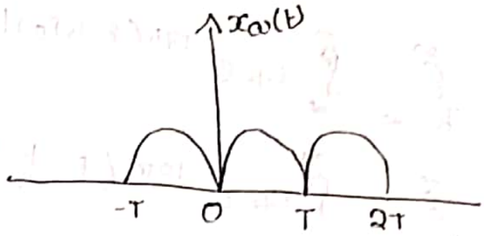
# DTFT Problems (Assignment-2).

1) Consider the Full wave rectified sinusoid

a) determine the spectrum $X_a(F)$.




Sol $$x_a(t) = \sum_{k=-\infty}^{\infty} C_k e^{j2\pi k F_0 t}$$

Let $F_0 = \frac{1}{T}$

$$= \sum_{k=-\infty}^{\infty} C_k e^{j2\pi \frac{kt}{T}}$$

$$C_k = \frac{1}{T}\int_0^T A \sin\left(\frac{\pi t}{T}\right) e^{-j2\pi k t/T} \cdot dt$$

$$= \frac{1}{2j \cdot T}\int_0^T \left(e^{\frac{j\pi t}{T}} - e^{\frac{-j\pi t}{T}}\right) e^{-j2\pi k \frac{t}{T}} dt$$

$$= \frac{A}{2jT}\int_0^T \left(e^{j\pi \frac{t}{T}} \cdot e^{-j2\pi k \frac{t}{T}}\right) - \left(e^{\frac{-j\pi t}{T}} \cdot e^{-j2\pi k \frac{t}{T}}\right) dt$$

$$= \frac{A}{2j \cdot T}\int_0^T e^{j\pi(1-2k)\frac{t}{T}} - e^{-j\pi(1-2k)\frac{t}{T}} \cdot dt$$

$$= \frac{A}{2jT}\left[\int_0^T e^{j\pi\left(1-2k\frac{t}{T}\right)} dt - \int_0^T e^{-j\pi(1-2k)\frac{t}{T}} dt\right]$$

$$= \frac{A}{2j \cdot T}\left[\frac{e^{j\pi(1-2k)\frac{t}{T}}}{j\pi(1-2k)\frac{1}{T}}\right]_0^T - \left[\frac{e^{-j\pi(1-2k)\frac{t}{T}}}{-j\pi(1-2k)\frac{1}{T}}\right]_0^T$$

$$= \frac{A}{2jT}\left[\frac{e^{j\pi(1-2k)} - e^0}{j\pi(1-2k)\cdot\frac{1}{T}} - \frac{e^{-j\pi(1-2k)} - e^0}{-j\pi(1-2k)\cdot\frac{1}{T}}\right]$$

$$= \frac{A}{2jT}\cdot\frac{T}{j\pi}\left[\frac{-1-1}{(1-2k)} + \frac{(-1-1)}{(1+2k)}\right]$$

$$= \frac{A}{2\pi}\left[\cancel{2}\left[\frac{1}{1-2k} + \frac{1}{1+2k}\right]\right]$$

$$= \frac{A}{\pi}\left[\frac{1+\cancel{2k}+1-\cancel{2k}}{1+\cancel{2k}-\cancel{2k}-4k}\right] = \frac{2A}{\pi(1-4k^2)}$$

$$X_a(F) = \int_{-\infty}^{\infty} x_a(t)\, e^{-j2\pi Ft} \cdot dt$$

$$= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} C_k\, e^{j2\pi k F_0 t} \cdot e^{-j2\pi Ft}\, dt$$

$$= \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} C_k\, e^{-j2\pi(F-kF_0)t}\, dt$$

$$= \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} C_k\, e^{-j2\pi(F-\frac{k}{T})t}\, dt$$

$$\boxed{X_a(F) = \sum_{k=-\infty}^{\infty} C_k\, \delta\left(F-\frac{k}{T}\right)} \checkmark$$

b) Compute the Power of the signal

$$P_x = \frac{1}{T}\int_0^T x_a^2(t)\, dt$$

$$= \frac{1}{T}\int_0^T \left(A \sin\frac{\pi t}{T}\right)^2 dt$$

$$= \frac{A^2}{T}\int_0^T \sin^2\frac{\pi}{T} t \cdot dt$$

$$= \frac{A^2}{T}\int_0^T \frac{1-\cos^2(\frac{\pi}{T})t}{2}\, dt$$

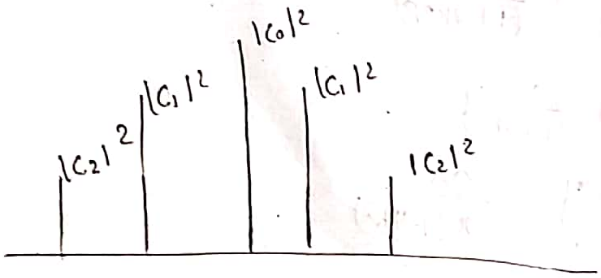
$$= \frac{A^2}{T}\int_0^T \frac{1}{2} - \frac{\cos 2(\frac{\pi}{T})t}{2}$$

$$= \frac{A^2}{T}\left[\frac{T}{2} - \left[\frac{\cos(2\pi)}{2} - 0\right]\right]$$

$$= \frac{A^2}{T} \cdot \frac{T}{2} - 0$$

$$\boxed{P_x = \frac{A^2}{2}}$$

c) Plot the Power spectral densit



d) check the validity of Parseval's relation for the given

Sol $P_x = \sum_{K=-\infty}^{\infty} |C_K|^2$

$$= \sum_{K=-\infty}^{\infty} \left( \frac{2A}{\pi(1-4k^2)} \right)^2$$

$$= \frac{4A^2}{\pi^2} \sum_{K=-\infty}^{\infty} \frac{1}{(4k^2-1)^2}$$

$$= \frac{4A^2}{\pi^2} \left[ \frac{1}{(4k^2-1)^2} \Big|_{k=0} + 2 \sum_{k=1}^{\infty} \frac{1}{(4k^2-1)^2} \right]$$

$$= \frac{4A^2}{\pi^2} \left[ 1 + \frac{2}{3^2} + \frac{2}{15^2} + \cdots \right]$$

$$= \frac{4A^2}{\pi^2} [1.231] = 0.498 A^2 = 0.5A^2 \approx \frac{A^2}{2}$$

4.2 Compute and sketch the magnitude and phase spectrum for the following signals

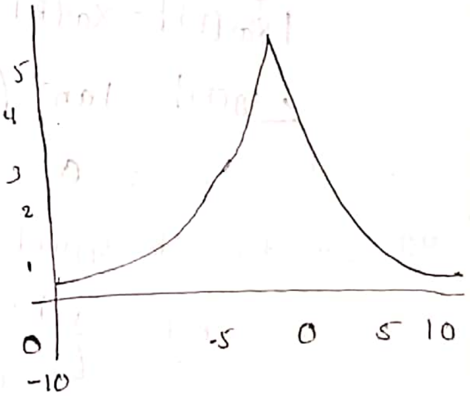
a) $x_a(t) = \begin{cases} Ae^{-at} & t \geq 0 \\ 0, & t < 0 \end{cases}$

$$x_a(F) = \int_0^{\infty} Ae^{-at} e^{-j2\pi Ft} \, dt$$

$$= A \int_0^{\infty} e^{-(a+j2\pi F)t} \, dt$$

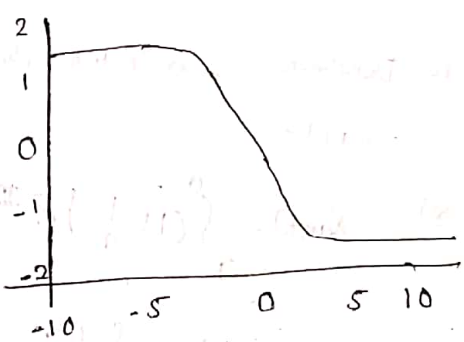
$$= A \left[ \frac{e^{-(a+2\pi Fj)t}}{-(a+2\pi Fj)} \right]_0^{\infty}$$

$$= A \cdot \frac{1}{a+j2\pi F}$$




$$|X_a(F)| = \frac{A}{\sqrt{a^2+(2\pi F)^2}}$$

$$\angle |X_a(F)| = -\tan^{-1}\left(\frac{2\pi F}{a}\right)$$



b) $x_a(t) = Ae^{-a|t|}$

Sol

$$X_a(F) = \int_{-\infty}^{\infty} Ae^{-a|t|} e^{-j2\pi Ft} dt$$

$$= \int_{-\infty}^{0} Ae^{at} e^{-j2\pi Ft} + \int_{0}^{\infty} Ae^{-at} e^{-j2\pi Ft}$$

