_1{}^* + \mathbf{V}_2\mathbf{I}_2{}^* + \mathbf{V}_3\mathbf{I}_3{}^* \\
&= \mathbf{V}_1\mathbf{I}_1{}^* + \frac{N_2}{N_1}\mathbf{V}_1\mathbf{I}_2{}^* + \frac{N_3}{N_1}\mathbf{V}_1\mathbf{I}_3{}^* \\
&= \frac{\mathbf{V}_1}{N_1}\left[N_1\mathbf{I}_1 + N_2\mathbf{I}_2 + N_3\mathbf{I}_3\right]^* \\
&= 0
\end{aligned}
\tag{8.10}
$$

The interpretation to be made here is that the ideal transformer can neither absorb real nor reactive power. An example should clarify these properties.

Arbitrarily select two base values V_{1base} and S_{1base}. Require base values for windings 2 and 3 to be:

$$V_{2base} = \frac{N_2}{N_1}V_{1base} \tag{8.11a}$$

$$V_{3base} = \frac{N_3}{N_1}V_{1base} \tag{8.11b}$$

and

$$S_{1base} = S_{2base} = S_{3base} = S_{base} \tag{8.12}$$

By definition,

$$I_{1base} = \frac{S_{base}}{V_{1base}} \tag{8.13a}$$

$$I_{2base} = \frac{S_{base}}{V_{2base}} \tag{8.13b}$$

$$I_{3base} = \frac{S_{base}}{V_{3base}} \tag{8.13c}$$

It follows that

$$I_{2base} = \frac{N_1}{N_2}I_{1base} \tag{8.14a}$$

$$I_{3base} = \frac{N_1}{N_3}I_{1base} \tag{8.14b}$$

Recall that a per-unit value is the actual value divided by its appropriate base. Therefore:

$$\frac{\mathbf{V}_1}{V_{1base}} = \frac{\left(N_1/N_2\right)\mathbf{V}_2}{V_{1base}} \tag{8.15a}$$

and

$$\frac{\mathbf{V}_1}{V_{1\text{base}}} = \frac{(N_1/N_2)\mathbf{V}_2}{(N_1/N_2)V_{2\text{base}}} \tag{8.15b}$$

or

$$\mathbf{V}_{1\text{pu}} = \mathbf{V}_{2\text{pu}} \tag{8.15c}$$

indicates per-unit values. Similarly,

$$\frac{\mathbf{V}_1}{V_{1\text{base}}} = \frac{(N_1/N_3)\mathbf{V}_3}{(N_1/N_3)V_{3\text{base}}} \tag{8.16a}$$

or

$$\mathbf{V}_{1\text{pu}} = \mathbf{V}_{3\text{pu}} \tag{8.16b}$$

Summarizing:

$$V_{1\text{pu}} = V_{2\text{pu}} = V_{3\text{pu}} \tag{8.17}$$

Divide Eq. (8.9) by N_1

$$\mathbf{I}_1 + \frac{N_2}{N_1}\mathbf{I}_2 + \frac{N_3}{N_1}\mathbf{I}_3 = 0$$

Now divide through by $I_{1\text{base}}$

$$\frac{\mathbf{I}_1}{I_{1\text{base}}} + \frac{(N_2/N_1)\mathbf{I}_2}{I_{1\text{base}}} + \frac{(N_3/N_1)\mathbf{I}_3}{I_{1\text{base}}} = 0$$

$$\frac{\mathbf{I}_1}{I_{1\text{base}}} + \frac{(N_2/N_1)\mathbf{I}_2}{(N_2/N_1)I_{2\text{base}}} + \frac{(N_3/N_1)\mathbf{I}_3}{(N_3/N_1)I_{3\text{base}}} = 0$$

Simplifying to

$$\mathbf{I}_{1\text{pu}} + \mathbf{I}_{2\text{pu}} + \mathbf{I}_{3\text{pu}} = 0 \tag{8.18}$$

Equations (8.17) and (8.18) suggest the basic scaled equivalent circuit, shown in Fig. 8.3. It is cumbersome to carry the pu in the subscript past this point: no confusion should result, since all quantities will show units, including pu.

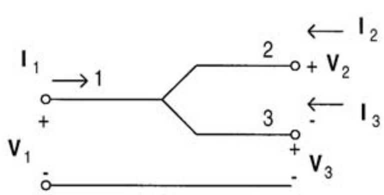


FIGURE 8.3 Single-phase ideal transformer.

Example 2:

The 3-winding single-phase transformer of Fig. 8.1 is rated at 13.8 kV/138kV/4.157 kV and 50 MVA/40 MVA/10 MVA. Terminations are as followings:

13.8 kV winding:	13.8 kV Source
138 kV winding:	35 MVA load, pf = 0.866 lagging
4.157 kV winding:	5 MVA load, pf = 0.866 leading

Using S_{base} = 10 MVA, and voltage ratings as bases,

(a) Draw the pu equivalent circuit.
(b) Solve for the primary current, power, and power, and power factor.

Solution:

(a) See Fig. 8.4.

(b,c)
$$S_2 = \frac{35}{10} = 3.5 \text{ pu} \qquad \mathbf{S}_2 = 3.5\angle{+30^\circ} \text{ pu}$$

$$S_3 = \frac{5}{10} = 0.5 \text{ pu} \qquad \mathbf{S}_3 = 0.5\angle{-30^\circ} \text{ pu}$$

$$V_1 = \frac{13.8}{13.8} = 1.0 \text{ pu} \qquad \mathbf{V}_1 = \mathbf{V}_2 = \mathbf{V}_3 = 1.0\angle 0^\circ \text{ pu}$$

$$\mathbf{I}_2 = \left(\frac{\mathbf{S}_2}{\mathbf{V}_2}\right)^* = 3.5\angle{-30^\circ} \text{ pu}$$

$$\mathbf{I}_3 = \left(\frac{\mathbf{S}_3}{\mathbf{V}_3}\right)^* = 0.5\angle{+30^\circ} \text{ pu}$$

All values in Per-Unit Equivalent Circuit:

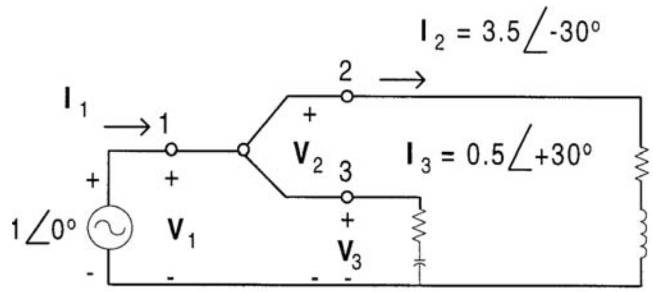


FIGURE 8.4 Per-unit circuit.

$$\mathbf{I}_1 = \mathbf{I}_2 + \mathbf{I}_3 = 3.5\angle{-30^\circ} + 0.5\angle{+30^\circ} = 3.464 - j1.5 = 3.775\angle{-23.4^\circ} \text{ pu}$$

$$\mathbf{S}_1 = \mathbf{V}_1\mathbf{I}_1{}^* = 3.775\angle{+23.4^\circ} \text{ pu}$$

$$S_1 = 3.775(10) = 37.75 \text{ MVA; pf} = 0.9177 \text{ lagging}$$

$$I_1 = 3.775\left(\frac{10}{0.0138}\right) = 2736 \text{ A}$$

Per-Unit Scaling Extended to Three-Phase Systems

The extension to three-phase systems has been complicated to some extent by the use of traditional terminology and jargon, and a desire to normalize phase-to-phase and phase-to-neutral voltage simultaneously. The problem with this practice is that it renders Kirchhoff's voltage and current laws invalid in some circuits. Consider the general three-phase situation in Fig. 8.5, with all quantities in SI units.

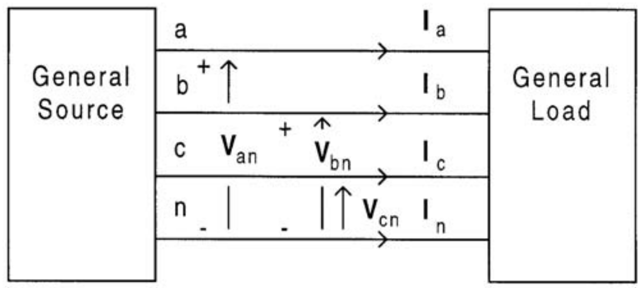


FIGURE 8.5 General three-phase system.

Define the complex operator:

$$\mathbf{a} = 1\angle 120°$$

The system is said to be balanced, with sequence abc, if:

$$\mathbf{V}_{bn} = \mathbf{a}^2\mathbf{V}_{an}$$

$$\mathbf{V}_{cn} = \mathbf{a}\mathbf{V}_{an}$$

and

$$\mathbf{I}_b = \mathbf{a}^2\mathbf{I}_a$$

$$\mathbf{I}_c = \mathbf{a}\mathbf{I}_a$$

$$-\mathbf{I}_n = \mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0$$

Likewise:

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn}$$

$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \mathbf{a}^2\mathbf{V}_{ab}$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \mathbf{a}\mathbf{V}_{ab}$$

If the load consists of wye-connected impedance:

$$\mathbf{Z}_y = \frac{\mathbf{V}_{an}}{\mathbf{I}_a} = \frac{\mathbf{V}_{bn}}{\mathbf{I}_b} = \frac{\mathbf{V}_{cn}}{\mathbf{I}_c}$$

The equivalent delta element is:

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y$$

To convert to per-unit, define the following bases:

$S_{3\phi base}$ = The three-phase apparent base at a specific location in a three-phase system, in VA.
V_{Lbase} = The line (phase-to-phase) rms voltage base at a specific location in a three-phase system, in V.

From the above, define:

$$S_{base} = S_{3\phi_{base}} \big/ 3 \tag{8.19}$$

$$V_{base} = V_{L_{base}} \big/ \sqrt{3} \tag{8.20}$$

It follows that:

$$I_{base} = S_{base} / V_{base} \tag{8.21}$$

$$Z_{base} = V_{base} / I_{base} \tag{8.22}$$

An example will be useful.

Example 3:

Consider a balanced three-phase 60 MVA 0.8 pf lagging load, sequence abc operating from a 13.8 kV (line voltage) bus. On bases of $S_{3\phi base}$ = 100 MVA and V_{Lbase} = 13.8 kV:

(a) Determine all bases.
(b) Determine all voltages, currents, and impedances, in SI units and per-unit.

Solution:

(a) $$S_{base} = \frac{S_{3\phi base}}{3} = \frac{100}{3} = 33.33 \text{ MVA}$$

$$V_{base} = \frac{V_{Lbase}}{\sqrt{3}} = \frac{13.8}{\sqrt{3}} = 7.967 \text{ kV}$$

$$I_{base} = \frac{S_{base}}{V_{base}} = 4.184 \text{ kA}$$

$$Z_{base} = \frac{V_{base}}{I_{base}} = 1.904\ \Omega$$

(b) $$\mathbf{V}_{an} = 7.967\angle 0° \text{ kV} \qquad (1.000\angle 0° \text{ pu})$$

$$\mathbf{V}_{bn} = 7.967\angle -120° \text{ kV} \qquad (1.000\angle -120° \text{ pu})$$

$$\mathbf{V}_{cn} = 7.967\angle +120° \text{ kV} \qquad (1.000\angle +120° \text{ pu})$$

$$S_a = S_b = S_c = \frac{S_{3\phi}}{3} = \frac{60}{3} = 20 \text{ MVA} \qquad (0.60 \text{ pu})$$

$$\mathbf{S}_a = \mathbf{S}_b = \mathbf{S}_c = 16 + j12 \text{ MVA} \qquad (0.48 + j0.36 \text{ pu})$$

$$\mathbf{I}_a = \left(\frac{\mathbf{S}_a}{\mathbf{V}_{an}}\right) = 2.510\angle -36.9° \text{kA} \qquad (0.6000\angle -36.9° \text{ pu})$$

$$\mathbf{I}_b = 2.510\angle -156.9°\,\text{kA} \quad (0.6000\angle -156.9°\ \text{pu})$$

$$\mathbf{I}_c = 2.510\angle 83.1°\,\text{kA} \quad (0.6000\angle 83.1°\ \text{pu})$$

$$\mathbf{Z}_Y = \frac{\mathbf{V}_{an}}{\mathbf{I}_a} = 3.174\angle +36.9° = 2.539 + j1.904\ \Omega \quad (1.33 + j1.000\ \text{pu})$$

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y = 7.618 + j5.713\ \Omega \quad (4 + j3\ \text{pu})$$

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = 13.8\angle 30°\ \text{kV} \quad (1.732\angle 30°\ \text{pu})$$

$$\mathbf{V}_{bc} = 13.8\angle -90°\ \text{kV} \quad (1.732\angle -90°\ \text{pu})$$

$$\mathbf{V}_{ca} = 13.8\angle 150°\ \text{kV} \quad (1.732\angle 150°\ \text{pu})$$

Converting voltages and currents to symmetrical components:

$$\begin{bmatrix} \mathbf{V}_0 \\ \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} \mathbf{V}_{an} \\ \mathbf{V}_{bn} \\ \mathbf{V}_{cn} \end{bmatrix} = \begin{bmatrix} 0\ \text{kV} & (0\ \text{pu}) \\ 7.967\angle 0°\ \text{kV} & (1\angle 0°\ \text{pu}) \\ 0\ \text{kV} & (0\ \text{pu}) \end{bmatrix}$$

$$\mathbf{I}_0 = 0\ \text{kA} \quad (0\ \text{pu})$$

$$\mathbf{I}_1 = 2.510\angle -36.9°\ \text{kA} \quad (0.6\angle -36.9°\ \text{pu})$$

$$\mathbf{I}_2 = 0\ \text{kA} \quad (0\ \text{pu})$$

Inclusion of transformers demonstrates the advantages of per-unit scaling.

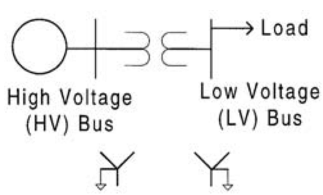


FIGURE 8.6 A three-phase transformer situation.

Example 4:

A 3ϕ 240 kV :15 kV transformer supplies a 13.8 kV 60 MVA pf = 0.8 lagging load, and is connected to a 230 kV source on the HV side, as shown in Fig. 8.6.

(a) Determine all base values on both sides for $S_{3\phi base}$ = 100 MVA. At the LV bus, V_{Lbase} = 13.8 kV.
(b) Draw the positive sequence circuit in per-unit, modeling the transformer as ideal.
(c) Determine all currents and voltages in SI and per-unit.

Solution:

(a) Base values on the LV side are the same as in Example 3.
The turns ratio may be derived from the voltage ratings ratios:

$$\frac{N_1}{N_2} = \frac{240/\sqrt{3}}{15/\sqrt{3}} = 16$$

$$\therefore \left(V_{base}\right)_{HV\ side} = \frac{N_1}{N_2}\left(V_{base}\right)_{LV\ side} = 16.00\left(7.967\right) = 127.5\ \text{kV}$$

$$\left(I_{base}\right)_{HV\ side} = \frac{S_{base}}{\left(V_{base}\right)_{HV\ side}} = \frac{33.33}{0.1275} = 261.5\ \text{A}$$

Results are presented in the following chart.

Bus	$S_{3\phi base}$ MVA	$V_{L\ base}$ kV	S_{base} MVA	I_{base} kA	V_{base} kV	Z_{base} ohm
LV	100	13.8	33.33	4.184	7.967	1.904
HV	100	220.8	33.33	0.2615	127.5	487.5

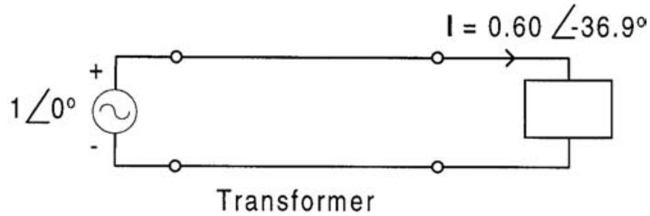


FIGURE 8.7 Positive sequence circuit.

(b) $$\mathbf{V}_{LV} = \frac{7.967\angle 0°}{7.967} = 1\angle 0°\ \text{pu}$$

$$S_{1\phi} = \frac{60}{3} = 20\ \text{MVA}$$

$$S_{1\phi} = \frac{20}{33.33} = 0.6\ \text{pu}$$

(c) All values determined in pu are valid on both sides of the transformer! To determine SI values on the HV side, use HV bases. For example:

$$\mathbf{V}_{an} = \left(1\angle 0°\right)127.5 = 127.5\angle 0°\ \text{kV}$$

$$\mathbf{V}_{ab} = \left(1.732\angle 30°\right)\left(127.5\right) = 220.8\angle 30°\ \text{kV}$$

$$\mathbf{I}_a = \left(0.6\angle -36.9°\right)\left(261.5\right) = 156.9\angle -36.9°\ \text{A}$$

Example 5:

Repeat the previous example using a 3ϕ 240 kV:15 kV Y Δ

Solution:

All results are the same as before. The reasoning is as follows.

The voltage ratings are interpreted as line (phase-to-phase) values *independent of connection* (wye or delta). Therefore the turns ratio remains:

$$\frac{N_1}{N_2} = \frac{240\ \sqrt{3}}{15/\sqrt{3}} = 16$$

As before:

$$\left(V_{an}\right)_{\text{LV side}} = 7.967 \text{ kV}$$

$$\left(V_{an}\right)_{\text{HV side}} = 127.5 \text{ kV}$$

However, $\mathbf{V}_{an}$ is no longer in phase on both sides. This is a consequence of the transformer model, and not due to the scaling procedure. Whether this is important depends on the details of the analysis.

Per-Unit Scaling Extended to a General Three-Phase System

The ideas presented are extended to a three-phase system using the following procedure.

1. Select a three-phase apparent power base ($S_{\text{3ph base}}$), which is typically 1, 10, 100, or 1000 MVA. This base is valid at every bus in the system.
2. Select a line voltage base ($V_{\text{L base}}$), user defined, but usually the nominal rms line-to-line voltage at a user-defined bus (call this the "reference bus").
3. Compute

$$S_{\text{base}} = \left(S_{\text{3ph base}}\right)/3 \qquad \left(\text{Valid at every bus}\right) \tag{8.23}$$

4. At the reference bus:

$$V_{\text{base}} = V_{\text{L base}}/\sqrt{3} \tag{8.24}$$

$$I_{\text{base}} = S_{\text{base}}/V_{\text{base}} \tag{8.25}$$

$$Z_{\text{base}} = V_{\text{base}}/I_{\text{base}} = V_{\text{base}}^2/S_{\text{base}} \tag{8.26}$$

5. To determine the bases at the remaining busses in the system, start at the reference bus, which we will call the "from" bus, and execute the following procedure:

   Trace a path to the next nearest bus, called the "to" bus. You reach the "to" bus by either passing over (1) a line, or (2) a transformer.

   (1) The "line" case: $V_{\text{L base}}$ is the same at the "to" bus as it was at the "from" bus. Use Eqs. (8.2), (8.3), and (8.4) to compute the "to" bus bases.

   (2) The "transformer" case: Apply $V_{\text{L base}}$ at the "from" bus, and treat the transformer as ideal. Calculate the line voltage that appears at the "to" bus. This is now the new $V_{\text{L base}}$ at the "to" bus. Use Eqs. (8.2), (8.3), and (8.4) to compute the "to" bus bases.

Rename the bus at which you are located, the "from" bus. Repeat the above procedure until you have processed every bus in the system.

6. We now have a set of bases for every bus in the system, which are to be used for every element terminated at that corresponding bus. Values are scaled according to:

   per-unit value = actual value/base value

   where actual value = the actual complex value of S, V, Z, or I, in SI units (VA, V, Ω, A); base value = the (user-defined) base value (real) of S, V, Z, or I, in SI units (VA, V, Ω, A); per-unit value = the per-unit complex value of S, V, Z, or I, in per-unit (dimensionless).

Finally, the reader is advised that there are many scaling systems used in engineering analysis, and, in fact, several variations of per-unit scaling have been used in electric power engineering applications. There is no standard system to which everyone conforms in every detail. The key to successfully using any scaling procedure is to understand how all base values are selected at every location within the power system. If one receives data in per-unit, one must be in a position to convert all quantities to SI units. If this cannot be done, the analyst must return to the data source for clarification on what base values were used.

8.2 Symmetrical Components for Power System Analysis

Tim A. Haskew

Modern power systems are three-phase systems that can be balanced or unbalanced and will have mutual coupling between the phases. In many instances, the analysis of these systems is performed using what is known as "per-phase analysis." In this chapter, we will introduce a more generally applicable approach to system analysis know as "symmetrical components." The concept of symmetrical components was first proposed for power system analysis by C.L. Fortescue in a classic paper devoted to consideration of the general N-phase case (1918). Since that time, various similar modal transformations (Brogan, 1974) have been applied to a variety of power type problems including rotating machinery (Krause, 1986; Kundur, 1994).

