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Q1 Linear equations.

$$4x+y+z=6$$
$$x+3y+z=5$$
$$x+y+2z=4$$

a.) Backward subsitution.

$$\begin{bmatrix} 4 & 1 & 1 & 6 \\ 1 & 3 & 1 & 5 \\ 1 & 1 & 2 & 4 \end{bmatrix}$$ Converted to matrix form

$R_1 \leftrightarrow R_2$ swaping (ERO 1)

$$\begin{bmatrix} 1 & 3 & 1 & 5 \\ 4 & 1 & 1 & 6 \\ 1 & 1 & 2 & 4 \end{bmatrix}$$

Row operations

$R_2 \to R_2 - 4R_1$

$[4 \quad 1 \quad 1 \quad 6] - 4[1 \quad 3 \quad 1 \quad 5]$

$[4 \quad 1 \quad 1 \quad 6] - [4 \quad 12 \quad 4 \quad 20]$

$[0 \quad -11 \quad -3 \quad -14]$ - R_2 row answer

$R_3 \to R_3 - R_1$

$[1 \quad 1 \quad 2 \quad 4] - [1 \quad 3 \quad 1 \quad 5] = [0 \quad -2 \quad 1 \quad -1]$

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substitute R_2 and R_3 in original matrix 20259b05129

$$\begin{bmatrix} 1 & 3 & 1 & 5 \\ 0 & -11 & -3 & -14 \\ 0 & -2 & 1 & -1 \end{bmatrix}$$

$$R_3 \to R_3 - \frac{2}{11} R_2$$

$$\begin{bmatrix} 0 & -2 & 1 & -1 \end{bmatrix} - \frac{2}{11}\begin{bmatrix} 0 & -11 & -3 & -14 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -2 & 1 & -1 \end{bmatrix} - \begin{bmatrix} 0 & -2 & \frac{-6}{11} & \frac{-28}{11} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 - \frac{-6}{11} & -1 - \frac{28}{11} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & \frac{17}{11} & \frac{17}{11} \end{bmatrix}$$

<u>Replace R_3 in original</u>

$$\begin{bmatrix} 1 & 3 & 1 & 5 \\ 0 & -11 & -3 & -14 \\ 0 & 0 & \frac{17}{11} & \frac{17}{11} \end{bmatrix}$$

<u>Backward subsitution</u>

$$\frac{17}{11} z = \frac{17}{11}$$

$$\boxed{z = 1} \,//$$

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$x + 3y + z = 5$

$0 \quad -11y \quad -3z = -14$

$0 \quad -11y \quad -3\times 1 = -14$

$-11y = -14 + 3$

$\boxed{y = 1}$ //

From R_1

$4x + y + z = 6$

$4x + 1 + 1 = 6$

$4x = 6 - 2$

$\boxed{x = 1}$ //

$\boxed{x = 1, \quad y = 1, \quad z = 1}$ //

b.) Eigen decomposition method

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 2 \end{bmatrix} \qquad b = \begin{bmatrix} 6 \\ 5 \\ 4 \end{bmatrix}$$

Its a symmetric

$A = A^T$

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$$|A - \lambda I| = 0$$

$$\lambda^3 - s_1\lambda^2 + s_2\lambda - s_3 = 0$$

s_1 = sum of diagonal matrix

$4 + 3 + 2 = 9$

s_2 = sum of minus of diagnal elements.

$$\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 4 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 4 & 1 \\ 1 & 3 \end{vmatrix}$$

$= 6 - 1 + 8 - 1 + 12 - 1$

$s_2 = 23$

$s_3 = |A|$

$$= 4\begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix}$$

$= 4(6-1) - 1(2-1) + 1(1-3)$

$= 17$

$$\lambda^3 - 9\lambda^2 + 23\lambda - 17 = 0$$

solving above cubic ic calc equation mode

$\lambda_1 = 5.214319743$

$\lambda_2 = 2.460811127$

$\lambda_3 = 1.324869129$

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A = Eigen vector $\lambda_1 = 5.214319743$

$$(A - \lambda I)v = 0$$

$$= \begin{pmatrix} 4 - 5.214319743 & 1 & 1 \\ 1 & 3 - 5.214319743 & 1 \\ 1 & 1 & 2 - 5.214319743 \end{pmatrix}$$

