

Curve Sketching.

Even function:- If $f(-x) = f(x)$
 ↓ Eg:- $\cos x$, x^2 , etc.

Symmetric about y-axis

Odd function:- If $f(-x) = -f(x)$
 ↓ Eg:- $x^3 + x$, $\tan x$, etc.

Symmetric about origin

Periodicity: Repeating of graph pattern in certain interval

To test periodicity,

① Find period (k) by

$$k = \frac{d}{|a|}$$

$d \rightarrow$ Fundamental period

$a \rightarrow$ Constant term of angle

② Show, $f(x) = f(x+k)$

Fundamental period : (i) Of $\sin x$ & $\cos x$ is 2π

(ii) Of $\tan x$ is π .

Increasing function: A function $f(x)$ is said to be increasing in an interval (a, b) if for all $x_1, x_2 \in (a, b)$

$$x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$$

Decreasing function: A function $f(x)$ is said to be decreasing in an interval (a, b) if for all $x_1, x_2 \in (a, b)$

$$x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$$

$x_1, x_2 \in$ Given interval

Characteristics for sketching the curve of the function:

(i) Origin [Passes or not] [By keeping $x=0$, if $y=0$; passes]

(ii) Domain & Range [x & y -axes ma क्षीण space कोडे]

(iii) Symmetry [Even function - Symmetric about y-axis
 Odd function - Symmetric about origin]

(iv) Periodicity [For trigonometric function]

(v) Asymptote: [For Logarithmic, exponential, tangent]

↳ [If $x=a$, $y \rightarrow \infty$ (vertical asymptote)]

[If $y=a$, $x \rightarrow \infty$ (horizontal asymptote)]

(vi) Increasing and decreasing region.

(vii) Openness [Quadratic, $ax^2+bx+c=0$ $a < 0$ opens downward
 $a > 0$ opens upward]

(viii) Vertex [For quadratic, $(h, k) = \left(\frac{-b}{2a}, \frac{4ac-b^2}{4a}\right)$]

(ix) Points on axes

Quadratic function

Characteristics:-

- (i) Openness (ii) Origin (iii) Vertex (h, k) (iv) Line of symmetry ($x = h$)
($a > 0; a < 0$)
- (v) Cutting x -axis ($ax^2 + bx + c = 0$) (vi) Domain = $(-\infty, \infty)$
Range = $[k, \infty)$ if $a > 0$
 $(-\infty, k]$ if $a < 0$.

For cubic function:-

- (i) Origin (ii) Symmetry (iii) Increasing and decreasing parts.
- (iv) Points on axes. (v) Domain = $(-\infty, \infty)$
Range = $(-\infty, \infty)$

For sine and cosine function:-

- (i) Origin (ii) Amplitude [$y = b \sin ax$ or $y = b \cos ax$
 $b \rightarrow$ amplitude]
- (iii) Point on axes (iv) Period ($\text{IC} = \frac{\pi}{|a|}$) (v) Domain $(-\infty, \infty)$ or
given interval
Range $[-b, b]$
- (vi) Symmetry [Odd \rightarrow origin. Even \rightarrow y -axis]

For tangent function [$y = a \tan bx$]

- (i) Origin (ii) Domain = $(-\infty, \infty)$ or given interval; Range = $(-\infty, \infty)$
- (iii) Period ($\text{IC} = \frac{\pi}{|b|}$) (iv) Symmetry (origin)
- (v) Points on axes (vi) Asymptote.

For exponential function [$y = a \log x \Rightarrow x = a^y$]

- (i) Origin (ii) Asymptote (iii) Points where A +ve / A -ve lie above/below x / y axes.
- (iv) Points on axes (v) cuts x -axis; $y = a^x$
- (vi) Increasing & decreasing.

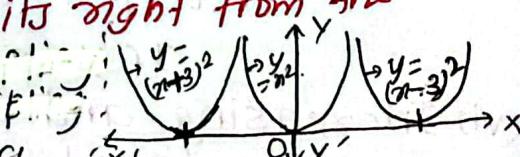
Horizontal shifting of curve: The graph of $f(x-a)$ is same as the graph of $f(x)$ moved to ' a ' units right from the graph of $y = f(x)$. Note,

$y = f(x-a)$, ($a > 0$), shifted a units right

$y = f(x+a)$, ($a > 0$), shifted a units left

Vertical shifting of curve:-

- (i) Graph of $y = f(x) + b$ ($b > 0$) is graph of $y = f(x)$ shifted b units upward.
- (ii) Graph of $y = f(x) - b$ ($b > 0$) is graph of $y = f(x)$ shifted b units downward.



Focal distance = $x+a$ } For horizontal parabola
Focal distance = $y+a$ } For vertical parabola.

For vertex \rightarrow Partial derivative ($w.r.t x$ & $w.r.t y$)

Axis \rightarrow Partial derivative

- Length of latus rectum $\Rightarrow y^2 = a k_0$ coeff. { keeping coeff. of y^2 1 }
 $\Rightarrow x^2 = b k_0$ coeff.

- Vertex is shortest distance from focus.

- Latus rectum is shortest focal chord

- If $(at_1^2, 2at_1)$ and $(at_2^2, 2at_2)$ are extremities of focal chord then $t_1 t_2 = -1$

& If these points subtend rt. angle at vertex then,

$$t_1 t_2 = -4$$

- Locus of mid-point of chord $\rightarrow y^2 = \frac{4ax}{3}$ passing through vertex $\rightarrow y^2 = \frac{2ay}{3} \leftarrow \frac{1}{2}$

- $y^2 = a$ \Rightarrow even in $y \rightarrow$ symmetrical about x -axis
 \Rightarrow even in $a \rightarrow$ y-axis
 \Rightarrow even in both \rightarrow both axes.
{ circle, ellipse, hyperbola }

Tangent at vertex, linear term = 0.

$\tan^{-1} \frac{dy}{dx}$ direction

- For tangent, $a^2 = 4ay$
 \downarrow
Take lines $\rightarrow y = mx + c$
from this $\hookrightarrow c = \frac{a}{m}$

Interchanging method.

- Parabola \rightarrow quadratic eqⁿ not linear.

- Slope = $\frac{dy}{dx}$ of tangent

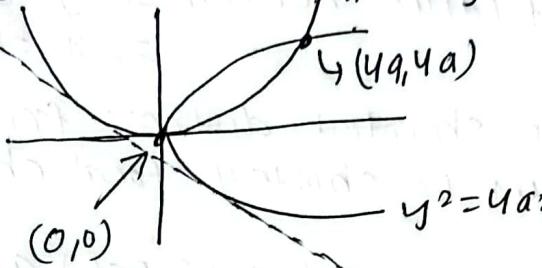
$$\text{Slope} = \frac{1}{\frac{dy}{dx}} \quad \left\{ \text{Normal} \right.$$

Angle b/w tangent from (x_1, y_1) to $y^2 = 4ax$

$$\hookrightarrow \tan^{-1} \left(\frac{\sqrt{y_1^2 - 4ax_1}}{x_1 + a} \right) \quad \begin{array}{l} \text{Root of s,} \\ \text{Direction.} \end{array}$$

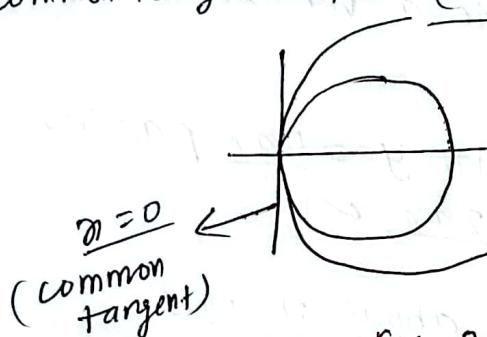
- Angle b/w tangents at end points of focal chord $\Rightarrow 90^\circ$
- If tangents drawn from parabola meet at direction then angle b/w them is 90°
- Common tangent b/w $y^2 = 4ax$ & $y^2 = 4bx$

$$a^{1/3}x + b^{1/3}y + a^{2/3}b^{2/3} = 0$$



- Max. normal from a point $\rightarrow 3$

- Common tangent b/w $(x-2a)^2 + y^2 = (2a)^2$ & $y^2 = 4ax$



circle

- If normal at point $(at_1^2, 2at_1)$ meet at another point $(at_2^2, 2at_2)$ on parabola $y^2 = 4ax$, then

$$t_2 = -t_1 - \frac{2}{t_1}$$

- Condition of tangency of line $y = mx + c$ to $y^2 = 4ax \Rightarrow c = a/m$

- Condition of tangency of $y = mx + c$ to $y^2 = 4a(x+a)$

↑ variable of line should be in this form

$$y = m(x+a) + (c-am)$$

$$\Rightarrow (c-am) = \frac{a}{m}$$

$$\Rightarrow c = am + \frac{a}{m}$$

Eqn of normal of parabola $y^2 = 4ax$ having slope m

$$\hookrightarrow y = mx - 2am - am^3$$

& POC is $(am^2, -2am)$ } $m \rightarrow$ slope of normal.

