

# 15

## Immittance Measurement

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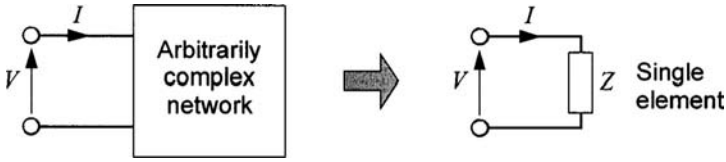
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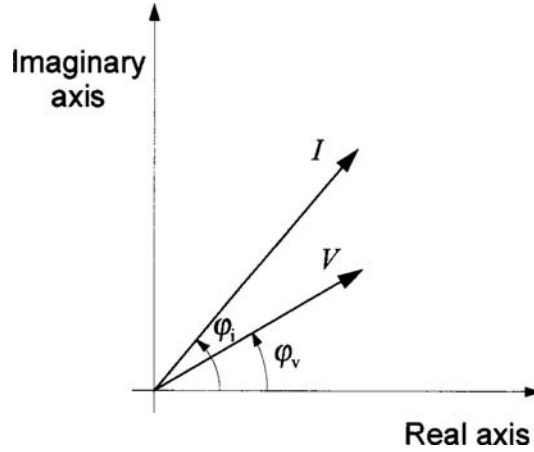
Electronic circuits consist of numerous elements that can be lumped, distributed, or a combination of both. The components are regarded as *lumped* if their size is much smaller than the signal wavelength. This condition holds for resistors, inductors, capacitors, transformers, diodes, transistors, or similar devices operating in printed circuits at frequencies up to a few hundred megahertz or even higher in small integrated circuits. In the microwave or millimeter-wave region, the elements and their connecting transmission lines must be considered as *distributed*. While in lumped circuits a change of voltage or current at one single point immediately affects these quantities at all other points, in distributed circuits the propagation properties now have to be taken into account. The same holds for long connecting cables even at lower frequencies.

To describe the effect of any element within an electronic circuit or of the connection of different circuits, the *immittance* is used as a characteristic quantity. It simply provides a relation of sinusoidal voltage and current at the terminals of the element as a function of frequency. The immittance therefore also characterizes arbitrarily complicated networks considered as one port. This is useful, since in practice the single elements are interconnected to networks. On the other hand, the elements themselves are not ideal. A resistor, for example, made of wound resistive wire, has parasitic components such as capacitance and inductance of winding and terminals. It must be represented by an equivalent circuit forming a complex network [1].

The word “immittance” was proposed by Bode [2] and is a combination of the words “impedance” and the reverse quantity called “admittance.” These terms do not only occur in electrodynamics but wherever wave propagation takes place — in acoustics as well as in elasticity. The emphasis of this chapter is on lumped networks and guided electromagnetic waves. Readers interested in more general propagation and scattering phenomena are referred to [3].



**FIGURE 15.1** An arbitrarily complex network can be replaced by its impedance for a given frequency without changing the electrical properties at the terminal.



**FIGURE 15.2** Voltage and current phasors in the complex plane.

## 15.1 Definitions

Assume a stable linear and time-invariant (LTI) network with only one port. Linearity and time independence are generally met for combinations of passive elements but also for active devices with small-signal driving under constant physical conditions (temperature, humidity, dimensions, etc.). In the steady state, a voltage  $v(t) = V_m \cos(\omega t + \phi_v)$  with amplitude  $V_m$  varying harmonically with the angular frequency  $\omega = 2\pi f$  which is applied to the terminal then only produces voltages and currents of the same frequency within the network (Figure 15.1). Using complex notation:

$$v(t) = \text{Re}\{V e^{j\omega t}\} \quad \text{with} \quad V = V_m e^{j\phi_v} \quad (15.1)$$

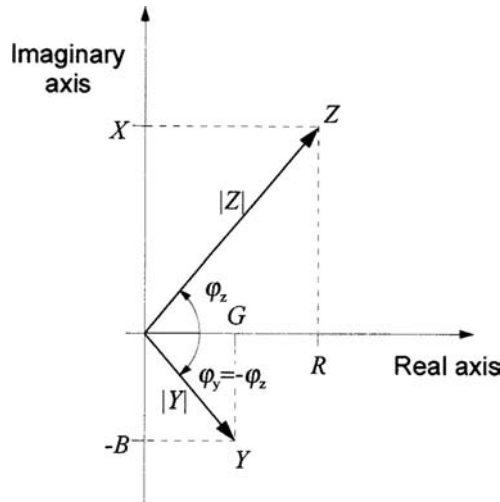
the current flowing into the network is given by:

$$i(t) = I_m \cos(\omega t + \phi_i) = \text{Re}\{I e^{j\omega t}\} \quad \text{with} \quad I = I_m e^{j\phi_i} \quad (15.2)$$

The phasors  $V$  and  $I$  are time independent and can be represented in the complex plane (Figure 15.2). Relating voltage and current at the terminal, the network is uniquely described by means of a complex frequency-dependent quantity, the impedance  $Z$ :

$$Z = \frac{V}{I} = \frac{V_m}{I_m} e^{j(\phi_v - \phi_i)} = |Z| e^{j\phi_z} \quad (15.3)$$

For a given frequency, an arbitrarily complex network within a circuit thus can be replaced by a single element without changing the electrical properties at the terminals. Sometimes it is more convenient to use the inverse of  $Z$ , the admittance  $Y$ :



**FIGURE 15.3** Representation of impedance and admittance in the complex plane showing the relations between rectangular and polar coordinates. Note that the units are different for each vector.

$$Y = \frac{1}{Z} = \frac{I}{V} = |Y|e^{j\phi_y} \quad \text{with} \quad \phi_y = \phi_i - \phi_v = -\phi_z \quad (15.4)$$

Both quantities are combined to form the word “immittance.” Figure 15.3 shows their representation in the complex plane. Equations 15.3 and 15.4 give the definition in polar coordinates. In data sheets, they are often written as:

$$|Z| \angle \phi_z, \quad |Y| \angle \phi_y \quad (15.5)$$

Using Euler’s identity  $e^{j\phi} = \cos \phi + j \sin \phi$ , one obtains in rectangular coordinates:

$$\begin{aligned} Z &= |Z| \cos \phi_z + j |Z| \sin \phi_z = R + jX \\ Y &= |Y| \cos \phi_y + j |Y| \sin \phi_y = G + jB \end{aligned} \quad (15.6)$$

From Figure 15.3, the following relations between rectangular and polar coordinate representation can be deduced immediately:

$$\begin{aligned} R &= |Z| \cos \phi_z & |Z| &= \sqrt{R^2 + X^2} & G &= |Y| \cos \phi_y & |Y| &= \sqrt{G^2 + B^2} \\ X &= |Z| \sin \phi_z & \phi_z &= \tan^{-1} \left( \frac{X}{R} \right) & B &= |Y| \sin \phi_y & \phi_y &= \tan^{-1} \left( \frac{B}{G} \right) \end{aligned} \quad (15.7)$$

The real parts are the resistance  $R$  and the conductance  $G$ . They indicate the losses within the network. The imaginary parts, which are termed reactance  $X$  and susceptance  $B$ , respectively, are a measure of the reactive energy stored in the network during one period. In general, all these quantities are frequency dependent.

Note that the correct sign of the imaginary parts must be used: the angle  $\phi$  is in the range of  $-180^\circ < \phi \leq 180^\circ$  and  $\phi < 0$  always corresponds to  $X, B < 0$ .

For elements with low losses, the loss angle  $\delta$  or loss factor  $D$  are often given instead of the phases  $\phi_z$  and  $\phi_y$ . They are always positive small quantities and tend to 0 for a lossless device

$$D = \tan \delta = \tan \left( \frac{\pi}{2} - |\varphi_z| \right) = \tan \left( \frac{\pi}{2} - |\varphi_y| \right) = \left| \frac{R}{X} \right| = \left| \frac{G}{B} \right| \quad (15.8)$$

The inverse quantity is the quality factor  $Q = 1/D$ . It involves a ratio of stored electric energy to power dissipated. A high  $Q$  indicates a nearly pure reactive component.

In high-power electronics, it is necessary to reduce losses on transmission lines and therefore avoid currents associated with reactive load components. To obtain a criterion for the application and efficiency of compensation techniques, a power factor is defined. From complex power representation:

$$P = VI^* = |P|(\cos \varphi + j \sin \varphi) \quad (15.9)$$

(the asterisk indicates the conjugate complex number) follows from Equations 15.3 and 15.4.

$$P = |I|^2 Z = |I|^2 |Z|(\cos \varphi_z + j \sin \varphi_z) = |V|^2 Y^* = |V|^2 |Y|(\cos \varphi_y - j \sin \varphi_y) \quad (15.10)$$

and since the effective power is given by the real part of  $P$ :

$$P_{\text{eff}} = \text{Re}\{P\} = |P|\cos \varphi \quad (15.11)$$

the power factor is:

$$\cos \varphi = \cos \varphi_z = \cos \varphi_y \quad (15.12)$$

In general, rms values are used for the phasors. Otherwise, a factor  $1/2$  has to be taken into account in Equations 15.9 and 15.10, since  $|P| = \frac{1}{2} V_m I_m$  for sinusoidal quantities.

It can also be seen from Equations 15.9 and 15.10 that the immittances are directly related to the apparent power:

$$|P| = |V||I| = |I|^2 |Z| = |V|^2 |Y| \quad (15.13)$$

## 15.2 Ideal Lumped Components

The immittances of the fundamental passive circuit elements are derived from their instantaneous voltage current relations using Equations 15.1 through 15.4 and the differentiation rules.

### Resistances

From Equation 15.14:

$$v(t) = Ri(t) \quad (15.14)$$

it follows  $V = RI$  and thus  $Z = R$  or  $Y = G$ . The immittance of a resistance is real and identical to its dc resistance or conductance.

### Inductances

Voltage and current are related via the differential equation:

$$v(t) = L \frac{di(t)}{dt} \quad (15.15)$$

with inductance  $L$ , from which follows that  $V = j\omega LI$  and

$$Z = j\omega L = jX_L, \quad Y = \frac{1}{j\omega L} = -j\frac{1}{X_L} = -jB_L \quad (15.16)$$

## Capacitances

From Equation 15.17:

$$i(t) = C \frac{dv(t)}{dt} \quad (15.17)$$

with capacitance  $C$ , it follows that  $I = j\omega CV$  and

$$Y = j\omega C = jB_C, \quad Z = \frac{1}{j\omega C} = -j\frac{1}{B_C} = -jX_C \quad (15.18)$$

The immittance of ideal inductors and capacitors is purely imaginary with different signs according to the phase shift of  $\pm 90^\circ$  between voltage and current. A general element or network is therefore called inductive or capacitive at a given frequency corresponding to the sign of the imaginary part of its impedance. Note, however, that the frequency dependence can be much more complicated than for these ideal elements and the impedance can even change several times between capacitive and inductive characteristic.

## 15.3 Distributed Elements

At high frequencies, the size of the elements may no longer be small compared to the signal wavelength. Propagation effects must then be taken into account and the components can no longer be described by means of simple lumped equivalent circuits. If at all possible, they are replaced by transmission line circuits, which are easier to characterize; they realize the required electrical properties more exactly within a defined frequency range.

### Transmission Lines

Assuming a simplifying transverse electromagnetic wave (TEM mode) with no field components in the propagation direction, voltages and currents can be uniquely defined and are given as solutions of the corresponding wave equations [4]:

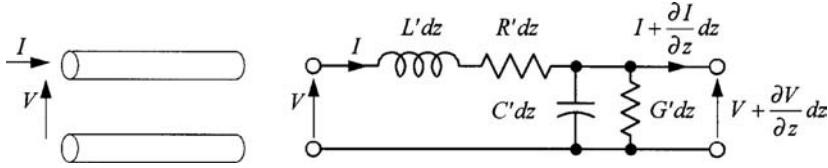
$$\frac{d^2 V}{dz^2} - \gamma^2 V = 0, \quad \frac{d^2 I}{dz^2} - \gamma^2 I = 0 \quad (15.19)$$

They vary along the line in the  $z$ -direction according to:

$$V(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}, \quad I(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \quad (15.20)$$

These solutions are sums of forward ( $e^{-\gamma z}$ ) and backward ( $e^{\gamma z}$ ) traveling waves with amplitudes  $V_0^+$ ,  $I_0^+$  and  $V_0^-$ ,  $I_0^-$  and a propagation constant:

$$\gamma = \sqrt{(R' + j\omega L')(G' + j\omega C')} \quad (15.21)$$



**FIGURE 15.4** Equivalent circuit of a differential length of transmission line. The wave equations can be obtained by simply applying Kirchhoff's laws to voltages and currents.

The equivalent circuit of the transmission line is shown in Figure 15.4. The energy storage in the electric field is accounted for by the distributed shunt capacitance  $C'$  per unit length, while the effect of the magnetic field is represented by the series inductance  $L'$  per unit length. The series resistance  $R'$  per unit length and the shunt conductance  $G'$  per unit length represent the power losses in the conductors and in the dielectric, respectively. The amplitudes of voltage and current are related by means of the characteristic impedance  $Z_0$ :

$$Z_0 = \frac{V^+}{I^+} = -\frac{V^-}{I^-} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \quad (15.22)$$

Of special interest for the use within a network is the input impedance  $Z_{in}$  of the transmission line. It depends also on the termination  $Z_L$  at the other end of the line. For a transmission line of length  $l$ , it is given by:

$$Z_{in} = Z_0 \frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \quad (15.23)$$

that is, a transmission line transforms the impedance  $Z_L$  into  $Z_{in}$  at the input.

A quantity more suitable to wave propagation and measurement at high frequencies is the reflection coefficient  $\Gamma$ . It is defined by the relation of the voltages associated with forward and backward traveling waves. At the end of the line, using  $V(l) = Z_L I(l)$ , one finds:

$$\Gamma = \frac{V_0^- e^{\gamma l}}{V_0^+ e^{-\gamma l}} = \frac{Z_L - Z_0}{Z_L + Z_0} \quad (15.24)$$

For devices that support quasi or strong non-TEM waves like microstrip lines, hollow waveguides, dielectric and optical waveguides, a voltage cannot be uniquely defined. That is why several definitions of the characteristic impedance  $Z_0$  exist [5].

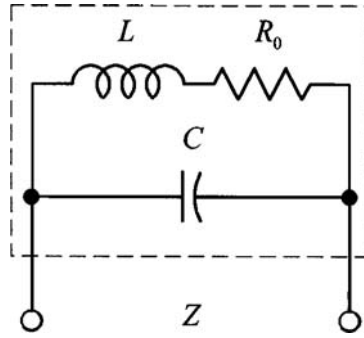
## 15.4 Interconnections and Graphical Representations

Since Kirchhoff's laws for voltages and currents also holds for complex quantities, the rules for series and parallel connections of resistances and susceptances in the dc case apply as well for immittances.

Series connection: 
$$Z = \sum_i Z_i \quad \frac{1}{Y} = \sum_i \frac{1}{Y_i} \quad (15.25)$$

Parallel connection: 
$$Y = \sum_i Y_i \quad \frac{1}{Z} = \sum_i \frac{1}{Z_i} \quad (15.26)$$

As an example, consider a simplified equivalent circuit of a resistor with the nominal value  $R_0$  (Figure 15.5). Gradually using the rules for series and parallel connection and the impedances for



**FIGURE 15.5** The simple equivalent circuit of a wire-wound resistor with nominal value  $R_0$ , inductance of the winding  $L$ , and capacitance of winding and terminal  $C$ . It is valid for a wide frequency range.

inductances (Equation 15.16) and capacitances (Equation 15.18), the impedance of the real resistor with parasitic elements as given leads to:

$$Z = \frac{R_0 + j\omega L}{1 - \omega^2 LC + j\omega R_0 C} \quad (15.27)$$

The magnitude and phase of  $Z/R_0$  as a function of  $\omega/\omega_0$  are shown in Figure 15.6 with  $\omega_0 = 1/\sqrt{LC}$  as the resonant frequency defined by the parasitic elements, which might be caused by the windings of a wire-wound resistor. The network is inductive for low ( $\phi_z > 0$ ) and capacitive for high frequencies. An alternative representation is to plot real and imaginary parts in the impedance plane with the frequency as parameter as indicated by the labels (Figure 15.7). This version, called the *locus*, is very suitable to see immittance changes caused by parameters like frequency or adjustable elements within the network. Note that both real and imaginary parts are parameter dependent and vary with frequency.

In high-frequency applications, one obtains the impedance more easily from the reflection coefficient. Rewriting Equation 15.24 in the form:

$$\Gamma = \frac{\bar{Z}_L - 1}{\bar{Z}_L + 1} \quad \text{with} \quad \bar{Z}_L = \frac{Z_L}{Z_0} \quad (15.28)$$

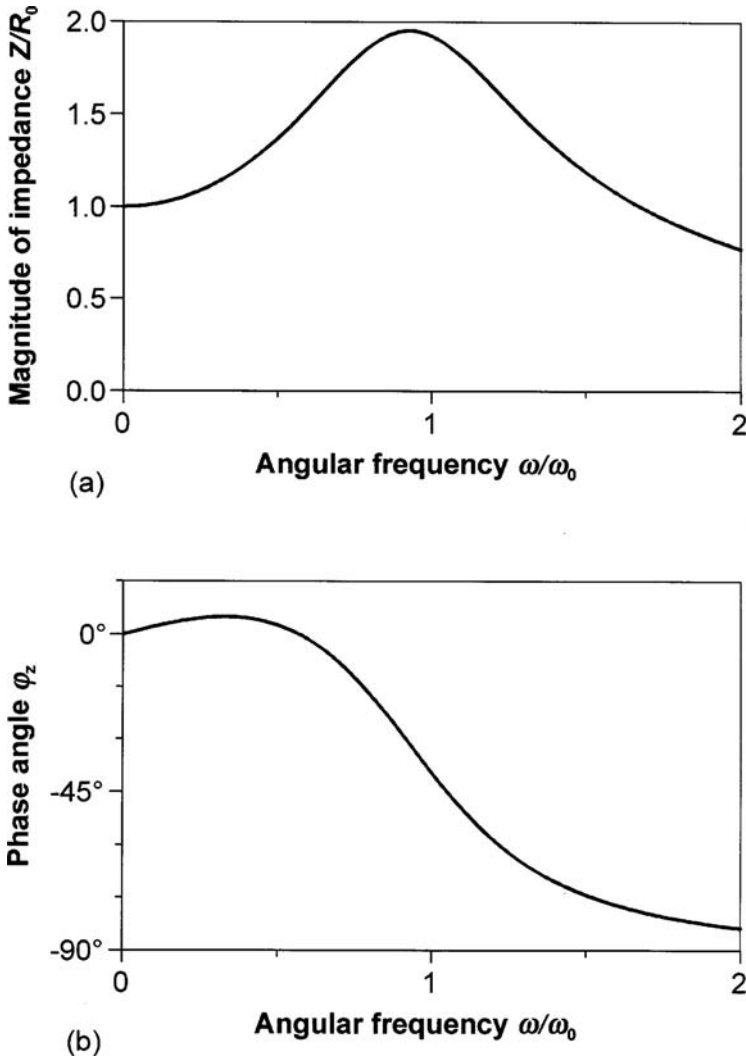
defines a transformation of which the graphical representation has been called the Smith chart (Figure 15.8). It can be regarded as two coordinate systems lying one on top of the other. The reflection coefficient is given in polar coordinates around the center, the circles give the real and imaginary part of the associated impedance. The Smith chart is very useful for solving transmission line and waveguide impedance matching problems [6].

## 15.5 Measurement Techniques

Since immittances are complex quantities, one must determine two parameters: magnitude and phase or real and imaginary part, described as vector measurements. There exist several techniques depending on frequency range and required accuracy [7].

### Current–Voltage Methods

A simple way to measure immittances follows directly from the defining Equation 15.3. Applying a well-known sinusoidal voltage to the terminal and measuring magnitude and phase of the current gives the desired quantity (Figure 15.1). However, the internal impedance  $Z_A$  of the ammeter should be known exactly and the unknown impedance is then given by:



**FIGURE 15.6** Normalized magnitude (a) and phase (b) of the impedance of a wire-wound resistor varying with frequency.  $\omega_0$  is the resonant frequency defined by the parasitic elements.

$$Z = \frac{V}{I} - Z_A \quad (15.29)$$

In practical applications, impedances below 1000  $\Omega$  are measured by passing a predetermined current through the unknown device and measuring the voltage across it. Phase angle information is obtained by comparing the relative phase between voltage and current by means of a phase detector [8].

A variant on this method using only the better practicable voltage measurements is shown in [Figure 15.9](#). The accurately known resistor  $R$  must be small compared to  $Z_x$  and to the internal resistance of  $V_2$ . One finds that:

$$Z_x = \left( \frac{V_1}{V_2} - 1 \right) R, \text{ or } Z_x \approx \frac{V_1}{V_2} R \text{ if } R \ll |Z_x| \quad (15.30)$$



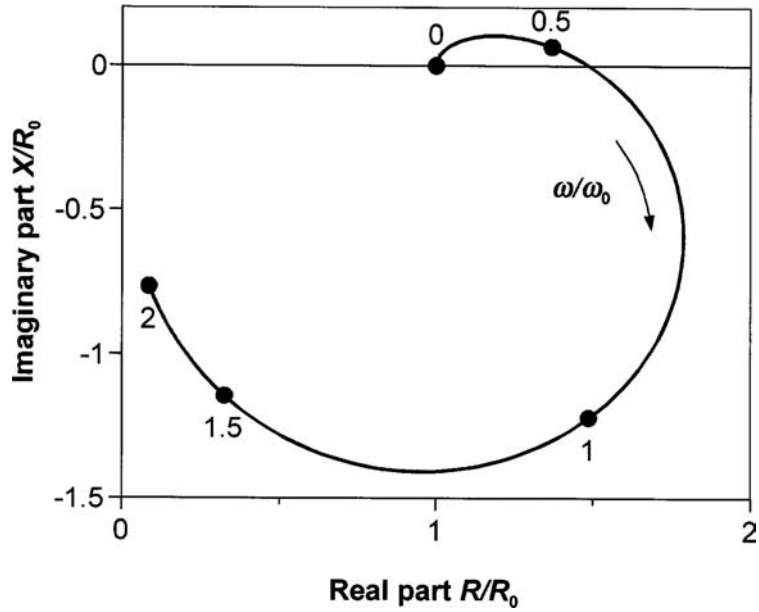


FIGURE 15.7 Normalized impedance of a wire-wound resistor in the complex plane. The arrow indicates the direction of increasing frequency.

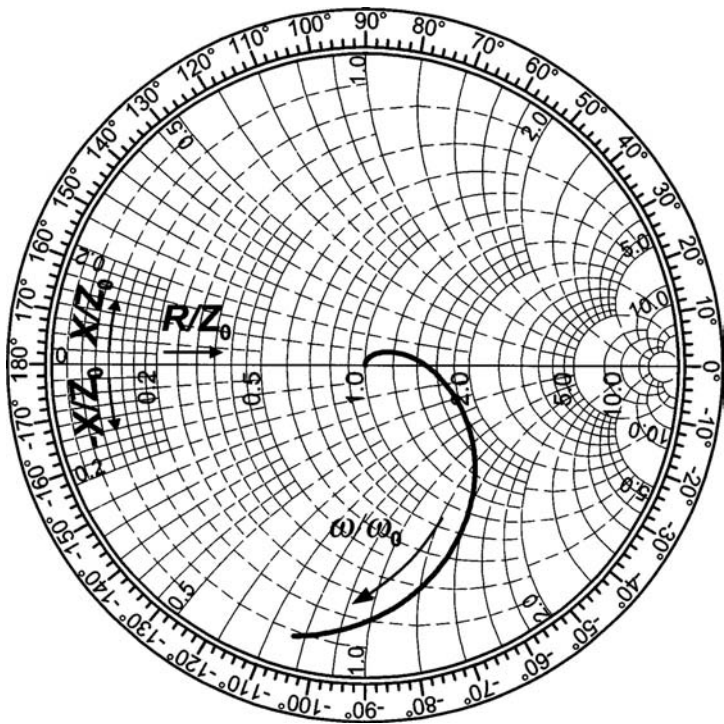
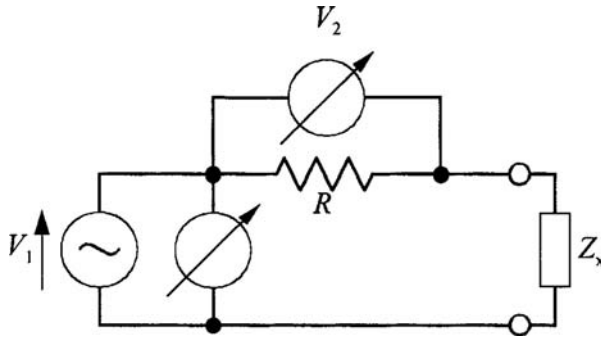
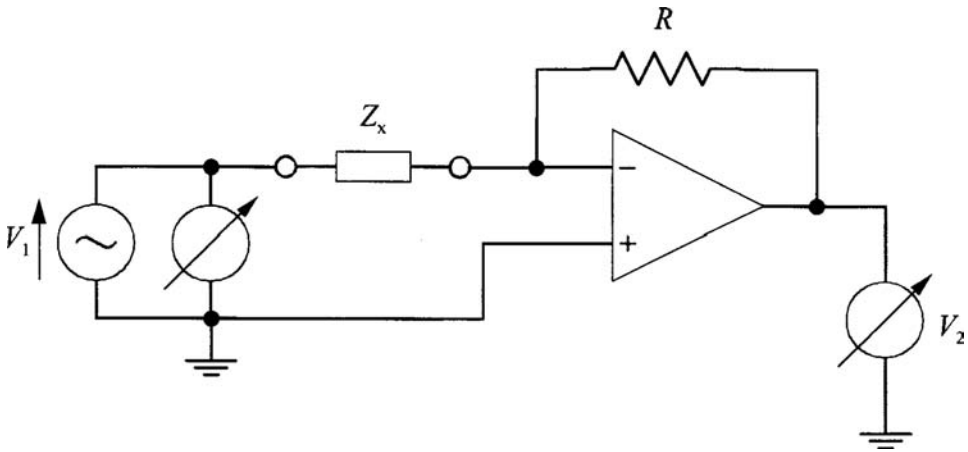


FIGURE 15.8 Smith chart representation of the impedance of a wire-wound resistor.



**FIGURE 15.9** Determination of an impedance  $Z_x$  by phase-sensitive voltage measurements, only using a well-known resistor  $R$ .



**FIGURE 15.10** Impedance measurement with an inverting operational amplifier circuit. Its advantages are high input and low output resistance.

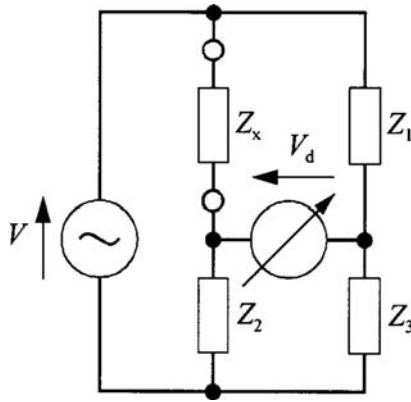
The measurement can be enhanced using an operational amplifier with high input and low output resistance in an inverting circuit (Figure 15.10). The unknown is then given by

$$Z_x = -\frac{V_1}{V_2} R \quad (15.31)$$

Practical implementations use operational amplifiers as part of an autobalancing bridge; see [7, 8].

## Bridge Methods

Alternating current bridges are low-cost standard laboratory devices to measure impedances over a wide frequency range from dc up to 300 MHz with very high precision (Figure 15.11). A comprehensive survey is given in [1]. Their main advantage is that only a zero indicator in the diagonal branch is necessary. For this reason, the internal impedance does not influence the accuracy and the null point can be detected with a high-sensitivity ac galvanometer as well as with headphones in the audio frequency range.



**FIGURE 15.11** Impedance measurement by bridge methods. The bridge is balanced when the voltage  $V_d$  across the diagonal branch is adjusted to zero by tuning  $Z_1$ ,  $Z_2$ , or  $Z_3$ .

If the bridge is balanced, the unknown immittance is given by:

$$Z_x = \frac{Z_1}{Z_3} Z_2 \quad \text{or} \quad Y_x = \frac{Z_3}{Z_1} Y_2 \quad (15.32)$$

Since the quantities are complex, Equation 15.32 involves the adjustment of two parameters: magnitude and phase:

$$|Z_x| = \left| \frac{Z_1}{Z_3} \right| |Z_2|, \quad \phi_x = \phi_1 - \phi_3 + \phi_2 \quad (15.33)$$

or real and imaginary parts, respectively.

An important property of an impedance bridge is the sensitivity  $\epsilon$ :

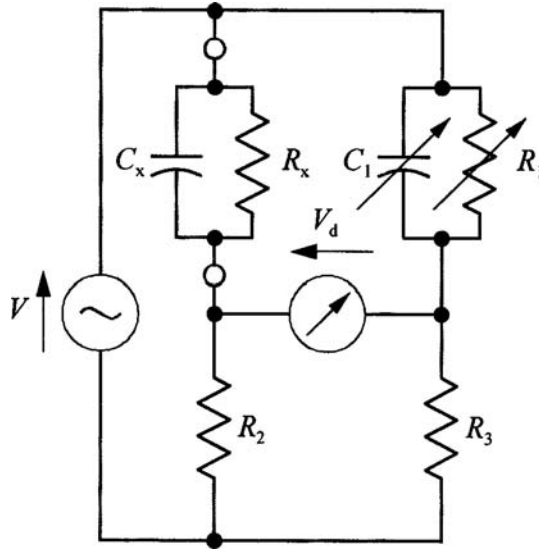
$$\epsilon = \left| \frac{\partial V_d}{\partial Z_x} \right| = V \frac{Z_2}{(Z_2 + Z_x)^2} \quad (15.34)$$

or (independent of  $Z_x$ )

$$\epsilon = V \frac{Z_3^2}{Z_2 (Z_1 + Z_3)^2} \quad (15.35)$$

in the vicinity of zero crossing when the bridge is balanced.

The precision of the measurement not only depends on the exact zero adjustment, which can be enhanced by choosing the elements and the voltage according to Equation 15.35 to obtain a high sensitivity, but also on the realization of  $Z_1 \dots Z_3$ . Mostly, these are connections of resistors and capacitors. Inductors are avoided because they always have a resistive component and it is difficult and expensive to manufacture inductors with exactly defined and reproducible electrical properties. There exist various types of bridges depending on how the elements are designed and interconnected. To choose the correct configuration, it must be known whether the unknown impedance is capacitive or inductive; otherwise, a zero adjustment is not always possible since the balancing condition cannot be fulfilled. Bridges are



**FIGURE 15.12** Wheatstone bridge for the capacitance and dissipation factor measurement of capacitors. The balancing condition is frequency independent. The resistor  $R_1$  and the capacitor  $C_1$  must be tuned successively until the bridge is balanced.

therefore principally used to measure capacitances and inductances as well as loss and quality factors of capacitors and coils. Since magnitude and phase conditions must be matched simultaneously, two elements must be tuned. To obtain a wide measurement range, the variable elements are designed as combinations of switchable and tunable capacitors and resistors. The sensitivity of the zero indicator can be changed for global search and final adjustment. Unfortunately, magnitude and phase cannot be adjusted independently of each other. If the balancing is performed by hand, a suitable strategy is to search the minimum voltage by tuning each element successively.

Frequently used bridges are the Wheatstone bridge (Figure 15.12) for the measurement of lossy capacitances, and the Hay bridge (Figure 15.13) to determine inductivity and quality factor of coils. Because of its symmetrical structure, the balancing condition for the Wheatstone bridge is simply:

$$R_x = \alpha R_1, \quad C_x = \alpha C_1, \quad \alpha = \frac{R_3}{R_2} \quad (15.36)$$

which is independent of frequency.

The measurement of a coil with the Hay bridge requires that:

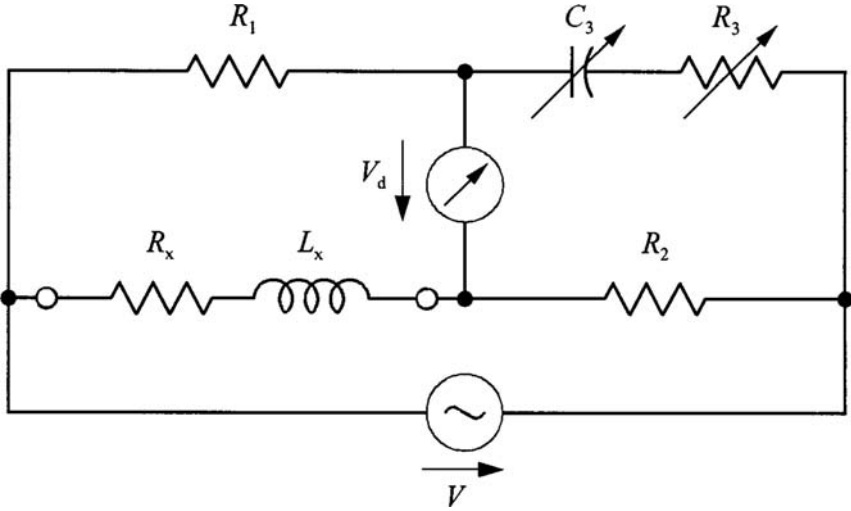
$$R_x + j\omega L_x = \frac{j\omega C_3 R_1 R_2}{1 + j\omega R_3 C_3} = \frac{\omega^2 C_3^2 R_1 R_2 R_3}{1 + (\omega R_3 C_3)^2} + j\omega \frac{C_3 R_1 R_2}{1 + (\omega R_3 C_3)^2} \quad (15.37)$$

from which the quality factor is obtained as:

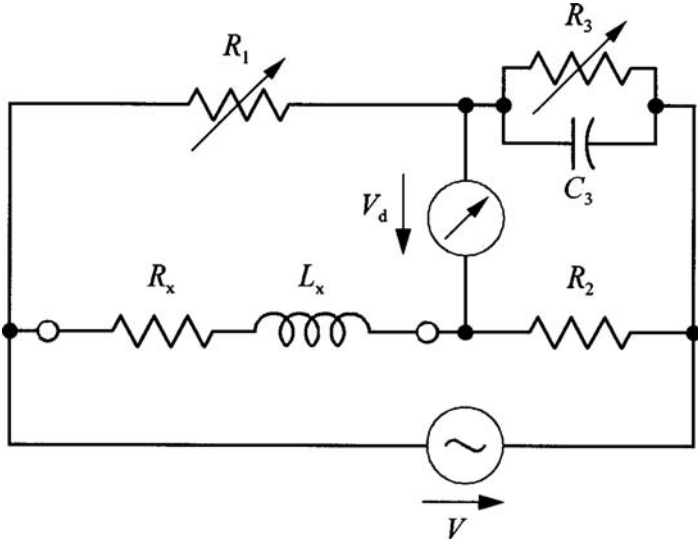
$$Q = \frac{\omega L_x}{R_x} = \frac{1}{\omega R_3 C_2} \quad (15.38)$$

The inductance of high- $Q$  coils can be determined frequency independent since

$$L_x \approx R_1 R_2 C_3 \quad \text{if } (\omega R_3 C_3)^2 \ll 1 \quad (15.39)$$



**FIGURE 15.13** Hay bridge for the measurement of the inductance and the quality factor of coils. If  $Q$  is sufficiently high, the inductance can be determined nearly frequency independent.

