7

Electrical Conductivity and Resistivity

Michael B. Heaney
Palo Alto Research Center

Electrical resistivity is a key physical property of all materials. It is often necessary to accurately measure the resistivity of a given material. The electrical resistivity of different materials at room temperature can vary by over 20 orders of magnitude. No single technique or instrument can measure resistivities over this wide range. This chapter describes a number of different experimental techniques and instruments for measuring resistivities. The emphasis is on explaining how to make practical measurements and avoid common experimental errors. More theoretical and detailed discussions can be found in the sources listed at the end of this chapter.

7.1 Basic Concepts

The *electrical resistivity* of a material is a number describing how much that material resists the flow of electricity. Resistivity is measured in units of ohm·meters (Ω m). If electricity can flow easily through a material, that material has low resistivity. If electricity has great difficulty flowing through a material, that material has high resistivity. The electrical wires in overhead power lines and buildings are made of copper or aluminum. This is because copper and aluminum are materials with very low resistivities (about 20 nΩ m), allowing electric power to flow very easily. If these wires were made of high resistivity material like some types of plastic (which can have resistivities about 1 EΩ m (1×10^{18} Ω m)), very little electric power would flow.

Electrical resistivity is represented by the Greek letter ρ. Electrical conductivity is represented by the Greek letter σ, and is defined as the inverse of the resistivity. This means a high resistivity is the same as a low conductivity, and a low resistivity is the same as a high conductivity:

$$\sigma \equiv \frac{1}{\rho} \tag{7.1}$$

This chapter will discuss everything in terms of resistivity, with the understanding that conductivity can be obtained by taking the inverse of resistivity. The electrical resistivity of a material is an intrinsic

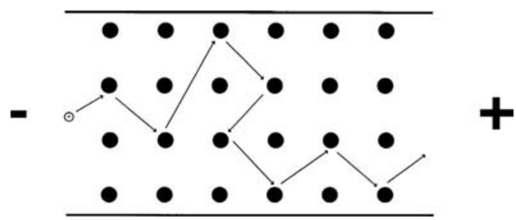

FIGURE 7.1 Simple model of electricity flowing through a material under an applied voltage. The white circle is an electron moving from left to right through the material. The black circles represent the stationary atoms of the material. Collisions between the electron and the atoms slow down the electron, causing electrical resistivity.

physical property, independent of the particular size or shape of the sample. This means a thin copper wire in a computer has the same resistivity as the Statue of Liberty, which is also made of copper.

7.2 Simple Model and Theory

Figure 7.1 shows a simple microscopic model of electricity flowing through a material [1]. While this model is an oversimplification and incorrect in several ways, it is still a very useful conceptual model for understanding resistivity and making rough estimates of some physical properties. A more correct understanding of the electrical resistivity of materials requires a thorough understanding of quantum mechanics [2].

On a microscopic level, electricity is simply the movement of electrons through a material. The smaller white circle in Figure 7.1 represents one electron flowing through the material. For ease of explanation, only one electron is shown. There are usually many electrons flowing through the material simultaneously. The electron tends to move from the left side of the material to the right side because an external force (represented by the large minus and plus signs) acts on it. This external force could be due to the voltage produced by an electrical power plant, or a battery connected to the material. As the electron moves through the material, it collides with the "stationary" atoms of the material, represented by the larger black circles. These collisions tend to slow down the electron. This is analogous to a pinball machine. The electron is like the metal ball rolling from the top to the bottom of a pinball machine, pulled by the force of gravity. The metal ball occasionally hits the pins and slows down. Just like in different pinball machines, the number of collisions the electron has can be very different in different materials. A material that produces lots of collisions is a high-resistivity material. A material that produces few collisions is a low-resistivity material.

The resistivity of a material can vary greatly at different temperatures. The resistivity of metals usually increases as temperature increases, while the resistivity of semiconductors usually decreases as temperature increases. The resistivity of a material can also depend on the applied magnetic field.

The discussion thus far has assumed that the material being measured is homogeneous and isotropic. Homogeneous means the material properties are the same everywhere in the sample. Isotropic means the material properties are the same in all directions. This is not always a valid assumption. A more exact definition of resistivity is the proportionality coefficient ρ relating a local applied electric field to the resultant current density:

$$\boldsymbol{E} \equiv \rho \boldsymbol{J} \tag{7.2}$$

where $\boldsymbol{E}$ is the electric field (V/m), $\boldsymbol{J}$ is the current density (A m^{-2}), and ρ is a proportionality coefficient (Ω m). Equation 7.2 is one form of Ohm's law. Note that $\boldsymbol{E}$ and $\boldsymbol{J}$ are vectors, and ρ is, in general, a tensor. This implies that the current does not necessarily flow in the direction of the applied electric field. In this chapter, isotropic and homogeneous materials are assumed, so ρ is a scalar (a single number).

Now consider the bar-shaped sample shown in Figure 7.2. The electric field $\boldsymbol{E}$ is given by the voltage V divided by the distance l over which the voltage is applied:

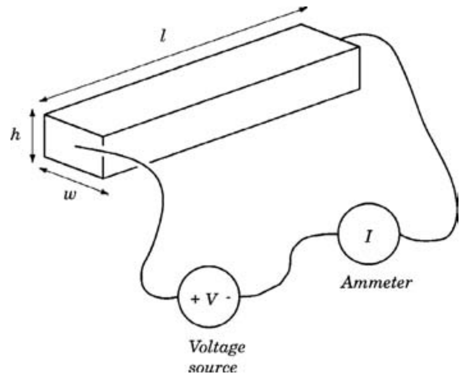

FIGURE 7.2 A two-point technique for measuring the resistivity of a bar of material. The voltage source applies a voltage across the bar, and the ammeter measures the current flowing through the bar.

$$\boldsymbol{E} \equiv \frac{V}{l} \tag{7.3}$$

The current density $\boldsymbol{J}$ is given by the current $\boldsymbol{I}$**,** divided by the cross-sectional area A through which the current flows:

$$\boldsymbol{J} \equiv \frac{\boldsymbol{I}}{A} \tag{7.4}$$

where the area A in Figure 7.2 is equal to the width w times the height h. Combining Equations 7.2, 7.3, and 7.4 and rearranging gives:

$$V = \frac{I\rho l}{A} \tag{7.5}$$

Now define a new quantity called "resistance" R with the definition:

$$R \equiv \frac{\rho l}{A} \tag{7.6}$$

Combining Equations 7.5 and 7.6 then gives:

$$I = \frac{V}{R} \tag{7.7}$$

where I is the current in amps (A) flowing through the sample, V is the voltage in volts (V) applied across the sample, and R is the resistance in ohms (Ω) of the sample. Equation 7.7 is another form of Ohm's law.

Note that the resistance R can depend on the size and shape of the sample, while ρ is independent of the size or shape of the sample. For example, if the length l of the sample bar is doubled, the resistance will double but the resistivity will remain constant.

The quantitative relationship between the resistivity ρ and the simple microscopic model shown in Figure 7.1 is given by:

$$\rho = \frac{m}{ne^2\tau} \tag{7.8}$$

where m is the mass of an electron, n is the number of electrons per unit volume carrying current in the material, e is the electric charge on an electron, and τ is the average time between collisions of an electron with the stationary atoms of the material. If there were more electrons per unit volume, they could carry more current through the material. This would lower the resistivity. If the electric charge on the electrons were greater, then the applied voltage would pull harder on the electrons, speeding them up. This would lower the resistivity. If the average time between collisions with the stationary atoms were longer, then the electrons could get through the material quicker. This would lower the resistivity. If electrons could be made more massive, they would move slower and take longer to get through the material. This would increase the resistivity.

7.3 Experimental Techniques for Measuring Resistivity

Two-Point Technique

The resistivity of a material can be obtained by measuring the resistance and physical dimensions of a bar of material, as shown in Figure 7.2. In this case, the material is cut into the shape of a rectangular bar of length l, height h, and width w. Copper wires are attached to both ends of the bar. This is called the two-point technique, since wires are attached to the material at two points. A voltage source applies a voltage V across the bar, causing a current I to flow through the bar. (Alternatively, a current source could force current through the sample bar, while a voltmeter in parallel with the current source measures the voltage induced across the sample bar.) The amount of current I that flows through the bar is measured by the ammeter, which is connected in series with the bar and voltage source. The voltage drop across the ammeter should be negligible. The resistance R of the bar is given by Equation 7.8a:

$$R = \frac{V}{I} \tag{7.8a}$$

where R = Resistance in Ω
V = Voltage in volts
I = Current in amps

The physical dimensions can be measured with a ruler, a micrometer, or other appropriate instrument. The two-point resistivity of the material is then:

$$\rho \equiv \frac{Rwh}{l} \tag{7.9}$$

where ρ is the resistivity in Ω m, R is the measured resistance in Ω, and w, h, and l are the measured physical dimensions of the sample bar in meters.

