Mustererkennung/Machine Learning - Assignment 9

In [1]:

```
import numpy as np
import pandas as pd
from sklearn.model_selection import train_test_split
from sklearn.utils import shuffle
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.preprocessing import StandardScaler
from sklearn.preprocessing import OneHotEncoder
from sklearn.metrics import accuracy_score
%matplotlib inline
```

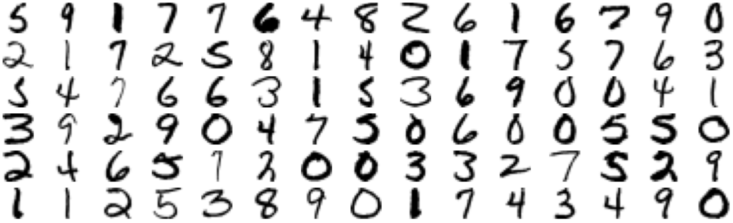
Excercise Solution: Multilayer-Perceptron (MLP) with Backpropogation

Splitting the data into training/test and according to their class memberships

In [2]:

```
training_data = np.array(pd.read_csv('zip.train', sep=' ', header=None),dtype=np.float
test_data = np.array(pd.read_csv('zip.test', sep =' ',header=None),dtype=np.float128)

X_train, y_train = training_data[:,1:-1], training_data[:,0]
X_test, y_test = test_data[:,1:], test_data[:,0]
def show_numbers(X):
    num_samples = 90
    indices = np.random.choice(range(len(X)), num_samples)
    sample_digits = X[indices]

    fig = plt.figure(figsize=(20, 6))

    for i in range(num_samples):
        ax = plt.subplot(6, 15, i + 1)
        img = 255 - sample_digits[i].reshape((16, 16))
        plt.imshow(img, cmap='gray')
        plt.axis('off')
show_numbers(X_train)
```

```
/home/suresh/.local/lib/python3.8/site-packages/IPython/core/pylabtools.py:132: UserWa
rning: Casting input data from 'float128' to 'float64' for imshow
  fig.canvas.print_figure(bytes_io, **kw)
```



In [3]:

```
#y_labels in form of vector  for digits for multiclass
enc = OneHotEncoder()
# 0 -> (1, 0, 0, 0), 1 -> (0, 1, 0, 0), 2 -> (0, 0, 1, 0), 3 -> (0, 0, 0, 1)
y_OH_train = enc.fit_transform(np.expand_dims(y_train,1)).toarray()
y_OH_test = enc.fit_transform(np.expand_dims(y_test,1)).toarray()
#print( X_train)
```

In [4]:

```
#Data Normalization/standardization
scaler = StandardScaler()
scaler.fit(X_train)
X_train=scaler.transform(X_train)
X_test=scaler.transform(X_test)
```

