Question-1(a)

Objective function: Maximisation of the total unit profit

Let
X_1 = number of Product-1
X_2 = number of Product-2
X_3 = number of Product-3
X_4 = number of Product-4
X_5 = number of Product-5

MAX : $380X_1 + 520X_2 + 390X_3 + 580X_4 + 455X_5$

Such that
Resource-1 : $1X_1 + 1X_2 + 1X_3 + 0X_4 + 0X_5 \leq 200$
Resource-2 : $0X_1 + 6X_2 + 3X_3 + 4X_4 + 0X_5 \leq 1566$
Resource-3 : $0X_1 + 0X_2 + 6X_3 + 5X_4 + 9X_5 \leq 2880$
Resource-4 : $1X_1 + 7X_2 + 0X_3 + 0X_4 + 3X_5 \leq 336$
Resource-5 : $11X_1 + 0X_2 + 6X_3 + 0X_4 + 8X_5 \leq 1256$
Resource-6 : $11X_1 + 10X_2 + 0X_3 + 6X_4 + 0X_5 \leq 2700$
Non-negativity : $X_1, X_2, X_3, X_4, X_5 \geq 0$

Question-1(b)

Create a spreadsheet model for this problem. Store the model in your Excel workbook [FamilyName-YourStudentId-2ndSem2022FIT5097.xlsx] and name your spreadsheet something like (e.g.) 'ProductsAndResources'

Refer to sheet "Products and Resources" for the working

Question-1(c)

Solve the problem - using Microsoft Excel Solver. Generate the Sensitivity report for the problem and name your spreadsheet (e.g.) 'Qu 1 ProductsAndResources Sensitivity Rep'.

Refer to the sheet "Qu-1 ProductsAndResources SensR" for the sensitivity analysis report.

<u>Question-1(d)</u>

What is the optimal production plan (X1, X2, X3, X4, X5) and the associated profit? Refer to your answers to any of a), b) and/or c) above as appropriate

Resources	**Optimal Solution obtained**
Resource-1	46
Resource-2	1566
Resource-3	2880
Resource-4	336
Resource-5	1256
Resource-6	2700

The associated profit is **285800**

<u>Question-1(e)</u>

Following on from the end of part 1d), it is proposed to make a new product which uses exactly 1 (i.e., exactly one) of each resource. This new product would sell for $125. Should we make this? If we should make it, by how much could we lower the price and yet it would still be worthwhile to make it? If we should not make it, how much more profitable would it need to be for us to make it? Show all working and explain

Refer to the sheet Qu-1(e) new product for the working.

<u>Explanation:</u>

Using Solver and on reducing the price manually we arrive at the following results:

Unit price of new product	Corresponding Total Profit
$125	$286035
$124	$285995
$123	$285955
$122	$285915
$121	$285875
$120	**$285835**
$119	$285795

Yes, we should make the product. The total unit profit is increased by $235 if the new product (Product-6) for $125 is made.

On reducing the unit price from $125 by one dollar each time, the total unit profit also decreases by $40. The lowest unit price can be $120 and not less than this, as its corresponding total profit would be $285835 which is close to the original profit $285800 (without the new product). This means that we could lower the price to $120 for it to be worthwhile to make it and not lesser than this.

Before adding the new product, the total unit profit was **$285800** and **on adding the new product** (Product-6) whose price is $125, the new total unit profit is **$286035** which gives a difference of $235.

If we should not make it, we would need the total profit to be $235 more than the original total unit profit for us to make it

<u>Question-1(f)</u>

How much would profit change by, if we were to have an extra 3 of Resource1? Show all working and explain.

Refer to the sheet Qu-1(f) and Qu-1(f) Sensitivity Report for working

<u>Explanation:</u>

On changing the availability to 203 (200 + 3 because of the extra 3) in Resource – 1, there is no change in the total unit profit. The unit profit still remains the same i.e. $285800 because only 46 is used out of 200 when the extra 3 is not added. On adding the extra 3 for Resource-1, there is no change because the shadow price is 0 in the sensitivity analysis report. This means that the resource is in excess supply and thus, there is no change in the profit amount.

<u>Question-1(g)</u>

It is proposed to make the following changes. The profitability of Product1 increases by 2, the profitability of Product2 decreases by 16, the profitability of Product4 decreases by 16, and the profitability of Product5 increases by 6. (4 marks) Can you say whether or not the optimal production plan (X1, X2, X3, X4, X5) will change from the value in part 1d)? Show your working and explain your answer.

