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1. Basic Properties of the integers

→ Divisibility & primality

$Z = \{\ldots -2, -1, 0, 1, 2 \ldots\}$. for $a, b \in Z$, a divides b if $az = b$ for some $z \in Z$.

$a \mid b \Rightarrow$ a is divisor of b

$\Rightarrow$ b is multiple of a

$\Rightarrow$ b is divisible by a.

$a \nmid b \Rightarrow$ a doesn't divide b.

→ **Theorem 1** - $\forall\ a, b, c \in Z$,

i) $a \mid a$, $1 \mid b$, & $a \mid 0$.

ii) $0 \mid b$ iff $b = 0$.

iii) $a \mid b$ iff $-a \mid b$ & iff $a \mid -b$

iv) $a \mid b$ & $a \mid c \Rightarrow a \mid (b+c)$

v) $a \mid b$ & $b \mid c \Rightarrow a \mid c$

→ **Proof** - i) $a \cdot 1 = a$; $\therefore a \mid a$

$1 \cdot b = b$; $\therefore 1 \mid b$

$a \cdot 0 = 0$; $\therefore a \mid 0$

$\Bigg\}$ def^n of divisibility $az = b$.

ii) $b = 0$, then $0 \cdot 0 = 0$; $\therefore 0 \mid b$.

→ **Note** - If $a \mid b$ & $b \neq 0$, then $1 \leq |a| \leq |b|$

If $az = b \neq 0$, then $a \neq 0$ & $z \neq 0$,

$|a| \geq 1$, $|z| \geq 1$, so, $|a| \leq |a||z| = |b|$.

→ **Theorem 2** - $\forall\ a, b \in Z$, $a \mid b$ & $b \mid a$ iff $a = \pm b$.

$\forall\ a \in Z$, $a \mid 1$ iff $a = \pm 1$.

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→ Proof - If $a = \pm b$, then $a | b$ & $b | a$.
Assume $a | b$ & $b | a$.
TP :- $a = \pm b$.
If $a = 0$, then $b = 0$ & vice versa.
Assume both $a, b \neq 0$.
$a | b \Rightarrow |a| \leq |b|$
$b | a \Rightarrow |b| \leq |a|$.
Thus $|a| = |b|$.
$\therefore a = \pm b$.
Similarly, put $b = 1$, we get $a | 1$.

→ Product of any 2 non-zero integers is again non-zero
Using cancellation laws,- $a, b, c \in \mathbb{Z}$, such that $a \neq 0$ & $ab = ac$. then $\Rightarrow a(b-c) = 0$.
Since $a \neq 0$, $b - c = 0$; $\therefore b = c$.

→ Primes & composites
Let $n \rightarrow$ +ve integer. so, 1 & n divide n.
→ If $n > 1$, & no other +ve int besides 1 & n divide n, then n is prime. eg - 2, 3, 5, 7 etc.
→ If $n > 1$, but n is not prime, then n is composite. eg - 4, 6, 8, 10 etc.
1 is neither prime nor composite.
n is composite iff $n = ab \cdot (1 < a < n$ & $1 < b < n)$.
eg - 14 is composite (product of 2 smaller int 2×7)
Every composite no. can be written as the product of 2 or more primes.
eg - $299 = 13 \times 23$.

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→ To check if a no. is prime or not.

i) Small no.s

1) find its factors.

2) check no. of factors.

3) If no. is >2, then not prime.

eg - 36 = 2 × 3 × 2 × 3

Factors = 1, 2, 3, 4, 6, 9, 12, 18, 36.

∴ not prime.

ii) Large no..

1) check units place.

If 0, 2, 4, 6, 8 → not prime.

2) sum of digits.

If divisible by 3, → not prime.

3) step 1 & 2 → false, then find square root.

4) divide no. by all prime nos. below sq root.

5) If divisible → not prime.

not divisible → prime.

eg - 26577

unit digit ×.

Sum = 27.

divisible by 3

not prime.

→ 2311

unit digit ×

Sum = 7 (not divisible by 3)

~~sqrt~~ square root = 48.0728

prime no. till 48 → 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47 (not divisible by any).

→ prime.

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→ How to determine the number of divisors of an int.

① Factorize the integer.

→ write in terms of its prime factors.

eg - $24 = \underbrace{2 \times 2 \times 2}_{} \times 3$.

→ write an exponential expression for each prime factor.

eg - $24 = 2^3 \times 3^1$.

② To find no. of factors -

$$\boxed{d(n) = (a+1)(b+1)(c+1)}$$

$d(n)$ → number

a, b, c → exponents

eg - $24 = 2^3 \times 3^1$

$d(24) = (3+1)(1+1)$

$d(24) = 4 \times 2$

$d(24) = 8$

∴ 24 has 8 divisors / factors.

2311 is a prime no.

2 divisors - 1 & 2311