

ML0101EN-Reg-Multiple-Linear-Regression-Co2-py-v1

October 16, 2019

Multiple Linear Regression

About this Notebook

In this notebook, we learn how to use scikit-learn to implement Multiple linear regression. We download a dataset that is related to fuel consumption and Carbon dioxide emission of cars. Then, we split our data into training and test sets, create a model using training set, Evaluate your model using test set, and finally use model to predict unknown value

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```

0.0.1 Importing Needed packages

```
[1]: import matplotlib.pyplot as plt
import pandas as pd
import pylab as pl
import numpy as np
%matplotlib inline
```

0.0.2 Downloading Data

To download the data, we will use !wget to download it from IBM Object Storage.

```
[2]: !wget -O FuelConsumption.csv https://s3-api.us-gio.objectstorage.softlayer.net/
    ↪cf-courses-data/CognitiveClass/ML0101ENv3/labs/FuelConsumptionCo2.csv
```

```
--2019-10-15 22:32:05-- https://s3-api.us-gio.objectstorage.softlayer.net/cf-
courses-data/CognitiveClass/ML0101ENv3/labs/FuelConsumptionCo2.csv
Resolving s3-api.us-gio.objectstorage.softlayer.net (s3-api.us-
gio.objectstorage.softlayer.net)... 67.228.254.193
Connecting to s3-api.us-gio.objectstorage.softlayer.net (s3-api.us-
gio.objectstorage.softlayer.net)|67.228.254.193|:443... connected.
```

```
HTTP request sent, awaiting response... 200 OK
Length: 72629 (71K) [text/csv]
Saving to: 'FuelConsumption.csv'
```

```
FuelConsumption.csv 100%[=====>] 70.93K --.-KB/s in 0.04s
```

```
2019-10-15 22:32:05 (1.66 MB/s) - 'FuelConsumption.csv' saved [72629/72629]
```

Did you know? When it comes to Machine Learning, you will likely be working with large datasets. As a business, where can you host your data? IBM is offering a unique opportunity for businesses, with 10 Tb of IBM Cloud Object Storage: [Sign up now for free](#)

Understanding the Data

0.0.3 FuelConsumption.csv:

We have downloaded a fuel consumption dataset, **FuelConsumption.csv**, which contains model-specific fuel consumption ratings and estimated carbon dioxide emissions for new light-duty vehicles for retail sale in Canada. [Dataset source](#)

- **MODELYEAR** e.g. 2014
- **MAKE** e.g. Acura
- **MODEL** e.g. ILX
- **VEHICLE CLASS** e.g. SUV
- **ENGINE SIZE** e.g. 4.7
- **CYLINDERS** e.g. 6
- **TRANSMISSION** e.g. A6
- **FUELTYPE** e.g. z
- **FUEL CONSUMPTION in CITY**(L/100 km) e.g. 9.9
- **FUEL CONSUMPTION in HWY** (L/100 km) e.g. 8.9
- **FUEL CONSUMPTION COMB** (L/100 km) e.g. 9.2
- **CO2 EMISSIONS** (g/km) e.g. 182 -> low -> 0

Reading the data in

```
[4]: df = pd.read_csv("FuelConsumption.csv")

# take a look at the dataset
df.head()
```

```
[4]:  MODELYEAR  MAKE      MODEL VEHICLECLASS  ENGINE SIZE  CYLINDERS  \
0      2014  ACURA      ILX      COMPACT      2.0          4
1      2014  ACURA      ILX      COMPACT      2.4          4
2      2014  ACURA  ILX HYBRID      COMPACT      1.5          4
3      2014  ACURA      MDX 4WD  SUV - SMALL      3.5          6
4      2014  ACURA      RDX AWD  SUV - SMALL      3.5          6

      TRANSMISSION FUELTYPE  FUELCONSUMPTION_CITY  FUELCONSUMPTION_HWY  \
0              AS5        Z                   9.9                   6.7
```

1	M6	Z	11.2	7.7
2	AV7	Z	6.0	5.8
3	AS6	Z	12.7	9.1
4	AS6	Z	12.1	8.7

	FUELCONSUMPTION_COMB	FUELCONSUMPTION_COMB_MPG	CO2EMISSIONS
0	8.5	33	196
1	9.6	29	221
2	5.9	48	136
3	11.1	25	255
4	10.6	27	244

Lets select some features that we want to use for regression.

```
[5]: cdf = df[['ENGINE SIZE', 'CYLINDERS', 'FUELCONSUMPTION_CITY', 'FUELCONSUMPTION_Hwy', 'FUELCONSUMPTION_COMB', 'FUELCONSUMPTION_COMB_MPG', 'CO2EMISSIONS']]
print(df) # 1067 ROWS 13 COLUMN
print(cdf) # 1067 ROWS 6 COLUMN
cdf.head(1067)
```

	MODELYEAR	MAKE	MODEL	VEHICLECLASS	ENGINE SIZE	CYLINDERS	\
0	2014	ACURA	ILX	COMPACT	2.0	4	
1	2014	ACURA	ILX	COMPACT	2.4	4	
2	2014	ACURA	ILX HYBRID	COMPACT	1.5	4	
3	2014	ACURA	MDX 4WD	SUV - SMALL	3.5	6	
4	2014	ACURA	RDX AWD	SUV - SMALL	3.5	6	
...	
1062	2014	VOLVO	XC60 AWD	SUV - SMALL	3.0	6	
1063	2014	VOLVO	XC60 AWD	SUV - SMALL	3.2	6	
1064	2014	VOLVO	XC70 AWD	SUV - SMALL	3.0	6	
1065	2014	VOLVO	XC70 AWD	SUV - SMALL	3.2	6	
1066	2014	VOLVO	XC90 AWD	SUV - STANDARD	3.2	6	

	TRANSMISSION	FUELTYPE	FUELCONSUMPTION_CITY	FUELCONSUMPTION_Hwy	\
0	AS5	Z	9.9	6.7	
1	M6	Z	11.2	7.7	
2	AV7	Z	6.0	5.8	
3	AS6	Z	12.7	9.1	
4	AS6	Z	12.1	8.7	
...	
1062	AS6	X	13.4	9.8	
1063	AS6	X	13.2	9.5	
1064	AS6	X	13.4	9.8	
1065	AS6	X	12.9	9.3	
1066	AS6	X	14.9	10.2	

	FUELCONSUMPTION_COMB	FUELCONSUMPTION_COMB_MPG	CO2EMISSIONS
0	8.5	33	196

1	9.6	29	221
2	5.9	48	136
3	11.1	25	255
4	10.6	27	244
...
1062	11.8	24	271
1063	11.5	25	264
1064	11.8	24	271
1065	11.3	25	260
1066	12.8	22	294

[1067 rows x 13 columns]

	ENGINE SIZE	CYLINDERS	FUELCONSUMPTION_CITY	FUELCONSUMPTION_HWY	\
0	2.0	4	9.9	6.7	
1	2.4	4	11.2	7.7	
2	1.5	4	6.0	5.8	
3	3.5	6	12.7	9.1	
4	3.5	6	12.1	8.7	
...	
1062	3.0	6	13.4	9.8	
1063	3.2	6	13.2	9.5	
1064	3.0	6	13.4	9.8	
1065	3.2	6	12.9	9.3	
1066	3.2	6	14.9	10.2	

	FUELCONSUMPTION_COMB	CO2EMISSIONS
0	8.5	196
1	9.6	221
2	5.9	136
3	11.1	255
4	10.6	244
...
1062	11.8	271
1063	11.5	264
1064	11.8	271
1065	11.3	260
1066	12.8	294

[1067 rows x 6 columns]

[5]:

	ENGINE SIZE	CYLINDERS	FUELCONSUMPTION_CITY	FUELCONSUMPTION_HWY	\
0	2.0	4	9.9	6.7	
1	2.4	4	11.2	7.7	
2	1.5	4	6.0	5.8	
3	3.5	6	12.7	9.1	
4	3.5	6	12.1	8.7	
...	

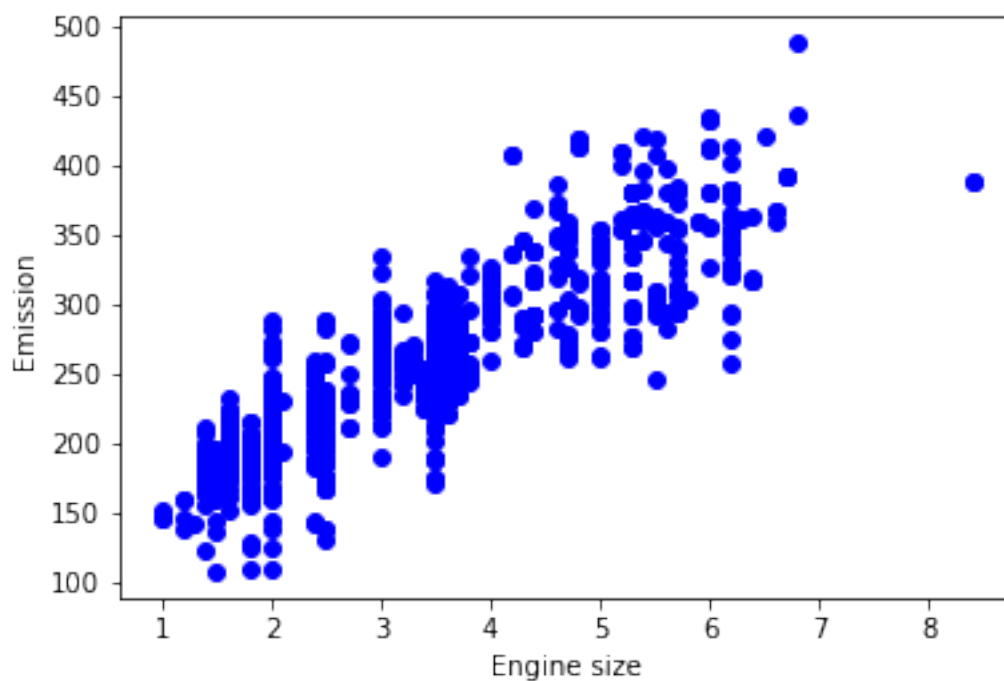
1062	3.0	6	13.4	9.8
1063	3.2	6	13.2	9.5
1064	3.0	6	13.4	9.8
1065	3.2	6	12.9	9.3
1066	3.2	6	14.9	10.2

	FUELCONSUMPTION_COMB	CO2EMISSIONS
0	8.5	196
1	9.6	221
2	5.9	136
3	11.1	255
4	10.6	244
...
1062	11.8	271
1063	11.5	264
1064	11.8	271
1065	11.3	260
1066	12.8	294

[1067 rows x 6 columns]

Lets plot Emission values with respect to Engine size:

```
[17]: plt.scatter(cdf.ENGINESIZE, cdf.CO2EMISSIONS, color='blue')
plt.xlabel("Engine size")
plt.ylabel("Emission")
plt.show()
```



Creating train and test dataset Train/Test Split involves splitting the dataset into training and testing sets respectively, which are mutually exclusive. After which, you train with the training set and test with the testing set. This will provide a more accurate evaluation on out-of-sample accuracy because the testing dataset is not part of the dataset that have been used to train the data. It is more realistic for real world problems.

This means that we know the outcome of each data point in this dataset, making it great to test with! And since this data has not been used to train the model, the model has no knowledge of the outcome of these data points. So, in essence, it's truly an out-of-sample testing.