Refer to sheet Qu-1(g) for the working

<u>Explanation:</u>

The profit has reduced from **$285800** to **$280328** because of the changes in the profitability in Product – 1, 2, 4 and 5 respectively. There is no change in the optimal production in comparison with the part-1d because there is no change in the resource constraints or the availability constraint. Hence, there is no change in the optimal production because of this.

Question-1(h)

Continuing from the end of part d), suppose that at most 4 products can be made (i.e., that at most 4 of the Xi have non-zero values). What is the optimal production plan (X1, Page 5 of 9 X2, X3, X4, X5) and the associated profit? Show your working and explain your answer.

Refer to sheet Qu-1(h) in the excel file.

Explanation:

Objective function: Maximisation of the total unit profit

Let
X_1 = number of Product-1
X_2 = number of Product-2
X_3 = number of Product-3
X_4 = number of Product-4
X_5 = number of Product-5

MAX : $380X_1 + 520X_2 + 390X_3 + 580X_4 + 455X_5$

Such that
Resource-1 : $1X_1 + 1X_2 + 1X_3 + 0X_4 + 0X_5 \leq 200$
Resource-2 : $0X_1 + 6X_2 + 3X_3 + 4X_4 + 0X_5 \leq 1566$
Resource-3 : $0X_1 + 0X_2 + 6X_3 + 5X_4 + 9X_5 \leq 2880$
Resource-4 : $1X_1 + 7X_2 + 0X_3 + 0X_4 + 3X_5 \leq 336$
Resource-5 : $11X_1 + 0X_2 + 6X_3 + 0X_4 + 8X_5 \leq 1256$
Resource-6 : $11X_1 + 10X_2 + 0X_3 + 6X_4 + 0X_5 \leq 2700$
Non-negativity : $X_1, X_2, X_3, X_4, X_5 \geq 0$

The optimal production plan is to find the profit with the usage of **Binary and side constraints.** Firstly, we make sure that the Xi values is set to binary using the solver. Further, we must focus on the fact that at most 4 products can be made which means that at most 4 should be non-zero values. To do that, we use the linking and binary constraints, sum the binary values and give a condition that the sum should be less than or equal to 4. This would result in us getting 4 non-zero values in Xi and one value with zero and the corresponding profit as **$285800**

<u>Question – 1(i)</u>

Continuing from the end of part d), suppose that at most 3 products can be made (i.e., that at most 3 of the Xi have non-zero values). What is the optimal production plan (X1, X2, X3, X4, X5) and the associated profit? Show your working and explain your answer.

(Refer to the sheet Qu-1(i) for working)

<u>Explanation:</u>

The optimal production plan is to find the profit with the usage of **Binary and side constraints.**

A solver condition is passed to keep the Xi values as integers. The second thing to focus is that at most 3 products can be made which means that at most 3 should be non-zero values. To achieve that, we are using the linking and binary constraints, and then sum the binary values. A solver parameter is given that the sum should be less than or equal to 3. This would result in us getting 3 non-zero values in Xi and two values with zero. The corresponding profit is **$284895**.

<u>Question – 1(j)</u>

Continuing from the end of part d), suppose that at most 2 products can be made (i.e., that at most 2 of the Xi have non-zero values). What is the optimal production plan (X1, X2, X3, X4, X5) and the associated profit? Show your working and explain your answer.

Refer to sheet Qu-1(j) for the working

Explanation:

Just like the previous two questions, a solver condition is passed to keep the Xi values as integers. The optimal production plan is to find the profit with the usage of **Binary and side constraints.** The second thing to focus is that at most 2 products can be made which means that at most 2 should be non-zero values. To achieve that, we are using the linking and binary constraints, and then sum the binary values. A solver parameter is given that the sum should be less than or equal to 2. This would result in us getting 2 non-zero values in Xi and three values with zero. The corresponding profit is **$273190**.

<u>Question-1(k):</u>

Continuing from the end of part d), suppose that at most 1 product can be made (i.e., that at most 1 of the Xi has non-zero values). What is the optimal

production plan (X1, X2, X3, X4, X5) and the associated profit? Show your working and explain your answer.

