Birla Institute of Technology & Science, Pilani
Work Integrated Learning Program Division
First Semester 2025-2026

Assignment 1

Course No.   : AIMLC ZC416/ DSECLZC416
Course Title   : Mathematical Foundations for Machine Language/
                        Mathematical Foundations for Data Science
Weightage   : 10%

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Instructions to Students:
1. All parts of a question should be answered consecutively. Each answer should start from a fresh page.
2. You must submit the scanned copy of your hand written assignment in pdf format. Assignments containing types answers will not be accepted.
3. The last date to submit the Assignment is 18-12-2025. No extension will be given after that.

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Q1.

Consider the following system of linear equations:

$$4x + y + z = 6$$
$$x + 3y + z = 5$$
$$x + y + 2z = 4.$$

Solve this system using **three different methods**:

(a) Form the augmented matrix corresponding to the system, reduce it to **echelon form**, and determine the solution using **backward substitution**. [0.5M]

(b) Compute the **eigen-decomposition** of the coefficient matrix, and then use this decomposition to solve the given system of equations. [1.5M]

(c) Compute the **Cholesky decomposition** of the coefficient matrix, and use this factorization to solve the system. [1.5M]

(d) Compare the three methods above and comment on **for this problem which method is more efficient in terms of computational cost**. [0.5M]

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Q2.

Define an inner product on $\mathbb{R}^4$ by

$$\langle x, y \rangle = x^T M y,$$

where

$$M = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 3 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}, x, y \in \mathbb{R}^4.$$

Consider the vectors

$$u = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}, v = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}, w = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix}.$$

Answer the following:

(a) Verify that the function $\langle \cdot, \cdot \rangle$ defines an **inner product** on $\mathbb{R}^4$. [1M]

(b) Compute the inner products $\langle u, v \rangle$, $\langle u, w \rangle$, and $\langle v, w \rangle$. [1M]

(c) Compute the norms $\| u \|, \| v \|, \| w \|$ [1M]

with respect to this inner product.

(d) Show that the vectors u, v, and w are **linearly independent** in $\mathbb{R}^4$. [1M]

(e) Apply the **Gram–Schmidt process** (with the given inner product) to the set $\{u, v, w\}$ to obtain an **orthonormal set** in $\mathbb{R}^4$. [2M]