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# Computer Networks I

# Signal Modulation Techniques (Analog to Analog)

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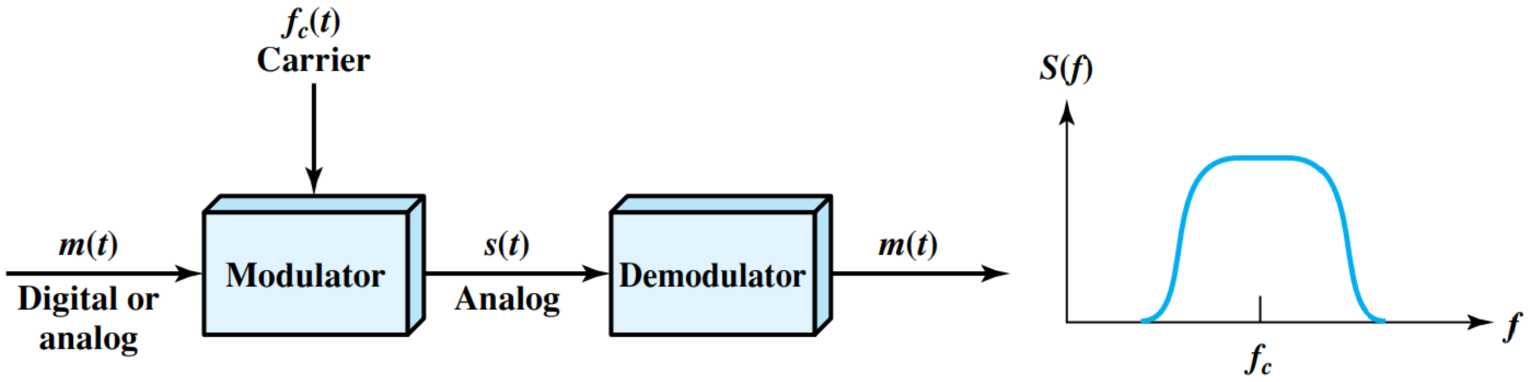
# Analog Data → Analog Signals




- Two principle techniques:
  - Amplitude modulation
  - Angle modulation
    - Frequency modulation
    - Phase modulation

# Time Domain ⇔ Frequency Domain

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft}\, dt$$

**Linearity property:** $x_1(t) \leftrightarrow X_1(f),\ x_2(t) \leftrightarrow X_2(f) \equiv a_1 x_1(t) + a_2 x_2(t) \leftrightarrow a_1 X_1(f) + a_2 X_2(f)$

$$a_1 x_1(t) + a_2 x_2(t) \leftrightarrow \int_{-\infty}^{\infty} \{a_1 x_1(t) + a_2 x_2(t)\} e^{-j2\pi ft}\, dt$$

$$= \int_{-\infty}^{\infty} a_1 x_1(t) e^{-j2\pi ft}\, dt + \int_{-\infty}^{\infty} a_2 x_2(t) e^{-j2\pi ft}\, dt = a_1 X_1(f) + a_2 X_2(f)$$

# Time Domain ⇔ Frequency Domain

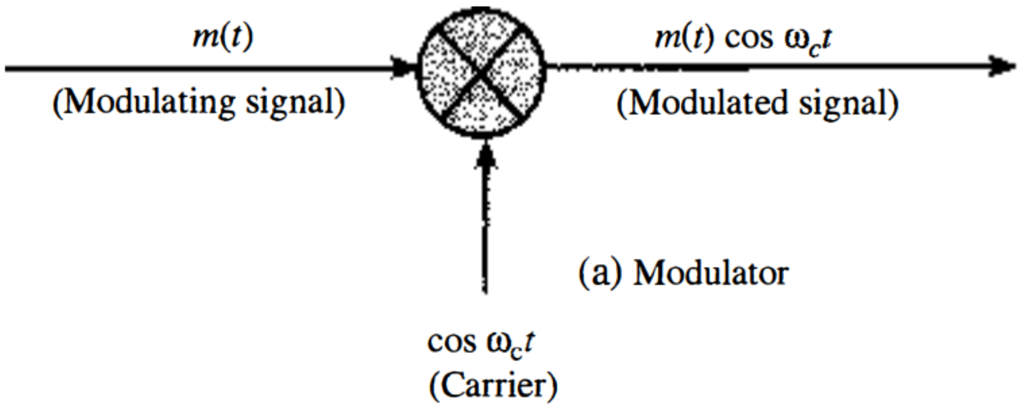
$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt$$

**Frequency shifting property:** $x(t) \leftrightarrow X(f) \equiv e^{j2\pi f_c t}x(t) \leftrightarrow X(f - f_c)$

$$e^{j2\pi f_c t}x(t) \leftrightarrow \int_{-\infty}^{\infty} e^{j2\pi f_c t}x(t)e^{-j2\pi ft}\,dt = \int_{-\infty}^{\infty} x(t)e^{-j2\pi(f-f_c)t}\,dt = X(f - f_c)$$

# DSB-SC Modulation

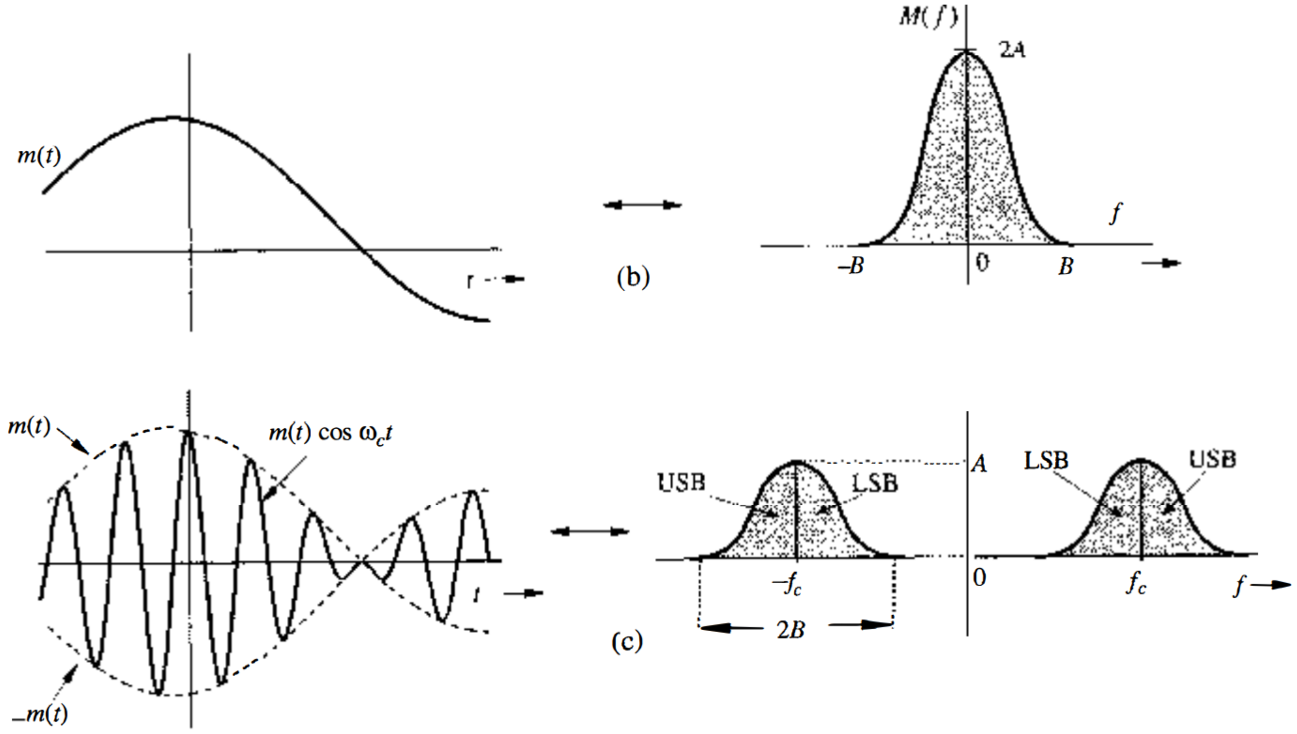
**Transmitted signal:** $m(t)\cos(2\pi f_c t)$




$$\mathbf{F}[m(t)\cos(2\pi f_c t)] = \mathbf{F}\left\{\left(\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}\right) m(t)\right\}$$

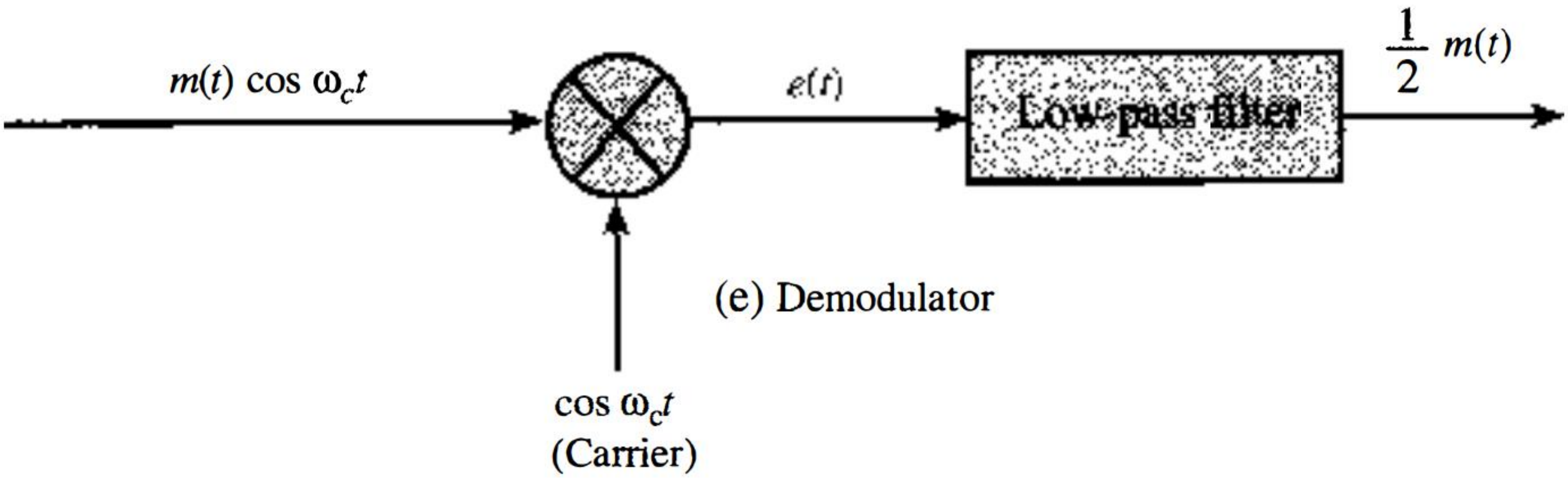
$$= \frac{1}{2}\left[\mathbf{F}\left\{e^{j2\pi f_c t} m(t)\right\} + \mathbf{F}\left\{e^{-j2\pi f_c t} m(t)\right\}\right] = \frac{1}{2}\left[M(f - f_c) + M(f + f_c)\right]$$

# DSB-SC Modulation




$$\mathbf{F}[m(t)\cos(2\pi f_c t)] = \frac{1}{2}[M(f - f_c) + M(f + f_c)]$$

# DSB-SC Demodulation

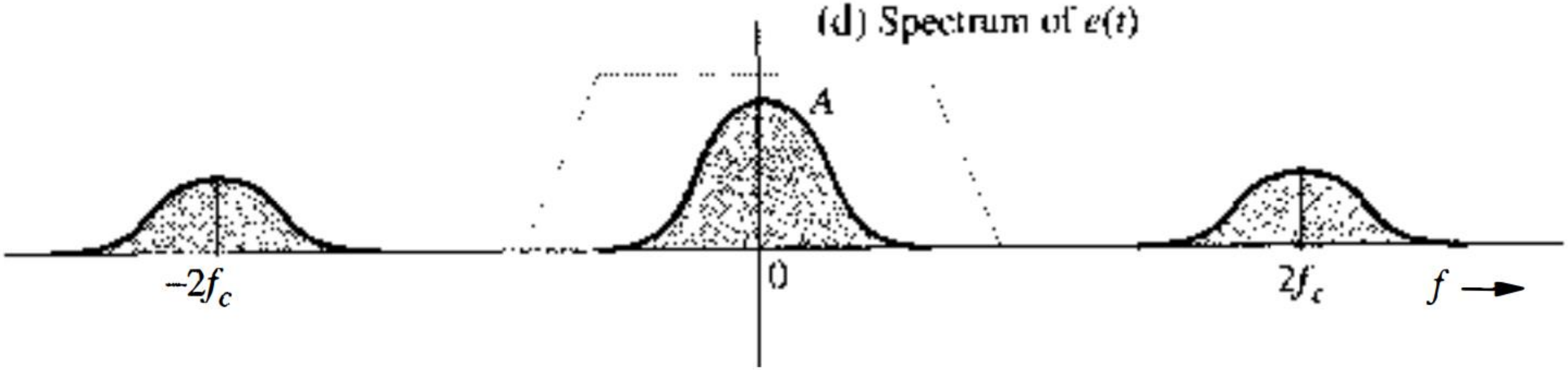


(e) Demodulator

$$\mathbf{F}[m(t)\cos^2(2\pi f_c t)] = \mathbf{F}\left\{\left(\frac{1+\cos(2\pi 2f_c t)}{2}\right)m(t)\right\}$$

$$= \frac{1}{2}M(f) + \frac{1}{4}[M(f-2f_c) + M(f+2f_c)]$$

# DSB-SC Modulation




$$\mathbf{F}[m(t)\cos^2(2\pi f_c t)] = \mathbf{F}\left\{\left(\frac{1+\cos(2\pi 2f_c t)}{2}\right)m(t)\right\}$$

$$= \frac{1}{2}M(f) + \frac{1}{4}[M(f-2f_c) + M(f+2f_c)]$$

# Angle Modulation

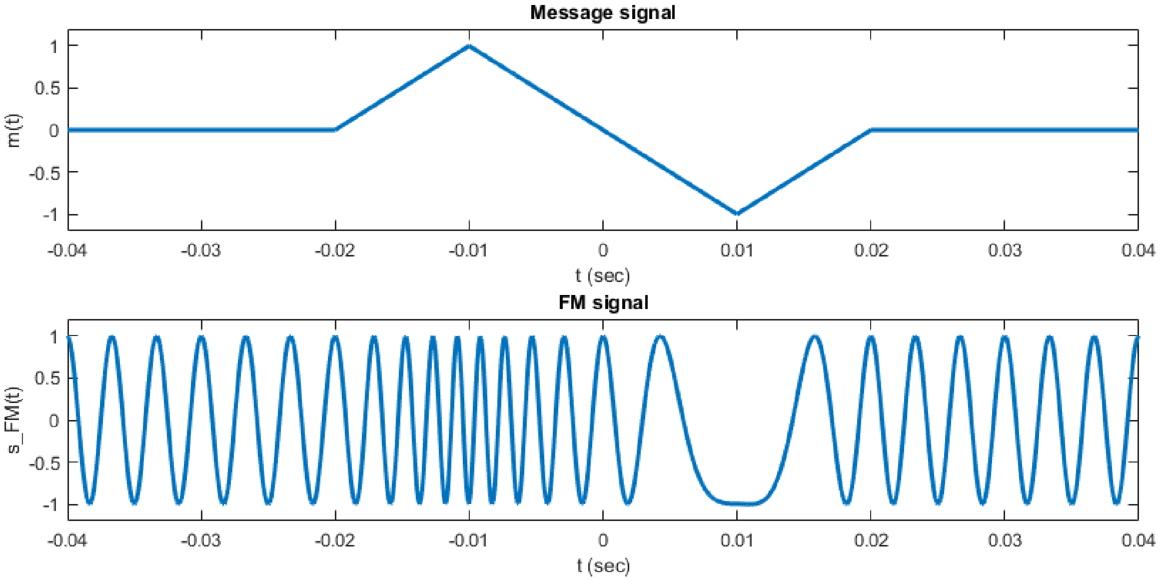
$$s(t) = A\cos\theta(t)$$

- Angle modulation:
  - Frequency modulation
  - Phase modulation

# Angle Modulation (Frequency Modulation)

$$s(t) = A\ \cos\theta(t)$$

$$w_i(t) = 2\pi f_c + k_f m(t) \qquad \therefore s_{FM}(t) = \cos((2\pi f_c + k_f m(t))t)$$

