ILA_CH1_01

Problem Set 1.1

1a) ↳ $\vec{u} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$

↳ $\vec{v} = 3\vec{u}$

↳ All linear combinations of $\vec{u}$ & $\vec{v}$ is $c\vec{u} + d\vec{v}$ = ~~all~~ line passing through origin $(0,0,0)$ & $(1,2,3)$

1b) ↳ $\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$

↳ $c\vec{u} + d\vec{v} = \begin{bmatrix} c \\ 2d \\ 3d \end{bmatrix}$ = Plane in $\mathbb{R}^3$.

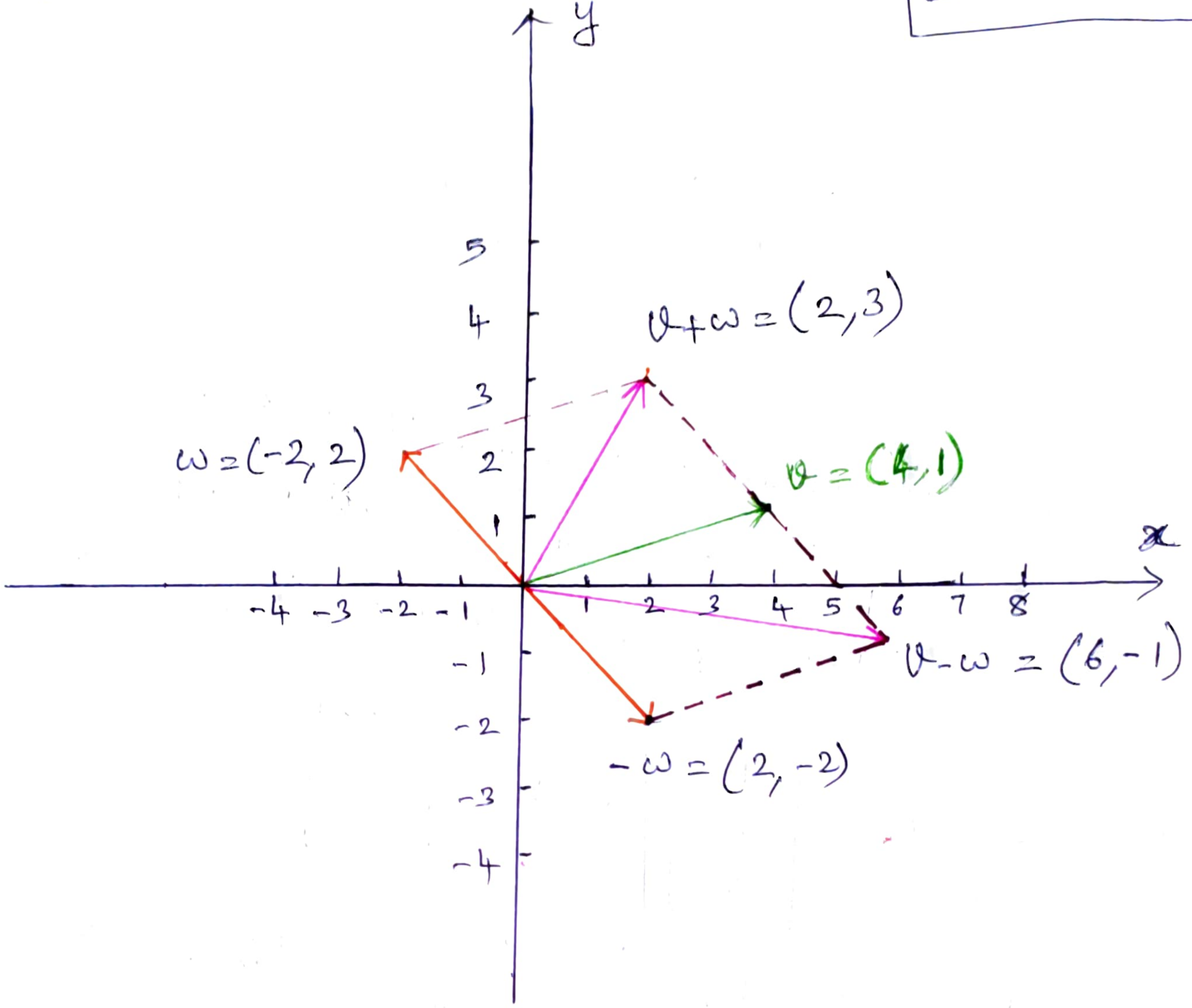
1c) ↳ $\vec{u} = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$

↳ All three vectors are independent

↳ Linear Combination =

$$\begin{bmatrix} 2c + 2e \\ 2d + 2e \\ 2d + 3e \end{bmatrix} \Rightarrow \text{fills all of } \mathbb{R}^3.$$

2) ~~Draw~~



3) $\vec{v} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

4) $3\vec{v} + \vec{w} = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$, $c\vec{v} + d\vec{w} = \begin{bmatrix} 2c + d \\ c + 2d \end{bmatrix}$

5) ↳ $\vec{u} + \vec{v} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix} = -\vec{w}$ ⇒ $\vec{u} + \vec{v} + \vec{w} = 0$

⇒ $2\vec{u} + 2\vec{v} + \vec{w} = (\vec{u} + \vec{v} + \vec{w}) + (\vec{u} + \vec{v}) = -\vec{w}$

↳ Since $\vec{w} = -(\vec{u} + \vec{v})$, $\vec{w}$ lies in the plane formed by linear combinations of $\vec{u}$ & $\vec{v}$.

6) $$c\vec{v} + d\vec{w} = \begin{bmatrix} c \\ -2c+d \\ c-d \end{bmatrix}$$

$\Rightarrow$ Components add upto zero.

$$c\vec{v} + d\vec{w} = \begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix} \Rightarrow c = 3, d = 9$$

$$c\vec{v} + d\vec{w} = \begin{bmatrix} 3 \\ 3 \\ 6 \end{bmatrix}$$ is not possible because the components do not add upto zero.

7) a) $c = 0, d = 0 \Rightarrow \vec{x} = c\begin{bmatrix} 2 \\ 1 \end{bmatrix} + d\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

b) $c = 0, d = 1 \Rightarrow \vec{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

c) $c = 0, d = 2 \Rightarrow \vec{x} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

d) $c = 1, d = 0 \Rightarrow \vec{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

e) $c = 1, d = 1 \Rightarrow \vec{x} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$

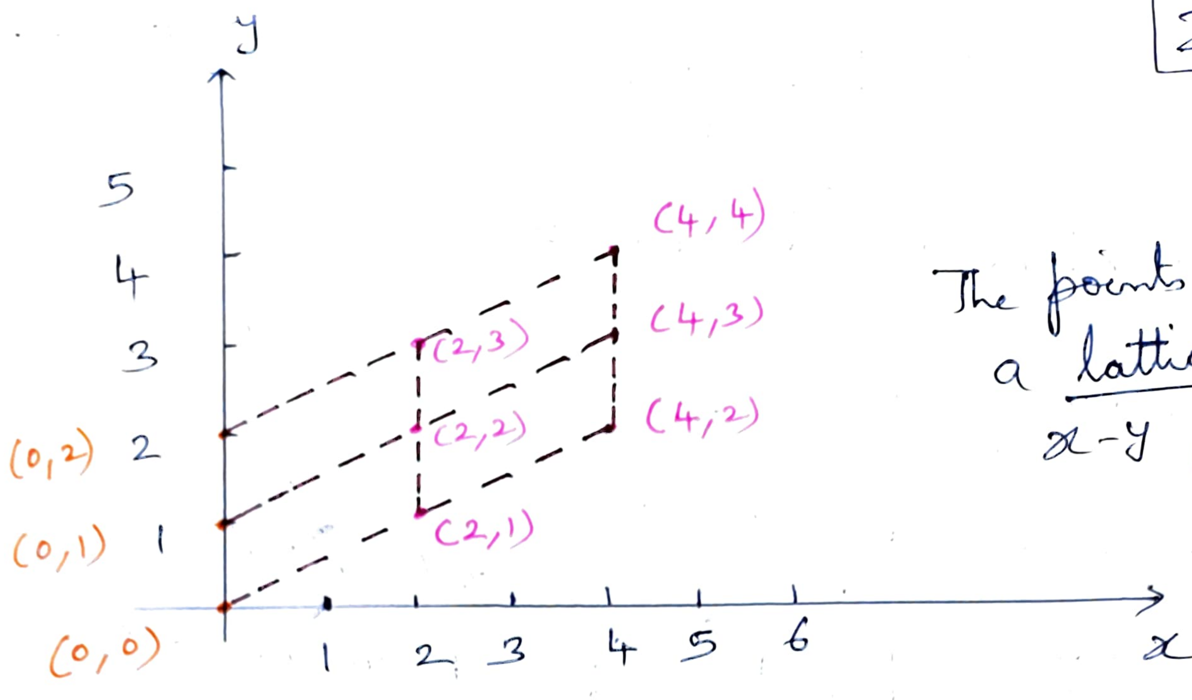
f) $c = 1, d = 2 \Rightarrow \vec{x} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

g) $c = 2, d = 0 \Rightarrow \vec{x} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$

h) $c = 2, d = 1 \Rightarrow \vec{x} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

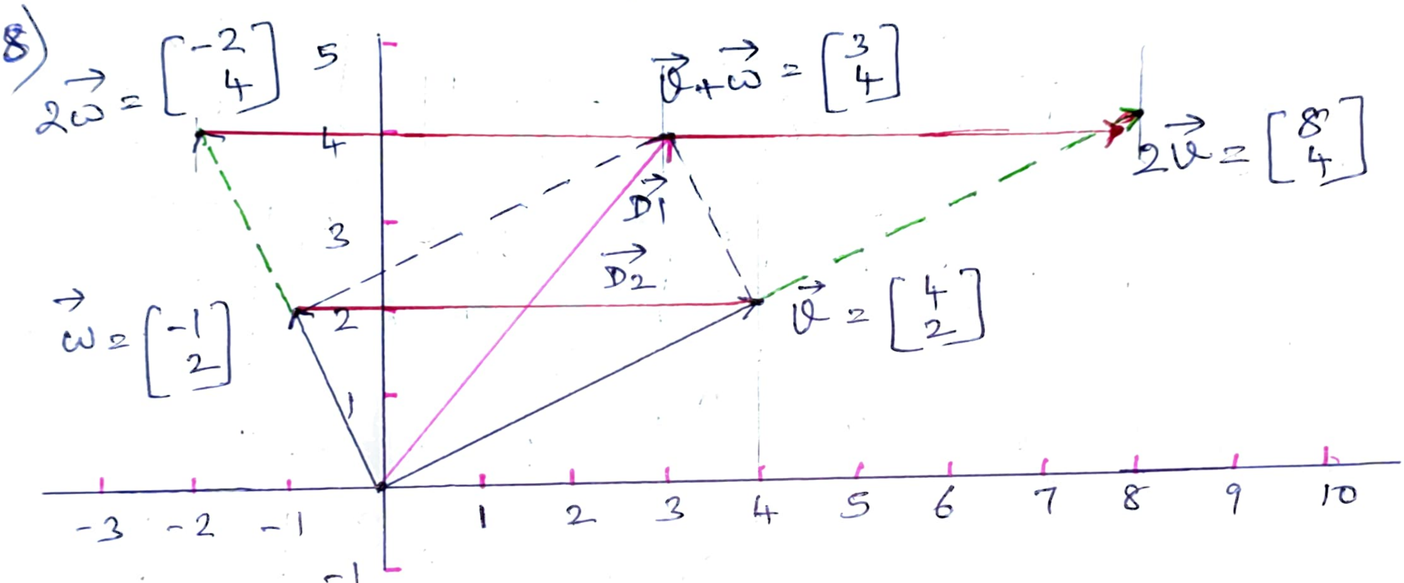
i) $c = 2, d = 2 \Rightarrow \vec{x} = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

(next page)



The points form a lattice in the x-y plane

8)



$$\vec{D_1} = \vec{v} + \vec{\omega} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

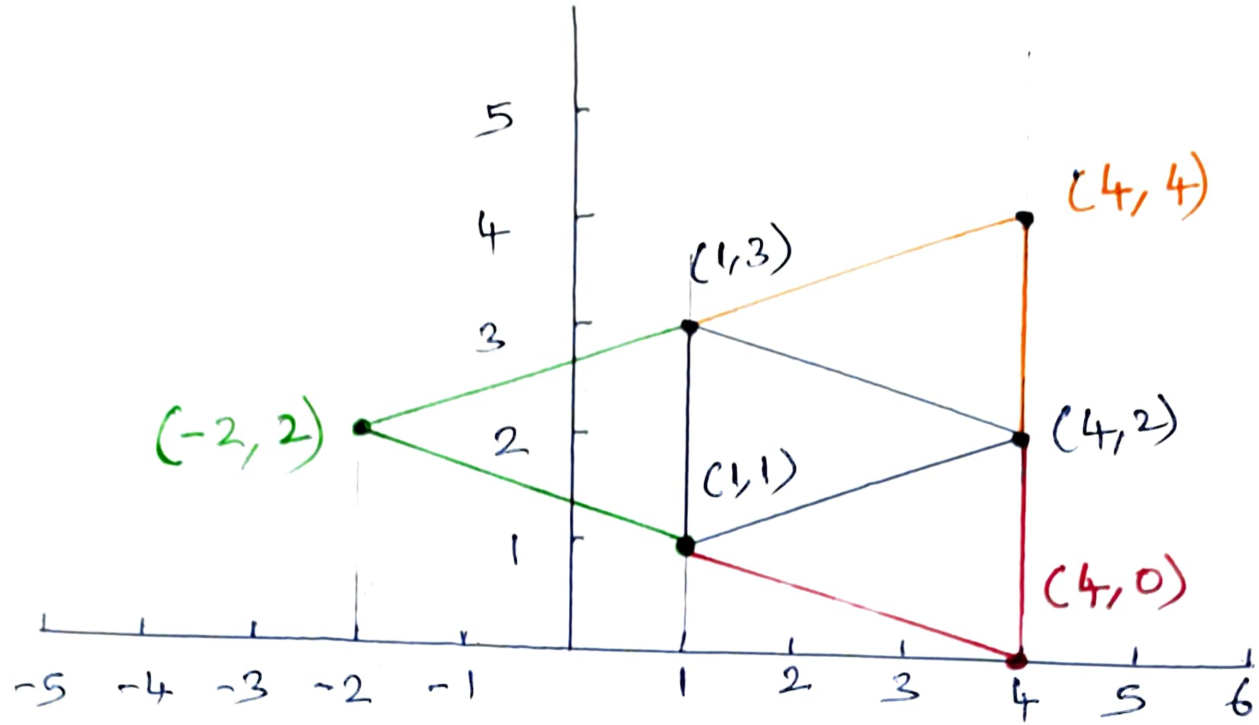
$$\vec{D_2} = \begin{cases} \vec{v} - \vec{\omega} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \\ \text{or} \\ \vec{\omega} - \vec{v} = \begin{bmatrix} -5 \\ 0 \end{bmatrix} \end{cases}$$

depending on the direction we consider

$$\vec{D_1} + \vec{D_2} = \begin{cases} 2\vec{v} = \begin{bmatrix} 8 \\ 4 \end{bmatrix} \\ \text{or} \\ 2\vec{\omega} = \begin{bmatrix} -1 \\ 4 \end{bmatrix} \end{cases}$$

depending on the direction we consider for $\vec{D_2}$.

9)



(−2, 2), (4, 0) & (4, 4) are the three possible values for the fourth corner.

⟵ END OF PROBLEMS (1 − 9) ⟶