The case for per-phase analysis can be made by considering the simple three-phase system illustrated in Fig. 8.8. The steady-state circuit response can be obtained by solution of the three loop equations presented in Eq. (8.27a) through (8.27c). By solving these loop equations for the three line currents, Eq. (8.28a) through (8.28a) are obtained. Now, if we assume completely balanced source operation (the impedances are defined to be balanced), then the line currents will also form a balanced three-phase set. Hence, their sum, and the neutral current, will be zero. As a result, the line current solutions are as presented in Eq. (8.29a) through (8.29c).

$$\bar{V}_a - \bar{I}_a\left(R_S + jX_S\right) - \bar{I}_a\left(R_L + jX_L\right) - \bar{I}_n\left(R_n + jX_n\right) = 0 \tag{8.27a}$$

$$\bar{V}_b - \bar{I}_b\left(R_S + jX_S\right) - \bar{I}_b\left(R_L + jX_L\right) - \bar{I}_n\left(R_n + jX_n\right) = 0 \tag{8.27b}$$

$$\bar{V}_c - \bar{I}_c\left(R_S + jX_S\right) - \bar{I}_c\left(R_L + jX_L\right) - \bar{I}_n\left(R_n + jX_n\right) = 0 \tag{8.27c}$$

$$\bar{I}_a = \frac{\bar{V}_a - \bar{I}_n\left(R_n + jX_n\right)}{\left(R_s + R_n\right) + j\left(X_s + X_n\right)} \tag{8.28a}$$

$$\bar{I}_b = \frac{\bar{V}_b - \bar{I}_n\left(R_n + jX_n\right)}{\left(R_s + R_n\right) + j\left(X_s + X_n\right)} \tag{8.28b}$$

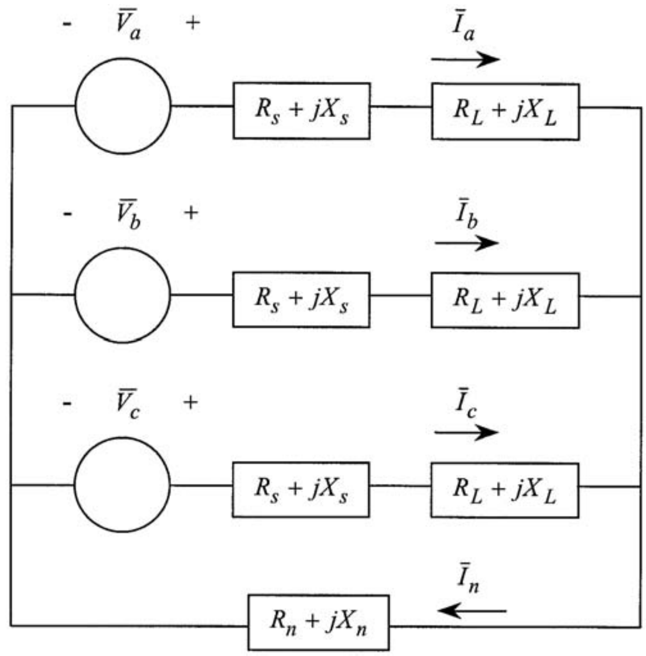


FIGURE 8.8 A simple three-phase system.

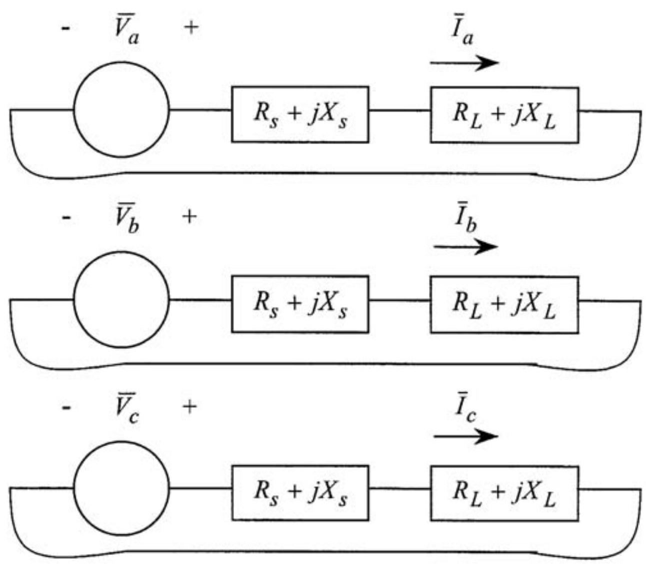


FIGURE 8.9 Decoupled phases of the three-phase system.

$$\bar{I}_c = \frac{\bar{V}_c - \bar{I}_n\left(R_n + jX_n\right)}{\left(R_s + R_n\right) + j\left(X_s + X_n\right)} \tag{8.28c}$$

$$\bar{I}_a = \frac{\bar{V}_a}{\left(R_s + R_n\right) + j\left(X_s + X_n\right)} \tag{8.29a}$$

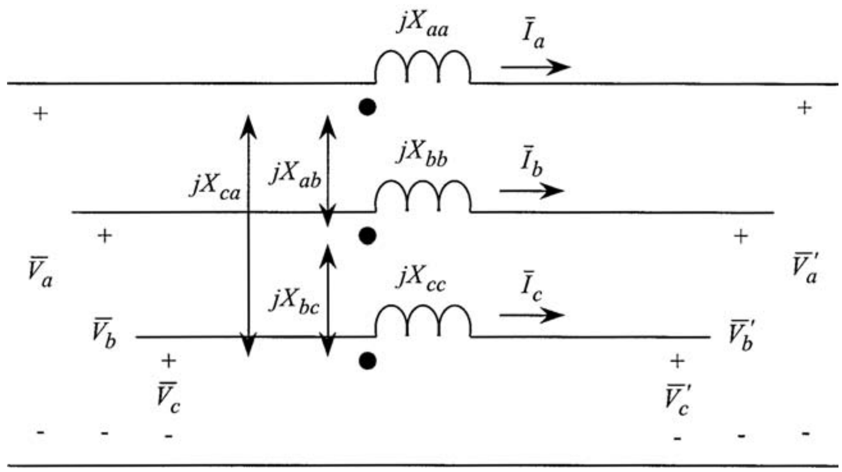


FIGURE 8.10 Mutually coupled series impedances.

$$\bar{I}_b = \frac{\bar{V}_b}{\left(R_s + R_n\right) + j\left(X_s + X_n\right)} \tag{8.29b}$$

$$\bar{I}_c = \frac{\bar{V}_c}{\left(R_s + R_n\right) + j\left(X_s + X_n\right)} \tag{8.29c}$$

The circuit synthesis of Eq. (8.29a) through (8.29c) is illustrated in Fig. 8.9. Particular notice should be taken of the fact the response of each phase is independent of the other two phases. Thus, only one phase need be solved, and three-phase symmetry may be applied to determine the solutions for the other phases. This solution technique is the per-phase analysis method.

If one considers the introduction of an unbalanced source or mutual coupling between the phases in Fig. 8.8, then per-phase analysis will not result in three decoupled networks as shown in Fig. 8.9. In fact, in the general sense, no immediate circuit reduction is available without some form of reference frame transformation. The symmetrical component transformation represents such a transformation, which will enable decoupled analysis in the general case and single-phase analysis in the balanced case.

Fundamental Definitions

Voltage and Current Transformation

To develop the symmetrical components, let us first consider an arbitrary (no assumptions on balance) three-phase set of voltages as defined in Eq. (8.30a) through (8.30c). Note that we could just as easily be considering current for the purposes at hand, but voltage was selected arbitrarily. Each voltage is defined by a magnitude and phase angle. Hence, we have six degrees of freedom to fully define this arbitrary voltage set.

$$\bar{V}_a = V_a \angle \theta_a \tag{8.30a}$$

$$\bar{V}_b = V_b \angle \theta_b \tag{8.30b}$$

$$\bar{V}_c = V_c \angle \theta_c \tag{8.30c}$$

We can represent each of the three given voltages as the sum of three components as illustrated in Eq. (8.31a) through (8.31c). For now, we consider these components to be completely arbitrary except for their sum. The 0, 1, and 2 subscripts are used to denote the zero, positive, and negative sequence components of each phase voltage, respectively. Examination of Eq. (8.31a-c) reveals that 6 degrees of freedom exist on the left-hand side of the equations while 18 degrees of freedom exist on the right-hand side. Therefore, for the relationship between the voltages in the abc frame of reference and the voltages in the 012 frame of reference to be unique, we must constrain the right-hand side of Eq. (8.31).

$$\overline{V}_a = \overline{V}_{a_0} + \overline{V}_{a_1} + \overline{V}_{a_2} \tag{8.31a}$$

$$\overline{V}_b = \overline{V}_{b_0} + \overline{V}_{b_1} + \overline{V}_{b_2} \tag{8.31b}$$

$$\overline{V}_c = \overline{V}_{c_0} + \overline{V}_{c_1} + \overline{V}_{c_2} \tag{8.31c}$$

We begin by forcing the a_0, b_0, and c_0 voltages to have equal magnitude and phase. This is defined in Eq. (8.32). The zero sequence components of each phase voltage are all defined by a single magnitude and a single phase angle. Hence, the zero sequence components have been reduced from 6 degrees of freedom to 2.

$$\overline{V}_{a_0} = \overline{V}_{b_0} = \overline{V}_{c_0} \equiv \overline{V}_0 = V_0 \angle \theta_0 \tag{8.32}$$

Second, we force the a_1, b_1, and c_1 voltages to form a balanced three-phase set with positive phase sequence. This is mathematically defined in Eq. (8.33a-c). This action reduces the degrees of freedom provided by the positive sequence components from 6 to 2.

$$\overline{V}_{a_1} = \overline{V}_1 = V_1 \angle \theta_1 \tag{8.33a}$$

$$\overline{\mathrm{V}}_{\mathrm{b}_1} = \mathrm{V}_1 \angle \left(\theta_1 - 120^\circ\right) = \overline{\mathrm{V}}_1 \bullet 1 \underline{/-120^\circ} \tag{8.33b}$$

$$\overline{\mathrm{V}}_{\mathrm{c}_1} = \mathrm{V}_1 \angle \left(\theta_1 + 120^\circ\right) = \overline{\mathrm{V}}_1 \bullet 1 \underline{/+120^\circ} \tag{8.33c}$$

And finally, we force the a_2, b_2, and c_2 voltages to form a balanced three-phase set with negative phase sequence. This is mathematically defined in Eq. (8.34a-c). As in the case of the positive sequence components, the negative sequence components have been reduced from 6 to 2 degrees of freedom.

$$\overline{V}_{a_2} = \overline{V}_2 = V_2 \angle \theta_2 \tag{8.34a}$$

$$\overline{\mathrm{V}}_{\mathrm{b}_2} = \mathrm{V}_2 \angle \left(\theta_2 + 120^\circ\right) = \overline{\mathrm{V}}_2 \bullet 1 \underline{/+120^\circ} \tag{8.34b}$$

$$\overline{\mathrm{V}}_{\mathrm{c}_2} = \mathrm{V}_2 \angle \left(\theta_2 - 120^\circ\right) = \overline{\mathrm{V}}_2 \bullet 1 \underline{/-120^\circ} \tag{8.34c}$$

Now, the right- and left-hand sides of Eq. (8.31a) through (8.31c) each have 6 degrees of freedom. Thus, the relationship between the symmetrical component voltages and the original phase voltages is unique. The final relationship is presented in Eq. (8.35a) through (8.35c). Note that the constant "a" has been defined as indicated in Eq. (8.36).

$$\overline{V}_a = \overline{V}_0 + \overline{V}_1 + \overline{V}_2 \tag{8.35a}$$

$$\overline{V}_b = \overline{V}_0 + \overline{a}^2\overline{V}_1 + \overline{a}\,\overline{V}_2 \tag{8.35b}$$

$$\overline{V}_c = \overline{V}_0 + \overline{a}\,\overline{V}_1 + \overline{a}^2\overline{V}_2 \tag{8.35c}$$

$$\overline{a} = 1\angle 120° \tag{8.36}$$

Equation (8.35) is more easily written in matrix form, as indicated in Eq. (8.37) in both expanded and compact form. In Eq. (8.37), the [T] matrix is constant, and the inverse exists. Thus, the inverse transformation can be defined as indicated in Eq. (8.38). The over tilde (~) indicates a vector of complex numbers.

$$\begin{bmatrix}\overline{V}_a\\ \overline{V}_b\\ \overline{V}_c\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\ 1 & \overline{a}^2 & \overline{a}\\ 1 & \overline{a} & \overline{a}^2\end{bmatrix}\begin{bmatrix}\overline{V}_0\\ \overline{V}_1\\ \overline{V}_2\end{bmatrix}$$
$$\tilde{V}_{abc} = \left[\overline{T}\right]\tilde{V}_{012} \tag{8.37}$$

$$\begin{bmatrix}\overline{V}_0\\ \overline{V}_1\\ \overline{V}_2\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \overline{a} & \overline{a}^2\\ 1 & \overline{a}^2 & \overline{a}\end{bmatrix}\begin{bmatrix}\overline{V}_a\\ \overline{V}_b\\ \overline{V}_c\end{bmatrix}$$
$$\tilde{V}_{012} = \left[\overline{T}\right]^{-1}\tilde{V}_{abc} \tag{8.38}$$

Equations (8.39) and (8.40) define an identical transformation and inverse transformation for current.

$$\begin{bmatrix}\overline{I}_a\\ \overline{I}_b\\ \overline{I}_c\end{bmatrix} = \begin{bmatrix}1 & 1 & 1\\ 1 & \overline{a}^2 & \overline{a}\\ 1 & \overline{a} & \overline{a}^2\end{bmatrix}\begin{bmatrix}\overline{I}_0\\ \overline{I}_1\\ \overline{I}_2\end{bmatrix}$$
$$\tilde{I}_{abc} = \left[\overline{T}\right]\tilde{I}_{012} \tag{8.39}$$

$$\begin{bmatrix}\overline{I}_0\\ \overline{I}_1\\ \overline{I}_2\end{bmatrix} = \frac{1}{3}\begin{bmatrix}1 & 1 & 1\\ 1 & \overline{a} & \overline{a}^2\\ 1 & \overline{a}^2 & \overline{a}\end{bmatrix}\begin{bmatrix}\overline{I}_a\\ \overline{I}_b\\ \overline{I}_c\end{bmatrix}$$
$$\tilde{I}_{012} = \left[\overline{T}\right]^{-1}\tilde{I}_{abc} \tag{8.40}$$

Impedance Transformation

In order to assess the impact of the symmetrical component transformation on systems impedances, we turn to Fig. 8.10. Note that the balanced case has been assumed. Kirchhoff's Voltage Law for the circuit dictates equations Eq. (8.41a-c), which are written in matrix form in Eq. (8.42) and even more simply in Eq. (8.43).

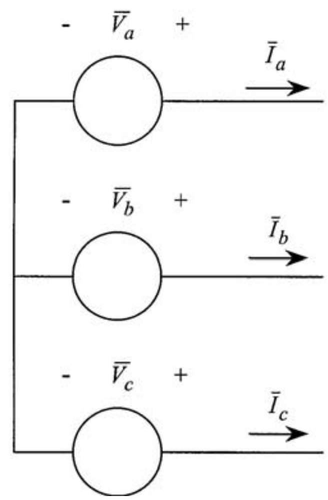


FIGURE 8.11 Three-phase wye-connected source.

$$\overline{V}_a - \overline{V}_a' = jX_{aa}\,\overline{I}_a + jX_{ab}\,\overline{I}_b + jX_{ca}\,\overline{I}_c \tag{8.41a}$$

$$\overline{V}_b - \overline{V}_b' = jX_{ab}\,\overline{I}_a + jX_{bb}\,\overline{I}_b + jX_{bc}\,\overline{I}_c \tag{8.41b}$$

$$\overline{V}_c - \overline{V}_c' = jX_{ca}\,\overline{I}_a + jX_{bc}\,\overline{I}_b + jX_{cc}\,\overline{I}_c \tag{8.41c}$$

$$\begin{bmatrix} \overline{V}_a \\ \overline{V}_b \\ \overline{V}_c \end{bmatrix} - \begin{bmatrix} \overline{V}_a' \\ \overline{V}_b' \\ \overline{V}_c' \end{bmatrix} = j\begin{bmatrix} X_{aa} & X_{ab} & X_{ca} \\ X_{ab} & X_{bb} & X_{bc} \\ X_{ca} & X_{bc} & X_{cc} \end{bmatrix}\begin{bmatrix} \overline{I}_a \\ \overline{I}_b \\ \overline{I}_c \end{bmatrix} \tag{8.42}$$

$$\tilde{V}_{abc} - \tilde{V}_{abc}' = \left[\overline{Z}_{abc}\right]\tilde{I}_{abc} \tag{8.43}$$

Multiplying both sides of Eq. (8.43) by $[\overline{T}]^{-1}$ yields Eq. (8.44). Then, substituting Eq. (8.38) and (8.39) into the result leads to the sequence equation presented in Eq. (8.45). The equation is written strictly in the 012 frame reference in Eq. (8.46) where the sequence impedance matrix is defined in Eq. (8.47).

$$\left[\overline{T}\right]^{-1}\tilde{V}_{abc} - \left[\overline{T}\right]^{-1}\tilde{V}_{abc}' = \left[\overline{T}\right]^{-1}\left[\overline{Z}_{abc}\right]\tilde{I}_{abc} \tag{8.44}$$

$$\tilde{V}_{012} - \tilde{V}_{012}' = \left[\overline{T}\right]^{-1}\left[\overline{Z}_{abc}\right]\left[\overline{T}\right]\tilde{I}_{012} \tag{8.45}$$

$$\tilde{V}_{012} - \tilde{V}_{012}' = \left[\overline{Z}_{012}\right]\tilde{I}_{012} \tag{8.46}$$

$$\left[\overline{Z}_{012}\right] = \left[\overline{T}\right]^{-1}\left[\overline{Z}_{abc}\right]\left[\overline{T}\right] = \begin{bmatrix} \overline{Z}_{00} & \overline{Z}_{01} & \overline{Z}_{02} \\ \overline{Z}_{10} & \overline{Z}_{11} & \overline{Z}_{12} \\ \overline{Z}_{20} & \overline{Z}_{21} & \overline{Z}_{22} \end{bmatrix} \tag{8.47}$$

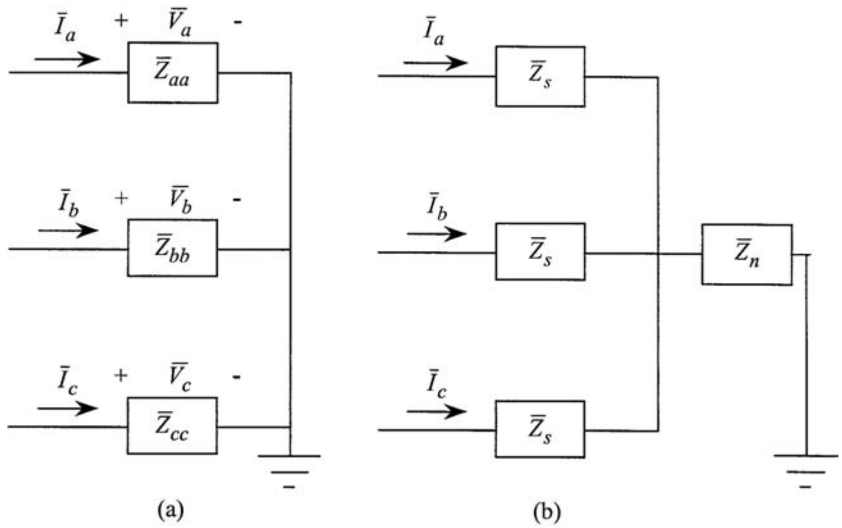


FIGURE 8.12 Three-phase impedance load model.

Power Calculations

The impact of the symmetrical components on the computation of complex power can be easily derived from the basic definition. Consider the source illustrated in Fig. 8.11. The three-phase complex power supplied by the source is defined in Eq. (8.48). The algebraic manipulation to Eq. (8.48) is presented, and the result in the sequence domain is presented in Eq. (8.49) in matrix form and in Eq. (8.50) in scalar form.

$$\bar S_{3\phi} = \bar V_a \bar I_a^* + \bar V_b \bar I_b^* + \bar V_c \bar I_c^* = \tilde V_{abc}^T \tilde I_{abc}^* \tag{8.48}$$

$$\bar S_{3\phi} = \tilde V_{abc}^T \tilde I_{abc}^* = \left\{ \left[\bar T\right] \tilde V_{012} \right\}^T \left\{ \left[\bar T\right] \tilde I_{012} \right\}^*$$

$$= \tilde V_{012}^T \left[\bar T\right]^T \left[\bar T\right]^* \tilde I_{012}^*$$

$$\left[\bar T\right]^T \left[\bar T\right]^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar a^2 & \bar a \\ 1 & \bar a & \bar a^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar a & \bar a^2 \\ 1 & \bar a^2 & \bar a \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\bar S_{3\phi} = 3 \tilde V_{012}^T \tilde I_{012}^* \tag{8.49}$$

$$\bar S_{3\phi} = 3 \left\{ \bar V_0 \bar I_0^* + \bar V_1 \bar I_1^* + \bar V_2 \bar I_2^* \right\} \tag{8.50}$$

Note that the nature of the symmetrical component transformation is not one of power invariance, as indicated by the multiplicative factor of 3 in Eq. (8.50). However, this will prove useful in the analysis of

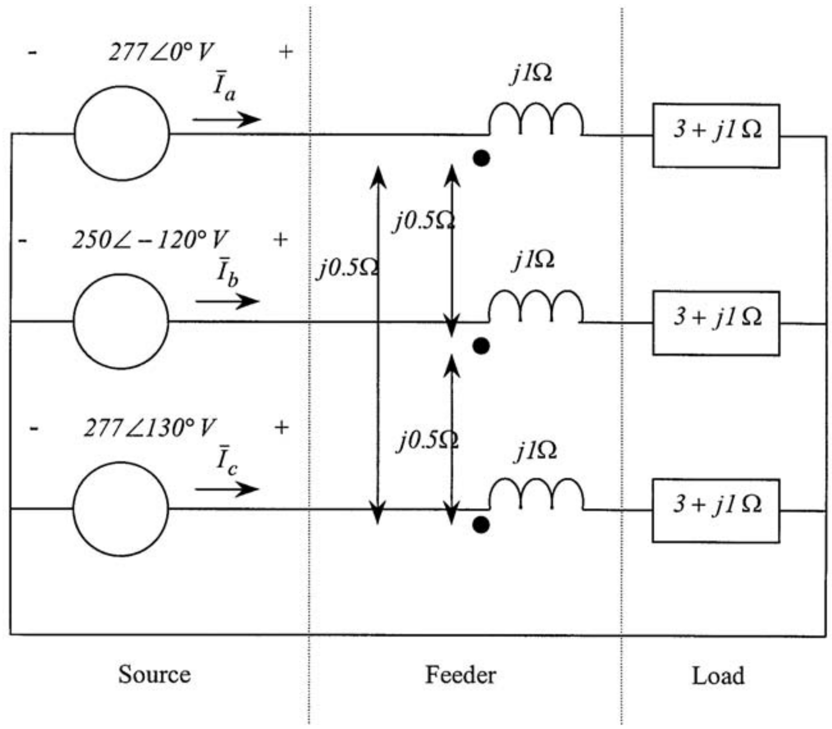


FIGURE 8.13 Power system for Example 1.

balanced systems, which will be seen later. Power invariant transformations do exist as minor variations of the one defined herein. However, they are not typically employed, although the results are just as mathematically sound.

System Load Representation

System loads may be represented in the symmetrical components in a variety of ways, depending on the type of load model that is preferred. Consider first a general impedance type load. Such a load is illustrated in Fig. 8.12a. In this case, Eq. (8.43) applies with $\tilde{V}'_{abc} = 0$ due to the solidly grounded Y connection. Therefore, the sequence impedances are still correctly defined by Eq. (8.47). As illustrated in Fig. 8.12a, the load has zero mutual coupling. Hence, the off-diagonal terms will be zero. However, mutual terms may be considered, as Eq. (8.47) is general in nature. This method can be applied for any shunt-connected impedances in the system.

If the load is Δ-connected, then it should be converted to an equivalent Y-connection prior to the transformation (Irwin, 1996; Gross, 1986). In this case, the possibility of unbalanced mutual coupling will be excluded, which is practical in most cases. Then, the off-diagonal terms in Eq. (8.47) will be zero, and the sequence networks for the load will be decoupled. Special care should be taken that the zero sequence impedance will become infinite because the Δ-connection does not allow a path for a neutral current to flow, which is equivalent to not allowing a zero sequence current path as defined by the first row of matrix Eq. (8.40). A similar argument can be made for a Y-connection that is either ungrounded or grounded through an impedance, as indicated in Fig. 8.12b. In this case, the zero sequence impedance will be equal to the sum of the phase impedance and three times the neutral impedance, or, $\bar{Z}_{00} = \bar{Z}_Y + 3\bar{Z}_n$. Notice should be taken that the neutral impedance can vary from zero to infinity.

The representation of complex power load models will be left for the section on the application of balanced circuit reductions to the symmetrical component transformation.

TABLE 8.1 Summary of the Symmetrical Components in the General Case

Quantity	Transformation Equations: abc ⇒ 012	Transformation Equations: 012 ⇒ abc
Voltage	$\begin{bmatrix}\bar{V}_0\\\bar{V}_1\\\bar{V}_2\end{bmatrix}=\frac{1}{3}\begin{bmatrix}1&1&1\\1&\bar{a}&\bar{a}^2\\1&\bar{a}^2&\bar{a}\end{bmatrix}\begin{bmatrix}\bar{V}_a\\\bar{V}_b\\\bar{V}_c\end{bmatrix}$ $\tilde{V}_{012}=\left[\bar{T}\right]^{-1}\tilde{V}_{abc}$	$\begin{bmatrix}\bar{V}_a\\\bar{V}_b\\\bar{V}_c\end{bmatrix}=\begin{bmatrix}1&1&1\\1&\bar{a}^2&\bar{a}\\1&\bar{a}&\bar{a}^2\end{bmatrix}\begin{bmatrix}\bar{V}_0\\\bar{V}_1\\\bar{V}_2\end{bmatrix}$ $\tilde{V}_{abc}=\left[\bar{T}\right]\tilde{V}_{012}$
Current	$\begin{bmatrix}\bar{I}_0\\\bar{I}_1\\\bar{I}_2\end{bmatrix}=\frac{1}{3}\begin{bmatrix}1&1&1\\1&\bar{a}&\bar{a}^2\\1&\bar{a}^2&\bar{a}\end{bmatrix}\begin{bmatrix}\bar{I}_a\\\bar{I}_b\\\bar{I}_c\end{bmatrix}$ $\bar{I}_{012}=\left[\bar{T}\right]^{-1}\bar{I}_{abc}$	$\begin{bmatrix}\bar{I}_a\\\bar{I}_b\\\bar{I}_c\end{bmatrix}=\begin{bmatrix}1&1&1\\1&\bar{a}^2&\bar{a}\\1&\bar{a}&\bar{a}^2\end{bmatrix}\begin{bmatrix}\bar{I}_0\\\bar{I}_1\\\bar{I}_2\end{bmatrix}$ $\bar{I}_{abc}=\left[\bar{T}\right]\bar{I}_{012}$
Impedance	$\left[\bar{Z}_{012}\right]=\left[\bar{T}\right]^{-1}\left[\bar{Z}_{abc}\right]\left[\bar{T}\right]$	
Power	$\bar{S}_{3\phi}=\bar{V}_a\bar{I}_a^*+\bar{V}_b\bar{I}_b^*+\bar{V}_c\bar{I}_c^*=\tilde{V}_{abc}^T\tilde{I}_{abc}^*$ $\bar{S}_{3\phi}=3\left\{\bar{V}_0\bar{I}_0^*+\bar{V}_2\bar{I}_2^*+\bar{V}_3\bar{I}_3^*\right\}=3\tilde{V}_{012}^T\tilde{I}_{012}^*$	

Summary of the Symmetrical Components in the General Three-Phase Case

The general symmetrical component transformation process has been defined in this section. Table 8.1 is a short form reference for the utilization of these procedures in the general case (i.e., no assumption of balanced conditions). Application of these relationships defined in Table 8.1 will enable the power system analyst to draw the zero, positive, and negative sequence networks for the system under study. These networks can then be analyzed in the 012 reference frame, and the results can be easily transformed back into the abc reference frame.

Example 1:

The power system illustrated in Fig. 8.13 is to be analyzed using the sequence networks. Find the following:

(a) three line currents
(b) line-to-neutral voltages at the load
(c) three-phase complex power output of the source

Solution:

The sequence voltages are computed in Eq. (8.51). The sequence impedances for the feeder and the load are computed in Eqs. (8.52) and (8.53), respectively. The sequence networks are drawn in Fig. 8.14.

$$\begin{bmatrix}\bar{V}_0\\\bar{V}_1\\\bar{V}_2\end{bmatrix}=\frac{1}{3}\begin{bmatrix}1&1&1\\1&\bar{a}&\bar{a}^2\\1&\bar{a}^2&\bar{a}\end{bmatrix}\begin{bmatrix}255\angle 0^\circ\\250\angle -120^\circ\\277\angle 130^\circ\end{bmatrix}=\begin{bmatrix}8.8\angle -171^\circ\\267.1\angle 3^\circ\\24.0\angle -37^\circ\end{bmatrix}V \tag{8.51}$$

$$\left[\bar{Z}_{012}\right]=\left[\bar{T}\right]^{-1}\begin{bmatrix}j1&j0.5&j0.5\\j0.5&j1&j0.5\\j0.5&j0.5&j1\end{bmatrix}\left[\bar{T}\right]=\begin{bmatrix}j2&0&0\\0&j0.5&0\\0&0&j0.5\end{bmatrix}\Omega \tag{8.52}$$

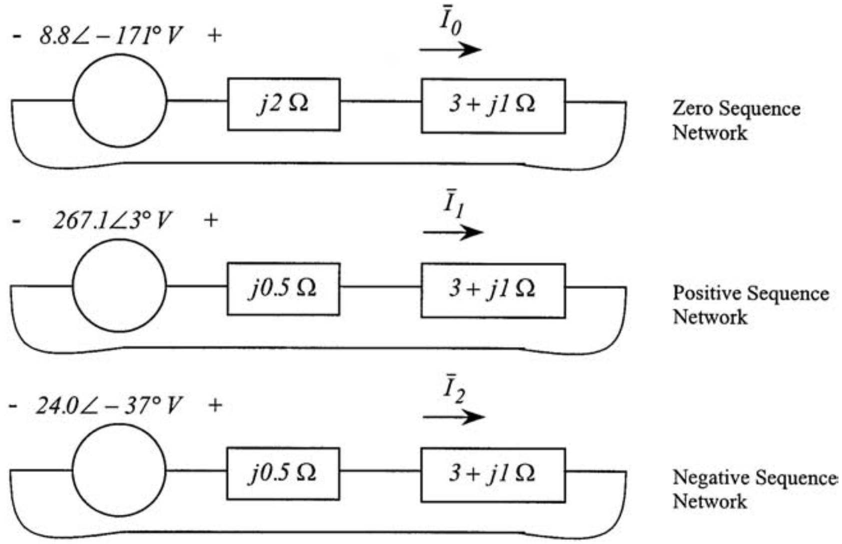


FIGURE 8.14 Sequence networks for Example 1.

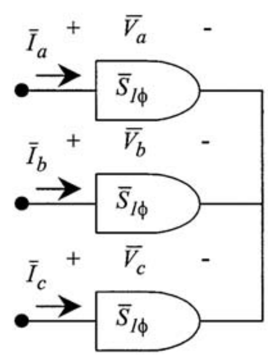


FIGURE 8.15 Balanced complex power load model.

$$\left[\bar{Z}_{012}\right]=\left[\bar{T}\right]^{-1}\begin{bmatrix}3+j1 & 0 & 0\\ 0 & 3+j1 & 0\\ 0 & 0 & 3+j1\end{bmatrix}\left[\bar{T}\right]=\begin{bmatrix}3+j1 & 0 & 0\\ 0 & 3+j1 & 0\\ 0 & 0 & 3+j1\end{bmatrix}\Omega \tag{8.53}$$

The sequence currents are computed in Eq. (8.54a-c). In Eq. (8.55), the sequence currents and sequence load impedances are used to compute the zero, positive, and negative sequence load voltages.

$$\bar{I}_0=\frac{8.8\angle-171^\circ}{3+j\left(1+2\right)}=2.1\angle 144^\circ\ A \tag{8.54a}$$

$$\bar{I}_1=\frac{267.1\angle 3^\circ}{3+j\left(1+0.5\right)}=79.6\angle-24^\circ\ A \tag{8.54b}$$

$$\bar{I}_2=\frac{24.0\angle-37^\circ}{3+j\left(1+0.5\right)}=7.2\angle-64^\circ\ A \tag{8.54c}$$

$$\begin{bmatrix} \bar{V}_0 \\ \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} = \left[\bar{Z}_{012}\right]\tilde{I}_{012} = \begin{bmatrix} 3+j1 & 0 & 0 \\ 0 & 3+j1 & 0 \\ 0 & 0 & 3+j1 \end{bmatrix} \begin{bmatrix} 2.1\angle 144° \\ 79.6\angle -24° \\ 7.2\angle -64° \end{bmatrix}$$

$$= \begin{bmatrix} 6.6\angle 162° \\ 251.7\angle -6° \\ 22.8\angle -46° \end{bmatrix} V \tag{8.55}$$

The three line currents can be computed as illustrated in Eq. (8.56), and the line-to-neutral load voltages are computed in Eq. (8.57). The three-phase complex power output of the source is computed in Eq. (8.58).

$$\begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a}^2 & \bar{a} \\ 1 & \bar{a} & \bar{a}^2 \end{bmatrix} \begin{bmatrix} 2.1\angle 144° \\ 79.6\angle -24° \\ 7.2\angle -64° \end{bmatrix} = \begin{bmatrix} 83.2\angle -27° \\ 73.6\angle -147° \\ 82.7\angle 102° \end{bmatrix} A \tag{8.56}$$

$$\begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \bar{a}^2 & \bar{a} \\ 1 & \bar{a} & \bar{a}^2 \end{bmatrix} \begin{bmatrix} 6.6\angle 162° \\ 251.7\angle -6° \\ 22.8\angle -46° \end{bmatrix} = \begin{bmatrix} 263.0\angle -9° \\ 232.7\angle -129° \\ 261.5\angle 120° \end{bmatrix} V \tag{8.57}$$

$$\bar{S}_{3\phi} = 3\left\{\bar{V}_0\bar{I}_0^* + \bar{V}_1\bar{I}_1^* + \bar{V}_2\bar{I}_2^*\right\} = 57.3 + j29.2\ kVA \tag{8.58}$$

Reduction to the Balanced Case

When the power system under analysis is operating under balanced conditions, the symmetrical components allow one to perform analysis on a single-phase network in a manner similar to per-phase analysis, even when mutual coupling is present. The details of the method are presented in this section.

Balanced Voltages and Currents

Consider a balanced three-phase source operating with positive phase sequence. The voltages are defined below in Eq. (8.59). Upon computation of Eq. (8.38), one discovers that the sequence voltages that result are those shown in Eq. (8.60).

$$\tilde{V}_{abc} = \begin{bmatrix} V_a\angle\theta_a \\ V_a\angle\left(\theta_a - 120°\right) \\ V_a\angle\left(\theta_a + 120°\right) \end{bmatrix} \tag{8.59}$$

$$\tilde{V}_{012} = \begin{bmatrix} 0 \\ V_a\angle\theta_a \\ 0 \end{bmatrix} \tag{8.60}$$

In Eq. (8.61), a source is defined with negative phase sequence. The sequence voltages for this case are presented in Eq. (8.62).

$$\tilde{V}_{abc} = \begin{bmatrix} V_a\angle\theta_a \\ V_a\angle\left(\theta_a + 120°\right) \\ V_a\angle\left(\theta_a - 120°\right) \end{bmatrix} \tag{8.61}$$

$$\tilde{V}_{012} = \begin{bmatrix} 0 \\ 0 \\ V_a \angle \theta_a \end{bmatrix} \tag{8.62}$$

These results are particularly interesting. For a balanced source with positive phase sequence, only the positive sequence voltage is non-zero, and its value is the a-phase line-to-neutral voltage. Similarly, for a balanced source with negative phase sequence, the negative sequence voltage is the only non-zero voltage, and it is also equal to the a-phase line-to-neutral voltage. Identical results can be shown for positive and negative phase sequence currents.

Balanced Impedances

In the balanced case, Eq. (8.42) is valid, but Eq. (8.63a-b) apply. Thus, evaluation of Eq. (8.47) results in the closed form expression of Eq. (8.64a). Equation (8.64b) extends the result of Eq. (8.64a) to impedance rather than just reactance.

$$X_{aa} = X_{bb} = X_{cc} \equiv X_s \tag{8.63a}$$

$$X_{ab} = X_{bc} = X_{ca} \equiv X_m \tag{8.63b}$$

$$\begin{aligned} \left[\bar{Z}_{012}\right] = \left[\bar{T}\right]^{-1}\left[\bar{Z}_{abc}\right]\left[\bar{T}\right] &= \begin{bmatrix} X_s + 2X_m & 0 & 0 \\ 0 & X_s - X_m & 0 \\ 0 & 0 & X_s - X_m \end{bmatrix} \\ &= \begin{bmatrix} Z_{00} & 0 & 0 \\ 0 & Z_{11} & 0 \\ 0 & 0 & Z_{22} \end{bmatrix} \end{aligned} \tag{8.64a}$$

$$\left[\bar{Z}_{012}\right] = \begin{bmatrix} \bar{Z}_s + 2\bar{Z}_m & 0 & 0 \\ 0 & \bar{Z}_s - \bar{Z}_m & 0 \\ 0 & 0 & \bar{Z}_s - \bar{Z}_m \end{bmatrix} = \begin{bmatrix} Z_{00} & 0 & 0 \\ 0 & Z_{11} & 0 \\ 0 & 0 & Z_{22} \end{bmatrix} \tag{8.64b}$$

Balanced Power Calculations

In the balanced case, Eq. (8.58) is still valid. However, in the case of positive phase sequence operation, the zero and negative sequence voltages and currents are zero. Hence, Eq. (8.65) results. In the case of negative phase sequence operation, the zero and positive sequence voltages and currents are zero. This results in Eq. (8.66).

$$\begin{aligned} \bar{S}_{3\phi} &= 3\left\{\bar{V}_0\,\bar{I}_0^* + \bar{V}_1\,\bar{I}_1^* + \bar{V}_2\,\bar{I}_2^*\right\} \\ &= 3\bar{V}_1\,\bar{I}_1^* = 3\bar{V}_a\,\bar{I}_a^* \end{aligned} \tag{8.65}$$

$$\begin{aligned} \bar{S}_{3\phi} &= 3\left\{\bar{V}_0\,\bar{I}_0^* + \bar{V}_1\,\bar{I}_1^* + \bar{V}_2\,\bar{I}_2^*\right\} \\ &= 3\bar{V}_2\,\bar{I}_2^* = 3\bar{V}_a\,\bar{I}_a^* \end{aligned} \tag{8.66}$$

Examination of Eqs. (8.65) and (8.66) reveals that the nature of complex power calculations in the sequence networks is identical to that performed using per-phase analysis (i.e., the factor of 3 is present). This feature of the symmetrical component transformation defined herein is the primary reason that power invariance is not desired.

Balanced System Loads

When the system loads are balanced, the sequence network representation is rather straightforward. We shall first consider the impedance load model by referring to Fig. 8.12a, imposing balanced impedances, and allowing for consideration of a neutral impedance, as illustrated in Fig. 8.12b. Balanced conditions are enforced by Eq. (8.67a-b). In this case, the reduction is based on Eq. (8.64). The result is presented in Eq. (8.68). Special notice should be taken that the mutual terms may be zero, as indicated on the figure, but have been included for completeness in the mathematical development.

$$\bar{Z}_{aa} = \bar{Z}_{bb} = \bar{Z}_{cc} \equiv \bar{Z}_s \tag{8.67a}$$

$$\bar{Z}_{ab} = \bar{Z}_{bc} = \bar{Z}_{ca} \equiv \bar{Z}_m \tag{8.67b}$$

$$\left[\bar{Z}_{012}\right] = \begin{bmatrix} \bar{Z}_s + 2\bar{Z}_m + 3\bar{Z}_n & 0 & 0 \\ 0 & \bar{Z}_s - \bar{Z}_m & 0 \\ 0 & 0 & \bar{Z}_s - \bar{Z}_m \end{bmatrix} = \begin{bmatrix} Z_{00} & 0 & 0 \\ 0 & Z_{11} & 0 \\ 0 & 0 & Z_{22} \end{bmatrix} \tag{8.68}$$

The balanced complex power load model is illustrated in Fig. 8.15. The transformation into the sequence networks is actually defined by the results presented in Eqs. (8.65) and (8.66). In positive phase sequence systems, the zero and negative sequence load representations absorb zero complex power; in negative phase sequence systems, the zero and positive sequence load representations absorb zero complex power. Hence, the zero complex power sequence loads are represented as short-circuits, thus forcing the sequence voltages to zero. The non-zero sequence complex power load turns out to be equal to the single-phase load complex power. This is defined for positive phase sequence systems in Eq. (8.69) and for negative phase sequence systems in Eq. (8.70).

$$\bar{S}_1 = \bar{S}_{1\phi} \tag{8.69}$$

$$\bar{S}_2 = \bar{S}_{1\phi} \tag{8.70}$$

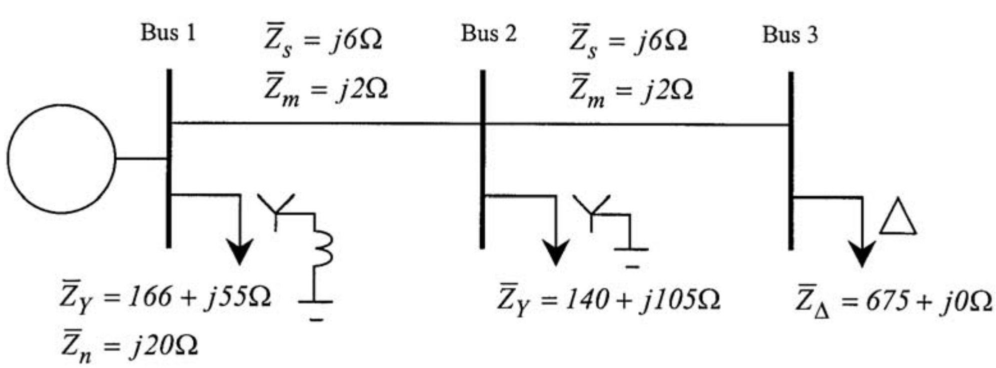


FIGURE 8.16 Balanced power system for Example 2.