- POC \Rightarrow Tangent.
- Eqn of given parametric } Anyhow Parameter ~~exists~~
- form

- No. of tangents } Given point (x_1, y_1)
- outside $\rightarrow 2$ } $y_1^2 - 4ax_1 > 0$
- on $\rightarrow 1$ } $y_1^2 - 4ax_1 = 0$
- inside $\rightarrow 0$ } $y_1^2 - 4ax_1 < 0$

- The polar of (x_1, y_1) w.r.t. $x^2 + y^2 = a^2$ is

$$xx_1 + yy_1 = a^2$$

Tangent \nrightarrow Formula lekhne
at the point

$\rightarrow \log_e K \Rightarrow$ is not defined for $K \leq 0$

\rightarrow NO. of solutions: NO of points where graphs cut each other

$f(x) = g(x) \Rightarrow$ curves are $y = f(x)$ & $y = g(x)$

$\rightarrow n_{C_\delta} = n_{C_S} \Rightarrow \delta = S \text{ or } \delta + S = n$

$$\cancel{n_{C_\delta} + n_{C_{\delta-1}} = {}^{n+1}C_\delta}$$

$$\cancel{\frac{n_{C_\delta}}{n_{C_{\delta-1}}} = \frac{n-\delta+1}{\delta}}$$

$$n_{C_0} + n_{C_1} + n_{C_2} + \dots + n_{C_n} = 2^n$$

Selecting one or more object out of $n = 2^n - 1$.

\Rightarrow If only two ways for each object.

$$n_{P_\delta} = \delta! \cdot {}^nC_\delta$$

Derivatives

Some useful derivatives:

$$\textcircled{1} \frac{d}{dx} [f\{g(x)\}] = f'\{g(x)\} g'(x)$$

$$\textcircled{2} \frac{d}{dx} x^n = n x^{n-1} \left[\text{Eg: } \frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}} \text{ & } \frac{d}{dn} \left(\frac{1}{n} \right) = -\frac{1}{n^2} \right]$$

$$\textcircled{3} \frac{d}{dx} [f(x)]^n = n \{f(x)\}^{n-1} \cdot f'(x)$$

$$\textcircled{4} \frac{d}{dx} e^{f(x)} = e^{f(x)} \cdot f'(x)$$

$$\textcircled{5} \frac{d}{dx} a^{f(x)} = a^{f(x)} \cdot f'(x) \log a$$

$$\textcircled{6} \frac{d}{dx} \log f(x) = \frac{1}{f(x)} \cdot f'(x)$$

$$\textcircled{7} \frac{d}{dx} \log x = \frac{1}{x}, \text{ for } x > 0$$

$$\textcircled{8} \frac{d}{dx} |\log x| = \frac{1}{x}, \text{ for } x \neq 0$$

$$\textcircled{9} \frac{d}{dx} (\log_a x) = \frac{d}{dx} \left(\frac{\log x}{\log a} \right) = \frac{1}{x \log a}, \text{ for } x > 0$$

$$\textcircled{10} \frac{d}{dx} |x| = \frac{x}{|x|} \text{ or } \frac{|x|}{x}$$

Derivatives of trigonometric functions:

$$\textcircled{1} \frac{d}{dx} \sin x = \cos x \quad \textcircled{2} \frac{d}{dx} \cos x = -\sin x \quad \textcircled{3} \frac{d}{dx} \tan x = \sec^2 x$$

$$\textcircled{4} \frac{d}{dx} \sec x = \sec x \tan x \quad \textcircled{5} \frac{d}{dx} \cosec x = \cosec x \cot x \quad \textcircled{6} \frac{d}{dx} \cot x = -\operatorname{cosec}^2 x$$

Derivatives of inverse trigonometric functions:

$$\textcircled{1} \frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}, \quad \frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$$

$$\textcircled{2} \frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}, \quad \frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2}$$

$$\textcircled{3} \frac{d}{dx} \sec^{-1} x = \frac{1}{x \sqrt{x^2-1}}, \quad \frac{d}{dx} \cosec^{-1} x = \frac{-1}{x \sqrt{x^2-1}}$$

Derivative of hyperbolic functions:

$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \frac{e^x + e^{-x}}{2}$$

Ex:

$$1. \frac{d}{dx} \sinh x = \cosh x$$

$$2. \frac{d}{dx} \cosh x = \sinh x$$

$$3. \frac{d}{dx} \tanh x = \operatorname{sech}^2 x$$

$$④ \frac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \tanh x$$

$$⑤ \frac{d}{dx} \operatorname{cosech} x = -\operatorname{cosech} x \coth x$$

$$⑥ \frac{d}{dx} \coth x = -\operatorname{cosech}^2 x$$

$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

↳ provided that this limit exists and is finite

⑦ $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$

⑧ $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$

⑨ $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

⑩ $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \quad \left. \begin{array}{l} \text{If } y = f(t) \\ t = f(x) \end{array} \right\}$

Derivative of parametric functions
If $x = \phi(t)$ and $y = \psi(t)$, then

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Partial derivative:

For $f(x, y) = 0$

$$\frac{\partial y}{\partial x} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$$

For u-substitution

$$\sqrt{u} \quad e^u \quad \frac{1}{u}$$

$$u^x$$

$$\frac{1}{\sqrt{u}}$$

$$\frac{1}{(u)^2}$$

$$v + C$$

$$\log u$$

$$\sin u, \cos u$$

Formulae:-

Integration (Anti-Derivatives)

$$1. \int x^n dx = \frac{x^{n+1}}{n+1} + C \quad [n \neq -1]$$

$$2. \int e^x dx = e^x + C$$

$$3. \int e^{ax} dx = \frac{e^{ax}}{a} + C$$

$$4. \int \frac{1}{x} dx = \log x + C$$

$$5. \int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C$$

$$6. \int \frac{1}{ax+b} dx = \frac{1}{a} \log(ax+b) + C$$

$$7. \int a^x dx = \frac{a^x}{\log a} + C$$

$$8. \int \sin x dx = -\cos x + C$$

$$9. \int \sin ax dx = -\frac{\cos ax}{a} + C$$

$$10. \int \cos x dx = \sin x + C$$

$$11. \int \tan x dx = \sec^2 x + C$$

$$12. \int \sec^2 x dx = \tan x + C$$

$$13. \int \sec^2 ax dx = \frac{\tan ax}{a} + C$$

$$14. \int \csc^2 x dx = -\cot x + C$$

$$15. \int \sec x \cdot \tan x dx = \sec x + C$$

$$16. \int \csc^2 x dx = -\cot x \cdot dx = -\csc x + C$$

$$17. \int \tan x dx = \log(\sec x) + C$$

$$18. \int \cot x dx = \log(\sin x) + C$$

$$19. \int \sec x dx = \log(\sec x + \tan x) + C$$

$$20. \int \csc x dx = \log(\csc x - \cot x) + C$$

$$\frac{\log(a^x)}{a} = \frac{a^x}{a} = b^x$$

$$21. \int 1 dx = x + C$$

$$22. \int e^{(ax+b)} dx = \frac{1}{a} e^{(ax+b)} + C$$

$$23. \int k f(x) dx = k \int f(x) dx$$

$$24. \int [f(x) \pm g(x)] dx = \int f(x) dx \pm \int g(x) dx$$

$$25. \int (uv) dx =$$

$$u \int v dx - \int \left(\frac{du}{dx} \int v dx \right) dx$$

+ C, this

is known as
integration by

parts: $u \rightarrow$ easily differentiable

$v \rightarrow$ easily integrable

26. For, (i) $a^2 - x^2$ integrable

$$x = a \sin \theta$$

$$(ii) x^2 - a^2$$

$$x = a \sec \theta$$

$$(iii) x^2 + a^2$$

$$x = a \tan \theta$$

For u-substitution

$$\sqrt{u} \quad e^u \quad \frac{1}{u}$$

$$\frac{1}{\sqrt{u}} \quad \frac{1}{(u)^2}$$

$$\log u$$

$$\sin u, \cos u$$

Integration by parts:-

$$\int_{\text{I}} uv \, dv = u \int v \, du - \int \left[\frac{du}{dn} \right] v \, dn$$

$u \rightarrow 1^{\text{st}}$ func.

$v \rightarrow 2^{\text{nd}}$ function

$u \rightarrow$ easily derivable

$v \rightarrow$ easily integrable

Order of choosing 1st f(x)

$\begin{cases} I = \text{Inverse func} \\ L = \text{Logarithmic func} \\ A = \text{Algebraic func} \\ T = \text{Trigo (func)} \\ E = \text{Expo. func} \end{cases}$

* If inverse function & logarithmic function

has no second product then take "I" as second product.

* If one function is a^n , $n \in \mathbb{N}$, then

$$\int a^n v \, dn = a^n \int v \, dn - F(n)$$

$\rightarrow F(n)$ is obtained by applying the rule (derivative of I) X (Integration of II)

on every previous term obtained & writing them in alternate sign.

$$\text{eg: } \int n^3 e^{an} \, dn = \underline{\underline{(n^3)}} \left(\frac{e^{an}}{a} \right) \Theta \left(\frac{(3n^2)(e^{an})}{a^2} \right) \Theta \left(\frac{(6n)(e^{an})}{a^3} \right) - \underline{(6)} \left(\frac{e^{an}}{a^4} \right) + \dots$$

Integral of type $\int \sin^m n \cos^n n \, dn$

\hookrightarrow (i) m -odd $\Rightarrow \cos n = t \rightarrow$ power odd x gain constant & put gano.

(ii) n -odd $\Rightarrow \sin n = t \rightarrow$ power odd x gain constant & put gano.

