

DEPARTMENT OF APPLIED MATHEMATICS AND COMPUTATIONAL SCIENCES

MSc CYBER SECURITY

20XC14 DIGITAL SYSTEM DESIGN

WORKSHEET 3

CONTEXT: Logic Gates and Boolean Algebra

1. Draw a logic circuit for the following functions:

(a) $F(X, Y, Z) = XY' + XZ' + YZ$

(b) $F(X, Y, Z) = (X + Y')(X' + Z')(Y + Z)$

2. Use Boolean theorems to simplify the following expressions:

(a) $X + X + X$

(b) $XY + XY$

(c) YYY

(d) $X + XY$

(e) $XY' + Y'$

(f) $(X + Y)Y'$

(g) $(XY) + (XY)'$

(h) $X'Y' + XY$

3. Simplify the following functions:

(a) $F(X, Y, Z) = XY + X'Y + XZ$

(b) $F(X, Y, Z) = (X + Y)(X' + Y + Z)$

(c) $F(X, Y, Z) = XY'Z + XYZ + Y'Z$

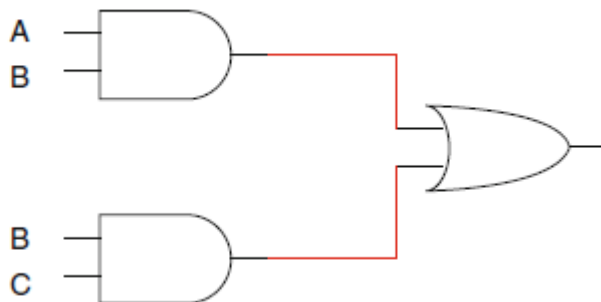
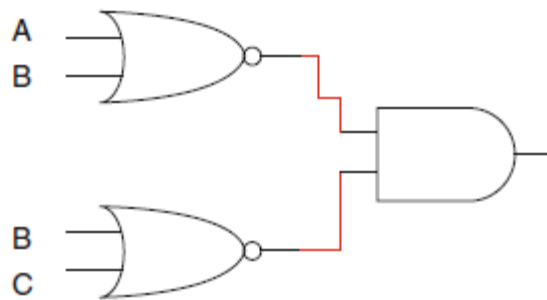
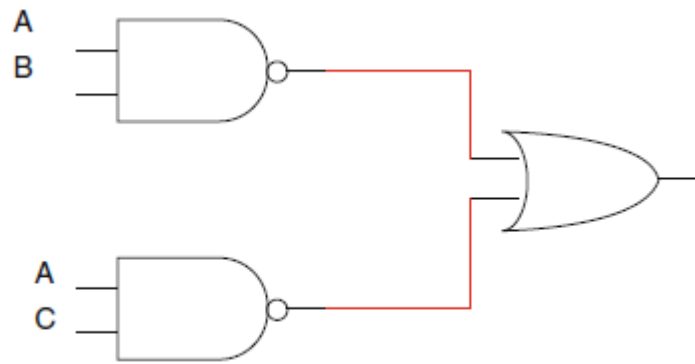
(d) $F(X, Y, Z) = XY + X'YZ$

(e) $F(X, Y, Z) = X'Y + XYZ'$

(f) $F(X, Y, Z) = (XY) + (X + Y + Z)'X + YZ$

(g) $F(X, Y, Z) = (XY)' + (X + Y + Z)'$

4. Show the output of the following logic circuits:



5. Simplify the following functions:

(a) $F(X, Y, Z) = YZ + (X + Y)' + (XYZ)'$

(b) $F(X, Y, Z) = (X + Y + Z)' (X + Y)$

CONTEXT: SOP and POS expressions

1. *Demonstrate by means of truth tables the validity of the following identities:
(a) DeMorgan's theorem for three variables: $\overline{XYZ} = \overline{X} + \overline{Y} + \overline{Z}$
(b) The second distributive law: $X + YZ = (X + Y)(X + Z)$
(c) $\overline{X}Y + \overline{Y}Z + X\overline{Z} = X\overline{Y} + Y\overline{Z} + \overline{X}Z$
2. *Prove the identity of each of the following Boolean equations, using algebraic manipulation:
(a) $\overline{X}\overline{Y} + \overline{X}Y + XY = \overline{X} + Y$
(b) $\overline{A}B + \overline{B}\overline{C} + AB + \overline{B}C = 1$
(c) $Y + \overline{X}Z + X\overline{Y} = X + Y + Z$
(d) $\overline{X}\overline{Y} + \overline{Y}Z + XZ + XY + Y\overline{Z} = \overline{X}\overline{Y} + XZ + Y\overline{Z}$
3. +Prove the identity of each of the following Boolean equations, using algebraic manipulation:
(a) $ABC + B\overline{C}\overline{D} + BC + \overline{C}D = B + \overline{C}D$
(b) $WY + \overline{W}Y\overline{Z} + WXZ + \overline{W}X\overline{Y} = WY + \overline{W}X\overline{Z} + \overline{X}Y\overline{Z} + X\overline{Y}Z$
(c) $A\overline{D} + \overline{A}B + \overline{C}D + \overline{B}C = (\overline{A} + \overline{B} + \overline{C} + \overline{D})(A + B + C + D)$
4. +Given that $A \cdot B = 0$ and $A + B = 1$, use algebraic manipulation to prove that

$$(A + C) \cdot (\overline{A} + B) \cdot (B + C) = B \cdot C$$

5

Simplify the following Boolean expressions to expressions containing a minimum number of literals:

- (a) $\overline{A}\overline{C} + \overline{A}BC + \overline{B}C$
- (b) $\overline{(A + B + C)} \cdot \overline{ABC}$
- (c) $ABC + AC$
- (d) $\overline{A}\overline{B}D + \overline{A}\overline{C}D + BD$
- (e) $\overline{(\overline{A} + B)(\overline{A} + \overline{C})}(\overline{ABC})$

6.

*Reduce the following Boolean expressions to the indicated number of literals:

- (a) $\overline{X}\overline{Y} + XYZ + \overline{X}Y$ to three literals
- (b) $X + Y(Z + \overline{X} + \overline{Z})$ to two literals
- (c) $\overline{W}X(\overline{Z} + \overline{Y}Z) + X(W + \overline{W}YZ)$ to one literal
- (d) $(AB + \overline{A}\overline{B})(\overline{C}\overline{D} + CD) + \overline{AC}$ to four literals

7.

*Find the complement of the following expressions:

(a) $A\bar{B} + \bar{A}B$

(b) $(\bar{V}W + X)Y + \bar{Z}$

(c) $WX(\bar{Y}Z + Y\bar{Z}) + \bar{W}\bar{X}(\bar{Y} + Z)(Y + \bar{Z})$

(d) $(A + \bar{B} + C)(\bar{A}\bar{B} + C)(A + \bar{B}\bar{C})$

8.

*Obtain the truth table of the following functions, and express each function in sum-of-minterms and product-of-maxterms form:

(a) $(XY + Z)(Y + XZ)$

(b) $(\bar{A} + B)(\bar{B} + C)$

(c) $WX\bar{Y} + WX\bar{Z} + WXZ + Y\bar{Z}$

9. For the Boolean functions E and F , as given in the following truth table:

X	Y	Z	E	F
0	0	0	0	1
0	0	1	1	0
0	1	0	1	1
0	1	1	0	0
1	0	0	1	1
1	0	1	0	0
1	1	0	1	0
1	1	1	0	1

(a) List the minterms and maxterms of each function.

(b) List the minterms of \bar{E} and \bar{F} .

(c) List the minterms of $E + F$ and $E \cdot F$.

(d) Express E and F in sum-of-minterms algebraic form.

(e) Simplify E and F to expressions with a minimum of literals.

10.

*Convert the following expressions into sum-of-products and product-of-sums forms:

(a) $(AB + C)(B + \bar{C}D)$

(b) $\bar{X} + X(X + \bar{Y})(Y + \bar{Z})$

(c) $(A + B\bar{C} + CD)(\bar{B} + EF)$

11.

Draw the logic diagram for the following Boolean expressions. The diagram should correspond exactly to the equation. Assume that the complements of the inputs are not available.

(a) $XYZ + \overline{X}\overline{Y} + \overline{X}\overline{Z}$

(b) $B(\overline{A}\overline{C} + AC) + \overline{D}(A + \overline{B}C)$

(c) $X\overline{Y}(\overline{W} + \overline{Z}) + \overline{W}Y(\overline{X} + \overline{Z}) + WY(\overline{X} + Z)$