Department of **P**hysics
Indian **I**nstitute of **T**echnology **M**adras

M.Sc. Entrance Test 2023

Date: June 1, 2023 Time: 11:30 AM– 12:30 PM EAT (2:00 PM-3:00 PM IST)

Instructions

1. This test contains three parts, viz. Parts I, II and III.
2. Part I contains ten questions, Part II contains five, and Part III contains five questions.
3. All the questions in Parts I and II are of the multiple choice type.
4. However, note that, while only one amongst the four choices is correct in Part I, more than one choice can be correct in Part II. In these two parts, clearly write the question number then write your chosen answers. No workings are required.
5. Part III requires you to provide brief answers including workings.
6. There are no negative marks.
7. Each correct answer in Part I will be awarded one mark. No marks will be given if you give more than one choice.
8. Each question in Part II carries two marks. But, you will have to give *all* the correct answers in a particular question to get the two marks. No marks will be awarded if you give an incorrect answer.
9. Each question in Part III carries two marks.
10. You can use a calculator.
11. Please scan your answers as a single pdf file and send to libby@physics.iitm.ac.in at the end of the test.

Part I

1. The eigenvalues of

$$A = \begin{pmatrix} 3 & i & 0 \\ -i & 3 & 0 \\ 0 & 0 & 6 \end{pmatrix},$$

are

(A) 2, 4 and 6
(B) $2i$, $4i$ and 6
(C) $2i$, 4 and 8
(D) 0, 4 and 8

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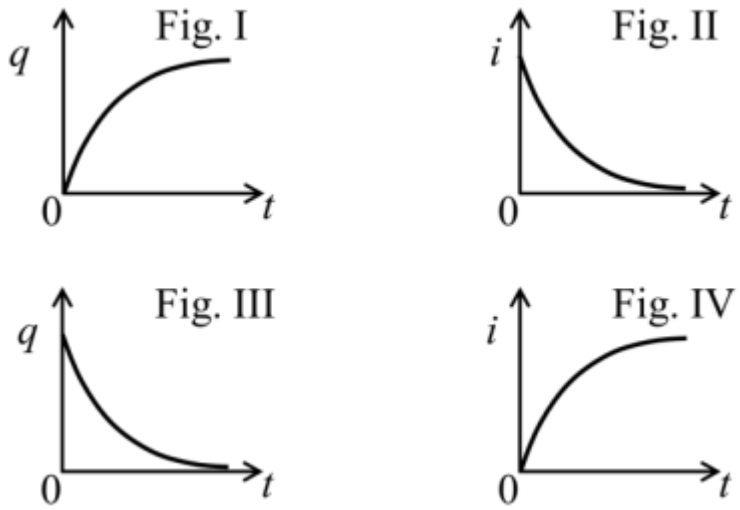
2. During the charging of a capacitor C in a series RC circuit, the typical variations in the magnitude of the charge $q(t)$ deposited on one of the capacitor plates, and the current $i(t)$ in the circuit, respectively are best represented by



(A) Fig. I and Fig. IV
(B) Fig. I and Fig. II
(C) Fig. III and Fig. IV
(D) Fig. III and Fig. II

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3. If the motion of a particle is described by $x = 5\cos(8\pi t)$, $y = 5\sin(8\pi t)$ and $z = 5t$, then the trajectory of the particle is

(A) circular
(B) elliptical
(C) helical
(D) spiral

4. Isothermal compressibility is given by

   (A) $\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$

   (B) $\frac{1}{P}\left(\frac{\partial P}{\partial V}\right)_T$

   (C) $-\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$

   (D) $-\frac{1}{P}\left(\frac{\partial P}{\partial V}\right)_T$

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5. The function $f(z) = \frac{8z}{z^2+9}$ is continuous everywhere except at

   (A) $z = 0$

   (B) $z = \pm 9$

   (C) $z = \pm 9i$

   (D) $z = \pm 3i$

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6. The Fermi-Dirac distribution function $[n(\epsilon)]$ is given by which of the following. Here $k_{\rm B}$, T and $\epsilon_{\rm F}$ denote the Boltzmann constant, temperature and Fermi energy respectively.

   (A) $n(\epsilon) = \frac{1}{e^{\left(\frac{1}{k_{\rm B}T}[\epsilon-\epsilon_{\rm F}]\right)}-1}$

   (B) $n(\epsilon) = \frac{1}{e^{\left(\frac{1}{k_{\rm B}T}[\epsilon_{\rm F}-\epsilon]\right)}-1}$

   (C) $n(\epsilon) = \frac{1}{e^{\left(\frac{1}{k_{\rm B}T}[\epsilon-\epsilon_{\rm F}]\right)}+1}$

   (D) $n(\epsilon) = \frac{1}{e^{\left(\frac{1}{k_{\rm B}T}[\epsilon_{\rm F}-\epsilon]\right)}+1}$

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7. Consider a hemi-spherical surface of radius R which is placed in the uniform electric field $\mathbf{E}$. If the field $\mathbf{E}$ is parallel to the axis of the hemi-spherical surface, then the electric flux passing through the hemi-spherical surface is

   (A) $\pi\, R^2\, |\mathbf{E}|$

   (B) $2\,\pi\, R^2\, |\mathbf{E}|$

   (C) $\pi\, R\, |\mathbf{E}|$

   (D) $2\,\pi\, R^3\, |\mathbf{E}|$

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8. Which of the following is an impossible magnetic field $\vec{\mathbf{B}}$?