In [5]:

```
class Sigmoid:
    def activation(z):
        return 1 / (1 + np.exp(-z))
    def gradient(z):
        return Sigmoid.activation(z) * (1 - Sigmoid.activation(z))
class Relu:
    def activation(z):
        z[z < 0] = 0
        return z
    def gradient(x):
        x[x<=0] = 0
        x[x>0] = 1
        return x

class softmax:
    def activation(Z):
        expZ = np.exp(Z - np.max(Z))
        return expZ / expZ.sum(axis=0, keepdims=True)
class Multilayer_perceptron:
    def __init__(self, total_layer=2,dimensions=None, activations=None, learning_rate
        """
        parameters
        1. total_layer: no of layers including input layer, hidden layers and output
        2. dimensions: Dimensions of the neural net. (no of input, no of nodes in hid
        3. activations:Activations functions for each layer.
        4. learning_rate: learning

        """

        self.n_layers = total_layer
        self.loss = None
        self.learning_rate = learning_rate
        self.sizes=dimensions

        # Weights and biases are initiated by index. For a one hidden layer net you w
        self.w = {}
        self.b = {}

        # Activations are also initiated by index. For the example we will have activa
        self.activations = {}

        for i in range(len(dimensions) - 1):
            #self.w[i + 1] = np.ones((dimensions[i+1], dimensions[i]))
            limit   = 1 / np.sqrt(dimensions[i])
            self.w[i + 1] = np.random.uniform(-limit, limit,(dimensions[i+1], dimensio
            self.b[i + 1] = np.zeros((dimensions[i + 1],1))
            self.activations[i + 2] = activations[i]

    def _feed_forward(self, x):
        """
        Execute a forward feed through the network.

         x:input data vectors.
        return:  Node outputs and activations per layer.
                 The numbering of the output is equivalent to the layer numbers.
        """

        # w(x) + b
        z = {}

        # activations: f(z)
```

```
        a = {1: x.T}  # First layer has no activations as input. The input x is the i

        for i in range(1, self.n_layers):
            # current layer = i
            # activation layer = i + 1
            #print(np.dot( self.w[i],a[i]))
            z[i + 1] = np.dot( self.w[i],a[i]) + self.b[i]
            #print(z[i+1])
            a[i + 1] = self.activations[i + 1].activation(z[i + 1])
            #print(a[i+1])
        #print(z,a)
        return z, a
    #backpropogation function
    def back_propogation(self, x, y):
        self.Z,self.A=self._feed_forward(x)
        self.dW = {}
        self.dB = {}
        self.dZ = {}
        self.dA = {}
        L = self.n_layers
        #print(self.A[L],self.A[L].shape)
        #print(y,y.shape)
        #gradient of error in respect of z  and z=wx+b
        self.dZ[L] = (self.A[L] - y.T)
        #print(self.A[L],y,self.dZ[L])
        for k in range(L, 1, -1):
          #previous value in chain rule
          delta=np.dot(self.w[k-1].T,self.dZ[k])
          #print('iter',k)
          #self.A[k-1]=np.array(self.A[k-1]).reshape(self.A[k-1].shape[0],1)
          self.dW[k-1] = np.dot(self.dZ[k],self.A[k-1].T)*(1/self.total_samples)
          #print('dw',self.dW[k-1])
          self.dB[k-1] = np.sum(self.dZ[k], axis=1, keepdims=True)*(1/self.total_sampl
          #print('dB',self.dB[k-1])
          # assuming activation function start from in between 1st layer and 2nd laye
          if(k>2):
              self.dZ[k-1] = delta*self.activations[k-1].gradient(self.A[k-1])

    def fit(self, X, Y, epochs=100, display_loss=True,test_data=None):

        if display_loss:
          loss = {}
          train_acc={}
          test_acc={}
        self.total_samples = X.shape[0]
        for epoch in range(epochs):
          dW = {}
          dB = {}
          for i in range(self.n_layers - 1):
            dW[i+1] = np.zeros((self.sizes[i+1],self.sizes[i]))
            dB[i+1] = np.zeros((self.sizes[i+1],1))
          #for x, y in zip(X, Y):
          X,Y=self.shuffle_data(X, Y)
          self.back_propogation(X, Y)
          for i in range(self.n_layers-1):
              dW[i+1] += self.dW[i+1]
              dB[i+1] += self.dB[i+1]


          #print(total_samples)
          for i in range(self.n_layers-1):
            #self.b[i+1]=self.b[i+1].reshape(1,-1)
            #print(dB[i+1].shape)
            self.w[i+1] -= self.learning_rate * dW[i+1]
            self.b[i+1] -= self.learning_rate * dB[i+1]
```

```
          if display_loss:
            #Y_pred = self.predict(X)
            loss[epoch] = self.cross_entropy(Y)
            train_acc[epoch]=self.accuracy(X,Y)
            test_acc[epoch]=self.accuracy(test_data[0],test_data[1])
            if epoch%500==0:
                print("epoch",epoch,'loss:',loss[epoch],' Training Accuracy:',train_ac
        #loss plot
        if display_loss:
           self.plot_loss_acc([loss,(train_acc,test_acc)])

    #prediction method
    def predict(self, X):
        lin_sum,output = self._feed_forward(X)
        return output[self.n_layers]
    #cross entropy calculation function
    def cross_entropy(self,Y):
        return  -np.mean(Y * np.log(self.A[self.n_layers].T))
    def accuracy(self,X,Y):
        Y_pred_test = self.predict(X)
        Y_pred_test = np.argmax(Y_pred_test.T,1)
        Y = np.argmax(Y, axis=1)
        return accuracy_score(Y_pred_test, Y)
    def plot_loss_acc(self,data):
        fig, axs = plt.subplots(1,2,figsize=(15, 10))
        y_label=['loss','Training Accuracy','Test Accuracy']
        for kk, (ax,yy) in enumerate(zip(axs.reshape(-1),data)):
            if(kk==1):
                ax.plot(yy[0].values(),label=y_label[kk])
                ax.plot(yy[1].values(), label=y_label[kk+1])
                ax.legend(loc='upper left')
                ax.set(xlabel='Epochs', ylabel='accuracy')
            else:
                ax.plot(yy.values())
                ax.set(xlabel='Epochs', ylabel=y_label[kk])
        #fig.delaxes(axs[1][1])
        plt.show()
    def shuffle_data(self,X, y, seed=None):
        """ Random shuffle of the samples in X and y """
        if seed:
            np.random.seed(seed)
        idx = np.arange(X.shape[0])
        np.random.shuffle(idx)
        return X[idx], y[idx]
```

