

CS203A: Mathematics for Computer Science III

Quiz 2 (Set A)

Name: _____

Date of Examination: 9.4.2024

Roll No: _____

Time duration: 45 minutes

Total Marks: 50

Instructions

- Write down your set number, name and roll number on each of your answer sheets.
 - **Tie the question paper with your answer-scripts;** the question paper must be on the top followed by the answer scripts. Any answer-script without the question paper attached will not be graded.
 - Present the complete solution neatly and clearly. No marks will be given on the final answer if the the steps to reach it are not provided.
 - Untidy and illegible answers will not be graded.
 - You do not need to do complex calculations. Leave the answer in a state that a (scientific) calculator can be used on your final expression to arrive at the answer.
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Q.1) An eight-sided die is marked with numbers 1,2,...8. A game is played by rolling the die until an 8 appears on the uppermost face. At this point the game ends. What is the smallest value of n for which the probability of the game ending before the n^{th} roll (but not counting the n^{th} roll) is more than $\frac{3}{4}$.

Q.2) Two identical biased coins are tossed together, and the outcome is recorded. After a large number of trials it is observed that the probability that both coins land showing heads is 0.36. What is the probability that both coins land showing tails?

Q.3) Find the value of c such that $f(x) = c \cdot (1/2)^x$, where $x \in \mathbb{N}$ is a PMF.

Q.4) If $E[X] = -2$ and $\text{Var}(X) = 1$. Find $E((1 - X)^2)$ and $\text{Var}(2 - 5X)$.

Q.5) If X be a Poisson random variable with parameter λ . Find the probability $P[X \text{ is even}]$.

Q.6) Light bulbs are installed successively into a socket. Assuming that each light bulb has a mean life of 2 months with a standard deviation of 0.25 months, what is the probability that 40 bulbs last at least 7 years?

Q.7) Let X be a Gaussian random variable with mean μ and variance σ^2 . Let $Y = \max(X, 0)$, where $\max(a, b)$ is the maximum of a and b . What is the PDF of Y ?

Q.8) If X is a random variable s.t $E[X] = 3$ and $E[X^2] = 13$. Prove that $P(-2 < X < 8) > 3/4$.

Q.9) Find the MGF of a Binomial random variable with parameters p (probability of a trial success) and n (number of trials).

Q.10) A coin is weighted such that its probability of landing on heads is 20%. Suppose the coin is flipped 20 times, find the bound for the probability it lands on head atleast 16 times, using both the Markov and the Chebyshev inequalities.