

CS203: Mathematics for Computer Science III

Quiz 1

Duration: 40 minutes

Marks: 120

Name:

Roll number:

Instructions:

- Please write only the final answer in the box provided.
- You may keep the answer in a "reasonable" form (as a fraction over numbers or factorials). You should not waste time in tedious calculations that could have been done with a calculator.
- There are 12 questions on 2 pages.

1. The coefficients a and c of the equation $ax^2 + 4x + c = 0$ are determined by throwing a six-sided fair dice. Find the probability that the equation formed will have real roots.

2. How many different rearrangements are there of the letters in the word **TATARS** where the two A's are never adjacent.

3. Let X be a random variable that takes any of the values -1 , 0 , and 1 with probabilities 0.4 , 0.1 and 0.5 respectively. Calculate $E[2X^2 + 5X + 1]$.

4. You have a 10 litre bucket, initially holding two litres of water. Everyday, you toss a fair coin: if it lands head, you add 1 litre of water to the bucket; if it is tails, you drain out 1 litre of water from it. You end the game if the bucket ever becomes empty or full. What is the probability that the bucket will eventually fill up completely?

5. You toss a dice 10 times. What is the probability of getting 1 instance of one, 2 instances of two and 3 instances of three in these throws?

6. Let X be an indicator random variable on the event A , i.e. it is $X=1$ when A happens and $X=0$ when A does not happen. Can you relate the $E[X]$ and $P(A)$?

7. If $P(A|B) > P(A)$, then how do $P(B|A)$ and $P(B)$ compare? Write the answer as $P(B|A) \circledast P(B)$, where $\circledast \in >, <, \geq, \leq, =$ or mention that *nothing can be said about this relation*.

8. Suppose 36% of families own a dog, 30% of families own a cat, and 22% of the families that have a dog also have a cat. A family is chosen at random and found to have a cat. What is the probability they also own a dog?

9. A random variable $X \sim \text{Binomial}(5, p)$ such that $P(X = 2) = 2 \cdot P(X = 3)$. What is the value of p ?

10. Let X be a geometric random variable with parameter $p = \frac{1}{6}$. What is $P(X \geq 6)$?

11. A student did not study for a quiz. As the quiz is a multiple-choice (MCQ) exam with exactly one correct option for 4 provided options (with no negative marking), the student randomly marks one of the choices with uniform probability. How many questions must the student answer such that the student obtains non-zero marks with a probability greater than $\frac{1}{2}$?

12. Let X be a random variable that counts the number of throws of an unbiased dice until three sixes are obtained. Find the probability mass function (PMF) of X .