$$= \int_{0}^{\infty} Ae^{-at} e^{j2\pi Ft} + \int_{0}^{\infty} Ae^{-at} e^{-j2\pi Ft}$$

$$= A \cdot \int_{0}^{\infty} e^{-(-j2\pi F + a)t} + A \int_{0}^{\infty} e^{-(a+j2\pi F)t}$$

$$= A \cdot \left[ \frac{e^{-(a-j2\pi F t)}}{-(a-j2\pi F)} \right]_0^{\infty} + A \left[ \frac{e^{-(a+j2\pi F)t}}{-(a+j2\pi F)} \right]_0^{\infty}$$

$$= A \left[ \frac{1}{a-j2\pi F} \right] + A \left[ \frac{1}{a+j2\pi F} \right]$$

$$= \frac{Aa + Aj2\pi F + Aa - Aj2\pi F}{a^2 - (j2\pi F)^2} = \frac{2aA}{a^2 + (2\pi F)^2}$$

$$|X_a(F)| = X_a(F)$$

$$\angle X_a(F) = \tan^{-1}\left(\frac{0}{\text{Same}}\right)$$

$$= 0$$

4.3 Consider the signal

$$x(t) = \begin{cases} 1 - \frac{|t|}{\tau}, & |t| \le \tau \\ 0, & \text{elsewhere} \end{cases}$$

a) Determine and sketch its magnitude and phase spectrum $|X_a(F)|$ and $X_a(F)$.

Sol

$$X_a(F) = \int_{T}^{0} \left(1 + \frac{t}{\tau}\right) e^{-j2\pi Ft} dt + \int_{0}^{T} -\frac{1}{\tau} e^{-j2\pi Ft} dt$$

Let $y(t) = \left\{ \frac{1}{T};\ -\tau \le t \le \tau \right\}$

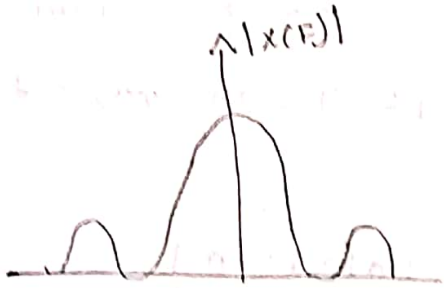
$$x(F) = \int_{-T}^{0} \frac{1}{\tau} e^{-j2\pi Ft} dt + \int_{0}^{T} -\frac{1}{\tau} e^{-j2\pi Ft} dt$$

$$= -2 \frac{\sin^2 \pi Ft}{j\pi Ft}$$

$$X(F) = \frac{1}{j2\pi F} Y(F)$$

$$= \tau\left(\frac{\sin \pi F\tau}{\pi F\tau}\right)^2$$

$$|X(F)| = \tau\left(\frac{\sin \pi F\tau}{\pi F\tau}\right)^2$$



$$\angle x_a(F) = 0$$

$$\tau \cdot \left(\frac{\sin \pi F\tau}{\pi F\tau}\right)^2$$

Let $F = 1/\tau$

$$\left(\frac{\sin \pi t}{\pi t}\right)^2 = \sin^2 c(t)$$

b) Create a periodic signal $x_p(t)$ with fundamental period $T_p \geq 2\tau$ so that $x(t) = x_p(t)$ for $|t| \leq T_p/2$. What are the Fourier coefficients $c_k$ for the signal $x_p(t)$?

Sol

$$c_k = \frac{1}{T_p} \int_{-T_p/2}^{T_p/2} x(t)\, e^{-j2\pi kt/T_p}\, dt$$

$$= \frac{1}{T_p} \int_{-\tau}^{0} \left(1 + \frac{t}{\tau}\right) e^{-j2\pi kt/T_p}\, dt + \int_{0}^{t} \left(1 - \frac{t}{\tau}\right) e^{-j2\pi kt/T_p}\, dt$$

$$= \frac{\tau}{T_p}\left[\frac{\sin \pi k \tau/T_p}{\pi k \tau/T_p}\right]^2$$

c) Using the results in parts a and b show that $c_k = \left(\frac{1}{T_p}\right) x_a\left(\frac{k}{T_p}\right)$

Sol

$$\frac{1}{T_p} \cdot x_a\left(\frac{k}{T_p}\right)$$

$$\frac{1}{T_p} \cdot \tau \cdot \left(\frac{\sin \pi \cdot \frac{k}{T_p} \tau}{\pi \frac{k}{T_p} \cdot \tau}\right)^2$$

$$= \frac{\tau}{T_p}\left(\frac{\sin \pi k\tau/T_p}{\pi k\tau/T_p}\right)^2 = c_k$$

$$\therefore c_k = \frac{1}{T_p} x_a\left(\frac{k}{T_p}\right)$$ hence proved.

4.4 Consider the Following Periodic signal.

$$x(n) = \{ \ldots\ldots 1, 0, 1, 2, 3, 2, 1, 0, 1 \ldots \}$$

a) sketch the signal $x(n)$ and its magnitude and Phase spectrum.

(a)

1 0 1 2 3 2 1 0 1

$N = 6$

$$C_K = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{kn}{N}}$$

$$= \frac{1}{6} \sum_{n=0}^{5} x(n) e^{\frac{-j2\pi kn}{6}}$$

For $n=0 \rightarrow x(0) \cdot e^{\frac{-j2\pi k}{6}(0)} = 3 \times 1 = 3$

$n=1 \rightarrow x(1)\, e^{\frac{-j2\pi k}{6}(1)} = 2e^{\frac{-j\pi 2k}{6}}$

$n=2 \rightarrow x(2)\, e^{\frac{-j2\pi k}{6}(2)} = e^{\frac{-j2\pi k}{3}}$

$n=3 \rightarrow x(3)\, e^{\frac{-j2\pi k}{6}(3)} = 0$

$n=4 \rightarrow x(4)\, e^{\frac{-j2\pi k \cdot 4}{6}} = 1 \cdot e^{\frac{-j2\pi k}{3}}$

$n=5 \rightarrow x(5)\, e^{\frac{-j2\pi k}{6} \cdot 5} = 2 \cdot e^{\frac{-j10\pi k}{6}}$

$$= \frac{1}{6}\left[3 + 2e^{\frac{-j\pi 2k}{6}} + e^{\frac{-j2\pi k}{3}} + 0 + e^{\frac{-j4\pi k}{3}} + 2e^{\frac{-j10\pi k}{6}}\right.$$

For $k=0$

$$= \frac{1}{6}[3+2+1+0+1+2]$$

$$= \frac{1}{6}\{9\} = \frac{9}{6}$$

For $k=1$

$$= \frac{1}{6}\left[3 + 2e^{-j\pi \frac{2}{6}} + e^{-j\frac{2\pi}{3}} + 0 + e^{-j\frac{4\pi}{3}} + 2e^{\frac{-j10\pi}{6}}\right]$$

$$= \frac{1}{6}\left[3 + 2\left(\cos\left(\frac{\pi}{3}\right) - j\sin\left(\frac{\pi}{3}\right) + \cos\left(\frac{2\pi}{3}\right) - j\sin\left(\frac{2\pi}{3}\right) \ldots\right.\right.$$

$$= \frac{4}{6}$$

similarly

For $k=2$; $C_2 = 0$

$k=3$; $C_3 = \frac{1}{6}$

$k=4$; $C_4 = 0$

$K=5;\ C_5=\frac{4}{6}$

b) using the results in part (a) verify Parsevals by computing the power in the time and frequency domains.

sol

$$P_t = \frac{1}{6}\sum_{n=0}^{5}|x(n)|^2$$

$$= \frac{1}{6}\left[1^2+0^2+1^2+2^2+3^2+2^2\right]$$

$$= \frac{1}{6}\left[1+1+4+9+4\right] = \frac{19}{6}$$

$$P_F = \sum_{n=0}^{5}|c(n)|^2$$

$$= \left(\frac{9}{6}\right)^2+\left(\frac{4}{6}\right)^2+0^2+\left(\frac{1}{6}\right)^2+0^2+\left(\frac{4}{6}\right)^2$$

$$= \frac{114}{36} = \frac{19}{6}.$$

4.5 consider the signal

$$x(n) = 2+2\cos\left(\frac{\pi n}{4}\right)+\cos\left(\frac{\pi n}{2}\right)+\frac{1}{2}\cos\left(\frac{3\pi n}{4}\right)$$

a) Determine and sketch its PDS.