(a) Optimize width (the number of neurons in a hidden layer; it is usually the same for all of them) and depth of the network. Try to find a setting that trains in a reasonable time. Plot the loss.

Answer: We tried with different architecture of neural network like number of layers , and number of neurons in each layer, however we found best results with 1 layer having 16 neurons with sigmoid activation function(hidden layer) and softmax activation function(output layer).

Neural Network Architecture 1: Hidden Layer=1, 16 neurons, Sigmoid (hidden layer), Softmax(output layer)

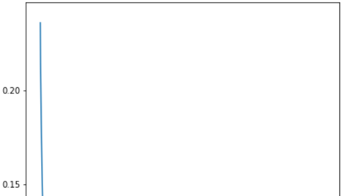
In [6]:

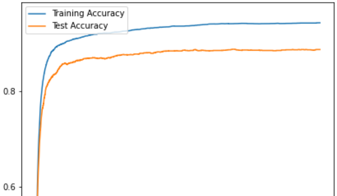
```
#dimenstions=(nodes_inputlayer,nodes_hidden_layer1,nodes_hidden_layer2, ....., nodes_h
dimenstions=(X_train.shape[1], 16, len(np.unique(y_train)))
#activations=(activation_function_of_hidden_layer1,activation_function_of_hidden_layer
activations_funct_list= ( Sigmoid, softmax)
mlp = Multilayer_perceptron(total_layer=len(dimenstions),dimensions=dimenstions, activ
```