$$= \begin{pmatrix} -1.214319743 & 1 & 1 \\ 1 & -2.214319743 & 1 \\ 1 & 1 & -3.214319743 \end{pmatrix}$$

let $v = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$

$$-1.214319743x + y + z = 0$$
$$x - 2.214319743y + z = 0$$
$$x + y - 3.214319743z = 0$$

solve the above equation

$$y = 1.214319743 - z$$

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$$x - 2.214319743(1.214319743x - z) + z = 0$$

$$3.214319742z = 1.68889218x$$

$$z = 0.524275609x$$

$$y = 1.214319743x - 0.5244275609x$$

$$y = 0.6888921821x$$

Eigen vector V_1 $(x, 0.6888921821x, 0.5244275609x)$

$$\|V_1\| = \sqrt{1^2 + 0.6888921821^2 + 0.524427560 9^2}$$

$$= 1.32312001$$

$$V_1 = \frac{1}{\|V_1\|} = \frac{1}{1.32312001}(1, 0.6888921821, 0.524427609)$$

$$V_1 = (0.7557893408, 0.5206573681, 0.3971125498)$$

Eigen vector $\lambda_2 = 2.460811127$

$$(A - \lambda_2 I)v = 0 = \begin{pmatrix} 4-2.460811127 & 1 & 1 \\ 1 & 3-2.46081127 & 1 \\ 1 & 1 & 2-2.46031112 \end{pmatrix}$$

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$$= \begin{pmatrix} 1.539188873 & 1 & 1 \\ 1 & 0.539188873 & 1 \\ 1 & 1 & -0.460811127 \end{pmatrix}$$

solve equation by sussiting.

1) $1.539188873x + y + z = 0$

2) $x + 0.539188873y + z = 0$

3) $x + y - 0.460811127z = 0$.

$x + 0.539188873(1.539188873x - z) + z = 0$

$x - 0.8299135138x - 0.539188873 + z = 0$

$0.1700864862x + 0.460811127z = 0$

$$z = \frac{-0.1700864862x}{0.460811127}$$

$z = -0.3691023854x$

$y = z - 1.539188873x$

$= +0.3691023854x - 1.539188873x$

$= -1.170086488$

$V_2 = (x, -1.170086488x \quad -0.3691023854x)$

$$\|V_2\| = \sqrt{1^2 + (-1.170086488)^2 + (-0.3691023854)^2}$$

$= 1.582826257$

$V_2 = (0.6317812808, -0.7392387401, -0.2331919778)$

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Eigen vector corresponding to $\lambda_3 = 1.324869129$

$$(A - \lambda_3 I)v = \begin{pmatrix} 4-1.324869129 & 1 & 1 \\ 1 & 3.1324869129 & 1 \\ 1 & 1 & 2-1.324869129 \end{pmatrix}$$

$$= \begin{pmatrix} 2.675130871 & 1 & 1 \\ 1 & 1.675130871 & 1 \\ 1 & 1 & 0.675130871 \end{pmatrix}$$

subsitity is equation

1.) $2.675130871x + y + z = 0$

2.) $x + 1.675130871y + z = 0$

3.) $x + y + 0.675130871 = 0$

$$x + 1.675130871(-2.67513087x - z) + z = 0$$

$$x - 4.481194182 - 1.657130871z + z = 0$$

$$z = \frac{3.481194187x}{-0.67513087} = -5.15632517x$$

$$y = -z - 2.675130871x$$

$$= 5.515632517x - 2.675130871x =$$

$$= 2.481194303x$$

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$V_3 (x, 2.48119403x, -5.15632174x)$

$$||V_3|| = \sqrt{1^2 + 2.48119403^2 + (-5.15632174)^2}$$

$$= 5.808959844$$

$V_3 = (0.172147859, 0.427132287, -0.887650338)$

Eigen vector and eigen decomposition

$$A = PDP$$

$P = [v_1 \quad v_2 \quad v_3]$, $D = diag(\lambda_1 + \lambda_2 \lambda_3)$

set $x = P_z$

$$Ax = PDP^T (P_z)$$

$$P^T PDZ = P^T b$$

$$D_z = P^T b$$

$z = D^{-1}(P^T b)$, $x = P_z$

hence $y = P^T b$ $(y_i = v_i^T b)$

$y_1 = v_1^T b = (0.7557893408 \times 6) + (0.5206573681 \times 5)$
$+ (0.397112548 \times 4)$

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$= 8.726473077$

$y_2 = V_2^T b = (0.6317812808 \times 6) + (-0.7392387401 \times 5)$
$+ (-0.233199778 \times 4)$

$= -0.8382739 4$

$y_3 = V_3^T b = (0.1721478596) + (0.42713228 \times 5) +$
(-0.8876503389×4)

$= -0.3820527666$

$z_i = y_i / \lambda_i$

$z_1 = y_1/\lambda_1 = \dfrac{8.726473077}{5.214319743} = 1.673559257$

$z_2 = y_2/\lambda_2 = \dfrac{-0.8882 7394}{2.460811127} = -0.\cancel{34}0649443 1$

$z_3 = y_3/\lambda_3 = \dfrac{0.3820527666}{1.324869127} = 0.2883701931$

$x = P_2 = z_1 V_1 + z_2 V_2 + z_3 V_3$

$x = 1.673559257 \begin{pmatrix} 0.7555789 3408 \\ 0.5206573681 \\ 0.3971125498 \end{pmatrix}$

$- (0.3406494431) \begin{pmatrix} 0.6317812808 \\ 0.7392387401 \\ -0.2331919778 \end{pmatrix}$

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$$+0.2883701931\begin{pmatrix}-0.172147859\\-0.427132287\\0.8876503887\end{pmatrix}$$

$$=\begin{pmatrix}1.264858248\\0.8713509581\\0.6645913838\end{pmatrix}+\begin{pmatrix}-0.2152159415\\0.2518212651\\0.07943671737\end{pmatrix}+\begin{pmatrix}-0.04964231134\\-0.1231722201\\0.2559719141\end{pmatrix}$$

$$x=\begin{pmatrix}0.9999999952\\1.000000003\\1.000000015\end{pmatrix}$$

Approximated

$$x=\begin{pmatrix}1\\1\\1\end{pmatrix}$$

c.) Cholesky decomposition $Ax = b$

$$Ax = b\begin{pmatrix}4&1&1\\1&3&1\\1&1&2\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix}=\begin{pmatrix}6\\5\\4\end{pmatrix}=b$$

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As matrix A is symmetric, cholesky factorization can be applied.

$$A = LL^T, \quad L = \text{lower triangular matrix}$$

$$L = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix}$$

$$LL^T = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \begin{pmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{pmatrix}$$

$$= \begin{pmatrix} l_{11}^2 & l_{11} l_{21} & l_{11} 3_1 \\ l_{21} l_{11} & l_{21}^2 + l_{22}^2 & l_{21} l_{31} + l_{22} + l_{32} \\ l_{31} l_{11} & l_{32} + l_{33} l_{22} & l_{31}^2 + l_{32}^2 + l_{33}^2 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 2 \end{pmatrix}$$

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Sussitute a_{ij} in l_{ij}

$$a_{11} = l_{11}^2 = 4$$

$$l_{11} = 2$$

$$a_{21} = l_{21} l_{11} = 1$$

$$l_{21} = \frac{1}{2}$$

$$a_{31} = l_{31} l_{11} = 1$$

$$l_{31} = \frac{1}{2}$$

$$a_{22} = l_{21}^2 + l_{22}^2 = 3$$

$$l_{22}^2 = 3 - \left(\frac{1}{2}\right)^2 = 3 - \frac{1}{4} = \frac{11}{4}$$

$$l_{22} = \frac{\sqrt{11}}{2}$$

$$a_{32} = l_{3} l_{21} + l_{32} l_{22} = 1$$

$$l_{32} = \frac{1 - l_{31} l_{21}}{l_{22}} = \frac{1 - 1/4}{\frac{\sqrt{11}}{2}}$$

$$= \frac{3}{2\sqrt{11}}$$

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$$a_{33} = l_{31}^2 + l_{32}^2 + l_{33}^2 = 2$$

$$l_{33}^2 = 2 - \frac{1}{4} - \frac{9}{44}$$

$$l_{33}^2 = \frac{88 - 11 - 9}{44}$$

$$l_{33}^2 = \frac{68}{44} = \frac{17}{11}$$

$$l_{33} = \sqrt{17/11}$$

hence
$$L = \begin{pmatrix} 2 & 0 & 0 \\ \frac{1}{2} & \frac{\sqrt{11}}{2} & 0 \\ \frac{1}{2} & \frac{3}{2\sqrt{11}} & \sqrt{17/11} \end{pmatrix}$$

