W4

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Computer Networks I

Error Correction

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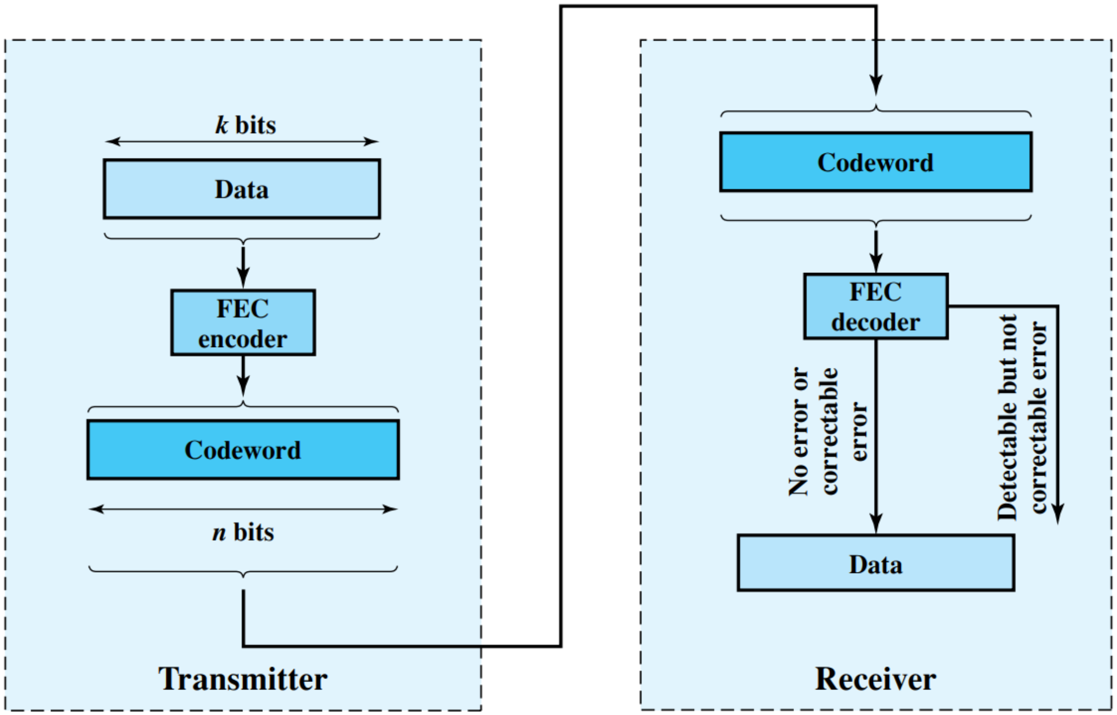
Error Correction

- Backward error correction: Correction of detected errors usually requires data blocks to be retransmitted
- Not appropriate for some wireless applications:
  - The bit error rate (BER) on a wireless link can be quite high, which would result in a large number of retransmissions
  - Propagation delay is very long compared to the transmission time of a single frame

Error Correction

- Forward error correction: Need to correct errors on basis of bits received



Error Correction

- Hamming distance:
  - $d(v_1, v_2)$ – Hamming distance in between any two binary sequences v_1 and v_2
    - $d(v_1, v_2)$ is the number of bits in which v_1 and v_2 disagree

Error Correction

- Codeword of n bits = m message bits + r check bits

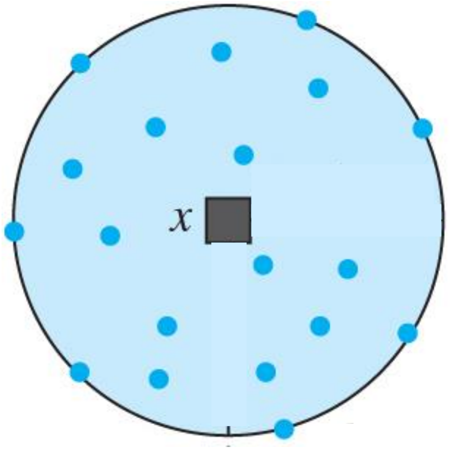
- Valid/legal and invalid/illegal codeword:
  - Total possible codewords with n bits is 2^n
  - All 2^m data messages are legal → each one of them has 1 legal codeword
  - All remaining codewords are invalid

0000000000

0000011111

1111100000

1111111111



Error Correction

- Given an algorithm for calculating the check bits
  - We can construct the list of valid codewords

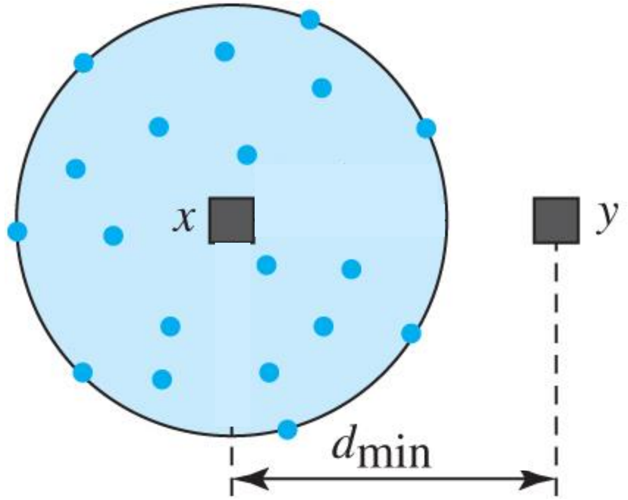
- Hamming distance of the code: Smallest Hamming distance in between any two valid codewords

0000000000

0000011111

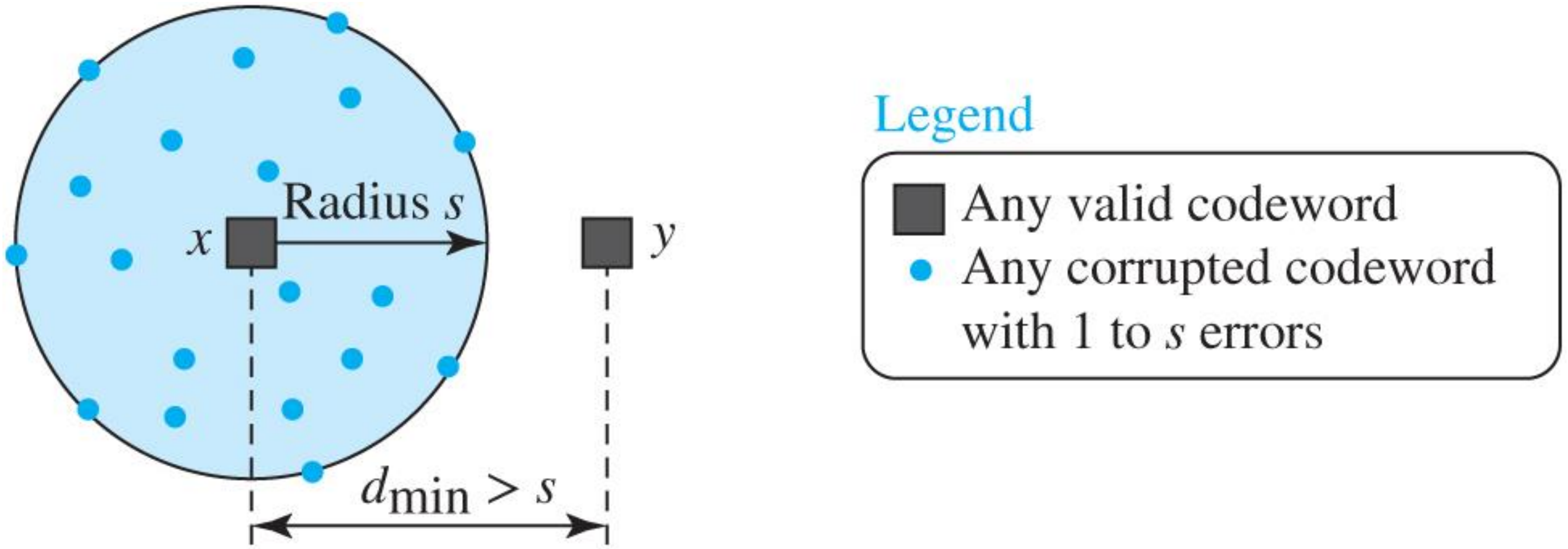
1111100000

1111111111



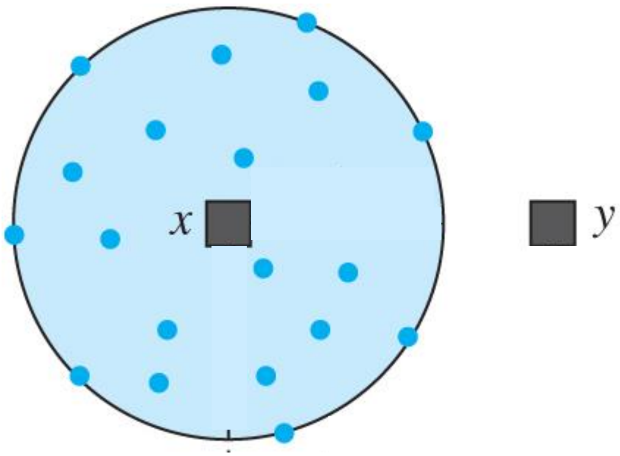
Error Correction

- To detect s bit errors, we need a distance $s + 1$ code
  - With such a code, there is no way that a s single bit error can change a valid codeword to another valid codeword



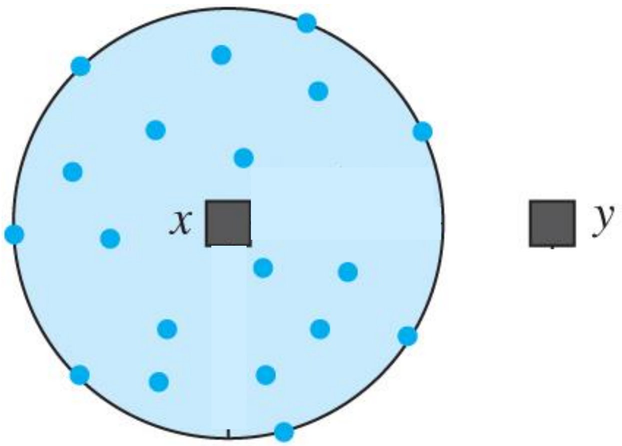
Error Correction

- To correct s bit errors, we need a distance $2s + 1$ code
  - With such a code, the legal codewords are so far apart that even s changes, the original codeword is still closer, than any other codeword



Error Correction

- To correct s bit errors, we need a distance $2s + 1$ code
  - Original codeword can be uniquely detected based on the assumption that a larger number of errors are less likely → time consuming search



Error Correction

- Codeword of n bits = m message bits + r check bits

- We want to design a code that allows all single bit error to be corrected
  - Each of the 2^m legal messages → there is n illegal codewords at a distance 1 from it
  - As there are 2^n total number of bit patterns

$$2^m(n+1) \leq 2^n \qquad \therefore n+1 \leq 2^{n-m} = 2^r \qquad \therefore m+r+1 \leq 2^r$$

Error Correction

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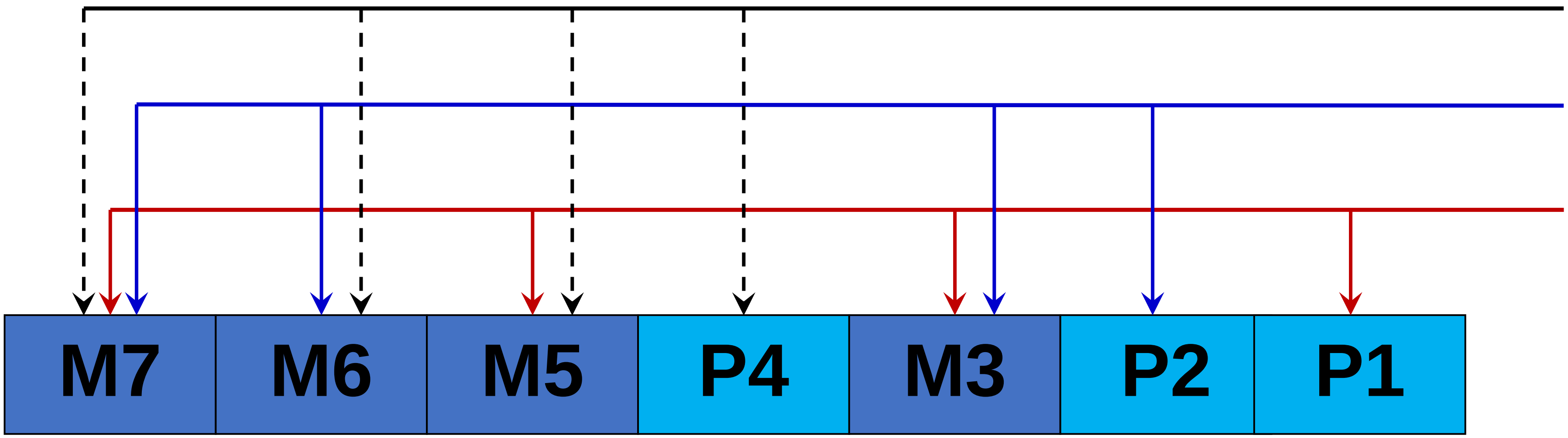
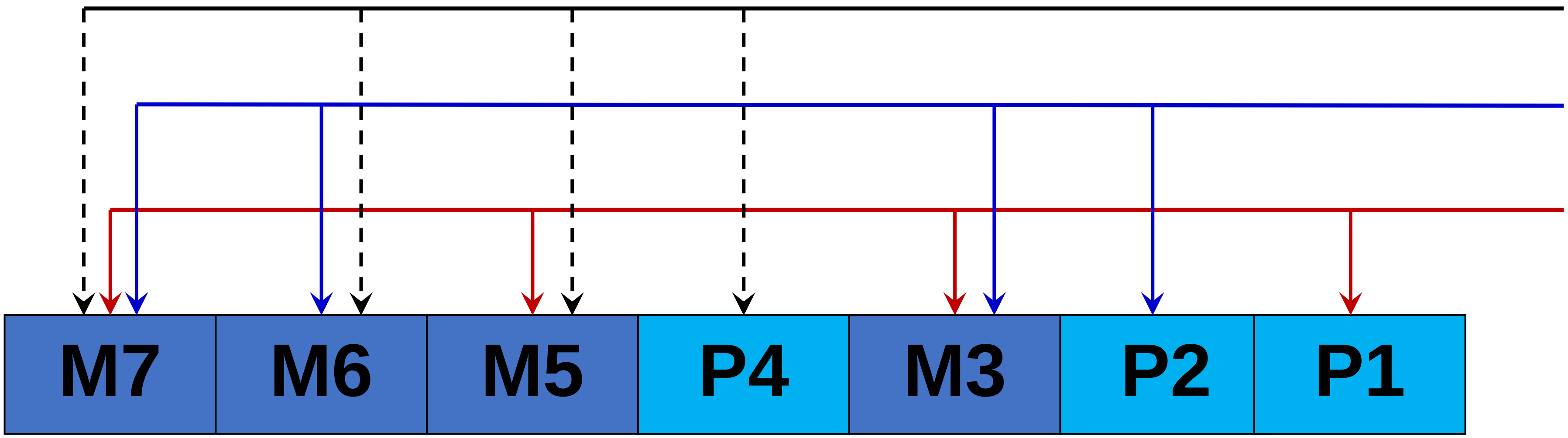
  - So, given m, this put a lower bound on the check bits that are needed to correct single bit errors
    - For m = 4, r = 3
    - For m = 7, r = 4

Hamming Code



parity place is decide by 2^i ,
where i = { 0,1,2, ... , r }

Hamming Code



Error Correction Codes

- Convolution codes: GSM mobile phone system, satellite communications, 802.11
- Reed-Solomon code: DSL, data over cable, satellite communications, CDs
- Low-density parity check: Digital video broadcasting, Ethernet, 802.11

Summary

- Different error correction techniques discussed:
  - Backward error correction
  - Forward error correction
  - Hamming distance
  - Hamming code