Linear Algebra Quiz – Full Solutions

Q1. Row space and column space of a matrix have the same dimension because both equal the rank of the matrix. Answer: C

Q2. Gram–Schmidt process converts linearly independent vectors into orthogonal vectors spanning the same subspace. Answer: a

Q3. From $\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$, trace = 6 and determinant = 6. Answer: c

Q4. Maximum number of linearly independent vectors equals the dimension of the space, which is 5. Answer: a

Q5. Eigenvector corresponding to eigenvalue –1 satisfies A x = –x. Option (b) satisfies this. Answer: b

Q6. For ■x,y■ = x■Ay to be an inner product, A must be symmetric and positive definite. Answer: d

Q7. RREF requires leading 1s with zeros above and below and pivots moving right. Option (b) satisfies all conditions. Answer: b

Q8. Frobenius norm squared equals sum of squares of singular values: 25 + 4 + 1 = 30. Answer: a

Q9. The vectors satisfy a linear relation, hence are linearly dependent. Answer: d

Q10. Vectors are orthogonal and each has unit norm, hence orthonormal. Answer: b

Q11. A real symmetric matrix is positive definite iff all eigenvalues are positive. Answer: a

Q12. Using trace, determinant, and principal minors gives $\lambda^3 - 27\lambda^2 + 167\lambda - 285 = 0$. Answer: c

Q13. Comparing scalar multiples gives a = –6, b = –4, c = –9. Answer: b

Q14. Trace = p + q = –5 and determinant = pq + 3 = 3 ⇒ pq = 0, so p + q + pq = –5. Answer: c

Q15. Matrix (b) does not have orthonormal columns, so it is not orthogonal. Answer: b

Q16. 10A – 5B has order 4 × m, so its transpose has order m × 4. Answer: a

Q17. Solving linear combination equations gives m = 5. Answer: c

Q18. Set {(x,y): x ≥ 0, y ≥ 0} is not closed under scalar multiplication, so not a subspace. Answer: d

Q19. Distance = $||x^2 - x|| = \text{sqrt}(\int_{-1}^{1}(x^2 - x)^2\,dx) = 4\sqrt{15} / 15$. Answer: d

Q20. Solving a(1,2,3)+b(2,5,8) = (3,7,11) gives a = b = 1. Answer: c