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Noise Measurement

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W. Marshall Leach, Jr.
Georgia Institute of Technology

This chapter describes the principal sources of electric noise and discusses methods for the measurement of noise. The notations for voltages and currents correspond to the following conventions: dc quantities are indicated by an upper-case letter with upper-case subscripts, e.g., V_{BE}. Instantaneous small-signal ac quantities are indicated by a lower-case letter with lower-case subscripts, e.g., v_n. The mean-square value of a variable is denoted by a bar over the square of the variable, e.g., $\overline{v_n^2}$, where the bar indicates an arithmetic average of an ensemble of functions. The root-mean-square or rms value is the square root of the mean-square value. Phasors are indicated by an upper-case letter and lower case subscripts, e.g., V_n. Circuit symbols for independent sources are circular, symbols for controlled sources are diamond shaped, and symbols for noise sources are square. In the evaluation of noise equations, Boltzmann's constant is $k = 1.38 \times 10^{-23}$ J K^{-1} and the electronic charge is $q = 1.60 \times 10^{-19}$ C. The standard temperature is denoted by T_0 and is taken to be $T_0 = 290$ K. For this value, $4kT_0 = 1.60 \times 10^{-20}$ J and the thermal voltage is $V_T = kT_0/q = 0.025$ V.

18.1 Thermal Noise

Thermal noise or *Johnson noise* is generated by the random collision of charge carriers with a lattice under conditions of thermal equilibrium [1–7]. Thermal noise in a resistor can be modeled by a series voltage source or a parallel current source having the mean-square values

$$\overline{v_t^2} = 4kTR\Delta f \tag{18.1}$$

$$\overline{i_t^2} = \frac{4kT\Delta f}{R} \tag{18.2}$$

where R is the resistance and Δf is the bandwidth in hertz (Hz) over which the noise is measured. The equation for $\overline{v_t^2}$ is commonly referred to as the *Nyquist formula.* Thermal noise in resistors is independent of the resistor composition.

The *crest factor* for thermal noise is the ratio of the peak value to the rms value. A common definition for the peak value is the level that is exceeded 0.01% of the time. The amplitude distribution of thermal noise is modeled by a gaussian or normal probability density function. The probability that the instantaneous value exceeds 4 times the rms value is approximately 0.01%. Thus, the crest factor is approximately 4.

18.2 Spectral Density

The *spectral density* of a noise signal is defined as the mean-square value per unit bandwidth. For the thermal noise generated by a resistor, the voltage and current spectral densities, respectively, are given by:

$$S_v(f) = 4kTR \tag{18.3}$$

$$S_i(f) = \frac{4kT}{R} \tag{18.4}$$

Because these are independent of frequency, thermal noise is said to have a uniform or flat distribution. Such noise is also called *white noise.* It is called this by analogy to white light, which also has a flat spectral density in the optical band.

18.3 Fluctuation Dissipation Theorem

Consider any system in thermal equilibrium with its surroundings. If there is a mechanism for energy in a particular mode to leak out of that mode to the surroundings in the form of heat, then energy can leak back into that mode from the surrounding heat by the same mechanism. The fluctuation dissipation theorem of quantum mechanics states that the average energy flow in each direction is the same. Otherwise, the system would not be in equilibrium.

Mathematically, the fluctuation dissipation theorem states, in general, that the generalized mean-square force $\overline{\Im^2}$ acting on a system in the frequency band from f_1 to f_2 is given by:

$$\overline{\Im^2} = 4kT\int_{f_1}^{f_2} \mathrm{Re}\left[Z(f)\right]df \tag{18.5}$$

where Re $[Z(f)]$ is the real part of the system impedance $Z(f)$ and f is the frequency in hertz (Hz). For a mechanical system, the generalized force is the mechanical force on the system and the impedance is force divided by velocity. For an electric system, the generalized force is the voltage and the impedance is the ratio of voltage to current.

Equation 18.1 is a statement of the fluctuation dissipation theorem for a resistor. The theorem can be used to calculate the mean-square thermal noise voltage generated by any two-terminal network containing resistors, capacitors, and inductors. Let $Z(f)$ be the complex impedance of the network. The mean-square open-circuit thermal noise voltage is given by:

$$\overline{v_t^2} = 4kT\int_{f_1}^{f_2} \mathrm{Re}\big[Z(f)\big]\mathrm{d}f \simeq 4kT\ \mathrm{Re}\big[Z(f)\big]\Delta f \tag{18.6}$$

where $\Delta f = f_2 - f_1$ and the approximation holds if Re $[Z(f)]$ is approximately constant over the band.

18.4 Equivalent Noise Resistance and Conductance

A mean-square noise voltage can be represented in terms of an *equivalent noise resistance* [8]. Let $\overline{v_n^2}$ be the mean-square noise voltage in the band Δf. The noise resistance R_n is defined as the value of a resistor at the standard temperature T_0 = 290 K that generates the same noise. It is given by:

$$R_n = \frac{\overline{v_n^2}}{4kT_0\Delta f} \tag{18.7}$$

A mean-square noise current can be represented in terms of an *equivalent noise conductance*. Let $\overline{i_n^2}$ be the mean-square noise current in the band Δf. The noise conductance G_n is defined as the value of a conductance at the standard temperature that generates the same noise. It is given by:

$$G_n = \frac{\overline{i_n^2}}{4kT_0\Delta f} \tag{18.8}$$

18.5 Shot Noise

Shot noise is caused by the random emission of electrons and by the random passage of charge carriers across potential barriers [1–7]. The shot noise generated in a device is modeled by a parallel noise current source. The mean-square shot noise current in the frequency band Δf is given by:

$$\overline{i_{sh}^2} = 2qI\Delta f \tag{18.9}$$

where I is the dc current through the device. This equation is commonly referred to as the *Schottky formula.* Like thermal noise, shot noise is white noise and has a crest formula of approximately 4.

18.6 Flicker Noise

The imperfect contact between two conducting materials causes the conductivity to fluctuate in the presence of a dc current [1–7]. This phenomenon generates what is called *flicker noise* or *contact noise.* It is modeled by a noise current source in parallel with the device. The mean-square flicker noise current in the frequency band Δf is given by:

$$\overline{i_f^2} = \frac{K_f I^m \Delta f}{f^n} \tag{18.10}$$

where K_f is the flicker noise coefficient, I is the dc current, m is the flicker noise exponent, and $n \simeq 1$. Other names for flicker noise are *1/f noise* (read "one-over-f-noise"), *low-frequency noise,* and *pink noise.* The latter comes from the optical analog of pink light, which has a spectral density that increases at lower frequencies.

18.7 Excess Noise

In resistors, flicker noise is caused by the variable contact between particles of the resistive material and is called *excess noise.* Metal film resistors generate the least excess noise, carbon composition resistors generate the most, with carbon film resistors lying between the two. In modeling excess noise, the flicker noise exponent has the value $m = 2$. The mean-square excess noise current is often written as a function of the *noise index NI* as follows:

$$\overline{i_{ex}^2} = \frac{10^{NI/10}}{10^{12}\ln 10} \times \frac{I^2 \Delta f}{f} \tag{18.11}$$

where I is the dc current through the resistor. The noise index is defined as the number of μA of excess noise current in each decade of frequency per A of dc current through the resistor. An equivalent definition is the number of μV of excess noise voltage in each decade of frequency per volt of dc drop across the resistor. In this case, the mean-square excess noise voltage generated by the resistor is given by:

$$\overline{v_{ex}^2} = \frac{10^{NI/10}}{10^{12}\ln 10} \times \frac{V^2 \Delta f}{f} \tag{18.12}$$

where $V = IR$ is the dc voltage across the resistor.

18.8 Burst Noise

Burst noise or *popcorn noise* is caused by a metallic impurity in a *pn* junction [4]. When amplified and reproduced by a loudspeaker, it sounds like corn popping. When viewed on an oscilloscope, it appears as fixed amplitude pulses of randomly varying width and repetition rate. The rate can vary from less than one pulse per minute to several hundred pulses per second. Typically, the amplitude of burst noise is 2 to 100 times that of the background thermal noise.

18.9 Partition Noise

Partition noise occurs when the charge carriers in a current have the possibility of dividing between two or more paths. The noise is generated in the resulting components of the current by the statistical process of partition [9]. Partition noise occurs in BJTs where the current flowing from the emitter into the base can take one of two paths. The recombination of injected carriers in the base region corresponds to the current flow in one path. This current flows in the external base lead. The current carried to the collector corresponds to the current flow in the second path. Because the emitter current exhibits full shot noise, the base and collector currents also exhibit full shot noise. However, the base and collector noise currents are correlated because they have equal and opposite partition components. Partition noise in the BJT can be accounted for if all shot noise is referred to two uncorrelated shot noise current sources, one from base to emitter and the other from collector to emitter [10]. This noise model of the BJT is described here.

18.10 Generation–Recombination Noise

Generation–recombination noise in a semiconductor is generated by the random fluctuation of free carrier densities caused by spontaneous fluctuations in the generation, recombination, and trapping rates [7]. In BJTs, it occurs in the base region at low temperatures. The generation–recombination gives rise to fluctuations in the base spreading resistance which are converted into a noise voltage due to the flow of a base current. In junction FETs, it occurs in the channel at low temperatures. Generation–recombination causes fluctuations of the carrier density in the channel, which gives rise to a noise voltage when a drain current flows. In silicon junction FETs, the effect occurs below 100 K. In germanium junction FETs, it occurs at lower temperatures. The effect does not occur in MOS FETs.

18.11 Noise Bandwidth

The *noise bandwidth* of a filter is the bandwidth of an ideal filter having a constant passband gain which passes the same rms noise voltage, where the input signal is white noise [1–7]. The filter and the ideal filter are assumed to have the same gains. Let $A_v(f)$ be the complex voltage gain transfer function of a filter, where f is the frequency in Hz. Its noise bandwidth B_n is given by:

$$B_n = \frac{1}{A_{vo}^2} \int_0^\infty \left| A_v(f) \right|^2 df \tag{18.13}$$

where A_{vo} is the maximum value of $|A_v(f)|$. For a white noise input voltage with the spectral density $S_v(f)$, the mean-square noise voltage at the filter output is $\overline{v_{no}^2} = A_{vo}^2 S_v(f) B_n$.

Two classes of low-pass filters are commonly used in making noise measurements. The first has n real poles, all with the same frequency, having the magnitude-squared transfer function given by:

$$\left| A_v(f) \right|^2 = \frac{A_{vo}^2}{\left[1 + \left(f/f_0 \right)^2 \right]^n} \tag{18.14}$$

where f_0 is the pole frequency. The second is an n-pole Butterworth filter having the magnitude-squared transfer function given by:

$$\left| A_v(f) \right|^2 = \frac{A_{vo}^2}{1 + \left(f/f_3 \right)^{2n}} \tag{18.15}$$

where f_3 is the –3 dB frequency. Table 18.1 gives the noise bandwidths as a function of the number of poles n for $1 \le n \le 5$. For the real-pole filter, B_n is given as a function of both f_0 and f_3. For the Butterworth filter, B_n is given as a function of f_3.

TABLE 18.1 Noise Bandwidth B_n of Low-Pass Filters

Number of poles	Slope dB/dec	Real pole B_n		Butterworth B_n
1	20	1.571 f_0	1.571 f_3	1.571 f_3
2	40	0.785 f_0	1.220 f_3	1.111 f_3
3	60	0.589 f_0	1.155 f_3	1.042 f_3
4	80	0.491 f_0	1.129 f_3	1.026 f_3
5	100	0.420 f_0	1.114 f_3	1.017 f_3

18.12 Noise Bandwidth Measurement

The noise bandwidth of a filter can be measured with a white noise source with a known voltage spectral density $S_v(f)$. Let $\overline{v_n^2}$ be the mean-square noise output voltage from the filter when it is driven by the noise source. The noise bandwidth is given by:

$$B_n = \frac{\overline{v_o^2}}{A_{vo}^2 S_v(f)} \tag{18.16}$$

If the spectral density of the source is not known, the noise bandwidth can be determined if another filter with a known noise bandwidth is available. With both filters driven simultaneously, let $\overline{v_{o1}^2}$ and $\overline{v_{o2}^2}$ be the two mean-square noise output voltages, B_{n1} and B_{n2} the two noise bandwidths, and A_{vo1} and A_{vo2} the two maximum gain magnitudes. The noise bandwidth B_{n2} is given by:

$$B_{n2} = B_{n1} \frac{\overline{v_{o2}^2}}{\overline{v_{o1}^2}} \left(\frac{A_{vo1}}{A_{vo2}} \right)^2 \tag{18.17}$$

The white noise source should have an output impedance that is low enough so that the loading effect of the filters does not change the spectral density of the source.

18.13 Spot Noise

Spot noise is the rms noise in a band divided by the square root of the noise bandwidth. For a noise voltage, it has the units $V/\sqrt{Hz}$, which is read "volts per root Hz." For a noise current, the units are $A/\sqrt{Hz}$. For white noise, the spot noise in any band is equal to the square root of the spectral density. Spot noise measurements are usually made with a bandpass filter having a bandwidth that is small enough so that the input spectral density is approximately constant over the filter bandwidth. The spot noise voltage at a filter output is given by $\sqrt{(\overline{v_{no}^2}/B_n)}$, where $\overline{v_{no}^2}$ is the mean-square noise output voltage and B_n is the filter noise bandwidth. The spot noise voltage at the filter input is obtained by dividing the output voltage by A_{vo}, where A_{vo} is the maximum value of $|A_v(f)|$.

A filter that is often used for spot noise measurements is a second-order bandpass filter. The noise bandwidth is given by $B_n = \pi B_3/2$, where B_3 is the –3 dB bandwidth. A single-pole high-pass filter having a pole frequency f_1 cascaded with a single-pole low-pass filter having a pole frequency f_2 is a special case of bandpass filter having two real poles. Its noise bandwidth is given by $B_n = \pi(f_1 + f_2)/2$. The –3 dB bandwidth in this case is $f_1 + f_2$, not $f_2 - f_1$.

18.14 Addition of Noise Voltages

Consider the instantaneous voltage $v = v_n + i_n R$, where v_n is a noise voltage and i_n is a noise current. The mean-square voltage is calculated as follows:

$$\overline{v^2} = \overline{\left(v_n + i_n R\right)^2} = \overline{v_n^2} + 2\rho\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}}\,R + \overline{i_n^2}R^2 \tag{18.18}$$

where ρ is the *correlation coefficient* defined by:

$$\rho = \frac{\overline{v_n i_n}}{\sqrt{\overline{v_n^2}}\sqrt{\overline{v_n^2}}} \tag{18.19}$$

For the case $\rho = 0$, the sources are said to be uncorrelated or independent. It can be shown that $-1 \le \rho \le 1$.

In ac circuit analysis, noise signals are often represented by phasors. The square magnitude of the phasor represents the mean-square value at the frequency of analysis. Consider the phasor voltage $V = V_n + I_n Z$, where V_n is a noise phasor voltage, I_n is a noise phasor current, and $Z = R + jX$ is a complex impedance. The mean-square voltage is given by:

$$\begin{aligned}\overline{v^2} &= \overline{\left(V_n + I_n R\right)\left(V_n^* + I_n^* Z^*\right)} \\ &= \overline{v_n^2} + 2\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}}\,\mathrm{Re}\left(\gamma Z^*\right) + \overline{i_n^2}\left|Z\right|^2\end{aligned} \tag{18.20}$$

where the * denotes the complex conjugate and γ is the *complex correlation coefficient* defined by:

$$\gamma = \gamma_r + j\gamma_i = \frac{\overline{V_n I_n^*}}{\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}}} \tag{18.21}$$

It can be shown that $|\gamma| \le 1$.