   (A) $\vec{\mathbf{B}} = 3x^2z^2\hat{\mathbf{e}}_x - 2xz^3\hat{\mathbf{e}}_z$

   (B) $\vec{\mathbf{B}} = -2xy\hat{\mathbf{e}}_x + yz^2\hat{\mathbf{e}}_y + \left(2yz - \frac{z^3}{3}\right)\hat{\mathbf{e}}_z$

   (C) $\vec{\mathbf{B}} = (xz + 4y)\,\hat{\mathbf{e}}_x - yx^3\hat{\mathbf{e}}_y + \left(x^3z - \frac{z^2}{2}\right)\hat{\mathbf{e}}_z$

   (D) $\vec{\mathbf{B}} = -6xz\hat{\mathbf{e}}_x + 3yz^2\hat{\mathbf{e}}_y$

9. For a quantum particle confined inside a cubic box of side L, the ground state energy is given by E_0. The energy of the first excited state is

   (a) $2E_0$

   (b) $\sqrt{2}E_0$

   (c) $3E_0$

   (d) $6E_0$

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10. A unit vector perpendicular to the plane containing $\vec{\mathbf{A}} = \hat{\mathbf{e}}_x + \hat{\mathbf{e}}_y - 2\hat{\mathbf{e}}_z$ and $\vec{\mathbf{B}} = 2\hat{\mathbf{e}}_x - \hat{\mathbf{e}}_y + \hat{\mathbf{e}}_z$

   (A) $\frac{1}{\sqrt{26}}\left(-\hat{\mathbf{e}}_x + 3\hat{\mathbf{e}}_y - 4\hat{\mathbf{e}}_z\right)$

   (B) $\frac{1}{\sqrt{19}}\left(-\hat{\mathbf{e}}_x + 3\hat{\mathbf{e}}_y - 3\hat{\mathbf{e}}_z\right)$

   (C) $\frac{1}{\sqrt{35}}\left(-\hat{\mathbf{e}}_x + 5\hat{\mathbf{e}}_y - 3\hat{\mathbf{e}}_z\right)$

   (D) $\frac{1}{\sqrt{35}}\left(-\hat{\mathbf{e}}_x - 5\hat{\mathbf{e}}_y - 3\hat{\mathbf{e}}_z\right)$

Part II

1. Which of the following statement(s) is/are true?

   (A) Newton's laws of motion and Maxwell's equations are both invariant under Lorentz transformations.

   (B) Newton's laws of motion and Maxwell's equations are both invariant under Galilean transformations.

   (C) Newton's laws of motion are invariant under Galilean transformations and Maxwell's equations are invariant under Lorentz transformations.

   (D) Newton's laws of motion are invariant under Lorentz transformations and Maxwell's equations are invariant under Galilean transformations.

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2. Consider a charged particle moving with the velocity $\mathbf{v}$ in a region containing electric ($\mathbf{E}$) and magnetic ($\mathbf{B}$) fields. If the particle remains unaccelerated, then

   (A) $\mathbf{E}$ must be perpendicular to $\mathbf{B}$

   (B) $\mathbf{v}$ must be perpendicular to $\mathbf{E}$

   (C) $\mathbf{v}$ must be perpendicular to $\mathbf{B}$

   (D) $|\mathbf{E}|$ must be equal to $|\mathbf{v}|\,|\mathbf{B}|$

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3. For an underdamped harmonic oscillator with velocity $v(t)$,

   (A) Rate of energy dissipation varies linearly with $v(t)$

   (B) The reduction in the oscillator frequency, compared to the undamped case, is independent of $v(t)$

   (C) Rate of energy dissipation varies as the square of $v(t)$

   (D) The amplitude decays exponentially

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4. The gradient of a scalar field $S(x, y, z)$ has the following characteristics

   (A) The line integral of a gradient is path dependent

   (B) Closed line integral of a gradient is zero

   (C) Gradient of S is a measure of the maximum rate of change in the field S

   (D) Gradient of S is a scalar quantity

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5. If P and Q are Hermitian matrices, which of the following is/are Hermitian?

   (A) $\mathsf{PQ} + \mathsf{QP}$

   (B) $i(\mathsf{PQ} - \mathsf{QP})$

   (C) $\mathsf{PQ} - \mathsf{QP}$

   (D) PQ

Part III

1. If the diameter of the Earth is increased by 4% without changing the mass, then the length of the day is how many hours? State any assumptions you make. **[2 marks]**

2. The wave function of a particle is $\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L}x\right)$ for $0 < x < L$ and $\psi(x) = 0$ elsewhere. What is the probability of finding the particle between $x = 0$ and $x = L/2$? **[2 marks]**

3. Two gases that have molecular diameters D_1 and D_2, and mean free paths λ_1 and λ_2, respectively, are trapped separately in identical containers. If $D_2 = 2D_1$, what is the ratio λ_1/λ_2? **[2 marks]**

4. Two transverse waves $y_1 = 5\cos(kx - \omega t)$ cm, and $y_2 = 5\cos(kx + \omega t)$ cm, travel on a string along the x-axis. If the speed of the point at $x = 0$ is zero at $t = 0$ s, 0.25 s and 0.5 s, what is the minimum frequency of the waves? **[2 marks]**

5. An infinitely long very thin straight wire carries uniform line charge density $8\pi \times 10^{-2}$ C/m. What is the magnitude of the electric displacement vector at a point located 20 mm away from the axis of the wire? **[2 marks]**