Pairwise Relatively Prime Integers :

The Positive integers $a_1 , a_2 , a_3 , . . . , an$ are pairwise relatively prime if Every pair of integers is relatively prime ; that is , $(ai , aj) = 1$, whenever $i \neq j$.

Corollary :

If the positive integers $a_1 , a_2 , a_3 , . . . , an$ are pairwise relatively prime , then the gcd $(a_1 , a_2 , a_3 , . . . , an) = 1$.

For Example :

Let the numbers be 4 , 27 , 35 as they are relatively prime (there is no any common divisors for the above three numbers) their gcd is 1 .

Theorem :

Let the numbers be $a_1, a_2, a_3, \dots, a_n$ be positive integers , where $n \geq 3$. Then $\gcd(a_1, a_2, a_3, \dots, a_n) = \gcd(\gcd(a_1, a_2, a_3, \dots, a_{n-1}), a_n)$.

Proof :

Let the $\gcd(a_1, a_2, a_3, \dots, a_n) = d$, $\gcd(a_1, a_2, a_3, \dots, a_{n-1}) = d'$ and let $d'' = \gcd(d', a_n)$.

We need to show that $d = d''$; $d \mid d''$ and $d'' \mid d$.

Since $d = \gcd(a_1, a_2, a_3, \dots, a_n)$, $d \mid a_i$ for $1 \leq i \leq n - 1$ and $d \mid d'$ which also holds

$d \mid \gcd(d', a_n) = d \mid d''$. $d'' = m * d$ ==➔ for some m belongs to Z

We also need to show that $d \mid d''$.

Since $d'' = \gcd(d', an)$ which means $d'' \mid d'$ and $d'' \mid an$; $d'' \mid d'$ - holds for $1 \leq i \leq n-1$. Thus d'' must divide d too . Hence $d'' \mid d$.

Hence we got $d \mid d''$ and $d'' \mid d$. Therefore $d = d''$

Then $\gcd(a_1, a_2, a_3, \ldots, an) = \gcd(\gcd(a_1, a_2, a_3, \ldots, a(n-1)), an)$

Hence Showed .