Calculus and its Applications
(Limits and Continuity - Functions and their Graphs(Part2))

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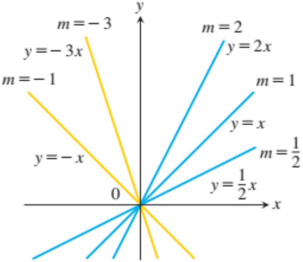
LIMITS AND CONTINUITY: Standard functions – Graphs - Limit - continuity - piecewise continuity - periodic - differentiable functions - Riemann sum - integrable functions - fundamental theorem of calculus

TEXT BOOKS:

1. Thomas G B Jr., Maurice D Wier, Joel Hass, Frank R. Giordano, Thomas' Calculus, Pearson Education, 2018.

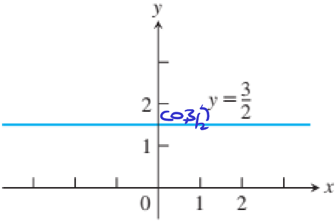
Linear function

A function of the form $f(x) = mx + b$, where m and b are fixed constants, is called a linear function.



(a)

lines through the origin with slope m



constant function with slope 0

Proportional to each other

Two variables y and x are proportional(to one another) if one is always a constant multiple of the other—that is, if $y = kx$ for some nonzero constant k.

If the variable y is proportional to the reciprocal $1/x$, then sometimes it is said that y is inversely proportional to x

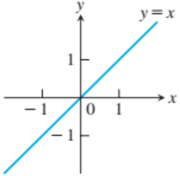
Power Functions

A function $f(x) = x^a$, where a is a constant, is called a power function. There are several cases

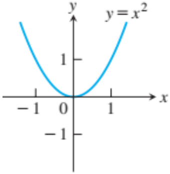
- $f(x) = x^a$, with $a = n$, a positive integer.
- $f(x) = x^a$ with $a = -1$ or $a = -2$
- $a = \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{2}{3}$

H/W

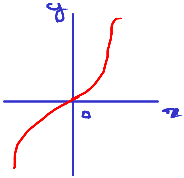
$a=1$
$\Rightarrow f(x) = x^1 = x$



$a=2$
$f(x) = x^2$



$a=3$
$f(x) = x^3$



$a=4$
$f(x) = x^4$

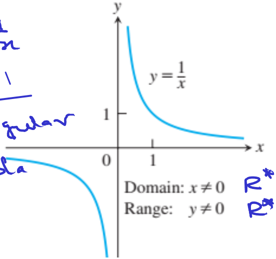
$f(x) = x^a$, a is $-$ve integer

$a = -1 \Rightarrow f(x) = x^{-1} = \frac{1}{x}$

$y = \frac{1}{x}$

$xy = 1$

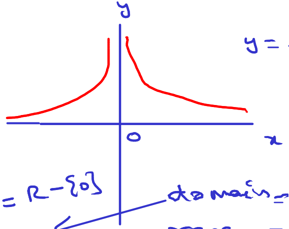
Rectangular hyperbola



R^*

R^*

$a = -2$ $\quad f(x) = \frac{1}{x^2}$



$y = \frac{1}{x^2}$

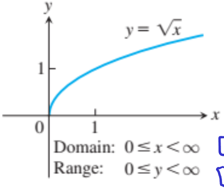
$R^* = R - \{0\}$

domain $= R - \{0\}$

range $= (0, \infty)$

$F(x) = x^a$, $a = \frac{1}{2}$

$f(x) = x^{1/2} = \sqrt{x}$



$[0, \infty)$

$[0, \infty)$

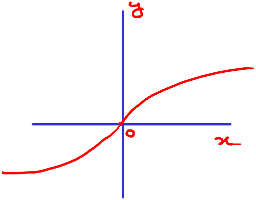
$y = \sqrt{x}$

$y^2 = x$

$\times$ $y^2 = (-1)$

$a = \frac{1}{3}$

$f(x) = x^{1/3}$



$y = \sqrt[3]{x}$

$y^3 = x$

$y^3 = (-1)$

$\exists\ y \in \mathbb{R} \ni y^3 = -1$

$a = 3/2$

$f(x) = x^{3/2}$

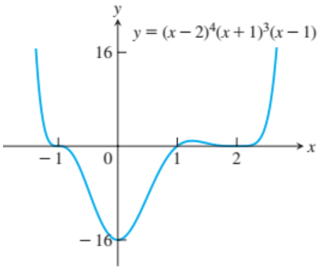
H.W

Polynomial functions

A function p is a polynomial if

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where n is a nonnegative integer and the numbers a_0, a_1, $a_2, \cdots$, a_n are real constants(called the coefficientsof the polynomial).



Rational functions

A rational function is a quotient or ratio $f(x) = p(x)/q(x)$, where p and q are polynomials. The domain of a rational function is the set of all real x for which $q(x) \neq 0$.

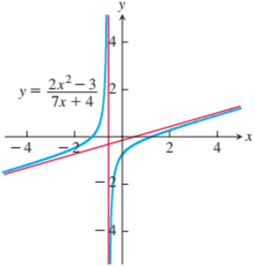
Example:

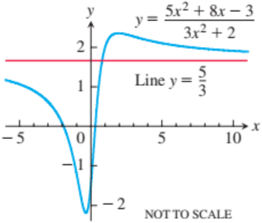
$$f(x) = \frac{2x^2 - 3}{7x + 4}$$

Domain $= \mathbb{R} - \{-4/7\}$

$7x+4=0$

$x = -4/7$





Algebraic functions:

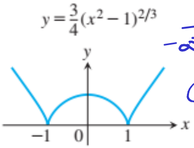
Any function constructed from polynomials using algebraic operations (addition, subtraction, multiplication, division, and taking roots) lies within the class of algebraic functions.

Example:

$$f(x) = \frac{3}{4}(x^2 - 1)^{2/3}$$

$x^2 - 1 = 0$

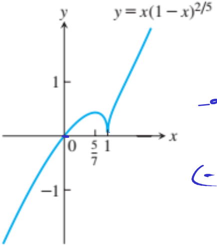
$x = \pm 1$



$-\infty \quad -1 \quad 1 \quad \infty$

$(-\infty, -1)$ $(-1, 1)$ $(1, \infty)$

$$f(x) = x(1 - x)^{2/5}$$



$x = 0, 1$

$-\infty \quad 0 \quad 1 \quad \infty$

$(-\infty, 0)$ $(0, 1)$ $(1, \infty)$

Trigonometric functions:

Sinx, cosx, cosecx secx tanx Cotx

odd even

H.W

θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\operatorname{cosec}\theta$	∞	2	$\sqrt{2}$	$2/\sqrt{3}$	1

Exponential functions:

why?

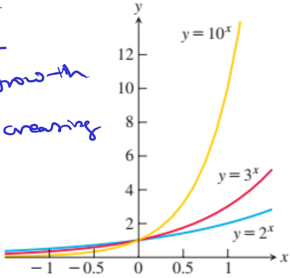
A function of the form $f(x) = a^x$, where $a \neq 0$ and $a \neq 1$, is called an exponential function (with base a). All exponential functions have domain $(-\infty, \infty)$ and range $(0, \infty)$, so an exponential function never assumes the value 0.

Graphs of exponential functions

$y = e^{x}$

growth

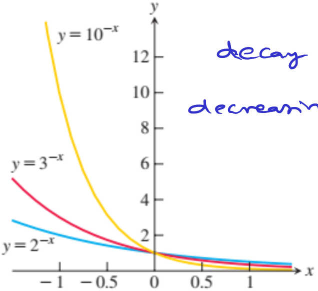
increasing



$y = e^{-x}$

decay

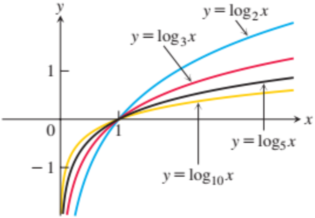
decreasing



Logarithmic functions:

A function of the form $f(x) = \log_a x$, where the base $a \neq 1$ is a positive constant. They are the inverse functions of the exponential functions. All logarithmic functions have domain $(0, \infty)$ and range $(-\infty, \infty)$, so an exponential function never assumes the value 0.

Graphs of logarithmic functions

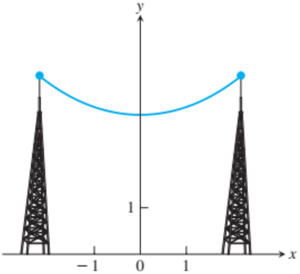


Domain (0,∞)

Transcendental functions:

These are functions that are not algebraic. They include the trigonometric, inverse trigonometric, exponential, and logarithmic functions, and many other functions as well.

Example: Catenary is one example of a transcendental function. Its graph has the shape of a cable, like a telephone line or electric cable, strung from one support to another and hanging freely under its own weight.



Practice problems:

Find the domain and range of the following functions:

- 1 $f(x) = 1 + x^2$, $D = (-\infty, \infty)$, range $= [1, \infty)$
- 2 $f(x) = \frac{1}{x+3} - 5$ $D = R - \{-3\}$, range $= (-\infty, \infty)$ (check !)
- 3 $f(x) = \sqrt{5x + 10}$ $D = [-2, \infty)$, range $= [0, \infty)$
- 4 $g(x) = \sqrt{x^2 - 3x}$
- 5 $g(t) = \frac{2}{t^2 - 16}$ $D = R - \{4, -4\}$ or $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$
- 6 $f(x) = \frac{x^2 - 3x - 4}{x+1}$

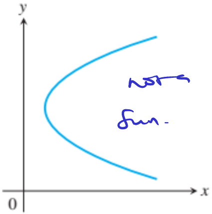
H.W

Practice problems:

Which of the following graphs are graphs of functions of x, and which are not? Give reasons for your answers

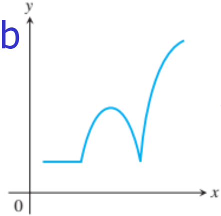
1

a



not a fun.

b



function

Hint:

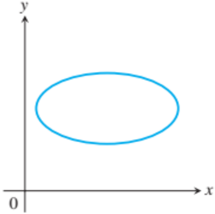
Vertical line test

2

a.

not a fun.

b.



not a fun.

Practice problems:

Specify the intervals over which the function is increasing and the intervals where it is decreasing.

- $y = -x^3$
- $y = x^2$
- $y = \frac{1}{|x|}$
- $y = \frac{1}{x}$
- $y = 2x - 5$

THANK YOU