Noise equations derived by phasor analysis can be converted easily into equations for real signals. However, the procedure generally cannot be done in reverse. For this reason, noise formulas derived by phasor analysis are the more general form.

18.15 Correlation Impedance and Admittance

The *correlation impedance* Z_γ and *correlation admittance* Y_γ between a noise phasor voltage V_n and a noise phasor current I_n are defined by [8]:

$$Z_\gamma = R_\gamma + jX_\gamma = \gamma\sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}} \tag{18.22}$$

$$Y_\gamma = G_\gamma + jB_\gamma = \gamma^*\sqrt{\frac{\overline{i_n^2}}{\overline{v_n^2}}} \tag{18.23}$$

where $\overline{v_n^2}$ is the mean-square value of V_n, $\overline{i_n^2}$ is the mean-square value of I_n, and γ is the complex correlation coefficient between V_n and I_n. With these definitions, it follows that $\overline{V_n I_n^*} = \overline{i_n^2}\, Z_\gamma = \overline{v_n^2}\, Y_\gamma^*$.

18.16 The $v_n - i_n$ Amplifier Noise Model

The noise generated by an amplifier can be modeled by referring all internal noise sources to the input [1–7], [11]. In order for the noise sources to be independent of the source impedance, two sources are required — a series voltage source and a shunt current source. In general, the sources are correlated.

Figure 18.1 shows the amplifier noise model, where V_s is the source voltage, $Z_s = R_s + jX_s$ is the source impedance, V_{ts} is the thermal noise voltage generated by R_s, $A_v = V_o/V_i$ is the complex voltage gain, and Z_i is the input impedance. The output voltage is given by:

$$V_o = \frac{A_v Z_i}{Z_s + Z_i}\left(V_s + V_{ts} + V_n + I_n Z_s\right) \tag{18.24}$$

The *equivalent noise input voltage* is the voltage in series with V_s that generates the same noise voltage at the output as all noise sources in the circuit. It is given by $V_{ni} = V_{ts} + V_n + I_n Z_s$. The mean-square value is:

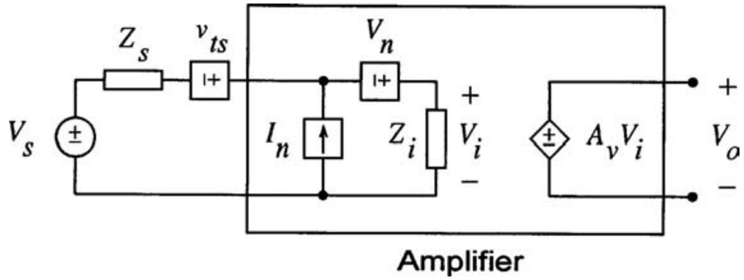

FIGURE 18.1 Amplifier $v_n - i_n$ noise model.

$$\overline{v_{ni}^2} = 4kTR_s B_n + \overline{v_n^2} + 2\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}}\,\mathrm{Re}\left(\gamma Z_s^*\right) + \overline{i_n^2}\left|Z_s\right|^2 \tag{18.25}$$

where B_n is the amplifier noise bandwidth and γ is the complex correlation between V_n and I_n. For $|Z_s|$ very small, $\overline{v_{ni}^2} \simeq \overline{v_n^2}$ and γ is not important. Similarly, for $|Z_s|$ very large, $\overline{v_{ni}^2} \simeq \overline{i_n^2}\,|Z_s|^2$ and γ is again not important.

When the source is represented by a Norton equivalent consisting of a source current i_s in parallel with a source admittance $Y_s = G_s + jB_s$, the noise referred to the input must be represented by an *equivalent noise input current.* The mean-square value is given by:

$$\overline{i_{ni}^2} = 4kTG_s B_n + \overline{v_n^2}\left|Y_2\right|^2 + 2\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}}\,\mathrm{Re}\left(\gamma Y_s\right) + \overline{i_n^2} \tag{18.26}$$

18.17 Measuring $\overline{v_{ni}^2}$, $\overline{v_n^2}$, and $\overline{i_n^2}$

For a given Z_s, the mean-square equivalent noise input voltage can be measured by setting $V_s = 0$ and measuring the mean-square noise output voltage $\overline{v_{no}^2}$. It follows that $\overline{v_{ni}^2}$ is given by:

$$\overline{v_{ni}^2} = \frac{\overline{v_{no}^2}}{\left|A_v\right|^2} \times \left|1 + \frac{Z_s}{Z_i}\right|^2 \tag{18.27}$$

To measure $\overline{v_n^2}$ and $\overline{i_n^2}$, $\overline{v_{no}^2}$ is measured with $Z_s = 0$ and with Z_s replaced by a large-value resistor. It follows that $\overline{v_n^2}$ and $\overline{i_n^2}$ are then given by:

$$\overline{v_n^2} = \frac{\overline{v_{no}^2}}{\left|A_v\right|^2} \quad \text{for } Z_s = 0 \tag{18.28}$$

$$\overline{i_n^2} = \left|\frac{1}{R_s} + \frac{1}{Z_i}\right|^2 \frac{\overline{v_{no}^2}}{\left|A_v\right|^2} \quad \text{for } Z_s = R_s \text{ and } R_s \text{ large} \tag{18.29}$$

18.18 Noise Temperature

The internal noise generated by an amplifier can be expressed as an equivalent *input-termination noise temperature* [12]. This is the temperature of the source resistance that generates a thermal noise voltage equal to the internal noise generated in the amplifier when referred to the input. The noise temperature T_n is given by:

$$T_n = \frac{\overline{v_{ni}^2}}{4kR_s B_n} - T \tag{18.30}$$

where $\overline{v_{ni}^2}$ is the mean-square equivalent input noise voltage in the band B_n, R_s is the real part of the source output impedance, and T is the temperature of R_s.

18.19 Noise Reduction with a Transformer

Let a transformer be connected between the source and the amplifier in Figure 18.1. Let n be the transformer turns ratio, R_1 the primary resistance, and R_2 the secondary resistance. The equivalent noise input voltage in series with the source voltage V_s has the mean-square value:

$$\begin{aligned}\overline{v_{ni}^2} &= 4kT\left(R_s + R_1 + \frac{R_2}{n^2}\right)\Delta f + \frac{\overline{v_n^2}}{n^2} \\ &\quad + 2\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}}\ \mathrm{Re}\left[\gamma\left(Z_s^* + R_1 + \frac{R_2}{n^2}\right)\right] + n^2\overline{i_n^2}\left|Z_s + R_1 + \frac{R_2}{n^2}\right|^2\end{aligned} \tag{18.31}$$

In general, $R_2/R_1 \propto n$, which makes it difficult to specify the value of n that minimizes $\overline{v_{ni}^2}$. For $R_1 + R_2/n^2 \ll |Z_s|$, it is minimized when:

$$n^2 = \frac{1}{|Z_s|}\sqrt{\frac{\overline{v_n^2}}{\overline{i_n^2}}} \tag{18.32}$$

18.20 The Signal-to-Noise Ratio

The *signal-to-noise ratio* of an amplifier is defined by:

$$\mathrm{SNR} = \frac{\overline{v_{so}^2}}{\overline{v_{no}^2}} \tag{18.33}$$

where $\overline{v_{so}^2}$ is the mean-square signal output voltage and $\overline{v_{no}^2}$ is the mean-square noise output voltage. The SNR is often expressed in dB with the equation SNR=10log $(\overline{v_{so}^2}/\overline{v_{no}^2})$. In measuring the SNR, a filter should be used to limit the bandwidth of the output noise to the signal bandwidth of interest. An alternative definition of the SNR that is useful in making calculations is:

$$\mathrm{SNR} = \frac{\overline{v_s^2}}{\overline{v_{ni}^2}} \tag{18.34}$$

where $\overline{v_s^2}$ is the mean-square signal input voltage and $\overline{v_{ni}^2}$ is the mean-square equivalent noise input voltage.

18.21 Noise Factor and Noise Figure

The *noise factor* F of an amplifier is defined by [1–8]:

$$F = \frac{\overline{v_{no}^2}}{\overline{v_{nos}^2}} \tag{18.35}$$

where $\overline{v_{\mathrm{no}}^2}$ is the mean-square noise output voltage with the source voltage zeroed and $\overline{v_{\mathrm{nos}}^2}$ is the mean-square noise output voltage considering the only source of noise to be the thermal noise generated by the source resistance R_{s}. The *noise figure* is the noise factor expressed in dB and is given by:

$$\mathrm{NF} = 10\log F \tag{18.36}$$

For the $v_{\mathrm{n}} - i_{\mathrm{n}}$ amplifier noise model, the noise factor is given by:

$$F = \frac{\overline{v_{\mathrm{ni}}^2}}{4kT R_{\mathrm{s}} B_{\mathrm{n}}} = 1 + \frac{\overline{v_{\mathrm{n}}^2} 2\sqrt{\overline{v_{\mathrm{n}}^2}}\sqrt{\overline{i_{\mathrm{n}}^2}}\,\mathrm{Re}\left(\gamma Z_{\mathrm{s}}^*\right) + \overline{i_{\mathrm{n}}^2}\left|Z_{\mathrm{s}}\right|^2}{4kT R_{\mathrm{s}} B_{\mathrm{n}}} \tag{18.37}$$

where B_{n} is the amplifier noise bandwidth. The value of Z_{s} that minimizes the noise figure is called the *optimum source impedance* and is given by:

$$Z_{\mathrm{so}} = R_{\mathrm{so}} + jX_{\mathrm{so}} = \left[\sqrt{1-\gamma_{\mathrm{i}}^2} - j\gamma_{\mathrm{i}}\right]\sqrt{\frac{\overline{v_{\mathrm{n}}^2}}{\overline{i_{\mathrm{n}}^2}}} \tag{18.38}$$

where $\gamma_{\mathrm{i}} = \mathrm{Im}\,(\gamma)$. The corresponding value of F is denoted by F_0 and is given by:

$$F_0 = 1 + \frac{\sqrt{\overline{v_{\mathrm{n}}^2}}\sqrt{\overline{i_{\mathrm{n}}^2}}}{2kTB_{\mathrm{n}}}\left(\gamma_r + j\sqrt{1-\gamma_{\mathrm{i}}^2}\right) \tag{18.39}$$

It follows that F can be expressed in terms of F_0 as follows:

$$F = F_0 + \frac{G_{\mathrm{n}}}{R_{\mathrm{ns}}}\left[\left(R_{\mathrm{s}} - R_{\mathrm{so}}\right)^2 + \left(X_{\mathrm{s}} - X_{\mathrm{so}}\right)^2\right] \tag{18.40}$$

where G_{n} is the noise conductance of I_{n} and R_{ns} is the noise resistance of the source. These are given by:

$$G_{\mathrm{n}} = \frac{\overline{i_{\mathrm{n}}^2}}{4kT_0 B_{\mathrm{n}}} \tag{18.41}$$

$$R_{\mathrm{ns}} = \frac{\overline{v_{\mathrm{ts}}^2}}{4kT_0 B_{\mathrm{n}}} = \frac{T R_{\mathrm{s}}}{T_0} \tag{18.42}$$

When the source is represented by a Norton equivalent consisting of a source current i_{s} in parallel with a source admittance $Y_{\mathrm{s}} = G_{\mathrm{s}} + jB_{\mathrm{s}}$, the *optimum source admittance* is given by:

$$Y_{\mathrm{so}} = G_{\mathrm{so}} + jB_{\mathrm{so}} = \left[\sqrt{1-\gamma_{\mathrm{i}}^2} + j\gamma_{\mathrm{i}}\right]\sqrt{\frac{\overline{i_{\mathrm{n}}^2}}{\overline{v_{\mathrm{n}}^2}}} \tag{18.43}$$

which is the reciprocal of Z_{so}. The noise factor can be written as:

$$F = F_0 + \frac{R_{\mathrm{n}}}{G_{\mathrm{ns}}}\left[\left(G_{\mathrm{s}} - G_{\mathrm{so}}\right)^2 + \left(B_{\mathrm{s}} - B_{\mathrm{so}}\right)^2\right] \tag{18.44}$$

where R_{n} is the noise resistance of V_{n} and G_{ns} is the noise conductance of the source. These are given by:

$$R_{\mathrm{n}} = \frac{\overline{v_{\mathrm{n}}^2}}{4kT_0B_{\mathrm{n}}} \tag{18.45}$$

$$G_{\mathrm{ns}} = \frac{\overline{i_{\mathrm{ts}}^2}}{4kT_0B_{\mathrm{n}}} = \frac{TG_{\mathrm{s}}}{T_0} \tag{18.46}$$

18.22 Noise Factor Measurement

The noise factor can be measured with a calibrated white noise source driving the amplifier. The source output impedance must equal the value of Z_s for which F is to be measured. The source temperature must be the standard temperature T_0. First, measure the amplifier noise output voltage over the band of interest with the source voltage set to zero. For the amplifier model of Figure 18.1, the mean-square value of this voltage is given by:

$$\overline{v_{\mathrm{no1}}^2} = \left|\frac{A_{\mathrm{vo}}Z_{\mathrm{i}}}{Z_{\mathrm{s}}+Z_{\mathrm{i}}}\right|^2 \left[4kT_0R_{\mathrm{s}}B_{\mathrm{n}} + \overline{v_{\mathrm{n}}^2} + 2\sqrt{\overline{v_{\mathrm{n}}^2}}\sqrt{\overline{i_{\mathrm{n}}^2}}\,\mathrm{Re}\left(\gamma Z_{\mathrm{s}}^*\right) + \overline{i_{\mathrm{n}}^2}\left|Z_{\mathrm{s}}\right|^2\right] \tag{18.47}$$

The source noise voltage is then increased until the output voltage increases by a factor r. The new mean square output voltage can be written as:

$$r^2\overline{v_{\mathrm{no1}}^2} = \left|\frac{A_{\mathrm{vo}}Z_{\mathrm{i}}}{Z_{\mathrm{s}}+Z_{\mathrm{i}}}\right|^2 \left[\left(S_{\mathrm{v}}(f) + 4kT_0R_{\mathrm{s}}\right)B_{\mathrm{n}} + \overline{v_{\mathrm{n}}^2} + 2\sqrt{\overline{v_{\mathrm{n}}^2}}\sqrt{\overline{i_{\mathrm{n}}^2}}\,\mathrm{Re}\left(\gamma Z_{\mathrm{s}}^*\right) + \overline{i_{\mathrm{n}}^2}\left|Z_{\mathrm{s}}\right|^2\right] \tag{18.48}$$

where $S_v(f)$ is the open-circuit voltage spectral density of the white noise source.

The above two equations can be solved for F to obtain:

$$F = \frac{S_{\mathrm{s}}(f)}{\left(r^2-1\right)4kT_0R_{\mathrm{s}}} \tag{18.49}$$

A common value for r is $\sqrt{2}$. The gain and noise bandwidth of the amplifier are not needed for the calculation. If a resistive voltage divider is used between the noise source and the amplifier to attenuate the input signal, the source spectral density $S_s(f)$ is calculated or measured at the attenuator output with it disconnected from the amplifier input.