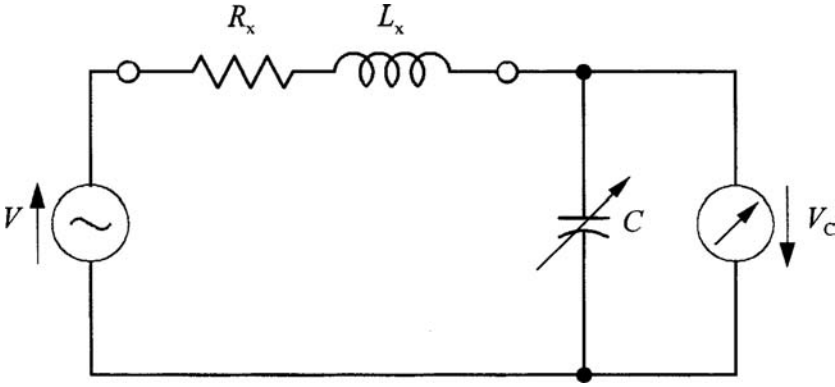


**FIGURE 15.14** Maxwell bridge with simple and frequency-independent balancing conditions. Despite these advantages, it is not recommended for high- $Q$  coils because of a very large  $R_1$  value.

A very interesting alternative is the Maxwell bridge (Figure 15.14), since it requires only resistors as variable elements, which can be manufactured with high precision. The balancing is frequency independent and leads to:

$$R_x = \frac{R_1 R_2}{R_3}, \quad L_x = R_1 R_2 C_3, \quad Q = \omega C R_1 \tag{15.40}$$

Nevertheless, the Hay bridge is preferred for high- $Q$  coils, because a very large value of  $R_1$  is required for the Maxwell bridge leading to a disadvantageous balancing [9].



**FIGURE 15.15** Coil as part of a resonance circuit to determine inductance and quality factor. The capacitor  $C$  is tuned to maximum voltage  $V_c$ .

### Resonant Method

Using the coil as part of a resonance circuit as in Figure 15.15 and tuning  $C$  to maximum voltage, the quality factor can be measured directly as:

$$Q = \left| \frac{V_{c, \max}}{V} \right| = \frac{1}{\omega R_x C} \quad (15.41)$$

The unknown inductance is then obtained from the test frequency by means of the resonance condition:

$$L_x = \frac{1}{\omega^2 C} \quad (15.42)$$

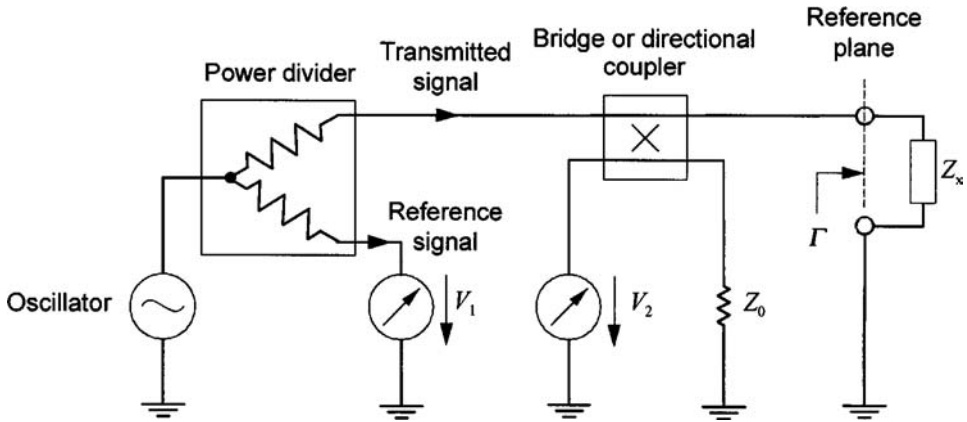
If a capacitor with sufficiently low losses is used,  $Q$  values as high as 1000 can be measured.

### Network Analysis Methods

#### Frequency Domain

In the case of distributed elements, measurements of currents and voltages depend on the position and are often not directly applicable to high-frequency devices like waveguides or microstrip lines. For that reason the determination of immittances is derived from measuring the reflection coefficient. Equation 15.23 shows the importance of defining a proper measurement plane. This is the cross-section of the line or waveguide perpendicular to the direction of propagation at a definite length  $l_0$ , where the reflection coefficient has to be measured. It can then be transformed along the line toward load or source using this relation or the Smith chart. Exact microwave measurements are very sophisticated and need a lot of practical experience. Further details can be found in the literature [5, 10–12].

Automated and precise immittance measurements over a wide frequency range are best carried out with a vector network analyzer [11]. Unfortunately, this is also the most expensive method. The principle of measurement is shown in Figure 15.16. A power divider splits the incident signal into a transmitted and a reference part. The directional bridge or coupler separates forward and backward traveling waves, and the reflected signal now appears in the branch with the phase-sensitive voltmeter  $V_2$ . Using a bridge with impedances matched to the line ( $Z_1 = Z_2 = Z_3 = Z_0$ ), the voltage in the diagonal branch is given by (Figure 15.11):



**FIGURE 15.16** Schematic of network analyzer measurements. The voltage ratio  $V_2/V_1$  of reflected wave and reference signal is proportional to the reflection coefficient  $\Gamma$ . The impedance  $Z_x$  can then be computed.

$$V_d = \frac{Z_x - Z_0}{2(Z_x + Z_0)} V = V_2 \quad (15.43)$$

and thus the reflection coefficient:

$$\Gamma = \frac{Z_x - Z_0}{Z_x + Z_0} = \alpha \frac{V_2}{V_1} \quad (15.44)$$

is directly proportional to the voltage ratio.

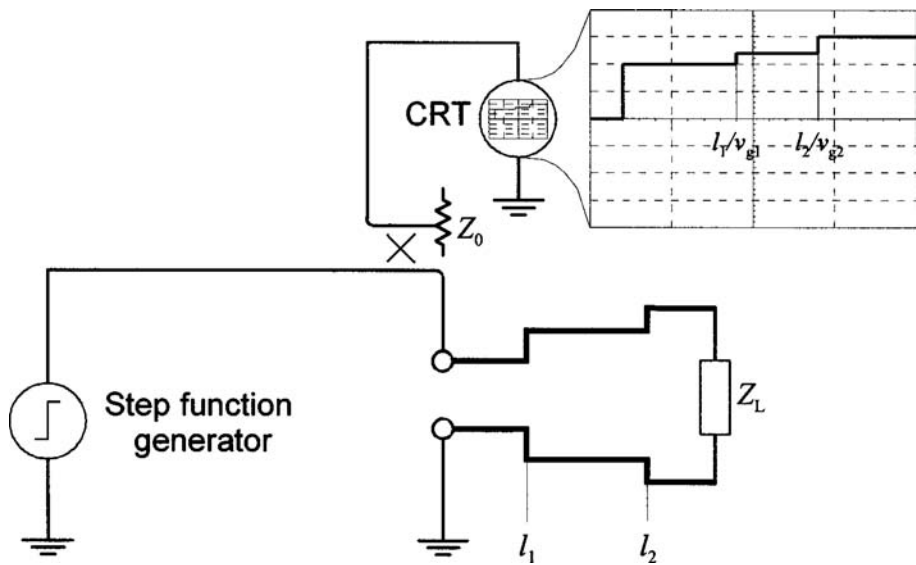
Network analyzers use an automatic error correction to eliminate the effect of internal and external couplers and junctions. Because of that, a calibration procedure with standard terminations is necessary. These terminations must be manufactured very precisely, since they define the measurement plane and determine the overall measurement error.

## Time Domain

It is often necessary to locate an impedance step, whether to find out the distance of a cable defect or to track down the origin of reflections within a connection. To this end, high-performance vector network analyzers have a Fourier transform procedure. But there also exist cheaper time domain reflectometers (TDR) [13, 14]. They use an incident step or impulse signal (Figure 15.17) and the reflected signal is separated by means of a directional coupler and displayed on a CRT in the time domain. From the shape of the signal, the impedance step can be localized by means of the time delay:

$$l = \frac{1}{2} v_g t \quad (15.45)$$

with  $v_g$  as signal or group velocity on the line varying from step to step. Characteristic and magnitude of the impedance can only be estimated, since phase information is usually not available. TDR measurements are restricted to the localization of impedance steps and not to be recommended for exact measurements. Moreover, additional pulse deformations occur in dispersive waveguides.



**FIGURE 15.17** Detection and measurement of impedance steps on a line or waveguide with a time domain reflectometer (TDR). Since phase information is usually not available, the characteristics and magnitudes of the impedances can only be estimated. Notice that the group or signal velocity  $v_g$  varies from step to step.

## 15.6 Instrumentation and Manufacturers

A broad range of instrumentation for measuring immittance is available. Some of these instruments are included in Table 15.1. Table 15.2 provides the names and addresses of some companies that produce immittance-measuring instrumentation.

**TABLE 15.1** Instruments for Immittance Measurements

Manufacturer	Model Number	Description
Agilent Technologies	E5100	Network analyzers 10 kHz–300 MHz
Agilent Technologies	ENA Series	Network analyzers 300 kHz–8.5 GHz
Agilent Technologies	PNA Series	Network analyzers 300 kHz–67 GHz
Agilent Technologies	8510C	Network analyzer systems 45 MHz–110 GHz
Agilent Technologies	4263B	LCR meter 100 Hz–100 kHz
Agilent Technologies	4284A	LCR meter 20 Hz–1 MHz
Agilent Technologies	4285A	LCR meter 75 kHz–30 MHz
Anritsu	ME7808A	Vector network analyzer 40 MHz–110 GHz
Fluke	PM 6303A	Automatic RCL meter
Fluke	PM 6304	Automatic RCL meter
Keithley	3321	LCZ meter, 4 test frequencies to 100 kHz
Keithley	3322	LCZ meter, 11 test frequencies to 100 kHz
Keithley	3330	LCZ meter 40 Hz–100 kHz (201 test frequencies)
Quadtech	1710	LCR meter 20 Hz–200 kHz
Quadtech	7000	LCR meter Series 10 Hz–2 MHz
Quadtech	1910	Inductance analyzer 20 Hz–1 MHz
Rohde & Schwarz	ZVR	Network analyzer
SST	SR715/720	LCR meter
TTi	LCR400	Precision LCR bridge
Voltech	ATi	Transformer tester



**TABLE 15.2** Companies Producing Immittance Measurement Equipment

Agilent Headquarters 395 Page Mill Rd. P.O. Box 10395 Palo Alto, CA 94303 Tel: (877) 4-Agilent www.agilent.com	Rohde & Schwarz, Inc. 7150-K Riverwood Drive Columbia, MD 21046 Tel: (410) 910-7800 www.rsd.de/www/dev_center.nsf/USA
Anritsu Co. 1155 East Collins Blvd. Richardson, TX 75081 Tel: (800) ANRITSU (267-4878) www.global.anritsu.com	SRS Stanford Research Systems 1290-D Reamwood Ave. Sunnyvale, CA 94089 Tel: (408) 744-9040 www.thinksrs.com
Fluke Corporation 6929 Seaway Boulevard P.O. Box 9090 Everett, WA 98206 Tel: (800) 44-FLUKE www.fluke.com	TTi (Thurlby Thandar Instruments Ltd.) Glebe Road Huntingdon Cambs. PE29 7DR U.K. Tel: +44-1480-412451 www.tti-test.com
Keithley Instruments, Inc. 28775 Aurora Road Cleveland, OH 44139 Tel: (800) 552-1115 www.keithley.com	Voltech Instruments, Inc. 11637 Kelly Road Suite 306 Fort Myers, FL 33908-2544 Tel: (239) 437-0494 www.voltech.com
QuadTech, Inc. 5 Clock Tower Place Suite 210 East Maynard, MA 01754 Tel: (800) 253-1230 www.quadtechinc.com	

## Defining Terms

**Admittance (Y):** The reciprocal of impedance.

**Immittance:** A response function for which one variable is a voltage and the other a current. Immittance is a general term for both impedance and admittance, used where the distinction is irrelevant.

**Impedance (Z):** The ratio of the phasor equivalent of a steady-state sine-wave voltage to the phasor equivalent of a steady-state sine-wave current. The real part is the *resistance*, the imaginary part is the *reactance*.

**Phasor:** A complex number, associated with sinusoidally varying electrical quantities, such that the absolute value (modulus) of the complex number corresponds to either the peak amplitude or root-mean-square (rms) value of the quantity, and the phase (argument) to the phase angle at zero time. The term “phasor” can also be applied to impedance and related complex quantities that are not time dependent.

**Reflection coefficient:** At a given frequency, at a given point, and for a given mode of propagation, the ratio of voltage, current, or power of the reflected wave to the corresponding quantity of the incident wave.

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