Appy forward subsition $Ly = b$

Backward substitution $L^T x = y$

L 1) $2y_1 = 6$

$$y_1 = \frac{6}{2} = 3$$

2) $\frac{1}{2}y_1 + \frac{\sqrt{11}}{2}y_2 = 5$

3) $\frac{1}{2}y_1 + \frac{3}{2\sqrt{11}}y_2 + \sqrt{\frac{17}{11}}y_3 = 4.$

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$$\frac{1}{2}(3) + \frac{\sqrt{11}}{2} y_2 = 5$$

$$\sqrt{11}/2 \, y_2 = 5 - 3/2$$

$$y_2 = \frac{7/2}{\frac{\sqrt{11}}{2}} = \frac{7}{\sqrt{11}}$$

third eq

$$\frac{1}{2}(3) + \frac{3}{2\sqrt{11}} \cdot \frac{7}{\sqrt{11}} + \sqrt{17/11}\, y_3 = 4$$

$$\frac{3}{2} + \frac{21}{22} + \sqrt{17/11}\, y_3 = 4$$

$$\sqrt{17/11}\, y_3 = 4 - \frac{27}{11}$$

$$y_3 = 4 - \frac{27}{11} \times \sqrt{11/17}$$

$$= \frac{17}{11} \times \sqrt{11/17}$$

$$y_3 = \sqrt{17/11}$$

$$y = \begin{pmatrix} 3 \\ 7/\sqrt{11} \\ \sqrt{17/11} \end{pmatrix}$$

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hence $L^T z = y$ $\therefore L^T = \begin{pmatrix} 2 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{\sqrt{11}}{2} & \frac{3}{2\sqrt{11}} \\ 0 & 0 & \sqrt{17/11} \end{pmatrix}$

Apply in the equation

1) $2x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3 = 3$

2) $\sqrt{11}/2\, x_2 + \frac{3}{2\sqrt{11}} x_3 = \frac{7}{\sqrt{11}}$

3.) $\sqrt{17/11}\, x_3 = \sqrt{17/11} = x_3 = 1$

$x_3 = 1$, apply in 2

$$\sqrt{11}/2\, x_2 + \frac{3}{2\sqrt{11}} \cdot 1 = \frac{7}{\sqrt{11}}$$

multiply bot sides by $\frac{2}{\sqrt{11}}$

$$x_2 + \frac{3}{2\sqrt{11}} \times \frac{2}{\sqrt{11}} = \frac{7}{\sqrt{11}} \cdot \frac{2}{\sqrt{11}}$$

$$x_2 + \frac{6}{22} = \frac{14}{11}$$

$$x_2 = \frac{14}{11} - \frac{6}{22} = 1$$

subsitiue in equation 1

$$2x_1 + \frac{1}{2}1 + \frac{1}{2}1 = 3$$

$\cancel{2x_1}$

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$$2x_1 + 1 = 3$$

$$x_1 = 1$$

$$(x_1 = 1, \quad x_2 = 1, \quad x_3 = 1)$$

as per equation.

$$(x = 1, \quad y = 1, \quad z = 1)$$

D) compare All three methods.

Method	Remarks	Cost
Backward Subsitution	suitable for small systems	Low
Eigen - Decomposition	It requires lot of compute, hence expensive.	High
cholesky Decomposition	For symmetric positive it is best method.	very efficient

cholesky decomposition is best since

the computional steps is very less and efficent,,

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Q2 Inner product of R^4:

$$(x,y) = x^T M y$$

$$M = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 3 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

vectors

$$u = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix} \quad v = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad w = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix}$$

a.) verify inner product

$$M^T = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 3 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$$

a1) $M = M^T$ which is symmetric.

hence $(x,y) = x^T M y = y^T M x = (y,x)$ //

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b)

Positive definiteness: $(x,x) = x^T Mx > 0$ for $x \neq 0$

Apply block diagonal

$$B_1 = \begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix} \quad B_2 = \begin{pmatrix} 4 & 1 \\ 1 & 2 \end{pmatrix}$$

$\det(B_1) = (6-1) = 5 > 0$, $\det(B_2) = (8-1) = 7 > 0$

c) no need check bilinearity, since builtin $x^T My$

$\boxed{\text{Based on a, b, c } \langle \cdot , \cdot \rangle \text{ is an inner product on } \mathbb{R}^4}$

B. compute the inner products (u,v) (u,w), (v,w)

Find mv, mw

$(x,y) = x^T(my)$

$$mv = \begin{bmatrix} 2(0) + -1(1) + 0\times0 + 0\times0 \\ (-1\times0) + (3\times1) + 0 + 0 \\ 0 \quad 0 \quad + 4\times1 + 1\times1 \\ 00 \quad 0 \quad + 1\times1 + 2\times1 \end{bmatrix} = \begin{pmatrix} -1 \\ 3 \\ 5 \\ 3 \end{pmatrix}$$

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the inner products:

$(u,v) = u^T(Mv)$

$u = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}$

$u^T = \begin{pmatrix} 1 & 2 & 0 & 1 \end{pmatrix}$

$(u,v) = u^T(Mv) = \begin{pmatrix} 1 & 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \\ 5 \\ 3 \end{pmatrix}$

$= -1 + 6 + 0 + 3$

$\boxed{(u,v) = 8}$

$(u,w) = u^T M w$

$= \begin{pmatrix} 1 & 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 3 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix}$

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$$= \begin{pmatrix} 1 & 2 & 0 & 1 \end{pmatrix} \begin{bmatrix} 2\times1 + -1\times0 + 0 + 0 \\ -1\times1 + 3\times0 + 0 + 0 \\ 0 + 0 + 4\times2 + 1\times0 \\ 0\times1 + 0 + 1\times2 + 2\times0 \end{bmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 8 \\ 2 \end{pmatrix}$$

$$= 2 + (-2) + 0 + 2$$

$$\boxed{(u,w) = 2}$$

$$\boxed{\langle u,w \rangle = u^T M w = 2}$$

$$vw = v^T M w$$

$$= \begin{pmatrix} 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 8 \\ 2 \end{pmatrix}$$

$$= 0 + (-1) + 8 + 2$$

$$\boxed{vw = v^T M w = 9}$$

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c.) norms $\|u\|, \|v\|, \|w\|$

with resfect to the given inner product

$$\|x\| = \sqrt{(x,x)}$$

$$(x,x) = x^T M x$$

$$(u,v) = u^T M u$$

$$u = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix} \quad u^T = \begin{bmatrix} 1 & 2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 3 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1, & 2, & 0, 1 \end{bmatrix} \begin{bmatrix} 2\times1 + -1\times2 + 0+0 \\ -1 + 6 + 0 + 0 \\ 0 + 0 + 0 + 1 \\ 0 + 0 + 0 + 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \\ 1 \\ 2 \end{bmatrix}$$

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$= [1\times 0 + 3\times 2 + 0 + 1\times 2)$

$= 12$

$(u,v) = ||u||^2 = 12$

$||u|| = \sqrt{12}$

$\boxed{||u|| = 2\sqrt{3}}$ //

$(v,v) = v^T M v$

$v = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad v^T = [0 \ 1 \ 1 \ 1]$

$= \begin{pmatrix} 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 3 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2\times 0 + -1 + 0 + 0 \\ 0 + 3 + 0 + 0 \\ 0 + 0 + 4 + 1 \\ 0 + 0 + 1 + 2 \end{pmatrix}$

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$$= \begin{pmatrix} 0 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \\ 5 \\ 3 \end{pmatrix}$$

$vv = (0 + 3 + 5 + 3)$

$vv = 11$

$$\boxed{\|v\| = \sqrt{11}}$$

//

$(w, w) = w^T M w$

$w = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix} \quad w^T = \begin{bmatrix} 1 & 0 & 2 & 0 \end{bmatrix}$

$$(w\,w) = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix}^T \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 3 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 & 0 \end{pmatrix}$$

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$$(w,w) = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix}^T \begin{pmatrix} 2+0+0+0 \\ -1+0+0+0 \\ 0+0+8+0 \\ 0+0+2+0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix}^T \begin{pmatrix} 2 \\ -1 \\ 8 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 2 & 0 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 8 \\ 2 \end{pmatrix}$$

$$= 2 + 0 + 16 + 0$$

$$(w,w) = 18$$

$$\|w\| = \sqrt{18}$$

$$\boxed{\|w\| = 3\sqrt{2}}$$ //

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$$\boxed{\|u\| = 2\sqrt{3}, \ \|v\| = \sqrt{11}, \ \|w\| = 3\sqrt{2}}$$ //.

d.) u, v, w are linearly independent

Definition used

$$\alpha u + \beta v + \vartheta w = 0$$

$$\alpha = \beta = \vartheta = 0$$

Given vectors

$$u = \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \quad v = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} \quad w = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix}$$

$$\alpha \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \vartheta \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

First co-ordinate

$$\alpha(1) + \beta(0) + \vartheta(1) = 0$$

$$\alpha + \vartheta = 0$$

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second co-ordinate

$$\alpha(2) + \beta(1) + \gamma(0) = 0$$

$$2\alpha + \beta = 0$$

Third co-ordinate

$$\alpha \cdot 0 + \beta_1 + \gamma(2) = 0$$

$$\beta + 2\gamma = 0$$

Fourth co-ordinate

$$\alpha \times 1 + \beta 1 + \gamma \times 0 = 0$$

$$\alpha + \beta = 0$$

let $\alpha = 0$

equation 1 $\alpha + \gamma = 0$, $0 + \gamma = 0$

ie $\gamma = 0$ (gamma is zero)

equation 2

$$2\alpha + \beta = 0$$

$$2 \times 0 + \beta = 0$$

ie $\beta = 0$ (beta is zero)

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hence

$$\alpha = \beta = \gamma = 0$$

$\boxed{\text{Therefore } u, v, w \text{ are Linearly indetendent in } R^4}$ //

e) Gram schmidt u, v, w produce an orthogonal set

orthogonal vectors a_1, a_2, a_3

normalized orthonormal vectors e_1, e_2, e_3

$a_1 = u$

Inner product $(a_1, a_1) = u, u = 12$

$$e_1 = \frac{a_1}{\sqrt{(a_1, a_1)}} = \frac{1}{\sqrt{12}} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix} = \frac{1}{2\sqrt{3}} \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

Step 2 a_2 = Projection of v into a_1

$$e_2 = \frac{u_2}{\|u_2\|}$$

$$\cancel{u_2 = v - \frac{(v, u_1)}{u_1, u_1} u_1}$$

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proj $a_1(v) = \frac{\langle v, a_1 \rangle}{\langle a_1, a_1 \rangle} a_1 = \frac{(v, u)}{(u, u)}$

$= \frac{8}{12} u$

$= \frac{2}{3} u$

$a_2 = v - \frac{2}{3} u$

$= \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} - \frac{2}{3} \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} \frac{2}{3} \\ \frac{4}{3} \\ 0 \\ \frac{2}{3} \end{pmatrix}$

$a_2 = \begin{pmatrix} -2/3 \\ -1/3 \\ 1 \\ 1/3 \end{pmatrix}$

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now $(a_2, a_2) = a_2^T M a_2$

$$= \begin{pmatrix} -2/3 & -1/3 & 1 & 1/3 \end{pmatrix} \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 3 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} -2/3 \\ -1/3 \\ 1 \\ 1/3 \end{pmatrix}$$

$$= \begin{pmatrix} -2/3 & -1/3 & 1 & 1/3 \end{pmatrix} \begin{pmatrix} 2 \times \frac{-2}{3} + -1 \times \frac{-1}{3} + 0 + 0 \\ -1 \times \frac{-2}{3} + 3 \times \frac{-1}{3} + 0 + 0 \\ 0 + 0 + 4 + \frac{1 \times 1}{3} \\ 0 + 0 + 1 + 2 \times \frac{1}{3} \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{2}{3} & \frac{-1}{3} & 1 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} -1 \\ \frac{-1}{3} \\ \frac{13}{3} \\ \frac{5}{3} \end{pmatrix}$$

$$= \left(\frac{-2}{3} \times -1 + \frac{-1}{3} \times \frac{-1}{3} + 1 \times \frac{13}{3} + \frac{1}{3} \times \frac{5}{3} \right)$$

$$= \frac{2}{3} + \frac{1}{9} + \frac{13}{3} + \frac{5}{9} = \frac{17}{3}$$

$$= \frac{17}{3}$$

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$\langle a_1, a_2 \rangle = 17/3$