Integrals of type $\int \sin^m n \cos^n n \, dn$, $\int \sin^m n \sin^n n \, dn$, $\int \cos^m n \cos^n n \, dn$

$$\text{use trigo identities: } (i) 2 \sin A \cos B = \sin(A+B) + \sin(A-B)$$

$$(ii) 2 \cos A \sin B = \sin(A+B) - \sin(A-B)$$

$$(iii) 2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$(iv) 2 \sin A \sin B = \cos(A-B) - \cos(A+B)$$

Standard integrals of Trigo.

$$\textcircled{1} \int \csc x dx = \log(\tan \frac{x}{2}) + C$$

$$\textcircled{2} \int \sec x dx = \log(\tan(\frac{\pi}{4} + \frac{x}{2})) + C$$

* Anyhow try to make \sec^2 in numerator and \tan^2 or \tan in denominator.

• linear term in deno:

$$\text{eg: } \int \frac{1}{a+b\sin x} dx, \int \frac{1}{a\sin x + b\cos x} dx, \int \frac{dx}{a+b\cos x}$$

$$\sin x = \frac{2\tan \frac{x}{2}}{1+\tan^2 \frac{x}{2}}, \cos x = \frac{1-\tan^2 \frac{x}{2}}{1+\tan^2 \frac{x}{2}}$$

$$\Rightarrow \tan \frac{x}{2} = t \text{ then, } 2dt = \sec^2 \frac{x}{2} dx$$

• square term in deno. (degree 2)

$$\text{eg: } \int \frac{1}{a+b\sin^2 x} dx, \int \frac{1}{a\sin^2 x + b\cos^2 x} dx$$

• divide num & deno by $\cos^2 x$ and put $\tan x = t \Rightarrow dt = \sec^2 x dx$.

Next, Use Standard integrals(I).

For integral of the form

$$\int \frac{dx}{a\sin x + b\cos x} \quad b \rightarrow \cos \frac{\alpha}{\text{coeff.}} \quad a \rightarrow \sin \frac{\alpha}{\text{coeff.}}$$

$$\text{Putting } a = r \cos \theta, b = r \sin \theta, r = \sqrt{a^2 + b^2}$$

$$\theta = \tan^{-1} \left(\frac{b}{a} \right)$$

$$I = \int \frac{dx}{r \sin(x \pm \theta)} = \frac{1}{\sqrt{a^2 + b^2}} \ln \tan \left(\frac{\pi}{2} \pm \frac{1}{2} \tan^{-1} \frac{b}{a} \right) + C$$

$$\# \textcircled{1} \int a^n dx = \frac{a^{n+1}}{\log a} + C \quad \# \int e^{ax} dx = \frac{e^{ax}}{a} + C$$

$$\textcircled{2} \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C = -\cos^{-1} x + C$$

$$\textcircled{3} \int \frac{dx}{1+x^2} = \tan^{-1} x + C = -\cot^{-1} x + C$$

$$\textcircled{4} \int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + C = -\cosec^{-1} x + C$$

$$\# \textcircled{5} \int \tan x dx = \ln |\sec x| + C = -\ln |\cos x| + C$$

$$\textcircled{6} \int \cot x dx = \ln |\sin x| + C$$

$$\textcircled{7} \int \sec x dx = \log(\sec x + \tan x) + C$$

$$= \log \tan \left(\frac{\pi}{4} + \frac{x}{2} \right) + C$$

$$\textcircled{8} \int \csc x dx = \log|\csc x - \cot x| + C$$

$$- \log \left(\tan \frac{x}{2} \right) + C$$

Hyperbolic function

$$\textcircled{1} \int \sinh x dx = \cosh x + C$$

$$\textcircled{2} \int \cosh x dx = \sinh x + C$$

$$\textcircled{3} \int \tanh x dx = \log(\cosh x) + C$$

$$\textcircled{4} \int \coth x dx = \log(\sinh x) + C$$

$$\textcircled{5} \int \operatorname{sech} x dx = \log \tan \left(\frac{x}{2} \right) + C$$

$$\textcircled{6} \int \operatorname{sech} x dx = 2 \tan^{-1} \left(\tanh \frac{x}{2} \right) + C$$

$$\textcircled{7} \int \operatorname{sech}^2 x dx = \tanh x + C$$

$$\textcircled{8} \int \operatorname{cosech}^2 x dx = -\coth x + C$$

$$\textcircled{9} \int \operatorname{sech} x \tanh x dx = -\operatorname{sech} x + C$$

$$\textcircled{10} \int \operatorname{cosech} x \coth x dx = -\operatorname{cosech} x + C$$

$$\# \sinh x = \frac{e^x - e^{-x}}{2}; \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\textcircled{11} \cosh^2 x - \sinh^2 x = 1$$

$$\textcircled{12} \cosh^2 x + \sinh^2 x = \cosh 2x$$

$$\textcircled{13} \sinh(-x) = -\sinh x$$

$$\cosh(-x) = \cosh x$$

$$\textcircled{14} \sinh 2x = 2 \sinh x \cdot \cosh x$$

$$\textcircled{15} \cosh(x+y) = \cosh x \cdot \cosh y + \sinh x \cdot \sinh y$$

$$\textcircled{16} \sinh(x+y) = \sinh x \cosh y + \cosh x \cdot \sinh y$$

Differentiate $\sinh x, \cosh x, \tanh x$

+ve
Other $\operatorname{sech} x, \operatorname{cosech} x, \coth x$
-ve

$$\# \log a^x = \frac{\log x}{\log a}$$

$$\# \log a^x = \frac{1}{\log x a}$$

$$\# \int \ln x dx = x \ln x - x + C$$

Standard integrals (I)

$$\text{D) } \int \frac{1}{x^2 + a^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C \quad \left\{ \begin{array}{l} \text{often} \\ \text{confused} \end{array} \right.$$

$$\text{E) } \int \frac{1}{a^2 - x^2} dx = \frac{1}{2a} \log\left(\frac{a+x}{a-x}\right) + C$$

$$\text{F) } \int \frac{1}{x^2 - a^2} dx = \frac{1}{2a} \log\left(\frac{x-a}{x+a}\right) + C \quad \left\{ \begin{array}{l} x^2 \text{ ko 3 terms} \\ \text{sign} \end{array} \right.$$

$$\text{G) } \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C \quad \left\{ \begin{array}{l} \text{often} \\ \text{confused} \end{array} \right.$$

$$\text{H) } \int \frac{1}{\sqrt{x^2 + a^2}} dx = \log\left(x + \sqrt{x^2 + a^2}\right) + C = \sinh^{-1}\left(\frac{x}{a}\right) + C \quad \left\{ \begin{array}{l} \text{sq. root x} \\ \text{unity} \end{array} \right.$$

$$\text{I) } \int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left(x + \sqrt{x^2 - a^2}\right) + C = \cosh^{-1}\left(\frac{x}{a}\right) + C$$

Integral of type $\int \frac{dx}{A \sqrt{B}}$ where A and B are linear or quadratic function of x.

~~I~~ (i) If B \rightarrow linear, substitution; $B = z^2$

~~II~~ (ii) A linear & B quadratic, $A = 1/z$

~~III~~ (iii) A and B both quadratic, $x = 1/t$

Standard integrals (II)

$$\text{A) } \int \sqrt{x^2 + a^2} dx = \frac{x}{2} \sqrt{x^2 + a^2} + \frac{a^2}{2} \log(x + \sqrt{x^2 + a^2}) + C$$

$$\text{B) } \int \sqrt{x^2 - a^2} dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \log(x + \sqrt{x^2 - a^2}) + C \quad \left\{ \begin{array}{l} \text{---} \\ \text{---} \\ \text{---} \end{array} \right. \quad \left\{ \begin{array}{l} \text{For type} \\ \int (m+n)(\sqrt{ax^2+bx+c})dx \end{array} \right.$$

$$\text{C) } \int \sqrt{a^2 - x^2} = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\# \int e^{ax} \cos bx dx = \frac{e^{ax}(a \cos bx + b \sin bx)}{a^2 + b^2} + C$$

$$\int e^{ax} \sin bx dx = \frac{e^{ax}(a \sin bx - b \cos bx)}{a^2 + b^2} + C$$

$$\# \int \frac{dx}{x \sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + C = -\frac{1}{a} \cosec^{-1}\left(\frac{x}{a}\right) + C$$

$$\# \text{ I) } \int e^{ax} \left[f(x) + \frac{f'(x)}{a} \right] dx = \frac{e^{ax} f(x)}{a} + C \quad \left\{ \begin{array}{l} \frac{d}{dx} \{ e^{ax} f(x) \} = a e^{ax} f(x) + \\ e^{ax} f'(x) \end{array} \right.$$

$$\hookrightarrow \int e^{ax} (f(x) + f'(x)) dx = e^{ax} f(x) + C$$

$$\text{II) } \int [x f'(x) + f(x)] dx = x f(x) + C$$

$$\# \text{ I) } \int [f(x)]^n f'(x) dx = \frac{[f(x)]^{n+1}}{n+1} + C \quad \left\{ \begin{array}{l} n \neq -1 \\ \text{---} \end{array} \right.$$

$$\text{II) } \int \frac{f'(x) dx}{f(x)} = \ln f(x) + C \quad \left\{ \begin{array}{l} n = -1 \\ \text{---} \end{array} \right.$$

