Calculus and its Applications
(Limits and Continuity - Functions and their Graphs)

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LIMITS AND CONTINUITY

1. Standard functions
2. Graphs
3. Limit
4. Continuity
5. Piecewise continuity
6. Periodic functions
7. Differentiable functions
8. Riemann sum
9. Integrable functions
10. Fundamental theorem of calculus

Invertible function: $f : X \to Y$ is invertible if there exist a function $g : Y \to X$ such that $g \circ f = I_X$ and $f \circ g = I_Y$. The function g is called the inverse of f and is denoted by f^{-1}.

f is invertible iff f is both 1-1 and onto.

Examples:

1. $f : X \to Y$ **defined by** $f(x) = 4x + 3$

f(x)=y

x=y-3/4 (inverse function)

2. $f : \mathbb{R} - \{-4/3\} \rightarrow \mathbb{R}$ **defined by** $f(x) = \frac{4x}{3x+4}$**.**
The inverse of f is
g

g=4x/(4-3x)

domain of g= $\mathbb{R} - \{4/3\}$

Problems to find domain and range

f:X-->Y

Determine the domain and associated ranges of the following functions

Function	Domain (x)	Range (y)
1 $y = x^2$	$\mathbb{R}$	$[0, \infty)$
2 $y = 1/x$	$\mathbb{R} - \{0\}$	$\mathbb{R} - \{0\}$
3 $y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
4 $y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
5 $y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$

4

$$4 - x \geq 0$$

5

$$1 - x^2 \geq 0$$

$(1+x)(1-x) \geq 0$

$1+x \geq 0 \quad 1-x \geq 0$

$x \geq -1 \quad 1 \geq x$

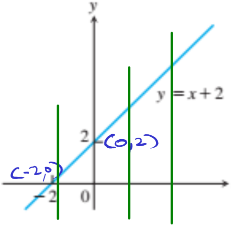
$x \geq -1$ & $x \leq 1$

$\Rightarrow x \in [-1, 1]$

Graphs of Functions

If f is a function with domain D, its graph consists of the points in the Cartesian plane whose coordinates are the input-output pairs for f. In set notation, the graph is

$$\{(x, f(x)) \mid x \in D\}$$



$x = 0$
$\Rightarrow y = 2$

FIGURE 1.3 The graph of $f(x) = x + 2$ is the set of points (x, y) for which y has the value $x + 2$.

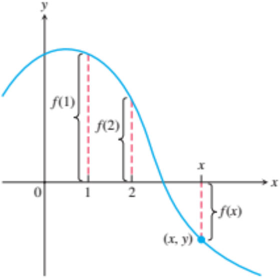
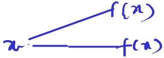
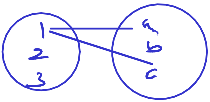


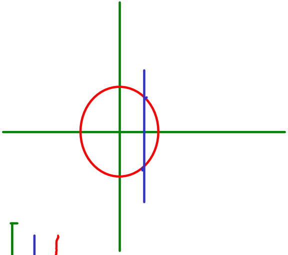
FIGURE 1.4 If (x, y) lies on the graph of f, then the value $y = f(x)$ is the height of the graph above the point x (or below x if $f(x)$ is negative).

Vertical and horizontal tests

A function f can have only one value $f(x)$ for each x in its domain, so no vertical line can intersect the graph of a function more than once.

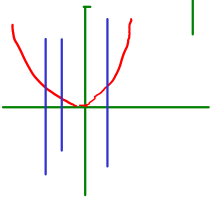


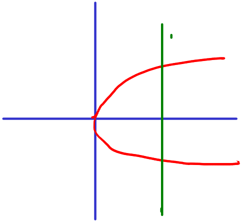




Check for other examples

function



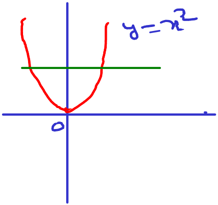


not a function

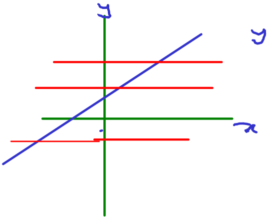
Vertical and horizontal tests

to check whether the function is 1−1

horizontal line should not intersect the graph of the function more than once



not 1−1



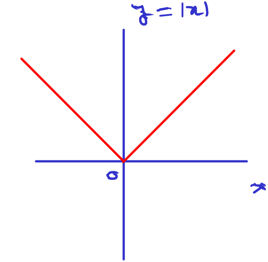
$y = x+1$

1−1 fun.

Piece-wise defined functions

A function which is described in pieces by using different formulas on different parts of its domain.

$$|x| = \begin{cases} x, & x \geq 0 \quad [0, \infty) \\ -x & x < 0 \quad (-\infty, 0) \end{cases}$$

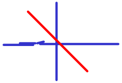


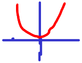
$$f(x) = \begin{cases} -1, & x < 0 \\ 0, & x = 0 \\ 1, & x > 0 \end{cases}$$

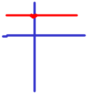
Draw the graph of $f(x)$ (H.W)

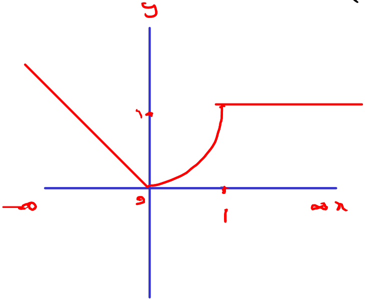
Piece-wise function - Example

$$f(x) = \begin{cases} -x, & x < 0 \\ x^2 & 0 \leq x \leq 1 \\ 1, & x > 1 \end{cases}$$









Greatest integer function

The function whose value at any number x is the greatest integer less than or equal to x is called the greatest integer function or the integer floor function. It is denoted by $\lfloor x \rfloor$

$f(x) = \lfloor x \rfloor$

$\lfloor 2.1 \rfloor = 2$

$\lfloor 2 \rfloor = 2$

$\lfloor 2.9 \rfloor = 2$

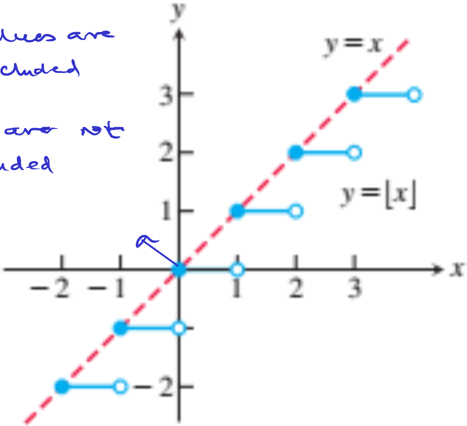
$\lfloor 2.99 \rfloor = 2$

$\lfloor -2.1 \rfloor = -3$

$\lfloor -2 \rfloor = -2$

black dot
→ values are
included

white dot
values are not
included



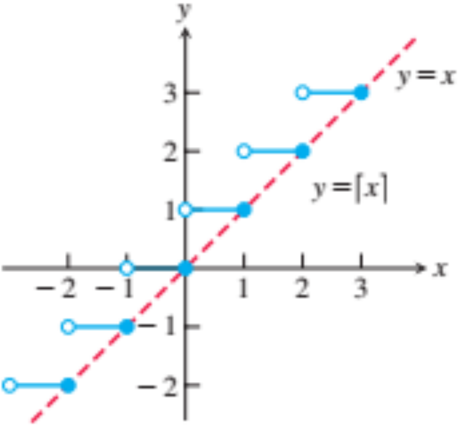
Least integer function

The function whose value at any number $x$is the smallest integer greater than or equal to $x$is called the least integer function or the integer ceiling function. It is denoted $\lceil x \rceil$

$\lceil 2.1 \rceil = 3$

$\lceil 2 \rceil = 2$

$\lceil -2.1 \rceil = -2$


y
x
3
2
1
−1
−2
−2
−1
1
2
3
y = x
y = [x]

Increasing and Decreasing Functions

Let f be a function defined on an interval I and let x_1 and x_2 be two distinct points in I.

- If $f(x_2) > f(x_1)$ whenever $x_1 < x_2$,then f is said to be increasing on I.
- If $f(x_2) < f(x_1)$ whenever $x_1 < x_2$,then f is said to be decreasing on I.

Even and Odd Functions

neither odd nor even

A function $y = f(x)$ is an

- **even function** of x if $f(-x) = f(x)$ eg: $y = x^2$, $y = \cos x$,
- **odd function** of x if $f(-x) = -f(x)$ eg: $y = x$, $y = \sin x$, $y = x^3$

for every x in the function's domain.

$f(x) = \tan x$? odd / even / neither odd nor even

Even and Odd Functions - Examples

THANK YOU