$\|a_2\| = \sqrt{17/3}$

$e_2 = \frac{a_2}{\|a_2\|} = \frac{a_2}{\sqrt{17/3}}$

$$e_2 = \sqrt{3/17} \begin{bmatrix} -\frac{2}{3} \\ -1/3 \\ 1 \\ 1/3 \end{bmatrix}$$

a_3 is result of orthogonalization of w against $a_1\ a_2$

$a_3 = w - \text{proj}_{a_1}(w) - \text{proj}_{a_2}(w)$

$\text{proj}_{a_1}(w) = \frac{(w, a_1)}{(a_1, a_1)} a_1 = \frac{w, u}{12} u = \frac{2}{12} u$

$= \frac{1}{6} u$

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$$\langle w, a_2 \rangle = w^T (M a_2)$$

$$= \begin{pmatrix} 1 & 0 & 2 & 0 \end{pmatrix} \begin{pmatrix} -1 \\ -1/3 \\ 13/3 \\ 5/3 \end{pmatrix}$$

$$= \left(1 \times -1 + 0 + \frac{2 \times 13}{3} + 0\right)$$

$$= \left(-1 + 0 + \frac{26}{3} + 0\right)$$

$$= 23/3$$

$$\text{proj}_{a_2}(w) = \frac{\langle w, a_2 \rangle}{a_2 \, a_2} a_2$$

$$= \frac{23/3}{17/3} a_2$$

$$= \frac{23}{17} \begin{pmatrix} -2/3 \\ -1/3 \\ 1 \\ 1/3 \end{pmatrix}$$

$$a_3 = w - \text{proj}_{a_1}(w) - \text{proj}_{a_2}(w)$$

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$$a_3 = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix} - \frac{1}{6}\begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix} - \frac{23}{17}\begin{pmatrix} \frac{-2}{3} \\ \frac{-1}{3} \\ 1 \\ 1/3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{1}{6} \\ \frac{2}{6} \\ 0 \\ 1/6 \end{pmatrix} - \begin{pmatrix} -46/51 \\ -23/51 \\ 23/17 \\ 23/51 \end{pmatrix}$$

$$= \begin{bmatrix} 59/34 \\ 2/17 \\ 11/17 \\ -21/34 \end{bmatrix}$$

$$e_3 = \frac{a_3}{\sqrt{a_3 \cdot a_3}}$$

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$$(a_3, a_3) = a_3^T M a_3$$

$$= \left(59/34,\ 2/17,\ 11/17,\ -21/34\right)\begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 3 & 0 & 0 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}\begin{pmatrix} 59/34 \\ 2/17 \\ 11/17 \\ -21/34 \end{pmatrix}$$

$$= \left(59/34\ \ 2/17\ \ 11/17\ \ -21/34\right)\begin{pmatrix} 2\times\frac{59}{34} - 1\times\frac{2}{17} + 0 + 0 \\ -1\times\frac{59}{34} + 3\times\frac{2}{17} + 0 + 0 \\ 0 + 0 + 4\times\frac{11}{17} + 1\times\frac{-21}{34} \\ 0 + 0 + 1\times\frac{11}{17} + 2\times\frac{-21}{34} \end{pmatrix}$$

$$= \left(59/34,\ 2/17\ \ 11/17\ \ -21/34\right)\begin{pmatrix} 57/17 \\ -47/34 \\ 67/34 \\ -10/17 \end{pmatrix}$$

$$= \left(\frac{59}{34}\times\frac{57}{17}\right) + \left(\frac{2}{17}\times\frac{-47}{34}\right) + \left(\frac{11}{17}\times\frac{67}{34}\right) + \left(\frac{-21}{34}\times\frac{-10}{17}\right)$$

$$= \frac{4216}{578}$$

$$= \frac{124}{17}$$

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$$e_3 = \frac{a_3}{\sqrt{a_3 . a_3}} = \frac{1}{\sqrt{124/17}} \begin{bmatrix} 59/34 \\ 2/17 \\ 11/17 \\ -21/34 \end{bmatrix}$$

orthonormal set with respect to Inner product is

$$\boxed{\begin{aligned} e_1 &= \frac{1}{2\sqrt{3}} \begin{bmatrix} 1 & 2 & 0 & 1 \end{bmatrix}^T \\ e_2 &\quad \sqrt{3/17} \begin{bmatrix} -2/3 & -1/3 & 1 & 1/3 \end{bmatrix}^T \\ e_3 &= \sqrt{\frac{17}{124}} \begin{bmatrix} 59/34 & 2/17 & 11/17 & -21/34 \end{bmatrix}^T \end{aligned}}$$

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