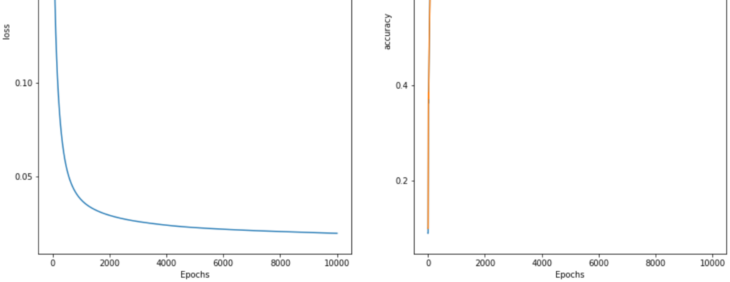
In [7]:

```
mlp.fit( X_train,y_OH_train, epochs=10000, display_loss=True,test_data=(X_test,y_OH_te
```

```
epoch 0 loss: 0.23562771682995468  Training Accuracy: 8.942531888629818  Test Accurac
y: 9.965122072745391
epoch 500 loss: 0.05384783461759707  Training Accuracy: 87.4365656288575  Test Accurac
y: 82.31190832087692
epoch 1000 loss: 0.03781895325598989  Training Accuracy: 90.0562337127966  Test Accura
cy: 85.74987543597409
epoch 1500 loss: 0.03241690274253408  Training Accuracy: 91.09861473048964  Test Accur
acy: 86.49725959143
epoch 2000 loss: 0.029522470578699153  Training Accuracy: 91.88040049375942  Test Accu
racy: 87.045341305431
epoch 2500 loss: 0.02760746927816763  Training Accuracy: 92.19585790700864  Test Accur
acy: 86.84603886397608
epoch 3000 loss: 0.026245943241190728  Training Accuracy: 92.47016870113839  Test Accu
racy: 87.593423019432
epoch 3500 loss: 0.025194061015126994  Training Accuracy: 92.90906597174599  Test Accu
racy: 87.7927254608869
epoch 4000 loss: 0.024309079102481233  Training Accuracy: 93.21080784528871  Test Accu
racy: 88.09167912306926
epoch 4500 loss: 0.023587018242810766  Training Accuracy: 93.45768756000548  Test Accu
racy: 88.39063278525163
epoch 5000 loss: 0.023024755899181088  Training Accuracy: 93.64970511589631  Test Accu
racy: 88.44045839561535
epoch 5500 loss: 0.02256055597558037  Training Accuracy: 93.75942943354822  Test Accur
acy: 88.5401096163428
epoch 6000 loss: 0.022157346761153596  Training Accuracy: 94.00630914826499  Test Accu
racy: 88.49028400597906
epoch 6500 loss: 0.021797549219961423  Training Accuracy: 94.14346454532986  Test Accu
racy: 88.5401096163428
epoch 7000 loss: 0.021474253227201166  Training Accuracy: 94.18461116444932  Test Accu
racy: 88.78923766816143
epoch 7500 loss: 0.021197033432662587  Training Accuracy: 94.18461116444932  Test Accu
racy: 88.58993522670653
epoch 8000 loss: 0.02093704043910084  Training Accuracy: 94.15718008503634  Test Accur
acy: 88.5401096163428
epoch 8500 loss: 0.02068204273267354  Training Accuracy: 94.22575778356878  Test Accur
acy: 88.5401096163428
epoch 9000 loss: 0.020430571100085403  Training Accuracy: 94.22575778356878  Test Accu
racy: 88.68958644743398
epoch 9500 loss: 0.020189115161869636  Training Accuracy: 94.3080510218077  Test Accur
acy: 88.73941205779771
```







In [8]:

```
import sklearn
Y_pred_train = mlp.predict(X_train)
print(Y_pred_train.T.shape)
Y_pred_train = np.argmax(Y_pred_train.T,1)

Y_pred_test = mlp.predict(X_test)
Y_pred_test = np.argmax(Y_pred_test.T,1)

print(Y_pred_train.shape,y_train.shape)
accuracy_train = accuracy_score(Y_pred_train, y_train)
accuracy_test = accuracy_score(Y_pred_test, y_test)

print("Training accuracy", round(accuracy_train, 2))
print("Test Dataset accuracy", round(accuracy_test, 2))
print(sklearn.metrics.classification_report(y_test, Y_pred_test))
```

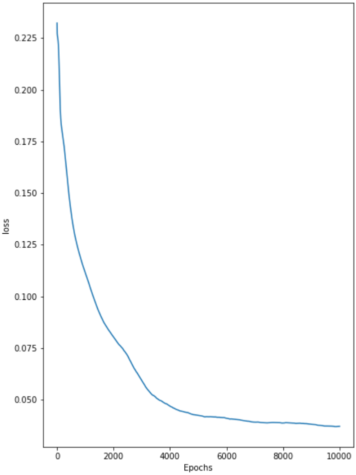
```
(7291, 10)
(7291,) (7291,)
Training accuracy 0.94
Test Dataset accuracy 0.89
              precision    recall  f1-score   support

         0.0       0.93      0.97      0.95       359
         1.0       0.97      0.94      0.96       264
         2.0       0.84      0.85      0.85       198
         3.0       0.86      0.88      0.87       166
         4.0       0.79      0.86      0.82       200
         5.0       0.86      0.80      0.83       160
         6.0       0.91      0.89      0.90       170
         7.0       0.88      0.89      0.89       147
         8.0       0.89      0.79      0.84       166
         9.0       0.88      0.89      0.89       177

    accuracy                           0.89      2007
   macro avg       0.88      0.88      0.88      2007
weighted avg       0.89      0.89      0.89      2007
```

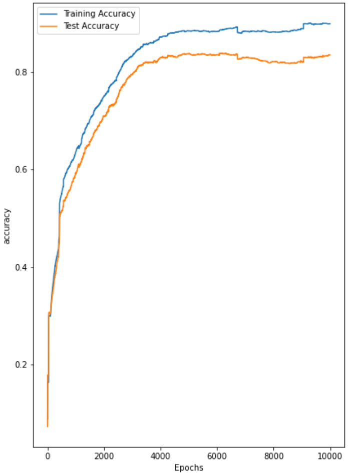
Neural Network Architecture 2: Hidden Layer=2, neurons in each hidden layer=16, Sigmoid Activation (hidden layer), Softmax(output layer)

In [18]:

```
#dimenstions=(nodes_inputlayer,nodes_hidden_layer1,nodes_hidden_layer2, ....., nodes_
dimenstions=(X_train.shape[1], 16,16, len(np.unique(y_train)))
#activations=(activation_function_of_hidden_layer1,activation_function_of_hidden_laye
activations_funct_list= ( Sigmoid, Sigmoid,softmax)
mlp1 = Multilayer_perceptron(total_layer=len(dimenstions),dimensions=dimenstions, act
mlp1.fit( X_train,y_OH_train, epochs=10000, display_loss=True,test_data=(X_test,y_OH_
```

```
epoch 0 loss: 0.23234757232659572  Training Accuracy: 8.846523110684405  Test Accurac
y: 7.324364723467862
epoch 500 loss: 0.14044484565345697  Training Accuracy: 54.99931422301467  Test Accura
cy: 51.569506726457405
<ipython-input-5-9b9c8a1fb92a>:3: RuntimeWarning: overflow encountered in exp
  return 1 / (1 + np.exp(-z))
epoch 1000 loss: 0.11140425427928147  Training Accuracy: 63.585242079275815  Test Accu
racy: 59.342301943198805
epoch 1500 loss: 0.09194209466151916  Training Accuracy: 70.18241667809629  Test Accur
acy: 65.6203288490284
epoch 2000 loss: 0.08030490178296298  Training Accuracy: 74.9828555753669  Test Accura
cy: 70.95166915794718
epoch 2500 loss: 0.07126757089183525  Training Accuracy: 79.44040597997531  Test Accur
acy: 74.98754359740907
epoch 3000 loss: 0.05925639682311391  Training Accuracy: 83.32190371691127  Test Accur
acy: 79.52167414050822
epoch 3500 loss: 0.05110381918383543  Training Accuracy: 85.77698532437252  Test Accur
acy: 82.01295465869457
epoch 4000 loss: 0.04686764792092947  Training Accuracy: 87.14853929502125  Test Accur
acy: 82.95964125560538
epoch 4500 loss: 0.044082922649294254  Training Accuracy: 88.01261829652996  Test Accu
racy: 83.10911808669657
epoch 5000 loss: 0.042419395893570945  Training Accuracy: 88.38293786860513  Test Accu
racy: 83.75685102142502
epoch 5500 loss: 0.04169286127323958  Training Accuracy: 88.36922232889864  Test Accur
acy: 83.70702541106128
epoch 6000 loss: 0.04098654195125314  Training Accuracy: 88.28692909065971  Test Accur
acy: 83.5575485799701
epoch 6500 loss: 0.040132085804207004  Training Accuracy: 88.91784391715814  Test Accu
racy: 83.5575485799701
epoch 7000 loss: 0.0391358307199845  Training Accuracy: 88.35550678919215  Test Accura
cy: 82.90981564524166
epoch 7500 loss: 0.0388665678366189  Training Accuracy: 88.32807570977917  Test Accura
cy: 82.41155954160439
epoch 8000 loss: 0.038739449942561524  Training Accuracy: 88.2046358524208  Test Accur
acy: 82.11260587942202
epoch 8500 loss: 0.03857817555891832  Training Accuracy: 88.27321355095323  Test Accur
acy: 81.81365221723966
epoch 9000 loss: 0.03807607681199798  Training Accuracy: 88.90412837745166  Test Accur
acy: 82.11260587942202
epoch 9500 loss: 0.03724910018627447  Training Accuracy: 89.79563845837333  Test Accur
acy: 82.95964125560538
```



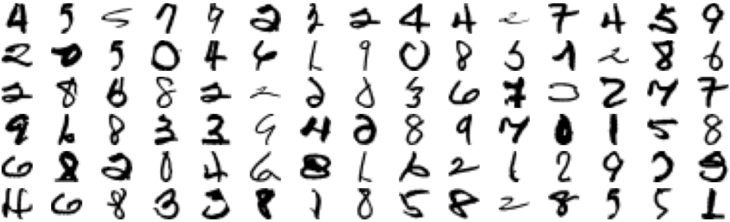


(b) Show some digits that are classified incorrectly.

In [10]:

```
Incorrectly_classfied_images=X_test[Y_pred_test!=y_test]
Incorrectly_classfied_images=scaler.inverse_transform(Incorrectly_classfied_images)
print('Incorrectly classfied Digits')
show_numbers(Incorrectly_classfied_images)
```

```
Incorrectly classfied Digits
```



(c) Plot your first weight layer as a grayscale image.

To scale the weights in range of grayscale image pixel(0,255) m=(m−rmin)/(rmax−rmin)*(tmax−tmin)+tmin will scale m linearly into [tmax, tmin] as desired. so here tmax=255, tmin=0 ,rmin denote the minimum of the range of your weights, rmax denote the maximum of the range of your weights

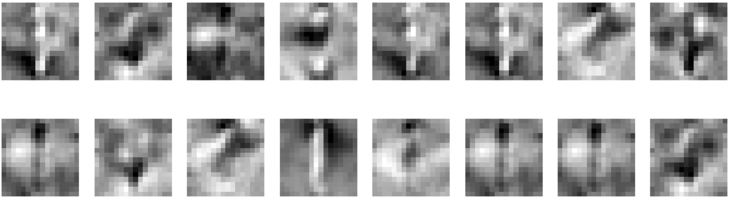
In [19]:

```
def show_numbers(X):
    num_samples = X.shape[0]
    print(num_samples)
    indices = np.random.choice(range(len(X)), num_samples)
    sample_digits = X[indices]

    fig = plt.figure(figsize=(20, 6))
    nu_images=np.round(num_samples/2)
    for i in range(num_samples):
        ax = plt.subplot(2,nu_images, i + 1)
        img = 255 - sample_digits[i].reshape((16, 16))
        plt.imshow(img, cmap='gray')
        plt.axis('off')
```

In [20]:

```
#rescalling values into the grayscale range 0 to 255
min_value=np.min(mlp.w[1])
max_value=np.max(mlp.w[1])
scaled_value=(mlp.w[1]-min_value)/(max_value-min_value)*255
#plt.imshow(scaled_value.T, cmap="gray")
#plt.show()
show_numbers(scaled_value)
```

```
16
<ipython-input-19-58b5dc5e4b46>:10: MatplotlibDeprecationWarning: Passing non-integers
as three-element position specification is deprecated since 3.3 and will be removed tw
o minor releases later.
  ax = plt.subplot(2,nu_images, i + 1)
```



In [ ]: