9

Capacitance and Capacitance Measurements

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A capacitor is a system of two conducting electrodes, having equal and opposite charges separated by a dielectric. The capacitance C of this system is equal to the ratio of the absolute value of the charge Q to the absolute value of the voltage between bodies as:

$$C = Q/V \tag{9.1}$$

where C = Capacitance in farad (F)
Q = Charge in coulomb (C)
V = Voltage (V)

The unit of capacitance, the farad, is a large unit; practical capacitors have capacitances in mirofarads (μF or 10^{-6} F), nanofarads (nF or 10^{-9} F), and picofarads (pF or 10^{-12} F). The unit conversions are shown in Table 9.1.

The capacitance C depends on the size and shape of charged bodies and their relative positions; examples are shown in Table 9.2. Generally, capacitance is inherent wherever an electrostatic field appears. In many electronic systems, it is necessary to deal with capacitances that are designed within the circuits and also with unwanted interference and stray capacitances that are introduced externally or internally at various stages of the circuits. For example, some sensors operate on capacitive principles, giving useful signals; in others, capacitance is inherent but undesirable. In many cases, cables and external circuits introduce additional capacitances that need to be accounted for the desirable operation of the system. In these cases, Table 9.2 is useful to identify and analyze possible sources of capacitances where charged bodies are involved.

In general, the capacitance can be determined by solving Laplace's equations $\nabla^2 V(x, y, z) = 0$ with appropriate boundary conditions. One type of boundary condition specifies the electrode voltages V_1 and V_2 of the plates. Laplace's equation yields to V and the electric field $E(x, y, z) = -\nabla V(x, y, z)$ between the electrodes. The charge of each electrode can also be obtained by integration of the flux density over each electrode surface as:

TABLE 9.1 Capacitance Unit Conversions

microfarads (μF)	nanofarads (nF)	picofarads (pF)
10^{-6} F	10^{-9} F	10^{-12} F
0.000001 μF	0.001 nF	1.0 pF
0.001 μF	1.0 nF	1000 pF
1.0 μF	1000 nF	1,000,000 pF

$$Q = \int \varepsilon(x,y,z) E(x,y,z) \mathrm{d}A \tag{9.2}$$

If the capacitor is made from two parallel plates, as shown in Figure 9.1, the capacitance value in terms of dimensions can be expressed by:

$$C = \varepsilon A/d = \varepsilon_r \varepsilon_0 A/d \tag{9.3}$$

where ε = Dielectric constant or permittivity
ε_r = Relative dielectric constant (in air, $\varepsilon_r = 1$)
ε_0 = Dielectric constant of vacuum (8.854188×10^{-12} F m^{-1})
d = Distance of the plates in m
A = Effective area of the plates in m^2

In arriving at Equation 9.3, the fringe field is neglected for small distances, d, between the plates.

Capacitances can also be expressed in terms of dielectric properties currents and voltages. Suppose that a uniform dielectric between two parallel plates has a resistance, R, which can be written as:

$$R = d\rho/A \tag{9.4}$$

where ρ is the specific resistance of the dielectric in Ωm. Then, using Equation 9.3 gives:

$$C = \varepsilon\rho/R \tag{9.5}$$

A voltage V across the capacitor causes a leakage current $I_l = V/R$ such that:

$$C = \varepsilon\rho I_l/V \tag{9.6}$$

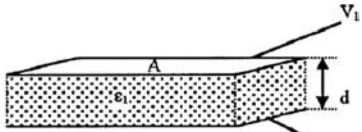
This indicates that the leakage current of a capacitor is proportional to its capacitance value.

As seen in Equations 9.3 and 9.4, the value of the capacitance is proportional to the permittivity of the dielectric material used. In the construction of capacitors, the permittivity of commonly used materials is given in Table 9.3.

9.1 Types of Capacitors

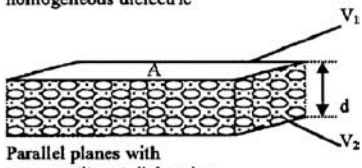
Commonly used fixed capacitors are constructed with air, paper, mica, polymers, and ceramic dielectric materials. A comprehensive list of common capacitors and their characteristics are given in Table 9.4 and Table 9.5 and a list of manufacturers is given in Table 9.6. Variable capacitors are generally made with air or ceramic dielectric materials. The capacitors used in electronic circuits can be classified as: low-loss, medium-loss, and high-tolerance capacitors.

TABLE 9.2 Capacitances of Various Electrode Systems



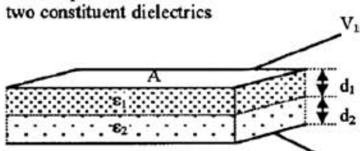
Parallel planes with homogeneous dielectric

$$C = \frac{\varepsilon A}{d}$$



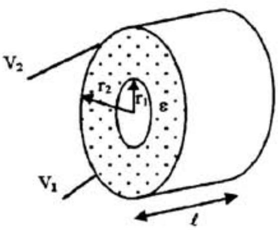
Parallel planes with two constituent dielectrics

$$C = \frac{(\varepsilon_1 r + \varepsilon_2)A}{(1+r)d}$$ where r value is the volumetric ratio



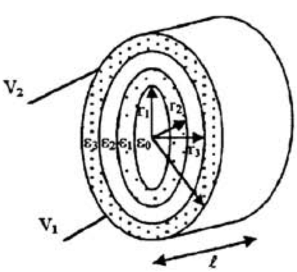
Parallel planes with two different dielectrics

$$C = \frac{\varepsilon A}{\dfrac{d_1}{\varepsilon_1} + \dfrac{d_2}{\varepsilon_2}}$$



Coaxial cylinders with single dielectric

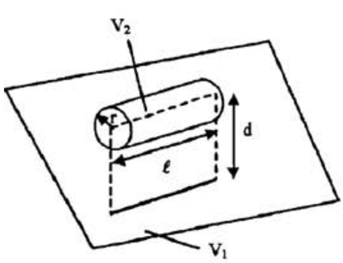
$$C = \frac{4\pi\varepsilon\ell}{\ln(r_2/r_1)}$$



Coaxial cylinders with three dielectrics

$$C = \frac{2\pi\varepsilon_0\ell}{\ln\left[\left(\dfrac{r_2}{r_1}\right)^{\frac{\varepsilon_0}{\varepsilon_1}} \times \left(\dfrac{r_3}{r_2}\right)^{\frac{\varepsilon_0}{\varepsilon_2}} \times \left(\dfrac{r_4}{r_3}\right)^{\frac{\varepsilon_0}{\varepsilon_3}}\right]}$$

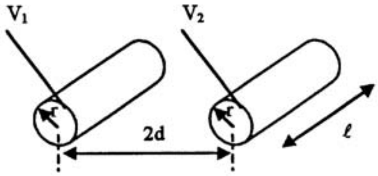
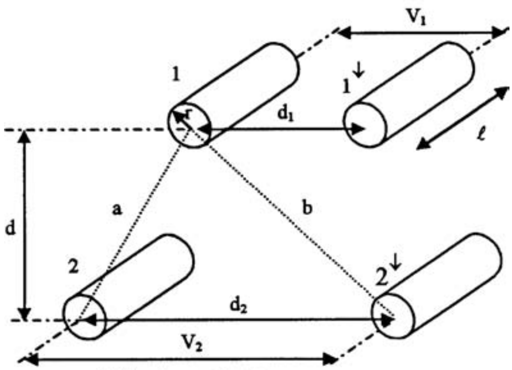
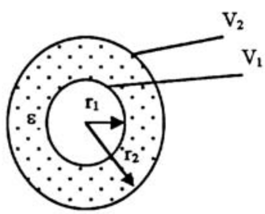
$$= \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$



Cylinder parallel with plane

$$C = \frac{\pi\varepsilon\ell}{\ln\left[\dfrac{d}{r} + \sqrt{\left(\dfrac{d}{r}\right)^2 - 1}\right]} = \frac{2\pi\varepsilon\ell}{\mathrm{Cosh}^{-1}\left(\dfrac{d}{r}\right)}$$

TABLE 9.2 (continued) Capacitances of Various Electrode Systems

 Parallel cylinders with equal radius	$C = \dfrac{\pi\varepsilon\ell}{\ln\left[\dfrac{d}{r} + \sqrt{\left(\dfrac{d}{r}\right)^2 - 1}\right]} = \dfrac{\pi\varepsilon\ell}{\mathrm{Cosh}^{-1}\left(\dfrac{d}{r}\right)}$
 Mutual capacitance	$C_{11} = \dfrac{\pi\varepsilon\ell}{\ln(d_1/r)} + \dfrac{\pi\varepsilon}{2}\dfrac{\ln(b/a)}{[\ln(a/r)]\ln(b/r)}$ $C_{21} = C_{12} = \dfrac{\pi\varepsilon\ell}{2}\dfrac{\ln(b/a)}{[\ln(a/r)\ln(b/r)]}$
 Concentric spheres	$C = \dfrac{4\pi\varepsilon r_1 r_2}{r_2 - r_1}$

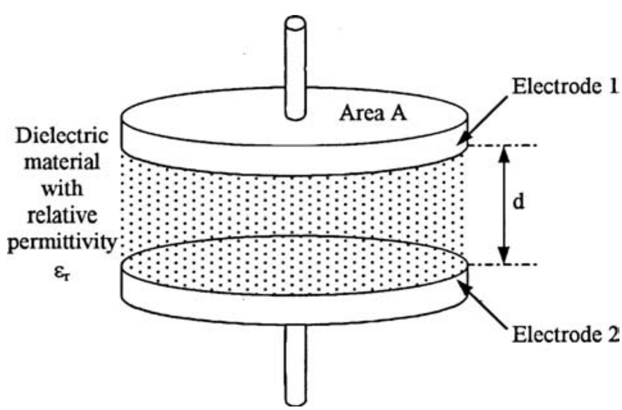


FIGURE 9.1 A typical capacitor made from two parallel plates. The capacitance between two charged bodies depends on the permittivity of the medium, the distance between the bodies, and the effective area. It can also be expressed in terms of the absolute values of the charge and the absolute values of the voltages between bodies.

TABLE 9.3 Permittivity (Dielectric Constants of Materials Used in Capacitors)

Material	Permittivity
Vacuum	1.0
Air	1.0006
Teflon	2.1
Polyethylene, etc.	2.0–3.0
Impregnated paper	4.0–6.0
Glass and mica	4.0–7.0
Ceramic (low *K*)	≤20.0
Ceramic (medium *K*)	80.0–100.0
Ceramic (high *K*)	≥1000.0

- *Low-loss capacitors* such as mica, glass, low-loss ceramic, and low-loss plastic film capacitors generally have a good capacitance stability. These capacitors are expensive and often selected in precision applications, e.g., telecommunication filters.
- *Medium-loss capacitors* have medium stability in a wide range of ac and dc applications. These are paper, plastic film, and medium-loss ceramic capacitors. Their applications include coupling, decoupling, bypass, energy storage, and some power electronic applications (e.g., motor starter, lighting, power line applications, and interference suppressions).
- *High-tolerance capacitors* such as aluminum and tantalum electrolytic capacitors deliver high capacitances. Although these capacitors are relatively larger in dimension, they are reliable and have longer service life. They are used in polarized voltage applications, radios, televisions, and other consumer goods, as well as military equipment and harsh industrial environments.

There are also specially designed capacitors (e.g., mica, glass, oil, gas, and vacuum). These capacitors are used particularly in high-voltage (35 kV) and high-current (200 A) applications.

In the majority of cases, the manufacturing process of the capacitors begins by forming one plate using metallization of one side of a flexible dielectric film. A foil such as aluminum is used as the other plate. The film/foil combination is rolled on a suitable core with alternate layers slightly extended and then heat-treated. In some cases, two-foil layers are divided by a dielectric film or paper impregnated with oil.

Generally, capacitors are two-terminal devices with one electrode as the ground terminal. However, if both terminals are separated from the common terminal, the additional capacitances between ground and electrodes might have to be taken into account. Usually, capacitance between electrodes and ground are small compared to the dominant capacitance between plates. Three-terminal capacitances exist and are manufactured in many different ways, as illustrated in Table 9.2.

As far as construction and materials and construction techniques are concerned, the capacitors can broadly be classified as: electrolytic, ceramic, paper, polymer, mica, variable capacitors, or integrated circuit capacitors.

Paper capacitors: Usually, paper capacitors are made with thin (5 to 50 μm in thickness) wood pulp. A number of sheets are used together to eliminate possible chemical and fibrous defects that may exist in each sheet. The paper sheets are placed between thin aluminum foils and convolutely wound, as shown in Figure 9.2. The moisture of the paper is removed at high-temperature vacuum drying before the capacitor is vacuum impregnated with oil, paraffin, or wax. The losses and self-inductance are sizeable and frequency dependent. The applications are usually restricted to low frequency and high voltages. When impregnated with silicone-oil, they can withstand voltages up to 300 kV.

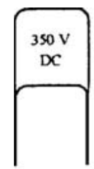
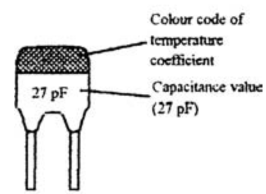
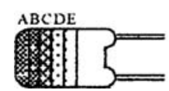
Electrolytic capacitors: This describes any capacitor in which the dielectric layer is formed by an electrolytic method. The electrolytic capacitors in dry foil form may be similar to construction to paper film capacitors; that is, two foil layers separated by an impregnated electrolyte paper spacer are rolled together. In this case, one of the plates is formed using metallization of one side of a flexible dielectric

TABLE 9.4 Characteristics of Common Capacitors

Capacitor Types	Range (F)	Tolerance (%)	Voltage Range (V)	Temperature Range (°C)	Temperature Coefficient (ppm/°C)	Frequency Range (Hz)	Permittivity ($\varepsilon/\varepsilon_0$ C^2/Nm^2)	Dielectric Strength (C/C_0)	Dissipation Factor (%)	Insulation Resistance (MΩ/μF)	Typical Average Failure Rates (fail per 10^6 h)
Mica, glass, porcelain, and Teflon											0.0133
Mica	5p–0.01μ	5	100–600	–55/125	–50	100Hz–10GHz	7.0	1000	0.001	2.5×10^4	
Glass	5p–1000p	5	100–600	–55/125	40	100Hz–10GHz	6.6	2500	0.001	10^6	
Porcelain	100p–0.1μ	5	50–400	–55/125	120				0.1	5×10^5	
Teflon	1000p–2μ	10	50–200	–70/250	–200				0.04	5×10^6	
Ceramic											
Low-Loss	100p–1 μ	10	50–400	–55/125	±30	100Hz–10GHz	5.7	200–300	0.02	5×10^3	0.11–0.008
Disk											
Multilayer											
Plate											
High Permittivity	10p–1μ		50–30000	–55/125		1kHz–1GHz	1000–7000	100			
Disk											
Multilayer											
Plate											
Paper											
Paper	0.1μ–10μ	10	200–1600	–55/125	±800	D.C.–1MHz	4.5	500–1000	1.0	5×10^3	0.002
Metallized paper											
Plastic											
Kapton	1000p–1μ	10		–55/220	100				0.3	10^5	0.05
Polyester/Mylar	1000p–50μ	10	50–600	–55/125	400	D.C.–10GHz	2.3	1000	0.75	10^5	

Parylene	5000p–1µ	10		–55/125	±100	D.C.–10GHz		1000	0.1	10^5	
Polysulfone	1000p–1µ	5		–55/150	80				0.3	10^5	
Polycarbonate											
Axial	100p–30µ	10	50–800	–55/125	±100	D.C.–10GHz	2.8		0.2	5×10^5	
Can											
Radial											
Polypropylene											
Axial	100p–50µ	10	100–800	–55/105	–200	D.C.–10GHz			0.2	10^5	
Can											
Radial											
Polystyrene											
Axial	10p–2.7µ	10	100–600	–55/85	–100	D.C.–10GHz	10		0.05	10^6	
Can											
Radial											
Electrolytic and solid											0.04
Aluminum	0.1µ–1.6	–10/100	3–600	–40/85	2500		8–10		10	100	
Tantalum											
Axial	0.1µ–1000µ	–10/100	6–100	–55/85	800	D.C.–1kHz	25–27		4.0	20	
Can											
Radial											
TiTiN Film	10p–200p	10	6–30	–55/125	100				0.01	10^6	
Oil	0.1µ–20µ		200–10000			D.C.–1MHz	2.4	1000	0.5		
Air/Vacuum	1p–100p		2000–3600				1.0				

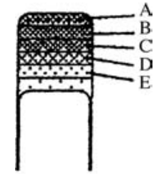
TABLE 9.5 Capacitor Specifications and Applications

Capacitor Types	Typical Commercial Specifications: Voltage A.C. V	Capacitance F	Tolerance %	Applications	Samples/codes
Mica, glass, porcelain, and Teflon				High temperature, low absorption, good in RF applications, and circuit requiring long-term stability	
Mica	350	2.2 p–1000 p	1		
Glass					
Porcelain					
Teflon					
Ceramic				Active filters, and high-density PCB applications, power-tuned circuits, coupling and decoupling of high-frequency circuits (PCB versions available)	
Low-loss					
Disk	100	1.8 p–470 p	±2		
Multilayer	63/50	10 p–1 µ	10		
Plate	100	390 p–4700 p	10		
High permittivity					
Disk					
Multilayer					
Plate					
Paper				General-purpose motor applications	
Paper					
Metallized paper					

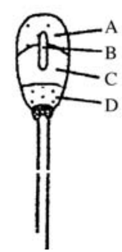
Color	α ppm/°C
Red Purple	100
Black	0
Brown	–33
Red	–75
Orange	–150
Yellow	–220
Green	–330
Blue	–470
Purple	–750

Color	Significant Figures A	Significant Figures B	Multiplier pF C	Tolerance % D	Voltage V E
Black	—	0	1	±20	
Brown	1	1	10		125
Red	2	2	10^2		160
Orange	3	3	10^3		250
Yellow	4	4	10		
Green	5	5	10^5		
Blue	6	6	10^6		
Purple	7	7	10^7		
Gray	8	8	—		
White	9	9	—	±10	

Plastic				Filters, timing and other high-stability applications, high quality, small low TC, tuned circuits, timing networks, stable oscillator circuits, resonance circuits and other high-performance pulse handling applications, phase shifting, pulse applications
Kapton polyester/ mylar	63	0.1 µ–68 µ	10	
Parylene				
Polysulfone				
Polycarbonate				
Axial	280	1 p–10 p	10	
Can	100	0.1 µ–68 µ	10	
Radial				
Polypropylene				
Axial	63	100 p–2200 p	5	
Can	400	150 p–1000 p	1	
Radial	1000	1 n–470 n	20	
Polystyrene				
Axial	450	47 p–680 p	1	
Can	1000	1 n–470 n	20	
Radial	160	10p–10000p	2.5	
Electrolytic and solid				General-purpose to high-performance applications, power supply filters, motor capacitors, switching circuits, high-voltage filter transmitters and long life applications
Aluminum	25	680 p–6800 p	20	
Tantalum				
Axial	6.3	6.8 µ–150 µ	20	
Can	35	6.8 µ–150 µ	20	
Radial	16	2.2 µ–68 µ	20	



Color	Significant Figures A	Significant Figures B	Multiplier pF C	Tolerance % D	Voltage V (D.C.) E
Black	0	0	1	±20	—
Brown	1	1	10		100
Red	2	2	10^2		250
Orange	3	3	10^3		—
Yellow	4	4	10^4		400
Green	5	5	10^5		—
Blue	6	6	—		630
Purple	7	7	—		—
Gray	8	8	10^8		—
White	9	9	10^9	±10	—



Color	Significant Figures A	Significant Figures B	Multiplier pF C	Voltage V D
Black	—	0	1	10
Brown	1	1	—	1.6
Red	2	2	—	4
Orange	3	3	—	40
Yellow	4	4	—	6.3
Green	5	5	—	16
Blue	6	6	—	—
Purple	7	7	10^{-3}	—
Gray	8	8	10^{-2}	25
White	9	9	10^{-1}	2.5

TABLE 9.6 List of Manufacturers

Bycap Inc. 5115 N. Ravenswood, Dept. T Chicago, IL 60640 Tel: (312) 561-4976 Fax: (312) 561-5095	Magnetek 902 Crescent Avenue Bridgeport, CT 06607 Tel: (800) 541-9997 Fax: (203) 335-2820
Chenelex Barr Road, P.O. Box 82 Norwich, NY 13815 Tel: (607) 344-3777 Fax: (607) 334-9076	Maxwell Laboratories Inc. 8888 Balboa Avenue San Diego, CA, 92123 Tel: (619) 576-7545 Fax: (619) 576-7545
Chicago Condenser 2900-T W. Chicago Ave. Chicago, IL 60622 Tel: (312) 227-7070 Fax: (312) 227-6646	Metuchen Capacitors Inc. 139 White Oak Lane Old Bridge, NJ 08857 Tel: (800) 679-0514 Fax: (800) 679-9959
Comet North America Inc. 11 Belden Avenue Norwalk, CT 06850 Tel: (203) 852-1231 Fax: (203) 838-3827	Murata Electronics Marketing Communications 2200 Lake Park Drive Smyrna, GA 30080 Tel: (800) 394-5592 Fax: (800) 4FAXCAT
Condenser Products 2131 Broad Street Brooksville, FL 34609 Tel: (800) 382-6874 Fax: (904) 799-0221	NWL Capacitors 204 Caroline Drive, P.O. Box 97 Snow Hill, NC 28580 Tel: (919) 747-5943 Fax: (919) 747-8979
CSI Capacitors 810 Rancheros Drive San Marcos, CA 92069-3009 Tel: (619) 747-4000 Fax: (619) 743-5094	Okaya Electric America Inc. 503 Wall Street Valparaiso, IN 46383 Tel: (219) 477-4488 Fax: (219) 477-4856
HVCO Inc. P.O. Drawer 223, 7137 Sycamore Ln. Cedarburg, WI 53012 Tel: (414) 375-0172 Fax: (414) 275-0173	

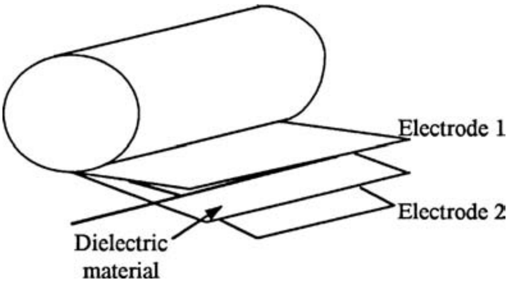


FIGURE 9.2 Construction of a typical capacitor. Dielectric material sheets are placed between electrode foils and convolutely wound. The moisture of the dielectric material is removed at high temperatures by vacuum drying before the capacitor is impregnated with oil, paraffin, or wax.

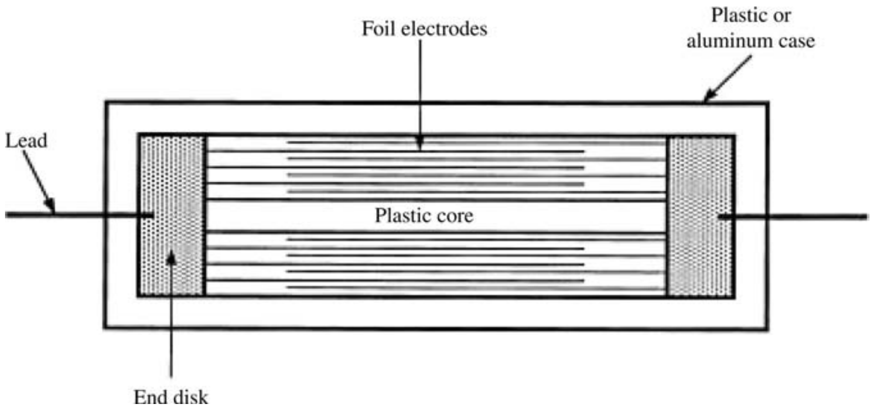


FIGURE 9.3 Construction of an electrolytic capacitor. The two foil-layer electrodes are separated by an impregnated electrolyte paper spacer and rolled together on a plastic core. Usually, a flexible metallized dielectric film is used as one of the plates and an ordinary foil is used for the other. The capacitor is then hermetically sealed in an aluminum or plastic can.

film. A foil (e.g., aluminum) is used as the plate. The capacitor is then hermetically sealed in an aluminum or plastic can, as shown in Figure 9.3. These capacitors can be divided into two main subgroups.

Tantalum electrolytic: The anode consists of sintered tantalum powder and the dielectric is Ta_2O_5, which has a high value of ε_r. A semiconductor layer MnO_2 surrounds the dielectric. The cathode made from graphite is deposited around MnO_2 before the capacitor is sealed. The form of a tantalum electrolytic capacitor includes a porous anode slug to obtain a large active surface. These capacitors are highly stable and reliable, with good temperature ranges, and are suitable for high-frequency applications.

Aluminum electrolytic capacitors: Aluminum foil is oxidized on one side as Al_2O_3. The oxide layer is the dielectric having a thickness of about 0.1 µm and a high electric field strength (7×10^5 Vmm^{-1}). A second layer acting as the cathode, made from etched Al-foil, is inserted. The two layers are separated by a spacer when the layers are rolled and mounted.

Electrolytic capacitors must be handled with caution, since in these capacitors the electrolytic is polarized. That is, the anode should always be positive with respect to the cathode. If not connected correctly, hydrogen gas will form; this damages the dielectric layer, causing a high leakage current or blow-up. These capacitors can be manufactured in values up to 1 F. They are used in not-so-critical applications such as coupling, bypass, filtering, etc. However, they are not useful at frequencies above 1 kHz or so.

Ceramic and glass capacitors: The dielectric is a ceramic material with deposited metals. They are usually rod or disk shaped. They have good temperature characteristics and are suitable in many high-frequency applications. There are many different types, such as (1): *Low K ceramic:* These capacitors are made with materials that contain a large fraction of titanium dioxide (TiO_2). The relative permittivity of these materials varies from 10 to 500, with negative temperature coefficient. The dielectric is TiO_2 + MgO + SiO_2, suitable in high-frequency applications in filters, tuned circuits, coupling and bypass circuits, etc. (2): *High K ceramic:* The dielectric contains a large fraction of barium titanate, $BaTiO_3$, mixed with $PbTiO_3$ or $PbZrO_3$ giving relative permittivity of 250 to 10,000. They have high losses, and also have high-voltage time dependence with poor stability. (3): *Miniature ceramic capacitors:* These are used in critical high-frequency applications. They are made in the ranges of 0.25 pF to 1 nF. (4): *Dielectric ceramic capacitors:* The material is a semiconducting ceramic with deposited metals on both sides. This arrangement results in two depletion layers that make up the very thin dielectric. In this way, high capacitances can be obtained. Due to thin depletion layers, only small dc voltages are allowed. They are used in small and lightweight equipment such as hearing aids.

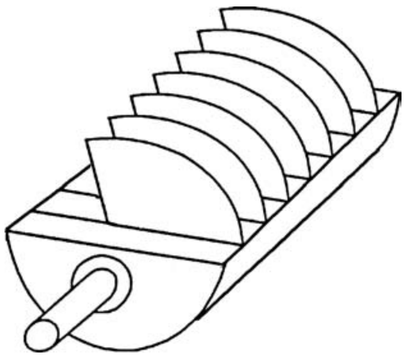


FIGURE 9.4 A variable capacitor. It consists of two assemblies of spaced plates positioned together by insulation members such that one set of plates can be rotated. The majority of variable capacitors have air as the dielectric. They are used mainly in adjustment of resonant frequency of tuned circuits in receivers and transmitters. By shaping the plates suitably, they can be made to be linear or logarithmic.

Glass capacitors are made with glass dielectric materials. The properties of glass dielectrics are similar to ceramic materials.

Polymer capacitors: Various polymers, such as polycarbonate, polystyrol, polystyrene, polyethylene, polypropylene, etc., are used as the dielectric. The construction is similar to that of paper capacitors. Polystyrene capacitors, in particular, are very stable, and are virtually frequency independent. They have low voltage ratings and are used in transistorized applications as tuning capacitors and capacitance standards.

Mica capacitors: A thin layer of mica, usually muscovite mica ($\geq$0.003 mm) are stapled with Cu-foil or coated with a layer of deposited silver. They are then vacuum impregnated and coated with epoxy. The field strength of these capacitors is very high (10^5 V mm^{-1}) and resistivity $\rho = 10^6$ to 10^{15} Ω m. These capacitors are available in values from 1.0 pF to several microfarads for high voltage (from 100 V to 2000 V) and high-frequency applications. They have tolerances between $\pm$20% and $\pm$0.5%.

Variable capacitors: These capacitors usually have air as the dielectric and consist of two assemblies of spaced plates positioned together by insulation members such that one set of plates can be rotated. A typical example of variable capacitors is given in Figure 9.4. Their main use is the adjustment of resonant frequency of tuned circuits in receivers and transmitters, filters, etc. By shaping the plates, various types of capacitances can be obtained, such as: *linear capacitance*, in which capacitance changes as a linear function of rotation, and *logarithmic capacitance*.

Variable capacitors can be grouped as: precision types, general-purpose types, transmitter types, trimmer types, and special types such as phase shifters.

Precision-type variable capacitors are used in bridges, resonant circuits, and many other instrumentation systems. The capacitance swing can be from 100 pF to 5000 pF. They have excellent long-term stability with very tight tolerances.

General-purpose type variable capacitors are used as tuning capacitors in radio and other broadcasting devices. They are available in many laws such as, straight line frequency, straight line wavelength, etc. The normal capacitance swing is from 400 pF to 500 pF. In some cases, a swing of 10 pF to 600 pF are available.

Transmitter-type variable capacitors are similar to general-purpose variable capacitors, but they are specially designed for high-voltage operations. The vanes are rounded and spaced wider to avoid flashover and excessive current leakages. The swing of these capacitors can go from few picofarads up to 1000 pF. In some cases, oil filling or compressed gases are used to increase operating voltages and capacitances.

Trimmer capacitors are used for coil trimming at intermediate radio frequencies. They can be air-spaced rotary types (2 pF to 100 pF), compression types (1.5 pF to 2000 pF), ceramic-dielectric rotary types (5 pF to 100 pF), and tubular types (up to 3 pF).

Sometimes, special type variable capacitors are produced for particular applications, such as differential and phase shift capacitors in radar systems. They are used for accurate measurement of time intervals, high-speed scanning circuits, transmitters and receivers, etc.

Integrated circuit capacitors: These are capacitors adapted for use in microelectronic circuits. They include some miniature ceramic capacitors, tantalum oxide solid capacitors, and tantalum electrolyte solid capacitors. The ceramic and tantalum oxide chips are unencapsulated and are fitted with end caps for direct surface mounting onto the circuit board. The beam-leaded tantalum electrolytic chips are usually attached by pressure bonding. Typical values of these capacitors are: 1 pF to 27 nF for temperature compensating ceramic, (100 to 3000 pF) for tantalum oxide, 390 pF to 0.47 μF for general-purpose ceramic, and (0.1 to 10 μF) for tantalum electrolyte. Operating voltages range from 25 to 200 V for ceramic, 12 to 35 V for tantalum electrolyte, and 12 to 25 V for tantalum oxide.

Integrated circuit capacitors are made mostly within MOS integrated circuits as monolayer capacitors containing tantalum or other suitable deposits. The plates of the capacitors of the integrated circuits are generally formed by two heavily doped polysilicon layers formed on a thick layer of oxide. The dielectric is usually made from a thin layer of silicon oxide. These capacitors are temperature stable, with a temperature coefficient of about 20 ppm/°C. Integrated circuit capacitive sensors are achieved by incorporating a dielectric sensitive to physical variables. Usually, the metallization layer formed on top of the dielectric forms a shape to provide access to measured physical variable to the dielectric.

Voltage variable capacitors: These capacitors make use of the capacitive effect of the reversed-biased p-n junction diode. By applying different reverse bias voltages to the diode, the capacitance can be changed. Hence, the name varicap or varactor diodes is given to theses devices. Varactors are designed to provide various capacitance ranges from a few picofarads to more than 100 pF. It is also possible to make use of high-speed switching silicon diodes as voltage variable capacitors. However, they are limited by the very low maximum capacitance available. Typical applications of these varactor diodes are in the tuning circuits in radio frequency receivers. Present-day varactor diodes operate into the microwave part of the spectrum. These devices are quite efficient as frequency multipliers at power levels as great as 25 W. The efficiency of a correctly designed varactor multiplier can exceed 50% in most instances. It is also worth noting that some Zener diodes and selected silicon power-supply rectifier diodes can work effectively as varactors at frequencies as high as 144 MHz. In the case of the Zener diode, it should be operated below its reverse breakdown voltage.

9.2 Characteristics of Capacitors

Capacitors are characterized by dielectric properties, break-down voltages, temperature coefficients, insulation resistances, frequency and impedances, power dissipation and quality factors, reliability and aging, etc. Typical characteristics of common capacitors are given in Table 9.4.

Dielectric properties: Dielectrics of capacitors can be made from polar or nonpolar materials. Polar materials have dipolar characteristics; that is, they consist of molecules whose ends are oppositely charged. This polarization causes oscillations of the dipoles at certain frequencies, resulting in high losses.

In general, capacitor properties are largely determined by the dielectric properties. For example, the losses in the capacitors occur due to the current leakage and the dielectric absorption. These losses are frequency dependent, as typified by Figure 9.5.

The dielectric absorption introduces a time lag during the charging and discharging of the capacitor, thus reducing the capacitance values at high frequencies and causing unwanted time delays in pulse circuits. The leakage current, on the other hand, prevents indefinite storage of energy in the capacitor. An associated parameter to leakage currents is the leakage resistance, which is measured in megohms, but usually expressed in megohm-microfarads or ohms-farads. The leakage resistance and capacitance introduces time constants that can vary from a few days for polystyrene to several seconds in some electrolytic capacitors. It is important to mention that the leakage current does not only depend on the properties of the dielectric materials, but also depends on the construction and structure of capacitors.

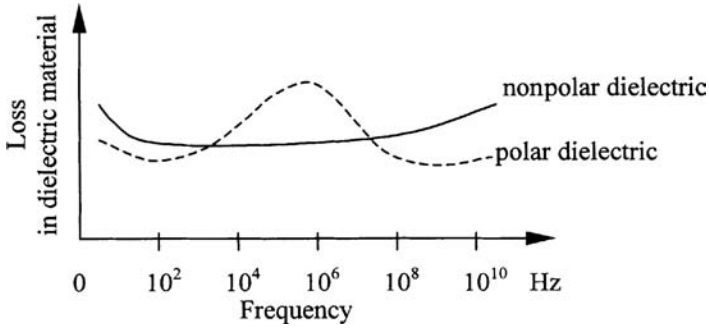


FIGURE 9.5 Frequency dependence of dielectric loss. The dielectric material of capacitors can be polar or nonpolar. In polar materials, polarization causes oscillations at certain frequencies, resulting in high losses. The dielectric losses introduce a time lag during the charging and discharging of the capacitor, thus reducing the capacitance values at high frequencies.

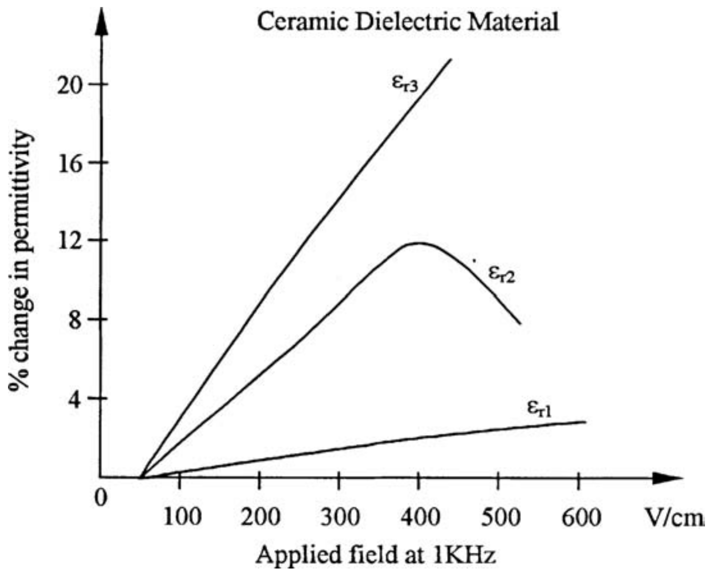


FIGURE 9.6 Changes in relative permittivity vs. field strength. The dielectric strength depends on the temperature, frequency, and applied voltage. Increases in the applied voltage cause higher changes in the dielectric strength. If the capacitor is subjected to high operating voltages, the electric field in the dielectric exceeds the breakdown value which can damage the dielectric permanently.

This is particularly true for capacitors having values less than 0.1 μF, having very thin dielectric materials between the electrodes.

Breakdown voltage: If the capacitor is subjected to high operating voltages, the electric field in the dielectric exceeds the breakdown value, which damages the dielectric permanently. The dielectric strength, which is the ability to withstand high voltages without changing properties, depends on the temperature, frequency, and applied voltage. An example of this dependence on the applied voltage is given in Figure 9.6. It is commonly known that the use of capacitors below their rated values increases the reliability and the expected lifetime. The standard voltage ratings of most capacitors are quoted by the manufacturers as 50, 100, 200, 400, and 600 V. Tantalum and electrolytic capacitors have ratings of 6, 10, 12, 15, 20, 25, 35, 50, 75, 100 V and higher.

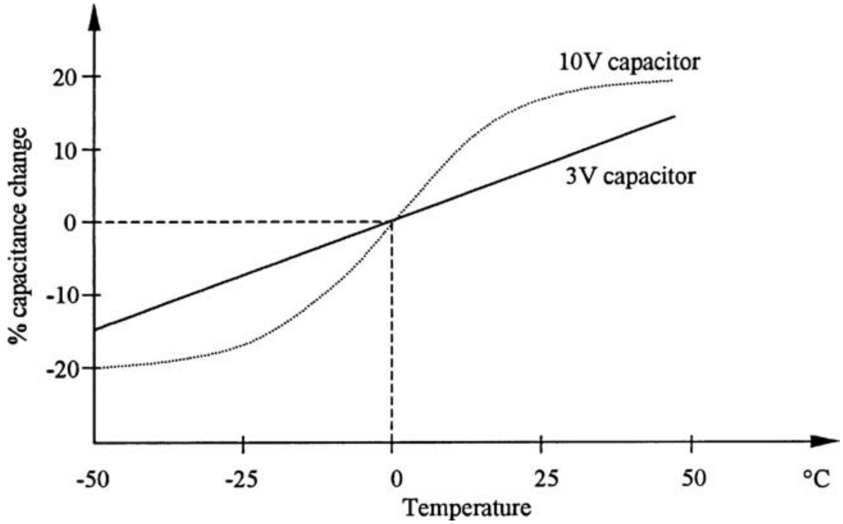


FIGURE 9.7 Temperature dependence of capacitors. The temperature characteristics of capacitors are largely dependent on the temperature properties of the dielectric materials used. The variations in capacitance due to temperature also depends on the type of capacitor and the operational voltage. The temperature coefficient of glass, teflon, mica, and polycarbonate are very small, and relatively high in ceramic capacitors.

Usually, values for surge voltages are given to indicate the ability of capacitors to withstand high transients. Typically, the surge voltages for electrolytic capacitors is 10% above the rated voltage, 50% for aluminum capacitors and about 250% for ceramic and mica capacitors.

The rated reverse voltages of electrolytic capacitors are limited to 1.5 V and, in some cases, to 15% of the rated forward voltages.

Temperature coefficient: The temperature characteristics of capacitors largely dependent on the temperature properties of the dielectric materials used, as given in Figure 9.7. The temperature coefficients of glass, teflon, mica, polycarbonate, etc. are very small, whereas in ceramic capacitors, they can be very high.

Insulation resistance: The insulation resistance of capacitors is important in many circuits. The insulation resistance is susceptible to temperature and humidity. For example, unsealed capacitors show large and rapid changes against temperature and humidity. For most capacitors, under high temperature conditions, the change in insulation resistance is an exponential function of temperature ($R_{T1} = R_{T2}e^{K(T1-T2)}$). The temperature dependence of insulation resistance of common capacitors is shown in Figure 9.8.

Frequency and impedance: Practical capacitors have increases in losses at very low and very high frequencies. At low frequencies, the circuit becomes entirely resistive and the dc leakage current becomes effective. At very high frequencies, the current passes through the capacitance and the dielectric losses become important. Approximate useable frequency ranges of capacitors are provided in Table 9.4.

An ideal capacitor should have an entirely negative reactance, but losses and inherent inductance prevents ideal operation. Depending on the construction, capacitors will resonate at certain frequencies due to unavoidable construction-based inductances. A typical impedance characteristic of a capacitor is depicted in Figure 9.9.

Power dissipation and quality factors: Ideally, a capacitor should store energy without dissipating any power. However, due to equivalent resistances, R_{eq}, some power will be dissipated. The power factor of a capacitor can be expressed as:

$$PF = \cos\theta = R_{eq} \Big/ \left|Z_{eq}\right| \tag{9.7}$$

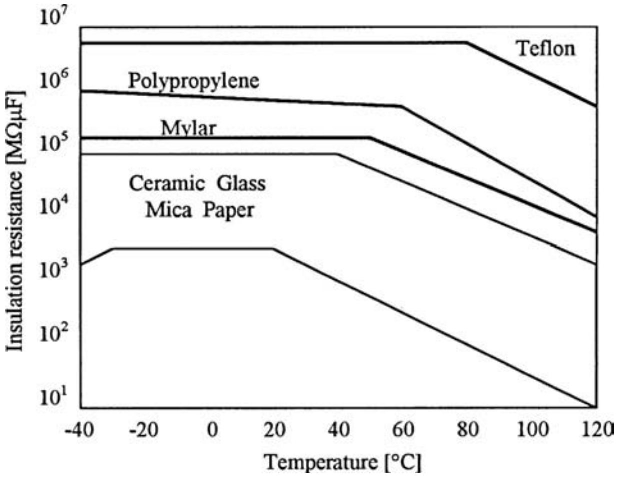


FIGURE 9.8 Temperature dependence of insulation resistance. The insulation resistance of many capacitors is not affected at low temperatures. However, under high temperature conditions, the change in insulation resistance can be approximated by an exponential relation. The insulation resistance is also susceptible to variations in humidity.

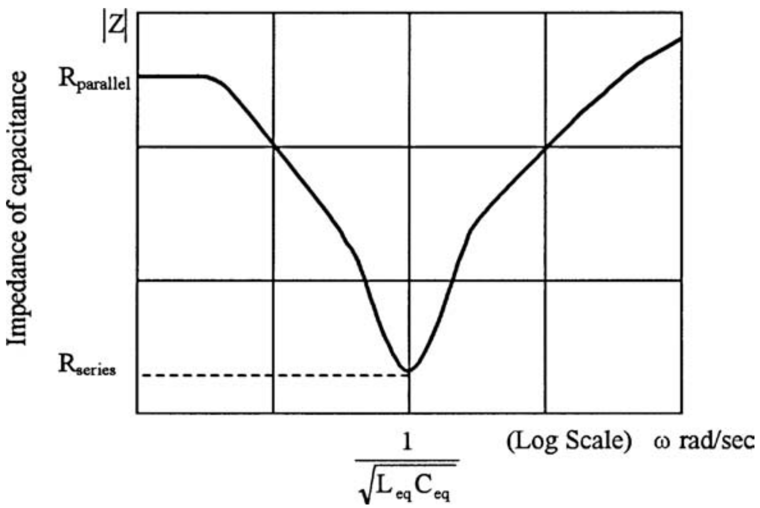


FIGURE 9.9 Frequency and impedance relation of capacitors. The losses and inherent inductance affects the ideal operation of capacitors and the capacitance impedance becomes a function of frequency. Depending on the construction, all capacitors will resonate at a certain frequency.

where θ = Phase angle
Z_{eq} = Equivalent total impedance

An important characteristic, the *dissipation factor* of capacitors, is expressed as:

$$DF = \tan\delta = R_{eq}/X_{eq} \tag{9.8}$$

where δ = Angle of loss
X_{eq} = Equivalent reactance

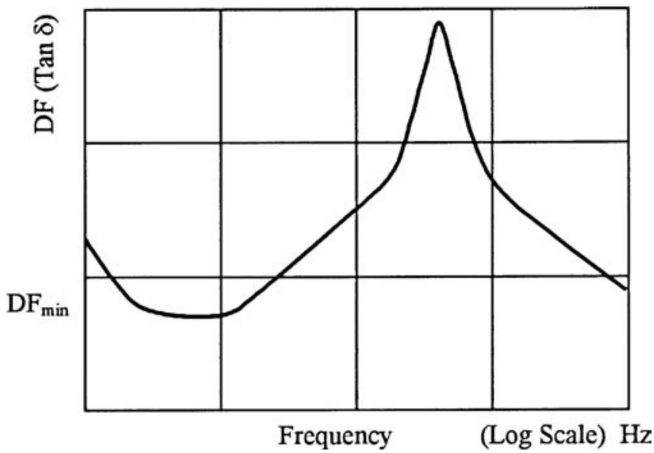


FIGURE 9.10 Power dissipation factors. In ideal operations, capacitors should store energy without dissipating power. Nevertheless, due to resistances, some power will be dissipated. The dissipation depends on frequency. The standard measurement of dissipation factor δ is determined by applying 1.0 Vrms at 1 kHz.

The dissipation factor depends on the frequency. Capacitors are designed such that this dependence is minimal. The measurement of dissipation factor δ is made at 1 kHz and 1.0 Vrms applied to the capacitor. A typical dissipation factor curve is depicted in Figure 9.10.

Selection of Capacitors and Capacitor Reliability

Capacitor Selection

Experience shows that a substantial part of component failures in electronic equipment is due to capacitors. The major cause of this can be attributed to the improper selection and inappropriate applications of capacitors. The following factors are therefore important criteria in the selection of capacitors in circuit applications: (1) the capacitance *values and tolerances* are determined by operating frequencies or by the value required for timing, energy storage, phase shifting, coupling, or decoupling; (2) the *voltages* are determined by the type and nature of the source, ac, dc, transient, surges, and ripples; (3) the *stability* is determined by operating conditions like temperature, humidity, shock, vibration, and life expectancy; (4) the *electrical properties* are determined by life expectancy, leakage current, dissipation factor, impedance, and self-resonant frequency; (5) the mechanical properties are determined by the types and construction, e.g., size, configuration, and packaging; and (6) the cost is determined by the types and physical dimensions of capacitors and the required tolerance.

Capacitor Reliability

Some of the common causes of capacitor failure are due to voltage and current overloads, high temperature and humidity, shock, vibration pressure, frequency effects, and aging. The voltage overload produces an excessive electric field in the dielectric that results in the breakdown and destruction of the dielectric. The current overload caused by rapid voltage variations results in current transients. If these currents are of sufficient amplitude and duration, the dielectric can be deformed or damaged, resulting in drastic changes in capacitance values, and thus leading to equipment malfunction. The high temperatures are mainly due to voltage and current overloads. The overheating and high temperatures accelerate the dielectric aging. This causes the plastic film to be brittle and also introduces cracks in the hermetic seals. The moisture and humidity due to severe operating environments cause corrosion, reduce the dielectric strength, and lower insulation resistances. The mechanical effects are mainly the pressure, variation, shock, and stress, which can cause mechanical damages of seals that result in electrical failures. Aging deteriorates the insulation resistance and affects the dielectric strength. The aging is usually determined by shelf-life; information about aging is supplied by the manufacturers.

TABLE 9.7 Standard Capacitors and Tolerances

Value (pF, nF, μF)\tolerance	5%	10%	20%
10	x	x	x
11	x		
12	x		
13	x		
15	x	x	x
16	x		
18	x	x	
20	x		
22	x	x	x
24	x		
27	x	x	
30	x		
33	x	x	x
36	x		
39	x		
43	x		
47	x	x	x
51	x		
56	x	x	x
62	x		
68	x	x	x
75	x		
82	x		
91	x		

Capacitor Standard Values and Tolerances

General-purpose capacitors values tend to be grouped close to each other in a bimodal distribution manner within their tolerance values. Usually, the tolerances of standard capacitors are 5%, 10%, and 20% of their values, as shown in Table 9.6. Nevertheless, tolerances of precision capacitors are much tighter — in the range of 0.25%, 0.5%, 0.625%, 1%, 2%, and 3% of the values. These capacitors are much more expensive than the standard range.

For capacitors in the small pF range, the tolerances can be given as ±1.5, ±1, ±0.5, ±0.25, and ±0.1 pF. Usually, low tolerance ranges are achieved by selecting manufactured items.

Standard capacitors are constructed from interleaved metal plates using air as the dielectric material. The area of the plates and distance between them are determined and constructed with precision. National Bureau of Standards maintains a bank of primary standard air capacitors that can be used to calibrate the secondary and working standards for laboratories and industry. Generally, smaller capacitance working standards are obtained from air capacitors, whereas larger working standards are made from solid dielectric materials. Usually, silver-mica capacitors are selected as working standards. These capacitors are very stable, have very low dissipation factors and small temperature coefficients, and have very little aging effect. They are available in decade mounting forms.

Capacitors as Circuit Components

The capacitor is used as a two-terminal element in electric circuits with the current–voltage relationship given by:

$$i(t) = C\mathrm{d}v(t)/\mathrm{d}t \tag{9.9}$$

where C is the capacitance.

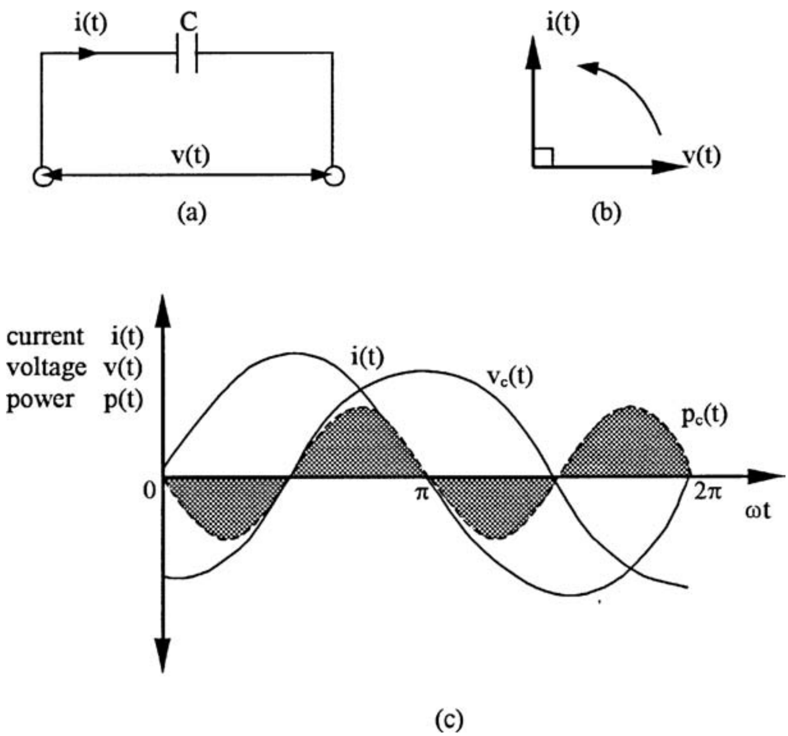


FIGURE 9.11 A capacitor as a two-terminal circuit element: (a) connection of a capacitor in electric circuits; (b) current–voltage relationship under sinusoidal operations; and (c) the power, voltage, and current relationships. The power has positive and negative values with twice the frequency of the applied voltage.

This element is represented by the circuit shape as shown in Figure 9.11(a).

From the $v(t)$, $i(t)$ relationship, the instantaneous power of this element can then be given by:

$$p(t) = v(t)i(t) \tag{9.10}$$

The stored energy in the capacitor at time t seconds can be calculated as:

$$\begin{aligned} w(t) &= \int Cv(t)\{\mathrm{d}v(t)/\mathrm{d}t\}\mathrm{d}t \\ &= \frac{[Cv^2(t)]}{2} \end{aligned} \tag{9.11}$$

If the voltage across the capacitor is changing in time, the energy stored in the capacitor in the time interval t_1 to t_2 can be found using Equation 9.11 as:

$$W = (1/2)C[v^2(t_2) - v^2(t_1)] \tag{9.12}$$

Although the voltage assumed different values in time interval t_1 to t_2, if the initial and final voltages are equal (e.g., $v(t_1) = v(t_2)$), the net energy stored in the capacitor will be equal to zero. This implies that the energy stored in the capacitor is returned to the circuit during the time interval; that is, the capacitor transforms the energy without dissipation. The stored energy is associated with the electrostatic field of the capacitor and, in the absence of the electrostatic field, it will be zero.

From the voltage–current relationship in Equation 9.9, it can be seen that a capacitor as a circuit element is a passive component. That is, if the voltage is known, the current can be immediately determined by differentiation. Conversely, if the current is known, then the voltage can be determined by integration. If the voltage $v(t)$ is considered as an input variable, and the current $i(t)$ as an output variable, then the behavior of the current in a certain time interval is completely determined by the behavior of the voltage in this interval. In this case, the solution of the differential equation has the forced component only. Here, the particular solution of the voltage–current differential equation coincides with a full solution and the Laplace transform gives the relationship:

$$I(s) = sCV(s) \tag{9.13}$$

From this, the input–output relationship yields the impedance of the capacitor as:

$$Z(s) = \frac{1}{sC}$$

or

$$Y(s) = sC \tag{9.14}$$

In the stationary condition, $s \to 0$, and $Z \to \infty$, and $Y \to 0$.

In the sinusoidal condition, $s = j\omega = 2\pi f$, where f = frequency and, hence:

$$\begin{aligned} Z(j\omega) &= \frac{1}{\{j\omega C\}} \\ &= \frac{-j}{\{\omega C\}} \end{aligned} \tag{9.15}$$

and

$$Y(j\omega) = j\omega C \tag{9.16}$$

The capacitor can then be characterized under sinusoidal conditions, by a reactance of $X_C = 1/\omega C$ measured in ohms with the current leading the voltage by 90°, as shown in the phasor diagram in Figure 9.11(b).

In sinusoidal operations, the instantaneous power $p(t) = v(t)\ i(t)$ can be calculated as:

$$v(t) = V_{max} \cos\omega t = \sqrt{2} V \cos\omega t \tag{9.17}$$

Using the relationship given by Equation 9.9, the current can be written as:

$$i(t) = C\,\mathrm{d}v/\mathrm{d}t = -\omega C \sqrt{2}\, V \sin\omega t \tag{9.18}$$

giving:

$$p(t) = v(t)i(t) = -2\omega C V^2 \sin\omega t \cos\omega t = \frac{V^2 \sin 2\omega t}{X_C} \tag{9.19}$$

This indicates that the average power is zero because of the $\sin 2\omega t$ term, but there is a periodic storage and return of energy and the amplitude of that power is V^2/X_c. The power, voltage, and current relationship in a capacitor is given in Figure 9.11(c).

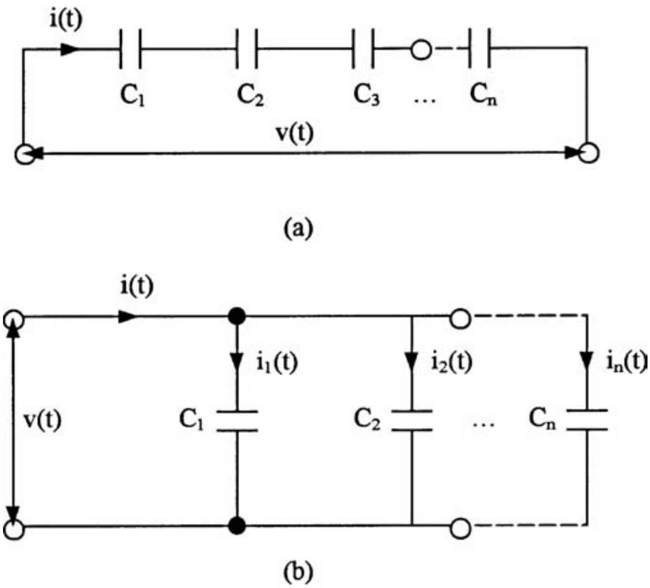


FIGURE 9.12 Series and parallel connection of capacitors: (a) series connection, and (b) parallel connection. In a series connection, the final capacitance value will always be smaller than the smallest value of the capacitor at the circuit element, whereas in parallel connection the final value is greater than the largest capacitance.

Series and Parallel Connection of Capacitors

The formulae for series and parallel connection of capacitors can be obtained from the general consideration of series and parallel connection of impedances as shown in Figures 9.12(a) and (b), respectively. For the series connection, the impedances are added such that:

$$\frac{1}{sC} = \frac{1}{sC_1} + \frac{1}{sC_2} + \cdots + \frac{1}{sC_n} \tag{9.20}$$

where C_1, C_2, … C_n are the capacitance of the capacitors connected in series as in Figure 9.12(a). The equivalent capacitance is then given by:

$$C = \left\{ \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n} \right\}^{-1} \tag{9.21}$$

The final capacitance value will always be smaller than the smallest value.

In a similar way, the equivalent capacitance of parallel connected capacitors is

$$C = C_1 + C_2 + \ldots + C_n \tag{9.22}$$

and the final value of C is always larger than the largest capacitance in the circuit.

Distributed Capacitances in Circuits

Since capacitance is inherent whenever an electric potential exists between two conducting surfaces, its effect will be most noticeable in coils and in transmission lines at high frequencies. In the case of coils, there are small capacitances between adjacent turns, between turns that are not adjacent, between terminal leads, between turns and ground, etc. Each of the various capacitance associated with the coil stores a quantity of electrostatic energy that is determined by the capacitance involved and the fraction of the

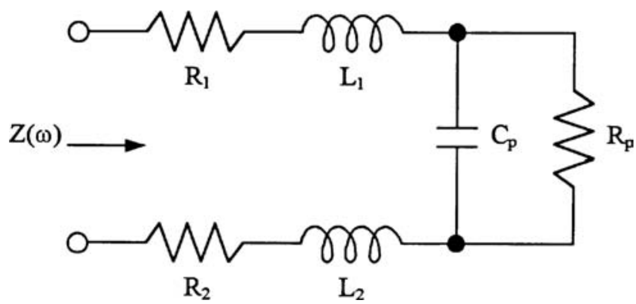


FIGURE 9.13 Capacitor equivalent circuit. A practical capacitor has resistance and inductances. Often, the electrical equivalent circuit of a capacitor can be simplified by a pure capacitance C_p and a parallel resistance R_p by neglecting resistances R_1, R_2 and inductances L_1, L_2. In low-leakage capacitors where R_p is high, the equivalent circuit can be represented by a series *RC* circuit.

total coil voltage that appears across it. The total effect is that the numerous small coil capacitances can be replaced by a single capacitor of appropriate size shunted across the coil terminals. This equivalent capacitance is called either the *distributed capacitance* or the *self-capacitance* of the coil, and it causes the coil to show parallel resonance effects under some conditions. In the case of a mismatched or unterminated transmission line, the distributed capacitance, together with the inductive effect, will create a phase difference between the voltage and current in the line. This phase difference depends on the type of termination and the electrical length of the line and, as a result, the input impedance of the line can effectively be an equivalent capacitor when its electrical length is less than a quarter wavelength for an open-circuit termination, or between a quarter wavelength and half a wavelength for a short-circuit termination.

Capacitor Equivalent Circuits

The electric equivalent circuit of a capacitor consists of a pure capacitance (C_p), plate inductances (L_1, L_2), plate resistances (R_1, R_2), and a parallel resistance R_p that represents the resistance of the dielectric or leakage resistance, as shown in Figure 9.13. The capacitors that have high leakage currents flowing through the dielectric have relatively low R_p values. Very low leakage currents are represented by extremely large R_p values. Examples of these two extremes are electrolytic capacitors that have high leakage current (low R_p), and plastic film capacitors, which have very low leakage current (high R_p). Typically, an electrolytic capacitor might easily have several microamperes of leakage current ($R_p < 1$ MΩ), while a plastic film capacitor could have a resistance greater than 100,000 MΩ.

It is usual to represent a low leakage capacitor (high R_p) by a series *RC* circuit, while those with high leakage (low R_p) are represented by parallel *RC* circuits. However, when a capacitor is measured in terms of the series *C* and *R* quantities, it is desirable to resolve them into parallel equivalent circuit quantities. This is because the (parallel) leakage resistance best represents the quality of the capacitor dielectric.

Capacitive Bridges and Measurement of Capacitance

Bridges are used to make precise measurements of unknown capacitances and associated losses in terms of some known external capacitances and resistances. Most commonly used bridges are: series-resistance-capacitance bridge, parallel-resistance-capacitance bridge, Wien bridge, and Schering bridge.

Series-Resistance-Capacitance Bridge

Figure 9.14 is a series-resistance-capacitance (*RC*) bridge, which is used for the comparison of a known capacitance with an unknown capacitance. The unknown capacitance is represented by C_x and R_x. A standard adjustable resistance R_1 is connected in series with a standard capacitor C_1. The voltage drop across R_1 balances the resistive voltage drop when the bridge is balanced. The additional resistor in series with C_x increases the total resistive component, so that small values of R_1 will not be required to achieve

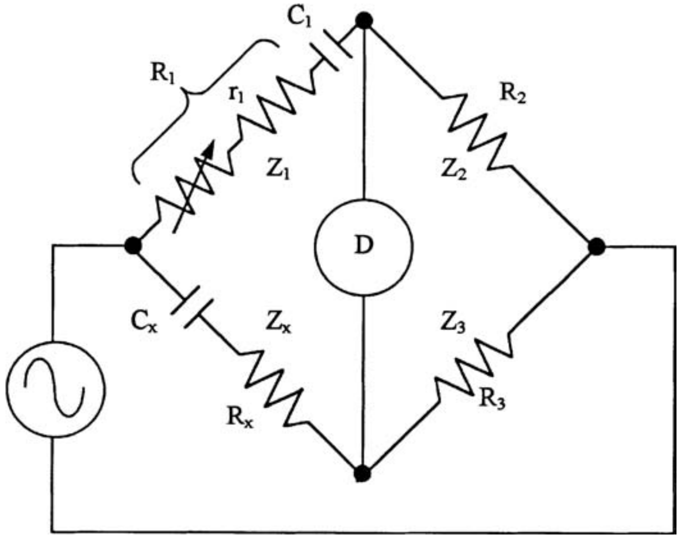


FIGURE 9.14 A series *RC* bridge. In these bridges, the unknown capacitance is compared with a known capacitance. The voltage drop across R_1 balances the resistive voltage drop in branch Z_2 when the bridge is balanced. The bridge balance is most easily achieved when capacitive branches have substantial resistive components. The resistors R_1 and either R_3 or R_4 are adjusted alternately to obtain the balance. This type of bridge is found to be most suitable for capacitors with a high-resistance dielectric, hence very low leakage currents.

balance. Generally, the bridge balance is most easily achieved when capacitive branches have substantial resistive components. To obtain balance, R_1 and either R_3 or R_4 are adjusted alternately. This type of bridge is found to be most suitable for capacitors with a high-resistance dielectric and hence very low leakage currents.

At balance:

$$Z_1 Z_3 = Z_2 Z_x \tag{9.23}$$

Substituting impedance values gives:

$$\left(R_1 - \frac{j}{\omega C_1}\right) R_3 = \left(R_x - \frac{j}{\omega C_2}\right) R_2 \tag{9.24}$$

Equating the *real* terms gives:

$$R_x = \frac{R_1 R_3}{R_2} \tag{9.25}$$

and equating *imaginary* terms gives:

$$C_x = \frac{C_1 R_2}{R_3} \tag{9.26}$$

An improved version of the series *RC* bridge is the *substitution bridge*, which is particularly useful to determine the values of capacitances at radio frequencies. In this case, a series-connected *RC* bridge is balanced by disconnecting the unknown capacitance C_x and resistance R_x, and replacing it by an adjustable standard capacitor C_s and adjustable resistor R_s. After having obtained the balance position, the unknown capacitance and resistance C_x and R_x are connected in parallel to the capacitor C_s. The capacitor C_s and

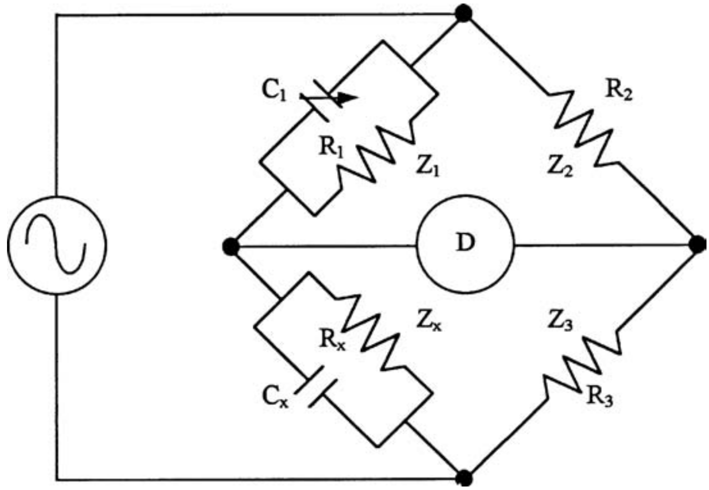


FIGURE 9.15 A parallel-resistance-capacitance bridge. The unknown capacitance is represented by its parallel equivalent circuit; C_x in parallel with R_x. The bridge balance is achieved by adjustment of R_1 and either R_3 or R_4. The parallel-resistance-capacitance bridge is found to be most suitable for capacitors with a low-resistance dielectric, hence relatively high leakage currents.

resistor R_s are adjusted again for the re-balance of the bridge. The changes in the ΔC_s and ΔR_s lead to unknown values as:

$$C_x = \Delta C_s \quad \text{and} \quad R_x = \Delta R_s \left(C_{s1}/C_x\right)^2 \tag{9.27}$$

where C_{s1} is the value of C_s in the initial balance condition.

The Parallel-Resistance-Capacitance Bridge

Figure 9.15 illustrates a parallel-resistance-capacitance bridge. In this case, the unknown capacitance is represented by its parallel equivalent circuit C_x in parallel with R_x. The Z_2 and Z_3 impedances are pure resistors with either or both being adjustable. The Z_1 is balanced by a standard capacitor C_1 in parallel with an adjustable resistor R_1. The bridge balance is achieved by adjustment of R_1 and either R_2 or R_3. The parallel-resistance-capacitance bridge is found to be most suitable for capacitors with a low-resistance dielectric, hence relatively high leakage currents. At balance:

$$\frac{1}{\left(\dfrac{1}{R_1} + j\omega C_1\right)} R_3 = \frac{1}{\left(\dfrac{1}{R_x} + j\omega C_x\right)} R_2 \tag{9.28}$$

Equating *real* terms gives:

$$R_x = \frac{R_3 R_1}{R_2} \tag{9.29}$$

and equating *imaginary* terms gives:

$$C_x = \frac{C_1 R_2}{R_3} \tag{9.30}$$

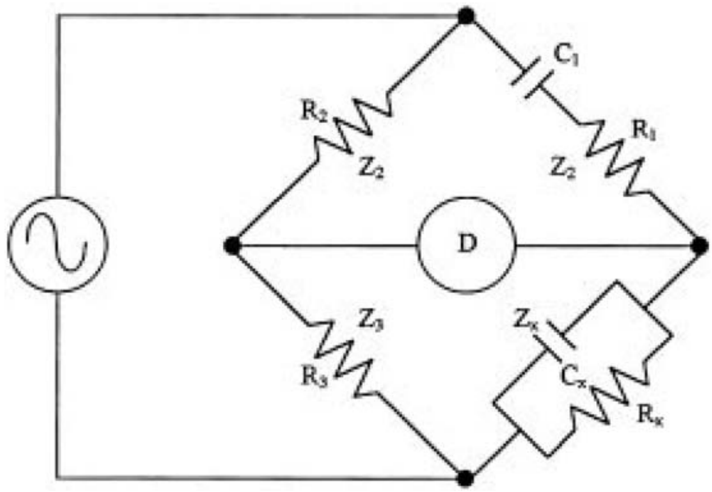


FIGURE 9.16 The Wien bridge. This bridge is used to compare two capacitors directly. It finds applications particularly in determining the frequency in *RC* oscillators. In some cases, capacitors C_1 and C_x are made equal and ganged together so that the frequency at which the null occurs varies linearly with capacitance.

The Wien Bridge

Figure 9.16 shows a Wien bridge. This is a special resistance-ratio bridge that permits two capacitances to be compared once all the resistances of the bridge are known. At balance, it can be proven that the unknown resistance and the capacitance are:

$$R_x = \frac{R_3\left(1+\omega^2 R_1^2 C_1^2\right)}{\omega^2 R_1 R_2 C_1^2} \tag{9.31}$$

and

$$C_x = \frac{C_1 R_2}{\left[R_3\left(1+\omega^2 R_1^2 C_1^2\right)\right]} \tag{9.32}$$

It can also be shown that:

$$\omega^2 = \frac{1}{R_1 C_1 R_x C_x} \tag{9.33}$$

As indicated in Equation 9.33, the Wien bridge has an important application in determining the frequency in *RC* oscillators. In frequency meters, C_1 and C_x are made equal and the two capacitors are ganged together so that the frequency at which the null occurs varies linearly with capacitances.

The Schering Bridge

Figure 9.17 illustrates the configuration of the Schering bridge. This bridge is used for measuring the capacitance, the dissipation factors, and the loss angles. The unknown capacitance is directly proportional to the known capacitance C_3. That is, the bridge equations are:

$$C_x = \frac{C_3 R_2}{R_1} \quad \text{and} \quad R_x = \frac{C_2 R_1}{C_3} \tag{9.34}$$

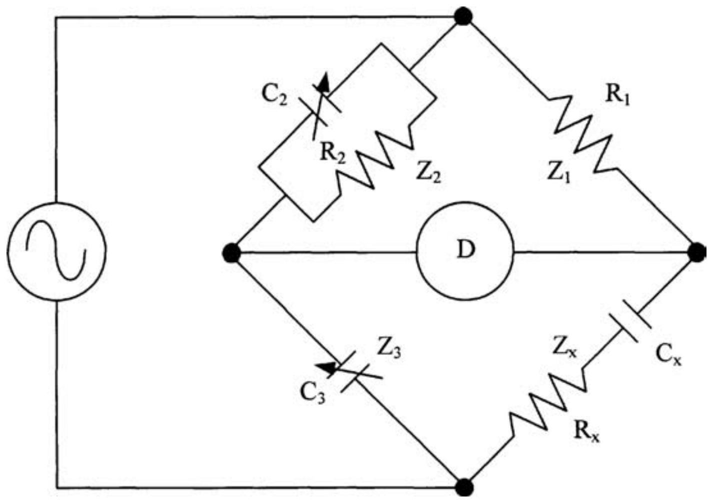


FIGURE 9.17 The Schering bridge. This bridge is particularly useful for measuring the capacitance, associated dissipation factors, and the loss angles. The unknown capacitance is directly proportional to the known capacitance C_1. The Schering bridge is frequently used as a high-voltage bridge with a high-voltage capacitor as C_1.

Usually, R_2 and R_3 are fixed, and C_2 and C_3 are made variable. Schering bridges are frequently used in highvoltage applications with high-voltage capacitor C_3. They are also used as high-frequency bridges since the use of two variable adjustment capacitors are convenient for precise balancing.

Further Information

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