Refer to sheet “Qu-1(k)” for the working
Explanation:
Just like the previous three questions, a solver condition is passed to keep the Xi values as integers. The optimal production plan is to find the profit with the usage of **Binary and side constraints.** The second thing to focus is that at most 1 product can be made which means that at most 1 should be non-zero value. To achieve that, we are using the linking and binary constraints, and then sum the binary values. A solver parameter is given that the sum should be less than or equal to 1. This would result in arriving at 1 non-zero values in Xi and 4 values with zero. The corresponding profit is **$226780**.

<u>Question-1(l):</u>
If Product4 is made then Product2 must be made.

Refer to sheet Qu-1(k) for the working
Explanation:
The first thing that we must make sure is that the Xi values are all integers. Here, they have said that if product-4 is made, then product-2 should be made. So, product 4 has a condition, but product 2 can be made irrespective of the fact if product 4 is made or not. The second thing is that, on making product 4, product 2 is also made, which makes the minimum availability as 2 (Product 2 and Product 4). So, to bring this into working, binary and linking constraints are used. The main solver condition to make sure “if product 4 is made, then product 2 is made” is by making sure that the binary value of the Product 4 is less than or equal to the binary value of Product 2 and the binary values sum is less than or equal to 2 because there are 2 products created on creating one (Product 4 and Product 2).
Thus,
Binary value (Product 4) <= Binary value (Product 2)
Binary value sum <= 2
Using these constraints and solver conditions, we can arrive to a profit of **$226080**

Question-1(m)

Return to the end of part 1d), but with the requirement that all the Xi must take integer values. We now introduce set-up costs (or start-up costs).

	Product 1	Product 2	Product 3	Product 4	Product 5
Start-up cost	**$10000**	**$12000**	**$14000**	**$16000**	**$18000**

What is the optimal production plan (X1, X2, X3, X4, X5) and the associated profit? Show your working and explain your answer.

Refer to sheet Qu-1(m) for the working

Explanation:

The Xi values are set to be integer in the Solver. To achieve the solution for the given question, the linking constraints and binary constraints are used.

The total profit would be the result of:

sumproduct(objective function, needed)–sumproduct(start-up cost, binary constraints)

By this, we arrive at a profit of **$215697**

Question-1(n)

We include the requirement that at most one of the values X1, X2, X3, X4, X5 is more than 200. What is the optimal production plan (X1, X2, X3, X4, X5) and the associated profit? Show your working and explain your answer.

Refer to sheet Qu-1(n) for the working

Explanation:

From the question, it is evident that at most only value can be more than 200. This means that all the values can be less that 200 or only value can be more than 200. To achieve this, count the values whose minimum is more than 200.

i.e

COUNT(MIN(needed values)>=200)

And in the solver, we add a condition that the count is <=1. By this, only one number can be more than 200. This gives us a total profit of **$215697**

Question-1(o)

If Product4 is produced then X4 has to be 150, 200, or 240 What is the optimal production plan (X1, X2, X3, X4, X5) and the associated profit? Show your working and explain your answer.

Refer to sheet Qu-1(o) for the working

Explanation
With the given condition, first we calculate the linking constraints and the corresponding binary constraints for the start-up cost. Then, we find the linking and binary constraints for product 4 separately . We create a new row called the total demand (for product 4) which are 150, 200 and 240. Once this is created, we find the sum of the binary constraints for product 4 and the sum of the total demand (150+200+240). In solver, we add the conditions that the sum of the total demand to be 0 and the sum of the binary values be less than or equal to 1.
The total profit would be
Sumproduct(objective function, needed)-sumproduct(start-up cost, overall binary constraints)
This gives us a profit of **$161906.**

Question-2: Network Modelling:

Question-2(a):
What is the shortest path from A to J? Show the path and the total distance.
Shortest path from A-J is: **A – C – G – J**
Shortest distance = **8**

Question-2(b):
What is the shortest path from B to K? Show the path and the total distance.
Shortest path from B – K is: **B – H – I – K**
Shortest distance = **13**

Question-2(c):
What is the shortest path from C to L? Show the path and the total distance.
Shortest path from C – L is: **C – G – J – L**
Shortest distance = **7**

Question—2(d)
What is the sum of your answers to 2(a), 2(b) and 2(c)?
8+13+7 = **28**

Question-2(e)
In each of 2(a), 2(b) and 2(c), we were moving 1 emergency vehicle (or 1 electric vehicle) from 1 starting point to 1 destination. In 2(d), we moved 3 emergency vehicle (or 3 electric vehicle) from 3 starting point to 3

destinations. If we require that the vehicles have to go from (A, B, C) to (J, K, L) with each destination having exactly one vehicle arrive there, what is the shortest total distance?
Shortest total distance = **26**

<u>**Question-2(f):**</u>
Is your answer to 2(e) longer, equal to, or less than your answer to 2(d)? Explain as clearly as you can - and why.
The answer to 2(e) is 26 and the answer to 2(d) is 28 which makes the sum of the individual shortest paths greater than the combined shortest path from A,B,C to J,K,L. The reason is that when the ambulances choose different routes to their respective destinations, the sum of their distances is less probably because there are certain factors to consider when the ambulances are leaving from A,B,C to J,K,L as a group having exactly one arrive there. Moreover, in the individual cases, the ambulance leaves from other source and arrives in a destination. In the combined one, three ambulances leave from three sources and arrive in three destinations. This changes the supply/demand and the routes too. This might be the reason that there is a difference between both these values.

<u>**Question-2(g):**</u>
For every edge, we introduce a modified cost. If there is flow of more than 1 along an edge, then any additional flow is at double the cost. As an example, the cost along the edge DE was given as 3 in the problems 2(a) to 2(f). We now modify this so that the 1st unit of flow along DF costs 3 but any other unit of flow would cost 6 per unit. Similarly, the cost along the edge DF was given as 6, but we now modify this so that the 1st unit of flow along DF costs 6 but any other unit of flow would cost 12 per unit. What is the shortest total cost?

Refer to sheet Qu-2(g) for working
Shortest total cost = **31**

<u>**Question-3: Inventory Modelling:**</u>

<u>**Question-3(a):**</u>
A product is required with monthly demand of 10,000. The cost of each product is $5, order costs are $100 per order, and storage costs are 0.1 = 10%. Showing calculations (both in .pdf and also in spreadsheet), what is the Economic Order Quantity (EOQ)?

Answer:
Given:
Monthly demand = 10,000
Annual demand (A) = 10,000 x 12 = 1,20,000
Product cost (c) = 5
Ordering cost (k) = 100
Storage cost = 10% = 0.1
EOQ = Q* = SQRT(2AK/ch)
= SQRT((2*120000*100)/(5*0.1))
= 6928.20323

EOQ (Q*) = 6928.20323

<u>Question-3(b)</u>
Suppose that the product cost reduces by 5% for orders of at least 8,800. • Showing calculations (both in .pdf and also in spreadsheet), what is the EOQ? • Also show calculations (both in .pdf and also in spreadsheet) for the optimal Q*.

Given:
Annual demand (A) = 120000
Product cost (c) = 5
C_1 = if (annual demand >=8800), then 5%(c)). Thus,
Discounted product cost (c_1) = 4.75
Ordering cost (k) = 100
Storage cost (h) = 10% = 0.1

EOQ = Q* = SQRT(2AK/c_1h)
= SQRT((2*120000*100)/(4.75*0.1))
= 7108.18653

EOQ = 7108.18653

Total Annual Cost = ((A*k)/Q*)+((Q* *c*h)/2)
= ((120000*100)/7108.18653)+((7108.18653*4.75*0.1)/2)
=3376.3886

Total Annual Cost = 3376.3886

<u>Question-3(c):</u>
Let us now return to the problem from part (a) - i.e., there is no discount for larger orders (as there was in part (b)). Assume that we are permitted to run out of stock, but there is a back-order penalty of $10 on each product for which we are out of stock for a month. • Showing calculations (both in .pdf and also in spreadsheet), what is the EOQ? • Also show calculations (both in .pdf and also in spreadsheet) for the optimal Q*.

Given:
Annual demand (A) = 120000
Product cost (c) = 5
Ordering cost (k) = 100
Storage cost (h) = 0.1
Backorder penalty (p) = 120
EOQ = Q* = SQRT((2Ak/ch)((p+ch)/p))
= SQRT(((2*120000*100)/(5*0.1))*((120+(5*0.1))/120))
= 6942.62198

EOQ (Q*) = 6942.62198