TABLE 8.2 Summary of the Symmetrical Components in the Balanced Case

Quantity	Transformation Equations: abc ⇒ 012	Transformation Equations: 012 ⇒ abc
Voltage	Positive Phase Sequence: $\begin{bmatrix}\overline{V}_0\\ \overline{V}_1\\ \overline{V}_2\end{bmatrix}=\left[\overline{T}\right]^{-1}\begin{bmatrix}\overline{V}_a\\ \overline{V}_b\\ \overline{V}_c\end{bmatrix}=\begin{bmatrix}0\\ \overline{V}_a\\ 0\end{bmatrix}$	Positive Phase Sequence: $\begin{bmatrix}\overline{V}_a\\ \overline{V}_b\\ \overline{V}_c\end{bmatrix}=\left[\overline{T}\right]\begin{bmatrix}\overline{V}_0\\ \overline{V}_1\\ \overline{V}_2\end{bmatrix}=\begin{bmatrix}\overline{V}_1\\ \overline{a}^2\overline{V}_1\\ \overline{a}\overline{V}_1\end{bmatrix}$
	Negative Phase Sequence: $\begin{bmatrix}\overline{V}_0\\ \overline{V}_1\\ \overline{V}_2\end{bmatrix}=\left[\overline{T}\right]^{-1}\begin{bmatrix}\overline{V}_a\\ \overline{V}_b\\ \overline{V}_c\end{bmatrix}=\begin{bmatrix}0\\ 0\\ \overline{V}_a\end{bmatrix}$	Negative Phase Sequence: $\begin{bmatrix}\overline{V}_a\\ \overline{V}_b\\ \overline{V}_c\end{bmatrix}=\left[\overline{T}\right]\begin{bmatrix}\overline{V}_0\\ \overline{V}_1\\ \overline{V}_2\end{bmatrix}=\begin{bmatrix}\overline{V}_2\\ \overline{a}\overline{V}_2\\ \overline{a}^2\overline{V}_2\end{bmatrix}$
Current	Positive Phase Sequence: $\begin{bmatrix}\overline{I}_0\\ \overline{I}_1\\ \overline{I}_2\end{bmatrix}=\left[\overline{T}\right]^{-1}\begin{bmatrix}\overline{I}_a\\ \overline{I}_b\\ \overline{I}_c\end{bmatrix}=\begin{bmatrix}0\\ \overline{I}_a\\ 0\end{bmatrix}$	Positive Phase Sequence: $\begin{bmatrix}\overline{I}_a\\ \overline{I}_b\\ \overline{I}_c\end{bmatrix}=\left[\overline{T}\right]\begin{bmatrix}\overline{I}_0\\ \overline{I}_1\\ \overline{I}_2\end{bmatrix}=\begin{bmatrix}\overline{I}_1\\ \overline{a}^2\overline{I}_1\\ \overline{a}\overline{I}_1\end{bmatrix}$
	Negative Phase Sequence: $\begin{bmatrix}\overline{I}_0\\ \overline{I}_1\\ \overline{I}_2\end{bmatrix}=\left[\overline{T}\right]^{-1}\begin{bmatrix}\overline{I}_a\\ \overline{I}_b\\ \overline{I}_c\end{bmatrix}=\begin{bmatrix}0\\ 0\\ \overline{I}_a\end{bmatrix}$	Negative Phase Sequence: $\begin{bmatrix}\overline{I}_a\\ \overline{I}_b\\ \overline{I}_c\end{bmatrix}=\left[\overline{T}\right]\begin{bmatrix}\overline{I}_0\\ \overline{I}_1\\ \overline{I}_2\end{bmatrix}=\begin{bmatrix}\overline{I}_1\\ \overline{a}\overline{I}_1\\ \overline{a}^2\overline{I}_1\end{bmatrix}$
Impedance	$\left[\overline{Z}_{012}\right]=\left[\overline{T}\right]^{-1}\left[\overline{Z}_{abc}\right]\left[\overline{T}\right]=\begin{bmatrix}\overline{Z}_s+2\overline{Z}_m+3\overline{Z}_n & 0 & 0\\ 0 & \overline{Z}_s-\overline{Z}_m & 0\\ 0 & 0 & \overline{Z}_2-\overline{Z}_m\end{bmatrix}$	
Power	$\overline{S}_{3\phi}=\overline{V}_a\overline{I}_a^*+\overline{V}_b\overline{I}_b^*+\overline{V}_c\overline{I}_c^*=3\overline{V}_a\overline{I}_a^*$ $\overline{S}_{3\phi}=\left\{\overline{V}_0\overline{I}_0^*+\overline{V}_1\overline{I}_1^*+\overline{V}_2\overline{I}_2^*\right\}=\begin{cases}3\overline{V}_1\overline{I}_1^* & \text{positive ph. seq.}\\ 3\overline{V}_2\overline{I}_2^* & \text{negative ph. seq.}\end{cases}$	

Summary of Symmetrical Components in the Balanced Case

The general application of symmetrical components to balanced three-phase power systems has been presented in this section. The results are summarized in a quick reference form in Table 8.2. At this point, however, power transformers have been omitted from consideration. This will be rectified in the next few sections.

Example 2:

Consider the balanced system illustrated by the one-line diagram in Fig. 8.16. Determine the line voltage magnitudes at buses 2 and 3 if the line voltage magnitude at bus 1 is 12.47 kV. We will assume positive phase sequence operation of the source. Also, draw the zero sequence network.

Solution:

The two feeders are identical, and the zero and positive sequence impedances are computed in Eqs. (8.71a) and (8.71b), respectively. The zero and positive sequence impedances for the loads at buses 1 and 2 are computed in Eq. (8.72a-b) through (8.73a-b), respectively. The Δ-connected load at bus 3 is converted to an equivalent Y-connection in Eq. (8.74a), and the zero and positive sequence impedances for the load are computed in Eq. (8.74b) and (8.74c), respectively.

$$\overline{Z}_{00_{feeder}}=\overline{Z}_s+2\overline{Z}_m=j6+2\left(j2\right)=j10\Omega \tag{8.71a}$$

$$\overline{Z}_{11_{feeder}} = \overline{Z}_s - \overline{Z}_m = j6 - j2 = j4\,\Omega \tag{8.71b}$$

$$\begin{aligned} \overline{Z}_{00_{bus1}} &= \overline{Z}_s + 2\overline{Z}_m + 3\overline{Z}_n \\ &= (166 + j55) + 2(0) + 3(j20) = 166 + j115\,\Omega \end{aligned} \tag{8.72a}$$

$$\overline{Z}_{11_{bus1}} = \overline{Z}_s - \overline{Z}_m = (166 + j55) - 0 = 166 + j55\,\Omega \tag{8.72b}$$

$$\begin{aligned} \overline{Z}_{00_{bus2}} &= \overline{Z}_s + 2\overline{Z}_m + 3\overline{Z}_n \\ &= (140 + j105) + 2(0) + 3(0) = 140 + j105\,\Omega \end{aligned} \tag{8.73a}$$

$$\overline{Z}_{11_{bus2}} = \overline{Z}_s - \overline{Z}_m = (140 + j105) - 0 = 140 + j105\,\Omega \tag{8.73b}$$

$$\overline{Z}_{Y_{bus3}} = \frac{\overline{Z}_\Delta}{3} = \frac{675 + j0}{3} = 225 + j0\,\Omega \tag{8.74a}$$

$$\overline{Z}_{00_{bus3}} = \overline{Z}_s + 2\overline{Z}_m + 3\overline{Z}_n = (225 + j0) + 2(0) + 3(\infty) \to \infty \tag{8.74b}$$

$$\overline{Z}_{11_{bus3}} = \overline{Z}_s - \overline{Z}_m = (225 + j0) - 0 = 225 + j0\,\Omega \tag{8.74c}$$

The zero and positive sequence networks for the system are provided in Figs. 8.17a and b. Note in the zero sequence network, that the voltage at bus 1 has been forced to zero by imposing a short-circuit to reference. For analysis, since the system is balanced, we need only concern ourselves with the positive sequence network. The source voltage at bus 1 is assumed to be the reference with a 0º phase angle. Note that the source voltage magnitude is the line-to-neutral voltage magnitude at bus 1. The positive sequence voltage at bus 2 can be found using the voltage divider, as shown in Eq. (8.75). Note here that the subscript numbers on the voltages denote the bus, not the sequence network. We assume that all voltages are in the positive sequence network. Again using the voltage divider, the positive sequence voltage at bus 3 can be found, as shown in Eq. (8.76). The requested line voltage magnitudes at buses 2 and 3 can be computed from the positive sequence voltages as shown in Eq. (8.77a-b).

$$\overline{V}_2 = 7200\angle 0^\circ \frac{\{(140 + j105)//(225 + j4)\}}{j4 + \{(140 + j105)//(225 + j4)\}} = 7095.9\angle -2^\circ V \tag{8.75}$$

$$\overline{V}_3 = 7095.9\angle -2^\circ \frac{225}{225 + j4} = 7094.8\angle -3^\circ V \tag{8.76}$$

$$V_{L_2} = \sqrt{3}\left|\overline{V}_2\right| = \sqrt{3}(7095.9) = 12{,}290.5V \tag{8.77a}$$

$$V_{L_3} = \sqrt{3}\left|\overline{V}_3\right| = \sqrt{3}(7094.8) = 12{,}288.6V \tag{8.77b}$$

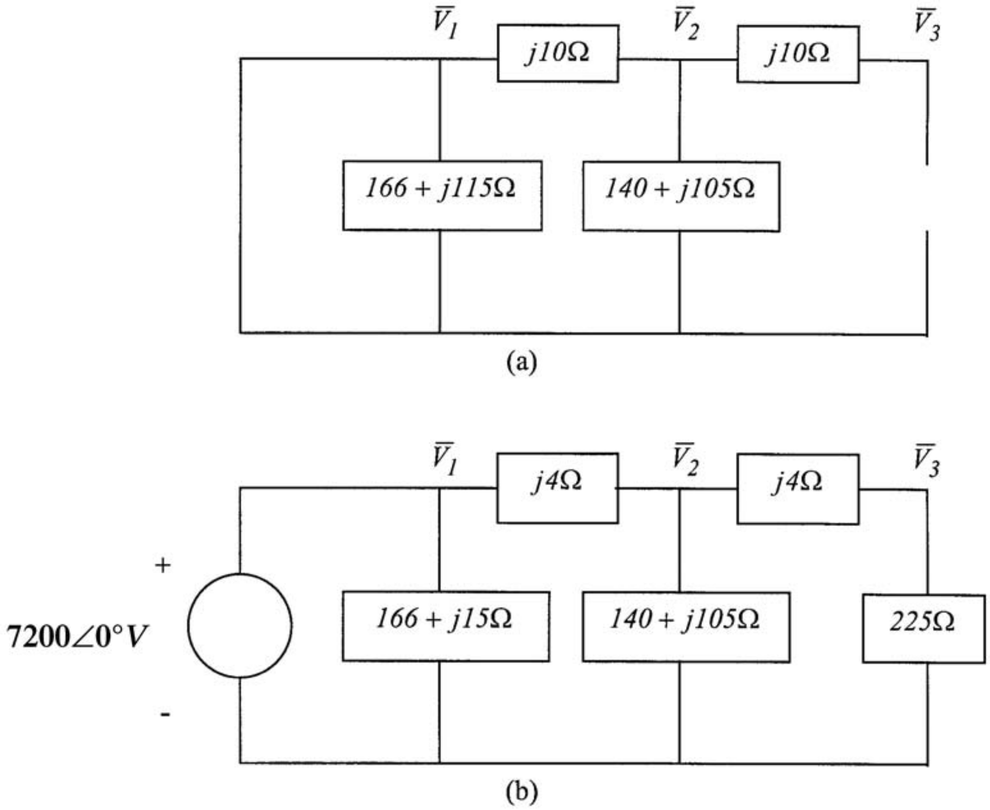


FIGURE 8.17 (a) Zero and (b) positive sequence networks for Example 2.

Sequence Network Representation in Per-Unit

The foregoing development has been based on the inherent assumption that all parameters and variables were expressed in SI units. Quite often, large-scale power system analyses and computations are performed in the per-unit system of measurement (Gross, 1986; Grainger and Stevenson, 1994; Glover and Sarma, 1989). Thus, we must address the impact of per-unit scaling on the sequence networks. Such a conversion is rather straightforward because of the similarity between the positive or negative sequence network and the a-phase network used in per-phase analysis (the reader is cautioned not to confuse the concepts of per-phase analysis and per-unit scaling). The appropriate bases are the same for each sequence network, and they are defined in Table 8.3. Note that the additional subscript "pu" has been added to denote a variable in per-unit; variables in SI units do not carry the additional subscripts.

Power Transformers

For the consideration of transformers and transformer banks, we will limit ourselves to working in the per-unit system. Thus, the ideal transformer in the transformer equivalent circuit can be neglected in the nominal case. The equivalent impedance of a transformer, whether it be single-phase or three-phase, is typically provided on the nameplate in percent, or test data may be available to compute equivalent winding and shunt branch impedances. Developing the sequence networks for these devices is not terribly complicated, but does require attention to detail in the zero sequence case. Of primary importance is the type of connection on each side of the transformer or bank.

The general forms of the per-unit sequence networks for the transformer are shown in Fig. 8.18. Notice should be taken that each transformer winding's impedance and the shunt branch impedance are all

TABLE 8.3 Per-Unit Scaling of Sequence Network Parameters

Quantity	Base Value	Scaling Relationship: Zero Sequence	Positive Sequence	Negative Sequence
Voltage	Line-to-Neutral Voltage Base: $V_{LN_{base}} = \frac{V_{L_{base}}}{\sqrt{3}}$	$\bar{V}_{0_{pu}} = \frac{\bar{V}_0}{V_{LN_{base}}}$	$\bar{V}_{1_{pu}} = \frac{\bar{V}_1}{V_{LN_{base}}}$	$\bar{V}_{2_{pu}} = \frac{\bar{V}_2}{V_{LN_{base}}}$
Current	Line Current Base: $I_{L_{base}} = \frac{S_{3\phi_{base}}}{\sqrt{3}V_{L_{base}}}$	$\bar{I}_{0_{pu}} = \frac{\bar{I}_0}{I_{L_{base}}}$	$\bar{I}_{1_{pu}} = \frac{\bar{I}_1}{I_{L_{base}}}$	$\bar{I}_{2_{pu}} = \frac{\bar{I}_2}{I_{L_{base}}}$
Impedance	Y-Impedance Base: $Z_{Y_{base}} = \frac{V_{L_{base}}^2}{S_{3\phi_{base}}}$	$\bar{Z}_{00_{pu}} = \frac{\bar{Z}_{00}}{Z_{Y_{base}}}$	$\bar{Z}_{11_{pu}} = \frac{\bar{Z}_{11}}{Z_{Y_{base}}}$	$\bar{Z}_{22_{pu}} = \frac{\bar{Z}_{22}}{Z_{Y_{base}}}$
Complex Power	Single-Phase Apparent Power Base: $S_{1\phi_{base}} = \frac{S_{3\phi_{base}}}{3}$		$\bar{S}_{1\phi_{pu}} = \frac{\bar{S}_{1\phi}}{S_{1\phi_{base}}} = \frac{\bar{S}_{3\phi}}{S_{3\phi_{base}}}$	

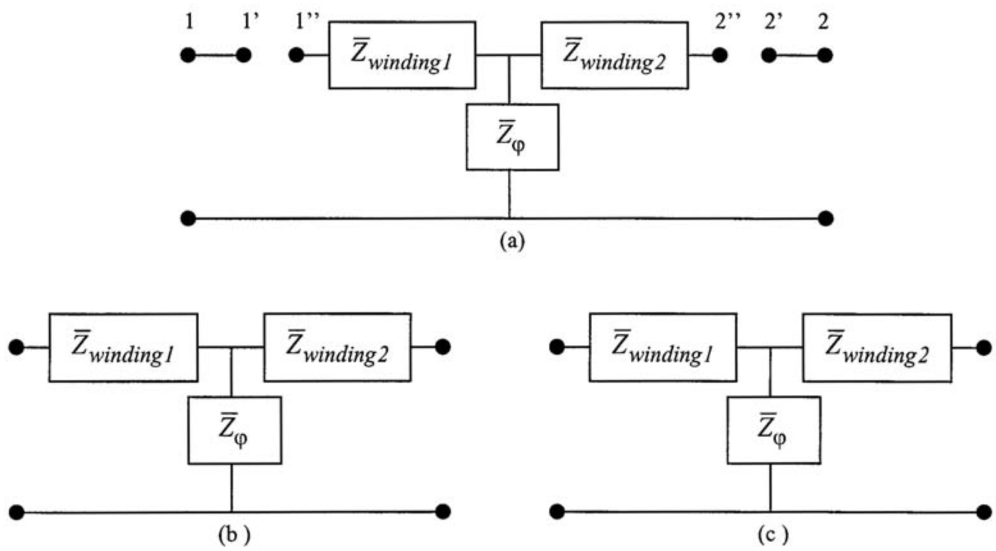
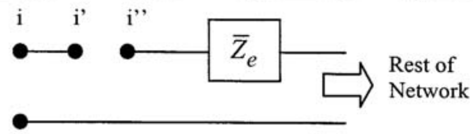
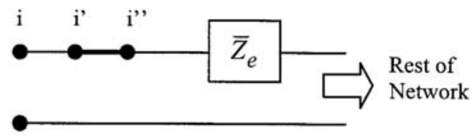
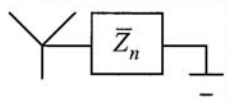
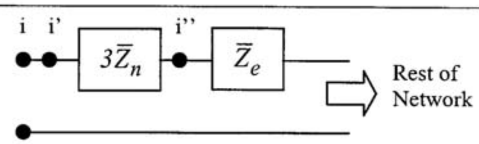
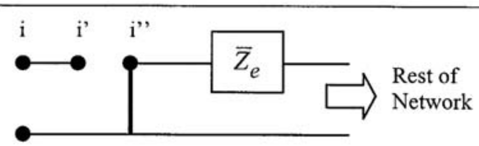


FIGURE 8.18 (a) Zero, (b) positive, and (c) negative sequence transformer networks.

modeled in the circuits. The sequence networks are of the presented form whether a three-phase transformer or a bank of three single-phase transformers is under consideration. Note that the positive and negative sequence networks are identical, and the zero sequence network requires some discussion. The "ith primed" terminals in the zero sequence network are terminated based on the type of connection that is employed for winding i. Details of the termination are presented in Table 8.4.

We must turn our attention to the calculation of the various impedances in the sequence networks as a function of the individual transformer impedances. The zero, positive, and negative sequence impedances are all equal for any transformer winding. Furthermore, the sequence impedances for any transformer winding are equal to the winding impedance expressed in per-unit on the system (not device)

TABLE 8.4 Power Transformer Zero Sequence Terminations.

Winding "i" Connection	Connection of Terminals	Schematic Representation
	Leave i' and i'' unconnected.	
	Short i' to i''.	
	Connect i' to i'' through $3\overline{Z}_n$.	
	Short i'' to reference.	

ratings. This is independent of the winding connection (Y or Δ), because of the per-unit scaling. If the sequence networks are to be drawn in SI units, then the sequence impedances for a Δ connection would be 1/3 of the transformer winding impedance. In the case of a three-phase transformer, where the phases may share a common magnetic path, the zero sequence impedance will be different from the positive and negative sequence impedances (Gross, 1986; Blackburn, 1993).

In many cases, a single equivalent impedance is provided on a transformer nameplate. Utilization of this value as a single impedance for the circuit model requires neglecting the shunt branch impedance, which is often justified. If open-circuit test data is not available, or just for the sake of simplicity, the shunt branch of the transformers may be neglected. This leads to the sequence networks illustrated in Fig. 8.19. Here again, care must be taken to place the equivalent transformer impedance in per-unit on the appropriate system bases. Derivation of the equivalent transformer impedance is most appropriately performed in a study focused on power transformers (Gross, 1986; Blackburn, 1993).

Example 3:

Consider the simple power system, operating with positive phase sequence, described by the one-line diagram presented in Fig. 8.20. Compute the line voltage at bus 1, and draw the zero sequence network.

Solution:

We begin by selecting system bases. For simplicity, we choose the system bases to be equal to the transformer ratings. In other words, the system apparent power base is chosen as 750 kVA (three times the single-phase transformer kVA rating), and the line voltage bases at buses 1 and 2 are chosen as 12,470 V (delta side) and 480 V (Y side), respectively. Thus, the transformer impedance provided for the transformer is unaltered when converted to the system bases, as illustrated in Eq. (8.78).

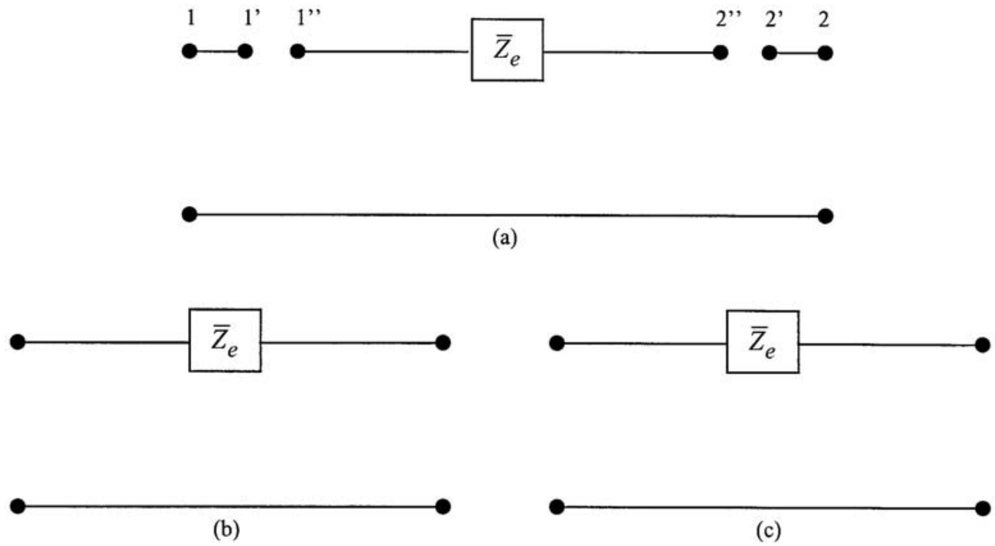


FIGURE 8.19 Reduced (a) zero, (b) positive, and (c) Negative sequence transformer networks.

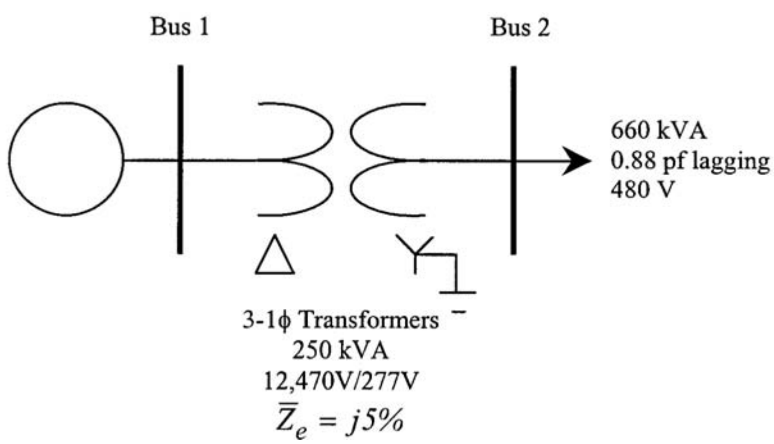


FIGURE 8.20 Power system with a transformer for Example 3.

$$\overline{Z}_e = (j0.05)\frac{Z_{transformer_{base}}}{Z_{Ysystem_{base}}} = (j0.05)\frac{\left(\dfrac{277^2}{250 \times 10^3}\right)}{\left(\dfrac{480^2}{750 \times 10^3}\right)} = j0.05 \tag{8.78}$$

Since balanced conditions are enforced, the load is a non-zero complex power in only the positive sequence network. The positive sequence load value is the single-phase load complex power. In per-unit, the three-phase and single-phase complex powers are equal, as indicated in Eq. (8.79).

$$\overline{S}_{1_{pu}} = \overline{S}_{1\phi_{pu}} = \overline{S}_{3\phi_{pu}} = \frac{\overline{S}_{3\phi}}{S_{3\phi_{base}}} = \frac{666\angle 28.4°}{750} = 0.88\angle 28.4° \tag{8.79}$$

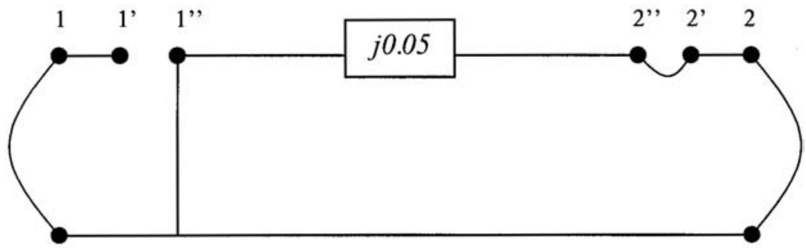


FIGURE 8.21 Zero sequence network for Example 3.

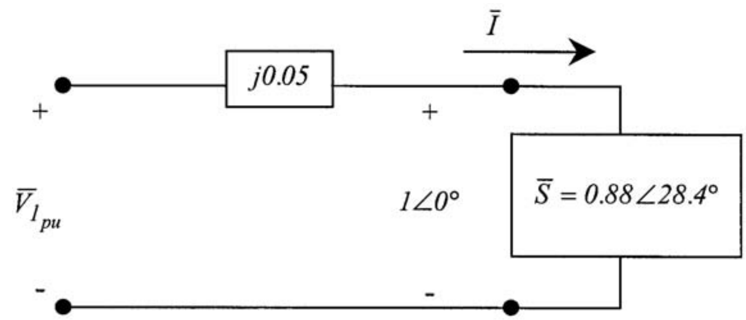


FIGURE 8.22 Positive sequence network for Example 3.

The positive sequence load voltage is the a-phase line-to-neutral voltage at bus 2. If we assume this to be the reference voltage with a zero degree phase angle, then we get 277 ∠ 0°V. In per-unit, this corresponds to unity voltage.

The zero and positive sequence networks are provided in Figs. 8.21 and 8.22, respectively. The line voltage at bus 1 is found by solution of the positive sequence network. The load current is computed from the load voltage and complex power in Eq. (8.80). The positive sequence per-unit voltage at bus 1 is computed in Eq. (8.81). The line voltage at bus 1 is computed from the bus 1 positive sequence voltage in Eq. (8.82). The positive sequence voltage magnitude at bus 1 is the per-unit line-to-neutral voltage magnitude at bus 1. In per-unit, the line and line-to-neutral voltages are equal. Thus, multiplying the per-unit positive sequence voltage magnitude at bus 1 by the line voltage base at bus 1 produces the line voltage at bus 1.

$$\bar{I} = \left(\frac{0.88\angle 28.4°}{1\angle 0°}\right) = 0.88\angle -28.4° \tag{8.80}$$

$$\bar{V}_{1_{pu}} = 1\angle 0° + \left(0.88\angle -28.4°\right)\left(j0.05\right) = 1.02\angle 2.2° \tag{8.81}$$

$$V_L = \left|\bar{V}_{1_{pu}}\right| V_{L_{base}}\left(bus1\right) = 1.02\left(12,470\right) = 12,719V \tag{8.82}$$

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8.3 Power Flow Analysis

L. L. Grigsby and Andrew Hanson

The equivalent circuit parameters of many power system components are described in other sections of this handbook. The interconnection of the different elements allows development of an overall power system model. The system model provides the basis for computational simulation of the system performance under a wide variety of projected operating conditions. Additionally, "post mortem" studies, performed after system disturbances or equipment failures, often provide valuable insight into contributing system conditions. This section discusses one such computational simulation, the power flow analysis.

Power systems typically operate under slowly changing conditions which can be analyzed using steady state analysis. Further, transmission systems operate under balanced or near balanced conditions allowing per-phase analysis to be used with a high degree of confidence in the solution. Power flow analysis computationally models these conditions and provides the starting point for most other analyses. For example, the small signal and transient stability effects of a given disturbance are dramatically affected by the "pre-disturbance" operating conditions of the power system. (A disturbance resulting in instability under heavily loaded system conditions may not have any adverse effects under lightly loaded conditions.) Additionally, fault analysis and transient analysis can also be impacted by the "pre-disturbance" operating point of a power system (although, they are usually affected much less than transient stability and small signal stability analysis).

The Power Flow Problem

Power flow analysis is fundamental to the study of power systems forming the basis for other anlayses. Power flow analyses play a key role in the planning of additions or expansions to transmission and generation facilities as well as establishing the starting point for many other types of power system analyses. In addition, power flow analysis and many of its extensions are an essential ingredient of the studies performed in power system operations. In this latter case, it is at the heart of contingency analysis and the implementation of real-time monitoring systems.

The power flow problem (also known as the load flow problem) can be stated as follows:

> For a given power network, with known complex power loads and some set of specifications or restrictions on power generations and voltages, solve for any unknown bus voltages and unspecified generation and finally for the complex power flow in the network components.

Additionally, the losses in individual components and the total network as a whole are usually calculated. Furthermore, the system is often checked for component overloads and voltages outside allowable tolerances.

This section addresses power flow computations for balanced networks (typically applicable to transmission voltage level systems). Positive sequence network components are used for the problem formulation presented here. In the solution of the power flow problem, the network element values are almost always taken to be in per-unit. Likewise, the calculations within the power flow analysis are typically in

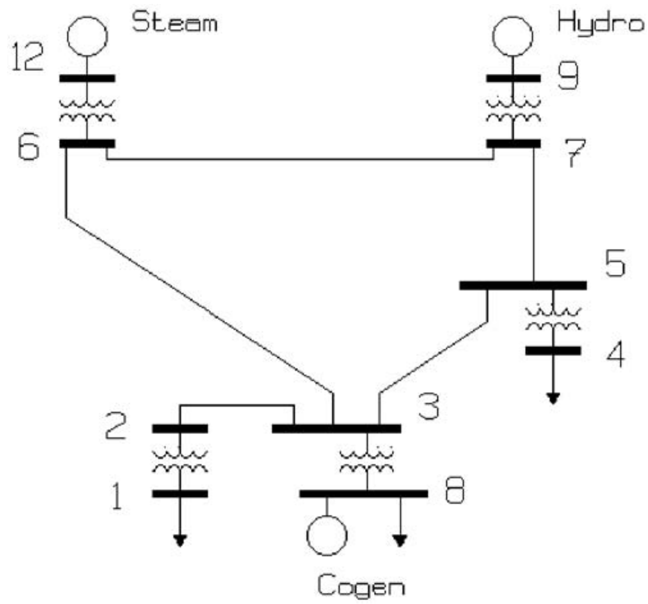

FIGURE 8.23 The one-line diagram of a power system.

per-unit. However, the solution is usually expressed in a mixed format. Solution voltages are usually expressed in per-unit; powers are most often given with kVA or MVA**.**

The "given network" may be in the form of a system map and accompanying data tables for the network components. More often, however, the network structure is given in the form of a one-line diagram (such as shown in Fig. 8.23).

Regardless of the form of the given network and how the network data is given, the steps to be followed in a power flow study can be summarized as follows:

1. Determine element values for passive network components.
2. Determine locations and values of all complex power loads.
3. Determine generation specifications and constraints.
4. Develop a mathematical model describing power flow in the network.
5. Solve for the voltage profile of the network.
6. Solve for the power flows and losses in the network.
7. Check for constraint violations.

Formulation of the Bus Admittance Matrix

The first step in developing the mathematical model describing the power flow in the network is the formulation of the bus admittance matrix. The bus admittance matrix is an n×n matrix (where n is the number of buses in the system) constructed from the admittances of the equivalent circuit elements of the segments making up the power system. Most system segments are represented by a combination of shunt elements (connected between a bus and the reference node) and series elements (connected between two system buses). Formulation of the bus admittance matrix follows two simple rules:

1. The admittance of elements connected between node k and reference is added to the (k, k) entry of the admittance matrix.
2. The admittance of elements connected between nodes j and k is added to the (j, j) and (k, k) entries of the admittance matrix. The negative of the admittance is added to the (j, k) and (k, j) entries of the admittance matrix.

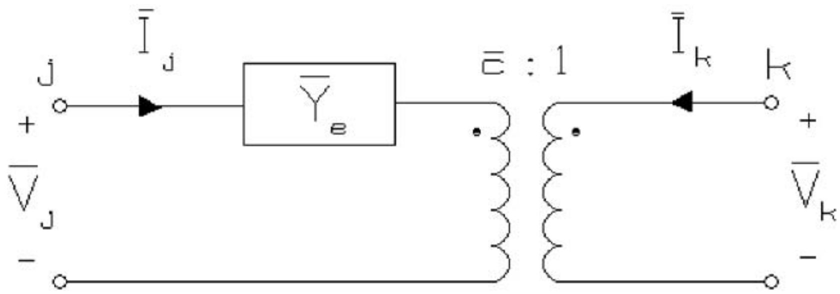

FIGURE 8.24 Off nominal turns ratio transformer.

Off nominal transformers (transformers with transformation ratios different from the system voltage bases at the terminals) present some special difficulties. Figure 8.24 shows a representation of an off nominal turns ratio transformer.

The admittance matrix base mathematical model of an isolated off nominal transformer is:

$$\begin{bmatrix} \bar{I}_j \\ \bar{I}_k \end{bmatrix} = \begin{bmatrix} \bar{Y}_e & -\bar{c}\bar{Y}_e \\ -\bar{c}^*\bar{Y}_e & |c|^2\bar{Y}_e \end{bmatrix} \begin{bmatrix} \bar{V}_j \\ \bar{V}_k \end{bmatrix} \tag{8.83}$$

where $\bar{Y}_e$ is the equivalent series admittance (refered to node j)
$\bar{c}$ is the complex (off nominal) turns ratio
$\bar{I}_j$ is the current injected at node j
$\bar{V}_j$ is the voltage at node j (with respect to reference)

Off nominal transformers are added to the bus admittance matrix by adding the corresponding entry of the isolated off nominal transformer admittance matrix to the system bus admittance matrix.

Formulation of the Power Flow Equations

Considerable insight into the power flow problem and its properties and characteristics can be obtained by consideration of a simple example before proceeding to a general formulation of the problem. This simple case will also serve to establish some notation.

A conceptual representation of a one-line diagram for a four bus power system is shown in Fig. 8.25. For generality, we have shown a generator and a load connected to each bus. The following notation applies:

$$\bar{S}_{G1} = \text{Complex complex power flow into bus 1 from the generator}$$

$$\bar{S}_{D1} = \text{Complex complex power flow into the load from bus 1}$$

Comparable quantities for the complex power generations and loads are obvious for each of the three other buses.

The positive sequence network for the power system represented by the one line diagram of Fig. 8.25 is shown in Fig. 8.26. The boxes symbolize the combination of generation and load. Network texts refer to this network as a five-node network. (The balanced nature of the system allows analysis using only the positive sequence network, reducing each three phase bus to a single node. The reference or ground represents the fifth node.) However, in power systems literature it is usually referred to as a four-bus network or power system.

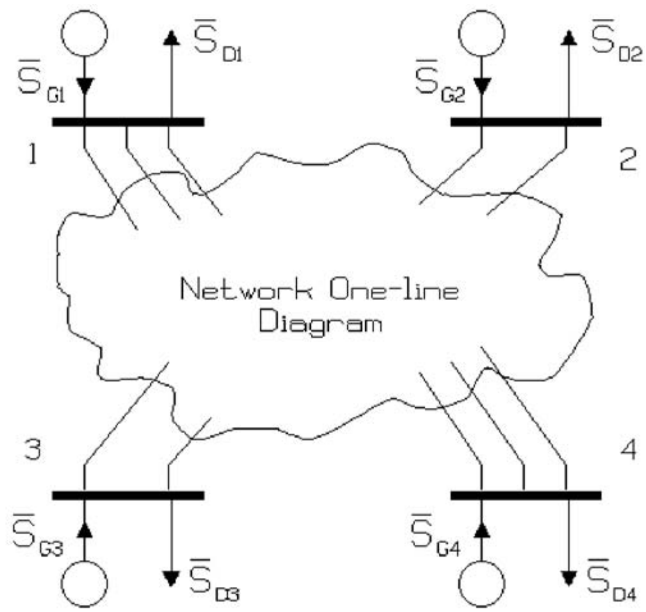

FIGURE 8.25 Conceptual one-line diagram of a four-bus power system.

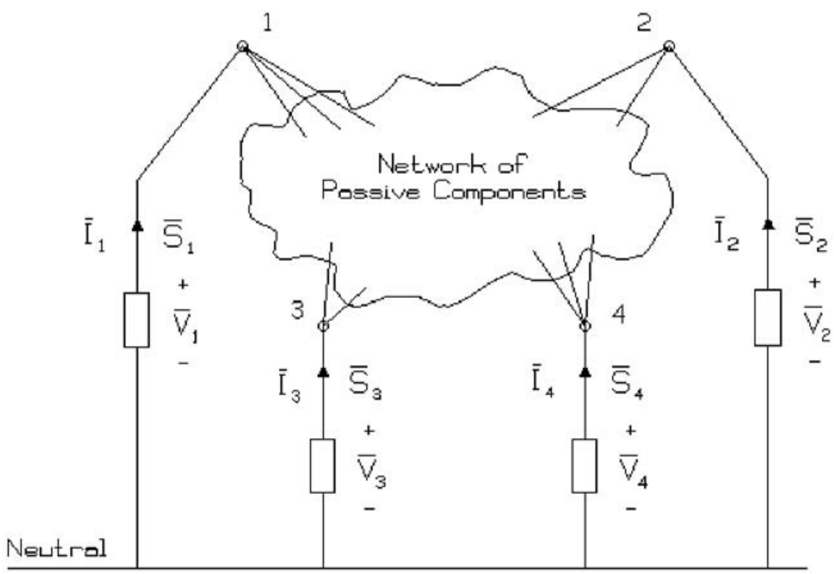

FIGURE 8.26 Positive sequence network for the system of Fig. 8.25.

For the network of Fig. 8.26, we define the following additional notation:

$$\bar{S}_1 = \bar{S}_{G1} - \bar{S}_{D1} = \text{Net complex power injected at bus 1}$$

$$\bar{I}_1 = \text{Net positive sequence phasor current injected at bus 1}$$

$$\bar{V}_1 = \text{Positive sequence phasor voltage at bus 1}$$

The standard node voltage equations for the network can be written in terms of the quantities at bus 1 (defined above) and comparable quantities at the other buses.

$$\bar{I}_1 = \bar{Y}_{11}\bar{V}_1 + \bar{Y}_{12}\bar{V}_2 + \bar{Y}_{13}\bar{V}_3 + \bar{Y}_{14}\bar{V}_4 \tag{8.84}$$

$$\bar{I}_2 = \bar{Y}_{21}\bar{V}_1 + \bar{Y}_{22}\bar{V}_2 + \bar{Y}_{23}\bar{V}_3 + \bar{Y}_{24}\bar{V}_4 \tag{8.85}$$

$$\bar{I}_3 = \bar{Y}_{31}\bar{V}_1 + \bar{Y}_{32}\bar{V}_2 + \bar{Y}_{33}\bar{V}_3 + \bar{Y}_{34}\bar{V}_4 \tag{8.86}$$

$$\bar{I}_4 = \bar{Y}_{41}\bar{V}_1 + \bar{Y}_{42}\bar{V}_2 + \bar{Y}_{43}\bar{V}_3 + \bar{Y}_{44}\bar{V}_4 \tag{8.87}$$

The admittances in Eqs. (8.84)–(8.87), are the ijth entries of the bus admittance matrix for the power system. The unknown voltages could be found using linear algebra if the four currents $\bar{I}_1 \ldots \bar{I}_4$ were known. However, these currents are not known. Rather, something is known about the complex power and voltage at each bus. The complex power injected into bus k of the power system is defined by the relationship between complex power, voltage, and current given by Eq. (8.88).

$$\bar{S}_k = \bar{V}_k \bar{I}_k^* \tag{8.88}$$

Therefore,

$$\bar{I}_k = \frac{\bar{S}_k^*}{\bar{V}_k^*} = \frac{\bar{S}_{Gk}^* - \bar{S}_{Dk}^*}{\bar{V}_k^*} \tag{8.89}$$

By substituting this result into the nodal equations and rearranging, the basic power flow equations for the four-bus system are given as Eqs. (8.90)–(8.93).

$$\bar{S}_{G1}^* - \bar{S}_{D1}^* = \bar{V}_1^*\left[\bar{Y}_{11}\bar{V}_1 + \bar{Y}_{12}\bar{V}_2 + \bar{Y}_{13}\bar{V}_3 + \bar{Y}_{14}\bar{V}_4\right] \tag{8.90}$$

$$\bar{S}_{G2}^* - \bar{S}_{D2}^* = \bar{V}_2^*\left[\bar{Y}_{21}\bar{V}_1 + \bar{Y}_{22}\bar{V}_2 + \bar{Y}_{23}\bar{V}_3 + \bar{Y}_{24}\bar{V}_4\right] \tag{8.91}$$

$$\bar{S}_{G3}^* - \bar{S}_{D3}^* = \bar{V}_3^*\left[\bar{Y}_{31}\bar{V}_1 + \bar{Y}_{32}\bar{V}_2 + \bar{Y}_{33}\bar{V}_3 + \bar{Y}_{34}\bar{V}_4\right] \tag{8.92}$$

$$\bar{S}_{G4}^* - \bar{S}_{D4}^* = \bar{V}_4^*\left[\bar{Y}_{41}\bar{V}_1 + \bar{Y}_{42}\bar{V}_2 + \bar{Y}_{43}\bar{V}_3 + \bar{Y}_{44}\bar{V}_4\right] \tag{8.93}$$

Examination of Eqs. (8.90)–(8.93) reveals that unless the generation equals the load at every bus, the complex power outputs of the generators cannot be arbitrarily selected. In fact, the complex power output of at least one of the generators must be calculated last since it must take up the unknown "slack" due to the, as yet uncalculated network losses. Further, losses cannot be calculated until the voltages are known. These observations are a result of the principle of conservation of complex power. (i.e., the sum of the injected complex powers at the four system buses is equal to the system complex power losses.)