sol $x(n) = 2+2\cos\frac{\pi n}{4}+\cos\frac{\pi n}{2}+\frac{1}{2}\cos\left(\frac{3\pi n}{4}\right)$

$$= 2+2\left[\frac{e^{\frac{j\pi n}{4}}+e^{\frac{-j\pi n}{4}}}{2}\right]+\frac{e^{\frac{j\pi n}{2}}+e^{\frac{-j\pi n}{2}}}{2}+\frac{1}{2}\left[\frac{e^{\frac{j3\pi n}{4}}+e^{\frac{-j3\pi n}{4}}}{2}\right]$$

$$= 2+e^{\frac{j\pi n}{4}}+e^{\frac{-j\pi n}{4}}+\frac{1}{2}e^{\frac{j\pi n}{2}}+\frac{1}{2}e^{\frac{-j\pi n}{2}}+\frac{1}{4}e^{\frac{j3\pi n}{4}}+\frac{1}{4}e^{\frac{-j3\pi n}{4}}$$

$\therefore N=8$

$$C_K = \frac{1}{8}\sum_{n=0}^{1} x(n)\, e^{-j\pi k\frac{n}{4}}$$

$$x(n) = \left\{\frac{11}{2},\ 2+\frac{3}{4}\sqrt{2},\ 1,\ 2-\frac{3}{4}\sqrt{2},\ \frac{1}{2},\ 2-\frac{3}{4}\sqrt{2},\ 1,\ 2+\frac{3}{4}\sqrt{2}\right\}$$

$C_0=2,\quad C_1=C_7=1,\quad C_2=C_6=\frac{1}{2},\quad C_3=C_5=\frac{1}{4},\ C_4=0.$

b) Evaluate the Power of the Signal.

Sol

$$P(t) = \sum_{n=0}^{N-1} |c_k|^2$$

$$= [2^2 + 1^2 + 1^2 + (-\tfrac{1}{2})^2 + (-\tfrac{1}{2})^2 + (-\tfrac{1}{4})^2 + (-\tfrac{1}{4})^2]$$

$$= [4 + 2 + \tfrac{1}{2} + \tfrac{1}{8}]$$

$$= \frac{32+16+4+1}{8}$$

$$\boxed{P(t) = \frac{53}{8}}$$

4.6 Determine and sketch the magnitude and phase spectra of the following periodic signals.

(a) $x(n) = 4 \sin \frac{\pi(n-2)}{3}$

Sol

$$= 4\left[\frac{e^{\frac{j\pi(n-2)}{3}} - e^{\frac{-j\pi(n-2)}{3}}}{2j}\right]$$

$$= 4\left[e^{3j\pi(n-2)} - e^{-3j\pi(n-2)}\right]$$

$N = 6$

$$c_k = \frac{1}{6} \sum_{n=0}^{5} x(n)\, e^{\frac{-j2\pi kn}{6}}$$

$$= \frac{4}{6} \sum_{n=0}^{5} \left[\frac{e^{\frac{j\pi(n-2)}{3}} - e^{\frac{-j\pi(n-2)}{3}}}{2j}\right] e^{\frac{-j2\pi kn}{6}}$$

$$\cancel{= \frac{4}{6} \sum_{n=0}^{5} e^{\frac{j\pi}{3}}[(n-2) - kn] - e}$$

$$= \frac{1}{\sqrt{3}}\left[-e^{-j2\pi k/3} - e^{-j\pi k/3} + e^{-j\pi k/3} + e^{-j2\pi k/3}\right]$$

$$= \frac{1}{\sqrt{3}}(-j2)\left[\sin\frac{2\pi k}{6} + \sin\frac{\pi k}{3}\right] e^{-j2\pi k/3}$$

$c_0 = 0 \quad c_1 = -j2e^{-j2\pi/3}, \quad c_2 = c_3 = c_4 = 0, \quad c_5 = c_1$

$\angle c_1 = \frac{5\pi}{6}$, $\angle c_5 = \frac{-5\pi}{6}$, $\angle c_1 = \angle c_2 = \angle c_3 = \angle c_4 = 0°$

b) $x(n) = \cos\left(\frac{2\pi}{3}\right)n + \sin\frac{2\pi}{5}n$

Sol $N = 15$

$\cos\frac{2\pi}{3}n$

$\frac{1}{2}\left[e^{j\frac{2\pi}{3}n} + e^{-j\frac{2\pi}{3}n}\right]$

$e^{-j\frac{2\pi}{3}n} = e^{-\frac{2j\pi kn}{N}}$

$k = \frac{N}{3} = 5$

$15 - 5 = 10$

$$C_{1k} = \begin{cases} \frac{1}{2} & ; k = 5, 10 \\ 0 & ; \text{otherwise} \end{cases}$$

$\sin\frac{2\pi}{5}n$

$\frac{1}{2j}\left[e^{j\frac{2\pi}{5}n} - e^{-j\frac{2\pi}{5}n}\right]$

$e^{-j\frac{2\pi}{5}n} = e^{-\frac{2j\pi kn}{N}}$

$-\frac{j2\pi n}{5} = \frac{-2j\pi kn}{N}$

$k = \frac{N}{5} = \frac{15}{5} = 3$

$15 - 3 = 12$

$$C_{2k} = \begin{cases} \frac{1}{2j} & ; k = 3 \\ -\frac{1}{2j} & ; k = 12 \\ 0 & ; \text{otherwise} \end{cases}$$

$$C_k = C_{1k} + C_{2k} = \begin{cases} \frac{1}{2j} & ; k = 3 \\ \frac{1}{2} & ; k = 5 \\ \frac{1}{2} & ; k = 10 \\ -\frac{1}{2j} & ; k = 12 \\ 0 & ; \text{otherwise} \end{cases}$$

(c) $x(n) = \cos\frac{2n}{3}n \cdot \sin\frac{2\pi}{5}n$

Sol

$$\cos a \cos b = \frac{\sin(a+b)}{2} - \frac{\sin(a-b)}{2}$$

$$= \frac{1}{2}\left[\sin\frac{10\pi n - 6\pi n}{15} - \sin\frac{10\pi n - 6\pi n}{15}\right]$$

$$= \frac{1}{2}\sin\frac{16\pi n}{15} - \frac{1}{2}\sin\frac{4\pi n}{15}$$

$$= \frac{1}{2}\left[\frac{e^{j\frac{16\pi n}{15}} - e^{-j\frac{16\pi n}{15}}}{2j}\right] - \frac{1}{2}\left[\frac{e^{j\frac{4\pi n}{15}} - e^{-j\frac{4\pi n}{15}}}{2j}\right]$$

$$= \frac{1}{4j}\left[e^{\frac{j16\pi n}{15}} - e^{\frac{-j16\pi n}{15}}\right] - \frac{1}{4j}\left[e^{\frac{4\pi n}{15}j} - e^{-j\frac{4\pi n}{15}}\right]$$

$$8 - \frac{16\pi n j}{15} = \frac{-2j\pi k n}{N} \qquad \frac{-j4\pi n}{15} = \frac{-2\pi k n}{N}$$

$$\frac{8}{15} = \frac{k}{N} \qquad 4 = 2k$$

$$k = 2$$

$$k = 8 \rightarrow \frac{1}{4j} \qquad -\frac{1}{4j} \Rightarrow k = 2$$

$$15 - 8 = 7 \rightarrow -\frac{1}{4j} \qquad 15 - 2 \Rightarrow 13 = \frac{1}{4j}$$

$$C_K = \begin{cases} \frac{1}{4j} & ; 8, 13 \\ -\frac{1}{4j} & ; 7, 2 \\ 0 & ; \text{Otherwise} \end{cases}$$

(b) $x(n) = \{\ldots\ldots -2, -1, \underset{\uparrow}{0}, 1, 2, -2, -1, 0, 1, 2, \ldots\}$.

$N = 5$

sol

$$C_K = \frac{1}{5}\sum_{n=0}^{4} x(n)\, e^{\frac{-j2\pi k n}{N}}$$

$$= \frac{1}{5}\sum_{n=0}^{4} x(n)\, e^{\frac{-j2\pi k n}{5}}$$

$$= \frac{1}{5}\left[0 + e^{\frac{-j2\pi k}{5}} + 2e^{\frac{-j4\pi k}{5}} - 2e^{\frac{-j6\pi k}{5}} - e^{\frac{-j8\pi k}{5}}\right]$$

$$= \frac{2j}{5}\left[-\sin\left(\frac{2\pi k}{5}\right) - 2\sin\left(\frac{4\pi k}{5}\right)\right]$$

For putting k values

$k = 0$; $C_0 = 0$

$k = 1$; $C_1 = \frac{2j}{5}\left[-\sin\frac{2\pi}{5} - 2\sin\frac{4\pi}{5}\right]$

$k = 2$; $C_2 = \frac{2j}{5}\left[-\sin\frac{4\pi}{5} - 2\sin\frac{8\pi}{5}\right]$

$C_3 = -C_2$

$C_4 = -C_1$

e) $x(n) = \{ -1, 2, 1, 2, -1, 0, -1, 2, 1, 2 \cdots \}$

$\overset{\uparrow}{}$ $\xleftrightarrow{n=6}$

Sol $N=6$

$$C_k = \frac{1}{6} \sum_{n=0}^{5} x(n)\, e^{-\frac{j2\pi kn}{N}}$$

Sol by substituting from 0 to 5 we get

$$= \frac{1}{6}\left[1 + 2e^{-\frac{j\pi k}{3}} - e^{-\frac{j2\pi k}{3}} - e^{-\frac{j4k}{3}} + 2e^{-\frac{j5\pi k}{3}}\right]$$

$$= \frac{1}{6}\left[1 + 4\cos\frac{\pi k}{3} - 2\cos\frac{2\pi k}{3}\right]$$

$C_0 = \frac{1}{2}$; $C_1 = \frac{2}{3}$; $C_2 = 0$; $C_3 = -\frac{5}{6}$, $C_4 = 0$, $C_5 = \frac{2}{3}$.

g) $x(n) = 1, \quad -\infty < n < \infty$

Sol $N = 1$

$C_k = x(0) = 1$
$(\infty)$

$C_0 = 1$

h) $x(n) = (-1)^n, \quad -\infty < n < \infty$.

Sol $N = 2$

$$C_k = \frac{1}{2} \sum_{n=0}^{1} x(n)\, e^{-j\pi nk}$$

$$= \frac{1}{2}(1 - e^{-j\pi k})$$

$C_0 = 0$ ; $C_1 = 1$

4.7 Determine the Periodic signals $x(n)$ with fundamental if their Fourier coefficients are given by

a) $C_k = \cos\frac{nk\pi}{4} + \sin\left(\frac{3k5}{4}\right)$

Sol

$$x(n) = \sum_{n=0}^{7} C_k\, e^{\frac{j2\pi nk}{N}}$$

Let

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$$C_k = e^{\frac{j2\pi pk}{N}}$$

$$\therefore \sum_{n=0}^{7} e^{\frac{j2\pi pk}{N}} \cdot e^{\frac{j2\pi nk}{N}}$$

$$\sum_{n=0}^{7} e^{\frac{j2\pi(p+n)k}{N}}$$

It gives 8 ; when $p=-n$

0 ; when $p \neq n$.

$$\therefore C_k = \frac{1}{2}\left[e^{\frac{j2\pi k}{8}} + e^{\frac{-j2\pi k}{8}}\right] - \frac{1}{2j}\left[e^{\frac{j6\pi k}{8}} - e^{\frac{-j6\pi k}{8}}\right]$$

$$\therefore x(n) = 4\delta(n+1) + 4\delta(n-1) - 4j\delta(n-1) - 4j\delta(n+3) + 4j\delta(n-3) \quad ; -3 \le n \le 5$$

b) $C_k = \begin{cases} \sin\frac{k\pi}{3}, & 0 \le k \le 6 \\ 0, & k=7 \end{cases}$

$C_0 = 0$; $C_1 = \frac{\sqrt{3}}{2}$ ; $C_2 = \frac{\sqrt{3}}{2}$ ; $C_3 = 0$ ; $C_4 = -\frac{\sqrt{3}}{2}$ ; $C_5 = \frac{\sqrt{3}}{2}$ & $C_6 = C_7 = 0$

$$x(n) = \sum_{k=0}^{7} C_k e^{\frac{j2\pi nk}{8}}$$

$$= \frac{\sqrt{3}}{2}\left[e^{\frac{j\pi n}{4}} + e^{\frac{j\pi n2}{4}} - e^{\frac{-j4\pi n}{4}} - e^{\frac{j5\pi n}{4}}\right]$$

$$= \sqrt{3}\left[\frac{\sin\pi n}{2} + \sin\frac{\pi n}{4}\right] \cdot e^{\frac{j\pi(3n-2)}{4}}$$

c) $C_k = \{\cdots 0, \frac{1}{4}, \frac{1}{2}, 1, 2, 1, \frac{1}{2}, \frac{1}{4}, 0 \cdots\}$

$$x(n) = \sum_{k=-3}^{4} C_k e^{\frac{j2\pi nk}{8}}$$

$$= 2 + e^{\frac{j\pi n}{4}} + e^{\frac{-j\pi n}{4}} + \frac{1}{2}e^{\frac{j\pi n}{2}} + \frac{1}{2}e^{\frac{-j\pi n}{2}} + \frac{1}{4}e^{\frac{j3\pi n}{4}} + \frac{1}{4}e^{\frac{-j3\pi n}{4}}$$

$$= 2 + 2\cos\frac{\pi n}{4} + \cos\frac{\pi n}{2} + \frac{1}{2}\cos\frac{3\pi n}{4}.$$

* Determine the signals having the following Fourier transform.

a) $X(\omega) = \begin{cases} 0 & 0 \le |\omega| \le \omega_0 \\ 1 & \omega_0 < |\omega| \le \pi \end{cases}$

Sol

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{\omega_0} X(\omega) e^{j\omega n} + \int_{\omega_0}^{\pi} X(\omega) e^{j\omega n} d\omega.\right.$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{\omega_0} 1 \cdot e^{j\omega n}\, d\omega + \int_{\omega_0}^{\pi} e^{j\omega n}\, d\omega\right] \quad -(1)$$

$$= \frac{1}{2\pi}\left[\frac{e^{-j\omega_0 n} - e^{-j\pi n}}{jn} + \frac{e^{j\pi n} - e^{j\omega_0 n}}{jn}\right]$$

$$= \frac{1}{2\pi}\left[2 \cdot \frac{e^{-j\omega_0 n} - e^{j\omega_0 n}}{2jn} + 2 \cdot \frac{e^{j\pi n} - e^{-j\pi n}}{2jn}\right]$$

$$= \frac{1}{2\pi}\left[\frac{2}{n} - \sin\omega_0 n + 2 \cdot \frac{\sin\pi n}{2jn}\right]$$

$$= -\frac{\sin\omega_0 n}{n\pi} \quad ; n \neq 0.$$

For $n = 0$

From eqn (1)

$$= \frac{1}{2\pi}(\pi - \omega_0) + \frac{1}{2\pi}(\pi - \omega_0)$$

$$= \frac{(\pi - \omega_0) + (\pi - \omega_0)}{2\pi}.$$

$$= \frac{2(\pi - \omega_0)}{2\pi} = (\pi - \omega_0) \quad ; \text{ when } n = 0$$

b) $X(\omega) = \cos^2 \omega$

$$= \left(\frac{e^{j\omega} + e^{-j\omega}}{2}\right)^2$$

$$= \frac{1}{4}\left[(e^{j\omega})^2 + 2 \cdot e^{j\omega} \cdot e^{-j\omega} + (e^{-j\omega})^2\right]$$

$$= \frac{1}{4}\left[e^{j2\omega} + 2 + e^{-j2\omega}\right]$$

$$= \frac{1}{4} e^{j2\omega} + \frac{1}{2} + \frac{1}{4} e^{-j2\omega}$$

$\downarrow$ I.FT

$$= \frac{1}{4}\delta(n+2) + \delta(n)\frac{1}{2} + \frac{1}{4}\delta(n-2)$$

$$= \frac{1}{4}\left[\delta(n+2) + 2\delta(n) + \delta(n-2)\right]$$

$$X(\omega) = \begin{cases} 1 \ ; & \omega_0 - \frac{\delta\omega}{2} \le |\omega| \le \omega_0 + \frac{\omega\delta_0}{2} \\ 0 \ ; & \text{elsewhere} \end{cases}$$

Sol

$$\omega_0 - \frac{\delta\omega}{2} \le \omega \le \omega_0 + \frac{\delta\omega}{2}$$

$$-\omega_0 + \frac{\delta\omega}{2} < -\omega < \omega_0 + \frac{\delta\omega}{2}$$

$$-\omega_t + \frac{\delta\omega}{3} \le \omega \le -\omega_0 - \frac{\delta\omega}{2}$$

consider limits $\omega_0 - \frac{\delta\omega}{2} \le \omega \le \omega_0 + \frac{\delta\omega}{2}$

$$= \frac{1}{2\pi}\int_{\omega_0 - \frac{\delta\omega}{2}}^{\omega_0 + \frac{\delta\omega}{2}} 1 \cdot e^{j\omega n}\, d\omega$$

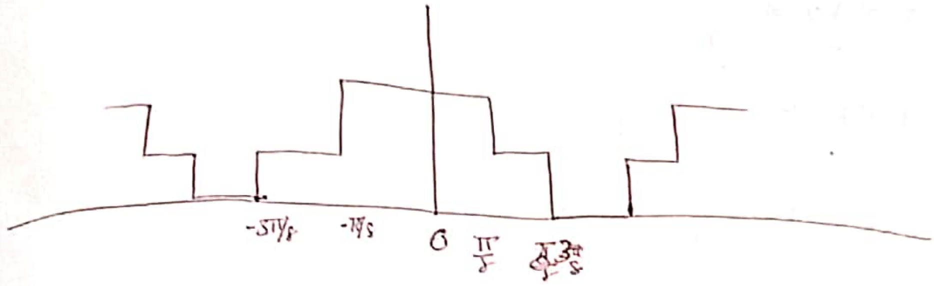
$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{\omega_0 - \frac{\delta\omega}{2}}^{\omega_0 + \frac{\delta\omega}{2}}$$

$$= \frac{2}{2\pi} \cdot \frac{e^{j\left(\omega_0 + \frac{\delta\omega}{2}\right)n} - e^{j\left(\omega_0 - \frac{\delta\omega}{2}\right)}}{2jn}$$

$$= \delta\omega \, \frac{2}{\pi}\left[\frac{\sin\left(\frac{\delta\omega}{2}\right)n}{\frac{n\delta\omega}{2}}\right] e^{jn\omega_0}$$

$$= \delta\omega \cdot \frac{2}{\pi} \cdot Sa\left(\frac{\delta\omega}{2}n\right) e^{jn\omega_0}$$

Q) The signal shown in fig




Sol

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$$

Let consider limits 0 to π

$$= 2\times\frac{1}{2\pi}\left[\int_{0}^{\pi/8} 2e^{j\omega n}\,d\omega + \int_{\pi/8}^{3\pi/8} e^{j\omega n}\,d\omega + \int_{6\pi/8}^{7\pi/8} e^{j\omega n}\,d\omega + \int_{7\pi/8}^{\pi} 2e^{j\omega n}\,d\omega\right]$$

$$= \frac{1}{\pi}\left[\int_{0}^{\pi/8} 2\cos\omega n\,d\omega + \int_{\pi/8}^{5\pi/8}\cos\omega n\,d\omega + \int_{6\pi/8}^{7\pi/8}\cos\omega n\,d\omega + \int_{7\pi/8}^{\pi} 2\cos\omega n\cdot d\omega.\right.$$

$$= \frac{1}{\pi}\left[\left[-2\sin\omega n\right]_{0}^{\pi/8} + (-\sin\omega n)_{\pi/8}^{3\pi/8} + \left(-\sin\omega n\right]_{-6\pi/8}^{-7\pi/8} + (-2\sin\omega n]_{7\pi/8}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\sin\frac{7\pi}{8}n - \frac{\sin\pi n}{8} + \sin\frac{6\pi}{8}n - \sin\frac{3\pi}{8}n\right]$$

* Consider the signal

$x(n) = \{1, 0, -1, 2, 3\}$ with Fourier transform $X(\omega) = X_R(\omega) +$

$j(X_I(\omega)$ signal $y(n)$ with Fourier transform.

$X(\omega) = X_I(\omega) + X_R(\omega)e^{j2\omega}.$

Sol

$$x_e(n) = \frac{x(n) + x(-n)}{2}$$

$$n=0: \quad x_e(n) = \frac{x(0)+x(-0)}{2} = \frac{2+2}{2} = 2$$

$$n=1 \quad ; \quad \frac{x(1)+x(-1)}{2} = \frac{3+(-1)}{2} = 1$$

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$n=-1$ ; $\frac{x(-1)+x(1)}{2} = 1$

$n=2$ ; $\frac{x(2)+x(-2)}{2} = 0$

$n=3$ ; $\frac{x(3)+x(3)}{2} = \frac{0+1}{2} = \frac{1}{2}$

$n=-2$ ; $\frac{x(-2)+x(2)}{2} = 0$

$n=-3$ ; $\frac{x(-3)+x(-3)}{2} = \frac{1+0}{2} = \frac{1}{2}$.

$x_e(n) = \{\frac{1}{2}, 0, 1, \underset{\uparrow}{2}, 1, 0, \frac{1}{2}\}$.

Similarly we get

$x_o(n) = \{\frac{1}{2}, 0, -2, 0, 2, 0, \frac{1}{2}\}$

From $x_e(n) = \frac{x(n) - x(-n)}{2}$

$$X_R(\omega) = \sum_{n=-3}^{3} x_e(n) e^{-jn\omega}$$

$$j X_I(\omega) = \sum_{n=-3}^{3} x_e(n) e^{-jn\omega}$$

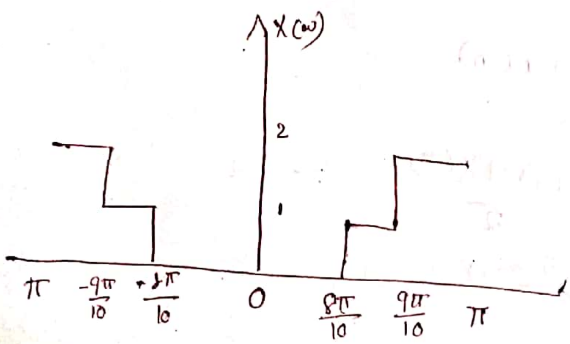
$$X(\omega) = X_I(\omega) + X_R(\omega) e^{j2\omega}$$

$= \frac{x_o(n)}{j} + x_o(n+2) \rightarrow$ from IFT of $X(\omega)$

$= -j\, x_o(n) + x_e(n+2)$

$= \{-\frac{1}{2}, 0, 1, \frac{-j}{2}, 2, 1+\frac{j}{2}, 0, \frac{1}{2} - j2, 0, \frac{j}{2}\}$

* Determine the signal $x(n)$ if its Fourier transform.



Sol (a)

$$x(n) = \frac{1}{2\pi}\left[\int_{-\pi}^{-\frac{9\pi}{10}} 2\cdot e^{jnw}\,dw + \int_{-9\pi/10}^{-8\pi/10} 1\cdot e^{jnw}\,dw + \int_{\frac{8\pi}{10}}^{9\pi/10} e^{jnw}\,dw + 2\int_{\frac{9\pi}{10}}^{\pi} e^{jnw}\,dw\right]$$

$$= \frac{1}{2\pi}\left[2\left(\frac{e^{jwn}}{jn}\right)_{\pi}^{-\frac{9\pi}{10}} + \left[\frac{e^{jwn}}{jn}\right]_{-\frac{9\pi}{10}}^{-\frac{8\pi}{10}} + \left[\frac{e^{jnw}}{jn}\right]_{\frac{8\pi}{10}}^{\frac{9\pi}{10}} + 2\left[\frac{e^{jnw}}{jn}\right]_{\frac{9\pi}{10}}^{\pi}\right]$$

$$= \frac{1}{2\pi j n}\left\{2\left[e^{-jn\frac{9\pi}{10}} - e^{-jn\pi}\right] + e^{jn\frac{8\pi}{10}} - e^{-jn\frac{9\pi}{10}} + e^{jn\frac{9\pi}{10}} - e^{jn\frac{8\pi}{10}}\right.$$

$$\left. + 2e^{jn\pi} - 2e^{jn\frac{9\pi}{10}}\right]$$

$$= \frac{1}{2\pi j n}\left[e^{-jn\frac{9\pi}{10}} - e^{+jn\frac{9\pi}{10}} - 2e^{-jn\pi} + 2e^{+jn\pi} + e^{\frac{-jn8\pi}{10}} - e^{\frac{jn8\pi}{10}}\right]$$

(b)

$$= \frac{1}{2n\pi}\left[\sin\frac{9\pi n}{10} + \sin\frac{4\cdot n\pi}{5}\right].$$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{0} x(w)\, e^{jwn}\,dw + \frac{1}{2\pi}\int_{0}^{\pi} x(w)\, e^{jwn}\,dw.$$

$$= \frac{1}{2\pi}\int_{\pi}^{0}\left(\frac{w}{\pi}+1\right) e^{jwn}\,dw + \frac{1}{2\pi}\int_{0}^{\pi}\frac{w}{\pi}\, e^{jwn}\,dw.$$

$$= \frac{1}{2\pi}\left[\left[\frac{w}{jn\pi}\cdot e^{jwn}\right]_{-\pi}^{\pi} + \left(\frac{e^{jwn}}{jn}\right)_{\pi}^{0}\right]$$

$$= \frac{1}{\pi n}\sin\frac{\pi n}{2}\, e^{\frac{-jn\pi}{2}}$$

13) given the F.T of the signal

$$x(n) = \begin{cases} 1 & ; -m \le n \le m \\ 0 & ; \text{otherwise} \end{cases}$$

was shown in

$$X(\omega) = 1 + 2\sum_{n=1}^{m} \cos \omega n$$

then show that the F.T of

$$x_1(n) = \begin{cases} 1 & ; 0 \le n \le m \\ 0 & ; \text{otherwise} \end{cases} \quad \text{is } X_1(\omega) = \frac{1-e^{j\omega(m+1)}}{1-e^{-j\omega}}$$

is

$$x_2(n) = \begin{cases} 1 & ; -m \le n \le -1 \\ 0 & ; \text{otherwise} \end{cases} \quad \text{is } X_2(\omega) = \frac{e^{j\omega} - e^{j\omega(n+1)}}{1-e^{j\omega}}.$$

