Problems

1) A discrete time signal $x[n]$ is defined as $x[n] = \begin{cases} 1+\frac{n}{3}, & -3 \ldots \\ 1, & 0 \le n \ldots \\ 0, & \text{elsewh}\ldots \end{cases}$

a) Determine its value and sketch the signal $x(n)$.

Sol

$1+\frac{n}{3}, \quad -3 \le n \le -1 \quad \Rightarrow$ at $n=-3 \quad 0$

$n=-2 \quad \frac{1}{3}$

$n=-1 \quad \frac{2}{3}$

From $0 \le n \le 3 \quad 1$

elsewhere 0

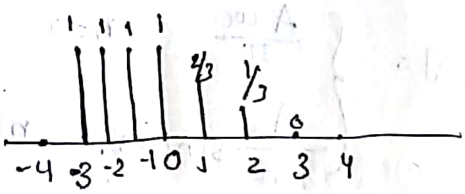
So $x[n] = \{\ldots\ldots 0, \frac{1}{3}, \frac{2}{3}, \underset{\uparrow_0}{1}, 1, 1, 1, 0, \ldots\}$.

b) Sketch its value and the signals that result if we:-

1) First fold $x[n]$ and then delay the resulting by four samples.

$x[n]$

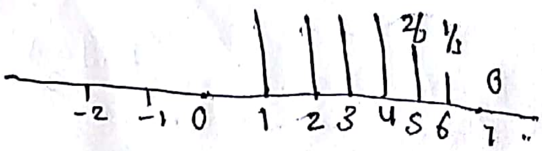
Folding $x[n] = x[-n]$



$x[-n+4) \rightarrow$ after delaying $(-3 \le -n+4 \le 2$

$-7 \le -n \le -2$

$+7 \ge n > -2$

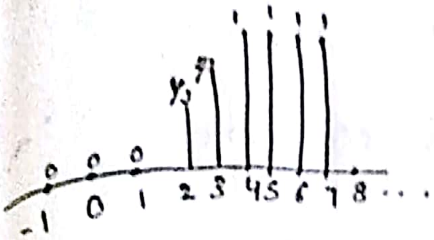


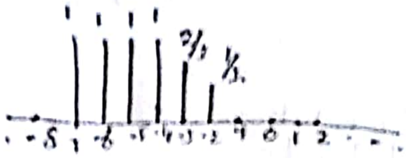
2) First delay $x[+n]$ by four samples then fold the resulting signal.

$x[n] \rightarrow$ delay by 4 samples

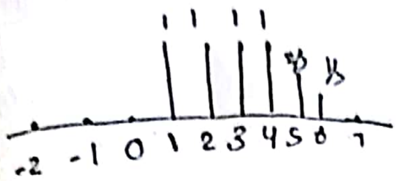
$x[n-4]$

Folding $x(n-4) \Rightarrow x(-n-4)$





d) Sketch the signal $x(-n+4)$



e) Compare the result.

→ By comparing results in part (b) and (c) we say that to get $x(-n+k)$ from $x[n]$ first we need to fold $x(n)$ which results in $x(-n)$ and then we need shift by k samples to right if $k>0$ (or) to left if $k<0$ results in $x(-n+k)$

c)

Yes $\quad x(n) = \frac{1}{3}\delta(n-2) + \frac{2}{3}\delta(n-1) + u(n) - u(n-4)$

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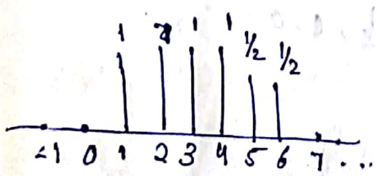
(2) A discrete

a) $x[n-2]$.

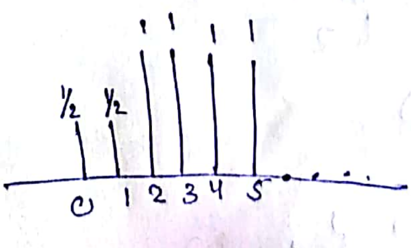


b) $x(4-n)$.

$-1 \le 4-n \le 4$

$-5 \le -n \le 0$

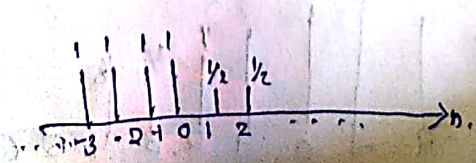
$5 \ge n \ge 0$



c) $x(n+2)$

$-1 \le n+2 \le 4$

$-3 \le n \le 2$

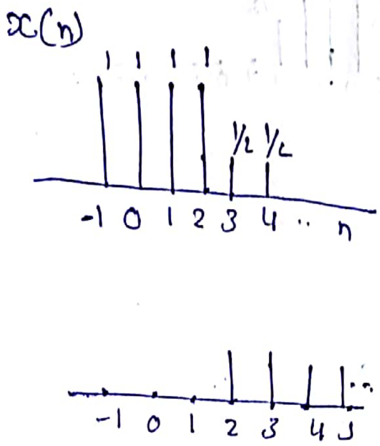


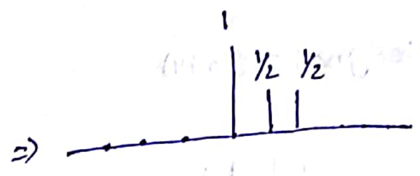
d) $x(n)\, u(2-n)$

$u(2-n) \Rightarrow 1 \quad 2-n \geq 0$

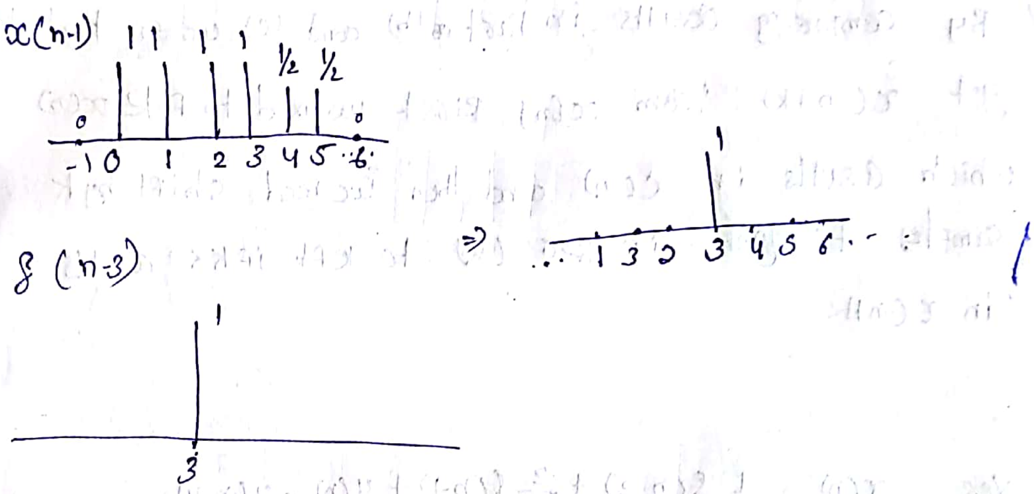
$-n \geq 0$

$n \geq 2 \quad x(n)\, u(2-n)$





e) $x(n-1)\,\delta(n-3)$.



f) $x(n^2)$.

$\Rightarrow x(n) = \{x(-2), x(-1), x(0), x(1), x(2), x(3), x(4) \cdots\}$

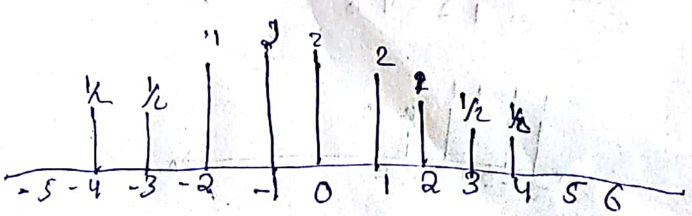
$x(n^2) = \{\ldots x(4), x(1), x(0), x(1), x(4), x(9), x(16) \ldots\}$

$= \{\ldots \frac{1}{2}, 1, 1, 1, \frac{1}{2}, 0, 0 \ldots\}$

g) even Part

$$x_e(n) = \frac{x(n)+x(-n)}{2}$$

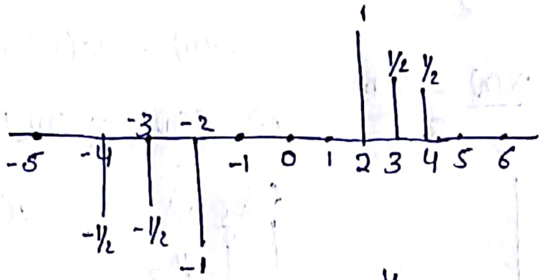
$x(n)+x(-n)$



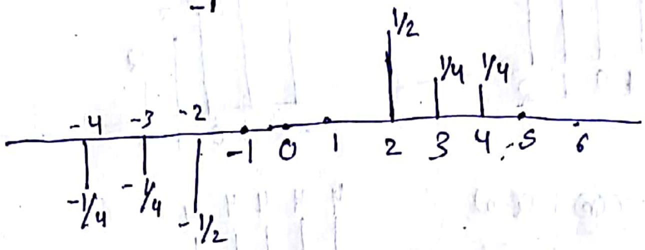
$\frac{x(n)+x(-n)}{2} \Rightarrow$

b) odd part :-

$x(n) - x(-n) =$



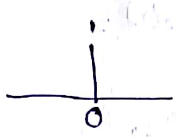
$\frac{x(n) - x(-n)}{2}$



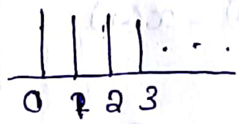
3)