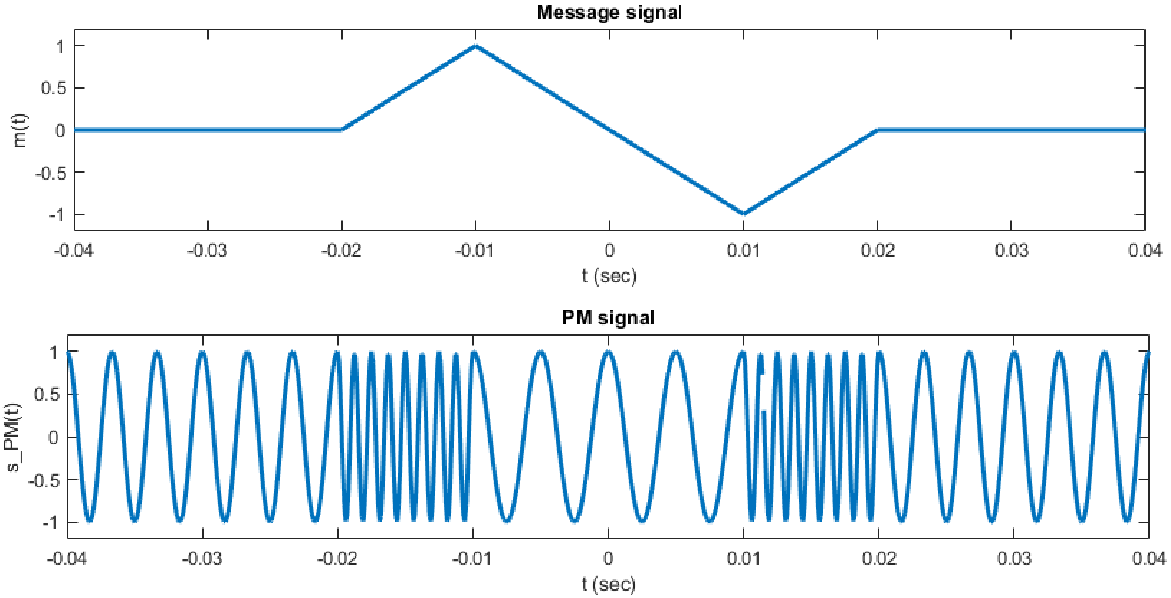


# Angle Modulation (Phase Modulation)

$$s(t) = A\ \cos\theta(t)$$

$$\theta(t) = 2\pi f_c t + k_p m(t)$$

$$\therefore s_{PM}(t) = \cos(2\pi f_c t + k_p m(t))$$



# AM vs FM/PM

- Amplitude modulation:
  - Bandwidth is less
  - More susceptible to noise → noise affects the amplitude

- Angle modulation:
  - Modulated signal requires greater bandwidth than AM
  - More robust against noise, interference

# Summary

- Modulation techniques (Analog data → Analog signals):
  - Different modulation techniques discussed
    - Amplitude modulation
    - Frequency modulation
    - Phase modulation
  - Comparison of the analog modulation techniques

# Extras

# DSB-TC Modulation

- The carrier is sent along with the message

**Transmitted signal:** $A\cos(2\pi f_c t) + \ m(t)\cos(2\pi f_c t)$

$$\mathbf{F}[A\cos(2\pi f_c t) + \ m(t)\cos(2\pi f_c t)] = \mathbf{F}[A\cos(2\pi f_c t)] + \mathbf{F}[m(t)\cos(2\pi f_c t)]$$
$$= \frac{A}{2}[\delta(f - f_c) + \delta(f + f_c)] + \ \frac{1}{2}[M(f - f_c) + M(f + f_c)]$$

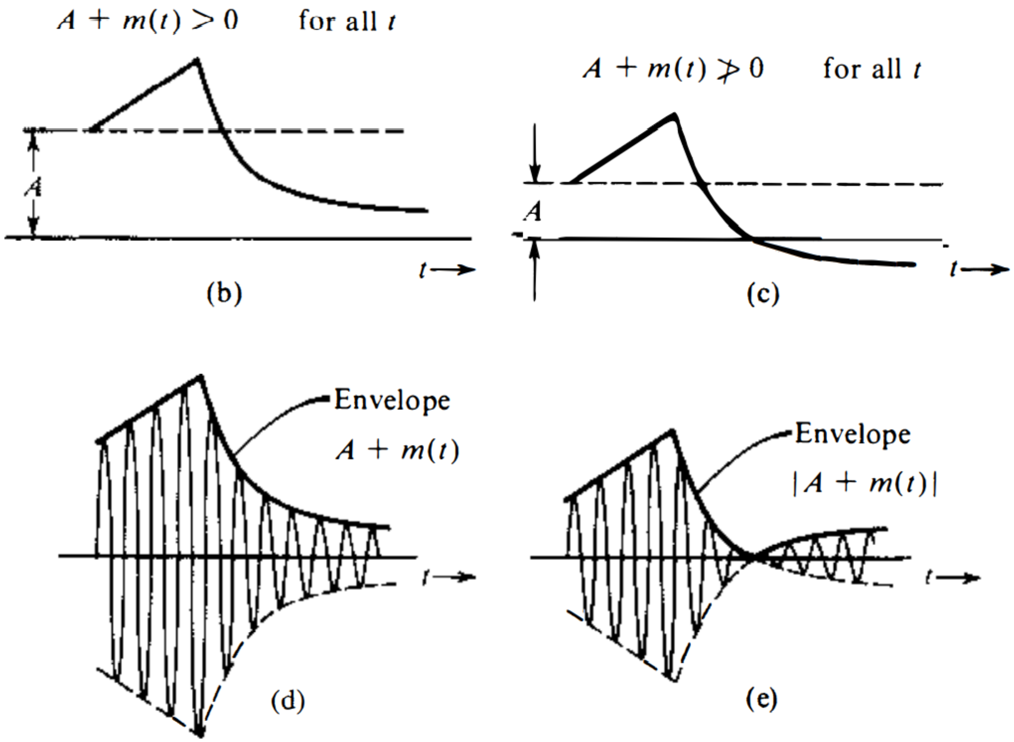
# DSB-TC Modulation

$$A + m(t) \geq 0$$

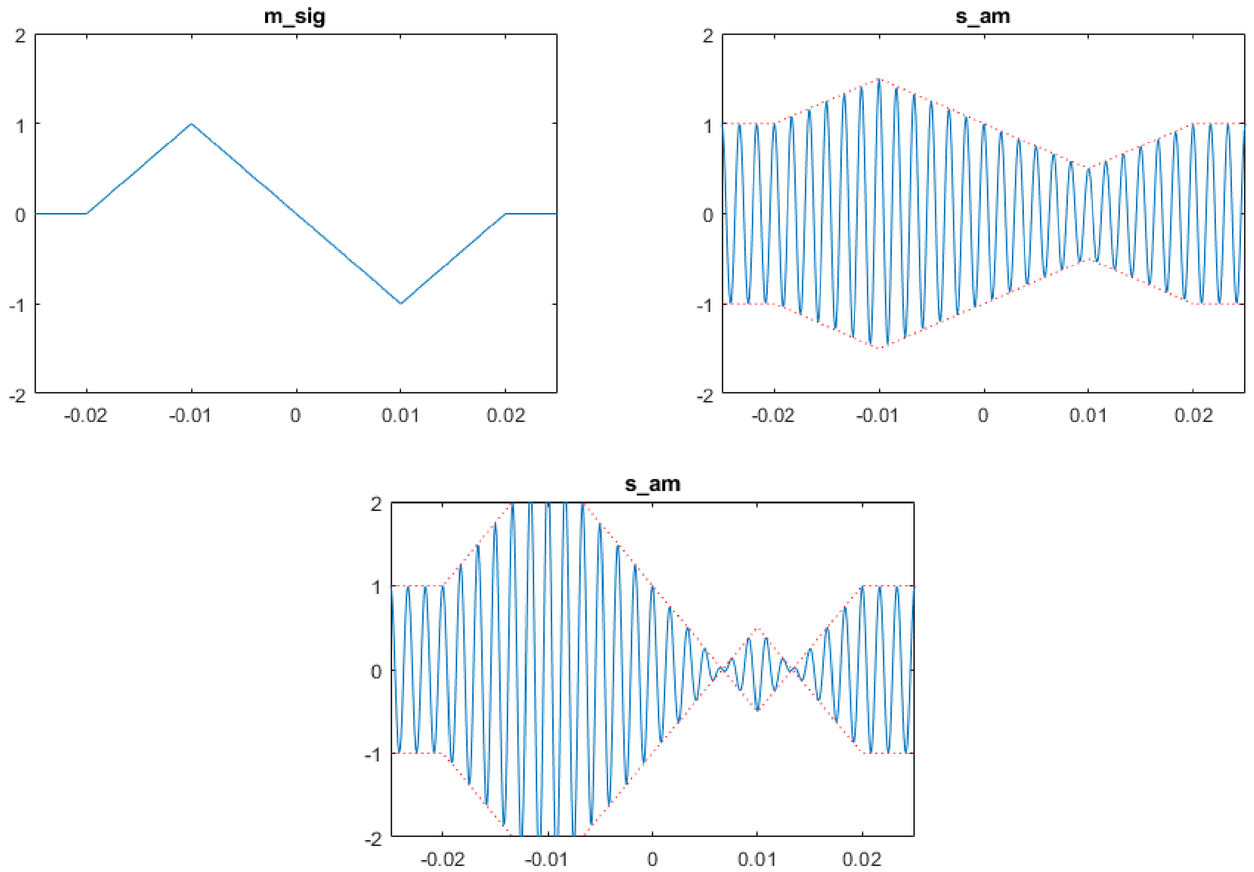
$$\therefore A \geq -m(t)$$