Sol

$$X_1(\omega) = \sum_{n=0}^{m} 1 \cdot e^{-j\omega n}$$

$$= 1 + e^{-j\omega} + e^{-j\omega 2} + e^{-j\omega 3} + \dots - \left[ \frac{e^{-j\omega(m+1)} + e^{-j\omega(m+2)} \dots}{\frac{e^{-j\omega(m+1)}}{1-e^{-j\omega}}} \right.$$

$$= \frac{1}{1-e^{j\omega}} - \frac{e^{-j\omega(m+1)}}{1-e^{-j\omega}}$$

$$= \frac{1-e^{-j\omega(m+1)}}{1-e^{-j\omega}}$$

$$X_2(\omega) = \sum_{n=-m}^{-1} e^{-j\omega n}$$

$$= \sum_{n=1}^{m} e^{j\omega n}$$

$$= \frac{1-e^{j\omega n}}{1-e^{j\omega}} e^{j\omega}$$

$$X(\omega) = X_1(\omega) + X_2(\omega)$$

$$= \frac{1-e^{j\omega(m+1)}}{1-e^{-j\omega}} + \frac{1-e^{j\omega m}}{1-e^{j\omega}} e^{j\omega}$$

$$= \frac{1+e^{j\omega} - e^{j\omega} - 1 - e^{-j\omega(m+1)} - e^{j\omega(m+1)} + e^{j\omega m} + e^{-j\omega m}}{2 - e^{-j\omega} - e^{j\omega}}$$

$$= \frac{2\cos\omega m - 2\cos\omega m(m+1)}{2 - 2\cos\omega}$$

$$= \frac{2\sin\left(\omega m + \frac{\omega}{2}\right) \cdot \cos\frac{\omega}{2}}{2\sin^2\left(\frac{\omega}{2}\right)}$$

$$= \frac{\sin\left(m+\frac{1}{2}\right)\omega}{\sin\left(\frac{\omega}{2}\right)}$$

proved that $1+2\sum_{n=1}^{m}\cos\omega n = \frac{\sin(n)+\frac{1}{2})\omega}{\sin(\omega/2)}$

14) Consider the signa $x(n) = \{-1, 2, -3, 2, -1\}$ consider the following quantities with Fourier transform. $X(\omega)$ : compute with explicitly computing $X(\omega)$:-

a) $X(0)$

Sol: $X(\omega) = \sum_{\omega=0} x(n) e^{-j\omega n}$

$x(0) = -3 \cdot e^{0} = -3$

b) $\angle X(\omega) = \pi$ for all $\omega$

c) $\int_{-\pi}^{\pi} X(\omega)\, d\omega$

$x(0) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, d\omega.$

$\int_{-\pi}^{\pi} X(\omega)\, d\omega = 2\pi\, x(0)$

$= 2\pi(-3) = -6\pi$

d) $x(\pi) = \sum_{n=-\infty}^{\infty} x(n) e^{-jn\omega}$

$= \sum_{n} e^{-jn\pi x} \cdot x(n)$

$= \sum_{n} \left[\cos(n\pi) - j\sin(n\pi)\right] x(n)$

$= \sum_n (-1)^n x(n)$

For $n=0$ : $(-1)^0 x(n) = 1 \cdot -3 = -3$

$n=1 : (-1)^1 x(1) \Rightarrow -1 \cdot 2 = 2$

$n=2 = (-1)^2 x(2) \Rightarrow -1 = -1$

$n=-1 = (-1)^{-1} x(1) \Rightarrow -2$

$n=-2 \ (-1)^2 x(2) \Rightarrow -1$

$\Rightarrow -3 \ -2 -1 \ -2 -1$

$= -9$

e) $\int_{-\pi}^{\pi} |X(\omega)|^2 \, d\omega$.

Sol $\frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\omega)|^2 = \sum_n |x(n)|^2$

$= (-1)^2 + (2)^2 + (-3)^2 + (2)^2 + (-1)^2$

$= 1+4+9+4+1 = 19.$

$\int_{-\pi}^{\pi} X|\omega|^2 = 2\pi \cdot 19 = 38\pi$

15) The center of gravity of a signal $x(n)$ is defined as

$$G = \frac{\sum_{n=-\infty}^{\infty} n x(n)}{\sum_{n=-\infty}^{\infty} x(n)}$$ and provides a measure

of the "time delay" of the signal.

(a) Express $C$ in terms of $X(\omega)$

we know $X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$

$X(0) = \sum_{n=-\infty}^{\infty} x(n) e^0$

$X(0) = \sum_{n=-\infty}^{\infty} x(n)$

we know from differentiation in 'ω' domain multiple 'n' with $x(n)$

$$n x(n) \xrightarrow{F.T} j \frac{dX(\omega)}{d\omega}$$

$$-j n x(n) \xleftrightarrow{F.T} \frac{dX(\omega)}{d\omega}$$

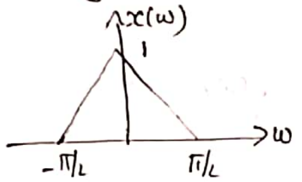
$$\frac{dX(\omega)}{d\omega} = \sum_{n=-\infty}^{\infty} -j n x(n) e^{-jn\omega} d\omega$$

$$= -j \sum_{n=-\infty}^{\infty} n x(n) e^{-jn\omega} d\omega$$

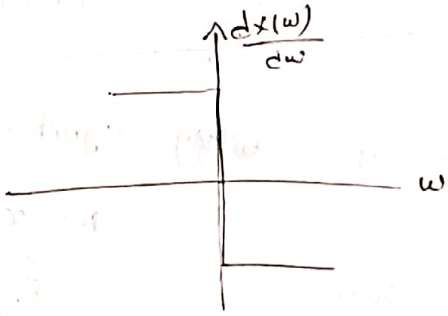
$$j \frac{dX(\omega)}{d\omega} = \sum_{n=-\infty}^{\infty} n x(n) e^{-jn\omega} d\omega$$

$$c = \frac{j \left. dX(\omega) \right|_{\omega=0}}{X(0)}$$

b) Compute c. for the signal $x(n)$ whose Fourier transform is shown




From given figure $X(0) = 1$




$$C = \frac{j dX(\omega)}{X(0)}$$

$$= \frac{0}{1}$$

$$= 0$$

16) Consider the F.T Pair

$$a^n u(n) \xleftrightarrow{FT} \frac{1}{1 - a e^{-j\omega}} \cdot |a| < 1$$

Use the differentiation in frequency theorem and induction to

$$x(n) = \frac{(n+l-1)!}{n!\,(l-1)!} a^n u(n) \xleftrightarrow{F} X(\omega) = \frac{1}{(1 - a e^{-j\omega})^l}$$

Sol

Let $l = k+1$

$$x(n) = \frac{(n+k+1-1)!}{n!\,(k+1-1)!} a^n u(n)$$

$$= \frac{(n+k)!}{n!\,k!} a^n u(n)$$

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$$= \frac{(n+k)\ (n+k-1)\,!}{n!\ (k-1)!} a^n u(n)$$

Let $$x_k(n) = \frac{(n+kr-1)}{n!(k-1)!} a^n u(n)$$

$$x_{k+1} = \sum_{n=-\infty}^{\infty} \frac{n+k}{k} x_k(n) e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left[\frac{n}{k} x_k(n) + x_k(n)\right] e^{-j\omega n}$$

$$= \frac{1}{k} \sum_{k=\infty}^{\infty} n x_k(n) e^{-j\omega n} + \sum_{n=-\infty}^{\infty} x_k(n) e^{-j\omega n}$$

$$= \frac{1}{k} \sum_{n=-\infty}^{\infty} n x_k(n) e^{-j\omega n} + X_k(\omega)$$

$$= \frac{1}{k} j \frac{dX_k(\omega)}{d\omega} + X_k(\omega)$$

$$\boxed{x_{k+1} = \frac{ae^{-j\omega}}{(1-ae^{j\omega})^{k+1}} + \frac{1}{(1-ae^{-j\omega})^k}}$$

) Let $x(n)$ be a a obitary signal,

(a) $x^*(n)$

$$\sum_{n=-\infty}^{\infty} x^*(n) e^{-j\omega n}$$

$$\sum_{n=-\infty}^{\infty} [x(n) e^{-j\omega n})^*$$

$X(-\omega)^*$

(b) $x^*(-n)$

$$\sum_{n=-\infty}^{\infty} x^*(-n) e^{-j\omega n}$$

Replace '$-n$' with '$n$'

$$\sum_{n=-\infty}^{\infty} x^*(n) e^{j\omega n}$$

$$\sum_{n=-\infty}^{\infty} (x(n) e^{-j\omega n})^*$$

$X(\omega)$

c) $y(n) = x(n) - x(n-1)$

$$= \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} - \sum_{n=\infty} x(n-1) e^{-j\omega n}$$

$$= X(\omega) - \sum_{n=-\infty}^{\infty} x(n-1) e^{-j\omega n}$$

Let $l = n-1$ [dummy variable]

$$= X(\omega) - \sum_{l=-\infty}^{\infty} x(l)\, e^{-j\omega(l+1)}$$

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$= X(\omega) - \sum_{l=-\infty}^{\infty} x(l)\ e^{-j\omega l} \cdot e^{-j\omega}$

Replac. $l$ by $n$

$= X(\omega) - e^{-j\omega} \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$

$= X(\omega) - e^{-j\omega}\ X(\omega)$

$= X(\omega) \left[1 - e^{-j\omega}\right]$

e) $y(n) = x(2n)$

$X(\omega) = \sum_{n=-\infty}^{\infty} x(2n)\, e^{-jn\omega}$

Let $l = 2n$

$= \sum_{K=-\infty}^{\infty} x(l)\, e^{-j\frac{n}{2}\cdot\omega}$

$= \sum_{l=-\infty}^{\infty} x(l)\, e^{-jl\frac{\omega}{2}}$

$= X\left(\frac{\omega}{2}\right)$

(f) $y(n) = \begin{cases} x(\frac{n}{2}) & ;\ 'n'\ \text{even} \\ 0 & ;\ 'n'\ \text{odd} \end{cases}$

$Y(\omega) = \sum_n x(\tfrac{n}{2})\, e^{jn\omega}$

Let $n = 2l$

$= \sum_n x(\frac{2l}{2})\ e^{j2l\omega}$

$= \sum_n x(l)\ e^{jl2\omega}$

$= X(2\omega)$

(8) Determine and sketch the F.T $x_1(\omega)$, $x_2(\omega)$ and $x_3(\omega)$ the For signals

(a) $x_1(n) = \{1, 1, \underset{\uparrow}{1}, 1, 1\}$, from $-2$ to $2$, arrow at $0$

$\sum_{n=-\infty}^{\infty} x_1(n)\ e^{-j\omega n}$

$\sum_{n=-2}^{2} x_1(n)\, e^{-j\omega n}$

For $n = -2$; $1 \cdot e^{j2\omega} = e^{j2\omega}$

$n = -1$; $1 \cdot e^{j\omega} = e^{j\omega}$

$n = 0$; $1 \cdot e^{0} = 1$

$n = 1$; $1 \cdot e^{-j\omega} = \frac{1}{e^{j\omega}}$

$n = 2$; $1 \cdot e^{-2j\omega} = \frac{1}{e^{2j\omega}}$

$= \cancel{2\cos(2\omega)}\ 1$

$= e^{j2\omega} + e^{j\omega} + 1\ e^{-j\omega} + e^{-j2\omega}$

$= 2\cos(2\omega) + 2\cos(\omega) + 1$

(b) $x_2(n) = \{1, 0, 1, 0, 1, 0, 1, 0, 1\}$
$\quad -4 \qquad \uparrow 1\ 2\ 3\ 4$

For $n = -2$; $\quad 1 \cdot x e^{j2\omega} = e^{j2\omega}$

$n = -4$; $\quad 1 \cdot e^{j4\omega} = e^{j4\omega}$

$n = 0$; $\quad 1 \times e^{0} = 1$

$n = 2$; $\quad 1 \times e^{-j\omega 2} = e^{-j2\omega}$

$n = 4$; $\quad 1 \times e^{-j\omega 4} = e^{-j4\omega}.$

$= e^{j2\omega} + e^{j4\omega} + 1 + e^{-j2\omega} + e^{-j4\omega}.$

$= 2\cos(2\omega) + 2\cos(4\omega) + 1$

c) $x_3(n) = \{1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1\}$
$\quad -6 \qquad \uparrow \qquad 6$

For $n = -6$; $\quad 1 \cdot e^{j6\omega n} = e^{j6n\omega}$

$n = -3$; $\quad 1 \cdot e^{j3n} = e^{j3n\omega}$

$n = 0$; $\quad 1 \cdot e^{j0} = e \cdot 1$

$n = 3$; $\quad 1 \times e^{-j3n} = e^{-j3\omega}$

$n = 6$; $\quad 1 \times e^{-j6\omega} = e^{-j6\omega}.$

$= e^{j6\omega} + e^{j3\omega} + 1 + e^{-j3\omega} + e^{-j6\omega}.$

$= 2\cos(6\omega) + 2\cos(3\omega) + 1$

d) is there relation.

Sol

$X_1(\omega) = 2\cos(2\omega) + 2\cos(\omega) + 1$

$X_2(\omega) = 2\cos(2\omega) + 2\cos(4\omega) + 1$

$X_2(\omega) = X_1(2\omega)$

$X_3(\omega) = 2\cos(6\omega) + 2\cos(8\omega) + 1$

$X_3(\omega) = X_1(3\omega).$

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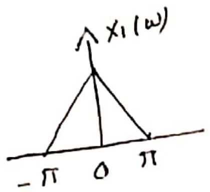
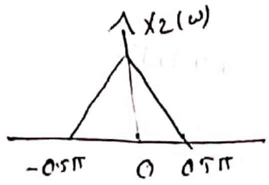
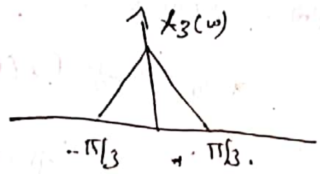
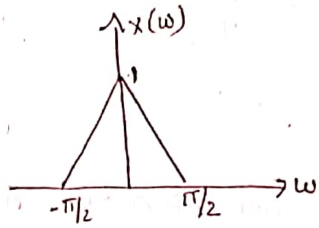



19 Let $x(n)$



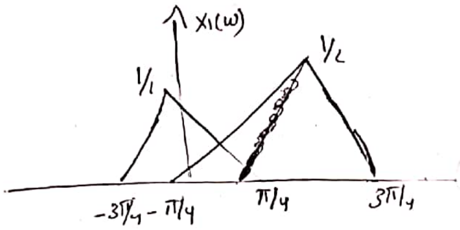

a) $x_1(n) = x(n) \cdot \cos\left(\frac{\pi n}{4}\right)$

sol

we know

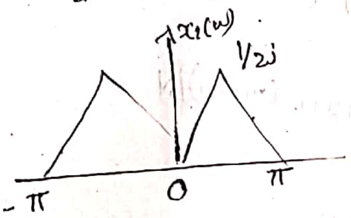
$$x(n)\cos(\omega_0 n) \xleftrightarrow{FT} \frac{1}{2}\left[X(\omega-\omega_0) + X(\omega+\omega_0)\right]$$

$$X_1(\omega) = \frac{1}{2}\left[X\left(\omega+\frac{\pi}{4}\right) + X\left(\omega-\frac{\pi}{4}\right)\right]$$



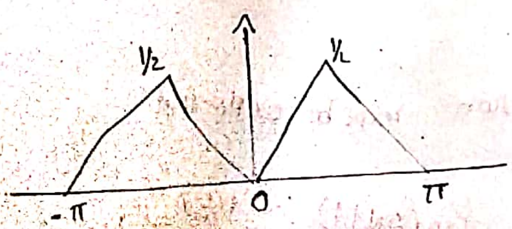

(b) $x_2(n) = x(n)\sin(\pi n/2)$

$$X_2(\omega) = \frac{1}{2j}\left[X\left(\omega+\frac{\pi}{2}\right) + X\left(\omega-\pi/2\right)\right]$$



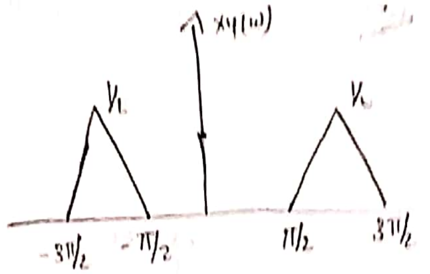

c) $x_3(n) = x(n)\cos\left(\frac{-n}{2}\pi\right)$

$$X_3(\omega) = \frac{1}{2}\left[X(\omega-\pi/2) + X(\omega+\pi/2)\right]$$



d) $x_4(n) = x(n)\cos(\pi n)$

$$X_4(\omega) = \frac{1}{2}\left[X(\omega-\pi) + X(\omega+\pi)\right]$$




20) Consider an aperiodic ...

$$y(n) = \sum_{l=-\infty}^{\infty} x(n-lN)$$

Sol

Given

$$y(n) = \sum_{l=-\infty}^{\infty} x(n-lN)$$

are given by

$$C_k^y = \frac{1}{N} X\left(\frac{2\pi}{N}k\right)$$

$$C_k^y = \frac{1}{N}\sum_{m=0}^{N-1} y(n) e^{-j2\pi n/N}$$

$$= \frac{1}{N}\sum_{n=0}^{N-1}\left[\sum_{l=-\infty}^{\infty} x(n-lN)\right] e^{-j2\pi kn/N}$$

$$= \frac{1}{N}\sum_{l=-\infty}^{\infty}\sum_{m=n}^{N-l-ln} x(m) e^{-j2\pi k(m+ln)/N}$$

But $\displaystyle\sum_{l=-\infty}^{\infty}\sum_{m=-ln}^{N-l-ln} x(m)\, e^{-j\omega(\omega+N)} = X(\omega)$

$$C_k^y = \frac{1}{N} X\left(\frac{2\pi k}{N}\right)$$

21) Prove that

$$X_N(\omega) = \sum_{n=-N}^{N} \frac{\sin\omega_c n}{\pi n} e^{-j\omega n} \text{ may be expressed.}$$

$$X_N(\omega) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} \frac{\sin(2N+1)(\omega-\theta/2)]}{\sin(\omega-\theta/2]} d\theta$$

Sol let $x_N(n) = \frac{\sin w_c n}{\pi n}$ ; $-N \leq n \leq N$

$= x(n)\, w(n)$

where $x(n) = \frac{\sin w_c n}{\pi n}$ $-\infty < n \leq \infty$

$w(n) = 1$ ; $-N < n \leq N$

$0$ ; otherwise

~~$X_N(w)$~~ $\frac{\sin w_c n}{\pi n} \xleftrightarrow{F} X(w)$

$= 1$ ; $|w| < w_c$

$0$ ; otherwise

$X_N(w) = X(w) * W(w)$

$= \int_{-\pi}^{\pi} X(\theta)\, W(w-\theta)\, d\theta$

$= \int_{-w_c}^{w_c} \frac{\sin(2N+1)(w-\theta)/2}{\sin(w-\theta)/2}\, d\theta$

22 A signal $x(n)$ has the following FT

$X(w) = \frac{1}{1-ae^{-jw}}$ then.

(a) $x(2n+1)$

Sol $\sum_{n=-\infty}^{\infty} x(2n+1) e^{-jwn}$

let $2n+1 = l$

$\sum_{l=-\infty}^{\infty} x(l)\, e^{-jw\left(\frac{l-1}{2}\right)}$

$\sum_{l=-\infty}^{\infty} x(l)\, e^{\frac{-jwl}{2}} \cdot e^{\frac{jw}{2}}$

$e^{\frac{jw}{2}} \sum_{l=-\infty}^{\infty} x(l)\, e^{\frac{-jwl}{2}}$

$e^{\frac{jw}{2}} \sum_{l=-\infty}^{\infty} x(n)\, e^{-\frac{jw}{2} n}$ Replace $l = n$.

$e^{\frac{jw}{2}} X\left(\frac{w}{2}\right) \Rightarrow e^{\frac{jw}{2}} \cdot \frac{1}{1-ae^{-j\frac{w}{2}}} \Rightarrow \frac{e^{\frac{jw}{2}}}{1-ae^{-j\left(\frac{w}{2}\right)}}$

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b) $e^{\frac{\pi j}{2}}, x(n+2)$

$e^{j2w} \cdot x(w - \frac{\pi j}{2})$

$x(n) \longleftrightarrow x(w)$

$x(n+2) \longleftrightarrow e^{j2w} x(w)$

$e^{\frac{\pi j}{2}} x(n+2) \longleftrightarrow e^{j2w} x(w - \frac{\pi j}{2})$

$\therefore e^{j2w} \cdot x(w - \frac{\pi j}{2})$.

c) $x(-2n)$

Sol

$x(n) \longleftrightarrow x(w)$

$x(2n) \longleftrightarrow x(\frac{w}{2})$

$x(-2n) \longleftrightarrow x(-\frac{w}{2})$

(d) $x(n) \cos(0.3\pi n)$

$x(n) \cos(w_0 n) \longleftrightarrow \frac{1}{2}[x(w+w_0) + x(w-w_0)]$

$x(n) \cos(0.3 n) \longleftrightarrow \frac{1}{2}[x(w+0.3\pi) + x(w-0.3\pi)]$

e) $x(n) * x(n-1)$

$x(w) \cdot e^{-jw} x(w)$

$x^2(w) e^{-jw}$

(f) $x(n) \times x(-n)$

$x(w) \; x^*(w)$

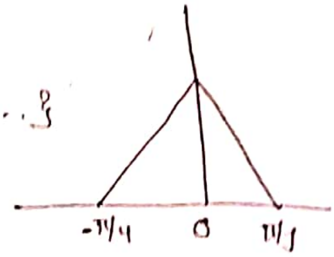
$\frac{1}{1-ae^{-jw}} \cdot \frac{1}{1-ae^{jw}}$

$$\frac{1}{1-ae^{jw}-ae^{-jw}+a^2e^0} = \frac{1}{1+a^2-2a\cos w}$$

Q3. From a discrete time signal $x(n)$ with F.T

Note that $y_1(n) = x(n)\,s(n)$

where $s(n) = \{\ldots\ 0,1,0,1,0,1,0,\ldots\}$




(a) $y_1(n) = \begin{cases} x(n) & , \text{'n' even} \\ 0 & , \text{'n' odd} \end{cases}$

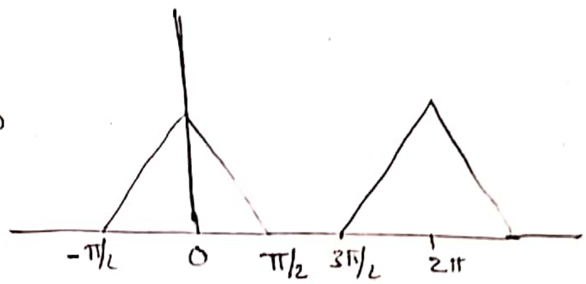
b) $y_2(n) = x(2n)$

$y_2(n) = x(2n)$

$Y_2(\omega) = \sum_n y_2(n) e^{-j\omega n}$

$= \sum_n x(2n)\, e^{-j\omega n}$

$= X\left(\frac{\omega}{2}\right)$



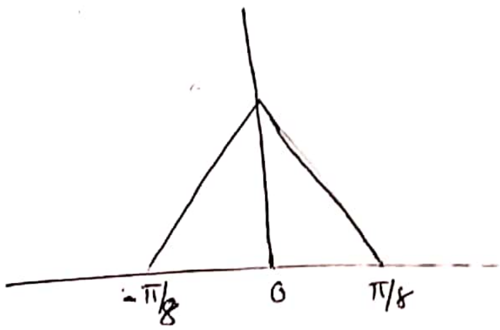

c) $y_3(n) = \begin{cases} x(n/2) & ; \text{'n' even} \\ 0 & ; \text{'n' odd} \end{cases}$

$= \sum_n y_3(n)\, e^{-j\omega n}$

$= \sum_{n=\text{even}} x\left(\frac{n}{2}\right) e^{-j\omega n}$

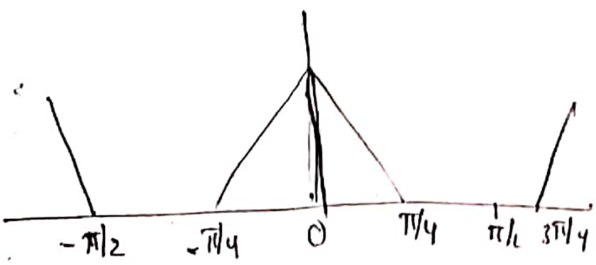
$= \sum_m x(m)\, e^{-j2\omega m}$

$= X(2\omega)$




(d) $y_1(n) = \begin{cases} y_2\left(\frac{n}{2}\right) & ; \text{'n' even} \\ 0 & ; \text{'n' odd} \end{cases}$

$X_1(\omega) = X_2(2\omega)$

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