Integrals reducible to standard forms. (when deno can't be factorized)

$$\text{1) If } \int \frac{dx}{ax^2 + bx + c} \quad \text{ii) } \int \frac{dx}{\sqrt{ax^2 + bx + c}} \quad \left\{ \begin{array}{l} \text{Only} \\ \text{quadratic} \end{array} \right.$$

\rightarrow Make coeff. of x^2 unity by taking coeff. of x^2 outside the quadratic. Then integrand is converted to S.I.L. $\left\{ \begin{array}{l} \text{make} \\ \text{perfect} \end{array} \right.$

2) Linear & Quadratic sq-term inside

$$\text{I) } \int \frac{mx + e}{ax^2 + bx + c} dx \quad \text{ii) } \int \frac{mx + e}{\sqrt{ax^2 + bx + c}} dx$$

Put: Numerator = p (Derivative of quadratic) + q

Caution while simplification

+ & -

Don't get confused in $a^2 + a!$

Caution while simplification

+ & -

For type $\int (m+n)(\sqrt{ax^2+bx+c})dx$

Put: linear function = p (Derivative of quadratic) + q

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$$\textcircled{1} \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{2R} = \frac{\Delta}{abc}$$

$$\Rightarrow \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

iff, $\angle A = 60^\circ \Rightarrow b^2 + c^2 - a^2 = bc$

$$\textcircled{2} \rightarrow \text{Projection Formula: } [a = b \cos C + c \cos B]$$

Used to find side of Δ when other two sides & their opp. angle are given.

$$\textcircled{3} \tan \frac{B-C}{2} = \frac{b-c}{b+c} \cot \frac{A}{2}$$

$$\begin{aligned} \bullet \frac{a}{\cos A} = \frac{b}{\cos B} = \frac{c}{\cos C} &\Rightarrow \text{eq. triangle} \\ \bullet \cos^2 A + \cos^2 B + \cos^2 C &= 1 \Rightarrow \text{Rt. angled triangle} \end{aligned}$$

$$\textcircled{4} \sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$\cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$$

$$\left. \begin{aligned} \tan \frac{A}{2} &= \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \frac{\Delta}{s(s-a)} \\ \therefore \Delta &= \sqrt{s(s-a)(s-b)(s-c)} \end{aligned} \right\} *$$

$$\rightarrow \Delta = \frac{1}{2} bc \sin A \Rightarrow \sin A = \frac{2\Delta}{bc}$$

$$\textcircled{5} \Delta = \frac{1}{2} \frac{a^2 \sin B \sin C}{\sin(B+C)}$$

$$\left. \begin{aligned} \text{if } A+B+C &= \pi \text{ or } 180^\circ \\ \text{In any triangle, } \cos A + \cos B + \cos C &= 1 + \frac{r}{R} \end{aligned} \right\}$$

$\rightarrow R \rightarrow$ Circumradius [Rad. of circumcircle of triangle]

$$\frac{1}{2R} = \frac{2\Delta}{abc} \Rightarrow R = \frac{abc}{4\Delta}$$

$\rightarrow r$: inradius [Rad. of incircle of triangle]

$$\textcircled{6} \cdot r = \frac{\Delta}{s}$$

$$\cdot r = (s-a) \tan \frac{A}{2} = (s-b) \tan \frac{B}{2} = (s-c) \tan \frac{C}{2}$$

$$\boxed{\begin{aligned} \text{Dimension:} \\ R, r, r_1, r_2, s \rightarrow L \\ \sin A, \cos A \dots \rightarrow 1 \\ \Delta \rightarrow L^2 \end{aligned}}$$

$$\textcircled{7} r = \frac{a \sin \frac{B}{2} \sin \frac{C}{2}}{\cos \frac{A}{2}} = \frac{b \sin \frac{A}{2} \sin \frac{C}{2}}{\cos \frac{B}{2}} = \frac{c \sin \frac{A}{2} \sin \frac{B}{2}}{\cos \frac{C}{2}}$$

$$\left. \begin{aligned} \text{In equilateral triangle,} \\ r : R : r_1 &= 1 : 2 : 3 \end{aligned} \right\}$$

$$\textcircled{8} r = UR \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

$\rightarrow r_1, r_2, r_3$: Radii of excircles of a triangle.

$$\textcircled{9} \cdot r_1 = \frac{\Delta}{s-a}; r_2 = \frac{\Delta}{s-b}; r_3 = \frac{\Delta}{s-c}$$

$$\cdot r_1 = s \tan \frac{A}{2}$$

$$\textcircled{10} \cdot r_1 = UR \sin \frac{A}{2} \cdot \cos \frac{B}{2} \cdot \cos \frac{C}{2}; r_2 = UR \cos \frac{A}{2} \sin \frac{B}{2} \cdot \cos \frac{C}{2};$$

$$r_3 = UR \sin \frac{C}{2} \cdot \cos \frac{A}{2} \cdot \cos \frac{B}{2}$$

$\left. \begin{aligned} \text{Assumption:} \\ \text{equilateral triangle} \\ \text{of side unit} \\ \text{+ Rt. angled triangle of side } 3, 4, 5 \end{aligned} \right\}$

→ Principle solution (α)
 $\sin \theta \rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}] ; \cos \theta \rightarrow [0, \pi] ; \tan \theta \rightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$

→ General Solution:-

I. $\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha$
 $\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha$
 $\tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha$

II. $\sin^2 \theta = \sin^2 \alpha$
 $\cos^2 \theta = \cos^2 \alpha$
 $\tan^2 \theta = \tan^2 \alpha$ $\Rightarrow \theta = n\pi \pm \alpha$

III. $\sin \theta = 0 \Rightarrow \theta = n\pi$
 $\cos \theta = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}$
 $\tan \theta = 0 \Rightarrow \theta = n\pi$

IV. $\sin \theta = -\sin \alpha \Rightarrow \theta = n\pi + (-1)^n(-\alpha)$
 $\tan \theta = \tan \alpha \Rightarrow \theta = n\pi + (-\alpha)$
 $\cos \theta = -\cos \alpha \Rightarrow \theta = 2n\pi + (\pi - \alpha)$

V. $\sin \theta = 1 \Rightarrow \theta = (4n+1)\frac{\pi}{2}$
 $\sin \theta = -1 \Rightarrow \theta = (4n-1)\frac{\pi}{2}$

$\cos \theta = 1 \Rightarrow \theta = 2n\pi$
 $\cos \theta = -1 \Rightarrow \theta = (2n+1)\pi$

→ Squaring while finding Roots \Rightarrow check the solutions to satisfy given equation.

- For $a \cos \theta + b \sin \theta = c$
- Divide by $\sqrt{a^2 + b^2}$
 - # $|c| > \sqrt{a^2 + b^2}$, no solution
 $|c| \leq \sqrt{a^2 + b^2}$, has solution.
- Given two variables & asked to find P.V & General value then.
Principle value (PV) of θ : b/w 0 & 2π .

If α lies in

1st quad, PV = α

2nd quad, PV = $\pi - \alpha$

3rd quad, PV = $\pi + \alpha$

4th quad, PV = $2\pi - \alpha$

& General value = $2n\pi + PV$.

→ Convert to sin & cos if question is complex.

→ $\sin^{-1} \alpha \Rightarrow$ Angle ; $\alpha = \sin \theta \rightarrow$ Real number.

I TF $f(\alpha)$	(α) Domain	($f(\alpha)$) Range rev
$\sin^{-1} \alpha$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\cos^{-1} \alpha$	$[-1, 1]$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\tan^{-1} \alpha$	$(-\infty, \infty)$	$(-\frac{\pi}{2}, \frac{\pi}{2})$
$\csc^{-1} \alpha$	$(-\infty, -1] \cup [1, \infty)$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$
$\sec^{-1} \alpha$	$(-\infty, -1] \cup [1, \infty)$	$[0, \pi] - \{\frac{\pi}{2}\}$
$\cot^{-1} \alpha$	$(-\infty, \infty)$	$(0, \pi)$

$\sin^{-1}(-\alpha) = -\sin^{-1} \alpha ; \cos^{-1}(-\alpha) = \pi - \cos^{-1} \alpha$
 $\tan^{-1}(-\alpha) = -\tan^{-1} \alpha ; \sec^{-1}(-\alpha) = \pi - \sec^{-1} \alpha$
 $\csc^{-1}(-\alpha) = -\csc^{-1} \alpha ; \cot^{-1}(-\alpha) = \pi - \cot^{-1} \alpha$

$\sin^{-1} \alpha + \cos^{-1} \alpha = \pi/2$
 $\tan^{-1} \alpha + \cot^{-1} \alpha = \pi/2$
 $\sec^{-1} \alpha + \csc^{-1} \alpha = \pi/2$

$2 \sin^{-1} \alpha = \sin^{-1}(2\alpha \sqrt{1-\alpha^2})$
 $2 \cos^{-1} \alpha = \cos^{-1}(2\alpha^2 - 1)$
 $2 \tan^{-1} \alpha = \tan^{-1}\left(\frac{2\alpha}{1-\alpha^2}\right) = \sin^{-1}\left(\frac{2\alpha}{1+\alpha^2}\right)$

→ Assumption: suitable value belonging to domain.

→ 'HAT' method: Using options to check answer.

→ Cast Rule

$$\rightarrow \cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\rightarrow \sin 2A = \frac{2\tan A}{1 + \tan^2 A}$$

$$\rightarrow \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A} \quad \left| \begin{array}{l} \tan 2A = \frac{2\tan A}{1 - \tan^2 A} \\ \text{(sine or cosine ko formula ka muni if } \therefore 1 + \tan^2 A \\ \text{sec}^2 \text{ aavnu paryo} \end{array} \right.$$

Quadrant $\rightarrow '+' \text{ or } '-'$

$$\rightarrow \sin 3A = 3\sin A - 4\sin^3 A$$

$$\rightarrow \cos 3A = 4\cos^3 A - 3\cos A$$

$$\rightarrow \tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A} \quad \left| \begin{array}{l} \text{sum of} \\ \text{one at a time} \\ - \text{sum of} \\ \text{three at a time} \\ 1 - \{ \text{sum of two at a time} \} \end{array} \right.$$

$$\rightarrow \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right) \cdot \cos\left(\frac{A-B}{2}\right)$$

$$\rightarrow \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right) - \cos\left(\frac{A-B}{2}\right)$$

$$\rightarrow \cos A - \cos B = 2\sin\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{B-A}{2}\right)$$

$$\rightarrow \sin(A+B) \cdot \sin(A-B) = \sin^2 A - \sin^2 B$$

$$\rightarrow \cos(A+B) \cdot \cos(A-B) = \cos^2 A - \sin^2 B$$

$$\rightarrow \text{If } A+B+C = \pi \text{ (or } 180^\circ)$$

$$\cdot \sin 2A + \sin 2B + \sin 2C = 4\sin A \cdot \sin B \cdot \sin C$$

$$\cdot \cos 2A + \cos 2B + \cos 2C = 1 - 4\cos A \cos B \cos C$$

$$\cdot \sin A + \sin B + \sin C = 4\cos\frac{A}{2} \cdot \cos\frac{B}{2} \cdot \cos\frac{C}{2}$$

$$\cdot \cos A + \cos B + \cos C = 1 + 4\sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$$

$$\cdot \tan A + \tan B + \tan C = \tan A \tan B \tan C$$

$$\cdot \cot A \cot B + \cot B \cot C + \cot C \cot A = 1$$

★ Assuming &
Getting values

★ Differentiation

★ $\sin x \leq 1$

If π difficult to
observe anything
in question
use this

reverse $(-) \rightarrow (+)$
 $\sin \rightarrow \cos A$

- If $\sec \theta + \tan \theta = a$,
 $\sec \theta - \tan \theta = \frac{1}{a}$
- If $\csc \theta + \cot \theta = a$,
 $\csc \theta - \cot \theta = \frac{1}{a}$

Domain	Range	$\sec^2 \theta - \tan^2 \theta = 1$
R	$[-1, 1]$	
$R - \frac{(2n+1)\pi}{2}$	$(-\infty, \infty)$	
$R - n\pi$	$(-\infty, 1] \cup [1, \infty)$	

• $c + \sqrt{a^2 + b^2} \geq a\sin \theta \pm b\cos \theta \geq c - \sqrt{a^2 + b^2}$

• If $z = a\sin \theta + b\cos \theta$ such that
 $ay = 1$,
then min. value of z is
obtained as
 $A \cdot M. \geq G \cdot M.$

$f(\theta)$	Range of $f(\theta)$	$T(f)$	$f(0)$	Domain	Range	$\sec^2 \theta - \tan^2 \theta = 1$
$\sin \theta$	R i.e. $(-\infty, \infty)$	$[-1, 1]$	0	\mathbb{R}	$[-1, 1]$	
$\tan \theta$	$R - \frac{(2n+1)\pi}{2}$	R	$(-\infty, \infty)$	$\sec \theta$	$R - \frac{(2n+1)\pi}{2}$	
$\csc \theta$	$R - n\pi$	$(-\infty, 1] \cup [1, \infty)$	0	$\cot \theta$	$R - n\pi$	$(-\infty, 1] \cup [1, \infty)$

- $c + \sqrt{a^2 + b^2} \geq a\sin \theta \pm b\cos \theta \geq c - \sqrt{a^2 + b^2}$
- $\sin^2 \theta + \csc^2 \theta \geq 2$
- $\cos^2 \theta + \sec^2 \theta \geq 2$

Complex Number

- $z = a + ib = (a, b)$ { $x\text{-axis} \rightarrow \text{real axis}$ } { $y\text{-axis} \rightarrow \text{imaginary axis}$ }
- Every real number is complex number (with imaginary part = 0)
- If $z = (a, b)$ & $w = (c, d)$, Then
- * $z = w$ iff $a = c$ and $b = d$.
 - * $z + w = (a+c, b+d)$ * $z - w = (a-c, b-d)$
↳ rep. as diagonal of 11^{th} row
 - * $z \cdot w = (a, b) \cdot (c, d)$
 - $z \cdot w = (ac - bd, ad + bc)$
 - * $\frac{z}{w} = \frac{(a+ib)(c-id)}{(c+id)(c-id)} = \left(\frac{ac+bd}{c^2+d^2} \right) + i \left(\frac{bc-ad}{c^2+d^2} \right)$

- Additive inverse of $z = (a, b)$ is $-z$ i.e. $-z = (-a, -b)$
- Additive identity is $(0, 0)$ { For all complex numbers }
- Multiplicative identity is $(1, 0)$
- Multiplicative inverse of $z = (a, b)$ is $1/z$.

$$\frac{1}{z} = \frac{1}{a+ib} = \frac{a-ib}{a^2+b^2} = \left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2} \right)$$

→ Conjugate of $z = a+ib$ is \bar{z} i.e. $\bar{z} = a-ib$.

→ $z = a+ib$ ↗ Purely real if $b=0$ → $\boxed{z = \bar{z}}$ ↗ Purely imaginary if $a=0$ → $\boxed{z = -\bar{z}}$

→ i : imaginary unit of complex number

- $i^0 = (0, 1)$
- $i^2 = (0, 1) \cdot (0, 1) = (0, -1) = -1$
- $i^{4n} = 1$ { where n is an integer }
- $i^{4n+2} = -1$
- $\frac{1}{i} = \frac{1}{i} \times \frac{i}{i} = -i$

• Sum of any four consecutive powers of i is zero.

→ Two complex numbers follows associative & distributive law of addition & multiplication and commutative law of addition ~~but not for multiplication~~

→ Properties of complex conjugate:

- X. (i) $(\bar{z}) = z$
- (ii) $z + \bar{z} = 2 \text{ Re}(z)$
- (iii) $z - \bar{z} = 2 \text{ Im}(z)$
- (iv) $\overline{z+w} = \bar{z} + \bar{w}$
- (v) $\overline{zw} = \bar{z} \cdot \bar{w}$

$$\begin{aligned} \text{(vi)} \quad \overline{z_1 z_2 z_3} &= \bar{z}_1 \cdot \bar{z}_2 \cdot \bar{z}_3 \\ \text{(vii)} \quad \overline{z_1 + z_2 + z_3} &= \bar{z}_1 + \bar{z}_2 + \bar{z}_3 \end{aligned}$$

$$\text{(viii)} \quad \left(\frac{\bar{z}}{w} \right)^* = \frac{\bar{z}}{\bar{w}}$$

* conjugate gets distributed to each complex number.

- # Modulus of complex number: $z = a+ib$
- ↳ $|z| = \sqrt{a^2+b^2}$
 - $|z_1| \rightarrow$ distance of z_1 from origin.
 - $|z_2 - z_1| \rightarrow$ dist. b/w z_1 & z_2
i.e. dist. of z_2 from z_1
- (i) $|z| = |\bar{z}| = |-z|$
- (ii) $|z| = 0$ iff $z = 0$
- (iii) $-|z_1| \leq \operatorname{Re}(z_1) \leq |z_1|$
- (iv) $-|z_1| \leq \operatorname{Im}(z_1) \leq |z_1|$
- (v) $(z_1 z_2) = |z_1| \cdot |z_2|$
- (vi) $|z^n| = |z|^n$
- (vii) $\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|} \quad (z_2 \neq 0)$
- (i) $i^i \rightarrow \text{Real no} \rightarrow e^{-\pi/2}$
- (x) $|z+w| \leq |z| + |w| \rightarrow \text{Triangle law of inequality}$
- (xi) $|z-w| \geq |z| - |w|$
- (xii) $|z|^2 = z \cdot \bar{z}$
- (xiii) $(z_1 + z_2)^2 + (z_1 - z_2)^2 = 2(|z_1|^2 + |z_2|^2)$
- (xiv) $|z_1 + z_2| = |z_1 - z_2| \Leftrightarrow \arg(z_1) - \arg(z_2) = \pi/2$
- (xv) $|z_1 + z_2| = |z_1| + |z_2| \Leftrightarrow \arg(z_1) = \arg(z_2)$

Polar Form of complex number

If $[z = x+iy]$ then, polar form of z is

Cartesian form
* $z = r(\cos\theta + i\sin\theta)$

$$r = \sqrt{x^2+y^2}$$

$$\tan\theta = y/x \Rightarrow \theta = \tan^{-1}(y/x)$$

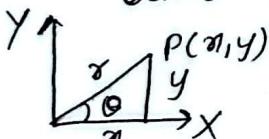


Table for arguments: (θ)

- $\theta \in [-\pi, \pi]$
- Argument of $z=0$ is not defined.

C is θ

$r = \sqrt{x^2+y^2}$

$\theta \rightarrow$ argument or amplitude.

Euler form

$$z = x+iy$$

$$z = re^{i\theta}$$

→ used to find big/big power of z .

$$e^{i\theta} = \cos\theta + i\sin\theta$$

# Quadrants		1st	2nd	3rd
Argument	0	$\pi - \theta$	$-\pi + \theta$	$4\pi - \theta$

For $z = x+iy$

$$\theta = \tan^{-1}(y/x)$$

$$\text{Let, } z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$$

$$z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$$

Then,

$$* z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$$

$$* \frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

Complex number	value of argument
$z = x, x > 0$	purely real $\rightarrow 0$
$z = -x, x < 0$	real $\rightarrow -\pi$
$z = iy, y > 0$	purely im. $\rightarrow \pi/2$
$z = -iy, y < 0$	im. $\rightarrow -\pi/2$ or $3\pi/2$
z_1, z_2	$\arg(z_1) + \arg(z_2)$
$\frac{z_1}{z_2}$	$\arg(z_1) - \arg(z_2)$
z^n	$n \arg(z)$
\bar{z}	$2\pi - \arg(z)$

De-Moivre's Theorem

For any integer n (positive or negative or zero),

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

& the n distinct roots are given by:

$$z_k = \sqrt[n]{r} (\cos\theta_k + i\sin\theta_k)$$

$$\text{where, } \theta_k = \frac{\theta + k \cdot 360^\circ}{n} \quad k = 0, 1, 2, \dots, (n-1)$$

Square roots of a complex number; $z = a+ib$

Let, $a+iy = \sqrt{a+ib}$

$$(a+iy)^2 = a+ib$$

then, $\sqrt{a^2} = \frac{\sqrt{a^2+b^2}}{2} + a$ and $\sqrt{y^2} = \frac{\sqrt{a^2+b^2}}{2} - a$

ay and b must have same sign

$\hookrightarrow b = -ve$; take a and y of alternate sign (opposite signs)

$b = +ve$; take a and y of same sign.

Cube roots of unity

$$z = \sqrt[3]{1}$$

$$z^3 - 1 = 0 \Rightarrow (z-1)(z^2+z+1) = 0$$

$$z = 1 \quad \text{or}, \quad z = \frac{-1 \pm \sqrt{3}i}{2}$$

$$\left. \begin{array}{l} z^2 + z + 1 = 0 \rightarrow \text{Roots} \\ -w \text{ & } -w^2 \end{array} \right\}$$

$$z^2 + z + 1 = 0 \rightarrow \text{Roots}$$

$$w \text{ & } w^2$$

$\checkmark 1, -\frac{1+\sqrt{3}i}{2}, -\frac{1-\sqrt{3}i}{2}$ i.e. $1, w, w^2$ & $w^2 = \bar{w}$

Properties of cube roots of unity

- $1+w+w^2=0 \Rightarrow w+w^2=-1$

- $w \cdot w^2 = w^3 = 1 \quad \text{Product of imaginary roots} = 1.$

- $w^{3n}=1$

Cube roots of unity forms an eq. triangle in argand plane

Cube roots of unity forms an eq. triangle in argand plane

Each im. cube root of unity is square of other

$$\text{i.e. } (w)^2 = w^2 \text{ & } (w^2)^2 = w.$$

$$\checkmark w^{3k} + w^{3k+1} + w^{3k+2} = 0, \quad k \in \mathbb{Z}.$$

\checkmark Fourth roots of unity $\rightarrow \pm 1, \pm i$

n^{th} roots of unity

\checkmark forms a regular polygon having n sides {eq. Δ, \square, \dots }

\checkmark sum of n roots = 0 i.e. $1+w+w^2+\dots+w^{n-1}=0$ ($n > 1$)

$$\checkmark$$
 Product $\rightarrow (-1)^{n-1}$

$$\checkmark$$
 Product $\rightarrow (-1)^{n-1}$ product of n roots = $\frac{1}{2} |z|^2$ as vertices = $\frac{1}{2} |z|^2$

Area of Δ formed by z, iz & $z+iz$ as vertices

Types of Relation

i) Reflexive Relation: $[(a, a) \in R, \text{ for every } a \in A] \rightarrow$ reflexive relation R in set A

e.g.: $A = \{1, 2, 3\}; R = \{(1, 1), (2, 2), (3, 3), (1, 3)\} \rightarrow$ reflexive relation.

ii) Symmetric Relation: $[(x, y) \in R \Rightarrow (y, x) \in R]$

e.g.: $A = \{1, 4, 5\}; R = \{(4, 5), (5, 4), (1, 1), (1, 4), (4, 1)\}$

iii) Antisymmetric relation: $[(x, y) \in R \text{ and } (y, x) \in R \Rightarrow x = y]$

iv) Transitive Relation: $[(x, y) \in R \text{ and } (y, z) \in R \Rightarrow (x, z) \in R]$

e.g.: $A = \{2, 4, 8\}; R = \{(2, 4), (4, 8), (2, 8)\} \because 2 < 4 \text{ and } 4 < 8 \Rightarrow 2 < 8$

v) Equivalence Relation: Reflexive, symmetric & transitive

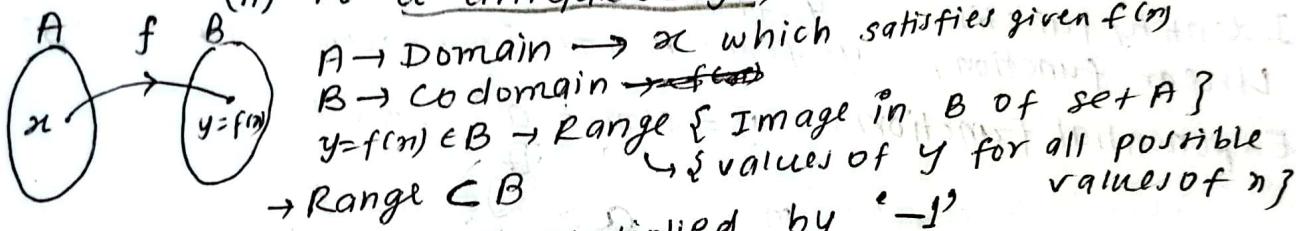
$R \subset A \times B$.

Function

~~From~~ Relation from set A to set B is said to be function if

(i) Every element of set A associates

(ii) To a unique (single) element of set B .



Inequality change (reverse) \rightarrow (i) Multiplied by -1 , (ii) reciprocal

Modulus function $\rightarrow |x| \geq 0$

$$y = |x| \begin{cases} x \text{ for } x \geq 0 \\ -x \text{ for } x < 0 \end{cases}$$

$$\begin{aligned} D_f &= R = (-\infty, \infty) \\ R_f &= [0, \infty) \\ &= R - (-\infty, 0) \end{aligned}$$

Equal function: $f(x) = g(x) \quad \{ \text{for all } x \in A \}$

(i) Domain of f = Domain of $g \Rightarrow D_f = D_g$.

(ii) $f(x) = g(x)$ for all $x \in D_f$.

~~If~~ Number of functions from finite set A into a finite set $B \rightarrow \{n(B)\}^{n(A)}$ ~~from~~ [TO]

Types of functions:-

(i) One to one or injection

↳ If diff. elements in A have diff. images in B .

i.e. For $x_1, x_2 \in A$

$x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$

or $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

~~If~~ Number of functions from a finite set A into a finite set $B \rightarrow \{n(B)\}^{n(A)}$

(ii) Onto or Surjective function.

↳ Every element of B has at least one pre-image in A , otherwise it is said to be into.

i.e. $f(A) = B \rightarrow$ onto functions. { B has every element has pre-image in A }
 $f(A) \subset B \rightarrow$ into function.

~~Every polynomial function $f: P \rightarrow P$ of degree odd is onto & constant with singleton co-domain \rightarrow onto function~~

→ eg: $f(x) = 3x + 2 \Rightarrow x = \frac{y-2}{3} \in Q \quad f: Q \rightarrow Q$ (Q - Rational)

$$f\left(\frac{y-2}{3}\right) = 3\left(\frac{y-2}{3}\right) + 2 = y \Rightarrow y \text{ is image of } x \text{ i.e. } \left(\frac{y-2}{3}\right)$$

⇒ f is onto function

(iii) Bijective function.

↳ Both one to one and onto i.e. Both injective & surjective.

• Constant function, $f(x) = c$ { singleton }

• Identity function, $y = f(x) = x$

• Linear function, $y = f(x) = mx + c$ { $m, c \rightarrow$ constant & $m \neq 0$ }

~~Exponential function, $y = f(x) = a^x$, $x \in R$~~

$a > 0 \text{ & } a \neq 1$

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

$$= \lim_{h \rightarrow 0} (1+h)^{1/h}$$

• Polynomial function, $y = f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a_n$

Polynomial function of degree n

$a_0 \neq 0$ & n is non-negative integer.

• Logarithmic function: inverse of exponential

If $y = a^x$, $a \neq 1$ & $a > 0$ its inverse

$$x = a^y$$

$$\Rightarrow y = \log_a x$$

• Reciprocal function: $y = f(x) = \frac{1}{x}$, $x \neq 0$

$$D_f = R - \{0\}$$

$$R_f = R - \{0\}$$

$xy = 1 \rightarrow$ Rect. hyperbola.

(iv) Inverse function,

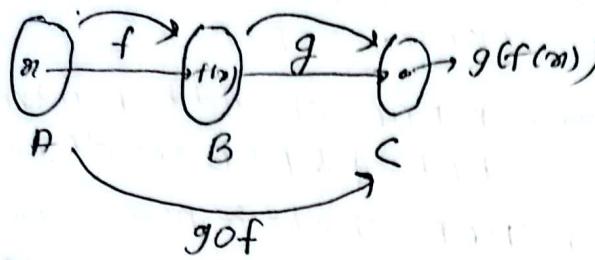
If $f: A \rightarrow B$; then $f^{-1}: B \rightarrow A$ is inverse function of f

~~Note ① For inverse function to exist, the function must be bijective & i.e. one-to-one & onto}~~

② If A and B are inverses of each other, then

: Domain of $A =$ Range of B } Can be used as trick as well.
: Range of $A =$ Domain of B }

Composite function
 Let; $f: A \rightarrow B$ and $g: B \rightarrow C$ then composite function of f and g denoted by gof is a function from A to C .
 i.e. $gof: A \rightarrow C$, defined by $(gof)(x) = g(f(x))$ for all $x \in A$



Even function
 $\hookrightarrow f(-x) = f(x)$
 Odd function
 $\hookrightarrow f(-x) = -f(x)$

Periodic function

- ① $\sin^n x, \cos^n x, \sec^n x, \csc^n x \rightarrow 2\pi \rightarrow n \text{ is odd}$
 $\pi \rightarrow n \text{ is even}$
- ② $\tan^n x, \cot^n x \rightarrow \pi \rightarrow n \text{ is even or odd.}$
- ③ $|\sin x|, |\cos x|, |\tan x|, |\sec x|, |\csc x|, |\cot x| \rightarrow \pi$
- ④ $|\sin x| + |\cos x|, |\tan x| + |\cot x|, |\sec x| + |\csc x| \rightarrow \pi/2$
- ⑤ $f(x)$ is periodic with period $\rightarrow T$, then $f(ax+b)$ is periodic with period $\frac{T}{|a|}$
- ⑥ If $f(x), g(x), h(x)$ are periodic with period T_1, T_2, T_3 then Period of $a f(x) \pm b g(x) \pm c h(x)$ is $\hookrightarrow \text{L.C.M. of } \{T_1, T_2, T_3\}$ {where. a, b, c are constants}

Note:

$$\text{LCM of } \frac{a}{a_1}, \frac{b}{b_1}, \frac{c}{c_1} = \frac{\text{LCM of } \{a, b, c\}}{\text{H.C.F. of } \{a_1, b_1, c_1\}}$$

Period \rightarrow can be checked from options. as well $A \times B \rightarrow$ cartesian product

If A contains m elements and B contains n elements. then number of elements in $A \times B$ is $m n$

No. of possible relations from set A to set B = 2^{mn} {Number of subsets-
 $n(A)$ }

No. of functions from finite set A to finite set B = $[n(B)]^{n(A)}$

$|a| < a \Rightarrow -a < x < a$ then $o((A \times B) \cap (B \times A)) = n^2$

$\emptyset \rightarrow$ empty set $\{0\} \rightarrow$ set containing only one element i.e. it's singleton set (finite set)

Power set of A: set of all subsets of A if $n(A)=n$.

\hookrightarrow No. of ~~sets~~ subsets in $P(A) = 2^n$

\hookrightarrow No. of proper subsets = $2^n - 1$ {except A}

\hookrightarrow No. of non-empty proper subset = $2^n - 2$ {except A & \emptyset }

Proper subset: If A is proper subset of B if
 $A \subset B$ and $A \neq B$

Equivalent set \rightarrow same no. of elements.

$\checkmark A \Delta B = (A - B) \cup (B - A)$ } set of non-common
= $(A \cup B) - (A \cap B)$ } elements of $A \& B$.

For $n(A \cup B)$ to be maximum, $n(A \cap B)$ should be minimum
 $n(A \cup B) \leq n(U)$ i.e. $n(A \cap B) = 0$ i.e. $A \cap B = \emptyset$ & $A \& B$ are disjoint sets.

For $n(A \cup B)$ to be min, $n(A \cap B)$ should be maximum
i.e. $A \subset B$.

~~# Idempotent law: $A \cup A = A$ & $A \cap A = A$~~

Identity law: $A \cup \emptyset = A$; $A \cap \emptyset = \emptyset$; $A \cup U = U$; $A \cap U = A$

De Morgan's law: $(\overline{A \cup B}) = (\overline{A} \cap \overline{B})$ & $(\overline{A \cap B}) = (\overline{A} \cup \overline{B})$

$\checkmark A - (B \cap C) = (A - B) \cup (A - C)$

$A - (B \cup C) = (A - B) \cap (A - C)$

$A \cap (B - C) = (A \cap B) - (A \cap C)$

~~# $x \notin B \cup C = x \notin B$ and $x \notin C$~~

~~$x \notin B \cap C = x \notin B$ or $x \notin C$~~

$|x| + y \leq |x| + |y|$; $|x - y| \geq |x| - |y|$; $|x|^2 = x^2$

$A \times B \neq B \times A$; $A \subseteq B$ then $A \times C \subseteq B \times C$ for any set C .

Relation on $A \Rightarrow$ Relation from A to A .

~~# If $n(A) = n$; Number of relation on $A = 2^{n \cdot n} = 2^{n^2}$~~

Relation $C(A \times B)$

$R = \{(x, y) : x \in A, y \in B\} \subseteq A \times B$

$R^{-1} = \{(y, x) : y \in B, x \in A\} \subseteq B \times A$

$(2^x)^2 = 2^{2x}$; 2^{x^2} if $2^{2^n} = 2^{n^2} \Rightarrow 2n = n^2$ or, $n^2 - 2n = 0$

$\sqrt{n^2} = |n|$ or $\pm n$.

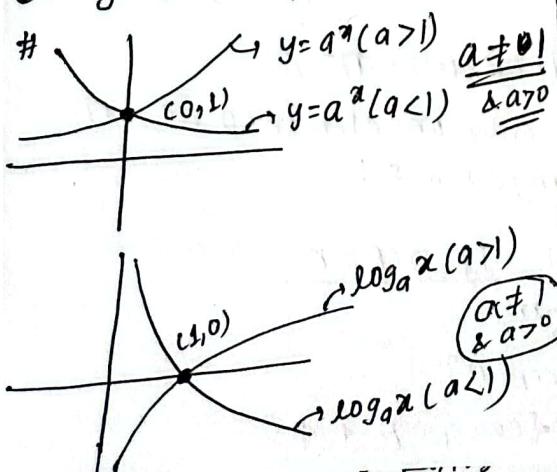
$\checkmark y = e^x \Rightarrow D_f = R$; $R_f = (0, \infty)$ } $y = a^x \Rightarrow D_f = R$; $R_f = (0, \infty)$, $a \neq 1$

$y = a^x$ (a > 1) $\frac{a \neq 0}{a > 0}$ } $\log_a a = 1$, $\log_a 1 = 0$ [a > 0, a ≠ 1]
 $\log_a b = \frac{1}{\log_b a}$ [base > 0 & ≠ 1]

$\log_a b = \log_c b - \log_c a = \frac{\log_c b}{\log_c a}$

$\log_a(xy) = \log_a x + \log_a y$ } $x > 0$,
 $\log_a(\frac{x}{y}) = \log_a x - \log_a y$ } $y > 0$,

$\log_{a^n}(x) = \frac{1}{n} \log_a x$ } base > 0 & ≠ 1
 $\log_{a^n}(x^m) = \frac{m}{n} \log_a x$



Binomial Theorem
 $n! = n \times (n-1)!$

$$(x+y)^n = {}^n C_0 x^n + {}^n C_1 x^{n-1} y^1 + \dots + {}^n C_r x^{n-r} y^r + \dots + {}^n C_n y^n$$

$n \rightarrow +ve$ integer value (Natural number) $\{0! = 1\}$

${}^n C_r \rightarrow$ Binomial coefficient: ${}^n C_r = \frac{n!}{(n-r)! r!}$ $\{1! = 1\}$

* ${}^5 C_2 = \frac{5 \times 4}{2 \times 1}$; ${}^6 C_3 = \frac{6 \times 5 \times 4}{3!}$ upto 3 numbers (lower index)
→ factorial of lower index.