Further examination of Eqs. (8.90)–(8.93) indicates that it is not possible to solve these equations for the absolute phase angles of the phasor voltages. This simply means that the problem can only be solved to some arbitrary phase angle reference.

In order to alleviate the dilemma outlined above, suppose $\bar{S}_{G4}$ is arbitrarily allowed to float or swing (in order to take up the necessary slack caused by the losses) and that $\bar{S}_{G1}$, $\bar{S}_{G2}$, and $\bar{S}_{G3}$ are specified (other cases will be considered shortly). Now, with the loads known, Eqs. (8.89)–(8.92) are seen as four simultaneous nonlinear equations with complex coefficients in five unknowns $\overline{V}_1$, $\overline{V}_2$, $\overline{V}_3$, $\overline{V}_4$, and $\bar{S}_{G4}$.

The problem of too many unknowns (which would result in an infinite number of solutions) is solved by specifying another variable. Designating bus 4 as the slack bus and specifying the voltage $\overline{V}_4$ reduces the problem to four equations in four unknowns. The slack bus is chosen as the phase reference for all phasor calculations, its magnitude is constrained, and the complex power generation at this bus is free to take up the slack necessary in order to account for the system real and reactive power losses.

The specification of the voltage $\overline{V}_4$, decouples Eq. (8.93) from Eqs. (8.90)–(8.92), allowing calculation of the slack bus complex power after solving the remaining equations. (This property carries over to larger systems with any number of buses.) The example problem is reduced to solving only three equations simultaneously for the unknowns $\overline{V}_1$, $\overline{V}_2$, and $\overline{V}_3$. Similarly, for the case of n buses, it is necessary to solve n – 1 simultaneous, complex coefficient, nonlinear equations.

Systems of nonlinear equations, such as Eqs. (8.90)–(8.92), cannot (except in rare cases) be solved by closed-form techniques. Direct simulation was used extensively for many years; however, essentially all power flow analyses today are performed using iterative techniques on digital computers.

Bus Classifications

There are four quantities of interest associated with each bus:

1. Real Power, P
2. Reactive Power, Q
3. Voltage Magnitude, V
4. Voltage Angle, δ

At every bus of the system, two of these four quantities will be specified and the remaining two will be unknowns. Each of the system buses may be classified in accordance with which of the two quantities are specified. The following classifications are typical:

Slack Bus — The slack bus for the system is a single bus for which the voltage magnitude and angle are specified. The real and reactive power are unknowns. The bus selected as the slack bus must have a source of both real and reactive power, since the injected power at this bus must "swing" to take up the "slack" in the solution. The best choice for the slack bus (since, in most power systems, many buses have real and reactive power sources) requires experience with the particular system under study. The behavior of the solution is often influenced by the bus chosen. (In the earlier discussion, the last bus was selected as the slack bus for convenience.)

Load Bus (P-Q Bus) — A load bus is defined as any bus of the system for which the real and reactive power are specified. Load buses may contain generators with specified real and reactive power outputs; however, it is often convenient to designate any bus with specified injected complex power as a load bus.

Voltage Controlled Bus (P-V Bus) — Any bus for which the voltage magnitude and the injected real power are specified is classified as a voltage controlled (or P-V) bus. The injected reactive power is a variable (with specified upper and lower bounds) in the power flow analysis. (A P-V bus must have a variable source of reactive power such as a generator.)

In all realistic cases, the voltage magnitude is specified at generator buses to take advantage of the generator's reactive power capability. Specifying the voltage magnitude at a generator bus requires a variable specified in the simple analysis discussed earlier to become an unknown (in order to bring the number of unknowns back into correspondence with the number of equations). Normally, the reactive power injected by the generator becomes a variable, leaving the real power and voltage magnitude as the specified quantities at the generator bus.

It was noted earlier that Eq. (8.93) is decoupled, and only Eqs. (8.90)–(8.92) need be solved simultaneously. Although not immediately apparent, specifying the voltage magnitude at a bus and treating the bus reactive power injection as a variable results in retention of, effectively, the same number of complex unknowns. For example, if the voltage magnitude of bus 1 of the earlier four-bus system is specified and the reactive power injection at bus 1 becomes a variable, Eqs. (8.90)–(8.92) again effectively have three complex unknowns. (The phasor voltages $\overline{V}_2$ and $\overline{V}_3$ at buses 2 and 3 are two complex unknowns and the angle δ_1 of the voltage at bus 1 plus the reactive power generation Q_{G1} at bus 1 result in the equivalent of a third complex unknown.)

Bus 1 is called a *voltage controlled bus* since it is apparent that the reactive power generation at bus 1 is being used to control the voltage magnitude. Typically, all generator buses are treated as voltage controlled buses.

Generalized Power Flow Development

The more general (n-bus) case is developed by extending the results of the simple four-bus example. Consider the case of an n-bus system and the corresponding n+1 node positive sequence network. Assume that the buses are numbered such that the slack bus is numbered last. Direct extension of the earlier equations (writing the node voltage equations and making the same substitutions as in the four-bus case) yields the basic power flow equations in the general form.

The Basic Power Flow Equations (PFE)

$$\overline{S}_k^* = P_k - jQ_k = \overline{V}_k^* \sum_{i=1}^{n} \overline{Y}_{ki} \overline{V}_i \qquad (8.94)$$

$$\text{for } k = 1, 2, 3, \ldots, n-1$$

and

$$P_n - jQ_n = \overline{V}_n^* \sum_{i=1}^{n} \overline{Y}_{ni} \overline{V}_i \qquad (8.95)$$

Equation (8.95) is the equation for the slack bus. Eq. (8.94) represents n-1 simultaneous equations in n-1 complex unknowns if all buses (other than the slack bus) are classified as load buses. Thus, given a set of specified loads, the problem is to solve Eq. (8.94) for the n-1 complex phasor voltages at the remaining buses. Once the bus voltages are known, Eq. (8.95) can be used to calculate the slack bus power.

Bus j is normally treated as a P-V bus if it has a directly connected generator. The unknowns at bus j are then the reactive generation Q_{Gj} and δ_j because the voltage magnitude, V_j, and the real power generation, P_{Gj}, have been specified.

The next step in the analysis is to solve Eq. (8.94) for the bus voltages using some iterative method. Once the bus voltages have been found, the complex power flows and complex power losses in all of the network components are calculated.

Solution Methods

The solution of the simultaneous nonlinear power flow equations requires the use of iterative techniques for even the simplest power systems. Although there are many methods for solving nonlinear equations, only two methods are discussed here.

The Newton-Raphson Method

The Newton-Raphson algorithm has been applied in the solution of nonlinear equations in many fields. The algorithm will be developed using a general set of two equations (for simplicity). The results are easily extended to an arbitrary number of equations.

A set of two nonlinear equations are shown in Eqs. (8.96) and (8.97).

$$f_1\left(x_1, x_2\right) = k_1 \tag{8.96}$$

$$f_2\left(x_1, x_2\right) = k_2 \tag{8.97}$$

Now, if $x_1^{(0)}$ and $x_2^{(0)}$ are inexact solution estimates and $\Delta x_1^{(0)}$ and $\Delta x_2^{(0)}$ are the corrections to the estimates to achieve an exact solution, Eqs. (8.96) and (8.97) can be rewritten as:

$$f_1\left(x_1^{(0)} + \Delta x_1^{(0)},\ x_2^{(0)} + \Delta x_2^{(0)}\right) = k_1 \tag{8.98}$$

$$f_2\left(x_1^{(0)} + \Delta x_1^{(0)},\ x_2^{(0)} + \Delta x_2^{(0)}\right) = k_2 \tag{8.99}$$

Expanding Eqs. (8.98) and (8.99) in a Taylor series about the estimate yields:

$$f_1\left(x_1^{(0)}, x_2^{(0)}\right) + \left.\frac{\partial f_1}{\partial x_1}\right|^{(0)} \Delta x_1^{(0)} + \left.\frac{\partial f_1}{\partial x_2}\right|^{(0)} \Delta x_2^{(0)} + \text{h.o.t.} = k_1 \tag{8.100}$$

$$f_2\left(x_1^{(0)}, x_2^{(0)}\right) + \left.\frac{\partial f_2}{\partial x_1}\right|^{(0)} \Delta x_1^{(0)} + \left.\frac{\partial f_2}{\partial x_2}\right|^{(0)} \Delta x_2^{(0)} + \text{h.o.t.} = k_2 \tag{8.101}$$

where the superscript (0) on the partial derivatives indicates evaluation of the partial derivatives at the initial estimate, and h.o.t. indicates the higher order terms.

Neglecting the higher order terms (an acceptable approximation if $\Delta x_1^{(0)}$ and $\Delta x_2^{(0)}$ are small), Eqs. (8.100) and (8.101) can be rearranged and written in matrix form:

$$\begin{bmatrix} \left.\frac{\partial f_1}{\partial x_1}\right|^{(0)} & \left.\frac{\partial f_1}{\partial x_2}\right|^{(0)} \\ \left.\frac{\partial f_2}{\partial x_1}\right|^{(0)} & \left.\frac{\partial f_2}{\partial x_2}\right|^{(0)} \end{bmatrix} \begin{bmatrix} \Delta x_1^{(0)} \\ \Delta x_2^{(0)} \end{bmatrix} = \begin{bmatrix} k_1 - f_1\left(x_1^{(0)}, x_2^{(0)}\right) \\ k_2 - f_2\left(x_1^{(0)}, x_2^{(0)}\right) \end{bmatrix} \tag{8.102}$$

The matrix of partial derivatives in Eq. (8.102) is known as the Jacobian matrix and is evaluated at the initial estimate. Multiplying each side of Eq. (8.102) by the inverse of the Jacobian yields an approximation of the required correction to the estimated solution. Since the higher order terms were neglected, addition of the correction terms to the original estimate will not yield an exact solution, but will often provide an improved estimate. The procedure may be repeated, obtaining sucessively better estimates until the estimated solution reaches a desired tolerance. Summarizing, correction terms for the ℓth iterate are given in Eq. (8.103) and the solution estimate is updated according to Eq. (8.104).

$$\begin{bmatrix} \Delta x_1^{(\ell)} \\ \Delta x_2^{(\ell)} \end{bmatrix} = \begin{bmatrix} \left.\frac{\partial f_1}{\partial x_1}\right|^{(\ell)} & \left.\frac{\partial f_1}{\partial x_2}\right|^{(\ell)} \\ \left.\frac{\partial f_2}{\partial x_1}\right|^{(\ell)} & \left.\frac{\partial f_2}{\partial x_2}\right|^{(\ell)} \end{bmatrix}^{-1} \begin{bmatrix} k_1 - f_1\left(x_1^{(\ell)}, x_2^{(\ell)}\right) \\ k_2 - f_2\left(x_1^{(\ell)}, x_2^{(\ell)}\right) \end{bmatrix} \tag{8.103}$$

$$x^{(\ell+1)} = x^{(\ell)} + \Delta x^{(\ell)} \tag{8.104}$$

The solution of the original set of nonlinear equations has been converted to a repeated solution of a system of linear equations. This solution requires evaluation of the Jacobian matrix (at the current solution estimate) in each iteration.

The power flow equations can be placed into the Newton-Raphson framework by separating the power flow equations into their real and imaginary parts and taking the voltage magnitudes and phase angles as the unknowns. Writing Eq. (8.103) specifically for the power flow problem:

$$\begin{bmatrix} \Delta\underline{\delta}^{(\ell)} \\ \Delta\underline{V}^{(\ell)} \end{bmatrix} = \begin{bmatrix} \left.\dfrac{\partial \underline{P}}{\partial \underline{\delta}}\right|^{(\ell)} & \left.\dfrac{\partial \underline{P}}{\partial \underline{V}}\right|^{(\ell)} \\ \left.\dfrac{\partial \underline{Q}}{\partial \underline{\delta}}\right|^{(\ell)} & \left.\dfrac{\partial \underline{Q}}{\partial \underline{V}}\right|^{(\ell)} \end{bmatrix}^{-1} \begin{bmatrix} \underline{P}(\text{sched}) - \underline{P}^{(\ell)} \\ \underline{Q}(\text{sched}) - \underline{Q}^{(\ell)} \end{bmatrix} \tag{8.105}$$

The underscored variables in Eq. (8.105) indicate vectors (extending the two-equation Newton-Raphson development to the general power flow case). The (sched) notation indicates the scheduled real and reactive powers injected into the system. $P^{(\ell)}$ and $Q^{(\ell)}$ represent the calculated real and reactive power injections based on the system model and the ℓth voltage phase angle and voltage magnitude estimates. The bus voltage phase angle and bus voltage magnitude estimates are updated, the Jacobian reevaluated, and the mismatch between the scheduled and calculated real and reactive powers evaluated in each iteration of the Newton-Raphson algorithm. Iterations are performed until the estimated solution reaches an acceptable tolerance or a maximum number of allowable iterations is exceeded. Once a solution (within an acceptble tolerance) is reached, P-V bus reactive power injections and the slack bus complex power injection may be evaluated.

Fast Decoupled Power Flow Solution

The fast decoupled power flow algorithm simplifies the procedure presented for the Newton-Raphson algorithm by exploiting the strong coupling between real power and bus voltage phase angles and reactive power and bus voltage magnitudes commonly seen in power systems. The Jacobian matrix is simplified by approximating the partial derivatives of the real power equations with respect to the bus voltage magnitudes as zero. Similarly, the partial derivatives of the reactive power equations with respect to the bus voltage phase angles are approximated as zero. Further, the remaining partial derivatives are often approximated using only the imaginary portion of the bus admittance matrix. These approximations yield the following correction equations:

$$\Delta\underline{\delta}^{(\ell)} = \left[B'\right]^{-1}\left[\underline{P}(\text{sched}) - \underline{P}^{(\ell)}\right] \tag{8.106}$$

$$\Delta\underline{V}^{(\ell)} = \left[B''\right]^{-1}\left[\underline{Q}(\text{sched}) - \underline{Q}^{(\ell)}\right] \tag{8.107}$$

where B′ is an approximation of the matrix of partial derviatives of the real power flow equations with respect to the bus voltage phase angles and B″ is an approximation of the matrix of partial derivatives of the reactive power flow equations with respect to the bus voltage magnitudes. B′ and B″ are typically held constant during the iterative process, eliminating the necessity of updating the Jacobian matrix (required in the Newton-Raphson solution) in each iteration.

The fast decoupled algorithm has good convergence properties despite the many approximations used during its development. The fast decoupled power flow algorithm has found widespread use since it is less computationally intensive (requires fewer computational operations) than the Newton-Raphson method.

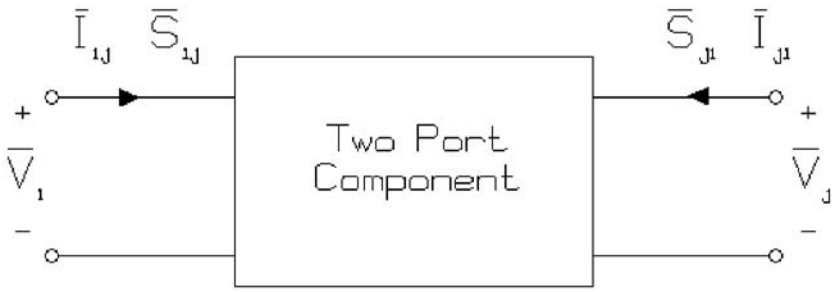

FIGURE 8.27 Typical power system component.

Component Power Flows

The positive sequence network for components of interest (connected between buses i and j) will be of the form shown in Fig. 8.27.

An admittance description is usually available from earlier construction of the nodal admittance matrix. Thus,

$$\begin{bmatrix} \bar{I}_i \\ \bar{I}_j \end{bmatrix} = \begin{bmatrix} \bar{Y}_a & \bar{Y}_b \\ \bar{Y}_c & \bar{Y}_d \end{bmatrix} \begin{bmatrix} \bar{V}_i \\ \bar{V}_j \end{bmatrix} \tag{8.108}$$

Therefore, the complex power flows and the component loss are:

$$\bar{S}_{ij} = \bar{V}_i \bar{I}_i^* = \bar{V}_i \left[\bar{Y}_a \bar{V}_i + \bar{Y}_b \bar{V}_i \right]^* \tag{8.109}$$

$$\bar{S}_{ji} = \bar{V}_j \bar{I}_j^* = \bar{V}_i \left[\bar{Y}_c \bar{V}_i + \bar{Y}_d \bar{V}_j \right]^* \tag{8.110}$$

$$\bar{S}_{loss} = \bar{S}_{ij} + \bar{S}_{ji} \tag{8.111}$$

The calculated component flows combined with the bus voltage magnitudes and phase angles provide extensive information about the power systems operating point. The pu voltage magnitudes may be checked to ensure operation within a prescribed range. The segment power flows can be examined to ensure no equipment ratings are exceeded. Additionally, the power flow solution may used as the starting point for other analyses.

An elementary discussion of the power flow problem and its solution are presented in this section. The power flow problem can be complicated by the addition of further constraints such as generator real and reactive power limits. However, discussion of such complications is beyond the scope of this section. The references provide detailed development of power flow formulation and solution under additional constraints.

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Further Information

The references provide clear introductions to the analysis of power systems. An excellent review of many issues involving the use of computers for power system analysis is provided in July 1974, *Proceedings of the IEEE* (Special Issue on Computers in the Power Industry). The quarterly journal *IEEE Transactions on Power Systems* provides excellent documentation of more recent research in power system analysis.

8.4 Fault Analysis in Power Systems

Charles A. Gross

A **fault** in an electrical power system is the unintentional and undesirable creation of a conducting path (a *short circuit*) or a blockage of current (an *open circuit*). The short-circuit fault is typically the most common and is usually implied when most people use the term *fault*. We restrict our comments to the short-circuit fault.

The causes of faults include lightning, wind damage, trees falling across lines, vehicles colliding with towers or poles, birds shorting out lines, aircraft colliding with lines, vandalism, small animals entering switchgear, and line breaks due to excessive ice loading. Power system faults may be categorized as one of four types: single line-to-ground, line-to-line, double line-to-ground, and balanced three-phase. The first three types constitute severe unbalanced operating conditions.

It is important to determine the values of system voltages and currents during faulted conditions so that protective devices may be set to detect and minimize their harmful effects. The time constants of the associated transients are such that sinusoidal steady-state methods may still be used. The method of symmetrical components is particularly suited to fault analysis.

Our objective is to understand how symmetrical components may be applied specifically to the four general fault types mentioned and how the method can be extended to any unbalanced three-phase system problem.

Note that phase values are indicated by subscripts, *a, b, c;* sequence (symmetrical component) values are indicated by subscripts 0, 1, 2. The transformation is defined by

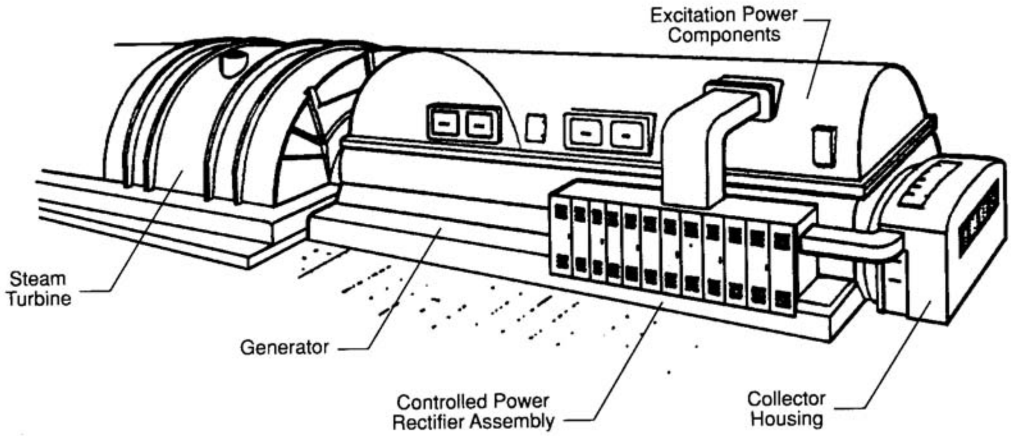
$$\begin{bmatrix} \overline{V}_a \\ \overline{V}_b \\ \overline{V} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \overline{V}_0 \\ \overline{V}_1 \\ \overline{V}_2 \end{bmatrix} = [T] \begin{bmatrix} \overline{V}_0 \\ \overline{V}_1 \\ \overline{V}_2 \end{bmatrix}$$

Simplifications in the System Model

Certain simplifications are possible and usually employed in fault analysis.

- Transformer magnetizing current and core loss will be neglected.
- Line shunt capacitance is neglected.
- Sinusoidal steady-state circuit analysis techniques are used. The so-called **DC offset** is accounted for by using correction factors.
- Prefault voltage is assumed to be $1\underline{/0^\circ}$ per-unit. One per-unit voltage is at its nominal value prior to the application of a fault, which is reasonable. The selection of zero phase is arbitrary and convenient. Prefault load current is neglected.