Now $m(t) \geq -m_p \quad \therefore A \geq m_p$

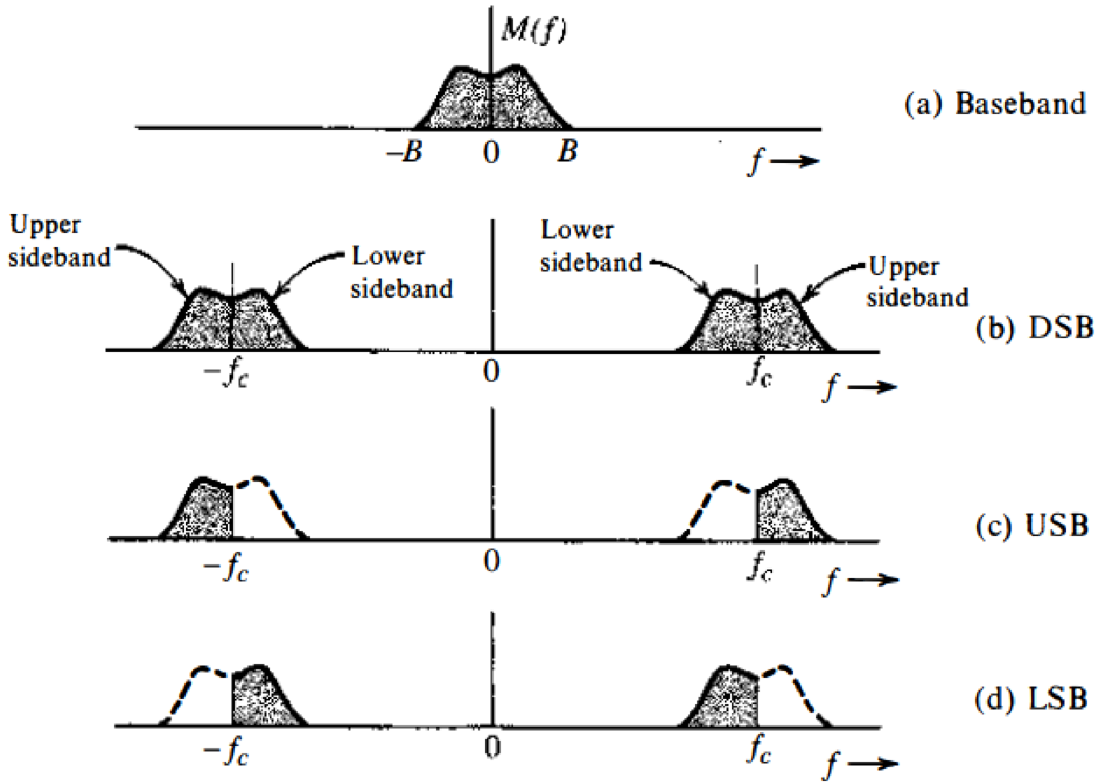
$$k_a = \frac{m_p}{A} \quad \therefore 0 \leq k_a \leq 1$$



# DSB-TC Modulation



# SSB Modulation



# Angle Modulation

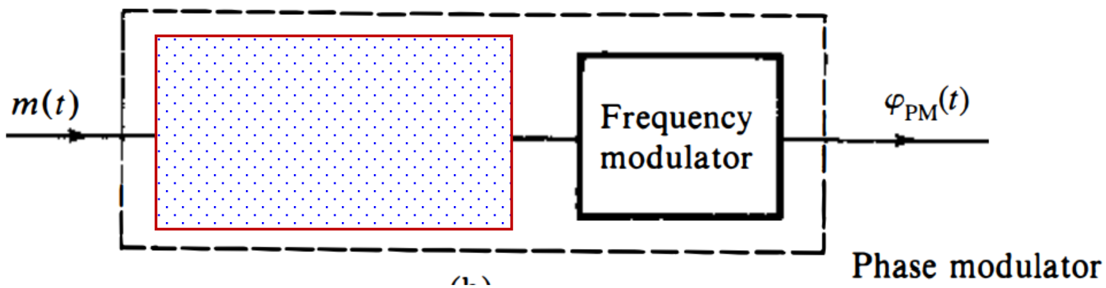
$$s(t) = A\ \cos\theta(t)$$

Instantaneous angular frequency is $w_i(t) = \frac{d\theta}{dt}$

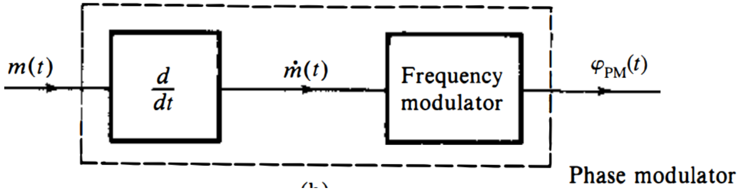
$w_i(t) = 2\pi f_c + k_f m(t)$ $\quad\therefore s_{FM}(t) = \cos((2\pi f_c + k_f m(t))t)$

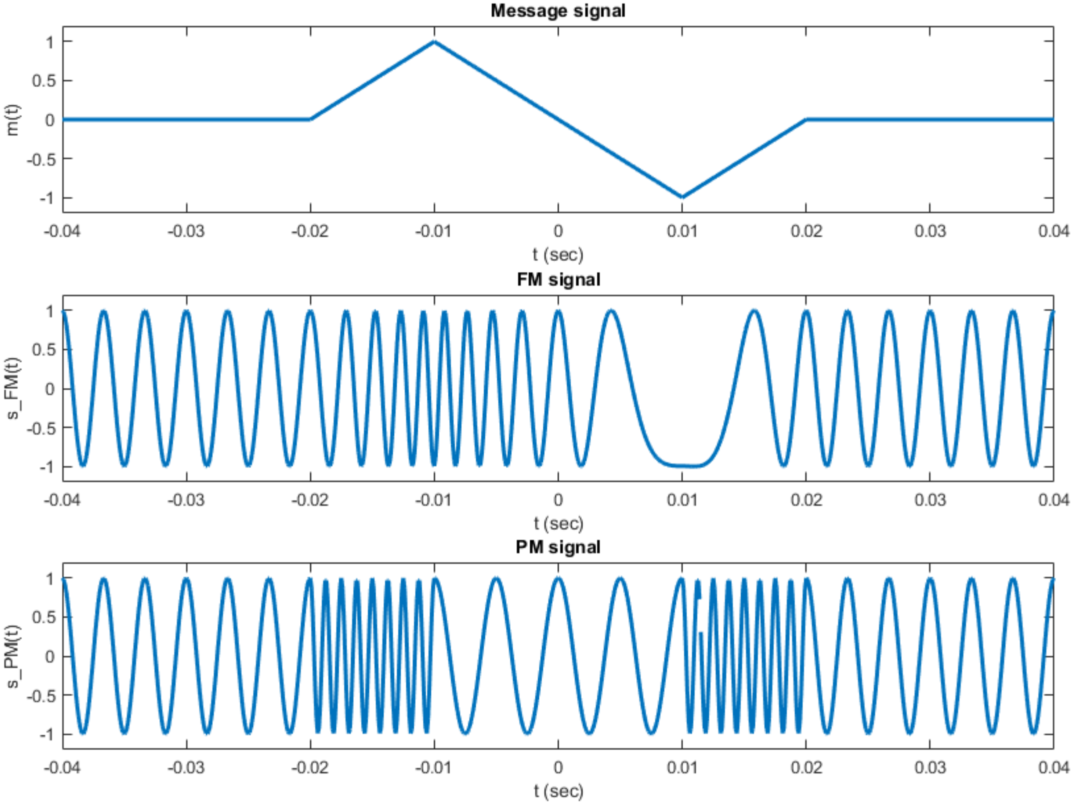
$\theta(t) = 2\pi f_c t + k_p m(t)$ $\quad\therefore s_{PM}(t) = \cos(2\pi f_c t + k_p m(t))$

Instantaneous angular frequency is $w_i(t) = \frac{d\theta}{dt} = 2\pi f_c + k_p \dot{m}(t)$



# Angle Modulation



# Angle Modulation