Longest Path for Knight on 32x32 Chess Board

<u>Problem importance:</u> A 32x32 chess board has 1024 cells/nodes, and a pure exploration approach requires searching up to depths of 1024. This combined that each node have a maximum exploration factor 8 child nodes make a direct search approach infeasible (8^1024). Tried applying some heuristics in addition to Wansdorff approach [1], but could not attain performance good enough for present available computing power.

<u>Background:</u> As proved by Conrada [3], for any chess board with of dimension (n), and *n>=6* there will always exit a Hamilton path [2] from any start *s* to any end *e*.
<u>Basic approach:</u> Using Conrada's theorem, the problem for *longest path* for given 32x32 chess board can now be rephrased as *finding Hamilton path*.

As finding Hamilton path for a 32x32 chess board is in itself very expensive following divide and conquer approach is applied.

<u>Basic concept:</u> ***The 32x32 board is divided into a set of 6x6, 6x8, 8x6 and 8x8 sub-boards, as shown figure-1. Now a Hamilton path computed in each of the sub-boards such that, these smaller path can then be stitched together (combined) to form one complete path covering almost all of 32x32 chess board cells.***

There are more details and scenarios to consider to this presumably simple solution. Some such questions are:
- How to divide 32x32 board into smaller boards such that start and end nodes fall in 6x6 sub-boards.
- How to find the start and end points of each sub-boards such that they can stitched with other paths.
- In what sequence should the path cross over these sub-boards.
- How to optimally find a Hamilton path within each sub-board.

<u>Here are two of the major scenarios:</u>
a) <u>Length of the longest path:</u> Observing that (i) there are equal number of black and white nodes/cells on a chessboard with even size dimensions, and (ii) knight's movement is such that at each step it jumps to a cell with a complimentary color. That is a knight in white cell will only reach cells of black color in single step. Hence, if *color* (*start cell*) == *color* (*end cell*), the longest possible path will only cover N-1 cells. Where N is total number of cells on the board. That is:

Length of longest path $= N$ *if (color (start) != color (end))*
$= N-1$ *if (color (start) == color (end))*

Each of these situations are handled separately when finding Hamilton paths within sub-boards

b) When start and end cells fall on different sub-boards, it is easy to find individual Hamilton paths within each sub-board and join them. But when both the start and end cells fall within same sub-board it requires additional. The Hamilton path within this common sub-board needs to be divided into two chucks one which forms the head of the final path, and other which forms the tail of the final path.

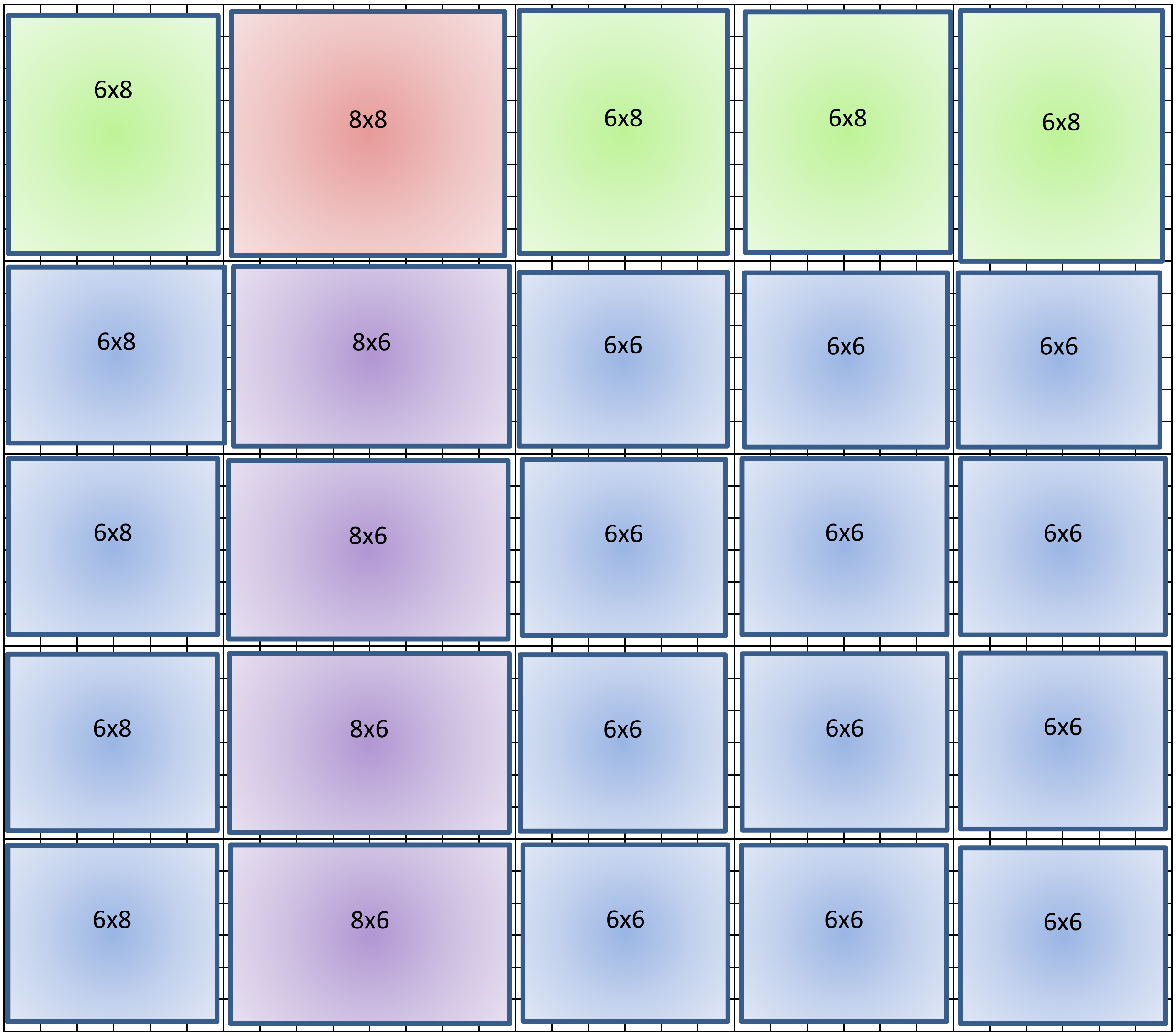

Figure-1: One of the scenario of dividing a 32x32 board into smaller sub-boards, such that each dimension of 32d is covered by (four of 6d + one of 8d). (Axel Conrada n.d.)

Reference:

[1] Warnsdorff Approach: http://www.slac.stanford.edu/cgi-wrap/getdoc/slac-pub-0261.pdf

[2] Hamilton Path: http://en.wikipedia.org/wiki/Hamiltonian_path

[3] “*Solution of the knight's Hamiltonian path problem on chessboards*“ Axel Conrada, Tanja Hindrichsb, Hussein Morsyc, Ingo Wegener http://www.sciencedirect.com/science/article/pii/0166218X9200170Q

Appendix:

Some notes on diagonal and side way stitching sub-boards: