PRML by Bishop – chapter 1, Section 1.2.4 – The Gaussian Distribution – Equation 1.55 / 1.56, Pg 27

1) μ_{ML} → Obtained by maximizing eqn (1.54) w.r.t μ.

$$\Rightarrow \frac{\partial}{\partial \mu}\left[\ln\left[p(\underline{x}\,|\,\mu,\sigma^2)\right]\right] = 0 \quad - (a)$$

$$\Rightarrow \frac{\partial}{\partial \mu}\left[\frac{-1}{2\sigma^2}\sum_{n=1}^{N}(x_n-\mu)^2\right] = 0 \quad - (b)$$

$$\Rightarrow \frac{1}{\sigma^2}\sum_{n=1}^{N}(x_n-\mu_{ML}) = 0 \quad - (c)$$

$$\Rightarrow \mu_{ML} = \frac{1}{N}\sum_{n=1}^{N} x_n \rightarrow (1.55)$$

2) σ^2_{ML} → Obtained by maximizing eqn (1.54) w.r.t σ^2

$$\Rightarrow \frac{\partial}{\partial \sigma^2}\left[\ln\left[p(\underline{x}\,|\,\mu,\sigma^2)\right]\right] = 0 \quad - (a)$$

$$\Rightarrow \frac{\partial}{\partial \sigma^2}\left[\frac{-1}{2\sigma^2}\sum_{n=1}^{N}(x_n-\mu)^2 - \frac{N}{2}\ln\sigma^2\right] = 0 \quad \hookrightarrow (b)$$

$$\Rightarrow \frac{1}{2\sigma_{ML}^4}\sum_{n=1}^{N}(x_n-\mu_{ML})^2 = \frac{N}{2\sigma_{ML}^2} \quad - (c)$$

$$\Rightarrow \sigma^2_{ML} = \frac{1}{N}\sum_{n=1}^{N}(x_n-\mu_{ML})^2 - (1.56)$$