• ${}^n C_r = {}^n C_{n-r}$ ${}^n C_1 = n$ ${}^n C_n = 1$, ${}^n C_0 = 1$

• General term of $(x+y)^n$

$$T_{r+1} = {}^n C_r x^{n-r} y^r \quad \left\{ \begin{array}{l} r \rightarrow \text{integer}; r \neq \text{negative or fraction} \\ r \leq n \end{array} \right.$$

$r \rightarrow (r+1)^{\text{th}}$ term eg: $r=5 \rightarrow 6^{\text{th}}$ term

$r=4 \rightarrow 5^{\text{th}}$ term

• In expansion of $(x+y)^n \rightarrow (n+1)^{\text{th}}$ term

~~p^{th}~~ term from end $\Rightarrow (n-p+2)^{\text{th}}$ term from Beginning

If coefficient of T_r, T_{r+1}, T_{r+2} of expansion $(1+x)^n$ are in A-P then

$$(n-2r)^2 = n+2$$

In G.P.: $S_n = \frac{a(r^n - 1)}{r-1}$ $\{n = \text{number of terms}\}$

$$\begin{aligned} \text{I} (1-x)^{-1} &= 1 + x + x^2 + x^3 + \dots + \infty \\ \text{II} (1+x)^{-1} &= 1 - x + x^2 - x^3 + \dots + \infty \end{aligned}$$

Middle term: $(x+y)^n$

$\rightarrow [n \rightarrow \text{even}] \rightarrow$ odd terms \rightarrow only 1 middle term $\{r \leq n\}$ $\{r \geq 0\}$ $\{r \leq n\}$ $\{r \geq 0\}$ $\{r \leq n\}$ $\{r \geq 0\}$

Middle term: $T_{\left(\frac{n}{2}\right) + 1}$

$$\begin{aligned} \text{III} (1-x)^{-2} &= 1 + 2x + 3x^2 + 4x^3 + \dots \\ \text{IV} (1+x)^{-2} &= 1 - 2x + 3x^2 - 4x^3 + \dots \\ \text{V} (1-x)^{-\frac{1}{2}} &= 1 + \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4} x^2 + \dots \end{aligned}$$

$\rightarrow [n \rightarrow \text{odd}] \rightarrow$ even terms \rightarrow Two middle terms

Middle terms: $T_{\left(\frac{n+1}{2}\right)}, T_{\left(\frac{n+1}{2} + 1\right)}$

(Q) Coefficient of x^5 in the expansion of $(1+x)^3(1-x^2)^7$

$$\textcircled{A} {}^7 C_5 - {}^5 C_3 \quad \textcircled{B} {}^{14} C_5 \quad \textcircled{C} {}^3 C_1 {}^7 C_2 - {}^3 C_3 {}^7 C_1 \quad \textcircled{D} {}^7 C_5 {}^5 C_1 - {}^7 C_2 {}^5 C_3$$

Greatest Binomial coefficient Numerically Greatest Coefficient & Numerically greatest term (for given value of variable)

→ Greatest Binomial coefficient \Rightarrow Binomial coeff. of middle terms.

- $(1+x)^n \rightarrow$ Greatest Binomial coefficient & same.
Numerically greatest term

~~$(x+y)^n$~~ : $m = \frac{n+1}{1 + \frac{|x|}{|y|}}$ → Numerically greatest coefficient ($x=y=1$)

$n \rightarrow$ index $m-1 \leq r \leq m$ { if $m \rightarrow$ decimal
 $r \rightarrow$ integral value b/w decimal values }

$m \rightarrow$ position of Numerically Greatest coefficient

No. of terms in the expansion

- $(n+y)^n \rightarrow (n+1)$ term

~~$(x+y+z)^n \rightarrow n+r-1 C_{r-1}$~~ { $r =$ no. of variables
here: $x, y, z \Rightarrow r=3$ }

Polynomial in single variable having same sign

Eg: $(4x^3 + 5x^2 + x + 1)^6$ { variable $\rightarrow x$
 x^0 sign $\rightarrow +$ }

~~No. of terms = $N = (\text{Higher degree} - \text{Lower degree}) \times \text{Index} + 1$~~

$N = (3-0) \times 6 + 1 = 19$ terms.

sum of coefficients: Put variables = 1

Eg: sum of coeff. in exp $(ax+by)^n$
sum of coeff = $(a+b)^n$ & For $x=y=1$.

$(x+y)^n = \sum_{r=0}^n n C_r x^{n-r} y^r$

$(1+x)^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \dots + c_n x^n$
 $(1-x)^n = c_0 - c_1 x + c_2 x^2 - c_3 x^3 + \dots + (-1)^n c_n x^n$

sum of binomial coefficients: $c_0 + c_1 + c_2 + c_3 + \dots + c_n = 2^n$

sum of even & odd coefficients: $c_0 + c_2 + c_4 + c_6 + \dots = c_1 + c_3 + c_5 + \dots$
 $= 2^n - 1$

$(1+\alpha)^n$
when, n is not natural number & n is fractional or negative

& $|x| < 1$ \hookrightarrow coeff. can't be expressed as nC_0, nC_1, \dots

$$\checkmark (1+\alpha)^n = 1 + \frac{n\alpha}{1!} + \frac{n(n-1)}{2!} \alpha^2 + \frac{n(n-1)(n-2)}{3!} \alpha^3 + \dots \text{ } \infty \text{-terms}$$

$$|x| < a \Rightarrow -a < x < a$$

Q) Coefficient of x^3 in expansion $\frac{(1+3x)^2}{1-2x}$

- (i) 50 (ii) 32 (iii) 48 (iv) 55

Sum of Product of Binomial coefficients of $C_0 C_2 + C_1 C_{2+1} + \dots$

= $\frac{(2n)!}{(n-2)! (n+2)!}$ $\begin{cases} \text{If difference of lower index is constant} \\ \text{or } x \rightarrow \text{difference of lower indices} \end{cases}$

eg: $\underset{\cancel{2}}{C_0} \underset{\cancel{2}}{C_2} + \underset{\cancel{2}}{C_1} \underset{\cancel{3}}{C_3} + \underset{\cancel{2}}{C_2} \underset{\cancel{4}}{C_4} + \dots + C_{n-2} + C_n$

$$= \frac{(2n)!}{(n-2)! (n+2)!}$$

★ Putting value of n to find sum

If given other constant in sum like a_1, a_2, \dots then false
 $n \geq 2$

$$\checkmark \underline{a_0 C_0 + a_1 C_1 + a_2 C_2 + a_3 C_3 + \dots + a_n C_n}$$

If a_0, a_1, a_2, a_3 are in A.P., then

$$= \underline{(2a_0 + nd) 2^{n-1}} \quad \begin{cases} a_0 \rightarrow \text{first term} \\ d \rightarrow \text{common difference} \end{cases}$$

$$\text{OR } S = (a_0 + a_n) 2^{n-1}$$

$$\checkmark \underline{(x+a)^n + (x-a)^n}$$

Total number of terms = $\left(\frac{n}{2} + 1\right)$ if n is even
(After simplification) & $\left(\frac{n+1}{2}\right)$ if n is odd

$$\checkmark \underline{(x+a)^n - (x-a)^n}$$

Total number of terms in the expansion after simplification
 $= \frac{n}{2}$ if n is even

$$= \frac{n+1}{2} \text{ if } n \text{ is odd}$$

$$\checkmark \text{ In expansion } \left(x^p + \frac{1}{x^q}\right)^n$$

For term independent of x , $\begin{cases} x = \frac{n \times p}{p+q} \end{cases}$

Ans: $C(n, r)$

$r \rightarrow -ve$ or fraction
 No term independent of x .

→ For $(x^p - \frac{1}{x^q})^n$: Independent term of n
 γ can be found in some way for other question of n^p but be cautious while finding coeff.
 γ Ans: $C(n, r)$ if r is even
 $-C(n, r)$ if r is odd.

| (8) The independent term in expansion

$$\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - \sqrt{x}} \right)^{10} \text{ is } \left(\frac{1}{(-ve)!} \right) = \text{considered 0.}$$

- Ⓐ $C(10, 0)$ Ⓑ $C(10, 3)$ Ⓒ $C(10, 10)$ Ⓓ $C(10, 4)$

Ⓑ $C(10, 4)$
 $\left\{ (-ve)! = \text{not defined}\right\}$

$$e^x = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{nx} = \lim_{n \rightarrow \infty} (1+n)^n$$

For $(ax^p \pm \frac{b}{x^q})$

To find coefficient of x^m in expansion

$$\gamma = \frac{n \times p - m}{p+q}$$

$$\boxed{\text{In AP} \quad S_n = \frac{n}{2} [2a + (n-1)d]}$$

• Next, use above γ to find coefficient.

$$\# e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \quad \left\{ e^{a+b} = e^a \cdot e^b \right\}$$

$$\# \log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \quad \left\{ \text{For } -1 < x \leq 1 \right\}$$

$$\# (a^x = 1 + \frac{x}{1!} \log_e a + \frac{x^2}{2!} (\log_e a)^2 + \dots) \rightarrow a > 0. \quad \left\{ \text{factorial} \rightarrow ! \Rightarrow \text{In exp. of } e^x \right\}$$

$$\# \log_e(1-x) = -\frac{x}{1} - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \dots \quad \left\{ \text{For } -1 \leq x < 1 \right\}$$

The coefficient of x^n in the expansion $\log_e(1+x+x^2)$ is

• $-2/n$ if n is multiple of 3

• $1/n$ if n is not multiple of 3

$$\# \log a - \log b = \log \left(\frac{a}{b}\right) \quad \# \log a + \log b = \log(a \cdot b)$$

$$\# \text{ In expansion } (n+1)(n+2)(n+3) \dots (x+n) \quad \text{coeff. of } x^{n-1} = \frac{n(n+1)}{2} \quad \log_e \left(\frac{1+x}{1-x}\right) = x \left(\frac{1}{1} + \frac{x^2}{2} + \frac{x^4}{3} + \dots \right) \quad \text{for } -1 < x < 1 \text{ i.e. } |x| < 1.$$

$$\# \text{ If the coeff. of } p^{\text{th}} \text{ and } q^{\text{th}} \text{ terms in the expansion of } (1+x)^n \text{ are equal, then} \quad \boxed{p+q=n+2} \quad \boxed{\text{In GP} \quad S_n = \frac{a(s^n-1)}{s-1}}$$

$$\# \frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \dots \quad \left\{ \begin{array}{l} \text{only even terms} \\ \text{don't forget } L^p \end{array} \right.$$