2 a) show that $\delta(n) = u(n) - u(n-1)$

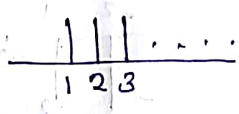
Sol we know $\delta(n) =$



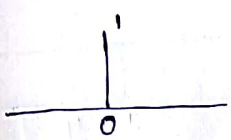
$u(n) =$



$u(n-1) =$



Now $u(n) - u(n-1)$

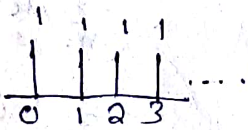


$\therefore \delta(n) = u(n) - u(n-1)$ Hence proved.

b) $u(n) = \sum_{K=-\infty}^{n} \delta(K) = \sum_{K=0}^{\infty} \delta(n-K)$

Sol

$u(n) =$



$\Rightarrow \sum_{K=-\infty}^{n} \delta(K) = u(n) = \begin{cases} 0, & n<0 \\ 1, & n \geq 0 \end{cases}$

$\sum_{K=0}^{\infty} \delta(n-K) = \begin{cases} 0, & n<0 \\ 1, & n \geq 0 \end{cases}$.

4) Show that any signal can be

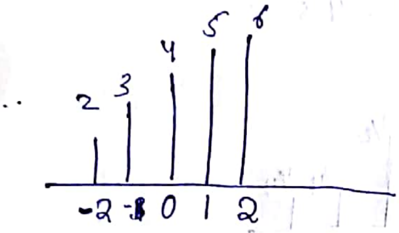
Sol

given $x(n) = \{2,3,\underset{\uparrow}{4},5,6\}$.

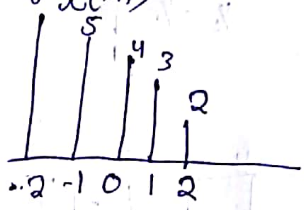
$x_e(n) = \dfrac{x(n) + x(-n)}{2}$ $x_e(n) = x_e(-n)$

$x_o(n) = -x_o(n)$

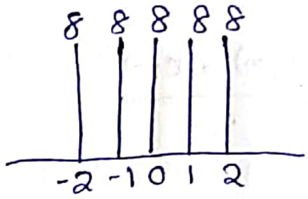
$x_o(n) = \dfrac{x(n) - x(-n)}{2}$ $\Rightarrow x(n) = x_e(n) + x_o(n)$



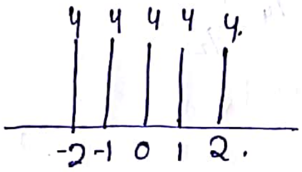
$x(-n)$



$x(n) + x(-n)$

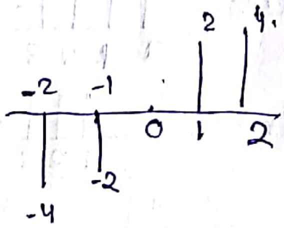


$x_e(n) = \dfrac{x(n) + x(-n)}{2} \Rightarrow$

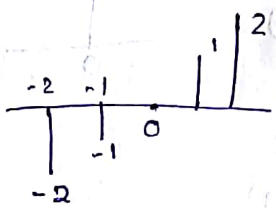


odd

$x(n) - x(-n)$



$x_o(n) =$



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5) show that energy.

Sol) First prove that

$\sum_{n=-\infty}^{\infty} x_e(n) + x_o(n) = 0 \Rightarrow \sum_{n=-\infty}^{\infty} x_e(n) + x_o(n) = \sum_{n=-\infty}^{\infty} x_e(-m)\, x_o(-m)$

$= -\sum_{n=-\infty}^{\infty} x_e(m)\, x_o(m)$

$= -\sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n)$

$= \sum_{n=-\infty}^{\infty} x_e(n)\, x_o(n)$

Properties (Power)

i) $\sum_{n=-\infty}^{\infty} x^2(n) = \sum_{n=-\infty}^{\infty} [x_e(n) + x_o(n)]^2$

$= \sum_{n=-\infty}^{\infty} x_e^2(n) + x_o^2(n) + 2x_e(n)x_o(n)$

$= E_e + E_o + 0$

$E = E_e + E_o$

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i) Consider

a) Det

given $y(n) = \tau[x(n)] = x[n^2]$

$x(n-k) \rightarrow y_1(n) = x[(n-k)^2]$

$= x[n^2 + k^2 - 2nk]$

$x(n-k) \neq y(n-k)$

so the given system is time variant.

b) To check

(1) $x(n) = \{ \ldots\ 0, 1, 1, 1, 1, 0, 0 \ldots \}$

2) $y[n] = \tau[x(n)] = x[n^2] \Rightarrow \{x(0), x(1), x(2^2), x(3^2), x(4^2) \ldots$

$\Rightarrow \{x(0), x(1), x(4), x(9) \ldots\ldots$

$y(n) = x(n^2) = \{ 1, 1, 0, 0, 0 \ldots$

$\uparrow$ 0 1 2 3 4.

3)

$y(n-2) = \{ \ldots\ 0, 0, 1, 1, 0, 0, 0 \ldots \}$

$\uparrow$

4)

$x(n-2) = \{ \ldots\ldots 0, 0, 1, 1, 1, 1, 0 \ldots \}$
$\uparrow$ 0 1 2 3 4 5 6.

5)

$y_2(n) = T[x_2(n-2)] = \{x(0), x(1), x(2), x(3), x(4), x(5), x(6)\}$

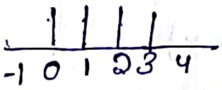
$= \{ \ldots 0, 1, 0, 0, \underset{\uparrow}{0}, 0, 1, 0 \ldots \}$

6)

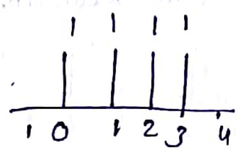
$y_2(n) \neq y(n-2) \Rightarrow$ system is time varient.

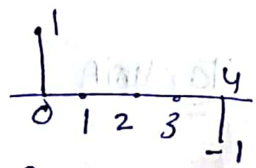
7) Repeat

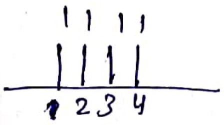
Sol

1) $x[n] =$  $= \{1, 1, 1, 1\}$.

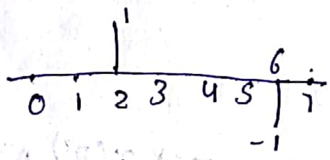
2) $y(n) = x(n) - x(n-1)$

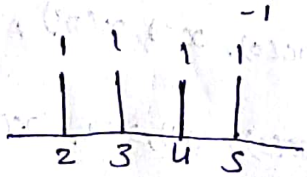
$x(n)$ 

$y(n) \Rightarrow x(n - x(n-1) =$ 

$x(n-1)$ 

$= \{0, \underset{\uparrow}{1}, 0, 0, 0, 1\}$

3) $y(n-2) \Rightarrow$  $\Rightarrow \{0, 0, \underset{\uparrow}{1}, 0, 0, 0, -1\}$

4) $x(n-2) \Rightarrow$  $\Rightarrow \{0, 0, 1, 1, 1, 1\}$

5) $y_2(n) = \{0, 0, 1, 0, 0, 0, -1\}$.

6) $y_2(n) = y(n-2) \Rightarrow$ system is invarient

8) Repeat

1) $y(n) = nx(n)$

$x(n) = \{\ldots\ 0,1,1,1,1,0\ \ldots\}$
$\quad\quad\quad\quad\uparrow$

2) $y(n) = \{\ldots\ 0,1,2,3,4\ \ldots\}$
$\quad\quad\quad\quad\uparrow$

3) $y(n-2) = \{\ldots\ 0,0,0,1,2,3,4\ldots\}$
$\quad\quad\quad\quad\uparrow$

4) $x(n-2) = \{\ldots\ 0,0,0,1,1,1,1\ldots\}$
$\quad\quad\quad\quad\uparrow$

5) $y_2(n) = T[x(n-2)] = \{\ldots\ 0,0,2,3,4,5\ldots\}$

6) $y_2(n) \neq y(n-2)$ => System is time varient.