If the noise bandwidth of the amplifier is known, its noise factor can be determined by measuring the mean-square noise output voltage $\overline{v_{no}^2}$ with the source voltage set to zero. The noise factor is given by:

$$F = \left|1+\frac{Z_{\mathrm{s}}}{Z_{\mathrm{i}}}\right|^2 \frac{\overline{v_{\mathrm{no}}^2}}{4kT_0R_{\mathrm{s}}B_{\mathrm{n}}A_{\mathrm{vo}}^2} \tag{18.50}$$

This expression is often used with $B_n = \pi B_3/2$, where B_3 is the –3 dB bandwidth. Unless the amplifier has a first-order low-pass or a second-order bandpass frequency response characteristic, this is only an approximation.

18.23 The Junction Diode Noise Model

When forward biased, a diode generates both shot noise and flicker noise [1–7]. The noise is modeled by a parallel current source having the mean-square value:

$$\overline{i_n^2} = 2qI\Delta f + \frac{K_f I^m \Delta f}{f} \tag{18.51}$$

where I is the dc diode current. A plot of $\overline{i_n^2}$ vs. f for a constant Δf exhibits a slope of –10 dB/decade for low frequencies and a slope of zero for higher frequencies.

Diodes are often used as noise sources in circuits. Specially processed zener diodes are marketed as solid-state noise diodes. The noise mechanism in these is called *avalanche noise,* which is associated with the diode reverse breakdown current. For a given breakdown current, avalanche noise is much greater than shot noise in the same current.

18.24 The BJT Noise Model

Figure 18.2(a) shows the BJT noise model [1–7]. The base spreading resistance r_x is modeled as an external resistor; v_{tx} is the thermal noise generated by r_x; i_{shb}, and i_{fb}, respectively, are the shot noise and flicker noise in the base bias current I_B; and i_{shc} is the shot noise in the collector bias current I_C. The sources have the mean-square values of:

$$\overline{v_{tx}^2} = 4kTr_x\Delta f \tag{18.52}$$

$$\overline{i_{shb}^2} = 2qI_B\Delta f \tag{18.53}$$

$$\overline{i_{fb}^2} = \frac{K_f I_B^m \Delta f}{f} \tag{18.54}$$

$$\overline{i_{shc}^2} = 2qI_C\Delta f \tag{18.55}$$

Let the resistances to signal ground seen looking out of the base and the emitter, respectively, be denoted by R_1 and R_2. The mean-square equivalent noise input voltages in series with the base or the emitter that generates the same collector noise current is given by:

$$\begin{aligned}\overline{v_{ni}^2} = {} & 4kT(R_1 + r_x + R_2)\Delta f + \left(2qI_B\Delta f + \frac{K_f I_B \Delta f}{f}\right)(R_1 + r_x + R_2)^2 \\ & + 2qI_C\Delta f\left(\frac{R_1 + r_x + R_2}{\beta} + \frac{V_T}{I_C}\right)^2\end{aligned} \tag{18.56}$$

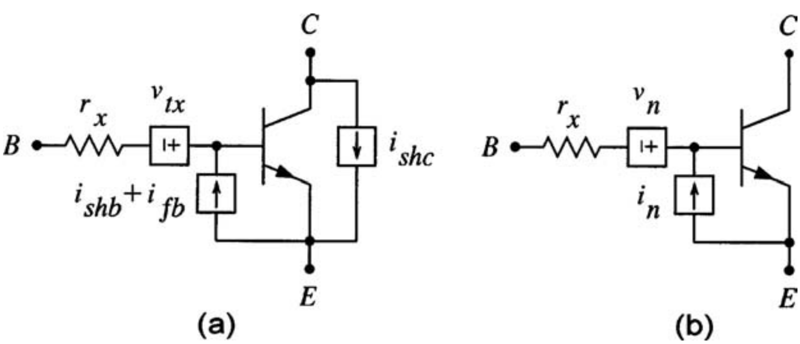

FIGURE 18.2 (a) BJT noise model. (b) BJT $v_n - i_n$ noise model.

At frequencies where flicker noise can be neglected, the value of I_C that minimizes $\overline{v_{ni}^2}$ is called the *optimum collector current.* It is given by:

$$I_{C_{opt}} = \frac{V_T}{R_1 + r_x + R_2} \times \frac{\beta}{\sqrt{1+\beta}} \tag{18.57}$$

The corresponding value of $\overline{v_{ni}^2}$ is given by:

$$\overline{v_{ni_{min}}^2} = 4kT\left(R_1 + r_x + R_2\right)\Delta f \times \frac{\sqrt{1+\beta}}{\sqrt{1+\beta}-1} \tag{18.58}$$

If N identical BJTs that are identically biased are connected in parallel, the equivalent noise input voltage is given by Equation 18.56 with r_x replaced with r_x/N, I_B replaced with NI_B, and I_C replaced with NI_C. In this case, R_1 and R_2, respectively, are the resistances to signal ground seen looking out of the parallel connected bases and the parallel connected emitters. For N fixed, the value of I_C that minimizes $\overline{v_{no}^2}$ is given by Equation 18.57 with R_1 replaced with NR_1 and R_2 replaced with NR_2. The corresponding value of $\overline{v_{ni_{min}}^2}$ is given by Equation 18.58 with r_x replaced with r_x/N. It follows that parallel connection of BJTs can be used to reduce the thermal noise of r_x, provided the devices are optimally biased.

The BJT $v_n - i_n$ noise model is given in Figure 18.2(b), where r_x is a noiseless resistor, for its thermal noise is included in v_n. The mean-square values of v_n and i_n and the correlation coefficient are given by:

$$\overline{v_n^2} = 4kTr_x\Delta f + 2kT\frac{V_T}{I_C}\Delta f \tag{18.59}$$

$$\overline{i_n^2} = 2qI_B\Delta f + \frac{K_f I_B^m \Delta f}{f} + \frac{2qI_C\Delta f}{\beta^2} \tag{18.60}$$

$$\rho = \frac{2kT\Delta f}{\beta\sqrt{\overline{v_n^2}}\sqrt{\overline{i_n^2}}} \tag{18.61}$$

where $\beta = I_C/I_B$ is the current gain. An alternative model puts r_x inside the BJT. For this model, the expressions for i_n and ρ are more complicated than the ones given here.

The $v_n - i_n$ noise model of Figure 18.1 does not have a noiseless resistor in series with its input. Before formulae that are derived for this model are applied to the BJT model of Figure 18.2(b), the formulae must be modified to account for the noiseless r_x. For the common-emitter amplifier, for example, the source resistance R_s would be replaced in the expression for $\overline{v_{ni}^2}$ by $R_s + r_x$ in all occurrences except in terms that represent the thermal noise of R_s.

The value of the base spreading resistance r_x depends on the method used to measure it. For noise calculations, r_x should be measured with a noise technique. A test circuit for measuring r_x is shown in Figure 18.3. The emitter bias current I_E and the collector bias voltage V_C are given by:

$$I_E = \frac{-V_{BE} - V_{EE}}{R_E} \tag{18.62}$$

$$V_C = V_{CC} - \alpha I_E R_C \tag{18.63}$$

where $\alpha = \beta/(1+\beta)$. Capacitors C_1 and C_2 should satisfy $C_1 \gg I_E/(2\pi f V_T)$ and $C_2 \gg 1/(2\pi f R_C)$, where f is the lowest frequency of interest. To minimize the noise contributed by R_C, R_F, and the op-amp, a low-noise op-amp should be used and R_F should be much larger than R_C. The power supply rails must be properly decoupled to minimize power supply noise.

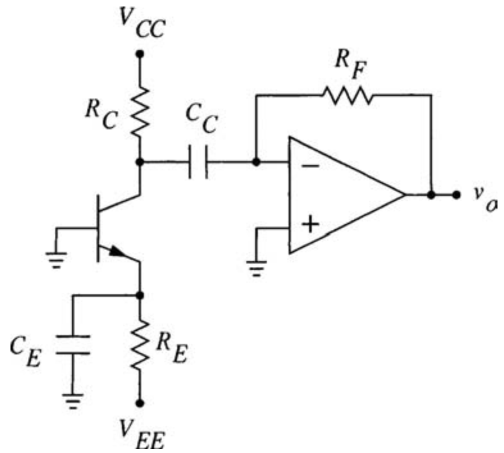

FIGURE 18.3 Test circuit for measuring r_x of a BJT.

To prevent flicker noise from affecting the data, the op-amp output voltage must be measured over a noise bandwidth where the spectral density is white. Denote the mean-square op-amp output voltage over the band B_n with the BJT in the circuit by $\overline{v_{no1}^2}$. Denote the mean-square voltage over the band B_n with the BJT removed by $\overline{v_{no2}^2}$. The base spreading resistance r_x can be obtained by solving:

$$r_x^2\left[\frac{A}{\beta^2}-2qI_BB_n\right]+r_x\left[\frac{2AV_T}{\beta I_C}-4kT\,B_n\right]+\frac{AV_T^2}{I_C^2}=0 \tag{18.64}$$

Where:

$$A=\frac{\overline{v_{no1}^2}-\overline{v_{no2}^2}}{R_F^2}-2qI_CB_n \tag{18.65}$$

The test circuit of Figure 18.3 can be used to measure the flicker noise coefficient K_f and the flicker noise exponent m. The plot of ($\overline{v_{no1}^2}$ – $\overline{v_{no2}^2}$) vs. frequency for a constant noise bandwidth Δf must be obtained, e.g., with a signal analyzer. In the white noise range, the slope of the plot is zero. In the flicker noise range, the slope is –10 dB per decade. The lower frequency at which ($\overline{v_{no1}^2}$ – $\overline{v_{no2}^2}$) is 3 dB greater than its value in the white noise range is the flicker noise corner frequency f_f. It can be shown that:

$$K_fI_B^m=\frac{\left(\overline{v_{no1}^2}-\overline{v_{no2}^2}\right)f_f}{2R_F^2\Delta f}\times\left(\frac{r_x}{\beta}+\frac{V_T}{I_C}\right)^2 \tag{18.66}$$

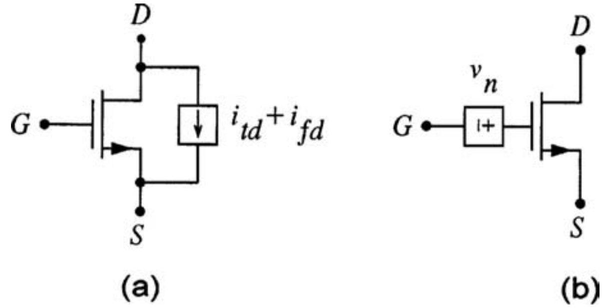

FIGURE 18.4 (a) FET noise model. (b) FET v_n noise model.

By repeating the measurements for at least two values of I_C, this equation can be used to solve for both K_f and m. Unless I_C is large, the r_x/β term can usually be neglected compared to the V_T/I_C term. The value of the flicker noise exponent is usually in the range $1 < m < 3$, but is often taken as unity. If it is assumed that $m = 1$, the value of K_f can be calculated by making the measurements with only one value of I_C.

18.25 The FET Noise Model

Figure 18.4(a) shows the MOSFET noise equivalent circuit, where i_{td} is the channel thermal noise current and i_{fd} is the channel flicker noise current [1–7]. The mean-square values of these currents are given by:

$$\overline{i_{td}^2} = \frac{8kT\Delta f}{3g_m} \tag{18.67}$$

$$\overline{i_{fd}^2} = \frac{K_f \Delta f}{4KfL^2C_{ox}} \tag{18.68}$$

where K is the transconductance parameter, $g_m = 2\sqrt{KI_D}$ is the transconductance, L is the effective length of the channel, and C_{ox} is the gate oxide capacitance per unit area.

Let the resistances to signal ground seen looking out of the gate and the source, respectively, be denoted by R_1 and R_2. The mean-square equivalent noise input voltage in series with either the gate or the source that generates the same drain noise current is given by:

$$\overline{v_{ni}^2} = 4kT(R_1 + R_2)\Delta f + \frac{4kT\Delta f}{3\sqrt{KI_D}} + \frac{K_f\Delta f}{4KL^2C_{ox}f} \tag{18.69}$$

where it is assumed that the MOSFET bulk is connected to its source in the ac circuit. If N identical MOSFETs that are identically biased are connected in parallel, the equivalent noise input voltage is given by Equation 18.69 with the exception that the second and third terms are divided by N.

The noise sources in Figure 18.4(a) can be reflected into a single source in series with the gate. The circuit is shown in Figure 18.4(b). The mean-square value of v_n is given by:

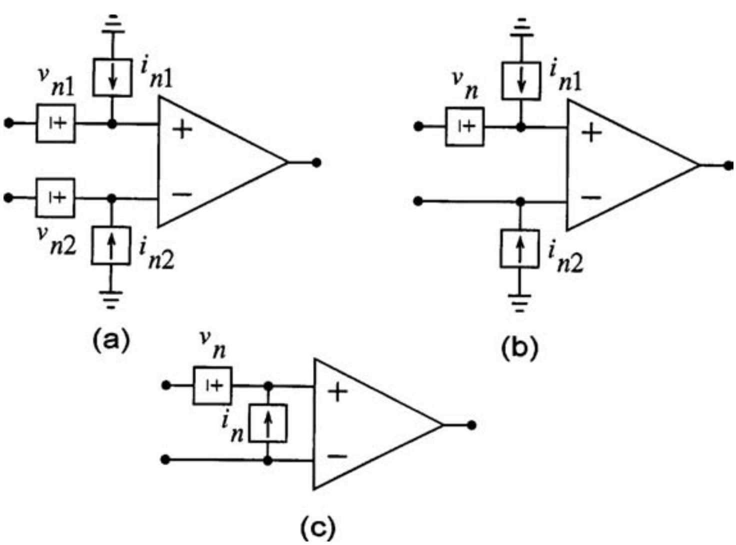

FIGURE 18.5 Op-amp noise models.

$$\overline{v_n^2} = \frac{8kT\Delta f}{3g_m} + \frac{K_f \Delta f}{4KfL^2C_{ox}} \tag{18.70}$$

The FET flicker noise coefficient K_f can be measured by replacing the BJT in Figure 18.3 with the FET. On a plot of $(\overline{v_{no1}^2} - \overline{v_{no2}^2})$ as a function of frequency for a constant noise bandwidth, the flicker noise corner frequency f_f is the lower frequency at which $(\overline{v_{no1}^2} - \overline{v_{no2}^2})$ is up 3 dB above the white noise level. A signal analyzer can be used to display this plot. The flicker noise coefficient is given by:

$$K_f = \frac{32kT\,K\,f_f L^2 C_{ox}}{3g_m} \tag{18.71}$$

The MOSFET circuits and equations also apply to the junction FET with the exception that the L_2 and C_{ox} terms are omitted from the formulae. This assumes that the junction FET gate-to-channel junction is reverse biased, which is the usual case. Otherwise, shot noise in the gate current must be modeled.

18.26 Operational Amplifier Noise Models

Variations of the $v_n - i_n$ amplifier noise model are used in specifying op-amp noise performance. The three most common models are given in Figure 18.5. In Figures 18.5(b) and (c), v_n can be placed in series with either input [4, 6]. In general, the sources in each model are correlated. In making calculations that use specified op-amp noise data, it is important to use the noise model for which the data apply.

18.27 Photodiode Detector Noise Model

Figure 18.6(a) shows the circuit symbol of a photodiode detector [4]. When reverse biased by a dc source, an incident light signal causes a signal current to flow in the diode. The diode small-signal noise model is shown in Figure 18.6(b), where i_s is the signal current (proportional to the incident light intensity), i_n is the diode noise current, r_d is the small-signal resistance of the reverse-biased junction, c_d is the small-signal junction capacitance, r_c is the cell resistance (typically <50 Ω), and v_{tc} is the thermal noise voltage generated by r_c. The noise current i_n consists of three components: shot noise i_{sh}, flicker noise i_f, and carrier generation–recombination noise i_{gr}. The first three have the mean-square values:

$$\overline{v_{tc}^2} = 4kTr_c\Delta f \tag{18.72}$$

$$\overline{i_{sh}^2} = 2qI_D\Delta f \tag{18.73}$$

$$\overline{i_f^2} = \frac{K_f I_D^m \Delta f}{f} \tag{18.74}$$

where I_D is the reverse-biased diode current. The carrier generation–recombination noise has a white spectral density up to a frequency determined by the carrier lifetime. Because the detector has a large output resistance, it should be used with amplifiers that exhibit a low input current noise.

18.28 Piezoelectric Transducer Noise Model

Figure 18.7(a) shows the circuit symbol of a piezoelectric transducer [4]. This transducer generates an electric voltage when a mechanical force is applied between two of its surfaces. An approximate equivalent circuit that is valid for frequencies near the transducer mechanical resonance is shown in Figure 18.7(b). In this circuit, C_e represents the transducer electric capacitance, while C_s, L_s, and R_s are chosen to have

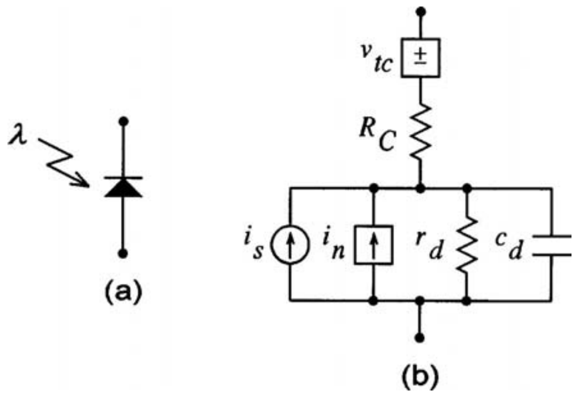

FIGURE 18.6 (a) Photodiode symbol. (b) Small-signal noise model of photodiode.

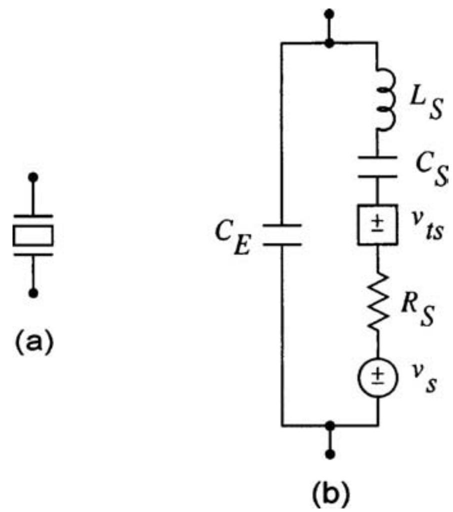

FIGURE 18.7 (a) Piezoelectric transducer symbol. (b) Noise model of piezoelectric transducer.

a resonant frequency and quality factor numerically equal to those of the transducer mechanical resonance. The source v_s represents the signal voltage, which is proportional to the applied force. The source v_{ts} represents the thermal noise generated by R_s. It has a mean-square value of:

$$\overline{v_{ts}^2} = 4kTR_s\Delta f \tag{18.75}$$

This noise component is negligible in most applications.

The piezoelectric transducer has two resonant frequencies: a short-circuit resonant frequency f_{sc} and an open-circuit resonant frequency f_{oc} given by:

$$f_{sc} = \frac{1}{2\pi\sqrt{L_s C_s}} \tag{18.76}$$

$$f_{oc} = \frac{1}{2\pi\sqrt{L_1 C_1}} \tag{18.77}$$

where $C_1 = C_s C_e/(C_s + C_e)$. It is normally operated at the open-circuit resonant frequency where the transducer output impedance is very high. For this reason, it is should be used with amplifiers that exhibit a low input current noise.

18.29 Parametric Amplifiers

A *parametric amplifier* is an amplifier that uses a time varying reactance to produce amplification [13]. In low-noise microwave parametric amplifiers, a reverse biased *pn* junction diode is used to realize a variable capacitance. Such diodes are called *varactors,* for variable reactance. The depletion capacitance of the reverse-biased junction is varied by simultaneously applying a signal current and a pump current at different frequencies. The nonlinear capacitance causes frequency mixing to occur between the signal frequency and the pump frequency. When the power generated by the frequency mixing exceeds the signal input power, the diode appears to have a negative resistance and signal amplification occurs. The only noise that is generated is the thermal noise of the effective series resistance of the diode, which is very small.

Figure 18.8 shows a block diagram of a typical parametric amplifier. The varactor diode is placed in a resonant cavity. A circulator is used to isolate the diode from the input and output circuits. A pump signal is applied to the diode to cause its capacitance to vary at the pump frequency. The filter isolates the pump signal from the output circuit. The idler circuit is a resonant cavity that is coupled to the diode cavity to reduce the phase sensitivity. Let the signal frequency be f_s, the pump frequency be f_p, and the resonant frequency of the idler cavity be f_i. In cases where $f_p = f_s + f_i$, the varying capacitance of the diode looks like a negative resistance and the signal is amplified. If $f_i = f_s$, the amplifier is called a *degenerate* amplifier. This is the simplest form of the parametric amplifier and it requires the lowest pump frequency and power to operate. For the *nondegenerate* amplifier, $f_p > 2f_s$. In both cases, the input and the output are at the same frequency. In the *up-converter* amplifier, $f_p = f_i - f_s$ and $f_p > 2f_s$. In this case, the varying capacitance of the diode looks like a positive resistance and the signal frequency output is not amplified. However, there is an output at the idler frequency that is amplified. Thus, the output frequency is higher than the input frequency. The conversion gain can be as high as the ratio of the output frequency to the input frequency.

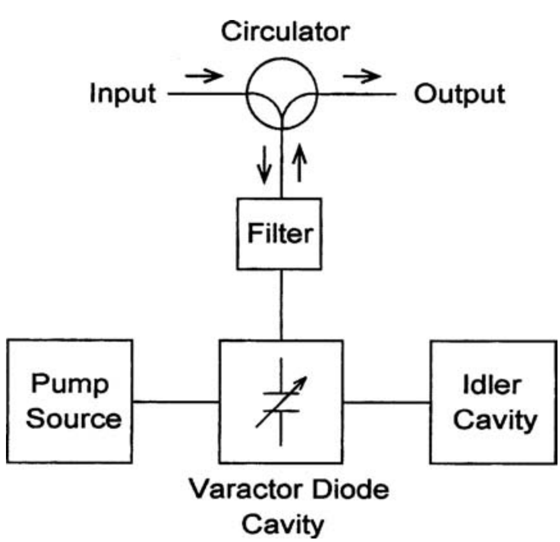

FIGURE 18.8 Block diagram of a typical parametric amplifier.

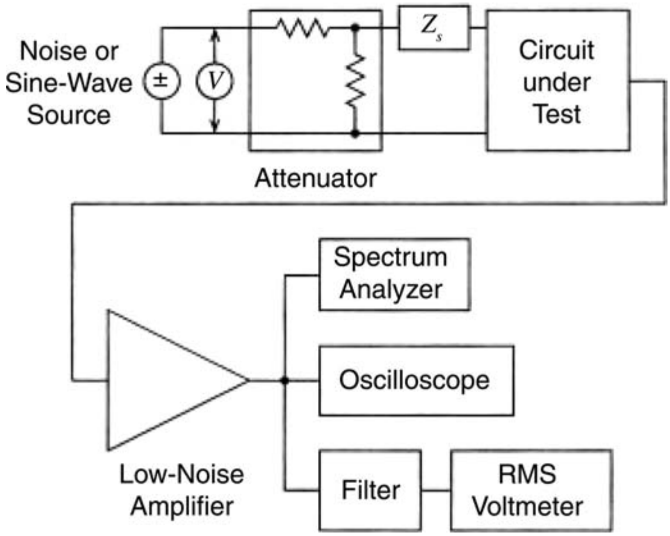

FIGURE 18.9 Noise measuring setup.

18.30 Measuring Noise

A typical setup for measuring noise is shown in Figure 18.9. To prevent measurement errors caused by signals coupling in through the ground and power supply leads, the circuit under test and the test set must be properly grounded and good power supply decoupling must be used [6]. For measurement schemes requiring a sine-wave source, an internally shielded oscillator is preferred over a function generator. This is because function generators can introduce errors caused by radiated signals and signals coupled through the ground system.

When making measurements on a high-gain circuit, the input signal must often be attenuated. Attenuators that are built into sources might not be adequately shielded, so that errors can be introduced by radiated signals. These problems can be minimized if a shielded external attenuator is used between the source and the circuit under test. Such an attenuator is illustrated in Figure 18.9. When a high attenuation is required, a multi-stage attenuator is preferred. For proper frequency response, the attenuator might require frequency compensation [4]. Unless the load impedance on the attenuator is large compared to its output impedance, both the attenuation and the frequency compensation can be a function of the load impedance.

Figure 18.9 shows a source impedance Z_s in series with the input to the circuit under test. This impedance is in series with the output impedance of the attenuator. It must be chosen so that the circuit under test has the desired source impedance termination for the noise measurements.

Because noise signals are small, a low-noise amplifier is often required to boost the noise level sufficiently so that it can be measured. Such an amplifier is shown in Figure 18.9. The noise generated by the amplifier will add to the measured noise. To correct for this, first measure the mean-square noise voltage with the amplifier input terminated in the output impedance of the circuit under test. Then subtract this from the measured mean-square noise voltage with the circuit under test driving the amplifier. The difference is the mean-square noise due to the circuit. Ideally, the amplifier should have no effect on the measured noise.

The noise voltage over a band can be measured with either a spectrum analyzer or with a filter having a known noise bandwidth and a voltmeter. The noise can be referred to the input of the circuit under test by dividing by the total gain between its input and the measuring device. The measuring voltmeter should have a bandwidth that is at least 10 times the noise bandwidth of the filter. The *voltmeter crest factor* is the ratio of the peak input voltage to the full-scale rms meter reading at which the internal meter circuits overload. For a sine-wave signal, the minimum voltmeter crest factor is $\sqrt{2}$. For noise measurements, a higher crest factor is required. For gaussian noise, a crest factor of 3 gives an error less than 1.5%. A crest factor of 4 gives an error less than 0.5%. To avoid overload on noise peaks caused by an inadequate crest factor, measurements should be made on the lower one-third to one-half of the voltmeter scale.

A true rms voltmeter is preferred over one that responds to the average rectified value of the input voltage but is calibrated to read rms. When the latter type of voltmeter is used to measure noise, the reading will be low. For gaussian noise, the reading can be corrected by multiplying the measured voltage by 1.13. Noise voltages measured with a spectrum analyzer must also be corrected by the same factor if the spectrum analyzer responds to the average rectified value of the input voltage but is calibrated to read rms.

Noise measurements with a spectrum analyzer require a knowledge of the noise bandwidth of the instrument. For a conventional analyzer, the bandwidth is proportional to frequency. When white noise is analyzed, the display exhibits a slope of +10 dB per decade. However, the measured voltage level at any frequency divided by the square root of the noise bandwidth of the analyzer is a constant equal to the spot-noise value of the input voltage at that frequency. Bandpass filters that have a bandwidth proportional to the center frequency are called *constant-Q filters.* For a second-order constant-Q filter, the noise bandwidth is given by $B_n = \pi f_0/2Q$, where f_0 is the center frequency and Q is the quality factor. The latter is given by $Q = f_0/B_3$, where B_3 is the –3 dB bandwidth. These equations are often used to estimate the noise bandwidth of bandpass filters that are not second order.

A second type of spectrum analyzer is called a *signal analyzer.* Such an instrument uses digital signal processing techniques to calculate the spectrum of the input signal as a discrete Fourier transform. The noise bandwidth of these instruments is a constant so that the display exhibits a slope of zero when white noise is the input signal.

Fairly accurate rms noise measurements can be made with an oscilloscope. A filter should be used to limit the noise bandwidth at its input. Although the procedure is subjective, the rms voltage can be estimated by dividing the observed peak-to-peak voltage by 6 [4]. One of the advantages of using the oscilloscope is that nonrandom noise that can affect the measurements can be identified, e.g., a 60 Hz hum signal.

Another oscilloscope method is to display the noise simultaneously on both inputs of a dual-channel oscilloscope that is set in the dual-sweep mode. The two channels must be identically calibrated and the sweep rate must be set low enough so that the displayed traces appear as bands. The vertical offset between the two bands is adjusted until the dark area between them just disappears. The rms noise voltage is then measured by grounding the two inputs and reading the vertical offset between the traces.

Defining Terms

Burst noise: Noise caused by a metallic impurity in a *pn* junction that sounds like corn popping when amplified and reproduced by a loudspeaker. Also called *popcorn noise.*

Crest factor: The ratio of the peak value to the rms value.

Equivalent noise input current: The noise current in parallel with an amplifier input that generates the same noise voltage at its output as all noise sources in the amplifier.

Equivalent noise input voltage: The noise voltage in series with an amplifier input that generates the same noise voltage at its output as all noise sources in the amplifier.

Equivalent noise resistance (conductance): The value of a resistor (conductance) at the standard temperature $T_0 = 290$ K that generates the same mean-square noise voltage (current) as a source.

Excess noise: Flicker noise in resistors.

Flicker noise: Noise generated by the imperfect contact between two conducting materials causing the conductivity to fluctuate in the presence of a dc current. Also called *contact noise, 1/f noise,* and *pink noise.*

Generation–recombination noise: Noise generated in a semiconductor by the random fluctuation of free carrier densities caused by spontaneous fluctuations in the generation, recombination, and trapping rates.

Noise bandwidth: The bandwidth of an ideal filter having a constant passband gain that passes the same rms noise voltage as a filter, where the input signal is white noise.

Noise factor: The ratio of the mean-square noise voltage at an amplifier output to the mean-square noise voltage at the amplifier output considering the thermal noise of the input termination to be the only source of noise.

Noise figure: The noise factor expressed in dB.

Noise index: The number of μA of excess noise current in each decade of frequency per A of dc current through a resistor. Also, the number of μV of excess noise voltage in each decade of frequency per V of dc voltage across a resistor.

Noise temperature: The internal noise generated by an amplifier expressed as an equivalent input-termination noise temperature.

Nyquist formula: Expression for the mean-square thermal noise voltage generated by a resistor.

Optimum source impedance (admittance): The complex source impedance (admittance) that minimizes the noise factor.

Parametric amplifier: An amplifier that uses a time-varying reactance to produce amplification.

Partition noise: Noise generated by the statistical process of partition when the charge carriers in a current have the possibility of dividing between two or more paths.

Shot noise: Noise caused by the random emission of electrons and by the random passage of charge carriers across potential barriers.

Schottky formula: Expression for the mean-square shot noise current.

Signal-to-noise ratio: The ratio of the mean-square signal voltage to the mean-square noise voltage at an amplifier output.

Spectral density: The mean-square value per unit bandwidth of a noise signal.

Spot noise: The rms noise in a band, divided by the square root of the noise bandwidth.

Thermal noise: Noise generated by the random collision of charge carriers with a lattice under conditions of thermal equilibrium. Also called *Johnson noise.*

Varactor diode: A diode used as a variable capacitance.

Voltmeter crest factor: The ratio of the peak input voltage to the full-scale rms meter reading at which the internal meter circuits overload.

White noise: Noise that has a spectral density that is flat, i.e., not a function of frequency.

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