In practice, measuring resistivity with a two-point technique is often not reliable. There is usually some resistance between the contact wires and the material, or in the measuring equipment itself. These additional resistances make the resistivity of the material measure higher than it really is. A second potential problem is modulation of the sample resistivity due to the applied current. This is often a possibility for semiconducting materials. A third problem is that contacts between metal electrodes and a semiconducting sample tend to have other electrical properties that give wrong estimates for the actual sample resistivity. The four-point measurement technique overcomes many of these problems.

Four-Point Technique

Figure 7.3 shows the four-point measurement technique on a bar of material. Four wires are attached to the sample bar as shown. A current source forces a constant current through the ends of the sample bar.

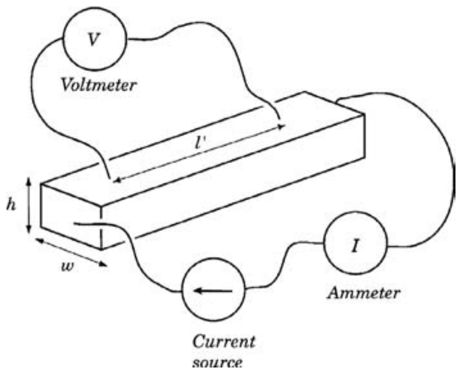

FIGURE 7.3 A four-point technique for measuring the resistivity of a bar of material. The current source forces a current through the bar, which is measured by a separate ammeter. The voltmeter measures the voltage across the middle of the bar.

A separate ammeter measures the amount of current I passing through the bar. A voltmeter simultaneously measures the voltage V produced across the inner part of the bar. (Alternatively, a voltage source could apply a voltage across the outer contacts, while an ammeter in series with this voltmeter measures the current flowing through the sample bar.)

The four-point resistivity of the material is then:

$$\rho = \frac{Vwh}{Il^1} \tag{7.10}$$

where ρ = Resistivity in Ωm
V = Voltage measured by the voltmeter in volts
w = Width of the sample bar measured in meters
h = Height of the sample bar measured in meters
I = Current the ammeter measures flowing through the sample in amperes
l^1 = Distance between the two points where the voltmeter wires make contact to the bar, measured in meters

Note that the total length l of the bar is not used to calculate the four-point resistivity: the length l^1 between the two inner contacts is used.

Common Experimental Errors

There are many experimental pitfalls to avoid when making resistivity measurements. The most common sources of error arise from doing a two-point measurement on a material that has any of the contact problems discussed earlier. For this reason, it is advisable to do four-point measurements whenever possible. This section describes experimental techniques to avoid errors in measuring resistivity:

1. The most difficult part of making resistivity measurements is often making good electric contacts to the sample. The general technique for making good electric contacts is to clean the areas of the sample where contacts are to be made with alcohol or an appropriate solvent, and then apply the contacts. If this does not work, try scraping the surface with a razor blade where contact is to be made, or cutting the sample to expose a fresh surface. Contacts can be made in many ways, such as using alligator clips, silver-paint, squeezing a wire against the material, soldering wires to the material, pressing small pieces of indium or a similar soft metal onto the contact areas, etc. Contacts can age: a good contact can become a bad contact over time. It might be necessary to make fresh contacts to a sample that has aged. There are many complications involved in the electrical

properties of contacts. Refer to the sources listed at the end of this chapter for more extensive discussions.

2. The measurement system should be calibrated before measuring any material samples. Calibration procedures are usually described in the equipment manuals.
3. The input resistance (or "impedance") of the voltmeter should be at least 10^5 higher than the resistance of the sample bar. The input impedance is usually listed in the equipment specifications. Note that some voltmeters and electrometers have a sufficiently high impedance between either of the inputs and ground, but not between the two inputs. In this case, it is necessary to use two voltmeters/electrometers (each with one input connected to ground and the other input connected to the sample bar). Measure the difference between them to obtain the voltage across the sample.
4. The measurement system should be tested before measuring any material samples. First test "short" with a thick copper wire or sheet in place of the sample. Then test "open" with nothing in place of the sample. Finally, test with a known, calibrated resistor whose resistance is within an order of magnitude of the sample resistance.
5. The geometry of the sample and electric contacts can be important. Contacts are often made by painting silver-paint or applying metal electrodes to the sample. If these contact areas are large or close to each other, this could reduce the accuracy of the resistivity measurement. It is best to make the two voltage contacts in a four-point measurement as small or thin as possible, and make the distance between inner electrodes much larger than the sample thickness. This also allows a more accurate estimate of the effective volume of the sample being probed.
6. It is critical that the four contacts to the sample bar in a four-point measurement are completely independent; there should be nothing other than the material of the bar connecting each of the four wires at the bar. For example, when pieces of indium are used to attach wires to a small sample, it is easy to smudge two adjacent indium pieces into one another. Those two contacts are no longer independent, and could easily cause errors. Visually inspect the contacts for this condition. If visual inspection is impractical, measure the resistance between the wires going to adjacent contacts. An unusually low resistance might indicate that two contacts are touching each other.
7. The applied voltage or current can cause heating of the material, which can change its resistivity. To avoid this problem, start with very small voltages or currents, and increase until the measured voltages and currents are at least 10 times larger than the random fluctuations of the meters. Then make sure the measured resistance is constant with time: the average resistance should not drift more than 10% in a few minutes.
8. Even if heating of the sample is not a problem, Ohm's law is not always obeyed. Many materials have a resistance that varies as the applied voltage varies, especially at higher voltages. Test for a linear relationship between current and voltage by measuring the resistance at several voltages on both sides of the measurement voltage. Whenever possible, make measurements in the linear (ohmic) region, where resistance is constant as voltage changes.
9. If one or both of the contacts to the voltmeter are bad, the voltmeter may effectively be disconnected from the material. In this situation, the voltmeter might display some random voltage unrelated to the voltage in the material. It might not be obvious that something is wrong, since this random voltage could accidentally appear to be a reasonable value. Check for this by setting the current source to zero amps and seeing if the voltmeter reading drops to zero volts. If it does not, try remaking the two inner contacts.
10. A critical check of a four-point measurement is to reverse the leads and remeasure the resistance. First, turn the current source to zero amps. Without disturbing any of the four contacts at the sample, swap the two sets of wires going to the voltmeter and the current source/ammeter. The two wires that originally plugged into the voltmeter should now plug into one terminal of the current source and one terminal of the ammeter. The two wires that originally plugged into the current source and ammeter should now plug into the voltmeter. Turn the current source on and remeasure the resistance. Note that current is now being forced to flow between the two inner

contact points on the sample, while the voltage is being measured between the two outer contacts on the sample. The two measured resistances should be within 10% of each other.

11. The resistivity of some materials can depend on how much light is hitting the material. This is especially a problem with semiconductors. If this is a possibility, try blocking all light from the sample during measurement.

Sheet Resistance Measurements

It is often necessary to measure the resistivities of thin films or sheets of various materials. If the material can be made into the form of a rectangle, then the resistivity can be measured just like the bar samples in Figure 7.2:

$$\rho \equiv \frac{Vwh}{Il} \tag{7.11}$$

where ρ = Sample resistivity in $\Omega\,\mathrm{m}$
V = Voltage measured by the voltmeter in volts
w = Width of the sample measured in meters
h = Thickness of the sample measured in meters
I = Current the ammeter measures flowing through the sample in amperes
l = Length of the film measured in meters

For the special case of a square film, the width w is equal to the length l, and Equation 7.11 becomes:

$$\rho(\textit{of square film}) \equiv \frac{Vh}{I} \tag{7.12}$$

The resistivity of a square film of material is called the "sheet resistivity" of the material, and is usually represented by the symbol ρ_s. The "sheet resistance" R_s is defined by:

$$R_s \equiv R(\textit{of square film}) = \frac{V}{I} \tag{7.13}$$

where V = Voltage measured by the voltmeter in volts
I = Current the ammeter measures flowing through the sample in amps

The units for sheet resistance are Ω, but people commonly use the units "Ω per square" or $\Omega/\square$. The sheet resistance is numerically equal to the measured resistance of a square piece of the material. Note that sheet resistance is independent of the size of the square measured, and it is not necessary to know the film thickness to measure sheet resistance. This makes sheet resistance a useful quantity for comparing different thin films of materials.

It is usually more convenient to measure thin-film samples of arbitrary shape and size. This is usually done by pressing four collinear, equally spaced contacts into a film whose length and width are both much greater than the spacing between contacts. In this situation, the sheet resistance is [3]:

$$R_s = 4.532\frac{V}{I} \tag{7.14}$$

where V = Voltage measured across the two inner contacts
I = Current applied through the two outer contacts

In many practical cases, the size of the thin-film sample will not be much greater than the spacing between the four-point contacts. In other cases, it might be necessary to measure a thin film near a corner or edge. In this situation, use geometric correction factors to accurately estimate the sheet resistance. These correction factors are available for the most commonly encountered sample geometries [3].

Instrumentation for Four-Point Resistivity Measurements

The resistivities of thin films of materials are often measured using commercial four-point probes. These probes generally have four equally spaced, collinear metal points that are pressed against the surface of the film. A current is applied between the outer two points, while the voltage is measured across the inner two points. These probes can also be used to measure the resistivity of bulk samples. Some companies that make probes and systems specifically for four-point resistivity measurements are listed in Table 7.1.

Instrumentation for High-Resistivity Measurements

Many materials such as rocks, plastics, and paper have very high resistivities, up to 1 EΩ m. The techniques described earlier for measuring resistivity are usually not reliable for these materials. In particular, it is often not possible to make a four-point measurement. One problem is that high voltages are needed to get any measurable current flowing through these materials. A second problem is that very long time constants prevent making steady-state measurements. A third problem is that the surfaces of these materials can often have significantly lower resistivity than the bulk, due to defects or other contamination. Measurements using the techniques described above then give falsely low values for the bulk resistivity. The best way to measure the resistivity of these materials is to use specialized commercial instruments. These are designed to separate out the bulk resistivity from the surface resistivity, and to minimize the many other problems encountered when measuring very high resistivities. Table 7.2 lists some companies that make high-resistivity measurement systems.

van der Pauw Technique

The four-point measurement technique described earlier has assumed the material sample has the shape of a rectangular thin film or a bar. There is a more general four-point resistivity measurement technique that allows measurements on samples of arbitrary shape, with no need to measure all the physical dimensions of the sample. This is the van der Pauw technique [4]. There are four conditions that must be satisfied to use this technique:

1. The sample must have a flat shape of uniform thickness.
2. The sample must not have any isolated holes.
3. The sample must be homogeneous and isotropic.
4. All four contacts must be located at the edges of the sample.

In addition to these four conditions, the area of contact of any individual contact should be at least an order of magnitude smaller than the area of the entire sample. For small samples, this might not be possible or practical. If sufficiently small contacts are not achievable, it is still possible to do accurate van der Pauw resistivity measurements, using geometric correction factors to account for the finite size of the contacts. See Ref. [5] for further details.

The inset illustration of Figure 7.4 illustrates one possible sample measurement geometry. A more common geometry is to attach four contacts to the four corners of a square-shaped sheet of the material.

The procedure for doing a van der Pauw measurement is as follows:

1. Define a resistance $R_{ij,kl} \ldots V_{kl}/I_{ij}$, where $V_{kl} \ldots V_k - V_l$ is the voltage between points k and l, and I_{ij} is the current flowing from contact i to contact j.
2. Measure the resistances $R_{21,34}$ and $R_{32,41}$. Define $R_>$ as the greater of these two resistances and $R_<$ as the lesser of these two resistances.
3. Calculate the ratio $R_>/R_<$ and find the corresponding value of the function $f(R_>/R_<)$ from Figure 7.4. Be careful to use the appropriate horizontal scale!

TABLE 7.1 Companies That Make Four-Point Resistivity Measurement Probes and Systems

Company and Comments
Creative Design Engineering, Inc. 20565 Elves Drive Cupertino, CA 95014 Tel: (408) 736-7273 Fax: (408) 738-3912
Creative Design Engineering makes manual and automatic four-point resistivity systems specially designed for both small and large semiconductor wafers.
Four Dimensions, Inc. 3138 Diablo Ave. Hayward, CA 94545 Tel: (510) 782-1843 Fax: (510-786-9321 http://www.4dimensions.com
Four Dimensions makes a variety of manual and automatic four-point probe systems for measurement of resistivity and resistivity mapping of flat samples such as semiconductor wafers.
Hewlett-Packard Company Test and Measurement Organization 5301 Stevens Creek Blvd. Santa Clara, CA 95052-8059 Tel: (800) 452-4844 Fax: (303) 754-4801 http://www.hp.com
Hewlett-Packard makes a variety of high-quality instruments useful for four-point measurements.
Jandel Engineering, Ltd. Grand Union House Leighton Road Linslade, Leighton Buzzard LU7 7LA U.K. Tel: (01525)-378554 Fax: (01525)-381945 http://www.getnet.com/~bridge/jandel.html
Jandel makes four-point probes useful for flat samples such as semiconductor wafers. They will build custom four-point probes for your particular needs. They also make a combined constant current source and digital voltmeter for resistivity measurements.
Keithley Instruments, Inc. 28775 Aurora Road Cleveland, OH 44139-1891 Tel: (440) 248-0400 Fax: (440) 248-6168 http://www.keithley.com
Keithley makes a wide variety of four-point measurement systems. They also have useful, free literature detailing techniques for making accurate resistivity measurements.
KLA-Tencor Corp. 1 Technology Drive Milpitas, CA 95035 Tel: (408) 875-3000 Fax: (408) 875-3030 http://www.kla-tencor.com
KLA-Tencor makes automated sheet resistance mapping systems designed for semiconductor wafers.

TABLE 7.1 (continued) Companies That Make Four-Point Resistivity Measurement Probes and Systems

Company and Comments
Lucas-Signatone Corp. 393-J Tomkins Ct. Gilroy, CA 95020 Tel: (408) 848-2851 Fax: (408) 848-5763 http://www.signatone.com
Signatone makes four-point resistivity measurement systems and a variety of four-point probe heads. They make a high-temperature, four-point probe head for temperatures up to 670 K.
Miller Design and Equipment, Inc. 2231-C Fortune Drive San Jose, CA 95131-1806 Tel: (408) 434-9544 Fax: (408) 943-1491
Miller Design makes semi-automatic resistivity probe systems, designed for semiconductor wafers.
Mitsubishi Chemicals Corp./Yuka Denshi Co., Ltd. Kyodo Bldg., 1-5 Nihonbashi Muromachi 4-chome Chuo-kyu, Tokyo 103 Japan Tel: 03-3270-5033 Fax: 03-3270-5036
Yuka Denshi makes a low-resistivity meter and a variety of four-point probe heads.
MMR Technologies, Inc. 1400 North Shoreline Blvd., # A5 Mountain View, CA 94043 Tel: (650) 962-9620 Fax: (650) 962-9647 http://www.mmr.com
MMR makes systems for four-point resistivity, Hall mobility, and Seebeck potential measurements over the temperature range 80 K to 400 K.
Napson Corporation Momose Bldg. 7F 2-3-6 Kameido Koto-kyu Tokyo 136 Japan QuadTech, Inc. 100 Nickerson Rd., Suite 3 Marlborough, MA 01752-9605 Tel: (800) 253-1230 Fax: (508) 485-0295 http://www.quadtechinc.com
QuadTech makes a four-point ohmmeter capable of measuring resistances from 1 μΩ to 2 MΩ.
Quantum Design 11578 Sorrento Valley Rd. San Diego, CA 92121-1311 Tel: (800) 289-6996 Fax: (619) 481-7410 http://www.quandsn.com
Quantum Design makes an automated system for measuring four-point resistivity, Hall mobility, and other properties over the temperature range 2 K to 400 K in magnetic fields up to 14 T.

TABLE 7.2 Companies That Make High-Resistivity Measurement Probes and Systems

Company and Comments
Hewlett-Packard Company Test and Measurement Organization 5301 Stevens Creek Blvd. Santa Clara, CA 95052-8059 Tel: (800) 452-4844 Fax: (303) 754-4801 http://www.hp.com
Hewlett-Packard makes high-resistance meters and specially designed resistivity test chambers.
Keithley Instruments, Inc. 28775 Aurora Road Cleveland, OH 44139-1891 Tel: (440) 248-0400 Fax: (440) 248-6168 http://www.keithley.com
Keithley makes special meters and resistivity test chambers for measuring high resistivities. They also have useful, free literature detailing techniques for making accurate resistivity measurements.
Mitsubishi Chemicals Corp./Yuka Denshi Co., Ltd. Kyodo Bldg., 1-5 Nihonbashi Muromachi 4-chome Chuo-kyu, Tokyo 103 Japan Tel: 03-3270-5033 Fax: 03-3270-5036
Yuka Denshi makes high-resistance meters and a variety of probes and resistivity test chambers.
Monroe Electronics, Inc. 100 Housel Avenue Lyndonville, NY 14098 Tel: (800) 821-6001 Fax: (716) 765-9330 http://www.monroe-electronics.com
Monroe Electronics makes portable and hand-held instruments for measuring surface resistivity, designed for testing antistatic materials.
QuadTech, Inc. 100 Nickerson Rd. Suite 3 Marlborough, MA 01752-9605 Tel: (800) 253-1230 Fax: (508) 485-0295 http://www.quadtechinc.com
QuadTech makes a high-resistance ohmmeter.

4. Calculate the resistivity ρ_a using:

$$\rho_a = \frac{\pi d(R_> + R_<) f(R_>/R_<)}{\ln 4} \tag{7.15}$$

where ρ_a = Resistivity in Ω m
d = Thickness of the sample in m
Resistances $R_>$ and $R_<$ are measured in Ω
ln4 = Approximately 1.3863

It is not necessary to measure the width or length of the sample.

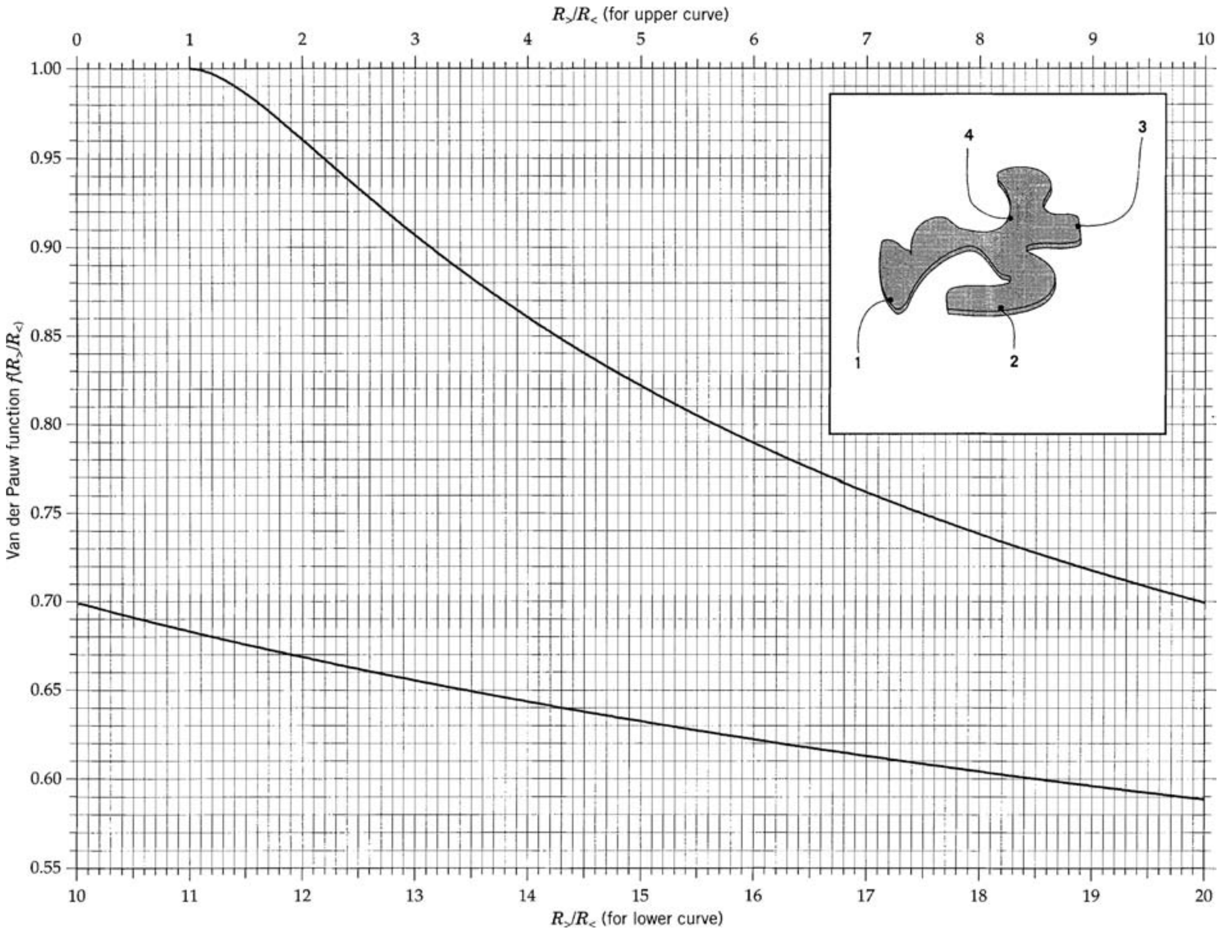

FIGURE 7.4 The van der Pauw technique. The inset shows one possible measurement geometry. The graph shows the function f $(R_>/R_<)$ needed to find the resistivity in Equation 7.15.

5. Switch the leads to measure $R_{43,12}$ and $R_{14,23}$. Repeat steps 3 and 4 to calculate ρ_b using these new values for $R_>$ and $R_<$. If the two resistivities ρ_a and ρ_b are not within 10% of each other, either the contacts are bad, or the sample is too nonuniform to measure reliably. Try making new contacts. If the two resistivities ρ_a and ρ_b are within 10% of each other, the best estimate of the material resistivity ρ is the average:

$$\rho = \frac{(\rho_a + \rho_b)}{2} \tag{7.16}$$

Note: The function $f(R_>/R_<)$ plotted in Figure 7.4 is defined by the transcendental equation:

$$f(R_>/R_<) \equiv \frac{-\ln 4(R_>/R_<)}{\left[1 + (R_>/R_<)\ln\left\{1 - 4^{-\left[(1+R_>/R_<)f\right]^{-1}}\right\}\right]} \tag{7.17}$$

Defining Terms

Conductance: The inverse of resistance.
Conductivity: The inverse of resistivity.
Contact resistance: The resistance between the surface of a material and the electric contact made to the surface.
Four-point technique: A method for measuring the resistivity of a material, using four electric contacts to the material, which avoids many contact resistance problems.
Resistance: The physical property of a particular piece of a material, quantifying the ease with which electricity can flow through it. Resistance will depend on the size and shape of the piece of material.
Resistivity: The intrinsic physical property of a material quantifying the ease with which electricity can flow through it. Resistivity will not depend on the size and shape of the piece of material. Higher resistivity means the flow of electricity is more difficult.
Sheet resistance: The resistance of a square thin film or sheet of material.
Two-point technique: A method for measuring the resistivity of a material, using two electric contacts to the material.
Van der Pauw technique: A method of measuring the four-point resistivity of an arbitrarily shaped material sample.

Acknowledgments

I thank Alison Breeze, John Clark, Kirsten R. Daehler, James M. E. Harper, Linda D. B. Kiss, Heidi Pan, and Shukri Souri for many useful suggestions.

References

1. P. Drude, Zur elektronentheorie der metalle, *Annalen der Physik,* 1, 566–613, 1900; 3, 369–402, 1900. See Ref. [2] for modern discussions of the Drude model and electrical conductivity.
2. N.W. Ashcroft and N.D. Mermin, *Solid State Physics*, Philadelphia, PA: Saunders College, 1976; C. Kittel, *Introduction to Solid State Physics,* 7th ed., New York: John Wiley & Sons, 1996.
3. L.B. Valdes, Resistivity measurements on germanium for transistors, *Proc. I.R.E.,* 42, 420–427, 1954.
4. L.J. van der Pauw, A method of measuring specific resistivity and Hall effect of discs of arbitrary shape, *Philips Res. Rep.,* 13, 1–9, 1958.
5. R. Chwang, B.J. Smith, and C.R. Crowell, Contact size effects on the van der Pauw method for resistivity and Hall coefficient measurement, *Solid-State Electron.,* 17, 1217–1227, 1974.

Further Information

H.H. Wieder, *Laboratory Notes on Electrical and Galvanomagnetic Measurements,* New York: Elsevier, 1979.

L.I. Maissel, Electrical properties of metallic thin films, 13-1 to 13-33, in L.I. Maissel and R. Glang (Eds.), *Handbook of Thin Film Technology,* San Francisco: McGraw-Hill, 1970.

D.C. Look, *Electrical Characterization of GaAs Materials and Devices,* New York: John Wiley & Sons, 1989.

D.C. Look, Bulk and contact electrical properties by the magneto-transmission-line method: application to GaAs, *Solid-State Electron.,* 30, 615–618, 1987.