

## NUMBER SYSTEMS - WORKSHEET 2

① a)  $(42)_{10} = 00101010_2$

b)  $-63_{10} =$

$$\begin{array}{r} 00111111 \\ 11000000 \\ \hline (+)1 \\ \hline 11000001 \end{array}$$

$-63_{10} = 11000001_2$

c)  $124_{10} = 01111100_2$

d)  $-128_{10} = 10000000_2$

e)  $133_{10} = 10000101_2$

② a)  $00000101_2$

b)  $1111010_2$

③ min:  $-2^{n-1}$

max:  $2^{n-1} - 1$

For 5 bits,

$$-2^{(5-1)} = -2^4 = -16$$

$$2^{(5-1)} - 1 = 2^4 - 1 = 15$$

5 bit 2's complement numbers  $> 0 = 15$

5 bit 2's complement number  $< 0 = -16$

④ No. of rows in the processor =  $2^8$  bits.

No. of columns in the processor =  $2^9$  bits.

$$\text{Total no. of bits} = 2^8 \times 2^9 = 2^{17} = 2^{10} \times 2^7$$

$$= 2^7 \text{ Kilobytes} = 128 \text{ Kilobytes.}$$

$$\begin{array}{l} \textcircled{5} \text{ a) } 10011\ 001 \Rightarrow 153_{10} \\ \underline{01000\ 100} \rightarrow 68_{10} \\ \underline{110111\ 01} \rightarrow 221_{10} \end{array}$$

$$\begin{array}{l} \text{b) } 11010010 \rightarrow 210_{10} \\ \underline{10110110} \rightarrow 182_{10} \end{array}$$

110001000 Result overflows.

$$\begin{array}{l} \text{c) } 1011011 \rightarrow 91 \\ - 101110 \rightarrow 46 \\ \hline 101101 \rightarrow 45 \end{array}$$

$$\begin{array}{l} \text{d) } 1001001 \rightarrow 73 \\ - 10101 \rightarrow 21 \\ \hline 0110100 \rightarrow 52 \end{array}$$

$$\begin{array}{r} \textcircled{6} \text{ a) } 1001_2 \times 1100_2 = \begin{array}{r} 1001 \times 1100 \\ \hline 0000 \\ 0000 \\ 1001 \\ 1001 \\ \hline 1101100 \end{array} \end{array}$$

$$= 1101100_2$$

$$b) 11011_2 \times 1001_2$$

$$\begin{array}{r} 11011 \times 1001 \\ \hline 11011 \\ 00000 \\ 00000 \\ 11011 \\ \hline 11110011 \end{array}$$

$$= 11110011_2 \rightarrow 243$$

$$c) 101101_2 / 11_2$$

$$\begin{array}{r} 1111 \\ \hline 11 \overline{) 101101} \\ \underline{11} \phantom{01} \\ 101 \\ \underline{11} \phantom{0} \\ 100 \\ \underline{010} \phantom{0} \\ 011 \\ \underline{11} \phantom{0} \\ 0 \end{array}$$

$$= 1111_2$$

$$d) 11111_2 / 110_2$$

$$\begin{array}{r} 1010.1 \\ \hline 110 \overline{) 111111} \\ \underline{110} \phantom{000} \\ 111 \\ \underline{110} \phantom{00} \\ 110 \\ \underline{110} \phantom{0} \\ 0 \end{array}$$