For hand calculations, neglect series resistance is usually neglected (this approximation will not be necessary for a computer solution). Also, the only difference in the positive and negative sequence networks is introduced by the machine impedances. If we select the subtransient reactance X_d'' for the



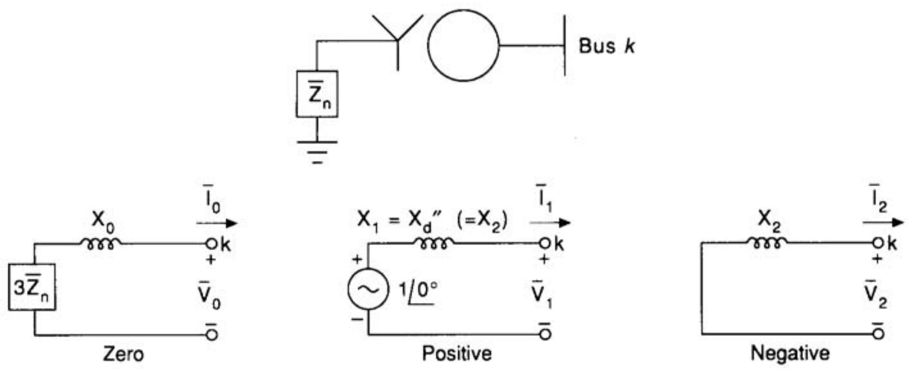


FIGURE 8.28 Generator sequence circuit models.

positive sequence reactance, the difference is slight (in fact, the two are identical for nonsalient machines). The simplification is important, since it reduces computer storage requirements by roughly one-third. Circuit models for generators, lines, and transformers are shown in Figs. 8.28, 8.29, and 8.30, respectively.

Our basic approach to the problem is to consider the general situation suggested in Fig. 8.31(a). The general terminals brought out are for purposes of external connections that will simulate faults. Note carefully the positive assignments of phase quantities. Particularly note that the currents flow *out of* the system. We can construct general *sequence* equivalent circuits for the system, and such circuits are indicated in Fig. 8.31(b). The ports indicated correspond to the general three-phase entry port of Fig. 8.31(a). The positive sense of sequence values is compatible with that used for phase values.

The Four Basic Fault Types

The Balanced Three-Phase Fault

Imagine the general three-phase access port terminated in a fault impedance ($\overline{Z}_f$) as shown in Fig. 8.32(a). The terminal conditions are

$$\begin{bmatrix} \overline{V}_a \\ \overline{V}_b \\ \overline{V}_c \end{bmatrix} = \begin{bmatrix} \overline{Z}_f & 0 & 0 \\ 0 & \overline{Z}_f & 0 \\ 0 & 0 & \overline{Z}_f \end{bmatrix} \begin{bmatrix} \overline{I}_a \\ \overline{I}_b \\ \overline{I}_c \end{bmatrix}$$

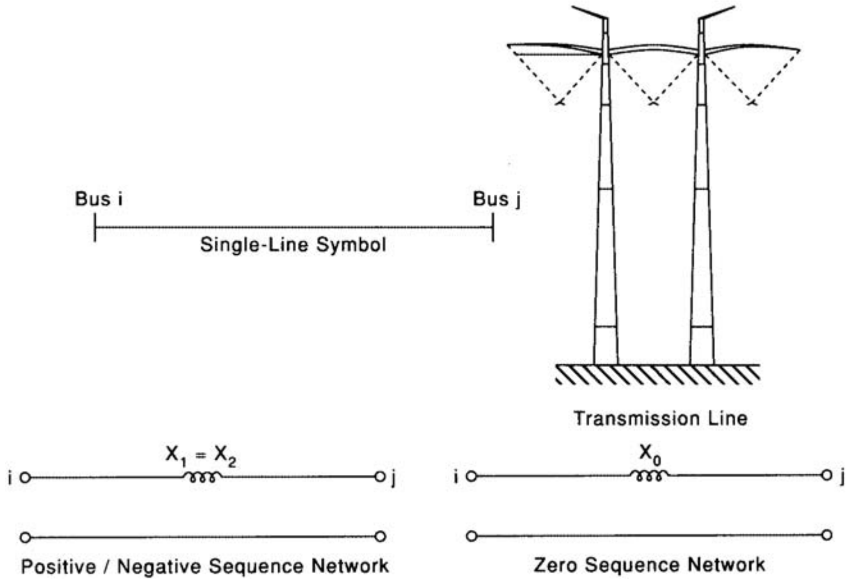


FIGURE 8.29 Line sequence circuit models.

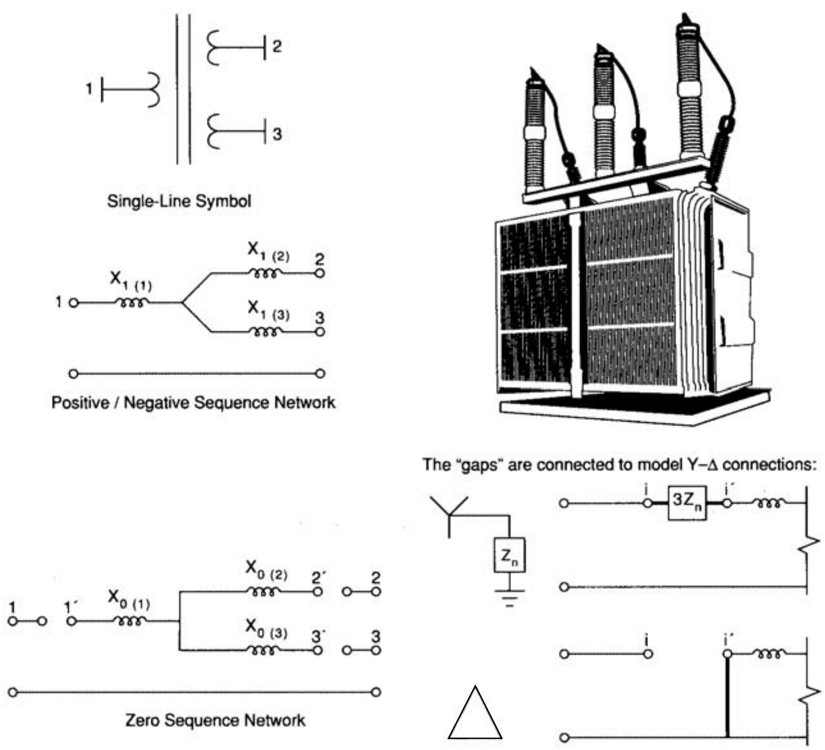


FIGURE 8.30 Transformer sequence circuit models.

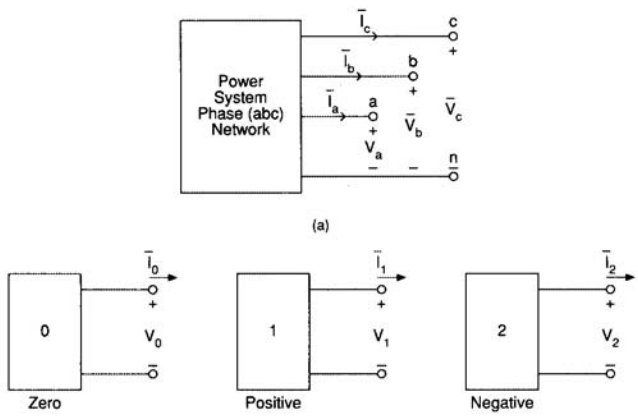


FIGURE 8.31 General fault port in an electric power system. (a) General fault port in phase (*abc*) coordinates; (b) corresponding fault ports in sequence (012) coordinates.

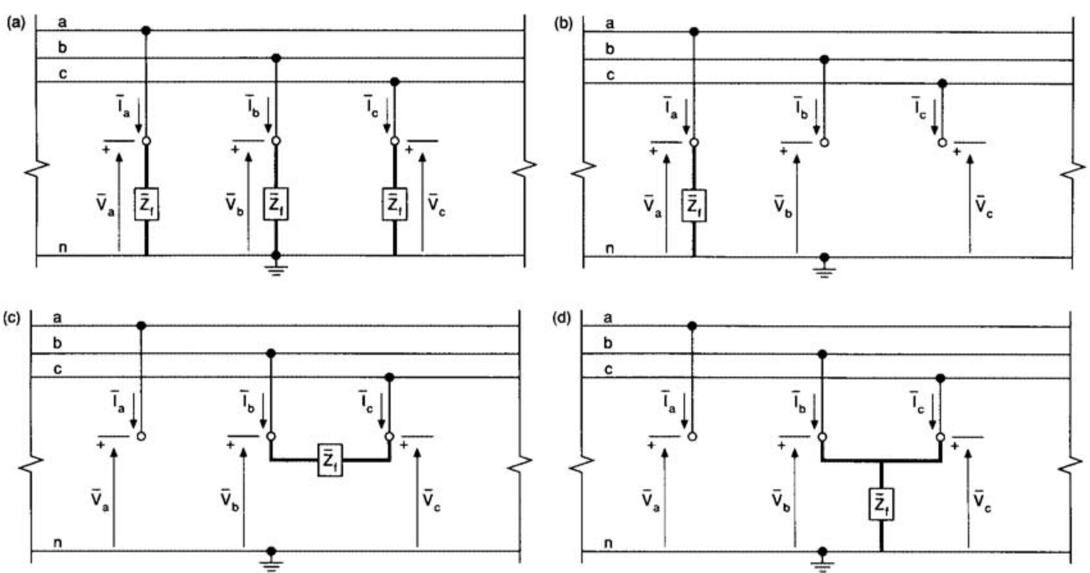


FIGURE 8.32 Fault types. (a) Three-phase fault; (b) single phase-to-ground fault; (c) phase-to-phase fault; (d) double phase-to-ground fault.

Transforming to $[Z_{012}]$,

$$[Z_{012}] = [T]^{-1}\begin{bmatrix} \overline{Z}_f & 0 & 0 \\ 0 & \overline{Z}_f & 0 \\ 0 & 0 & \overline{Z}_f \end{bmatrix}[T] = \begin{bmatrix} \overline{Z}_f & 0 & 0 \\ 0 & \overline{Z}_f & 0 \\ 0 & 0 & \overline{Z}_f \end{bmatrix}$$

The corresponding network connections are given in Fig. 8.33(a). Since the zero and negative sequence networks are passive, only the positive sequence network is nontrivial.

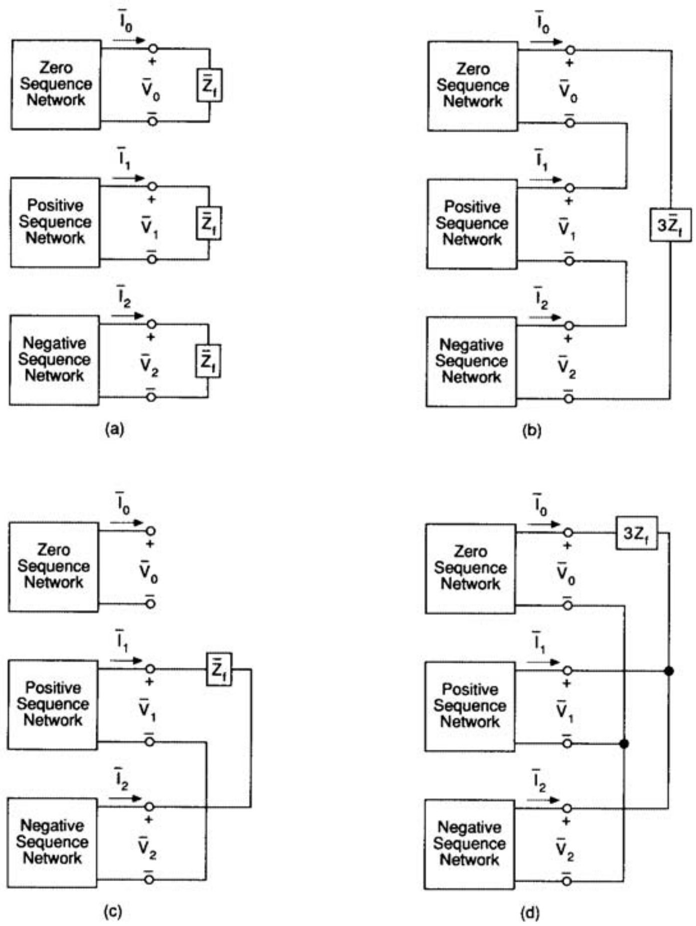


FIGURE 8.33 Sequence network terminations for fault types. (a) Balanced three-phase fault; (b) single phase-to-ground fault; (c) phase-to-phase fault; (d) double phase-to-ground fault.

$$\bar{V}_0 = \bar{V}_2 = 0 \tag{8.112}$$

$$\bar{I}_0 = \bar{I}_2 = 0 \tag{8.113}$$

$$\bar{V}_1 = \bar{Z}_f \bar{I}_1 \tag{8.114}$$

The Single Phase-to-Ground Fault

Imagine the general three-phase access port terminated as shown in Fig. 8.32(b). The terminal conditions are

$$\bar{I}_b = 0 \qquad \bar{I}_c = 0 \qquad \bar{V}_a = \bar{I}_a \bar{Z}_f$$

Therefore,

$$\bar{I}_0 + a^2 \bar{I}_1 + a\bar{I}_2 = \bar{I}_0 + a\bar{I}_1 + a^2 \bar{I}_2 = 0$$

or

$$\bar{I}_1 = \bar{I}_2$$

Also,

$$\bar{I}_b = \bar{I}_0 + a^2\bar{I}_1 + a\bar{I}_2 = \bar{I}_0 + (a^2 + a)\bar{I}_1 = 0$$

or

$$\bar{I}_0 = \bar{I}_1 = \bar{I}_2 \tag{8.113}$$

Furthermore, it is required that

$$\begin{aligned} \bar{V}_a &= \bar{Z}_f\bar{I}_a \\ \bar{V}_0 + \bar{V}_1 + \bar{V}_2 &= 3\bar{Z}_f\bar{I}_1 \end{aligned} \tag{8.114}$$

In general then, Eqs. (8.113) and (8.114) must be simultaneously satisfied. These conditions can be met by interconnecting the sequence networks as shown in Fig. 8.33(b).

The Phase-to-Phase Fault

Imagine the general three-phase access port terminated as shown in Fig. 8.32(c). The terminal conditions are such that we may write

$$\bar{I}_0 = 0 \qquad \bar{I}_b = -\bar{I}_c \qquad \bar{V}_b = \bar{Z}_f\bar{I}_b + \bar{V}_c$$

It follows that

$$\bar{I}_0 + \bar{I}_1 + \bar{I}_2 = 0 \tag{8.115}$$

$$\bar{I}_0 = 0 \tag{8.116}$$

$$\bar{I}_1 = -\bar{I}_2 \tag{8.117}$$

In general then, Eqs. (8.115), (8.116), and (8.117) must be simultaneously satisfied. The proper interconnection between sequence networks appears in Fig. 8.33(c).

The Double Phase-to-Ground Fault

Consider the general three-phase access port terminated as shown in Fig. 8.32(d). The terminal conditions indicate

$$\bar{I}_a = 0 \qquad \bar{V}_b = \bar{V}_c \qquad \bar{V}_b = (\bar{I}_b + \bar{I}_c)\bar{Z}_f$$

It follows that

$$\bar{I}_0 + \bar{I}_1 + \bar{I}_2 = \bar{0} \tag{8.118}$$

$$\bar{V}_1 = \bar{V}_2 \tag{8.119}$$

and

$$\overline{V}_0 - \overline{V}_1 = 3\overline{Z}_f \overline{I}_0 \tag{8.120}$$

For the general double phase-to-ground fault, Eqs. (8.118), (8.119), and (8.120) must be simultaneously satisfied. The sequence network interconnections appear in Fig. 8.33(d).

An Example Fault Study

Case: EXAMPLE SYSTEM
Run :
System has data for 2 Line(s); 2 Transformer(s);
4 Bus(es); and 2 Generator(s)

Transmission Line Data

Line	Bus	Bus	Seq	R	X	B	Srat
1	2	3	pos	0.00000	0.16000	0.00000	1.0000
			zero	0.00000	0.50000	0.00000	
2	2	3	pos	0.00000	0.16000	0.00000	1.0000
			zero	0.00000	0.50000	0.00000	

Transformer Data

Trans-former	HV Bus	LV Bus	Seq	R	X	C	Srat
1	2	1	pos	0.00000	0.05000	1.00000	1.0000
	Y	Y	zero	0.00000	0.05000		
2	3	4	pos	0.00000	0.05000	1.00000	1.0000
	Y	D	zero	0.00000	0.05000		

Generator Data

No.	Bus	Srated	Ra	Xd″	Xo	Rn	Xn	Con
1	1	1.0000	0.0000	0.200	0.0500	0.0000	0.0400	Y
2	4	1.0000	0.0000	0.200	0.0500	0.0000	0.0400	Y

Zero Sequence [Z] Matrix

0.0 + j(0.1144)	0.0 + j(0.0981)	0.0 + j(0.0163)	0.0 + j(0.0000)
0.0 + j(0.0981)	0.0 + j(0.1269)	0.0 + j(0.0212)	0.0 + j(0.0000)
0.0 + j(0.0163)	0.0 + j(0.0212)	0.0 + j(0.0452)	0.0 + j(0.0000)
0.0 + j(0.0000)	0.0 + j(0.0000)	0.0 + j(0.0000)	0.0 + j(0.1700)

Positive Sequence [Z] Matrix

0.0 + j(0.1310)	0.0 + j(0.1138)	0.0 + j(0.0862)	0.0 + j(0.0690)
0.0 + j(0.1138)	0.0 + j(0.1422)	0.0 + j(0.1078)	0.0 + j(0.0862)
0.0 + j(0.0862)	0.0 + j(0.1078)	0.0 + j(0.1422)	0.0 + j(0.1138)
0.0 + j(0.0690)	0.0 + j(0.0862)	0.0 + j(0.1138)	0.0 + j(0.1310)

The single-line diagram and sequence networks are presented in Fig. 8.34.

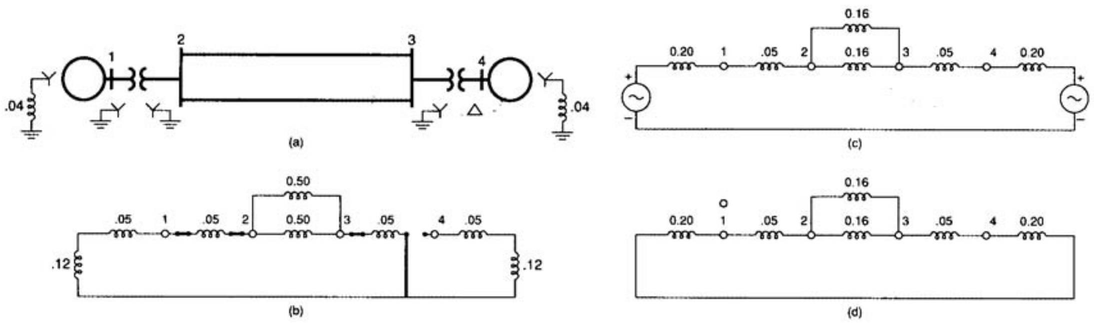


FIGURE 8.34 Example system. (a) Single-line diagram; (b) zero sequence network; (c) positive sequence network; (d) negative sequence network.

Suppose bus 3 in the example system represents the fault location and $\bar{Z}_f = 0$. The positive sequence circuit can be reduced to its Thévenin equivalent at bus 3:

$$E_{T1} = 1.0\underline{/0^\circ} \qquad \bar{Z}_{T1} = j0.1422$$

Similarly, the negative and zero sequence Thévenin elements are:

$$\begin{aligned} \bar{E}_{T2} &= 0 \qquad & \bar{Z}_{T2} &= j0.1422 \\ \bar{E}_{T0} &= 0 \qquad & Z_{T0} &= j0.0452 \end{aligned}$$

The network interconnections for the four fault types are shown in Fig. 8.35. For each of the fault types, compute the currents and voltages at the faulted bus.

Balanced Three-Phase Fault

The sequence networks are shown in Fig. 8.35(a). Obviously,

$$\bar{V}_0 = \bar{I}_0 = \bar{V}_2 = \bar{I}_2 = 0$$

$$\bar{I}_1 = \frac{1\underline{/0^\circ}}{j0.1422} = -j7.032; \quad \text{also } \bar{V}_1 = 0$$

To compute the phase values,

$$\begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} = [T]\begin{bmatrix} \bar{I}_0 \\ \bar{I}_1 \\ \bar{I}_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}\begin{bmatrix} 0 \\ -j7.032 \\ 0 \end{bmatrix} = \begin{bmatrix} 7.032\underline{/-90^\circ} \\ 7.032\underline{/150^\circ} \\ 7.032\underline{/30^\circ} \end{bmatrix}$$

$$\begin{bmatrix} \bar{V}_a \\ \bar{V}_b \\ \bar{V}_c \end{bmatrix} = [T]\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

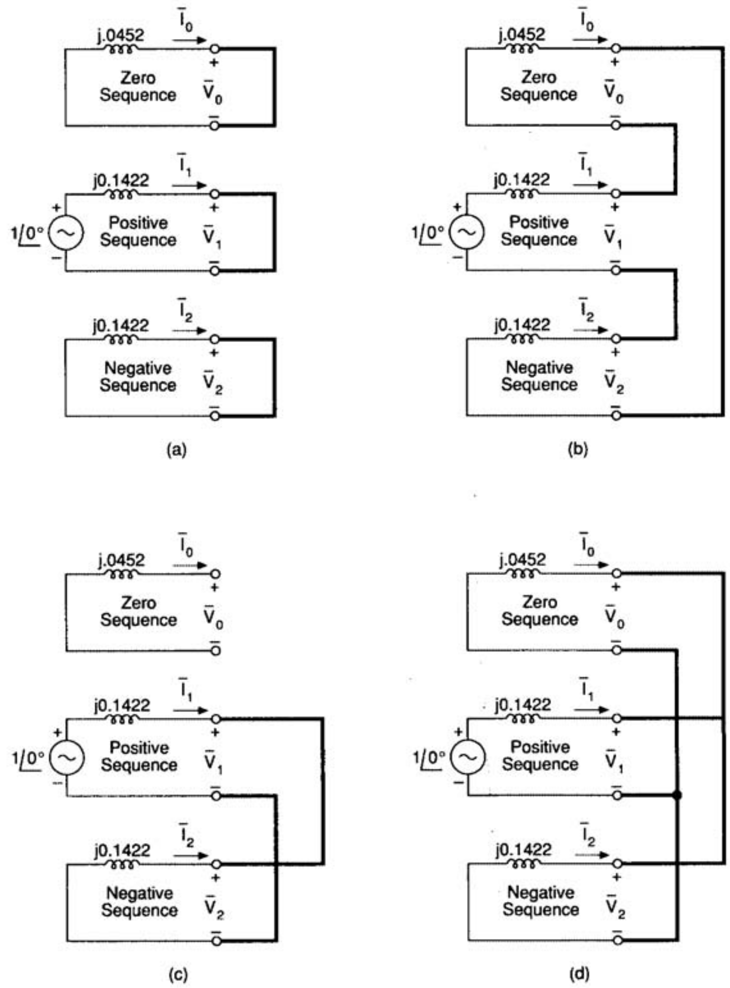


FIGURE 8.35 Example system faults at bus 3. (a) Balanced three-phase; (b) single phase-to-ground; (c) phase-to-phase; (d) double phase-to-ground.

Single Phase-to-Ground Fault

The sequence networks are interconnected as shown in Fig. 8.35(b).

$$\bar{I}_0 = \bar{I}_1 = \bar{I}_2 = \frac{1\angle 0^\circ}{j0.0452 + j0.1422 + j0.1422} = -j3.034$$

$$\begin{bmatrix} \bar{I}_a \\ \bar{I}_b \\ \bar{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -j3.034 \\ -j3.034 \\ -j3.034 \end{bmatrix} = \begin{bmatrix} -j9.102 \\ 0 \\ 0 \end{bmatrix}$$

The sequence voltages are

$$\overline{V}_0 = -j0.0452(-j3.034) = -1371$$
$$\overline{V}_1 = 1.0 - j0.1422(-j3.034) = 0.5685$$
$$\overline{V}_2 = -j0.1422(-j3.034) = -0.4314$$

The phase voltages are

$$\begin{bmatrix} \overline{V}_a \\ \overline{V}_b \\ \overline{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} -0.1371 \\ 0.5685 \\ -0.4314 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.8901 \underline{/-103.4°} \\ 0.8901 \underline{/-103.4°} \end{bmatrix}$$

Phase-to-phase and double phase-to-ground fault values are calculated from the appropriate networks [Figs. 8.35(c) and (d)]. Complete results are provided.

Faulted Bus	Phase a	Phase b	Phase c
3	G	G	G

Sequence Voltages

Bus	V0		V1		V2	
1	0.0000/	0.0	0.3939/	0.0	0.0000/	0.0
2	0.0000/	0.0	0.2424/	0.0	0.0000/	0.0
3	0.0000/	0.0	0.0000/	0.0	0.0000/	0.0
4	0.0000/	0.0	0.2000/	–30.0	0.0000/	30.0

Phase Voltages

Bus	Va		Vb		Vc	
1	0.3939/	0.0	0.3939/	–120.0	0.3939/	120.0
2	0.2424/	0.0	0.2424/	–120.0	0.2424/	120.0
3	0.0000/	6.5	0.0000/	–151.2	0.0000/	133.8
4	0.2000/	–30.0	0.2000/	–150.0	0.2000/	90.0

Sequence Currents

Bus to Bus		I0		I1		I2	
1	2	0.0000/	167.8	3.0303/	–90.0	0.0000/	90.0
1	0	0.0000/	–12.2	3.0303/	90.0	0.0000/	–90.0
2	3	0.0000/	167.8	1.5152/	–90.0	0.0000/	90.0
2	3	0.0000/	167.8	1.5152/	–90.0	0.0000/	90.0
2	1	0.0000/	–12.2	3.0303/	90.0	0.0000/	–90.0
3	2	0.0000/	–12.2	1.5152/	90.0	0.0000/	–90.0
3	2	0.0000/	–12.2	1.5152/	90.0	0.0000/	–90.0
3	4	0.0000/	–12.2	4.0000/	90.0	0.0000/	–90.0
4	3	0.0000/	0.0	4.0000/	–120.0	0.0000/	120.0
4	0	0.0000/	0.0	4.0000/	60.0	0.0000/	–60.0

Faulted Bus	Phase a	Phase b	Phase c
3	G	G	G

Phase Currents

Bus to Bus		Ia		Ib		Ic	
1	2	3.0303/	–90.0	3.0303/	150.0	3.0303/	30.0
1	0	3.0303/	90.0	3.0303/	–30.0	3.0303/	–150.0
2	3	1.5151/	–90.0	1.5151/	150.0	1.5151/	30.0
2	3	1.5151/	–90.0	1.5151/	150.0	1.5151/	30.0
2	1	3.0303/	90.0	3.0303/	–30.0	3.0303/	–150.0
3	2	1.5151/	90.0	1.5151/	–30.0	1.5151/	–150.0
3	2	1.5151/	90.0	1.5151/	–30.0	1.5151/	–150.0
3	4	4.0000/	90.0	4.0000/	–30.0	4.0000/	–150.0
4	3	4.0000/	–120.0	4.0000/	120.0	4.0000/	–0.0
4	0	4.0000/	60.0	4.0000/	–60.0	4.0000/	–180.0

Faulted Bus	Phase a	Phase b	Phase c
3	G	0	0

Sequence Voltages

Bus	V0		V1		V2	
1	0.0496/	180.0	0.7385/	0.0	0.2615/	180.0
2	0.0642/	180.0	0.6731/	0.0	0.3269/	180.0
3	0.1371/	180.0	0.5685/	0.0	0.4315/	180.0
4	0.0000/	0.0	0.6548/	–30.0	0.3452/	210.0

Phase Voltages

Bus	Va		Vb		Vc	
1	0.4274/	0.0	0.9127/	–108.4	0.9127/	108.4
2	0.2821/	0.0	0.8979/	–105.3	0.8979/	105.3
3	0.0000/	89.2	0.8901/	–103.4	0.8901/	103.4
4	0.5674/	–61.8	0.5674/	–118.2	1.0000/	90.0

Sequence Currents

Bus to Bus		I0		I1		I2	
1	2	0.2917/	–90.0	1.3075/	–90.0	1.3075/	–90.0
1	0	0.2917/	90.0	1.3075/	90.0	1.3075/	90.0
2	3	0.1458/	–90.0	0.6537/	–90.0	0.6537/	–90.0
2	3	0.1458/	–90.0	0.6537/	–90.0	0.6537/	–90.0
2	1	0.2917/	90.0	1.3075/	90.0	1.3075/	90.0
3	2	0.1458/	90.0	0.6537/	90.0	0.6537/	90.0
3	2	0.1458/	90.0	0.6537/	90.0	0.6537/	90.0
3	4	2.7416/	90.0	1.7258/	90.0	1.7258/	90.0
4	3	0.0000/	0.0	1.7258/	–120.0	1.7258/	–60.0
4	0	0.0000/	90.0	1.7258/	60.0	1.7258/	120.0

Faulted Bus	Phase a	Phase b	Phase c
3	G	0	0

Phase Currents

Bus to Bus		Ia		Ib		Ic	
1	2	2.9066/	–90.0	1.0158/	90.0	1.0158/	90.0
1	0	2.9066/	90.0	1.0158/	–90.0	1.0158/	–90.0
2	3	1.4533/	–90.0	0.5079/	90.0	0.5079/	90.0
2	3	1.4533/	–90.0	0.5079/	90.0	0.5079/	90.0
2	1	2.9066/	90.0	1.0158/	–90.0	1.0158/	–90.0
3	2	1.4533/	90.0	0.5079/	–90.0	0.5079/	–90 0
3	2	1.4533/	90.0	0.5079/	–90.0	0.5079/	–90 0
3	4	6.1933/	90.0	1.0158/	90.0	1.0158/	90.0
4	3	2.9892/	–90.0	2.9892/	90.0	0.0000/	–90.0
4	0	2.9892/	90.0	2.9892/	–90.0	0.0000/	90.0

Faulted Bus	Phase a	Phase b	Phase c
3	0	C	B

Sequence Voltages

Bus	V0		V1		V2	
1	0.0000/	0.0	0.6970/	0.0	0.3030/	0.0
2	0.0000/	0.0	0.6212/	0.0	0.3788/	0.0
3	0.0000/	0.0	0.5000/	0.0	0.5000/	0.0
4	0.0000/	0.0	0.6000/	–30.0	0.4000/	30.0

Phase Voltages

Bus	Va		Vb		Vc	
1	1.0000/	0.0	0.6053/	–145.7	0.6053/	145.7
2	1.0000/	0.0	0.5423/	–157.2	0.5423/	157.2
3	1.0000/	0.0	0.5000/	–180.0	0.5000/	–180.0
4	0.8718/	–6.6	0.8718/	–173.4	0.2000/	90.0

Sequence Currents

Bus to Bus		I0		I1		I2	
1	2	0.0000/	–61.0	1.5152/	–90.0	1.5152/	90.0
1	0	0.0000/	119.0	1.5152/	90.0	1.5152/	–90.0
2	3	0.0000/	–61.0	0.7576/	–90.0	0.7576/	90.0
2	3	0.0000/	–61.0	0.7576/	–90.0	0.7576/	90.0
2	1	0.0000/	119.0	1.5152/	90.0	1.5152/	–90.0
3	2	0.0000/	119.0	0.7576/	90.0	0.7576/	–90.0
3	2	0.0000/	119.0	0.7576/	90.0	0.7576/	–90.0
3	4	0.0000/	119.0	2.0000/	90.0	2.0000/	–90.0
4	3	0.0000/	0.0	2.0000/	–120.0	2.0000/	120.0
4	0	0.0000/	90.0	2.0000/	60.0	2.0000/	–60.0

Faulted Bus	Phase a	Phase b	Phase c
3	0	C	B

Phase Currents

Bus to Bus		Ia		Ib		Ic	
1	2	0.0000/	180.0	2.6243/	180.0	2.6243/	0.0
1	0	0.0000/	180.0	2.6243/	0.0	2.6243/	180.0
2	3	0.0000/	–180.0	1.3122/	180.0	1.3122/	0.0
2	3	0.0000/	–180.0	1.3122/	180.0	1.3122/	0.0
2	1	0.0000/	180.0	2.6243/	0.0	2.6243/	180.0
3	2	0.0000/	–180.0	1.3122/	0.0	1.3122/	180.0
3	2	0.0000/	–180.0	1.3122/	0.0	1.3122/	180.0
3	4	0.0000/	–180.0	3.4641/	0.0	3.4641/	180.0
4	3	2.0000/	–180.0	2.0000/	180.0	4.0000/	0.0
4	0	2.0000/	0.0	2.0000/	0.0	4.0000/	– 180.0

Faulted Bus	Phase a	Phase b	Phase c
3	0	G	G

Sequence Voltages

Bus	V0		V1		V2	
1	0.0703/	0.0	0.5117/	0.0	0.1177/	0.0
2	0.0909/	0.0	0.3896/	0.0	0.1472/	0.0
3	0.1943/	–0.0	0.1943/	0.0	0.1943/	0.0
4	0.0000/	0.0	0.3554/	–30.0	0.1554/	30.0

Phase Voltages

Bus	Va		Vb		Vc	
1	0.6997/	0.0	0.4197/	–125.6	0.4197/	125.6
2	0.6277/	0.0	0.2749/	–130.2	0.2749/	130.2
3	0.5828/	0.0	0.0000/	–30.7	0.0000/	–139.6
4	0.4536/	–12.7	0.4536/	–167.3	0.2000/	90.0

Sequence Currents

Bus to Bus		I0		I1		I2	
1	2	0.4133/	90.0	2.4416/	– 90.0	0.5887/	90.0
1	0	0.4133/	–90.0	2.4416/	90.0	0.5887/	–90.0
2	3	0.2067/	90.0	1.2208/	– 90.0	0.2943/	90.0
2	3	0.2067/	90.0	1.2208/	– 90.0	0.2943/	90.0
2	1	0.4133/	–90.0	2.4416/	90.0	0.5887/	–90.0
3	2	0.2067/	– 90.0	1.2208/	90.0	0.2943/	– 90.0
3	2	0.2067/	– 90.0	1.2208/	90.0	0.2943/	– 90.0
3	4	3.8854/	– 90.0	3.2229/	90.0	0.7771/	– 90.0
4	3	0.0000/	0.0	3.2229/	– 120.0	0.7771/	120.0
4	0	0.0000/	–90.0	3.2229/	60.0	0.7771/	–60.0

Faulted Bus	Phase a	Phase b	Phase c
3	0	G	G

Bus to Bus		Phase Currents Ia		Ib		Ic	
1	2	1.4396/	–90.0	2.9465/	153.0	2.9465/	27.0
1	0	1.4396/	90.0	2.9465/	–27.0	2.9465/	–153.0
2	3	0.7198/	–90.0	1.4733/	153.0	1.4733/	27.0
2	3	0.7198/	–90.0	1.4733/	153.0	1.4733/	27.0
2	1	1.4396/	90.0	2.9465/	–27.0	2.9465/	–153.0
3	2	0.7198/	90.0	1.4733/	–27.0	1.4733/	–153.0
3	2	0.7198/	90.0	1.4733/	–27.0	1.4733/	– 153.0
3	4	1.4396/	–90.0	6.1721/	–55.9	6.1721/	–124.1
4	3	2.9132/	–133.4	2.9132/	133.4	4.0000/	–0.0
4	0	2.9132/	46.6	2.9132/	–46.6	4.0000/	–180.0

Further Considerations

Generators are not the only sources in the system. All rotating machines are capable of contributing to fault current, at least momentarily. Synchronous and induction motors will continue to rotate due to inertia and function as sources of fault current. The impedance used for such machines is usually the transient reactance X'_d or the subtransient X''_d, depending on protective equipment and speed of response. Frequently, motors smaller than 50 hp are neglected. Connecting systems are modeled with their Thévenin equivalents.

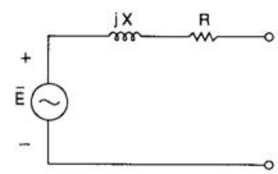


FIGURE 8.36 Positive sequence circuit looking back into faulted bus.

Although we have used AC circuit techniques to calculate faults, the problem is fundamentally transient since it involves sudden switching actions. Consider the so-called DC offset current. We model the system by determining its positive sequence Thévenin equivalent circuit, looking back into the positive sequence network at the fault, as shown in Fig. 8.36. The transient fault current is

$$i(t) = I_{ac}\sqrt{2}\cos(\omega t - \beta) + I_{dc}e^{-t/\tau}$$

This is a first-order approximation and strictly applies only to the three-phase or phase-to-phase fault. Ground faults would involve the zero sequence network also.

$$I_{ac} = \frac{E}{\sqrt{R^2 + X^2}} = \text{rms AC current}$$

$$I_{dc}(t) = I_{dc}e^{-t/\tau} = \text{DC offset current}$$

The maximum initial DC offset possible would be

$$\text{Max } I_{DC} = I_{max} = \sqrt{2}I_{AC}$$

The DC offset will exponentially decay with time constant τ, where

$$\tau = \frac{L}{R} = \frac{X}{\omega R}$$

The maximum DC offset current would be $I_{DC}(t)$

$$I_{DC}(t) = I_{DC} e^{-t/\tau} = \sqrt{2} I_{AC} e^{-t/\tau}$$

The *transient rms* current $I(t)$, accounting for both the AC and DC terms, would be

$$I(t) = \sqrt{I_{AC}^2 + I_{DC}^2(t)} = I_{AC}\sqrt{1 + 2e^{-2t/\tau}}$$

Define a multiplying factor k_i such that I_{AC} is to be multiplied by k_i to estimate the interrupting capacity of a breaker which operates in time T_{op}. Therefore,

$$k_i = \frac{I(T_{op})}{I_{AC}} = \sqrt{1 + 2e^{-2T_{op}/\tau}}$$

Observe that the maximum possible value for k_i is $\sqrt{3}$.

Example

In the circuit of Fig. 8.36, E = 2400 V, X = 2 Ω, R = 0.1 Ω, and f = 60 Hz. Compute k_i and determine the interrupting capacity for the circuit breaker if it is designed to operate in two cycles. The fault is applied at t = 0.

Solution:

$$\begin{aligned} I_{ac} &\cong \frac{2400}{2} = 1200 \text{ A} \\ T_{op} &= \frac{2}{60} = 0.0333 \text{ s} \\ \tau &= \frac{X}{\omega R} = \frac{2}{37.7} = 0.053 \\ k_i &= \sqrt{1 + 2e^{-2T_{op}/\tau}} = \sqrt{1 + 2e^{-0.0067/0.053}} = 1.252 \end{aligned}$$

Therefore,

$$I = k_i I_{ac} = 1.252(1200) = 1503 \text{ A}$$

The Thévenin equivalent at the fault point is determined by normal sinusoidal steady-state methods, resulting in a first-order circuit as shown in Fig. 8.36. While this provides satisfactory results for the steady-state component I_{AC}, the X/R value so obtained can be in serious error when compared with the rate of decay of $I(t)$ as measured by oscillographs on an actual faulted system. The major reasons for the discrepancy are, first of all, that the system, for transient analysis purposes, is actually high-order, and second, the generators do not hold constant impedance as the transient decays.

Summary

Computation of fault currents in power systems is best done by computer. The major steps are summarized below:

- Collect, read in, and store machine, transformer, and line data in per-unit on common bases.
- Formulate the sequence impedance matrices.
- Define the faulted bus and Z_f. Specify type of fault to be analyzed.
- Compute the sequence voltages.
- Compute the sequence currents.
- Correct for wye-delta connections.
- Transform to phase currents and voltages.

For large systems, computer formulation of the sequence impedance matrices is required. Refer to Further Information for more detail. Zero sequence networks for lines in close proximity to each other (on a common right-of-way) will be mutually coupled. If we are willing to use the same values for positive and negative sequence machine impedances,

$$[Z_1] = [Z_2]$$

Therefore, it is unnecessary to store these values in separate arrays, simplifying the program and reducing the computer storage requirements significantly. The error introduced by this approximation is usually not important. The methods previously discussed neglect the prefault, or load, component of current; that is, the usual assumption is that currents throughout the system were zero prior to the fault. This is almost never strictly true; however, the error produced is small since the fault currents are generally much larger than the load currents. Also, the load currents and fault currents are out of phase with each other, making their sum more nearly equal to the larger components than would have been the case if the currents were in phase. In addition, selection of precise values for prefault currents is somewhat speculative, since there is no way of predicting what the loaded state of the system is when a fault occurs. When it is important to consider load currents, a power flow study is made to calculate currents throughout the system, and these values are superimposed on (added to) results from the fault study.

A term which has wide industrial use and acceptance is the *fault level* or **fault MVA** at a bus. It relates to the amount of current that can be expected to flow out of a bus into a three-phase fault. As such, it is an alternate way of providing positive sequence impedance information. Define

$$\begin{aligned}\text{Fault level in MVA at bus } i &= V_{i_{\text{pu}_{\text{nominal}}}} I_{i_{\text{pu}_{\text{fault}}}} S_{3\phi_{\text{base}}} \\ &= (1)\frac{1}{Z_{ii}^1} S_{3\phi_{\text{base}}} = \frac{S_{3\phi_{\text{base}}}}{Z_{ii}^1}\end{aligned}$$

Fault study results may be further refined by approximating the effect of DC offset.

The basic reason for making fault studies is to provide data that can be used to size and set protective devices. The role of such protective devices is to detect and remove faults to prevent or minimize damage to the power system.

Defining Terms

DC offset: The natural response component of the transient fault current, usually approximated with a first-order exponential expression.

Fault: An unintentional and undesirable conducting path in an electrical power system.

Fault MVA: At a specific location in a system, the initial symmetrical fault current multiplied by the prefault nominal line-to-neutral voltage (×3 for a three-phase system).

Sequence (012) quantities: Symmetrical components computed from phase (*abc*) quantities. Can be voltages, currents, and/or impedances.

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Further Information

For a comprehensive coverage of general fault analysis, see Paul M. Anderson, *Analysis of Faulted Power Systems,* New York, IEEE Press, 1995. Also see Chapters 9 and 10 of *Power System Analysis* by C.A. Gross, New York: Wiley, 1986.