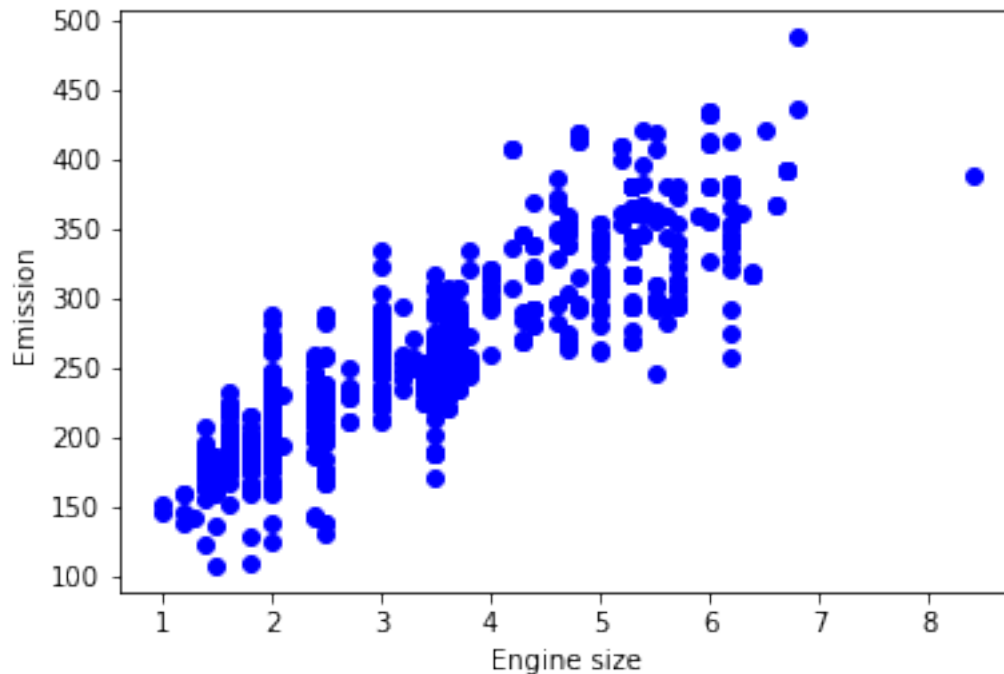
```
[8]: msk = np.random.rand(len(df)) < 0.8
      print(msk)
      train = cdf[msk]
      print(msk.shape)

      print(train.shape)
      test = cdf[~msk]
      print(test.shape)
```

```
[ True  True  True ... False  True  True]
(1067,)
(828, 6)
(239, 6)
```

Train data distribution

```
[9]: plt.scatter(train.ENGINESIZE, train.CO2EMISSIONS, color='blue')
      plt.xlabel("Engine size")
      plt.ylabel("Emission")
      plt.show()
```



Multiple Regression Model

In reality, there are multiple variables that predict the Co2emission. When more than one independent variable is present, the process is called multiple linear regression. For example, predicting co2emission using FUELCONSUMPTION_COMB, EngineSize and Cylinders of cars. The good thing here is that Multiple linear regression is the extension of simple linear regression model.

```
[15]: from sklearn import linear_model
regr = linear_model.LinearRegression()
x = np.asanyarray(train[['ENGINE_SIZE', 'CYLINDERS', 'FUELCONSUMPTION_COMB']])
y = np.asanyarray(train[['CO2EMISSIONS']])
regr.fit(x, y)
# The coefficients
print ('Coefficients: ', regr.coef_)
print ('Intercept: ',regr.intercept_)
```

```
Coefficients:  [[11.41629121  6.98492401  9.872258   ]]
Intercept:  [63.61863989]
```

As mentioned before, **Coefficient** and **Intercept** , are the parameters of the fit line. Given that it is a multiple linear regression, with 3 parameters, and knowing that the parameters are the intercept and coefficients of hyperplane, sklearn can estimate them from our data. Scikit-learn uses plain Ordinary Least Squares method to solve this problem.

Ordinary Least Squares (OLS) OLS is a method for estimating the unknown parameters in a linear regression model. OLS chooses the parameters of a linear function of a set of explanatory

variables by minimizing the sum of the squares of the differences between the target dependent variable and those predicted by the linear function. In other words, it tries to minimize the sum of squared errors (SSE) or mean squared error (MSE) between the target variable (y) and our predicted output (\hat{y}) over all samples in the dataset.

OLS can find the best parameters using the following methods: - Solving the model parameters analytically using closed-form equations - Using an optimization algorithm (Gradient Descent, Stochastic Gradient Descent, Newton's Method, etc.)

Prediction

```
[16]: y_hat= regr.predict(test[['ENGINE SIZE', 'CYLINDERS', 'FUELCONSUMPTION_COMB']])

x = np.asanyarray(test[['ENGINE SIZE', 'CYLINDERS', 'FUELCONSUMPTION_COMB']])
y = np.asanyarray(test[['CO2EMISSIONS']])

print("Residual sum of squares: %.2f"
      % np.mean((y_hat - y) ** 2))

# Explained variance score: 1 is perfect prediction
print('Variance score: %.2f' % regr.score(x, y))
```

Residual sum of squares: 534.79

Variance score: 0.85

explained variance regression score:

If \hat{y} is the estimated target output, y the corresponding (correct) target output, and Var is Variance, the square of the standard deviation, then the explained variance is estimated as follow:

$$\text{explainedVariance}(y, \hat{y}) = 1 - \frac{\text{Var}\{y - \hat{y}\}}{\text{Var}\{y\}}$$

The best possible score is 1.0, lower values are worse.

Practice

Try to use a multiple linear regression with the same dataset but this time use **FUEL CONSUMPTION in CITY** and **FUEL CONSUMPTION in HWY** instead of FUELCONSUMPTION_COMB. Does it result in better accuracy?

```
[33]: # write your code here

regr = linear_model.LinearRegression()
x = np.
    ↳ asanyarray(train[['ENGINE SIZE', 'CYLINDERS', 'FUELCONSUMPTION_CITY', 'FUELCONSUMPTION_HWY']])
y = np.asanyarray(train[['CO2EMISSIONS']])
regr.fit(x, y)
print('Coefficients: ', regr.coef_)
y_ = regr.
    ↳ predict(test[['ENGINE SIZE', 'CYLINDERS', 'FUELCONSUMPTION_CITY', 'FUELCONSUMPTION_HWY']])
x = np.
    ↳ asanyarray(test[['ENGINE SIZE', 'CYLINDERS', 'FUELCONSUMPTION_CITY', 'FUELCONSUMPTION_HWY']])
y = np.asanyarray(test[['CO2EMISSIONS']])
```



```
print("Residual sum of squares: %.2f"% np.mean((y_ - y) ** 2))
print('Variance score: %.2f' % regr.score(x, y))
```

Coefficients: `[[10.2622554 8.31345003 5.63232638 3.32971759]]`

Residual sum of squares: 529.91

Variance score: 0.88

Double-click [here](#) for the solution.

Want to learn more?

IBM SPSS Modeler is a comprehensive analytics platform that has many machine learning algorithms. It has been designed to bring predictive intelligence to decisions made by individuals, by groups, by systems – by your enterprise as a whole. A free trial is available through this course, available here: [SPSS Modeler](#)

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Thanks for completing this lesson!

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Saeed Aghabozorgi, PhD is a Data Scientist in IBM with a track record of developing enterprise level applications that substantially increases clients' ability to turn data into actionable knowledge. He is a researcher in data mining field and expert in developing advanced analytic methods like machine learning and statistical modelling on large datasets.

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