$$= 1010.1_2$$

7 a)  $16_{10} + 9_{10}$

$$16_{10} = 010000_2$$

$$9_{10} = 001001_2$$

$$\begin{array}{r} 010000 \\ 001001 \\ \hline 011001_2 = 25_{10} \end{array}$$

b)  $27_{10} + 31_{10}$

$$27_{10} = 011011_2$$

$$31_{10} = 011111_2$$

$$\begin{array}{r} 011011 \\ 011111 \\ \hline 111010_2 = 58_{10} \end{array}$$

c)  $-4_{10} + 19_{10}$

$$4 \Rightarrow \begin{array}{r} 000100 \\ 111011 \\ \hline \end{array}$$

$$-4 \Rightarrow \begin{array}{r} 111100 \\ \hline \end{array}$$

$$19_{10} + (-4)_{10}$$

$$010011$$

$$111100$$

$$\boxed{1} \begin{array}{r} 001111 \\ \hline \end{array}$$

$$\text{ans: } 001111_2 = 15_{10}$$

d)  $3_{10} + 32_{10}$

$$3_{10} = 000011_2$$

$$32_{10} = 100000_2$$

$$3_{10} + 32_{10} = 35_{10}$$

$$100011_2$$

⑧ a)  $9_{10} - 7_{10} = 9_{10} + (-7_{10}) =$

$$\begin{array}{r} 7_{10} = 00111 \\ \underline{11000} \\ 1 \\ \hline 11001 = -7_{10} \end{array}$$

$9_{10} + (-7_{10})$

$$\begin{array}{r} 01001 \rightarrow 9 \\ 11001 \rightarrow -7 \\ \hline 1 \overline{)00010} \rightarrow 4_{10} \end{array}$$

b)  $12_{10} - 15_{10} = 12_{10} + (-15)_{10}$

$$\begin{array}{r} 15_{10} = 01111 \\ \underline{10000} \\ 1 \\ \hline 10001 = -15_{10} \end{array}$$

$$\begin{array}{r} 12_{10} + (-15)_{10} \\ 01100 \rightarrow 12 \\ 10001 \rightarrow -15 \\ \hline 11101 \rightarrow -3_{10} \end{array}$$

c)  $-6_{10} - 11_{10} = -6_{10} + (-11)_{10}$

$$\begin{array}{r} 6_{10} \rightarrow 00110 \\ 11001 \\ \underline{1} \\ 11010 = -6_{10} \end{array}$$

$$\begin{array}{r} 11_{10} \rightarrow 01011 \\ 10100 \\ \underline{1} \\ 10101 \rightarrow -11_{10} \end{array}$$

$-6_{10} + (-11)_{10}$

$$\begin{array}{r} 11010 \\ 10101 \\ \hline 1 \overline{)00111} \end{array}$$

As range btw  $-16$  to  $15$ ,  
 $-17$  cannot be represented  
 in a 5 bit 2's complement

$$d) 4_{10} - 8_{10} = 4_{10} + (-8)_{10}$$

$$8_{10} \rightarrow \begin{array}{r} 01000 \\ 10111 \\ \hline 11000 \end{array} \rightarrow -8_{10}$$

$$\begin{array}{r} 00100 \\ 11000 \\ \hline 11100 \end{array} \rightarrow -4_{10}$$

$$(9a) 3 + (-7)$$

$$3 \rightarrow 0011$$

$$+(-7) \rightarrow \begin{array}{r} 1001 \\ \hline 1100 \end{array} \Rightarrow -4_{10}$$

$$7 \Rightarrow \begin{array}{r} 0111 \\ 1000 \\ \hline 1001 \end{array}$$

$$b) -3 + 7$$

$$7 \Rightarrow 0111$$

$$-3 \Rightarrow \begin{array}{r} 1101 \\ \hline 10100 \\ \hline 0100 \end{array}$$

$$= 0100$$

$$3 \Rightarrow \begin{array}{r} 0011 \\ 1100 \\ \hline 1101 \end{array} \Rightarrow -3$$

$$c) 11 + (-11)$$

$$11 \Rightarrow 01011$$

$$-11 \Rightarrow \begin{array}{r} 10101 \\ \hline 100000 \end{array}$$

$$11 + (-11) = 0$$

$$11 \Rightarrow \begin{array}{r} 01011 \\ 10100 \\ \hline 10101 \end{array} \Rightarrow -11$$



d)  $18 - (-3) = 18 + 3$

$18 \Rightarrow 10010$

$3 \rightarrow 00011$

$$\begin{array}{r} 10010 \\ + 00011 \\ \hline 10101 \end{array} \Rightarrow 21$$

(10) I disagree with both Ben and Alyssa;

Ben: All integers divisible by 6 have exactly 2 1's in their binary.

But,

$$30 = 011110$$

It has 4 1's in their binary.

Alyssa: All integers divisible by 6 have an even number of 1's in their binary.

But,

$$42 = 101010$$

It has 3 1's in their binary.