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1)

a) $y(n) = \cos[x(n)]$

(i) Static (only Present i/P)

(iv) only Present i/P => Casual

v) stable

(iii) $y(n) = \cos[x(n-n_0)]$

$y'(n) = \cos(x(n-n_0)]$

=> Time variant

(ii) $y_1(n) = \cos[x_1(n)]$

$y_2(n) = \cos(x_2(n)]$

$y(n) = \cos(x_1(n)] + \cos(x_2(n)]$

$y'(n) = \cos(x_1(n) + x_2(n)]$

=> Non linear

b) $y(n) = \sum_{k=-\infty}^{n+1} x(k)$

Dynamic (depends on Future values)

linear, Time invarient, non Casual (also depends on future value)

unstable.

c) $y(n) = x(n)\cos(w_0(n))$

Static, linear, Time Varient, Causal, Stable

$y[n] = x(n-n_0)\cos(w_0(n-n_0))$

$y'(n) = x(n-n_0)\ \cos w_0 n.$

d) $y(n) = x(-n+2)$

Dynamic

at $n=0 \Rightarrow y(0) = x(2)$

$y_1(n) = x_1(-n+2) + x_2(-n+2)$

$y_2(n) = (x + x_2(-n+2)) \Rightarrow x_1(-n+2) + x_2(-n+2)$

linear

Time invarient.

e) $y(n) = Trun[x(n)]$

Static, non linear, time invariant, Causal, stable.

f) $y(n) = Round[x(n)]$

Static, non linear, time invarient, Causal, Stable.

g) $y(n) = |x(n)|$

Static, non linear, timeinvarient, Causal, stable.

h) $y(n) = x(n)\, u(n)$

Static, linear, time variant, ~~non~~ causal, stable.

i) $y(n) = x(n) + n x(n+1)$

Dynamic, linear, time varient, non-causal, unstable

j) $y(n) = x(2n)$

Dynamic, linear, Time variant, non causal, stable.

k) $y(n) = \begin{cases} x(n); & \text{if } x(n) \geq 0 \\ 0; & \text{if } x(n) < 0 \end{cases}$

Static, linear, time varient, non-causal, stable

l) Dynamic, linear, Time varient, causal, stable.

m) static, non-linear, Time invarient, Causal, Stable

n) Static, linear, time varient, non Causal, stable.

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9)

Sol:

$$x(n) = x(n+N) \;\; \forall\, n \geq 0$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$y(n+N) = \sum_{k=-\infty}^{n+N} h(k)\, x(n+N-k)$$

$$y(n+N) = \sum_{k=n+1}^{n+N} h(k)x(n-k) + \sum_{k=-\infty}^{n} h(k)\, x(n-k)$$

$$y(n+N) = y(n) + \sum_{k=n+1}^{n+N} h(k)\, x(n-k)$$

For BIBO system, $\lim_{n\to\infty} |h(n)| = 0$

$$\lim_{n\to\infty} \sum_{k=n+1}^{n+N} h(k)\, x(n-k) = 0$$

$$\lim_{n\to\infty} y(n+N) = y(N)$$

$$\therefore y(N) = y(n+N).$$

9) $x(n) = x_0(n) + a u(n)$ $\quad x_0(n) \to$ bounded with $\lim_{n\to\infty} x_0(n) = 0$

$$\Rightarrow y(n) = a \sum_{K=0}^{\infty} h(K) u(n-K) + \sum_{K=0}^{\infty} h(K) x_0(n-K) = a\sum_{K=0}^{n} h(K) + y_0(n)$$

$$\Rightarrow \sum_n x_0^2(n) < \infty \Rightarrow \sum_n y_0^2(n) < \infty$$

hence $\lim_{n\to\infty} |y_0(n)| = 0 = a\sum_{K=0}^{n} h(K) = $ constant

Q)

$$y(n) = \sum_K h(K)\, x(n-K)$$

$$\sum_{-\infty}^{\infty} y^2(n) = \sum_{-\infty}^{\infty}\left[\sum_K h(K)\, x(n-K)\right]^2 = \sum_K \sum_l h(K) h(l) \sum_n x(n-K) x(n-l)$$

$$\sum_n x(n-K) x(n-l) \le \sum_n x^2(n) |h(l)|$$

BIBO stable system $\sum_K |h(K)| < \infty$.

Hence $E_y \le E_x$, so that $E_y < 0$ if $E_x < 0$.

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10 The following i/p-o/p pairs have.....

As this is a time-invarient system.

$y_2(n)$ should have only 3elements and

$y_3(n)$ should have 4 elements

So its linear.

11

Since $x_1(n) + x_2(n) = \delta(n)$

& system is linear, the impulse response of the system is

$$y_1(n) + y_2(n) = \{0, \underset{\uparrow}{3}, -1, 2, 1\}$$

if system we time invarient the response of $x_3(n)$ would

$$\{\underset{\uparrow}{3}, 2, 1, 3, 1\}$$

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12) The only aviolable information.

1(a)

Any linear combination of signal in the form of

$x_i(n)$; $i = 1, 2, \ldots N$.

because if we take $i = 1, 3$

$y_1(n) = x_1(n)$
$y_3(n) = x_3(n)$ $\Rightarrow y(n) = y_1(n) + y_3(n) = x_1(n) + x_3(n)$

$y(n) = x(n) + x_3(n)$

linear

1(b) Same

Sol

Any $a_i(n-k)$ where k is any integer, $i = 1, 2, \ldots N$

1st replace $n = n - n_0 \Rightarrow x_i(n - n_0 - k)$

$x(n)$ by $x(n - n_0) \Rightarrow x_i(n - k - n_0)$ [Time invariant.

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13 Show that the necessary

Sol

A system to be BIBO stable only when bounded o/p produce bounded i/p

$$y(n) = \sum_K h(k)\, x(n-k)$$

$$|y(n)| = \sum_K |h(k)|\,|x(n-k)|$$

$$= \sum_K |x(n-k)| \leq m_n \text{ [some constant]}$$

so $|y(n)| = m_n \sum_K |h(k)|$.

$|y(n)| < \infty$ for all n, if and only if $\sum_K |h(k)| < \infty$

so $\sum_{n=-\infty}^{\infty} |y(n)|$

$\Rightarrow$ A system to be BIBO stable only when bounded i/p produces bounded o/p.

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$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) \; ; \; n \leq n-k$$
$$k \geq 0$$
$$|y(n)| = \sum_{k=-\infty}^{\infty} |h(k)|\, |x(n-k)| \qquad k \geq 0$$

as $\sum_{k=-\infty}^{\infty} |x(n-k)| \leq m_n$ for some constant

$$|y(n)| = m_n \sum_{k=-\infty}^{\infty} |h(k)| \; ; \; n \leq n-k \; ; \; k \geq 0$$

$|y(n)|$ is $< \infty$ if & only if $\sum_{k=\infty}^{\infty} |h(k)| < \infty$

so $\sum_{n=-\infty}^{\infty} |h(n)| < \infty$.

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14) Show that

(a)

Sol if a system is causal output depends only on the present and past i/ps as $x(n) = 0$ for $x(n) = 0$ for $n < n_0$ then $y(n)$ also zero for $n < n_0$

(b)

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

For ~~is~~ finite impulse response

$h(n) = 0 \; , \; n < 0$ and $n \geq m$

so $y(n)$ reduces to $y(n) = \sum_{k=0}^{n-1} h(k)\, x(n-k)$

15 Show that

a)

Sol

For $a = 1$, $\sum_{n=m}^{N} a^n = N - m + 1$

For $a \neq 1$, $\sum_{n=m}^{N} a^n = \dfrac{a^m - a^{N+1}}{1-a}$

$$(1-a)^N \sum_{n=m}^{N} a^n = a^m + a^{m+1} - a^{m+1} + \ldots a^n - a^N - a^{N+1}$$
$$= a^m - a^{N+1}$$

b)

Sol.

For $n = 0$, $|a| < 1$ and $N \to \infty$

$$= \sum_{n=-\infty}^{\infty} a^n = \frac{1}{1-a} , |a|<1.$$

16) (a) if $y(n) = x(n) * h(n)$ . . .

Sol

$$y(n) = \sum_k h(k)\, x(n-k)$$

$$\sum_n y(n) = \sum_n \sum_k h(k)\, x(n-k)$$

$$\sum_n y(n) = \sum_k h(k) . \sum_{n=-\infty}^{\infty} x(n-k)$$

$$\sum_n y(n) = \sum \left(\sum_k h(k) \right) \left(\sum_n x(n) \right)$$

(b) Compute

(1) $x(n) = \{1, 2, 4\}$, $h(n) = \{1, 1, 1, 1, 1\}$.

Sol

$y(n) = \{1, 3, 7, 7, 7, 6, 4\}$

$\sum_n y(n) = 35$; $\sum_n x(n) = 7$, $\sum_n h(n) = 5$

$\therefore \sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$35 = 7 \times 5$

$35 = 35$

By Tabular

h(n) \ x(n)	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4
1	1	2	4

2) $x(n) = \{1, 2, -1\}$, $h(n) = x(n)$

Sol) $x(n) = \{1, 2, -1\}$, $h(n) = \{1, 2, -1\}$.

$y(n) = x(n) * h(n)$

$y(n) = \{1, 4, 2, -4, 1\}$

$\sum_n y(n) = 4$; $\sum_n x(n) = 2$, $\sum_n h(n) = 2$

$\sum_n y(n) = \sum_n (x) \sum h(n)$

$4 = 2 \times 2 = 4$

x(n) \ h(n)	1	2	-1
1	1	2	-1
2	2	4	-2
-1	-1	-2	1

$x(n) = \{0, 1, -2, 3, -4\}$, $h(n) = \{\frac{1}{2}, \frac{1}{2}, 1, \frac{1}{2}\}$.

$y(n) = \{0, \frac{1}{2}, \frac{-3}{2}, \frac{3}{2}, -2, 0, -\frac{5}{2}, 2\}$.

$\sum_n y(n) = -5$, $\sum_n x(n) = -2$, $\sum_n h(n) =$ [illegible]

$\sum_n y(n) = \sum_n x(n)\, h(n)$

$-5 = -5$

h(n) \ x(n)	0	+1	-2	3	-4
$\frac{1}{2}$	0	$+\frac{1}{2}$	-1	$\frac{3}{2}$	-2
$\frac{1}{2}$	0	$\frac{1}{2}$	-1	$\frac{3}{2}$	-2
1	0	1	-2	3	-4
$\frac{1}{2}$	0	$\frac{1}{2}$	-1	$\frac{3}{2}$	-2

5) $x(n) = \{1, 2, 3\}$, $h(n) = \{0, 0, 1, 1, 1, 1\}$.

Sol $y(n) = \{0, 0, 1, -1, 2, 2, 1, 3\}$

$\sum_n y(n) = 8$; $\sum_n x(n) = 2$; $\sum_n h(n) = 4$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$8 = 8$

h(n) \ x(n)	1	-2	3
0	0	0	0
0	0	0	0
1	1	-2	3
1	1	-2	3
1	1	-2	3
1	1	-2	3

6) $x(n) = \{0, 0, 1, 1, 1, 1\}$ $h(n) = \{1, -2, 3\}$

Sol $y(n) = \{\underset{\uparrow}{0}, 0, 1, -1, 2, 2, 1, 3\}$

$\sum_n y(n) = 8$; $\sum_n x(n) = 4$ $\sum_n h(n) = 2$

$\sum_n y(n) = \sum_n x(n) \sum h(o)$

$8 = 8$

h(n) \ x(n)	0	0	1	1	1	1
1	0	0	1	1	1	1
-2	0	0	-2	-2	-2	-2
3	0	0	3	3	3	3

7) $x(n) = \{0, 1, 4, -3\}$, $h(n) = \{\underset{\uparrow}{1}, 0, -1, -1\}$.

Sol $y(n) = \{0, 1, 4, -4, -5, -1, 3\}$.

$\sum_n y(n) = -2$; $\sum_n x(n) = -2$; $\sum_n h(n) = 1$

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$-2 = -2$

h(n) \ x(n)	0	1	4	-3
1	0	1	4	-3
0	0	0	0	0
-1	0	-1	-4	3
-1	0	-1	-4	3

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8) $x(n) = \{1, 1, 2\}$, $h(n) = u(n)$

$h(n) \backslash x(n)$	1	1	2
1	1	1	2
1	1	1	2
2	2	2	4

$y(n) = \{1, 2, 4, 3, 2\}$.

$\sum_n y(n) = \sum_n x(n) \sum h(n)$

$12 = 4 \times 3$

$12 = 12.$

9) $x(n) = \{1, 1, 0, 1, 1\}$, $h(n) = \{1, -2, -3, 4\}$.

$h(n) \backslash x(n)$	1	1	0	1	1
1	1	1	0	1	1
-2	-2	-2	0	-2	-2
-3	-3	-3	0	-3	-3
-4	-4	-4	0	-4	-4

Sol $y(n) = \{1, -1, -5, 2, 3, -5, 1, 4\}$.

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$0 = 0$

10) $x(n) = \{1, 2, 0, 2, 1\}$, $h(n) = x(n)$.

$h(n) \backslash x(n)$	1	2	0	2	1
1	1	2	0	2	1
2	2	4	0	4	2
0	0	0	0	0	0
2	2	4	0	4	2
1	1	2	0	2	1

Sol $y(n) = \{1, 2, 4, 4, 10, 4, 4, 4, 1\}$.

$\sum_n y(n) = \sum_n x(n) \sum_n h(n)$

$36 = 36$

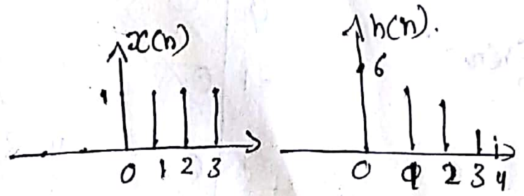
11) $x(n) = \left(\frac{1}{2}\right)^n u(n)$, $h(n) = \left(\frac{1}{4}\right)^n u(n)$

Sol $y(n) = \left[2\left(\frac{1}{2}\right)^n - \left(\frac{1}{4}\right)^n u(n)\right]$

$\sum_n y(n) = \frac{8}{3}$, $\sum_n h(n) = \frac{4}{3}$, $\sum_n x(n) = 2$

12) Compute and plot convolution.

a)

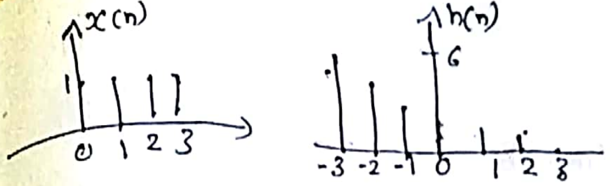


$h(n) \backslash x(n)$	1	1	1
6	6	6	6
5	5	5	5
4	4	4	4
3	3	3	3
2	2	2	2
1	1	1	1

Sol $x(n) = \{1, 1, 1, 1\}$, $h(n) = \{6, 5, 4, 3, 2, 1, 0\}$

$y(n) = x(n) * h(n)$

$y(n) = \{6, 11, 15, 18, 14, 10, 6, 3, 1\}$.



h(n) \ x(n)	1	1	1
6	6	6	6
5	5	5	5
4	4	4	4
3	3	3	3
2	2	2	2
1	1	1	1

$x(n) = \{1,1,1,1\}$ $h(n) = \{6,5,4,3,2,1,0\}$.

$y(n) = \{6, 11, 15, \underset{\uparrow}{18}, 14, 10, 6, 3, 1\}$.

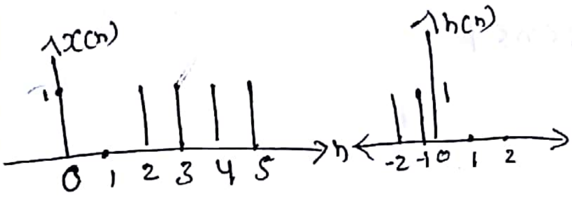
)

Sol) $x(n) = \{\underset{\uparrow}{0}, 0, 0, 1, 1, 1, 1\}$

$h(n) = \{1, 1, 0, 0, \underset{\uparrow}{0}\}$

$y(n) = \{0, 0, 0, 1, 2, 2, 2, 1, 0, 0, 0\}$.

)



h(n) \ x(n)	0	0	1	1	1	1
1	0	0	1	1	1	1
1	0	0	1	1	1	1
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

Sol $x(n) = \{\underset{\uparrow}{0}, 0, 1, 1, 1, 1\}$ $h(n) = \{1, 1, \underset{\uparrow}{0}, 0\}$

$y(n) = \{0, 0, 1, 2, 2, 2, 1, 0, 0, 0\}$.

18) Determine and stable.

a) graphically.

$x(n) = \{0, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3}, 2\}$

$h(n) = \{1, 1, 1, 1, 1\}$

$y(n) = \{0, \frac{1}{3}, 1, 2, \frac{10}{3}, 5, \frac{20}{3}, 6, 5, \frac{11}{3}, 2\}$.

h(n) \ x(n)	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{9}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
1	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2

b) analytically

Sol

$x[n] = \frac{1}{3} n[u(n) - u(n-7)]$

$h[n] = u(n+2) - u(n-3)$

$y[n] = \frac{1}{3} n[n\, u(n) - u(n-7)] * u(n+2) - u(n-3)$

$= \frac{1}{3} n\, u(n) * u(n+2) - \frac{1}{3} n(u(n) * u(n-3) - \frac{1}{3} n\, u(n-7))$

$* u(n+2) + \frac{1}{3} n(u(n-7) * u(n-3)).$

Sol

19)

Sol

$y(n) = \sum_{k=0}^{4} h(k)\, x(n-k)$

$x(n) = \{\alpha^{-3}, \alpha^{-2}, \alpha^{-1}, \underset{\uparrow}{1}, \alpha, \ldots \alpha^{5}\}$

$h(n) = \{\underset{\uparrow}{1}, 1, 1, 1, 1\}$

$y(n) = \sum_{k=0}^{4} x(n-k), -3 \le n \le 9$

$= 0$ otherwise

$\therefore$

$y(-3) = \alpha^{-3}$

$y(-2) = x(-3) + x(-2) = \alpha^{-3} + \alpha^{-2}$

$y(-1) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1}$

$y(0) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1$

$y(1) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha$

$y(2) = \alpha^{-3} + \alpha^{-2} + \alpha^{-1} + 1 + \alpha + \alpha^{2}$

$y(3) = \alpha^{-1} + 1 + \alpha + \alpha^{2} + \alpha^{3}$

$y(4) = \alpha^{4} + \alpha^{3} + \alpha^{2} + \alpha + 1$

$y(5) = \alpha + \alpha^{2} + \alpha^{3} + \alpha^{4} + \alpha^{5}$

$y(6) = \alpha^{2} + \alpha^{3} + \alpha^{4} + \alpha^{5}$

$y(7) = \alpha^{3} + \alpha^{4} + \alpha^{5}$

$y(8) = \alpha^{4} + \alpha^{5}$

$y(9) = \alpha^{5}$

1)

a) $131 \times 122 = 15982$

(b) $y(n) = \{15, 9, 8, 2\}$

$y(n) \backslash x(n)$	1	2	2
1	1	2	2
3	3	6	6
1	1	2	2

c) $(z^2+3z+1) * (2z^2+2z+1)$

$= 2z^4 + 8z^3 + 9z^2 + 5z + 1$

d) $1.31 \times 12.2 = 15.982$

e) These are different ways to perform convolution.

---

2)

a)

Sol $y[n] = x(n) * h(n)$

$= a^n u(n) * b^n u(n)$

$\neq [a^n * b^n] u(n)$

$y[n] = \sum_{k=0}^{n} a^k u(k) b^{n-k} u(n-k)$

$= b^n \sum_{k=0}^{n} a^k u(k) b^{-k}$

$= b^n \sum_{k=0}^{n} (ab)^{-k}$

if $a \neq b$ then $y(n) = \dfrac{b^{n+1} - a^{n+1}}{b-a} u(n)$

if $a = b \Rightarrow b^n (n+1) u(n)$

(b) $x(n) = \{1, 2, 1, 1\}$. $h(n) = \{1, -1, 0, 0, 1, 1\}$

$y(n) = \{1, 1, -1, 0, 0, 3, 3, 2, 1\}$

c) $x(n) = \{1, \underset{\uparrow}{1}, 1, 1, 1, 0, -1\}$

$h(n) = \{1, 2, \underset{\uparrow}{3}, 2, 1\}$

$y[n] = \{1, 3, 6, 8, 9, 8, 5, 1, -2, -2, 1\}$

$h(n) \backslash x(n)$	1	1	1	1	1	0	-1
1	1	1	1	1	1	0	-1
2	2	2	2	2	2	0	-2
3	3	3	3	3	3	0	-3
2	2	2	2	2	2	0	-2
1	1	1	1	1	1	0	-1

22)

(a)

22)

Express

Sol

we can express $\delta(n) = u(n) - u(n-1)$

$$h(n) = h(n) * \delta(n)$$
$$= h(n) * [u(n) - u(n-1)]$$
$$= \delta(n) - \delta(n-1)$$

then $y(n) = h(n) * x(n)$

$$= [\delta(n) - \delta(n-1)] * x(n)$$
$$= \delta(n) * x(n) - \delta(n-1) * x(n)$$

---

23)

23

Sol

$$y(n) = ny(n-1) + x(n), \ n \geq 0$$

$y_1(n) = ny_1(n-1) + x_1(n)$

$y_2(n) = ny_2(n-1) + x_2(n)$

$\Big\}\ \oplus \Rightarrow y(n) = ny_1(n-1) + x_1(n) + ny_2(n-1) + x_2(n)$

$y(n) = ny(n-1) + x(n)$

$x(n) = ax_1(n) + bx_2(n)$

$y(n) = ay_1(n) + by_2(n)$

Hence the system is linear

$\Rightarrow y(n-1) = (n-1)\, y(n-2) + x(n-1)$

delayed $\Rightarrow y(n-1) = ny(n-2) + x(n-1)$

so the system is time varient

$\Rightarrow$ if $x(n) = u(n)$, then $|x(n)| \leq 1$ For the bounded i/p and o/p is $y(0) = 6$, $y(1) = 2$, $y(2) = 5$, ...

unbounded so system is unstable.

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consider the signal $r(n) = a^n u(n), 0 < a < 1$

$\delta(n) = \delta(n) - a\, r(n-1)$

$\delta(n-k) = r(n-k) - a r(n-k-1)$

$x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k)$

$= \sum_{k=-\infty}^{\infty} x(k) \left[r(n-k) - a r(n-k-1) \right]$

$x(n) = \sum_{k=-\infty}^{\infty} x(k)\, r(n-k) - a \sum_{k=-\infty}^{\infty} x(k)\, r(n-k-1)$

$x(n) = \sum_{k=-\infty}^{\infty} x(k)\, \delta(n-k) - a \sum_{k=-\infty}^{\infty} x(k-1)\, \delta(n-k)$

$= \sum_{k=-\infty}^{\infty} \left[x(k) - a x(k-1)\, \delta(n-k) \right]$

Thus $c_k = x(k) - a x(k-1)$

(b) use the property.

Sol) $y(n) = T[x(n)]$

$= T \left[\sum_{k=-\infty}^{\infty} c_k\, \delta(n-k) \right]$

$= \sum_{k=-\infty}^{\infty} c_k\, T(\delta(n-k))$

$= \sum_{k=-\infty}^{\infty} c_k\, g(n-k)$

c)

Sol) $h(n) = T[\delta(n)]$

$h(n) = T[\delta(n) - a\delta(n-1)]$

$= g(n) - a g(n-1)$

---

20) Determine the zero input response ....

Sol) given $x(n) - 3y(n-1) - 4y(n-2) = 0$

with $x(n) = 0$

$-3y(n-1) - 4y(n-2) = 0 \quad (\div(-3))$

$y(n-1) + \frac{4}{3} y(n-2) = 0$

at $n=0$

$$y(-1) = -\frac{4}{3} y(-2)$$

at $n=1$

$$y(0) = -\frac{4}{3} y(-1) = \left(-\frac{4}{3}\right)^2 y(-2)$$

$$y(1) = \left(-\frac{4}{3}\right)^3 y(-2)$$

$$\vdots$$

$$y[k] = \left(-\frac{4}{3}\right)^{k+2} y(-2)$$

zero - input response.

2b) Determine the Particular Solution.

Sol

$$y[n] = \frac{5}{6} y(n-1) - \frac{1}{6} y(n-2) + x(n)$$

$$x(n) = y(n) - \frac{5}{6} y(n-1) + \frac{1}{6} y(n-2)$$

Characteristic equation is

$$\lambda^2 - \frac{5}{6}\lambda + \frac{1}{6} = 0; \quad \lambda = \frac{1}{2}, \frac{1}{3}$$

So $y_h(n) = C_1 \left(\frac{1}{2}\right)^n + C_2 \left(\frac{1}{3}\right)^n$

$$x(n) = 2^n u(n)$$

$$y_p(n) = K(2^n) u(n)$$

So $K(2^n)u(n) - K[5/6](2^{n-1}) u(n-1) + K(1/6)(2^{n-2}) u(n-2) = 2^n u(n)$

For $n=2$

$$4K - \frac{5K}{3} + \frac{K}{6} = 4$$

$$K = \frac{8}{5}$$

Total solution is

$$y_p(n) + y_h(n) = y(n)$$

$$y(n) = \frac{8}{5}(2^n) u(n) + C_1 \left(\frac{1}{2}\right)^n u(n) + C_2 (1/3)^n u(n)$$

Assume $y(-2) = y(-1) = 0$ so $y(0) = 1$

Then $y(1) = \frac{5}{6} y(0) + 2 = \frac{17}{6}$

So $\frac{8}{5} + C_1 + C_2 = 1$

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$C_1 + C_2 = \frac{3}{5}$ — (1)

$\frac{16}{5} + \frac{1}{2}C_1 + \frac{1}{3}C_2 = \frac{17}{6}$

$3C_1 + 2C_2 = \frac{-11}{5}$ — (2)

By solving 1 & 2

$C_1 = -1, \; C_2 = 2/5$

So the total solution is

$$y[n] = \left[\frac{8}{5}(2)^n - \left(\frac{1}{2}\right)^n + \frac{2}{5}\left(\frac{1}{3}\right)^n\right]u(n)$$

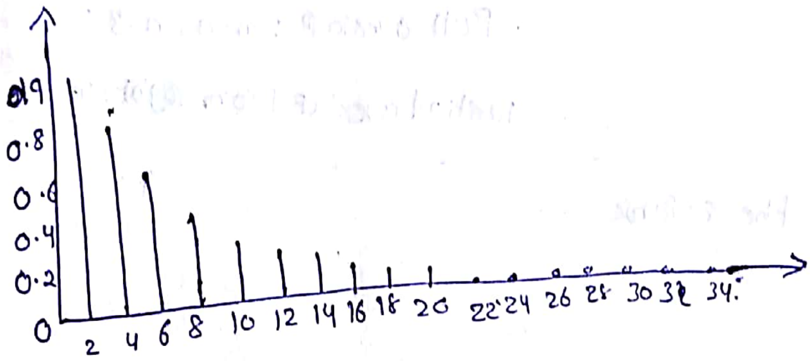
(ii) In the given equation.

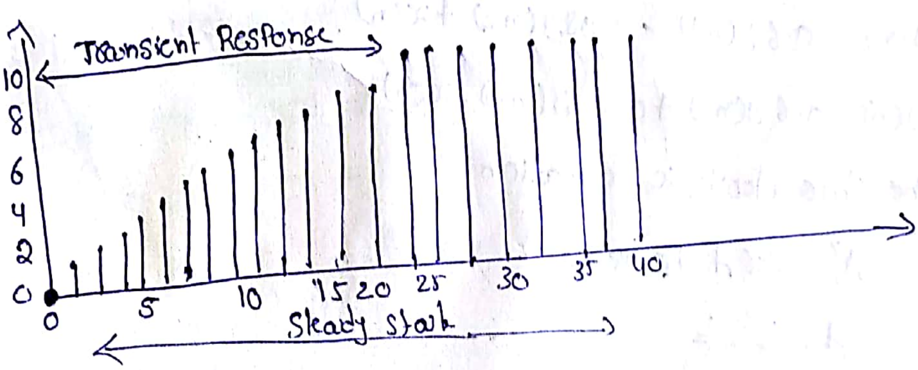
(a) At $y(-1) = 1$

The given equation $y(n) = (-a)^{n+1} + \frac{(1-(-a)^{n+1})}{1+a}$

$y(n) = y_{zi}(n) + y_{zs}(n)$

= Transient state.





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(a) $L_1 = N_1 + m_1$ & $L_2 = N_2 + m_2$

(b) Partial overlap from left:

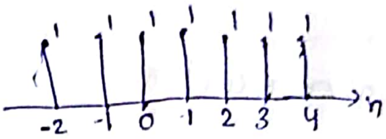
low N_1+m_1 high N_1+m_2-1

Full overlap: low N_1+m_2 high N_2+m_1

Partial overlap from right

low N_2+m_1+1 high N_2+m_2

c) $x(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1, 1, 1\}$



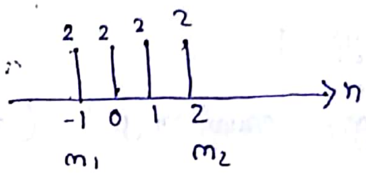
$h(n) = \{2, \underset{\uparrow}{2}, 2, 2\}$

$N_1 = -2$

$N_2 = 4$

$m_1 = -1$

$m_2 = 2$



Partial overlap from left: $n=-3$

$n=-1$, $L_1=-3$

Full overlap: $n=0$, $n=3$

Partial overlap from right: $n=4$, $n=6$ $L_2=$

3d Determine the response . . . .

Sol

$y(n) = 0.6\,y(n-1) - 0.08\,y(n-2) + x(n)$

$y(n) - 0.6\,y(n-1) + 0.08\,y(n-2) = x(n)$

The characteristic equation:

$\lambda^2 - 0.6\lambda + 0.08 = 0$

$\lambda = \frac{2}{5}, \frac{1}{5}$

$y(n) = C_1\left(\frac{1}{5}\right)^n + C_2\left(\frac{2}{5}\right)^n$

with $x(n) = \delta(n)$ the initial conditions are

$y(0) = 1$

$y(1) - 0.6\,y(0) = 0 \Rightarrow y(1) = 0.6$

$$c_1 + c_2 = 1 \Rightarrow \frac{1}{5}c_1 + \frac{1}{5}c_2 = \frac{1}{5}$$

$$\frac{1}{5}c_1 + \frac{2}{5}c_2 = 0.6 \Rightarrow \frac{1}{5}c_1 + \frac{2}{5}c_2 = 0.6$$

$$-\frac{1}{5}c_2 = \frac{2}{5}$$

$$c_2 = 2$$

$$\therefore 2 + c_1 = 1$$

$$c_1 = 1 - 2 = -1$$

$$\therefore h(n) = \left[-\left(\tfrac{1}{5}\right)^n + 2\left(\tfrac{2}{5}\right)^n\right] u(n)$$

the step response

$$s(n) = \sum_{k=0}^{n} h(n-k) \; ; n \geq 0$$

$$= \sum_{k=0}^{n} \left[2\left(\tfrac{2}{5}\right)^{n-k} - \left(\tfrac{1}{5}\right)^{n-k}\right]$$

$$= \left\{\frac{1}{0.12}\left\{\left(\tfrac{2}{5}\right)^{n+1} - 1\right] - \frac{1}{0.16}\left\{\left(\tfrac{1}{5}\right)^{n+1} - 1\right]\right\} u(n)$$

~~y(n)~~

31) considers a system.

Sol

$$h(n) = \{ \underset{\uparrow}{1}, \tfrac{1}{2}, \tfrac{1}{4}, \tfrac{1}{8}, \tfrac{1}{16} \}.$$

$$y(n) = \{ \underset{\uparrow}{1}, 2, 2.5, 3, 3, 3, 2, 1, 0 \}.$$

$$x(0)\,h(0) = y(0) \Rightarrow x(0) = 1$$

$$\frac{1}{2}x(0) + x(1) = y(1) \Rightarrow x(1) = \frac{3}{2}$$

by Continuing this Process

$$x(n) = \{ 1, \tfrac{3}{2}, \tfrac{3}{2}, \tfrac{7}{4}, \tfrac{3}{2}, \ldots \}.$$

32)

a) $h(n) = h_1(n) * [h_2(n) - h_3(n) * h_4(n)]$

b) $h_3(n) * h_4(n) = (n-1)\, u(n-2)$

$$h_2(n) - h_3(n) * h_4(n) = 2u(n) - \delta(n)$$

$$h_1(n) = \frac{1}{2}\delta(n) + \frac{1}{4}\delta(n-1) + \frac{1}{2}\delta(n-2)$$

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c) $x(n) = \{1, 0, 0, 3, 0, -4\}$
$\uparrow$

$y(n) = \{\frac{1}{2}, \frac{5}{4}, 2, \frac{25}{4}, \frac{13}{2}, 5, 2, 0, 0, \cdots\}.$

$$s(n) = u(n) * h(n)$$

$$s(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} h(n-k)$$

$$= \sum_{k=0}^{\infty} a^{n-k}$$

$$= \frac{a^{n+1}-1}{a-1}, \quad n \geq 0$$

For $x(n) = u(n+5) - u(n-10)$

$$s(n+5) - s(n-10) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10)$$

$$y(n) = x(n) * h(n) - x(n) * h(n-k)$$

$$y(n) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10) - \frac{a^{n+4}-1}{a-1} u(n+3) + \frac{a^{n-11}-1}{a-1} u(n-12)$$

33

Sol

$$x(n) = u(n+5) - u(n-10)$$

$$s(n+5) - s(n-10) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10)$$

$$y(n) = x(n) * h(n) - x(n) * h(n-2)$$

$$y(n) = \frac{a^{n+6}-1}{a-1} u(n+5) - \frac{a^{n-9}-1}{a-1} u(n-10)$$

$$- \frac{a^{n+4}-1}{a-1} u(n+3) + \frac{a^{n-11}-1}{a-1} u(n-12)$$

Sol

$$h(n) = [u(n) - u(n-m)]/m$$

$$s(n) = \sum_{k=-\infty}^{\infty} u(k)\, h(n-k)$$

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$s(n) = \begin{cases} \frac{n+1}{m}, & n<m \\ 1, & n \geq m \end{cases}$$

35

Sol

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0,\, n\,even}^{\infty} |a|^n.$$

$$= \sum_{n=0}^{\infty} |a|^{2n}$$

$$= \frac{1}{1-|a|^2}$$

stable if $|a|<1$.

36

Sol

$$x(n) = u(n) - u(n-10)$$

$h(n) = a^n u(n)$, the response $u(n)$ is

$$y_1(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k)$$

$$= \sum_{k=0}^{n} a^{n-k}$$

$$= a^n \sum_{k=0}^{n} a^{-k} = \frac{1-a^{n+1}}{1-a} u(n)$$

Then, $y(n) = y_1(n) - y_1(n-10)$

$$= \frac{1}{1-a}\left[(1-a^{n+1}) u(n) - (1-a^{n-1})\, u(n-10) \right].$$

38 Determine

Sol

$h(n) = (\frac{1}{2})^n u(n) \quad y_1(n) = \sum_{k=0}^{\infty} u(k)\, h(n-k)$

$= \sum_{k=0}^{n} (\frac{1}{2})^{n-k}$

$= (\frac{1}{2})^n \sum_{k=0}^{n} (1/2)^{-k}$

$= \frac{1-(\frac{1}{2})^{n+1}}{1-\frac{1}{2}} u(n).$

$\Rightarrow y(n) = y_1(n) - y_1(n-10)$

$= \frac{1}{1-\frac{1}{2}} \left[\left(1-(\frac{1}{2})^{n+1}\right) u(n) - \left(1-(\frac{1}{2})^{n-9}\right) u(n-10) \right]$

$= 2\left[1-(\frac{1}{2})^{n+1}\right] u(n) - 2\left[1-(\frac{1}{2})^{n-9} u(n-10)\right]$

(a) $a = 2$

$y(n) = \frac{1}{1-2} \left[\left(1-(2)^{n+1}\right) u(n) \right] + \left[\left(1-(2)^{n-9}\right) u(n-10) \right]$

39

Sol

a) $h(n) = h_1(n) * h_2(n) * h_3(n)$

$= [\delta(n) - \delta(n-1)] * u(n) \times h(n)$

$= [u(n) - u(n-1)] * h(n)$

$= \delta(n) * h(n) = h(n)$

b) not affected

40

Sol

a) $x(n)\, \delta(n-n_0) = x(n_0)$. Thus, only the value of $x(n)$ at $n = n_0$ is of interest

$x(n) * \delta(n-n_0) = x(n-n_0)$

Thus, we obtain the shifted version of the sequence $x(n)$

(b) $y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$

$y(n) = h(n) * x(n)$

Linearity :-

$x_1(n) \rightarrow y_1(n) = h(n) * x_1(n)$

$x_2(n) \rightarrow y_2(n) = h(n) * x_2(n)$

$x(n) = \alpha x_1(n) + \beta x_2(n) \Rightarrow y(n) = h(n) * x(n)$

$y(n) = h(n) * [\alpha x_1(n) + \beta x_2(n)]$

$y(n) = \alpha h(n) * x_1(n) + \beta h(n) * x_2(n)$

$$\boxed{y(n) = \alpha y_1(n) + \beta y_2(n)}$$

Time invariance

$x(n) \rightarrow y(n) = h(n) * x(n)$

$x(n-n_0) \rightarrow y_1(n) = h(n) * x(n-n_0)$

$= \sum_{n} h(k)\, x(n-n_0-k)$

$= y(n-n_0)$

c) $h(n) = \delta(n-n_0)$

---

41)

(a) $S(n) = -a_1 S(n-1) - a_2 S(n-2) \ldots a_N S(n-N) + b_0 V(n)$

b) $V(n) = \frac{1}{b_0} [S(n) + a_1 S(n-1) + a_2 S(n+2) + \ldots + a_N S(n-N)]$

44

$x(n) = \{1, 0, 0, 1, \ldots\}$

$y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

$y(0) = x(0) = 1$

$y(1) = \frac{1}{2} y(0) + x(1) + x(0) = \frac{1}{2} + 0 + 1 = \frac{3}{2}$

$y(2) = \frac{1}{2}\left(\frac{3}{2}\right) + 0 + 0 = \frac{3}{8}$

$y(4) = \frac{1}{2}\left(\frac{3}{8}\right) + 0 + 0 = \frac{3}{16}$

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$y(5) = \frac{1}{2}\left(\frac{3}{16}\right) + 0 + 0 = \frac{3}{32}$

$y(6) = \frac{1}{2}\left(\frac{3}{32}\right) + 0 + 0 = \frac{3}{64}$

$y(7) = \frac{1}{2}\left(\frac{3}{64}\right) + 0 + 0 = \frac{3}{128}$

$y(8) = \frac{1}{2}\left(\frac{3}{128}\right) + 0 + 0 = \frac{3}{256}$

$y(9) = \frac{1}{2}\left(\frac{3}{256}\right) + 0 + 0 = \frac{3}{512}$

$y(10) = \frac{1}{2}\left(\frac{3}{512}\right) + 0 + 0 = \frac{3}{1028}$

$\therefore\ y(n) = \left\{1, \frac{3}{2}, \frac{3}{4}, \frac{3}{8}, \frac{3}{16}, \frac{3}{32}, \frac{3}{64}, \frac{3}{128}, \frac{3}{256}, \frac{3}{512}, \frac{3}{1024}\right.$

b)

$y(n) = \frac{1}{2}y(n-1) + x(n) + x(n-1)$

c)

$x(n) = \{1, 1, 1, 1, \ldots\}$

$y(0) = x(0) = 1$

$y(1) = \frac{1}{2}(1) + 1 + 1 = \frac{5}{2}$

$y(2) = \frac{1}{2}\left(\frac{5}{2}\right) + 1 + 1 = \frac{13}{4}$

$y(3) = \frac{1}{2}\left(\frac{13}{4}\right) + 1 + 1 = \frac{29}{8}$

$\vdots$

$y(10) = \frac{1}{2}\left(\frac{2045}{512}\right) + 2 = 3.9970$

$y[n] = \left\{1, \frac{5}{2}, \frac{13}{4}, \frac{29}{8}, \frac{61}{16}, \frac{125}{32}, \frac{253}{64}, \frac{509}{128}, \frac{1021}{256}, \frac{2045}{512}, 3.9970 \ldots\right\}$

e) $h(n) = 0$ For $n < 0$ $\Rightarrow$ the system is causal

$\sum_{n=0}^{\infty} |h(n)| = 1 + \frac{3}{2}\left(1 + \frac{1}{2} + \frac{1}{4}\right)$

$= 1 + \frac{3}{2}(2)$

$= 4 \Rightarrow$ the system is stable.

45

a) $y(n) = ay(n-1) + bx(n)$

$\Rightarrow h(n) = ba^n u(n)$

$\Rightarrow \sum_{n=0}^{\infty} h(n) = \frac{b}{1-b} = 1 \quad (\because b = 1-a)$

b) $s(n) = \sum_{k=0}^{n} h(n-k) = b\left[\frac{1-a^{n+1}}{1-a}\right] u(n)$

$s(\infty) = \frac{b}{1-a} = 1$

$b = 1-a)$

c) $b = 1-a$ in both cases.

46

a) $y(n) = 0.8y(n-1) + 2x(n) + 3x(n-1)$

$y(n) - 0.8y(n-1) = 2x(n) + 3x(n-1)$

The characteristic equation is

$\lambda - 0.8 = 0$

$\lambda = 0.8$

$y_h(n) = c(0.8)^n$

let us first consider the response of the system.

$y(n) - 0.8y(n-1) = x(n)$

$\Rightarrow x(n) = \delta(n)$

since $y(0) = 1$, it flows that $c = 1$

Then, the impulse response of the original system is

$h(n) = 2(0.8)^n u(n) + 3(0.8)^{n-1} u(n-1)$

$h(n) = 2\delta(n) + 4.6\,(0.8)^{n-1} u(n-1)$

b) The inverse system is characteristic by the difference equation

$x(n) = -1.5x(n-1) + \frac{1}{2}y(n) - 0.4y(n-1)$

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47)

a) $y(n) = 0.9\,y(n-1) + x(n) + 2x(n-1) + 3x(n-2)$

For $x(n) = \delta(n) = \{\cdots 0, \dot{1}, 0, 0, 0 \cdots\}$

$y(0) = 0.9(0) + 1 + 0 + 0 = 1$

$y(1) = 0.9(1) + 0 + 2 + 0 = 2.9$

$y(2) = 0.9(2.9) + 0 + 0 + 3 = 5.61$

$y(3) = 0.9(5.61) + 0 + 0 + 0 = 5.049$

$y(4) = 0.9(5.049) + 0 + 0 + 0 = 4.5441$

$y(5) = 0.9(4.5441) + 0 + 0 + 0 = 4.08969$

$y(6) = 0.9(y(5) + x(6) + 2x(5) + 3x(4)) = 0.9(4.08696) + 0 + 0 = 3.68$

b)

$S(0) = y(0) = 1$

$S(1) = y(0) + y(1) = 3.91$

$S(2) = y(0) + y(1) + y(2) = 9.51$

$S(3) = y(0) + y(1) + y(2) + y(3) = 14.56$

$S(4) = y(0) + y(1) + y(2) + y(3) + y(4) = 19.16$

$S(5) = y(0) + y(1) + y(2) + y(3) + y(4) + y(5) = 23.19$

$S(6) = y(0) + y(1)$ " " + " + " + $y(6) = 26.87$

c) $h(n) = (0.9)^n u(n) + 2(0.9)^{n-1} u(n-1) + 3(0.9)^{n-2} u(n-2)$

$h(n) = \delta(n) + 2.9\,\delta(n-1) + 5.6(0.9)^{n-2} u(n-2)$

49

a) $h_1(n) = c_0\delta(n) + c_1\delta(n-1) + c_2\delta(n-2)$

$h_2(n) = b_2\delta(n) + b_1\delta(n-1) + b_0\delta(n-2)$

$h_3(n) = a_0\delta(n) + (a_1 + a_0 a_2)\delta(n-1) + a_1 a_2 \delta(n-2)$

b) let $a_0 = c_0$

$a_1 + a_2 c_0 = c_1$

$a_0 = a_1 = c_2$

Hence

$\frac{C_2}{a_2} + a_2 C_0 - C_1 = 0$

$\Rightarrow C_0 a_2 - C_1 a_2 + C_2 = 0$

For $C_0 \neq 0$, the quadratic has a real solutions if and only if $C_1^2 - 4C_0C_2 \geq 0$.

Sol

a) Sol $y(n) = \frac{1}{2} y(n-1) + x(n) + x(n-1)$

$y(n) - \frac{1}{2} y(n-1) = \delta(n)$, the solution of the system is

$h(n) = \left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{2}\right)^{n-1} u(n-1)$

b)

$h(n) * [\delta(n) + \delta(n-1)] = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{n-1} u(n-1)$

Sol

a) Convolution : $y_1(n) = x_1(n) * h_1(n)$

$y_1(n) = \{1, 3, 7, 7, 7, 6, 4\}$

	1	1	1	1	1
1	1	1	1	1	1
2	2	2	2	2	2
4	4	4	4	4	4

Correlation :-

$r_1(n) = x(n) * h(-n)$

$h(-n) = \{1, 1, 1, 1, \underset{\uparrow}{1}\}$

$x(n) = \{\underset{\uparrow}{1}, 2, 4\}$

$r_1(n) = \{1, 3, 7, 7, \underset{\uparrow}{7}, 6, 4\}$

b)

con

b) $x_2(n) = \{0, 1, -2, 3, 4\}$, $h_2(n) = \{\frac{1}{2}, 1, 2, 1, \frac{1}{2}\}$

Convolution : $y_2(n) = x_2(n) * h_2(n)$

$y_2(n) = \{0, \frac{1}{2}, 0, \frac{5}{2}, -2, \frac{1}{2}, -6, -5/2, -2\}$

$x(n)$ \ $h(n)$	$\frac{1}{2}$	1	2	1	$\frac{1}{2}$
0	0	0	0	0	0
1	$\frac{1}{2}$	1	2	1	$\frac{1}{2}$
-2	-1	-2	-4	-2	-1
3	$\frac{3}{2}$	3	6	3	$\frac{3}{2}$
-4	-2	-4	-8	-4	-2

Correlation

$r_1(n) = x(n) * h(n)$

$h(-n) = \{\frac{1}{2}, 1, 2, 1, \frac{1}{2}\}$

$x(n) = \{0, 1, -2, 3, -4\}$

~~$r_1(n) = \{1, 3, 7, 7, 7, 6, 4\}$.~~

$r_2(n) : \{0, \frac{1}{2}, 0, \frac{5}{2}, -2, \frac{1}{2}, -6, \frac{-5}{2}, -2\}$.

c) Convolution $\Rightarrow y_3(n) = x_3(n) * h_3(n)$

$y_3(n) = \{4, 11, 20, 30, 20, 11, 4\}$

$x_3(n)$ \ $h(n)$	4	3	2	1
1	4	3	2	1
2	8	6	4	2
3	12	9	6	3
4	16	12	8	4

Correlation $\Rightarrow$

$h_3(-n) = \{1, 2, 3, 4\}$

$x_3(n) = \{1, 2, 3, 4\}$

$r_3(n) = \{1, 4, 10, 20, 25, 24, 16\}$

$x_3(n)$ \ $h_3(-n)$	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

(d) Convolution $\Rightarrow y_4(n) = x_4(n) * h_4(n)$

$y_4(n) = \{1, 4, 10, 20, 25, 24, 16\}$

$x_4(n)$ \ $h_4(n)$	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

Correlation :-

$h_4(-n) = \{4, 3, 2, 1\}$

$x_4(n) = \{1, 2, 3, 4\}$

$r_4(n) : \{4, 11, 20, 30, 20, 11, 4\}$

$x_4(n)$ \ $h_4(n)$	4	3	2	1
1	4	3	2	1
2	8	6	4	2
3	12	9	6	3
4	16	12	8	4

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The 2000 stats

$x(n) = \{1, 3, 3, 1\}$

$y(n) = \{1, 4, 6, 4, 1\}$

the length of the i/p is, $L_1 = 4$

" " " " o/p is, $N = 5$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

length of the h(n) is L_2

$L_1 + L_2 - 1 = N$

$4 + L_2 - 1 = 5$

$3 + L_2 = 5$

$L_2 = 5 - 3 = 2$

$\therefore h(0) = y(0) = x(0) = 1$ $\quad h(n) = \{h(0), h(1)\}$

$3h(0) + h(1) = y(1) = 4$

$\Rightarrow 3(1) + h(1) = 4$

$3 + h(1) = 4$

$h(1) = 4 - 3 = 1$

$\therefore h(n) = \{1, 1\}.$

---

54)

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda^2 - 2\lambda - 2\lambda + 2 \times 2 = 0$

$\lambda(\lambda - 2) + (-2)(\lambda - 2) = 0$

$(\lambda - 2)(\lambda - 2) = 0$

$\lambda = 2, 2$ Hence $y_h(n) = c_1 2^n + c_2 n 2^n.$

The particular solution is $y_p(n) = K(-1)^n u(n)$

$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-1} u(n-1) = (-1)^n u(n) - (-1)^{n-1} u(n$

55

Sol

$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$

The characteristic equation is

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda^2 - 2\lambda - 2\lambda + 2\times 2 = 0$

$(\lambda - 2)(\lambda - 2) = 0$

$\lambda = 2, 2$

$\therefore y_h(n) = 2^n c_1 + n 2^n c_2$

when $x(n) = \delta(n)$

with $y(0) = 1$ & $y(1) = 3$,

$\Rightarrow c_1 = 1$

$\Rightarrow 2c_1 + 2c_2 = 3$

$2(c_1 + c_2) = 3$

$1 + c_2 = \frac{3}{2}$

$c_2 = \frac{3}{2} - 1 = \frac{1}{2}$

$\therefore h(n) = \left[2^n + \frac{1}{2} n 2^n\right] u(n)$

56

Sol

$x(n) = x(n) * \delta(n)$

$\Rightarrow x(n) = x(n) * [u(n) - u(n-1)]$

$\Rightarrow x(n) = x(n) * [u(n) - u(n-1)]$

$\Rightarrow x(n) = u(n) * [x(n) - x(n-1)]$

$= \sum_{k=-\infty}^{\infty} [x(k) - x(n-k)] u(n-k)$

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Sol Let $h(n)$ be the Impulse response of the system

$$S(k) = \sum_{m=-\infty}^{\infty} h(m)$$

$$\Rightarrow h(k) = S(k) - S(k-1)$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$\Rightarrow y(n) = \sum_{k=-\infty}^{\infty} [S(k) - S(k-1)]\, x(n-k)]$$

---

5)

a) $x(n) = \{1, 2, 1, 1\}$ $\quad x(0) = 1,\ x(1) = 2$

$x(2) = 1,\ x(3) = 1$

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l) = \sum_{n=0}^{3} x(n)\, x(n-l)$$

$r_{xx}(-3) = x(0)\ x(0-(-3)) = x(0)\, x(3) = (1)(1) = 2$

$r_{xx}(-2) = x(0)\ x(0-(-2)) = x(0)\ x(2) + x(1)\, x(3) = [1] + (1) + (1)(2) = 1 + 2 = 3.$

$r_{xx}(1) = x(0)\ x(1) + x(1)\, x(2) + x(2)\, x(3) = 2 + 2 + 1 = 5$

$$r_{xx}(0) = \sum_{n=0}^{3} x^2(n) = 7$$

also $r_{xx}(-l) = r_{xx}(l)$

$\therefore\ r_{xx}(l) = \{1, 3, 5, 7, 5, 3, 1\}$

b) $y(n) = \{1, 1, 2, 1\}$
$\quad\quad\quad\uparrow$

$$r_{yy}(l) = \sum_{n=-\infty}^{\infty} y(n)\, y(n-l)$$

$r_{yy}(l) = \{1, 3, 5, 7, 5, 3, 1\}.$

we observe that $y(n) = x(-n+3)$, which is equivalent to reversing the sequence of $x(n)$.

5) what is the normalized autocorrelation.

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$$

$$r_{xx}(l) = \begin{cases} 2N+1-|l| ; & -2N < l < 2N \\ 0 & ; \text{ otherwise} \end{cases}$$

$r_{xx}(0) \Rightarrow r_{xx}(0) = 2N+1 - |0| = 2N+1$

$\therefore$ The normalised autocorrelation is

$$\gamma_{xx}(l) = \begin{cases} \frac{1}{2N+1}(2N+1-|l|) ; & -2N \le l < 2N \\ 0, & \text{else.} \end{cases}$$

---

6) An audio signal . . . . . .

a) $r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n)\, x(n-l)$

$= \sum_{n=-\infty}^{\infty} (s(n) - r_1 s(n-k) + r_2 s(n-k_2)] * [s(n-l) + r_1 s(n-l-k_1) + r_{(}$

$= [1 + r_1^2 + r_2^2]\, r_{33}(l) + r_1 [r_{22}(1+k_1) + r_{23}(k+3) + r_2 (r_{33}(l+k_2) + r_{33}(l-k_2)] +$

$r_1 r_2 [r_{33}(l+k_1-x_2) + r_{33}(l + r_3 - k_1)]$

b) $r_{xx}(l)$ has Peaks at $l=0$, $\pm 1k$, $\pm k_2$ & $\pm (k_1+k_2)$

Suppose that $k_1 < k_2$. Then we can determine r & k_1

The Problem is determine r_2 & k_2 From the other Peaks

c) if $r_2 = 0$, the Peaks occur at $l = 0$ & $l = \pm k_1$

Then, it is easy obtain r_1 & k_1