

Binary Number Systems

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⇒ Analog System is called as continuous systems

⇒ Digital System is called as discrete systems.

5 Decimal System : $0 \rightarrow 9$ (highest digit = base - 1)
 $= 10 - 1$

$$(518.425)_{10} = 5 \times 10^2 + 1 \times 10^1 + 8 \times 10^0 + 4 \times 10^{-1} + 2 \times 10^{-2} + 5 \times 10^{-3}$$

10 Octal System : $0 \rightarrow 7$ (highest num = $8 - 1 = 7$)

$$\begin{aligned}(467)_8 &= 4 \times 8^2 + 6 \times 8 + 7 \times 8^0 \\ &= 256 + 48 + 7 \\ &= (311)_{10}\end{aligned}$$

$$\begin{aligned}15 (345)_8 &= 3 \times 8^2 + 4 \times 8 + 5 \times 8^0 \\ &= 192 + 32 + 5 \\ &= (229)_{10}\end{aligned}$$

20 Hexadecimal System : $0 \rightarrow 15$ (highest num = $16 - 1 = 15$)

$$(A)_{16} - (10)_{10}, (B)_{16} - (11)_{10}, (C)_{16} - (12)_{10}, (D)_{16} - (13)_{10}, (E)_{16} - (14)_{10}, (F)_{16} - (15)_{10}$$

$$\begin{aligned}25 (4AC2)_{16} &= 4 \times 16^3 + A \times 16^2 + C \times 16^1 + 2 \times 16^0 \\ &= 16384 + 2560 + 192 + 2 \\ &= (19138)_{10}\end{aligned}$$

$$\begin{aligned}30 (5CD4)_{16} &= 5 \times 16^3 + C \times 16^2 + D \times 16^1 + 4 \times 16^0 \\ &= \dots\end{aligned}$$

Binary System : \rightarrow (highest num = 2 - 1) 0, 1

\rightarrow 0, 1 are called bits.

\rightarrow 8 bits are one byte

$$\begin{aligned}(11011)_2 &= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ &= 16 + 8 + 2 + 1 \\ &= (27)_{10}\end{aligned}$$

Generalized form : \rightarrow

The General form for converting any number which is written in other systems into decimal

$$(a_r a_{r-1} \dots a_1 a_0 \dots)_{r^n} = a_r r^5 + a_{r-1} r^4 + a_{r-2} r^3 + a_{r-3} r^2 + a_{r-4} r^1 + a_{r-5} r^0 + a_{r-6} r^{-1} + a_{r-7} r^{-2} \dots$$

Decimal to Binary : \rightarrow

Find the remainders to the given number by 2.

Take remainders & write in reverse order.

Eg:

$$\begin{array}{r} 2 | 49 \\ 2 | 24 \\ 2 | 12 \\ 2 | 6 \\ 2 | 3 \end{array}$$

$$\begin{array}{r} 2 | 97 \\ 2 | 48 \\ 2 | 24 \\ 2 | 12 \\ 2 | 6 \\ 2 | 3 \end{array}$$

$$(49)_{10} = (110001)_2$$

$$(97)_{10} = (1100001)_2$$

$$(84 \cdot 63)_{10} =$$

Integer.

$$\begin{array}{r} 2(84 & 0 \\ 2(42 & 0 \\ 2(21 & 0 \\ 2(10 & 1 \\ 2(5 & 1 \\ 2(2 & 0 \\ 1 & \end{array}$$

$$1011000,$$

$$0 \cdot 63 \times 2 = 1 \cdot 26$$

fraction part

$$\begin{array}{l} 1 & 0 \cdot 26 \\ 0 & 0 \cdot 52 \\ 1 & 0 \cdot 04 \\ 0 & 0 \cdot 08 \\ 0 & 0 \cdot 16 \\ 0 & 0 \cdot 32 \end{array}$$

$$(84 \cdot 63)_{10} = (1011000 \cdot 101000)_2$$

$$(75 \cdot 54)_{10} =$$

Integer.

$$\begin{array}{r} 2(75 & 1 \\ 2(37 & 1 \\ 2(18 & 0 \\ 2(9 & 1 \\ 2(4 & 0 \\ 2(2 & 0 \\ 1 & \end{array}$$

$$1001011 \cdot$$

$$0 \cdot 54 \times 2 = 1 \cdot 08$$

fraction part

$$\begin{array}{l} 1 & 0 \cdot 08 \\ 0 & 0 \cdot 16 \\ 0 & 0 \cdot 32 \\ 0 & 0 \cdot 64 \\ 1 & 0 \cdot 28 \end{array}$$

$$10001$$

$$(75 \cdot 54)_{10} = (1001011 \cdot 10001)_2$$

Converting from diff. bases to diff. bases using decimal

$$()_n = ()_m$$

↓
convert
to base 10
using sevⁿ.



↓
using LCM.
convert from base 10
to m base.

$$(8A3)_{16} = (?)_8$$

$$\begin{aligned} 8A3 &= 8 \times 16^2 + A \times 16^1 + 3 \times 16^0 \\ &= 2048 + 160 + 3 \end{aligned}$$

$$(8A3)_{16} = (2211)_{10}$$

$$(2211)_{10} = (?)_8$$

$$\begin{array}{r} 2211 \cdot 3 \\ \hline 2764 \\ \hline 34 \end{array}$$

Interconversions without using decimal

5) hexadecimal to binary \rightarrow 4 bits of all numbers

$$(8 \ A \ 3)_{16} = ((1000 \ 10)(10.0011))_2 \\ = (9 \cdot 2 \ 4 \ 3)_8.$$

10) binary to octal \rightarrow 3 bit combination

$$1) (748)_8 = (\underline{\underline{1}}\underline{\underline{0}}\underline{\underline{0}}) \underline{\underline{1}\underline{\underline{1}}\underline{\underline{1}}\underline{\underline{0}}\underline{\underline{0}}\underline{\underline{0}}\underline{\underline{1}}\underline{\underline{1}}) \\ = (1 \ E \ 3)_{16}$$

$$2) (8A3.B2)_{16} = (\underline{\underline{1}}\underline{\underline{0}}\underline{\underline{0}} \cdot \underline{\underline{1}}\underline{\underline{0}}\underline{\underline{1}}\underline{\underline{0}}\underline{\underline{0}}\underline{\underline{1}}\underline{\underline{1}} \cdot \underline{\underline{1}}\underline{\underline{0}}\underline{\underline{1}}\underline{\underline{0}}\underline{\underline{1}}\underline{\underline{0}}\underline{\underline{0}}) \xleftarrow{\text{padding zero}} \xrightarrow{\text{padding zero}} \\ = (4 \ 2 \ 4 \ 3 \cdot 5 \ 4 \ 4.)_8$$

$$3) \underline{\underline{1}}\underline{\underline{0}}\underline{\underline{0}}\underline{\underline{1}} \cdot \underline{\underline{0}}\underline{\underline{0}}\underline{\underline{1}}\underline{\underline{1}} \cdot \underline{\underline{1}}\underline{\underline{0}}\underline{\underline{0}}\underline{\underline{1}} - \underline{\underline{0}}\underline{\underline{0}}\underline{\underline{0}}\underline{\underline{1}} \cdot \underline{\underline{1}}\underline{\underline{1}}\underline{\underline{0}}\underline{\underline{0}} \cdot \underline{\underline{1}}\underline{\underline{0}}\underline{\underline{0}}\underline{\underline{0}} \\ (4 \ 4 \ 7 \ 1 \cdot 0 \ 7 \ 1) \cdot \\ (9 \ 3 \ 9 \cdot 1 \ C8)_{16}$$

25)

30)

Bases \Rightarrow	Decimal <u>(10)</u>	Binary <u>(2)</u>	Octal <u>(8)</u>	Hexadecimal <u>(16)</u>
0	0	0000	0	0
1	1	0001	1	1
2	2	0010	2	2
3	3	0011	3	3
4	4	0100	4	4
5	5	0101	5	5
6	6	0110	6	6
10	A	0111	7	7
8	8	1000	10	8
9	9	1001	11	9
10	10	1010	12	A
11	11	1011	13	B
15	12	1100	14	C
13	13	1101	15	D
14	14	1110	16	E
15	15	1111	17	F

Binary Arithmetic →

$$\begin{array}{r}
 1010 \\
 + 1011 \\
 \hline
 10101
 \end{array}
 \quad \begin{array}{l}
 2 \rightarrow 10 \\
 3 \rightarrow 11 \\
 4 \rightarrow 100
 \end{array}$$

$$\begin{array}{r}
 111 \\
 10110 \\
 + 01111 \\
 \hline
 100101
 \end{array}
 \quad \begin{array}{l}
 110^{\text{10}} \\
 0111 \\
 \hline
 0111
 \end{array}$$

$$\begin{array}{r}
 0110^{\text{10}} \\
 - 0101 \\
 \hline
 0001
 \end{array}
 \quad \begin{array}{l}
 0101^{\text{10}} \\
 0111 \\
 \hline
 0101
 \end{array}$$

$$\begin{array}{r}
 1010 \\
 \times 101 \\
 \hline
 1010 \\
 000+ \\
 1010+ \\
 \hline
 110010
 \end{array}$$

→ Generally, computers follow addition to subtract numbers
 i.e., adding -ve numbers.
 these are complements.

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Complements :-

(i) Diminished Radix complement :- $(x-1)^k$ compliment
 The diminished Radix compliment of the num N with
 base x having n digits in it is expressed as

$$(x^n - 1) - N$$

Decimal

$(10-1) 9^k$ compliment

$$(492)_{10} =$$

$$\begin{aligned} 9^k \text{compliment} &= (10^3 - 1) - 492 \\ &= 1000 - 493 \\ &= 507 \end{aligned}$$

$$\begin{aligned} (2845)_{10} \\ = 10^4 - 1 - 2845 \\ = 10000 - 2845 \\ = 7154 \end{aligned}$$

Binary

$(2-1)$ 1st compliment

$$(10101)_2$$

$$= (2^5 - 1) - 10101$$

$$= 31 - 10101$$

$$= 11111 - 10101$$

$$= 01010$$

$$(011011)_2$$

$$= 2^6 - 1 - 011011$$

$$\leftarrow = 63 - 011011$$

$$= 100100$$

$$= 11111 - 011011$$

$$= 100100$$

Octal

\rightarrow 's complement
 $(456)_8$

$$= 321$$

$$= 8^3 - 1 - 456$$

$$= (511)_8 - (456)_8$$

$$= (777)_8 - (456)_8$$

$$= 321$$

Hexadecimal

$15'$ complement.

$$(SAB)_{16}$$

$$= 16^3 - 1 - SAB$$

$$4096 - SAB$$

$$= (1000 - 1) - SAB$$

$$= FFF - SAB_{16} \quad | 4096 - 0$$

$$= A54$$

$$\begin{array}{r} 16 \\ 768 \\ 16 \\ 0 \end{array}$$

iii) Diminished R.

(ii) Radix complement \rightarrow ($8'$ complement)

The Radix complement of the num N with base r having n digits in it is expressed as

$$(r^n) - N$$

Decimal

$10'$ complement

$$(243)_{10}$$

$$10' \text{ compli} = 10^3 - 243$$

$$= 757$$

Binary

2^5 complement

$$(1.00110)_2$$

$$= 2^6 - 100110$$

$$= 64 - 100110$$

$$= 1000000 - 100110$$

B.

$$\begin{array}{r} 0111110 \\ 1000000 \\ \hline 100110 \end{array} = 011010$$

$$011010$$

30

Octal

$$(672)_8$$

$$= 8^3 - 6 \cdot 8^2$$

$$= (512) - 6 \cdot 64$$

$$= 1000 - 6 \cdot 64$$

$$= 106$$

Hexadecimal

$$(4AD)_{16}$$

$$= 16^3 - 4 \cdot 16^2$$

$$= 4096 - 4 \cdot 256$$

$$= 1000 - 4 \cdot 16$$

$$\begin{array}{r} 1000 \\ - 4 \cdot 256 \\ \hline 1000 \\ - 1024 \\ \hline 16 \\ \hline 16 \\ - 16 \\ \hline 0 \end{array}$$

16 | 4096
16 | 256
16 | 16
16 | 0

B53

$$= 1353$$

Subtraction using radix compliment :-

$$x - y = x + (-y)$$

$-y \rightarrow$ converts to 2^k complement
and gets added to x

$$= x + (x^n - y) \text{ carry neglected}$$

$$= x - (y - x^n)$$

$$= x - y$$

$x^n \rightarrow$ comes as carry

so final carry is neglected

so we get actual result

$$\therefore 492 - 342$$

$$= 492 + (-342)$$

$$342 = 652 + 1 = 658$$

$$= 492 + 1000 - 658 + 1000 - 342$$

$$= 492 + 658$$

$$= 150$$

$$\begin{array}{r} 492 \\ 658 \\ \hline 150 \end{array}$$

149

*One
accy.*

$$342 - 492$$

$$\begin{array}{r} 342 \\ - 492 \\ \hline 508 \end{array}$$

$$342 + (-492)$$

$$342 + 508 = \boxed{850}$$

$$= \boxed{850} \rightarrow \text{Find Total Length of m.}$$

$$150$$

$$\begin{array}{r} 999 \\ - 850 \\ \hline 149 \end{array}$$

$$149 + 1 = 150$$

10 Binary

$$011100 + (-001010)$$

$$= 011100 + 110110$$

$$= \boxed{1010010}$$

$$\begin{array}{r} 111111 \\ - 001010 \\ \hline 1101011 \end{array}$$

$$0001010 \quad 110110$$

2's complement is

$$= 110101 + 1 = 110110$$

$$001010 - 011100$$

$$= 001010 + (-011100)$$

$$= 001010 + 100100$$

$$= \boxed{1011110}$$

$$= \boxed{0100110}$$

$$\begin{array}{r} 011100 \\ - 110110 \\ \hline 0010010 \end{array}$$

$$\begin{array}{r} 111111 \\ - 011100 \\ \hline 100100 \end{array}$$

$$\begin{array}{r} 111111 \\ - 011100 \\ \hline 100011 \end{array}$$

25

$$101110$$

$$010001+1$$

$$010010$$

$$6 - 001010$$

$$011112$$

$$10000000$$

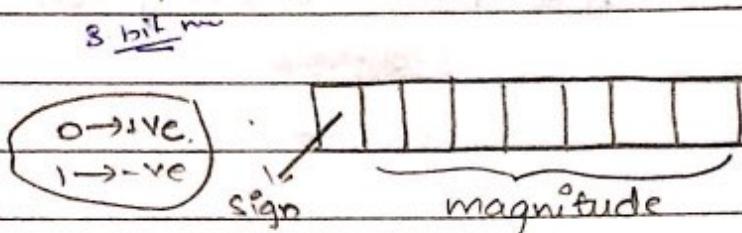
$$001010$$

$$110110$$

30

Representation of signal binary numbers

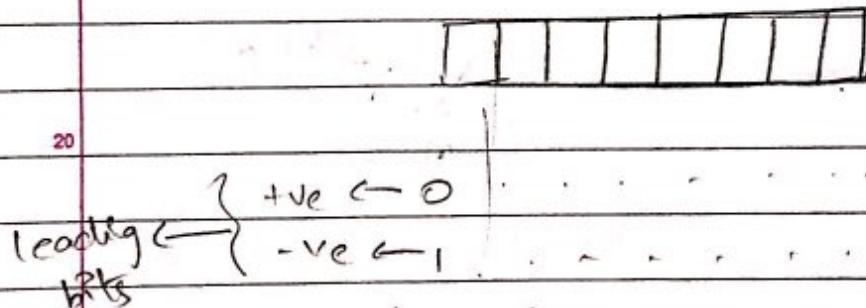
5 (i) Signed Magnitude method:-



10 The other 7 bits have 2^7 combination i.e 128 for each sign. We can write -1 to -127 , +1 to $+127$, $-0/+0$.

15 The maximum number which can be written is $+127$.

(ii) 1's Complement method:-



Here there is no -ve or +ve num. and bits are used as magnitude.

25 the -ve numbers are converted to 1's Compliment

(ii) 2^1 's Compliment method: →

$$-128 \rightarrow 0 \rightarrow +12 \rightarrow$$

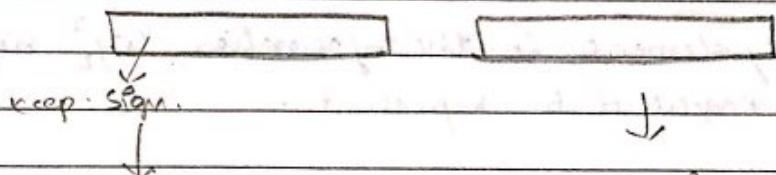
5



Sign extension

10

Sign extension: →



15

all 8 bits should
be written as
same for sign.

(most significant bit
is written)

20

25

30

Binary codes:-

→ Coding is the process of assigning a group of binary bits to represent multivalued icons of information.

→ The assignment of bit patterns to discrete elements of information is generally referred to as binary coding.

→ With an n no of bits 2^n elements of information can be created.

→ for n no of elements in the information $\log_2 n$ no of bits are required to represent.

(i) BCD Code:= It is also called 8421 code
(Binary coded decimal)

Decimal digits	(4 bits)	
	BCD bits	8421
0		0000
1		0001
2		0010
3		0011
4		0100
5		0101
6		0110
7		0111
8		1000
9		1001

25 in BCD is

$\begin{array}{r} 2/12 \\ 2/12 \\ 2/12 \\ 2/12 \end{array}$ 25 → 11001 → binary
 \hookrightarrow 00100101 → BCD

12 34
 $\begin{array}{r} 1 \\ 2 \\ 3 \\ 4 \end{array}$
 $\begin{array}{r} 0001 \\ 0010 \\ 0011 \\ 0100 \end{array}$

Add BCD Arithmetic :-

Addition :-

when result
10 4587 → 0100 0101 1000 0111 3253 → 0011 0010 0101 0011 $\begin{array}{r} 0111 \\ + 0011 \\ \hline 1000 \end{array}$ 0110 1110 1010 $\begin{array}{r} 0110 \\ + 0110 \\ \hline 0100 \end{array}$ 0000
fs beyond limit of 9. then add 6 to result. We will get actual result
15 4592 3953 $\begin{array}{r} 0100 \\ + 0011 \\ \hline 1000 \end{array}$ 0101 1110 1010 $\begin{array}{r} 0100 \\ + 0100 \\ \hline 0000 \end{array}$

4592
 3953

add
 6

when 10's addition is > 0
 10's addition is > 0
 or

1110 1010

20 2485
 6245

Subtraction :-

It is applied by changing 2nd num to 10's
 compliment and add it to actual num. (1st num)

25 0485 → 10485
 $\begin{array}{r} 0485 \\ - 0245 \\ \hline 0240 \end{array}$

+ 9 = 55 This addition is done
 by BCD adding.

neglect 0 2 4 0

1 1 0 1

1 0 5 1

0 0 3 1

decimal

Other binary codes :-

Slipping code
we add 1's
to complement

Decimal

8421

2421

84-2-1

Self
complementing
code

Weighted
code

Weighted
code

Decimal	<u>8421</u>	<u>2421</u>	<u>84-2-1</u>
0	0000	0000	0000 0000
1	0001	0001	0110 0111
2	0010	0010	0110 0110
3	0011	0011	0100 0101
4	0100	0100	1011 0100
5	0101	0101	1010 1011
6	0110	1100	1010 1010
7	0111	1101	1001 1001
8	1000	1110	1000 1000
9	1001	1111	1111 1111

15

add 3 to
BCD

Excess 3-code :-

Inverting
number
^{14 hot}

a got
redundant

co ill is

Set
complement
²⁵

Gray code :-

to avoid switching
of switches may
times and to avoid
confusion gray code
is used.

0	0000	0000
1	0001	0001
2	0011	0010
3	0010	0011
4	0110	0100
5	0111	0101
6	0101	0110
7	0000	0111
8	1000	1000
9	1001	1001
10	1111	1010
11	1110	1011
12	1010	1100
13	1011	1101
14	1001	1110
15	1000	1111

30

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Binary \rightarrow Gray code :- \rightarrow The 1st bit of any gray code is same as their binary.

$$B: 0 \ 1 \ 0 \ 0$$

$$G: 0$$

 \rightarrow Then compare 2 bits if equal, put 0, unequal = 1.

$$B: 0 \overset{\curvearrowleft}{1} 0 \ 0$$

$$G: 0 \overset{\curvearrowleft}{1}$$

$$B: 0 \overset{\curvearrowleft}{1} 0 \ 0$$

$$G: 0 \ 1 \ 1$$

$$B: 0 \overset{\curvearrowleft}{1} 0 \ 0$$

$$G: 0 \ 1 \ 1 \ 0$$

Gray \rightarrow Binary code :- \rightarrow The 1st bit is same.

$$G: 0 \ 1 \ 1 \ 0$$

$$B: 0$$

 \rightarrow Compare 2nd gray bit and 1st binary bit.

if equal = 0, unequal = 1.

$$G: 0 \ 1 \ 1 \ 0$$

$$B: 0 \ 1$$

 \rightarrow Repeat the same.

$$G: 0 \ 1 \ 1 \ 0$$

$$B: 0 \ 1 \ 0 \ 0$$

ASCII Code :-

- American standard code for information interchange
- It is 7-bit code
- $2^7 = 128$ combination can be used.



94 → printable characters

34 → non-printable.

upper lowercase

94 → $26 + 26 \rightarrow 52$ english characters

→ 0-9 → 10 digits.

↳ 32 → other printable characters.

ASCII Codes

0 → 30H

1 → 31H = 011 0001

9 → 39H = 011 1001

A → Z : 41H → 5A H

a → z : 61H → 7A H

⇒ later this code is extended to 8-bits.

we get 2^8 combination = 256

0 → 30H = 0011 0001

9 → 39H = 0011 1001

⇒ Keeping 1 in the 1st position they are allotted to other signs.

Error detection code:

- While sending the code, there might be some errors or disturbances in between. Then the error detection code is used.
- All the digits in the code are added. i.e. called parity (1st round adder)

Ex: 31H	even parity P _{1011 0001}	if no of ones are odd, one is added to make it even parity
011 0001	odd parity P _{0011 0001}	
011 1001	even. P _{0011 1001} odd. P _{1011 1001}	

- If a number is transmitted.

For odd parity

all no of ones are odd. Including the parity number.

For even parity

all no of ones are even including the parity number.

- If there are even parity for odd (or) vice versa error is occurred. (It identifies error)

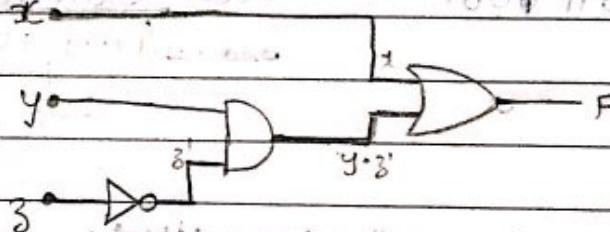
- But it can identify a single bit error.

$$(i) F = x + y \cdot z'$$

↓
OR AND

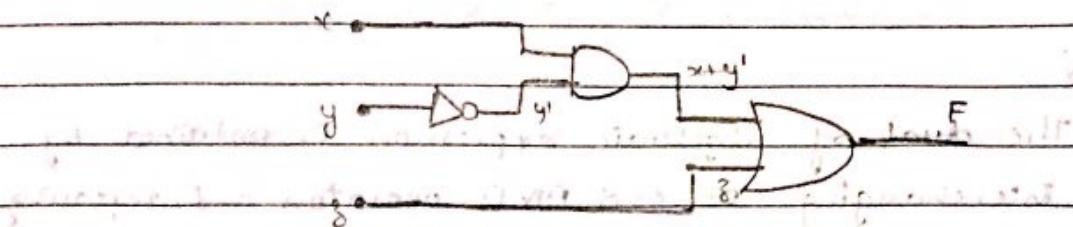
Truth Tables:-

$x \cdot y \cdot z$	$x + y \cdot z'$	F
0 0 0	1	0
0 0 1	0	1
0 1 0	1	0
0 1 1	0	0
1 0 0	1	1
1 0 1	0	0
1 1 0	1	1 + 1 = 1
1 1 1	0	0



$$(ii) F = (x + y') \cdot z$$

y'	x	y	z	$x + y'$	F
1	0	0	0	• 1	0
1	1	0	0	1	0
0	0	1	0	0	0
0	1	0	0	• 1	0
1	1	0	1	1	1
0	0	1	1	0	0
0	1	1	1	1	1
0	1	1	1	1	1



Canonical and standard forms:-

Boolean Algebra:-

These are the algebraic structures defined by a set of elements B together with 2 binary operators (\oplus) and (\odot) provided that the following postulates are satisfied:

(i) The structure is closed w.r.t the operators. these are ' \oplus ' and ' \odot ', [OR and AND]

(ii), the element 0 is an identity element w.r.t OR (\oplus) and the element 1 is an identity element w.r.t AND

(iii) The structure is commutative w.r.t OR (\oplus) and AND (\odot)

(iv) The operations OR and AND are distributive one on another.

(v) For every x belongs to B there exist an element x' which belongs to B such that

$$x + x' = 1$$

$$x \cdot x' = 0$$

(vi) There exists 2 elements x, y belonging to B such that $x \neq y$

(viii)

The dual of algebraic expression is obtained by interchanging OR and AND operators and replacing 1's by 0's and 0's by 1's.

4 Postulates of Boolean Algebra:-

$$(i) x + 0 = x$$

$$x \cdot 1 = x$$

$$(ii) x + x' = 1$$

$$x \cdot x' = 0$$

$$(iii) x + y = y + x$$

$$x \cdot y = y \cdot x$$

$$(iv) x(y+z) = xy + xz$$

$$x + (y + z) = (x + y)(x + z)$$

(v) .

Theorems of Boolean Algebra :-

$$(i) x + x = x$$

$$x \cdot x = x$$

$$(ii) x + 1 = 1$$

$$x \cdot 0 = 0$$

$$(iii) (x')' = x \quad (\text{Involution})$$

$$(iv) x + (y + z) = (x + y) + z \quad , \quad x \cdot (y \cdot z) = (x \cdot y) \cdot z$$

$$(vi) x(x+y)' = x \cdot y' \quad , \quad b(x \cdot y)' = x' + y'$$

$$(vii) x + xy = x \quad , \quad x(x+y) = x$$

$$x(1+xy) = x \cdot 1$$

$$1+1=1$$

$$0 \cdot 0 = 0$$

$$x(1+1)$$

$$0 \cdot 1 = 0$$

$$1 \cdot 1 = 1$$

Canonical and Standard forms:-

Canonical forms are used to represent a complete function.

These are 2 types - (i) min. terms
 (ii) max terms.

It consists of literals (variables x, y, z)

Min terms:-

$$0 \rightarrow x' \quad 1 \rightarrow x$$

min terms are sum of product of individual variables

Ex:	<u>x</u>	<u>y</u>	<u>z</u>	min terms
	0	0	0	$m_0 = x'y'z'$
	0	0	1	$m_1 = x'y'z$
	0	1	0	$m_2 = x'yz'$
	0	1	1	$m_3 = x'yz$
	1	0	0	$m_4 = xy'z'$
	1	0	1	$m_5 = xy'z$
	1	1	0	$m_6 = xyz'$
	1	1	1	$m_7 = xyz$

$$(x+y+z) = F = m_1 + m_3 + m_5 = \Sigma(m_1, m_3, m_5)$$

→ Add the min terms whose result is 1.

(iii) Max terms: →

$$x \rightarrow 0 \quad x' \rightarrow 1$$

⇒ These are addition of individual terms.

Eg:-

x	y	z	max term
0	0	0	$M_0 (x+y+z)$
0	0	1	$M_1 (x+y+z')$
0	1	0	$M_2 (x+y'+z)$
0	1	1	$M_3 (x+y'+z')$
1	0	0	$M_4 (x'+y+z)$
1	0	1	$M_5 (x'+y+z')$
0	1	0	$M_6 (x+y'+z)$
1	1	1	$M_7 (x+y'+z')$

$$(x+y+z)$$

$$F = M_0 M_1 M_2 = \bar{x} \bar{y} \bar{z} + x \bar{y} \bar{z} + \bar{x} y \bar{z} + x y \bar{z}$$

→ A complement of min term is its corresponding max term.

Standard to canonical conversion

$$\begin{aligned}
 F &= x + yz' \\
 &= x \cdot 1 \cdot 1 + 1 \cdot y \cdot z' \\
 &= x \cdot (y+y') \cdot (z+z') + (x+x') \cdot y \cdot z' \\
 &= xyz + \underline{xyz'} + xy'z + xy'z' + \underline{x'yz'} + \underline{x'y'z'} \\
 &= xyz + xyz' + xy'z + xy'z' + x'y'z'
 \end{aligned}$$

$$F = m_7 + m_6 + m_5 + m_{B_1} + m_2$$

$$\begin{aligned}
 F &= x + y z' \\
 &= (x+y)(x+z') \\
 &= (x+y+0)(x+0+z') \\
 &= (\cancel{x}+\cancel{y}+\cancel{z}\cdot\cancel{z'}) (\underline{x}+\cancel{y}\cdot\cancel{y}+\cancel{z}) \\
 &\Rightarrow (x+y)(x+z') + \cancel{x+y} \\
 &= (x+y+z)(x+y+z') \\
 &= (x+y+z)(x+y+z') (x+y) \\
 &= M_0 M_1 M_2
 \end{aligned}$$

Reducing the function :-

$$\text{iii) } F = xyz' + xy' + x'y z$$

$$= x(yz' + y' + yz)$$

$$= x(yz' + z) + y'$$

$$= x(y + y')$$

$$F = x$$

$$\text{iv) } F = \sum m_0 m_2 m_4 m_5 m_6$$

$$= x'y'z' + x'y'z + x'y'z + x'y'z + x'y'z$$

$$= x'(y'z' + yz') + x(y'z' + y'z + yz')$$

$$= x'(z'(y + y')) + x(y'(z' + z) + yz')$$

$$= x'z' + x(y' + yz')$$

$$= y'z' + xy' + x'y'z'$$

$$= x(y' + yz') + x'z'$$

$$= x(y' + y)(y' + z') + x'z'$$

$$= x(y' + z') + x'z'$$

$$= xy' + xz' + x'z'$$

$$= xy' + z'(x + y')$$

$$= xy' + z'$$

(OR)

$$= y'z'(x + x') + yz'(x + x') + xy'z$$

$$= y'z' + yz' + xy'z$$

$$= z' + xy'z$$

$$= (z' + z)(z' + xy')$$

$$= xy' + z'$$

$$F = \bar{x} + y_3'$$

	x	y	z	F	min term	Max term
5	0	0	0	0	m ₀	M ₀
	0	0	1	0	m ₄	M ₁
	0	1	0	1	m ₂	M ₂
	0	1	1	0	m ₃	M ₃
	1	0	0	1	m ₄	M ₄
10	1	0	1	1	m ₅	M ₅
	1	1	0	1	m ₆	M ₆
	1	1	1	1	m ₇	M ₇

$$F = m_2 + m_4 + m_5 + m_6 + m_7$$

$$= \Sigma(2, 4, 5, 6, 7)$$

$$= M_0 \cdot M_1 \cdot M_5$$

$$= \Pi(0, 1, 3)$$

To get Complement

$$F' = m_1 + M_3 + M_0$$

$$= \Sigma(1, 3, 0)$$

$$= \Pi(2, 4, 5, 6, 7)$$

$$F' = M_2 \cdot M_4 \cdot M_5 \cdot M_6 \cdot M_7$$

By using de-morgan's theorem.

$$F = m_2 + m_4 + m_5 + m_6 + m_7$$

$$F' = (m_2 + m_4 + m_5 + m_6 + m_7)'$$

$$= m_1' \cdot m_4' \cdot m_5' \cdot m_6' \cdot m_7'$$

$$= M_2 \cdot M_4 \cdot M_5 \cdot M_6 \cdot M_7$$

2. Gate-level Minimization

Camlin	Page
Date	/ /

→ The no of gates are reduced in designing circuits.

This is gate-level minimization.

→ Using boolean algebra also function can be reduced but it is complicated.

→ Other method to reduce gates and reduce complexity is Karnaugh map i.e. K' map.

→ K map

2 Variable K' map:-

$y \backslash x$	0	1
0	m_0 (00)	m_1 (01)
1	m_2 (10)	m_3 (11)

normal method

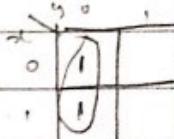
$$F = \sum (0, 2)$$

$$= m_0 + m_2$$

$$= \bar{x}y' + xy'$$

$$= y'$$

K' map



$$F = \bar{y}'$$

→ K map follows adjacency rules, put 1 at minimum places.

→ combine two adj numbers whose sum is same.

→ eliminate the variable who is changing its value among the combined squares

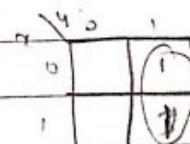
→ then, if the variable sum and 1 are same then put

the remaining squares have value 0 put x' if 1

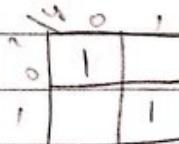
put x . // y' to "y".

$$\text{Ex. } F = \sum (1, 3)$$

$$F = \sum (0, 3)$$



$$F = y$$



$$F = \bar{x}y' + xy$$

→ XNOR & XOR cannot be reduced.

$$F = \Sigma(0, 1, 3)$$

$= m_0 \quad m_1 \quad m_3$

1	0	1
0	1	1
1	1	1

$$F = x' + y$$

3 variable K' map: It follows gray code.

y_2	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

$$F = \Sigma(1, 2, 5, 6)$$

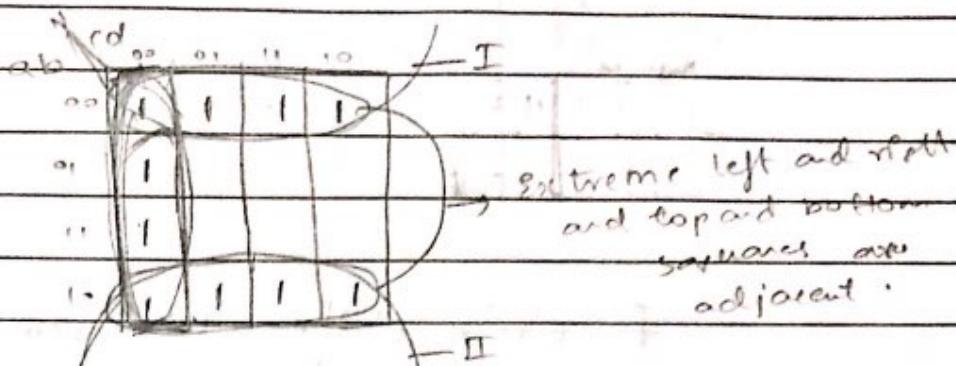
	y_2	00	01	11	10
0	0	1	1	1	1
1	1	1	1	1	1

$$F = y_2' z + y_2 z'$$

4-variable K' map: \rightarrow

ab	cd	00	01	11	10
00	0	m_0	m_1	m_3	m_2
01	1	m_4	m_5	m_7	m_6
11	1	m_{12}	m_{13}	m_{15}	m_{14}
10	0	m_8	m_9	m_{11}	m_{10}

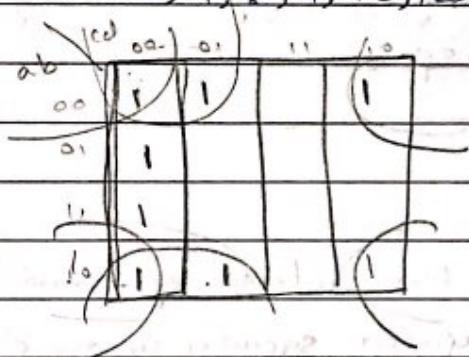
$$F = (0, 1, 2, 4, 8, 9, 10, 11, 12)$$



$$F = c'd' + b'$$

→ If a square having 1 is left out without any connection then take the mid term func exactly:-

$$F = \Sigma (0, 1, 2, 4, 8, 9, 10, 12)$$



$$F = c'*b' + c'd' + d'*b'$$

$$F = b'c' + c'd' + b'd'$$

ab	cd	00	01	11	10
00	1	1			
01	1	1	1		
11			1		
10	1				

$$F = \overline{c}\overline{a} + \overline{c}\overline{d} + ab + abcd$$

$$\begin{aligned} F &= \overline{a}\overline{c} + b\overline{c}\overline{d} + \overline{a}b\overline{d} + abcd \\ &= (\overline{a}\overline{c} + ac)(\overline{a}\overline{c} + bd) + d'(\overline{b}\overline{c} + a'b) \\ &= \end{aligned}$$

Implicant :-

It is the product term formed by combining adjacent squares in the k' map.

Prime Implicant :-

It is the product term formed by combining the maximum possible no of adjacent squares in the k' map.

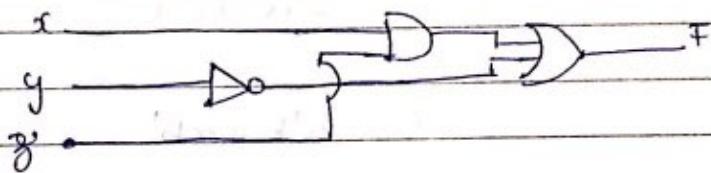
Essential prime Implicant :-

The Essential prime Implicant is the prime implicant in which ^{there exist} atleast there exist one mrs term which is not covered by other prime Implicants.

$$F(x,y,z,w) = \Sigma(0,1,2,3,8,9,10,11,14,15)$$

	00	01	11	10
00	1	1	1	1
01				
11			1	1
10	1	1	1	1

$$F = b' + ac = y' + xz$$



→ In sum of products $1^{\text{st}} \rightarrow \text{AND}$ $2^{\text{nd}} \rightarrow \text{OR}$
 → In product of sum $1^{\text{st}} \rightarrow \text{OR}$ $2^{\text{nd}} \rightarrow \text{AND}$

Reduce the function $F(x,y,z,w) = \Sigma(0,1,2,3,8,9,10,11,14,15)$

into product of sums.

$$F = \Sigma(0,1,2,3,8,9,10,11,14,15)$$

	00	01	11	10
00	0	0	0	0
01	0	0		
11				
10				

$$F = (\bar{F})' \\ = [y \cdot (x' + z')]'$$

$$\bar{F} = a'b + c'b$$

$$\bar{F} = x'y + yz'$$

$$\bar{F} = y(x+z)'$$

$$= y' + (x+z)'$$

$$= y' + (a'b)' \cdot (b')'$$

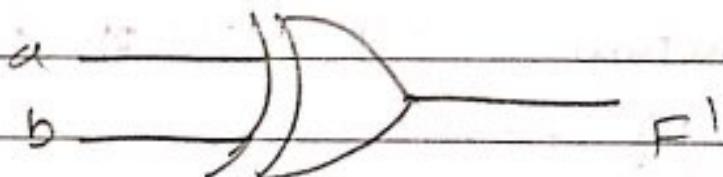
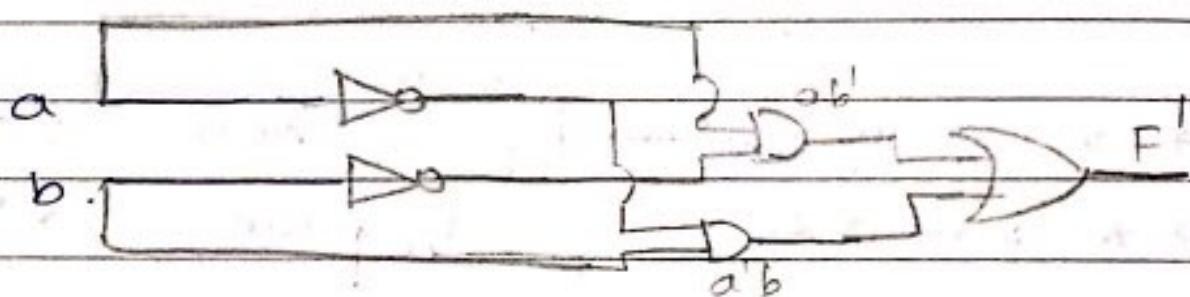
$$F = y' + xz$$

For the function $F(u, v, s, w) = \sum(D, 1, 2, 3, 12, 13, 14, 15)$
 get the reduced exp for \bar{F} using K-map
 and implement it using basic gates

$$F =$$

ab	00	01	11	10
00	1			
01	0	0	0	0
11				
10	0	0	0	0

$$\bar{F} = a'b + ab'$$



Don't care functions:-

In some functions some terms will not be cared.

In Binary to BCD conversion

numbers after 9 are don't cares.

$$\Phi = \Sigma(4, 5, 6, 7, 8, 9) \quad F = \Sigma(0, 1, 2, 3, 12, 13, 14, 15)$$

	00	01	11	10
00	1	1	1	1
01	X	X	X	X
11	1	1	1	1
10	X	X		

4 don't care at 1

$$F = x^1 + y$$

5-Variable K' map:-

A 5-variable K' map consists of 2 to 4 Variable K' maps, The MSB is 1 for one 4 variable K' map and it is 0 for other 4 Variable K' map.

bc \ de				bc \ de			
no	m ₁	m ₅	m ₆	m ₁₆	m ₁₄	m ₁₅	m ₁₇
0=0	m ₂	m ₅	m ₆	m ₂₀	m ₂₁	m ₂₃	m ₂₂
	m ₁₂	m ₁₃	m ₁₅	m ₁₁	m ₂₈	m ₂₉	m ₃₁
	m ₁₆	m ₁₇	m ₁₈	m ₁₉	m ₂₄	m ₂₅	m ₂₆
	m ₂₈	m ₂₉	m ₃₀	m ₃₁	m ₂₀	m ₂₁	m ₂₂

a=1

All the adjacency rules that are applicable for 4 variable K' map are also applicable in case of 5 variable K' map along with them each square in 1-variable K' map is adjacent to the corresponding square in the other 4 variable K' map.

$$F(a, b, c, d, e) = \Sigma (0, 1, 2, 5, 6, 7, 12, 15, 14, 15, 16, 19, 18, 19)$$

	de	30.81	30.79		W	30.80	30.81	30.77	30.75
bc	1	1	1		1	1	1	1	1
26.4	1	1	1		1	1	1	1	1
31	1	1			31				
37	1	1	1	1		11			
10					116				

$$f = b'c' + a'b'c + a'bc$$

Reduce F if id contains dot one of

$$\mathcal{P} = S(24, 25, 26, 27, 28, 29, 30, 31)$$

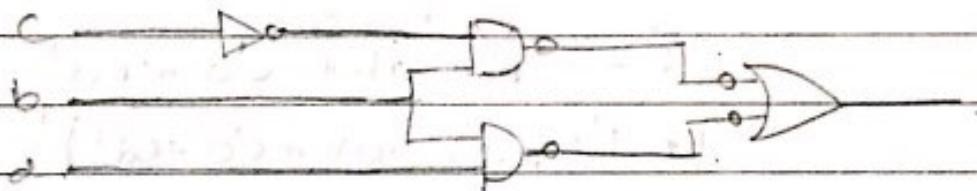
$$F = bc' + a'ce + bc +$$

Implement the complement of the function $F(a, b, c, d) = \sum(1, 2, 3, 6, 8, 9, 10, 11, 14)$ using NAND gates.

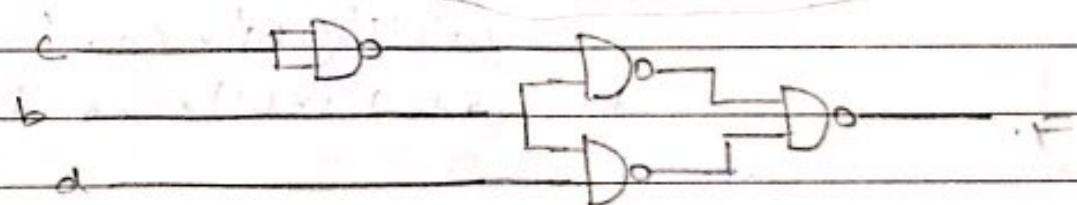
$a'c'$	$a'b$	ab'	$b'd$	$b'd'$	$c'd$	$c'd'$	$a'b'd'$
00	00	01	10	11	00	01	00
01	01	10	11	10	01	10	01
11	11	00	01	00	11	01	11
10	10	-	-	-	10	-	10

$$\bar{F} = c'b + bd$$

$$\bar{F} = bc' + bd.$$



($c' \cdot b$) \cdot ($d' \cdot b$) \rightarrow NOT or \rightarrow AND-NAND



Don't care functions:-

In some functions some terms will not be cared.

In Binary to BCD converter,

numbers after 9 are don't care.

Q. $\phi = \Sigma(4, 5, 6, 7, 10, 9)$ $F = \Sigma(0, 1, 2, 3, 12, 13, 14, 15)$

		3'2' 00 01 11 10				
		00	1	1	1	
		01	x	x	x	x
		11	1	1	1	1
		10	x	x		

4 don't care
as 1

15 $F = x^1 + y$

5-Variable K' map:-

Rule 1 A 5-variable K' map consists of 2 4-variable K' maps,
The MSB is 0 for one 4-variable K' map and
it is 1 for other 4-variable K' map.

		bc				de			
		m ₀	m ₁	m ₂	m ₃	m ₄	m ₅	m ₆	m ₇
a=0		m ₀	m ₁	m ₂	m ₃	m ₄	m ₅	m ₆	m ₇
		m ₅	m ₆	m ₇	m ₀	m ₁	m ₂	m ₃	m ₄
		m ₁₂	m ₁₃	m ₁₄	m ₁₅	m ₂₀	m ₂₁	m ₂₂	m ₂₃
		m ₁₆	m ₁₇	m ₁₈	m ₁₉	m ₂₄	m ₂₅	m ₂₆	m ₂₇
		m ₂₈	m ₂₉	m ₃₀	m ₃₁	m ₃₂	m ₃₃	m ₃₄	m ₃₅
a=1		m ₈	m ₉	m ₁₀	m ₁₁	m ₁₆	m ₁₇	m ₁₈	m ₁₉

Rule 2 All the adjacency rules that are applicable for 4 Variable K' map are also applicable in case of 5 Variable K' map along with them each square is 1-variable K' map adjacent to the corresponding square in the other 4 variable K' map.

Q. $F(a, b, c, d, e) = \Sigma(0, 1, 2, 5, 6, 7, 12, 13, 14, 15, 16, 17, 18, 19)$

	de	00	01	11	10		de	00	01	11	10
bc											
a'		1	1	1	1			1	1	1	1
01		1	1	1	1			01			
11		1	1	1	1			11			
10								10			

$$F = b'c' + a'bc + a'ce$$

Reduce F if it contains don't care ϕ

$$\phi = \Sigma(24, 25, 26, 27, 28, 29, 30, 31)$$

	de	00	01	11	10		de	00	01	11	10
bc											
a'		1	1	1	1			1	1	1	1
01		1	1	1	1			01			
11		1	1	1	1			11	1	1	1
10								10	X	X	X

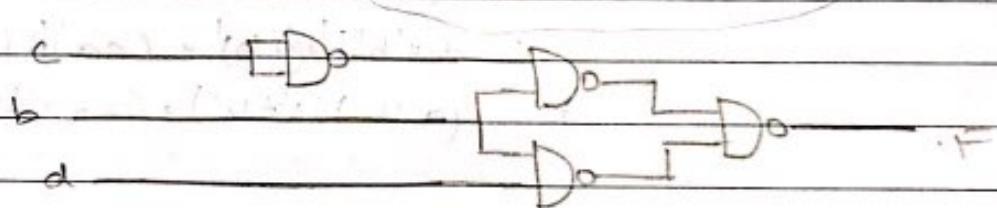
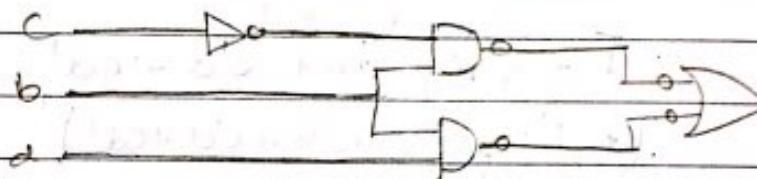
$$F = b'c' + a'ce + bc +$$

Implement the Complement of the func $F(a, b, c, d) = \sum(1, 2, 3, 6, 8, 9, 10, 11, 14)$ using NAND gates.

	ab	00	01	11	10
00	00
01	00	00	00	00	00
11	00	00	00	00	00
10

$$\bar{F} = \bar{c}'\bar{b} + \bar{b}\bar{d}$$

$$\bar{F} = \bar{b}\bar{c}' \oplus \bar{b}\bar{d}, =$$



Implement the funcⁿ using NOR gates

$$\begin{aligned}
 F &= (A \oplus B)(C \oplus D)' \\
 &= (AB' + A'B)(A + B')(A' + B) \\
 &\quad (C + D)(D' + C) \\
 &= (AB' + A'B)(C'D' + CD) \\
 &= AB'C'D' + A'B'C'D' + AB'C'D + A'B'CD
 \end{aligned}$$

$$F = m_8 + m_4 + m_{11} + m_7$$

		rd	rd	rd	rd
	ab	0	0	0	0
10	0d	0	0	1	0
	01	1	0	1	0
11	1d	0	0	0	0
10	10	1	0	1	0

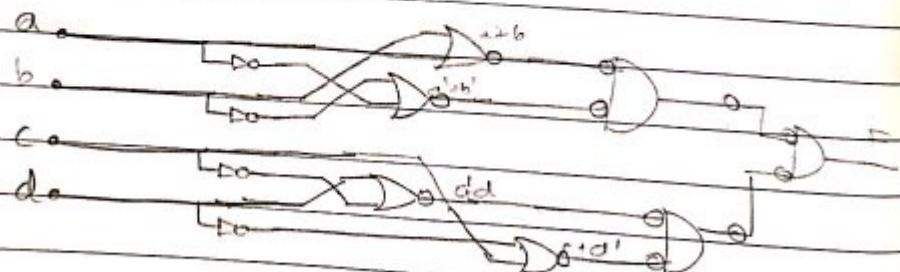
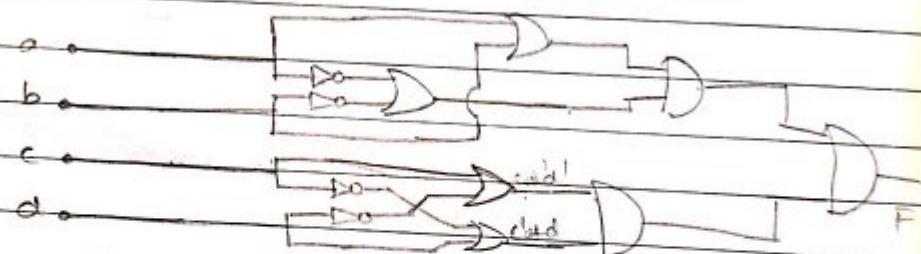
$$F = a'b' + ab + c'd + cd'$$

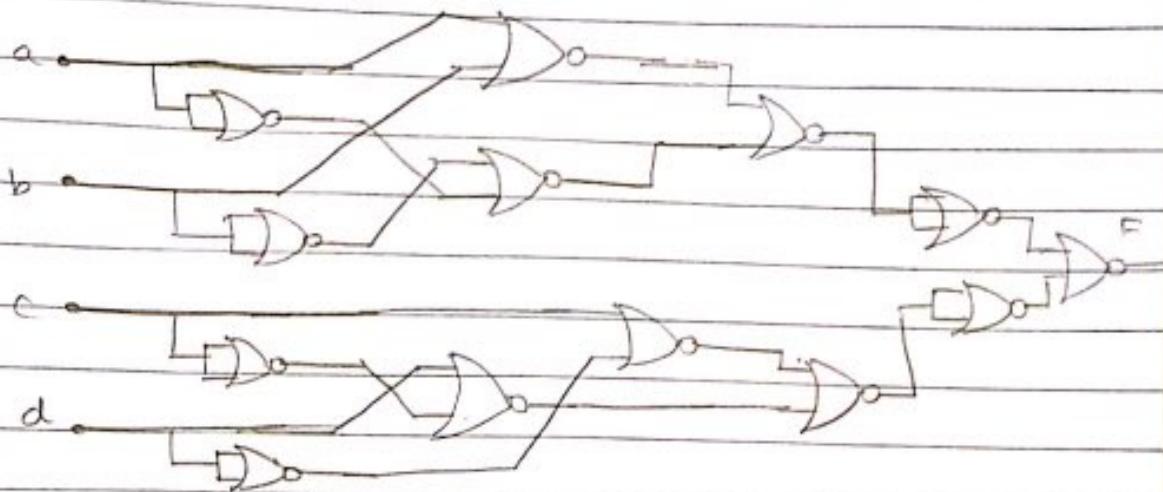
$$(F')' = (a'b' + ab + c'd + cd')'$$

$$= (a'b' + ab)' \cdot (c'd + cd')'$$

$$= (a'b')' \cdot (ab)' \cdot (cd)' \cdot (cd')'$$

$$F = (a+b)(a'+b') \cdot (c+d')(c'+d)$$





12.12.12

XOR properties:-

$$x \oplus y = xy' + x'y$$

(i) $x \oplus 0 = x$

(ii) $x \oplus 1 = x'$

(iii) $x \oplus x = 0$

(iv) $x \oplus x' = 1$

(v) $(x \oplus y)' = (x' \oplus y) = (x \oplus y)'$

(vi) $A \oplus B = B \oplus A$

(vii) $A \oplus (B \oplus C) = (A \oplus B) \oplus C$.

→ A multiple variable XOR operation is defined as odd function

→ An n variable XOR func" is an odd func" defined as

the logical sum of $\frac{2^n}{2}$ terms whose binary values have

an odd no of '1's.

$F(x, y, z)$

$m_0 \rightarrow m_7 \quad \text{no } 8^{th}$

$0 \rightarrow 000 \quad -0$

$1 \rightarrow 001 \quad -1 \rightarrow \text{odd}$

$2 \rightarrow 010 \quad -1 \rightarrow \text{odd}$

$3 \rightarrow 011 \quad -2$

$4 \rightarrow 100 \quad -1 \rightarrow \text{odd}$

$5 \rightarrow 101 \quad -2$

$6 \rightarrow 110 \quad -2$

$7 \rightarrow 111 \quad -3 \rightarrow \text{odd.}$

$F = \Sigma (1, 2, 4, 7) \quad \text{odd function}$

	y_2	y_1	y_0
x	0	1	1
	0	1	1
	1	1	1

$$F = xy'z' + x'y'z + x'yz + x'yz'$$

$$x(y'z' + yz) + x'(y'z + yz')$$

$$\Rightarrow (x \oplus y \oplus z)$$

$F = \Sigma (0, 3, 5, 6) \quad \text{even function}$

	y_2	y_1	y_0
x	0	1	1
	0	1	1
	1	1	1

$$F = (x \oplus y \oplus z)'$$

→ The function which have odd num of 1's in binary representation of their min term is called odd fun.

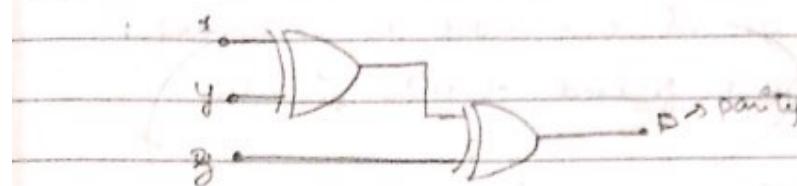
→ By far even function

→ Odd and Even functions can be implemented by XOR and XNOR gates.

Application of XOR:-)

Parity generator:- It generates parity at transmitter

Even 3-bit parity generator:-



The parity generator gives a
output - 1 if (odd num of 1's)
output - 0 if (even num of 1's)

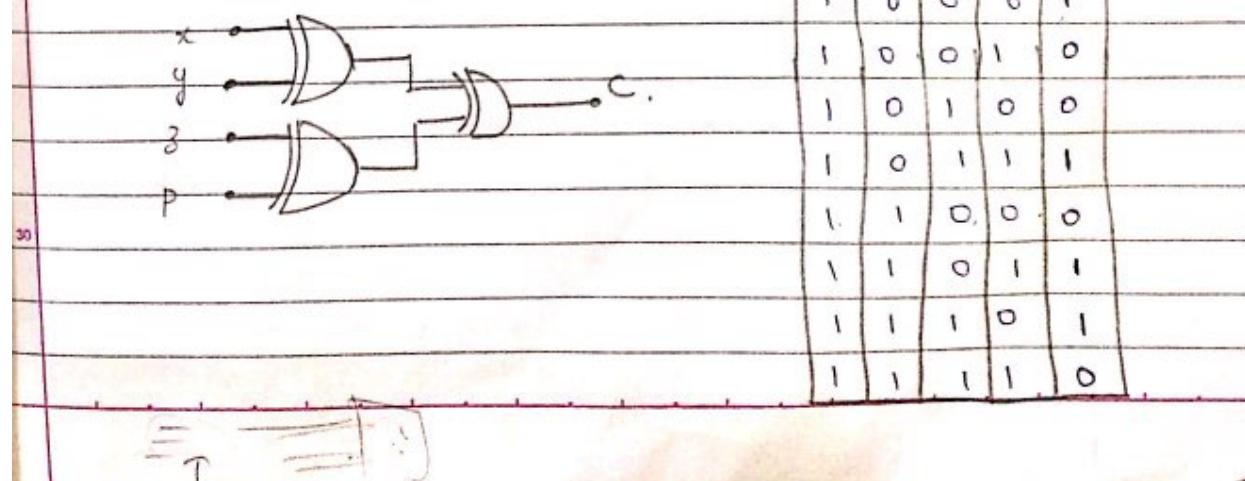
x	y	z	p
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	1	1

Even parity checker:-

At the receiving end, it checks the parity for
error detection.

x	y	z	p	c
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	1	0	1	0
0	1	1	1	0
1	0	0	0	1
0	1	0	0	0
0	1	1	0	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	0
1	1	0	1	1
1	1	1	0	1
1	1	1	1	0

$$C = x \oplus y \oplus z \oplus p.$$



The parity checker can be used for generation and checking of parity

If we ground one input to parity checker
its sum is used as parity generator

$$P=0.$$

$$x \oplus 0 = x.$$

$$x \oplus y \oplus z \oplus P = x \oplus y \oplus z \oplus 0$$

$$F = x \oplus y \oplus z.$$

Digital Circuits:-

These are 2 types

→ Combinational

→ Sequential.

Combinational :-

The o/p depends on the present i/p.

Sequential :-

→ The o/p depends not only depend on present i/p
but also on past i/p.

→ There is a storage element which is a memory
element which stores past i/p.

(i) Combinational Circuits.

Analysis procedure:-

(i) The 1st step is to make sure that the given circuit is combinational or not.

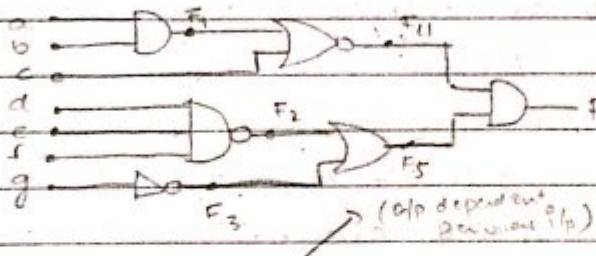
(ii) Label all the gate o/p that are function of i/p variables and determine the boolean function for each gate o/p.

(iii) Label the o/p's of the gates that are function of i/p variables and previously labelled gate and determine the boolean function of these gates.

(iv) repeat the above process until the final o/p of the circuit are obtained.

(v) By repeated substitution of previously defined function the final o/p boolean function can be obtained in terms of i/p variables.

Problem.



→ As there is no feedback or memory element it is a combinational circuit.

→ we name the intermediary outputs.

→ find intermediary function of i/p

$$F_1 = a \cdot b$$

$$F_2 = (a \cdot b \cdot c)' \cdot (d \cdot e \cdot f)'$$

$$F_3 = g'$$

$$F_4 = (F_1 + c)'$$

$$F_5 = F_2 + F_3$$

$$F = F_4 \cdot F_5$$

$$F_1 = (a \cdot b + c)'$$

$$F_5 = (d \cdot e \cdot f)' + g'$$

$$F = (ab + c)' \cdot [(def)' + g']$$

$$= (ab)' \cdot c' \cdot (defg)' \cdot [(de)' + f' + g']$$

$$= (a' + b') \cdot (c') \cdot [d' + e' + f' + g']$$

$$= (ab)' \cdot c' \cdot (defg)'$$

$$F = (ab + c + defg)$$

Designing Procedure:

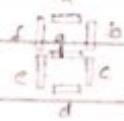
- (i) The design starts from converting the design objective into concrete specifications.
- ii) From specifications required no of inputs and outputs are to be determined and a symbol is attached to each of them.
- iii) Truth table that defines the relationship between the inputs and outputs is to be determined.
- iv) Simplified boolean function for each output as a function of input variables is to be obtained.
- v) Appropriate gates are to be chosen and the logic diagram is to be drawn.
- vi) Finally the correctness of the logic diagram is to be verified.

Code converter:-

For: BCD to 7 segment display decoder.

BCD

7 segment display
Inputs

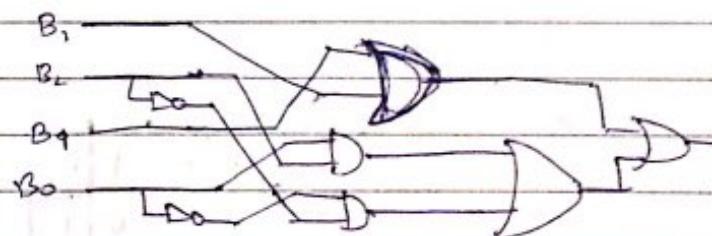


B_3, B_2, B_1, B_0	a	b	c	d	e	f	g
0 0 0 0	1	1	1	1	1	0	0
0 0 0 1	0	1	1	0	0	0	1
0 0 1 0	1	1	0	1	1	0	1
0 0 1 1	1	1	1	0	0	1	1
0 1 0 0	0	1	1	0	0	1	1
0 1 0 1	1	0	1	1	0	1	1
0 1 1 0	1	0	1	1	1	1	1
0 1 1 1	1	1	1	0	0	0	0
1 0 0 0	1	1	1	1	1	1	1
1 0 0 1	1	1	1	1	0	1	1

for a.

B_3, B_2	00	01	11	10
B_1, B_0	1	0	1	1
00	1	0	1	1
01	0	1	1	1
11	x	v	x	x
10	1	1	x	v

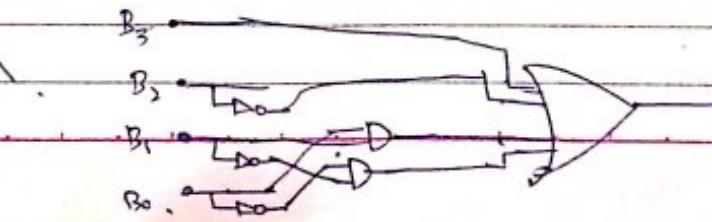
$$a = B'_0 B'_2 + B_1 + B_3 + B_0 B_2$$



for b.

B_3, B_2	00	01	11	10
B_1, B_0	1	1	1	0
00	1	1	1	0
01	1	0	1	0
11	x	x	x	x
10	1	1	x	x

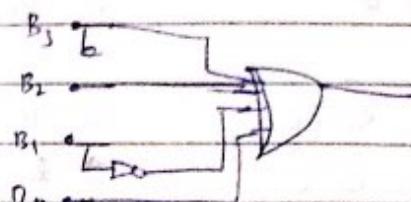
$$b = B'_1 B'_0 + B_1 B_0 + B'_2 + B_3$$



for C

$B_3 B_2$	00	01	11	10
00	1 1 1 0			
01	1 1 1 1			
11	x x	x x		
10	1 1	x x		

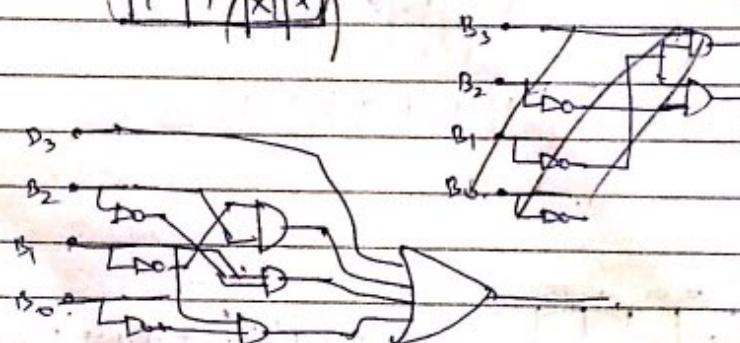
$$C = B_1' + B_0 + B_2 + B_3$$



for g

$B_3 B_2$	00	01	11	10
00	0 0 1 1			
01	1 1 0 1			
11	x x	x x		
10	1 1	x x		

$$g = B_0' B_1 + B_3' B_2' B_1 + B_1' B_2 + B_3 B_1$$



for d.

$B_2 B_1$	00	01	11	10
00	1 0 1 1			
01	0 1 0 1			
11	x x x x	x x		
10	1 1	x x		

$$d = B_0' B_1' B_2 + B_2' B_3' + B_0 B_1 + B_1' B_3 + B_1 B_2' B_3$$

for e

$B_0 B_1$	$B_2 B_3$	00	01	11	10
00	1	0	0	1	
01		0	0	1	
11	x	x	x	x	
10	1	0	x		

$$C = B_2 B_3 + B_1' B_3'$$

for f

$B_0 B_1$	$B_2 B_3$	00	01	11	10
00	1	0	0	0	
01	1	1	0	1	
11	x	x	x	x	
10	1	1	x	x	

$$f = B_2' B_3' + B_0 + B_1 B_2' + B_1 B_2 B_3'$$

Adder-Subtractor :-

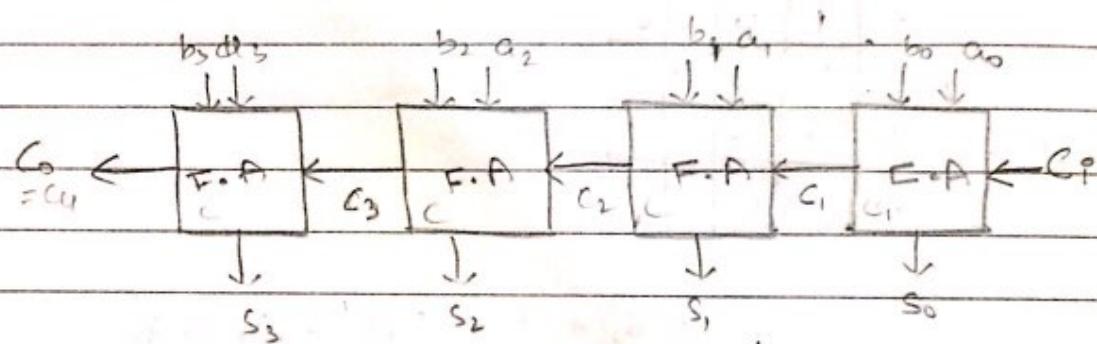
4-bit binary adder :-

$$A = a_3 \ a_2 \ a_1 \ a_0$$

$$B = b_3 \ b_2 \ b_1 \ b_0 \leftarrow \text{Carry In}$$

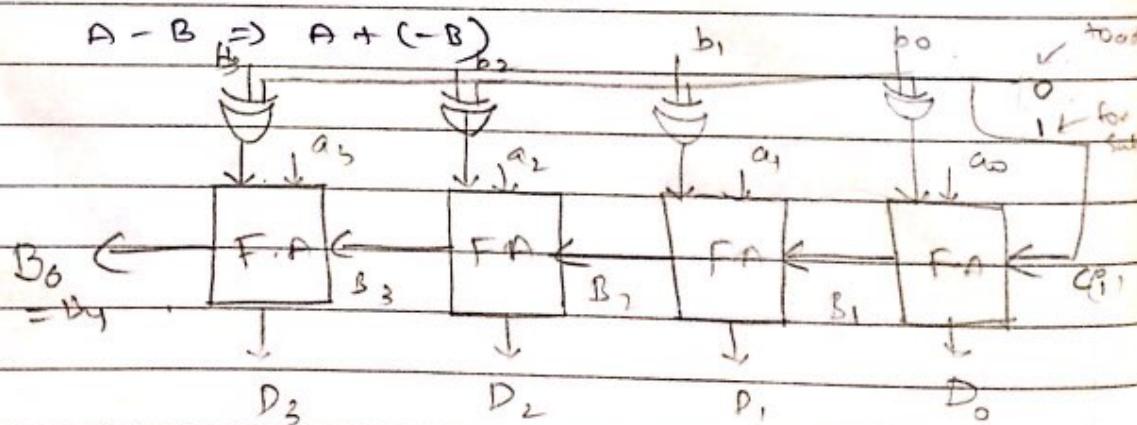
$$\begin{matrix} C_0 & a_3 & s_2 & s_1 & s_0 \\ \downarrow & & & & \\ \text{carry output} & & & & \end{matrix}$$

9 inputs \rightarrow 5 outputs



4-bit binary subtractor :-

$$A - B \Rightarrow A + (-B)$$



Generally it gets delay to get the off carry to avoid this there is another adder

Carry look ahead adder:-

In full adder

$$S_i = A_i \oplus B_i \oplus C_i$$

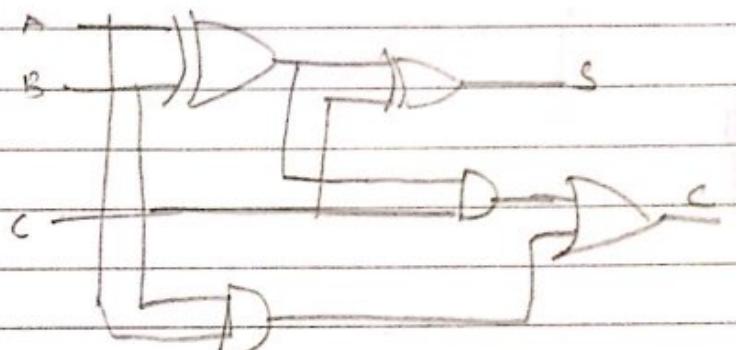
$$C_{i+1} = (A_i \oplus B_i)C_i + A_i \cdot B_i$$

$$P_i = A_i \oplus B_i$$

↳ carry propagator

$$G_i = A_i \cdot B_i$$

↳ carry generate



$$C_{i+1} = P_i C_i + G_i$$

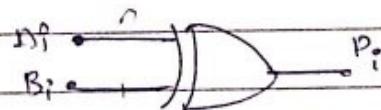
$$G = G_0 + P_i C_0$$

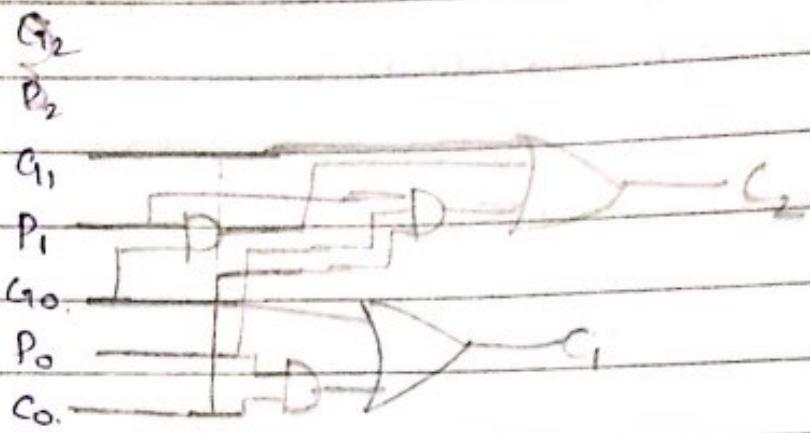
$$C_1 = G_1 + P_1 C_1$$

$$= G_1 + P_1 (G_0 + P_0 C_0)$$

$$C_2 = G_2 + P_2 (G_1 + P_1 G_0 + P_1 P_0 C_0)$$

$$C_3 = G_3 + P_3 (G_2 + P_2 G_1 + P_2 P_1 G_0 + P_3 P_2 P_0 C_0)$$

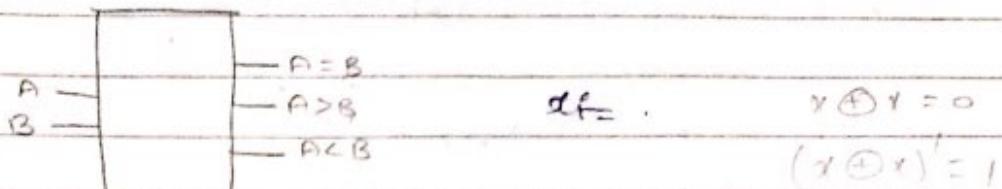




Magnitude comparator :-

1-bit \rightarrow 8 bits \rightarrow 4 bits

If checks or compares the ip values



for checking equality of ip we use XNOR gate

$$x_i' = (A_i \oplus B_i)' = A_i B_i + A_i' B_i'$$

$$A = A_3 \cdot A_2 \cdot A_1 \cdot A_0$$

$$B = B_3 \cdot B_2 \cdot B_1 \cdot B_0$$

$x_3 x_2 x_1 x_0 = 1$ for all bits if $A = B$.

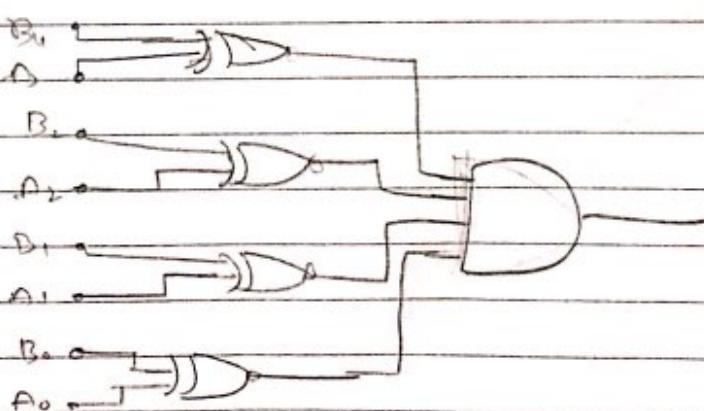
for $(A > B)$

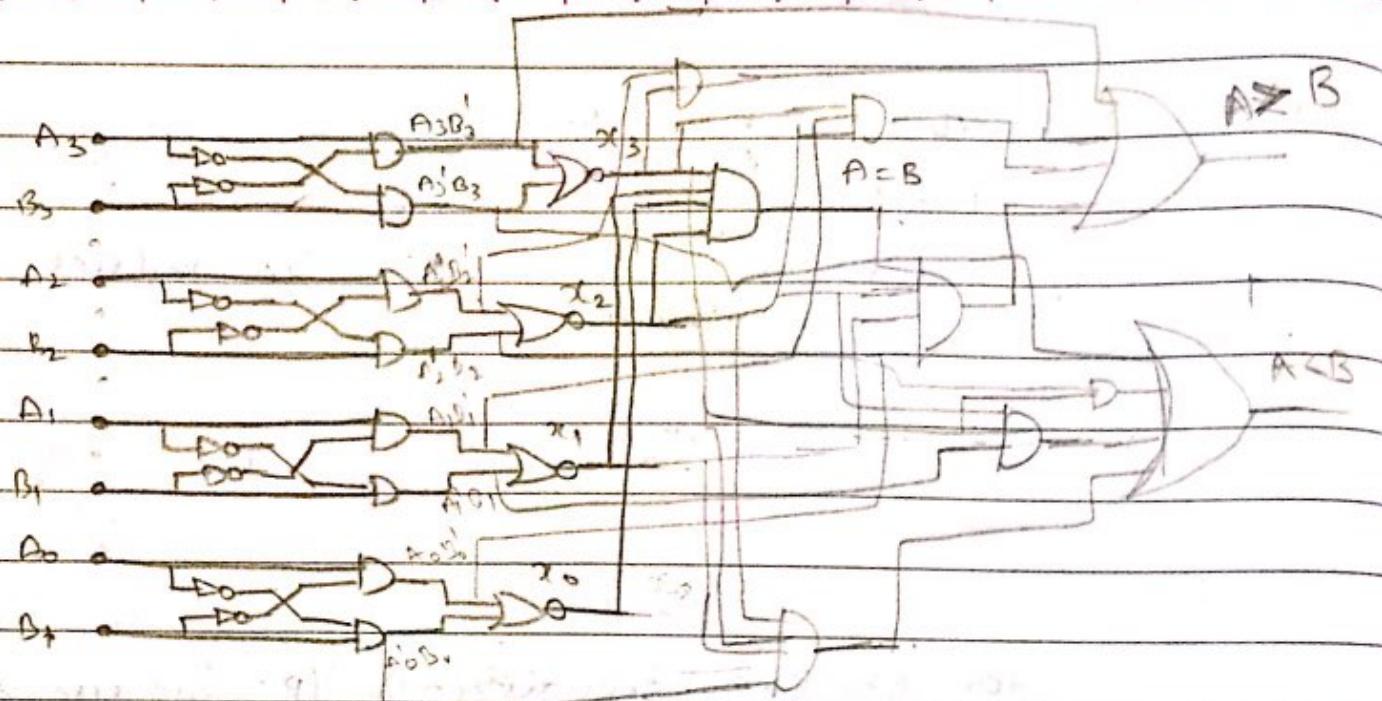
$$= A_3 B_3' + x_3 A_2 B_2' + x_2 A_1 B_1' + x_1 A_0 B_0' + x_3 x_2 x_1 A_0 B_0'$$

for $(A < B)$

$$= A_3' B_3 + x_3 A_2' B_2 + x_2 x_1 A_1' B_1 + x_3 x_2 x_1 A_0' B_0$$

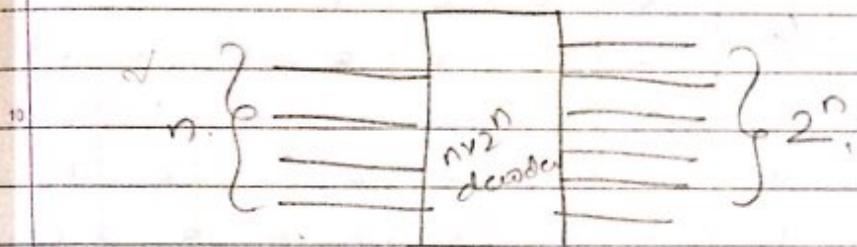
$(A = B)$





Decoder :- ($n \times 2^n$)

- Decoder is a logic circuit that converts binary information from n coded inputs to a maximum of 2^n unique outputs.
- It simply changes a code into set of signals.



2x4 decoder :-

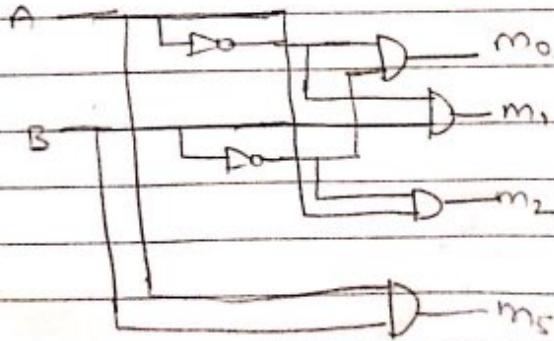
A	B	D ₃	D ₂	D ₁	D ₀
0	0	0	0	0	1
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0

$$D_0 = A'B' = m_0$$

$$D_1 = \bar{A}B = m_1$$

$$D_2 = AB' = m_2$$

$$D_3 = AB = m_3$$



3x8 decoder :-

A	B	C	D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

$$D_0 = A'B'C'$$

$$D_1 = A'B'C$$

$$D_2 = A'BC'$$

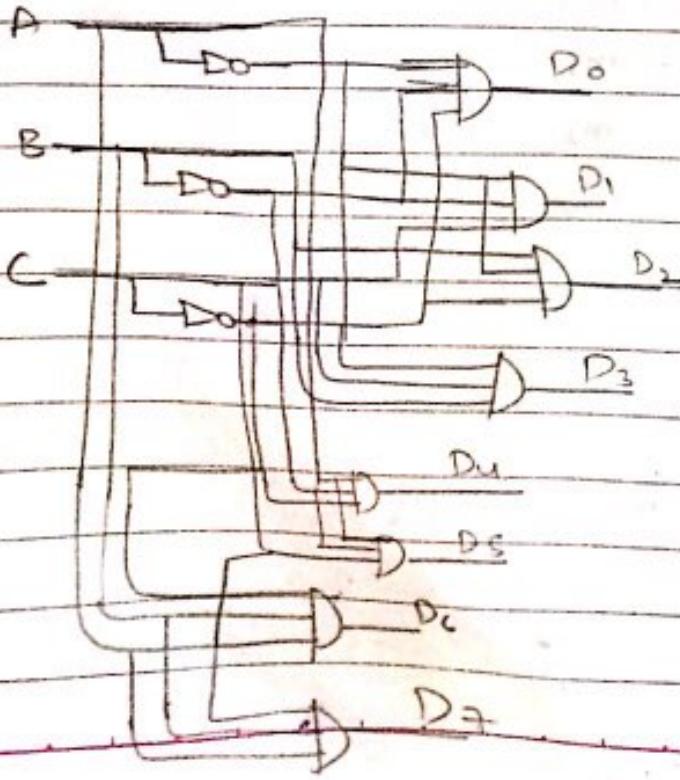
$$D_3 = A'B'C'$$

$$D_4 = AB'C'$$

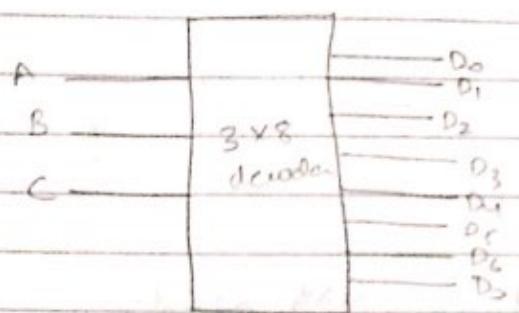
$$D_5 = AB'C$$

$$D_6 = ABC'$$

$$D_7 = ABC$$



Implement a full adder using decoder:-



10 Implement the func $F = \Sigma(0,1,4,5)$ using 3-to-8 decoder.

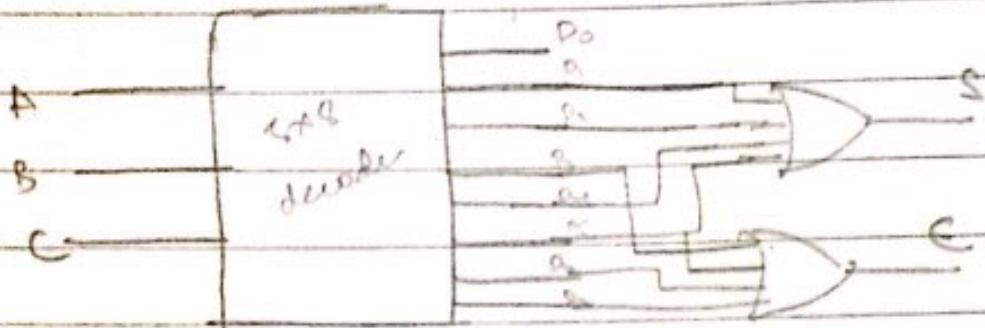


Implement a full adder using 3x8 decoder:-

	x1	y	z	s	c
20	0	0	0	0	0
	0	0	1	1	0
	0	1	0	0	0
	0	1	1	0	1
	1	0	0	1	0
25	1	0	1	0	1
	1	1	0	0	1
	1	1	1	1	1

$$\text{Sum} = \Sigma(1, 2, 4, 7)$$

$$\text{Carry} = \Sigma(3, 5, 6, 7)$$



→ If 4 digits exist the 4x8 decoder:

$$F = xy + yz' \text{ implement using } 3 \times 8 \text{ decoder.}$$

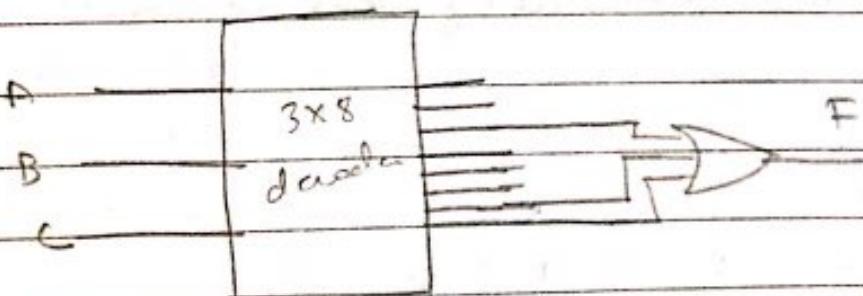
$$F = xy(z + z') + yz'(x + x')$$

$$= xyz + xyz' + x'yz' + x'yz$$

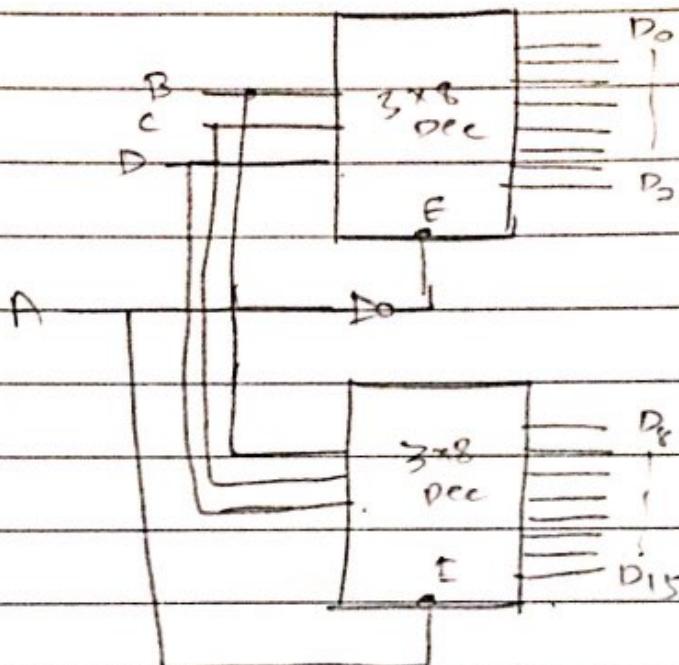
$$= xyz + xyz' + x'yz'$$

$$= m_7 + m_6 + m_2$$

$$F = \Sigma(2, 6, 7)$$



4x16 decoders using 3x8 decoders



D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇	D ₈	-	-	-	D ₁₅
1	0	0	0	-	-	0
0	1	0	0	-	-	-	-	-	.	.	.	0
0	0	1	0	*	*	*	*	*	*	*	*	0
0	0	0	1	-	-	-	-	-	-	-	-	0
1	1	1	1	1	1	1	1	1	1	1	1	1
0	0	0	0	-	-	-	-	-	-	-	-	1

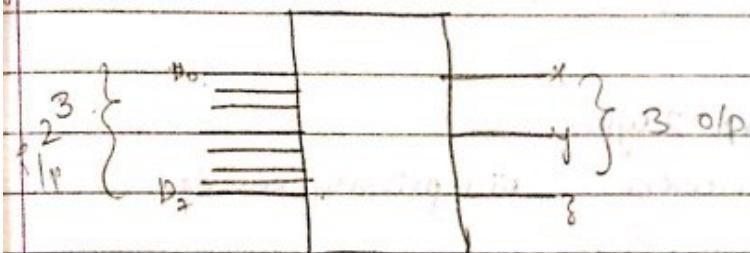
Encoder :- $(2^n \times 2^m)$

→ Encoder is a combinational circuit that generates a

specified code at its o/p such as binary or BCD ..

→ It changes set of signals into a code.

8 × 3 Encoder :-

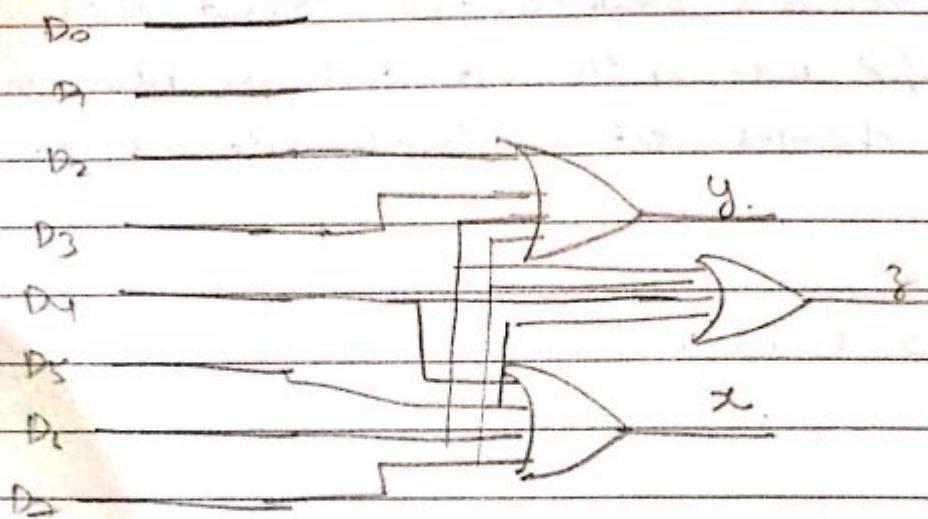


	D ₀	D ₁	D ₂	D ₃	D ₄	D ₅	D ₆	D ₇	X	Y	Z
0	0	0	0	0	0	0	0	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	1
2	0	0	1	0	0	0	0	0	0	1	0
3	0	0	0	1	0	0	0	0	0	0	1
4	0	0	0	0	1	0	0	0	1	0	0
5	0	0	0	0	0	1	0	0	1	0	0
6	0	0	0	0	0	0	1	0	1	1	0
7	0	0	0	0	0	0	0	1	1	1	1

$$x = \Sigma (4, 5, 6, 7)$$

$$y = \Sigma (2, 3, 6, 7)$$

$$z = \Sigma (1, 3, 5, 7)$$



→ Encoders are 2 types

(i) Binary encoder (ii) priority encoder.

→ Binary encoder is a multi i/p combinational logic circuit that converts the logic level '1' data at its i/p into an equivalent binary code at its o/p.

Limitations:-

• If we give more than one i/p at same time, it confuses to give o/p.

e.g.:

For $D_0 = 1$ & $D_1, D_2, D_3, D_4, D_5 = 0$

$y = 1, z = 0, x = 0$

⇒ We get 2 same o/p for 2 diff i/p.

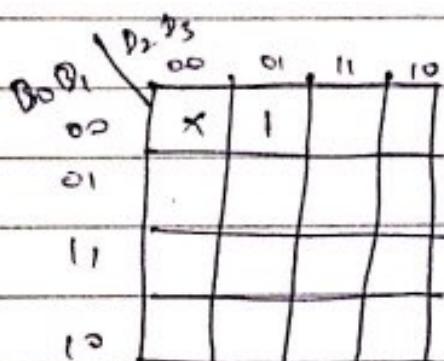
→ The operation of priority encoder solves this problem.

Priority encoder:-

This encoder only considers the High Value of the I/p having highest priority - The rest of all I/p with lower priority are lowered or ignored.

4x2 priority encoder:-

D ₀	D ₁	D ₂	D ₃	x	y	v	→ I/p's which are high
0	0	0	0	x'	x	0	
1	0	0	0	0	0	1	
x	1	0	0	0	1	1	
x	x	1	0	1	0	1	
x	x	x	1	1	1	1	



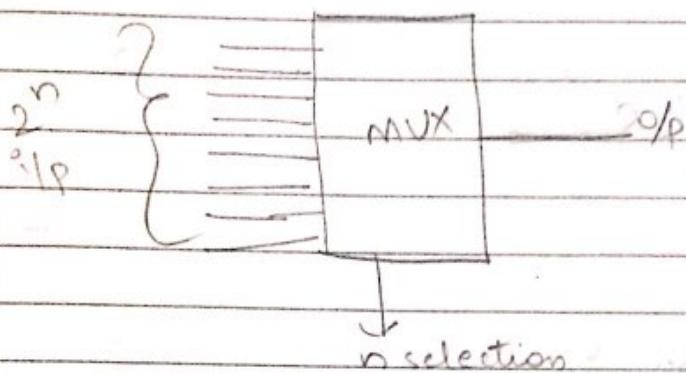
→ If two or more I/p are equal to 1 at the same time, the I/p having highest priority will take precedence.

Multiplexers :- (MUX)

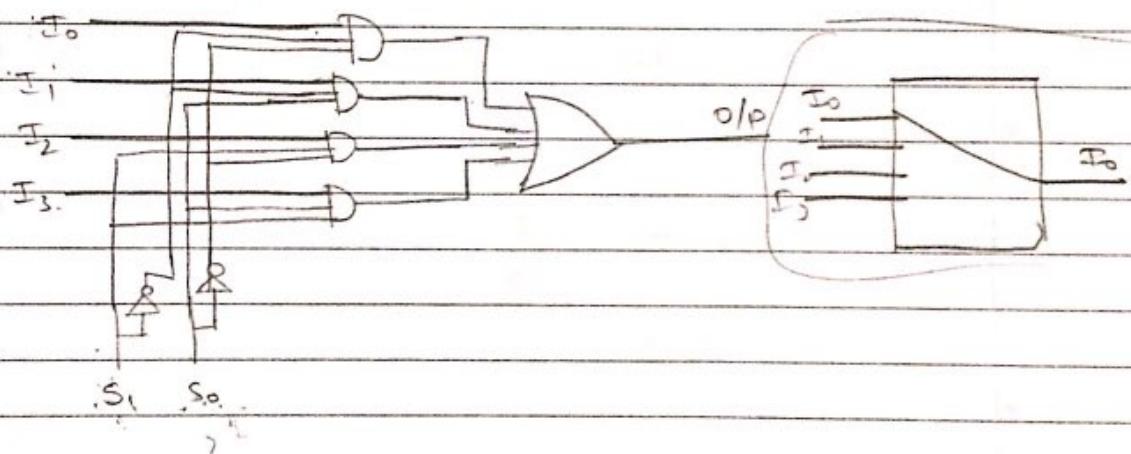
It is a combinational logic circuit designed to switch.

One of several input lines through a single common output line by application of a control signal.

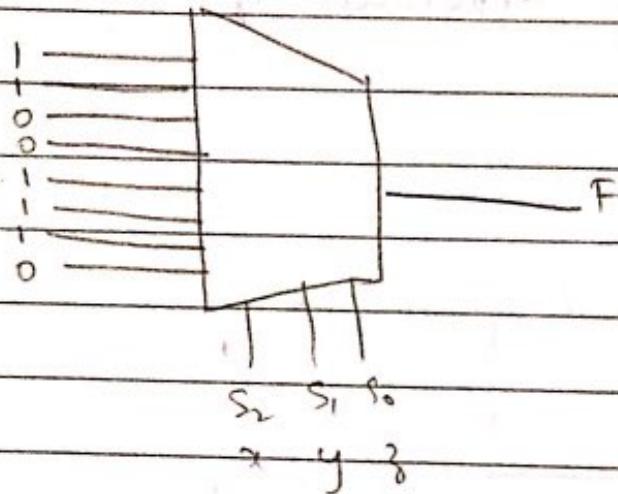
$2^n \rightarrow 1$



4x1 Mux :-



Implement the function $F = S(0,1,4,5,6)$ using 8x4 MUX



S_2	S_1	S_0	S
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Implement the funcⁿ : $x + y \cdot z'$ using 8x1 MUX

$x \quad y \quad z \quad z' \quad F$

0 0 0 1 0

0 0 1 0 0

0 1 0 1 1

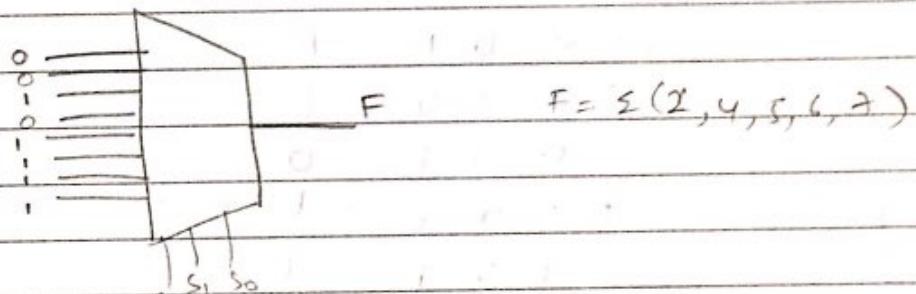
0 1 1 0 0

1 0 0 1 1

1 0 1 0 1

1 1 0 1 1

1 1 1 0 1



Implement above using 4x1 MUX

$x \quad y \quad z \quad F$

0 0 0 0 $F=0$

0 0 1 0 $F=z$

0 1 0 1 $F=1$

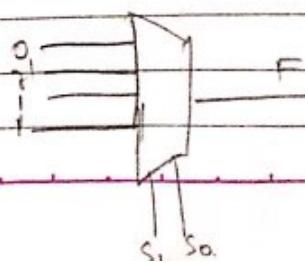
0 1 1 0 $F=1$

1 0 0 1 $F=1$

1 0 1 1 $F=1$

1 1 0 1 $F=1$

1 1 1 1 $F=1$



$$F = \Sigma(0, 1, 5, 7, 8, 9, 10, 12, 13)$$

8×1 MUX

a b c d. F

0 0 0 0 1 F = 1

0 0 0 1 1

0 0 1 0 0

0 0 1 1 0 F = 0

0 1 0 0 0

0 1 0 1 1 F = d

0 1 1 0 0

0 1 1 1 1 F = d

1 0 0 0 1

1 0 0 1 1 F = 1

1 0 1 0 1

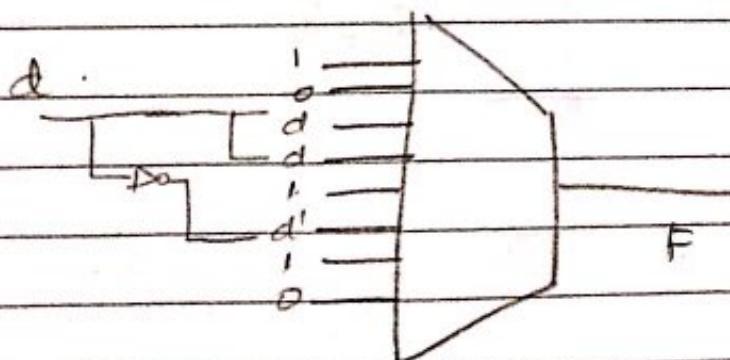
1 0 1 1 0 F = d'

1 1 0 0 1

1 1 0 1 1 F = 1

1 1 1 0 0

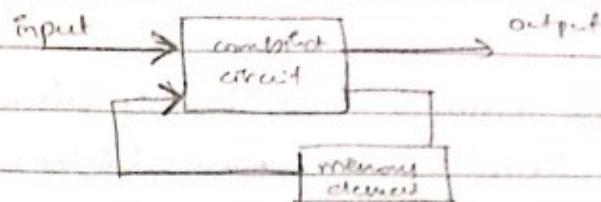
1 1 1 1 0 F = 0



3. Synchronous Sequential Logics

Camlin	Page
Date	/ /

(ii) Sequential Circuits



→ The o/p depends not only on present input but also on past i/p.
(stores the element in memory element and feedbacks the out)

→ There are 2 types

- Synchronous → clock is present (o/p changes when clock comes)
- asynchronous → no clock (diff to i/p, o/p changes)

Synchronous S.C

(i) The change in i/p signals can effect the memory element upon the activation of clock signal.

(ii) Memory elements are clocked flip flops (FF's).

(iii) The speed of the circuit depends upon the clock Speed.

(iv) Easier to design.

(v) Circuit operation is stable.

(vi)

Asynchronous S.C

(i) change in i/p signals can effect memory element at any instant of time.

(ii) Memory elements are either unclocked flip flops or time delay elements.

(iii) These circuits don't depend upon the clock. (so they are fast w.r.t S.S.C)

(iv) They are difficult to design.

(v) Circuit instability problems are present.

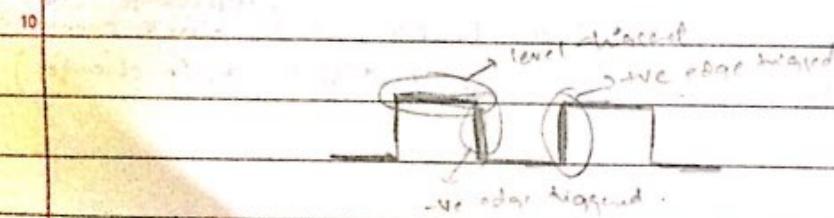
Storage Elements:-

Catch:- Storage elements that operate with signal levels

5 (Level-sensitive) are called catchs.
level - triggered

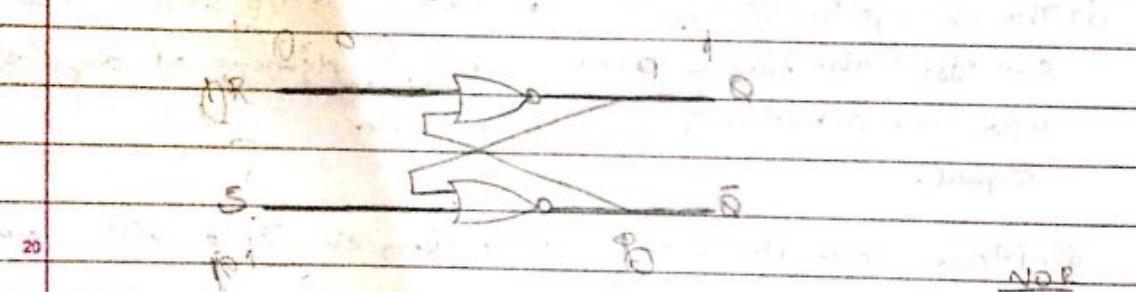
Flip Flop (FF):- Storage elements that operate with signal transition (Edge-sensitive) are called FF's.

edge - triggered



Logic Circuits:-

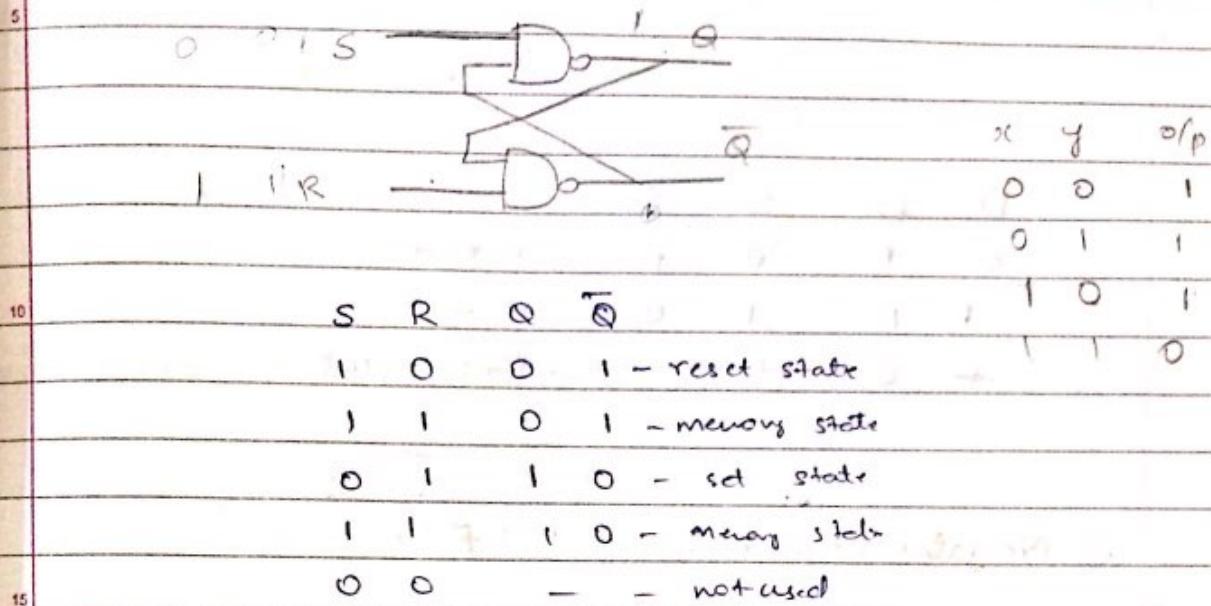
15 (i) S R latch with NOR gate:- (SR latch)



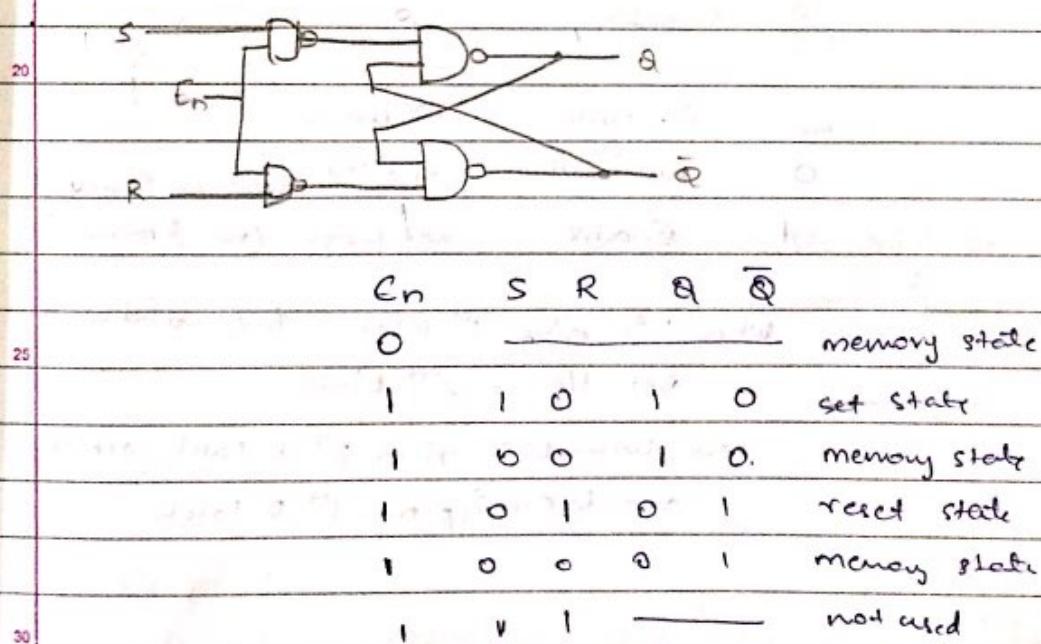
20

S	R	Q	\bar{Q}	X	Y	O/P
Setting One	0	1	0	0	1	0
Don't care	0	0	1	0	1	0
Set memory	0	0	0	0	0	0
25	0	1	0	1	0	1
Resetting Zero	0	0	1	1	0	0
	1	1	—	—	—	—
				because Q, \bar{Q} should be opposite		

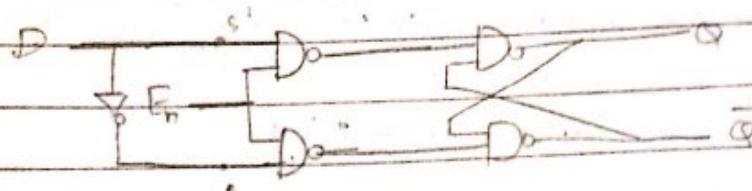
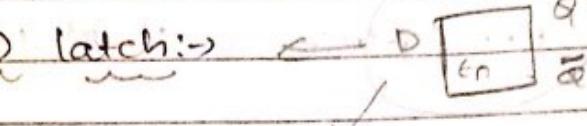
(ii) SR latch with NAND gate :- (S^1-R^1 latch)



(iii) SR latch with Enable Input :- (popular SR latch)



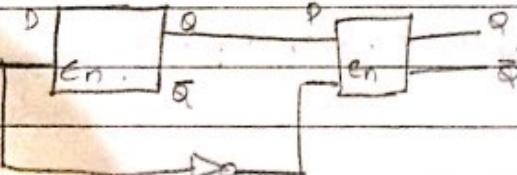
in 1 D latch :-



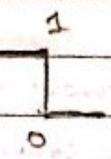
D	En	Q	Q̄
0	1	0	1
1	1	1	0
0	0	—	—

→ reset
→ set
→ memory state

in Negative Edge triggered D-F-F :-



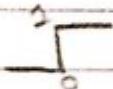
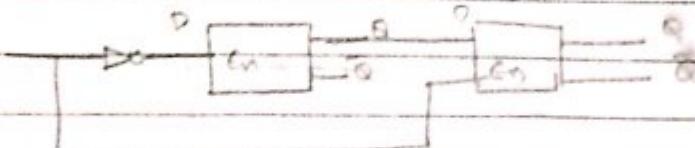
En	1 st D latch	2 nd D latch
0	not enable	Enabled. (But no internal)
1	enable	not enable (no internal)



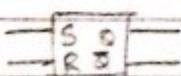
when 1 is given 1st D latch is enabled we get Q_p for 2nd D latch

now when 0 is given 2nd D latch activate and takes Q_p from 1st D latch

(vii) Positive Edge triggered D-F-F :-



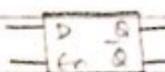
→ Logic Symbols :-



(i) SR latch



(ii) S' R' latch



(iii) D latch

(iv) positive triggered D-FF

(v) -ve triggered D-FF



(vi) positive triggered D-FF

(vii) -ve triggered D-FF

Setup time :-

Minimum time during which the D I/p must be maintained at a const value prior to the occurrence of the clock transition

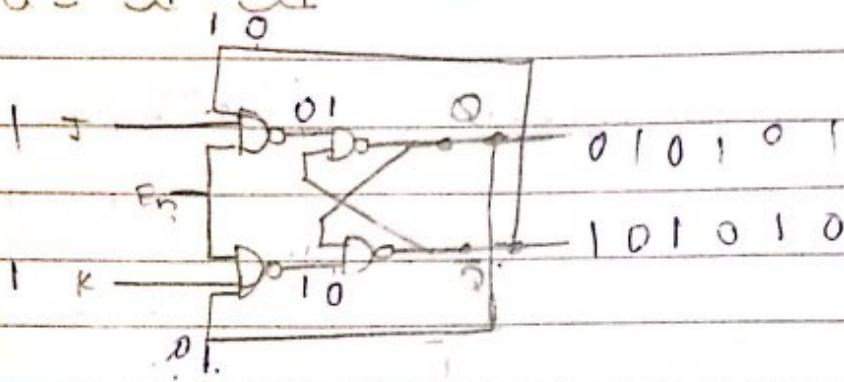
Hold time:-

Minimum time during which D I/p must not change after the application of the clock transition

Propagation delay:-

It is the interval b/w the edge of the trigger and the stabilization of the o/p to a new state.

J K Flip Flop



En	J	K	Q	\bar{Q}	
0					memory state
1	1	0	1	0	→ set state
1	0	0			memory state
1	0	1	0	1	reset state
1	1	1	1	0	set state

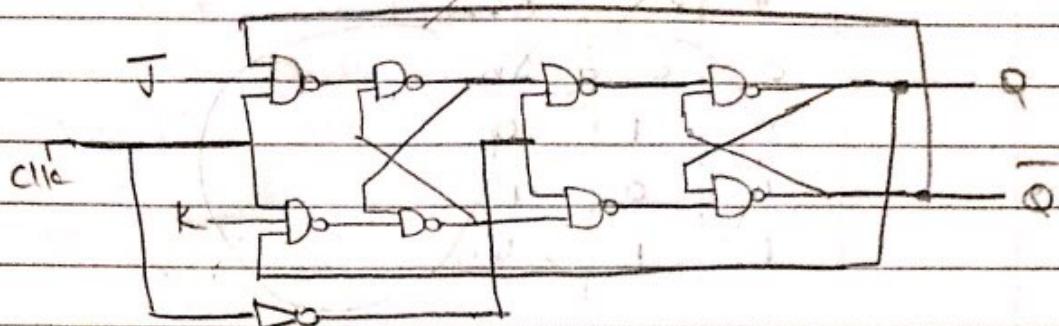
En	J	K	Q	\bar{Q}	
0					memory state
1	1	0	1	0	set state
1	0	0			memory state
1	0	1	0	1	reset state
1	1	1	1	0	set state

→ Race around condition

→ Race around is not used in digital circuit as the o/p is unpredictable. ((Raney))

→ This condition is rectified using master slave JK flip flop

Master slave J K Flip Flop:-



These are 2 stage JK FF.
a clock is used to enable the chip.

J	K	Q	\bar{Q}
0	1	0	1
1	0	1	0
0	0	—	—
1	1	—	—

→ when the the clock goes from high to low
the master is active and slave is inactive.
i.e When the clock goes high to low at that instant
the o/p will flip.
(→ Toggling the o/p)

the o/p is controlled by the clock.

$J \ K \ Q_{n+1}$

$0 \ 0 \ S_n$

$0 \ 1 \ . \ 0$

$1 \ 0 \ . \ 1$

$1 \ 1 \ \bar{Q}_n$

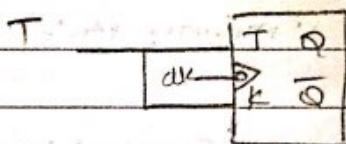
Characteristic Table:-

T	K	Q_{n+1} (next state)
0	0	$Q_n \rightarrow \text{memory}$
0	1	$0 \rightarrow \text{reset}$
1	0	$1 \rightarrow \text{set}$
1	1	$\textcircled{Q}_n \rightarrow \text{controlled complement}$ be toggling

The o/p will be flipped by using clock.

the clock controls the o/p.
(toggles)

T Flip Flop:- (Toggle flip flop)



Q_n	T	Q_{n+1}	T	Q
0	0	Q_n	clk	$\rightarrow Q$
1		\bar{Q}_n		

Excitation Table:-

We know the present state variables but should predict the ~~past state~~ values
~~ip~~ ilp values

Excitation Table for SR Latch:-

Q_n	Q_{n+1}	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

for S

Q_n	Q_{n+1}	Out
0	0	1
1	0	X

$$S = Q_{n+1}$$

for R

Q_n	Q_{n+1}	Out
0	1	X
1	0	0

$$R = \bar{Q}_{n+1}$$

Excitation table for D latch :-

Q_n	Q_{n+1}	D	Out
0	0	0	Q_n
0	1	1	
1	0	0	
1	1	1	

$$D = Q_{n+1}$$

Excitation table for T K FF :-

Q_n	Q_{n+1}	T	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

1, 0,
0, 0

for T

Q_n	Q_{n+1}	Out
0	0	1
1	X	Y

$$T = Q_{n+1}$$

for K

Q_n	Q_{n+1}	Out
0	0	X
1	1	0

$$K = \bar{Q}_{n+1}$$

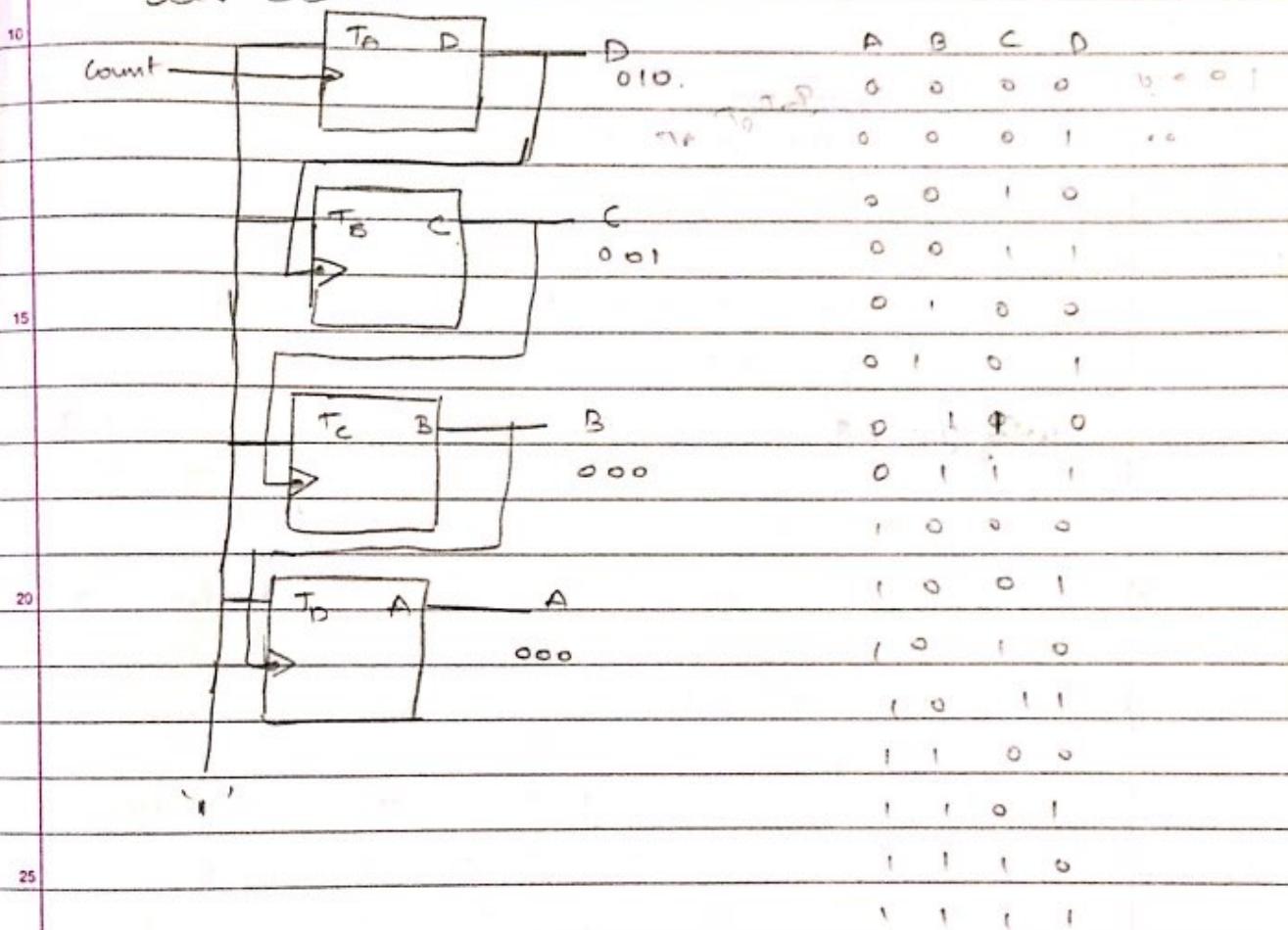
27/08/14

Counters:-

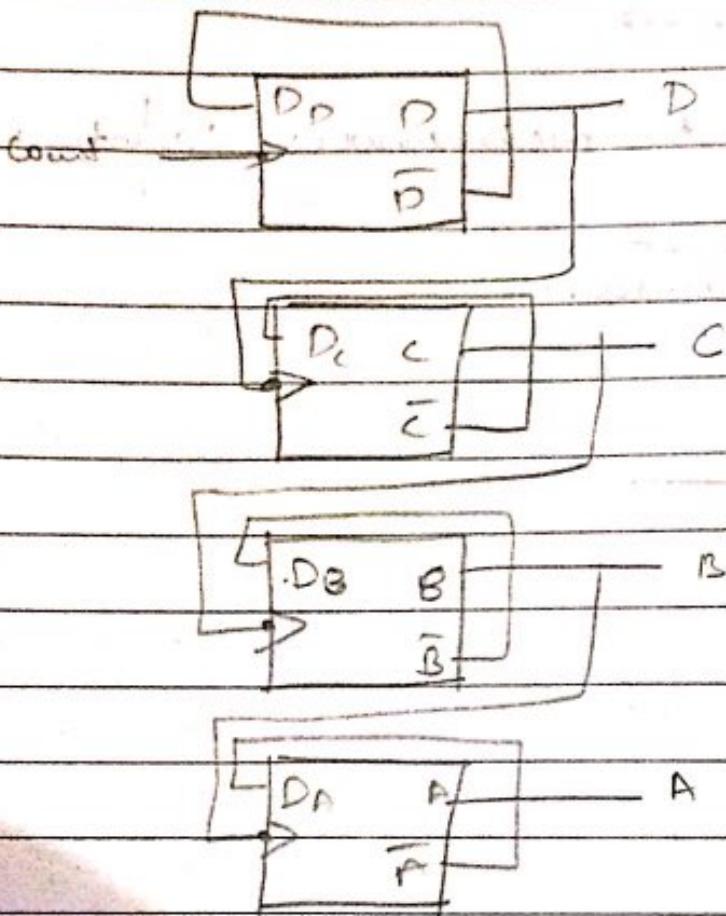
These are 2 types asynchronous and synchronous counter
(Ripple)

Ripple Counter:- (OR) Asynchronous Counter:-

- binary
1. four bit Ripple up counter:-
using TFF



using D FF

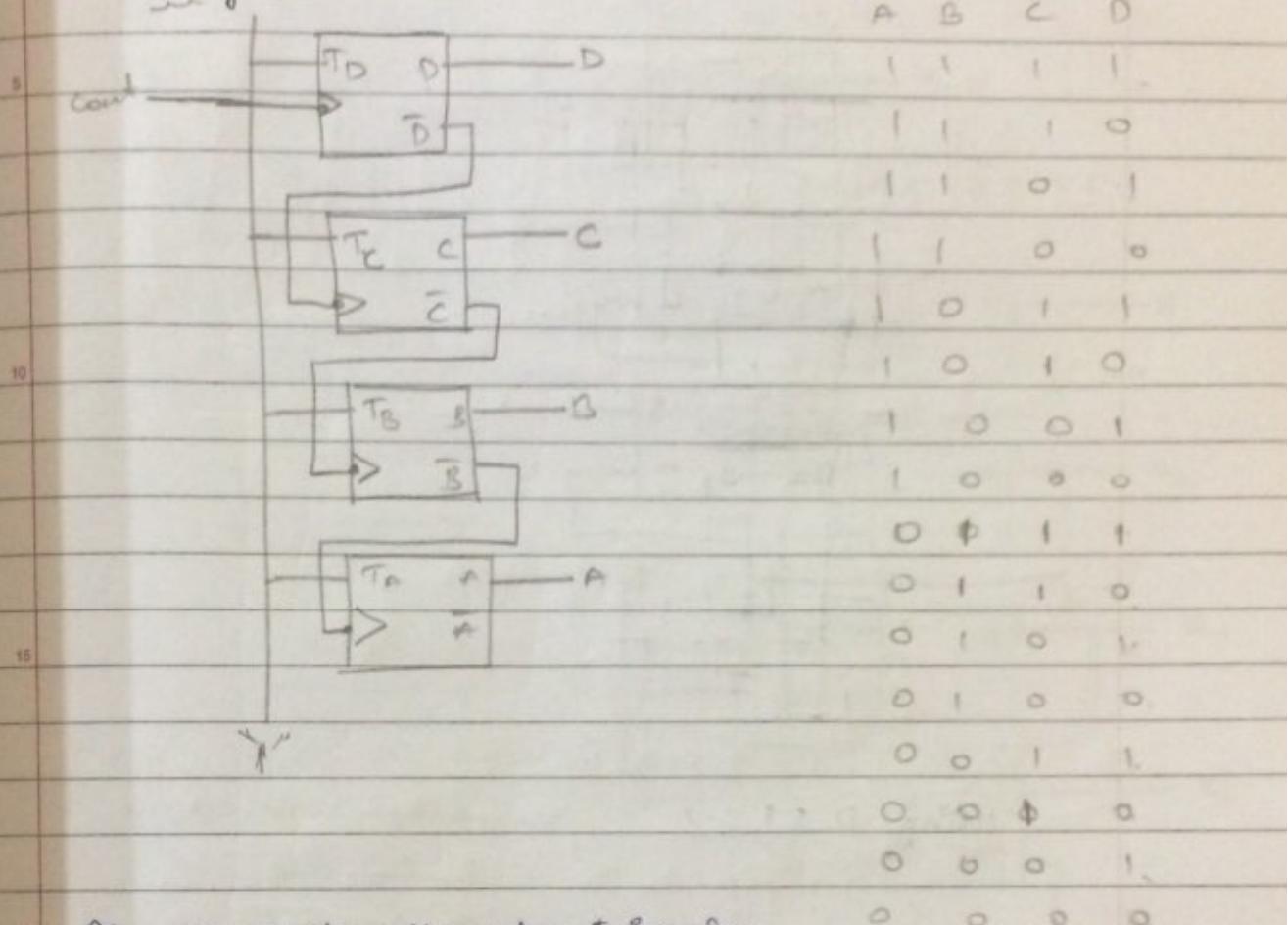


using 4K FF

23/06/21

2. 4-bit binary ripple down counter:-

using T FF



As we apply, -ve edge triggering

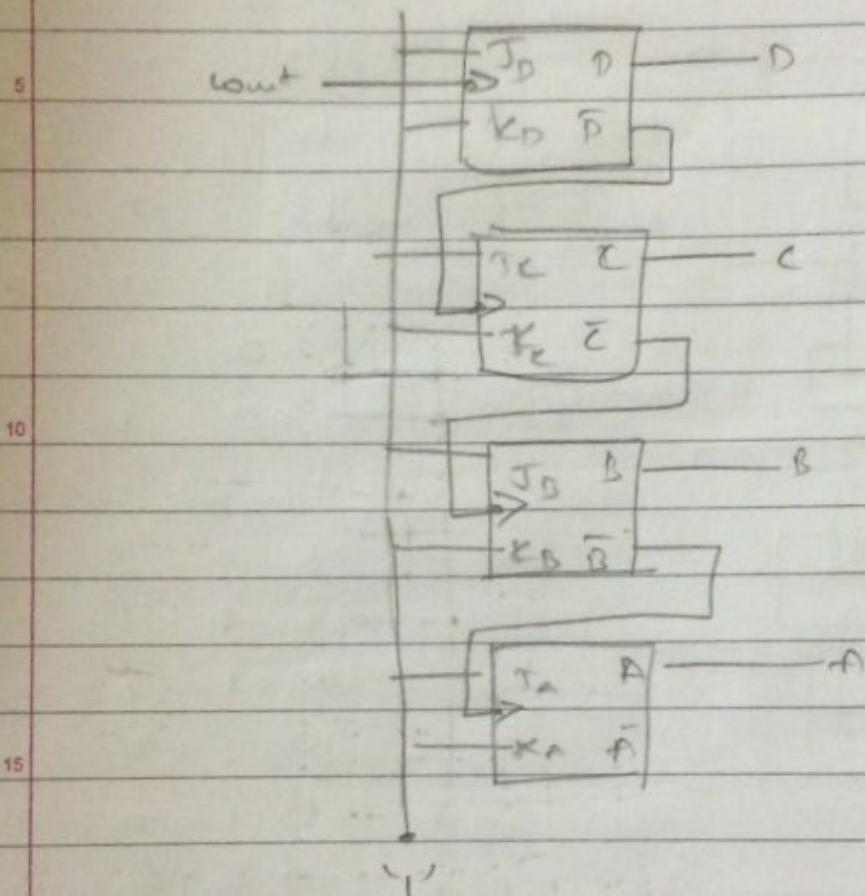
If we take clock as complement then it is down counter
If it is not complement then it is up counter.

for +ve edge triggering

clock as complement then it is up counter
not complement then it is down counter.

30

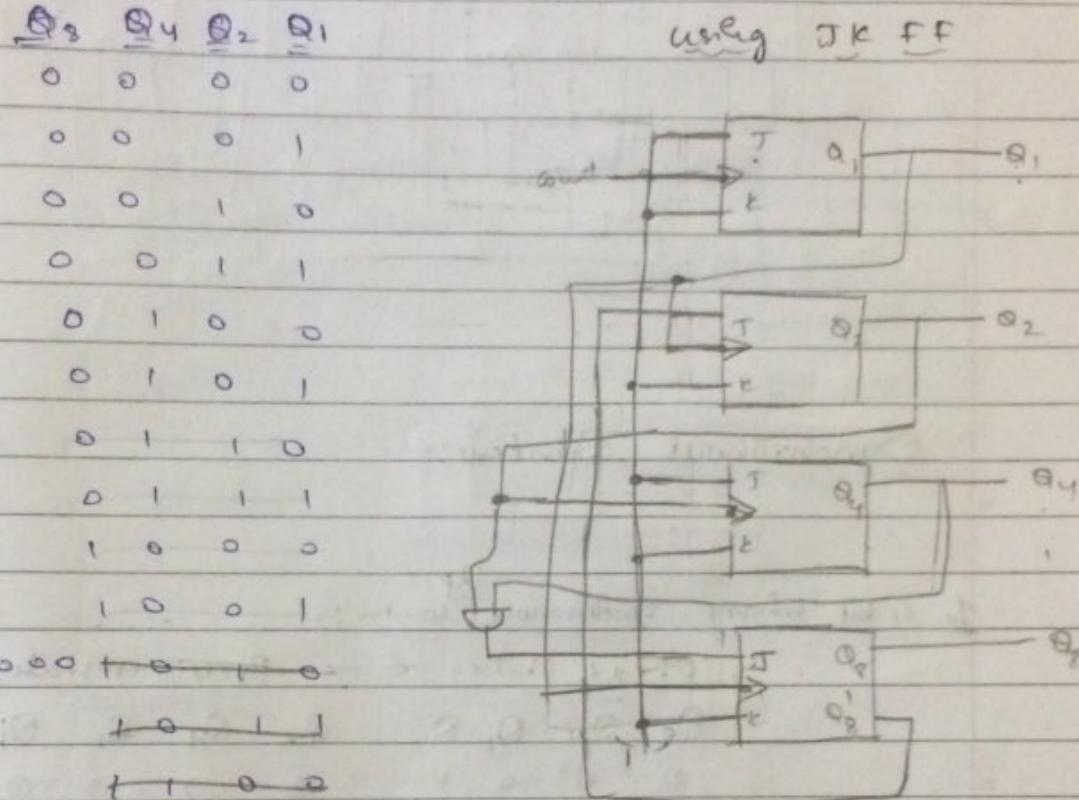
using JK FF



3- BCD Ripple up counter :- also called
Modulo 10

In binary counter we get 10, 11, 12 ... but for
BCD after 9 it goes to zero.

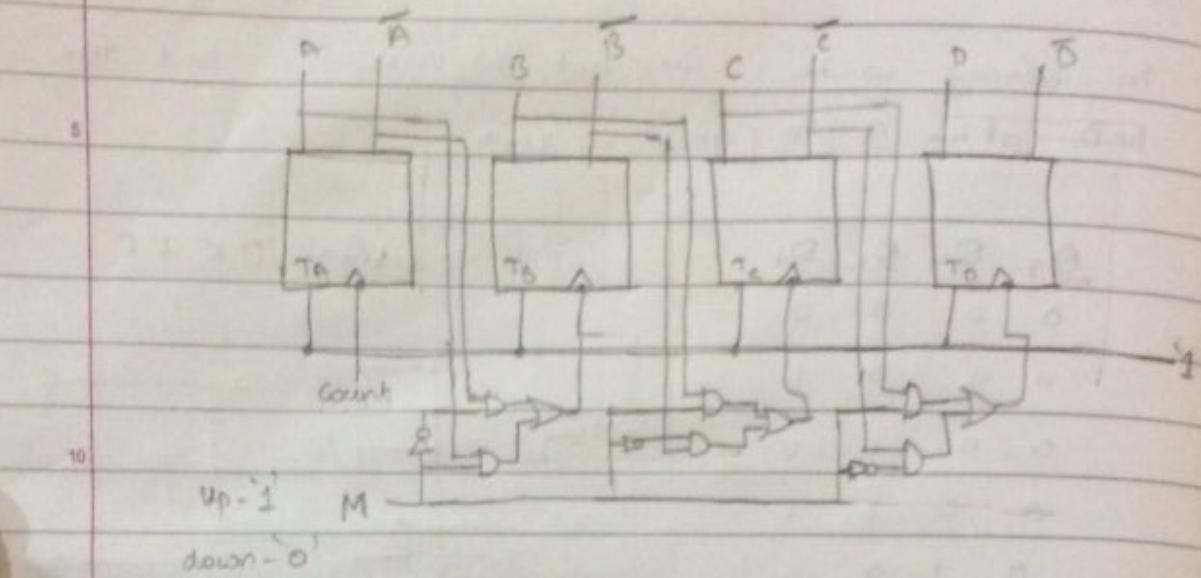
	<u>Q_3</u>	<u>Q_4</u>	<u>Q_2</u>	<u>Q_1</u>
10	0	0	0	0
	0	0	0	1
	0	0	1	0
	0	0	1	1
	0	1	0	0
	0	1	0	1
	0	1	1	0
15	0	1	1	1
	1	0	0	0
	1	0	0	1
	0000	+ 0	1 0	
	1 0	1 1		
20	1 1	0 0		
	1 1	0 1		
	1 1 1 0			
	1 1 1 1			



25

30

4. Binary ripple up-down counter:-



Synchronous counters:-

1. 4 bit binary synchronous counter:

Next state \leftarrow Previous state

	Q_3	Q_2	Q_1	Q_0		Q_3	Q_2	Q_1	Q_0
0	0	0	0	1	0	0	0	0	0
1	0	0	1	0	0	0	0	0	1
2	0	0	1	1	0	0	1	0	0
3	0	1	0	0	0	0	0	1	1
4	0	1	0	1	0	1	0	0	0
5	0	1	1	0	0	1	0	1	0
6	0	1	1	1	0	1	1	0	0
7	1	0	0	0	0	1	1	1	1
8	1	0	0	1	1	0	0	0	0
9	1	0	1	0	1	0	0	1	1
10	1	0	1	1	1	0	1	0	0
11	1	1	0	0	0	0	1	1	1
12	1	1	0	1	1	1	0	0	0
13	1	1	1	0	1	1	0	1	0
14	1	1	1	1	1	1	1	0	0
15	0	0	0	0	1	1	1	1	1

$T_8 \quad T_4 \quad T_2 \quad T_1$
 $0 \quad 0 \quad 0 \quad 1$
 $0 \quad 0 \quad 1 \quad 1$
 $0 \quad 0 \quad 0 \quad 1$
 $0 \quad 1 \quad 1 \quad 1$
 $0 \quad 0 \quad 0 \quad 1$
 $0 \quad 0 \quad 1 \quad 1$
 $0 \quad 0 \quad 0 \quad 1$
 $1 \quad 0 \quad 1 \quad 0$
 $0 \quad 0 \quad 0 \quad 1$
 $0 \quad 0 \quad 1 \quad 0$
 $0 \quad 0 \quad 0 \quad 1$
 $0 \quad 1 \quad 1 \quad 1$
 $0 \quad 0 \quad 0 \quad 1$
 $0 \quad 0 \quad 1 \quad 1$
 $0 \quad 0 \quad 0 \quad 1$
 $1 \quad 1 \quad 1 \quad 1$
 T_{12}

$Q_3 Q_4$	$Q_2 Q_1$						
00	00	00	00	00	00	00	00
01	01	01	01	01	01	01	01
10	10	10	10	10	10	10	10
11	11	11	11	11	11	11	11

$T_8 = Q_1$

$T_1 = 1$

20

$Q_3 Q_4$	$Q_2 Q_1$						
00	00	00	00	00	00	00	00
01	01	01	01	01	01	01	01
11	11	11	11	11	11	11	11
10	10	10	10	10	10	10	10

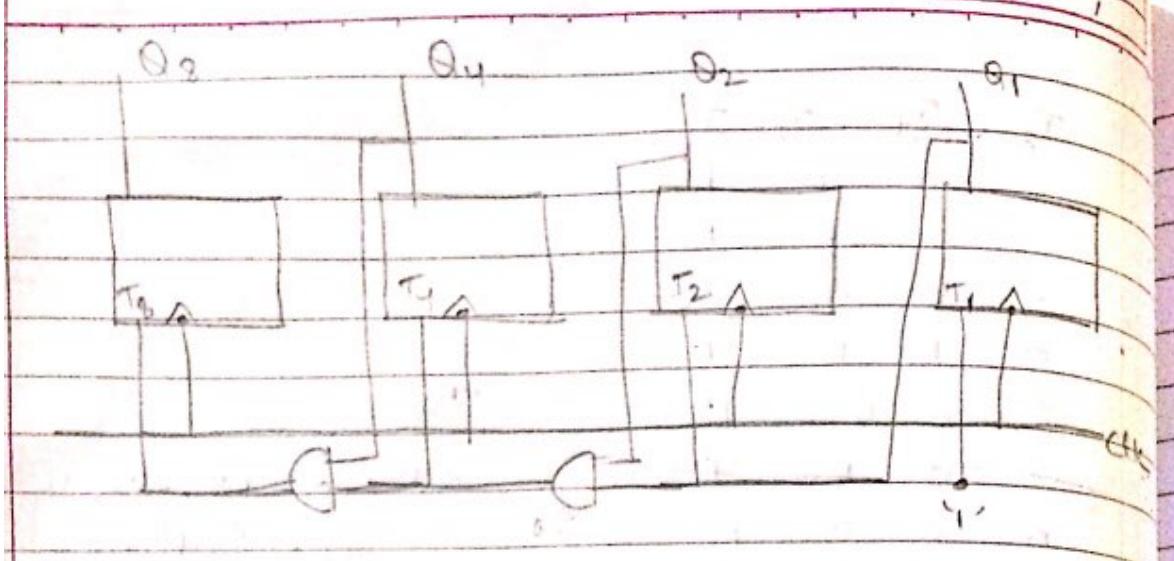
$Q_3 Q_4$	$Q_2 Q_1$						
00	00	00	00	00	00	00	00
01	01	01	01	01	01	01	01
4	00	00	00	00	00	00	00
10	00	00	00	00	00	00	00

$T_8 = Q_4 Q_2 Q_1$

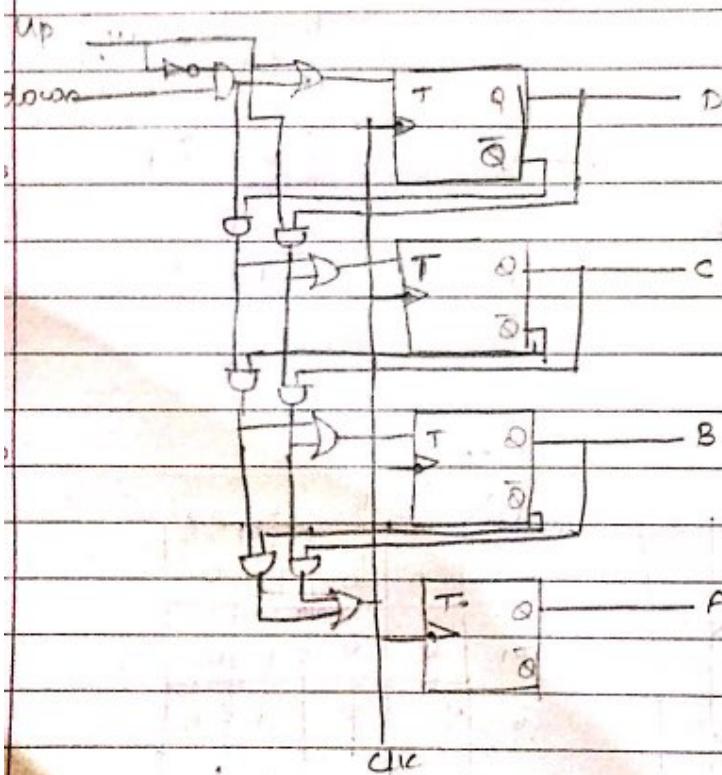
$T_4 = Q_2 Q_1$

30

$T_8 = Q_4 Q_2 Q_1$
$T_4 = Q_2 Q_1$
$T_2 = Q_1$
$T_1 = 1$



Q) 1 bit Synchronous up-down counter...



→ For up counter, 1 is given to Up irrespective of down.

→ For down counter, 0 is given to up and 1 to down.

3) Synchronous BCD counter ^{up}

P.S	N.S	FF flip
A B C D	A B C D	T _A T _B T _C T _D
0 0 0 0	0 0 0 1	0 0 0 1
0 0 0 1	0 0 1 0	0 0 1 1
0 0 1 0	0 0 1 1	0 0 0 1
0 0 1 1	0 1 0 0	0 1 1 1
0 1 0 0	0 1 0 1	0 0 0 1
0 1 0 1	0 1 1 0	0 0 1 1
0 1 1 0	0 1 1 1	0 0 0 1
0 1 1 1	1 0 0 0	1 1 1 1
1 0 0 0	1 0 0 1	0 0 0 1
1 0 0 1	0 0 0 0	1 0 0 1

T _B	A ¹ B C D	A B' C' D
00	0 0 0 0	0 0 0 0
01	0 0 0 1	1 0 0 0
11	X X X Y	X X X Y
10	Y Y Y X	Y Y Y X

$$T_D = 1$$

$$\begin{aligned}
 T_A &= A^1 B C D + A B^1 C^1 D \\
 &= D (A^1 B C + A B^1 C^1) \\
 &= D [(A^1 + A)(A^1 + B^1 C^1)(B C + B^1 C^1)] \\
 &= D [(A^1 + A^1 C^1)(B C + B^1)(B C + C^1)]
 \end{aligned}$$

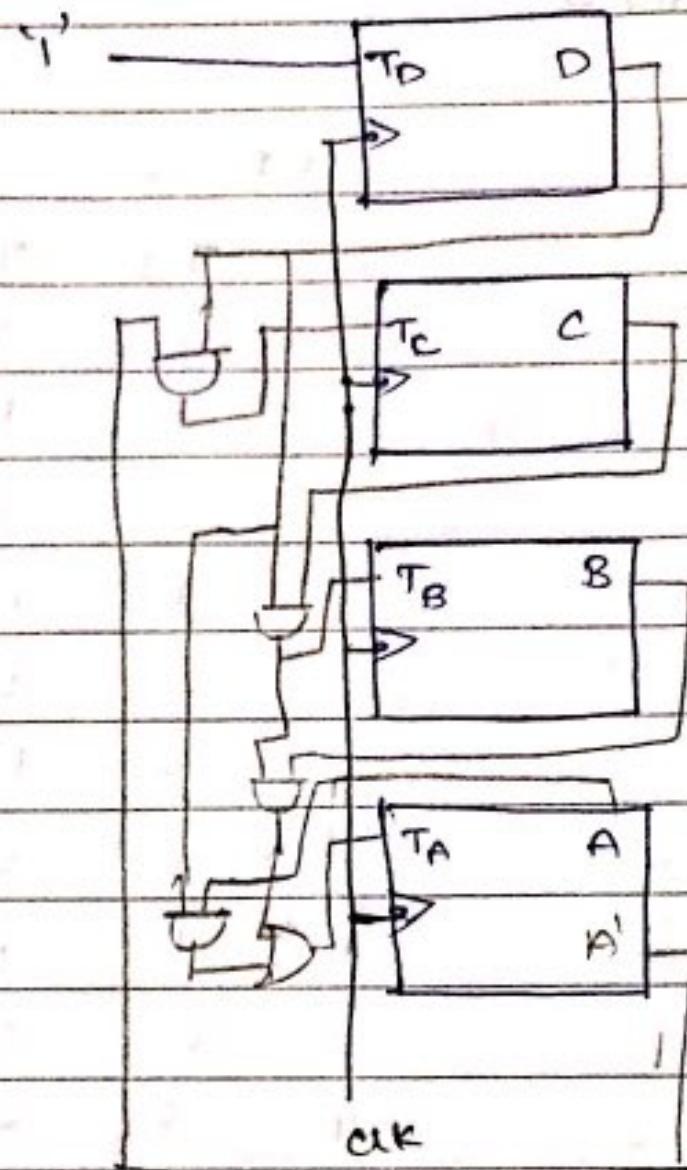
$$T_B = \neg A C D$$

T _C	A B	00 01 11 10
00	0 1 1 0	0 1 1 0
01	0 1 1 0	0 1 1 0
11	X X X Y	X X X Y
10	0 0 Y Y	0 0 Y Y

$$T_C = A^1 D$$

T _A	A B C D	00 01 11 10
00		
01		
11	X X X Y	X X X Y
10	1 X X X	1 X X X

$$T_A = A D + B C D$$

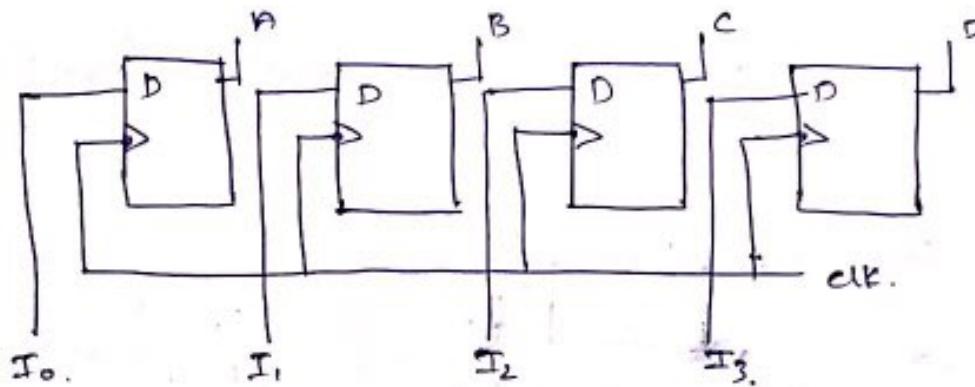


4. Registers and Memories

Registers:-

It is a group of flip-flops each one of which shares a common clock and is capable of storing ~~one-bit~~ n-bit information using n-FF's

4-bit register:-



→ As we apply the clock the value might change.

(There are 2 types
synchronous → $\text{clk} \uparrow$, load \uparrow
asynchronous → only load \uparrow)

→ So we use register with parallel load.

Take, that if

load is 0 O/p is ?/p (holds previous value)

load is 1 O/p is next value.

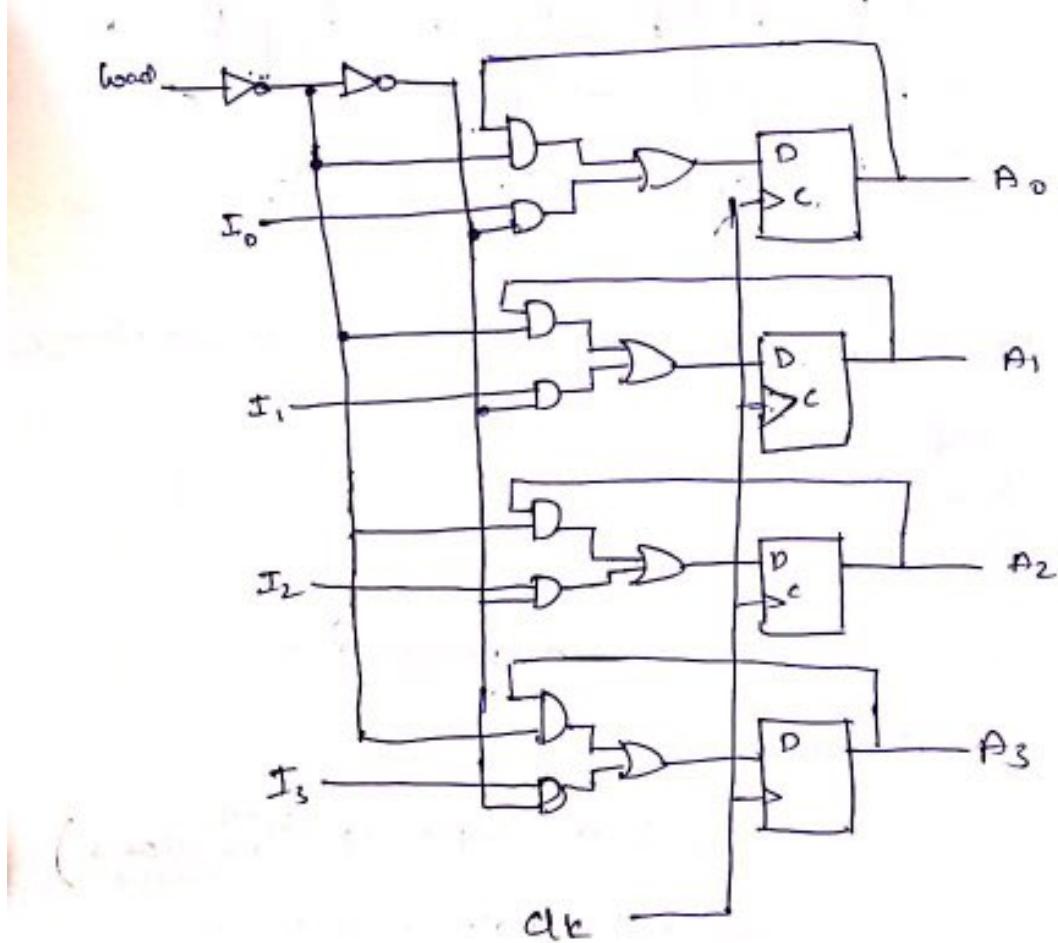
Register with Parallel Load:-

load I_i Q $_i$ D $_i$

0	0	0	0	}	load = 0 $D_i = Q_i$
0	0	1	1		
0	1	0	0		
0	1	1	1		

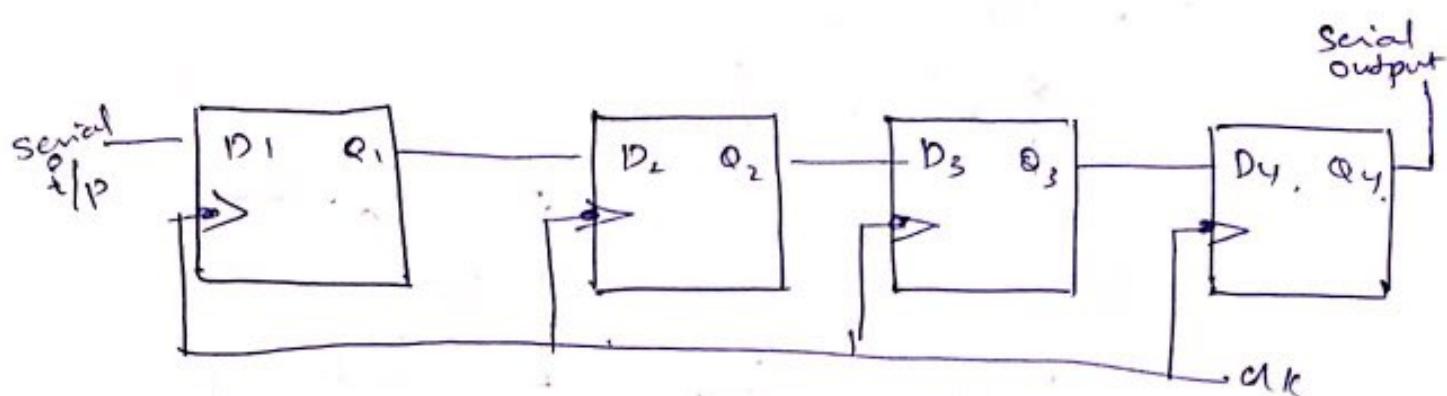
1	0	0	0	}	load = 1 $D_i = I_i$
1	0	1	0		
1	1	0	1		
1	1	1	1		

$$\text{Taking K-map } D_i = \text{load } I_i + \overline{\text{load}} Q_i$$

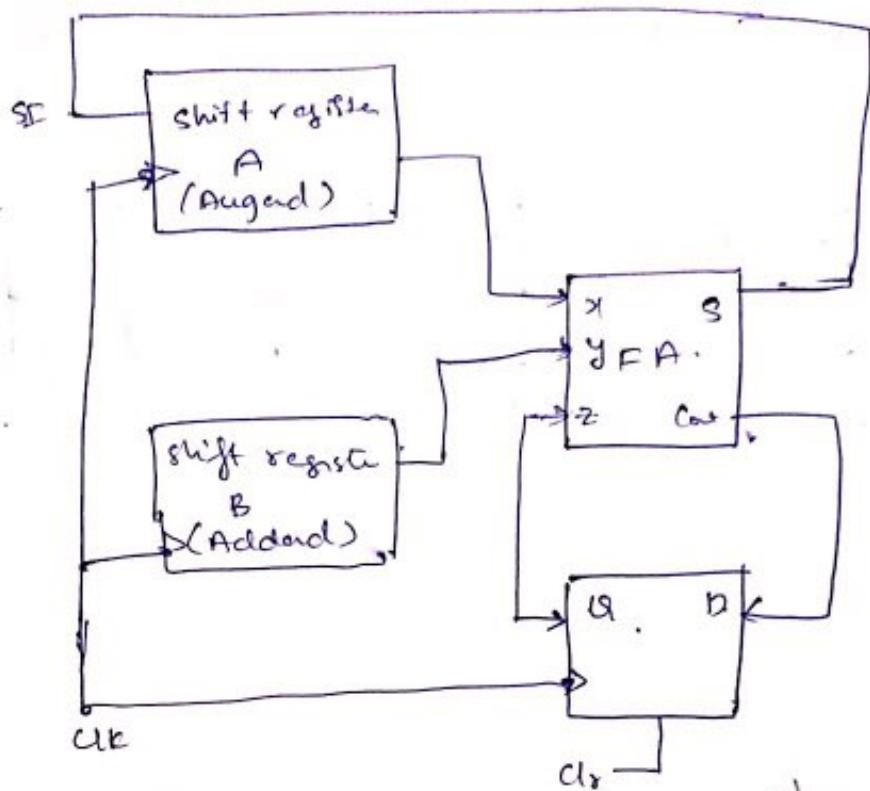


Shift register :-

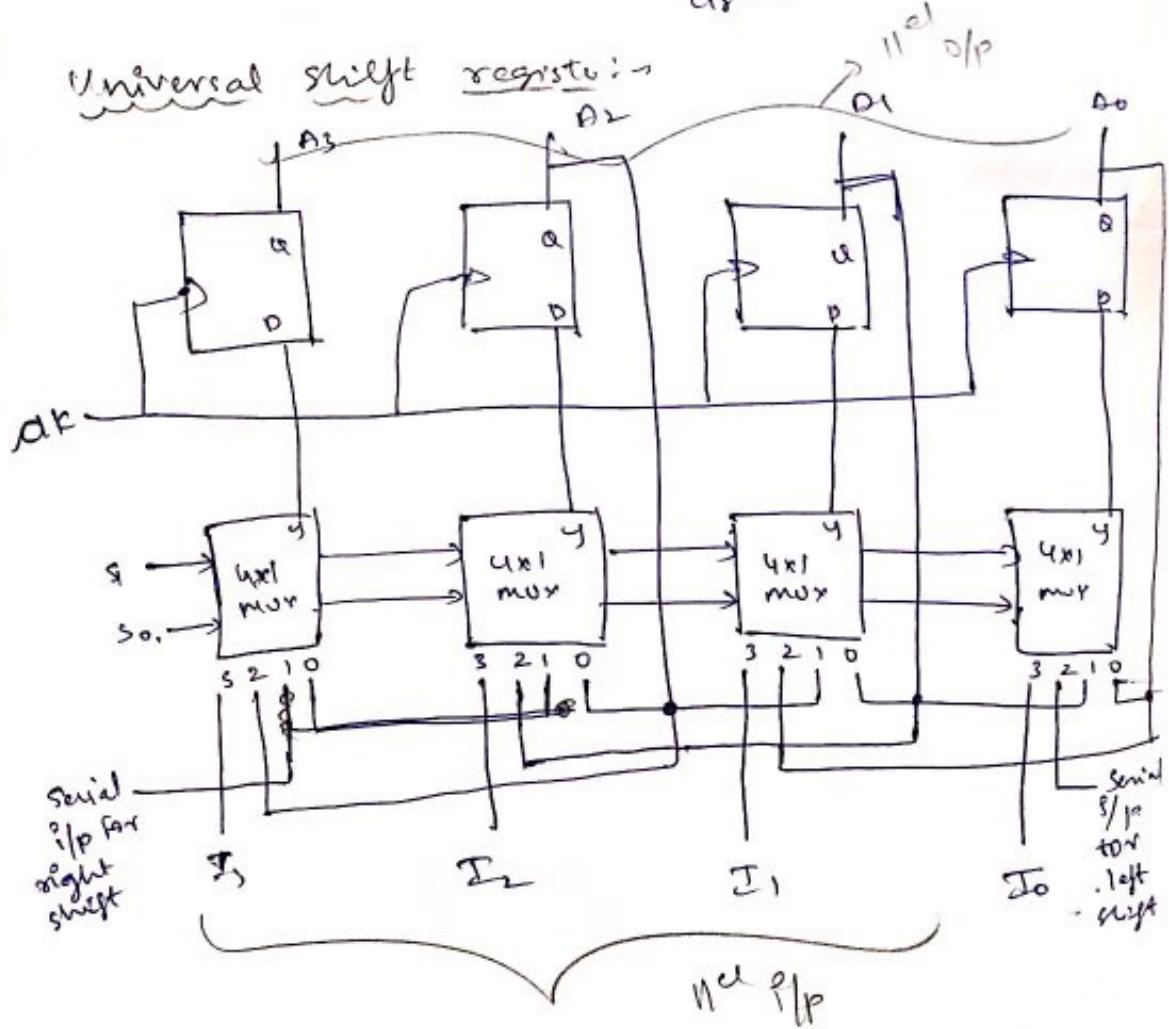
This register is capable of shifting memory to its neighborhood FF in the specified direction.



Serial adder :-



Universal shift register :-

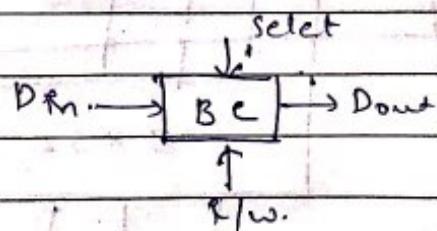
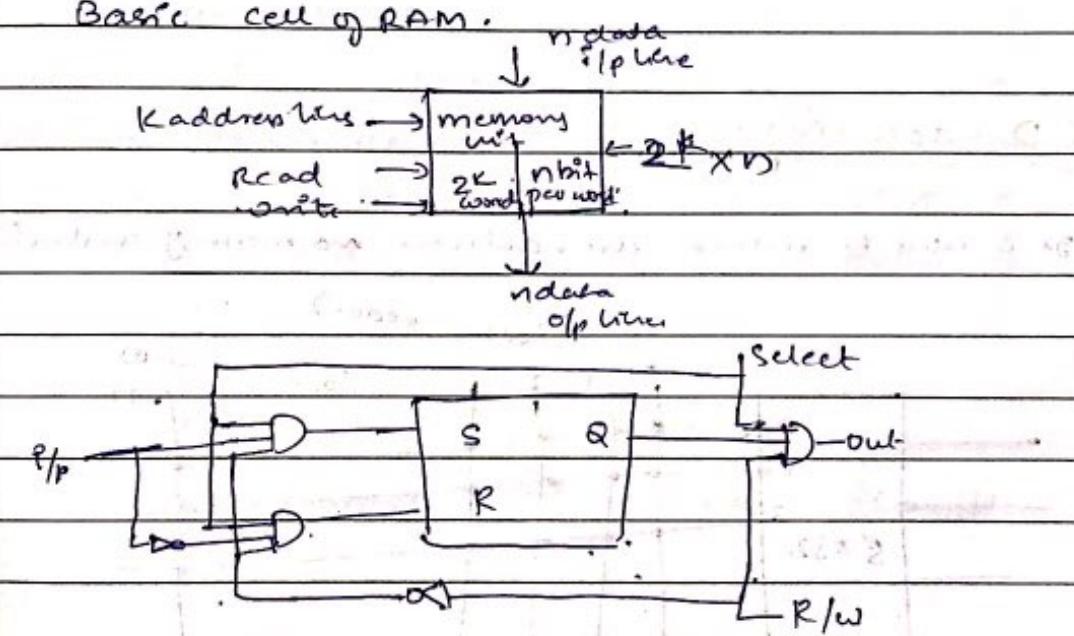


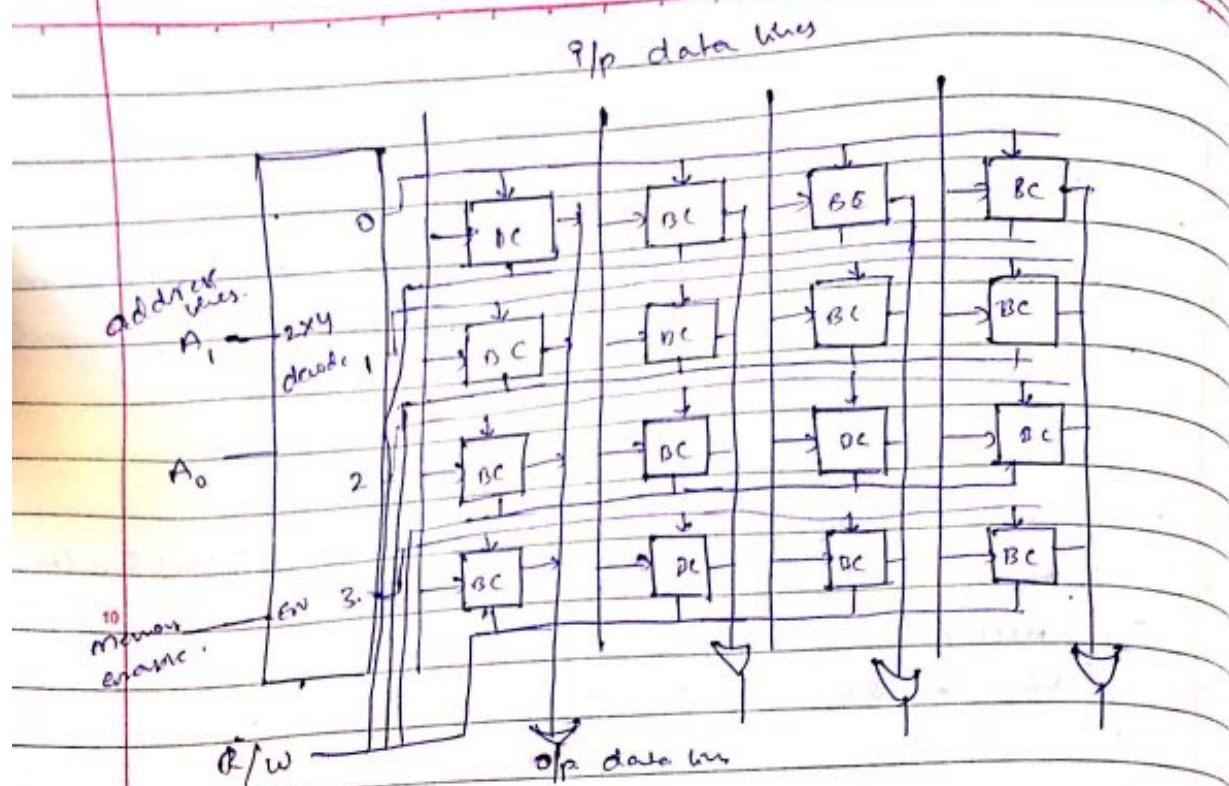
Memory :-

- A memory unit is a device to which binary information is transferred for storage and from which information is retrieved when needed.
- It is a collection of cells capable of storing a large quantity of binary information.
- We use 2 types of memories mainly in Digital System RAM and ROM.
 - RAM stores new information for late use where as ROM is a programmable logic device.
↳ only read.

Random Access Memory :- (RAM)

Basic cell of RAM.

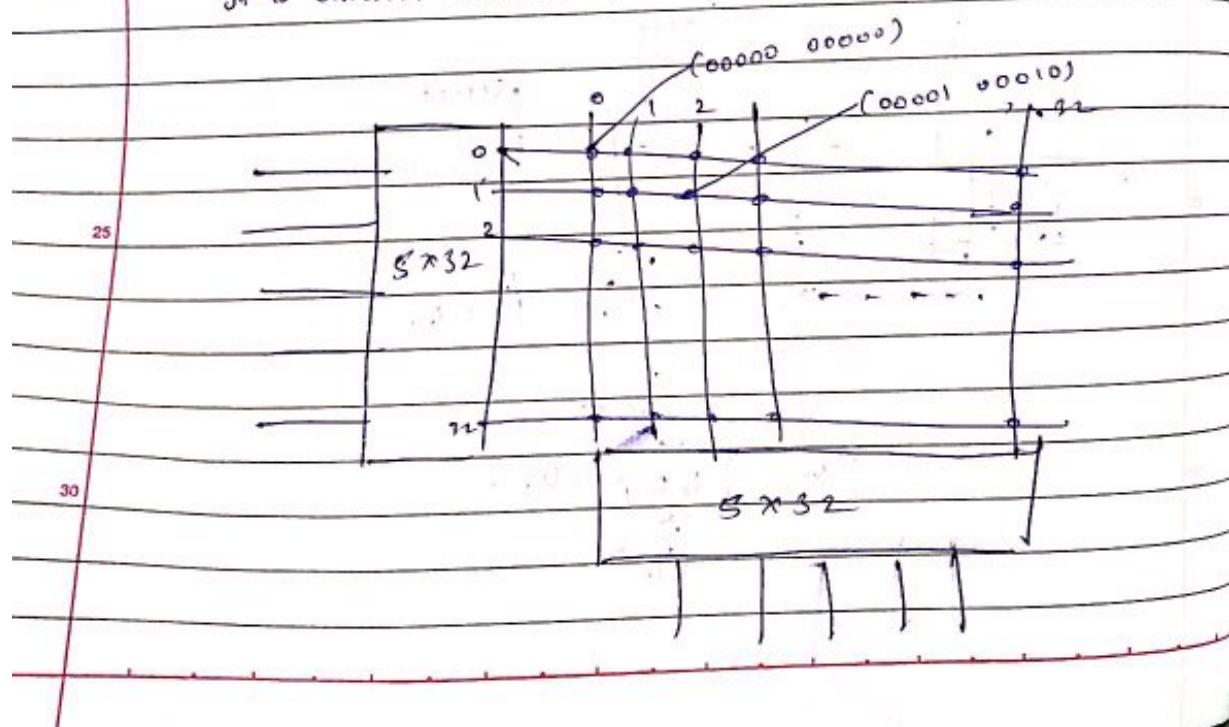




15 $\rightarrow 4 \times 4 \rightarrow 4$ memory locations in 2 rows and 4 bits in each row.

2 D - decoding :-

20 8 used to reduce the hardware and memory construction



we have 32×32 bits location

$$= 1024 \cdot \text{locations}$$

AND gates used as $64 \rightarrow 512$ AND gate
instead of $1024 \rightarrow 1024$ AND gate.

→ There are 2 types of RAMs.

SRAM

(Static)

DRAM.

(Dynamic)

i) stored bits in memory cells composed of FF

ii) Memory cells composed of capacitor and transistors

iii) Requires 1 Transistor to store 1 bit

iv) 1 Transistor and 1 capacitor to store 1 bit

v) more costly

vi) less costly

vii) faster access time

viii) lower access time

vii) more power consumption

vii) less power consumption

viii) low density per chip

vii) high density per chip

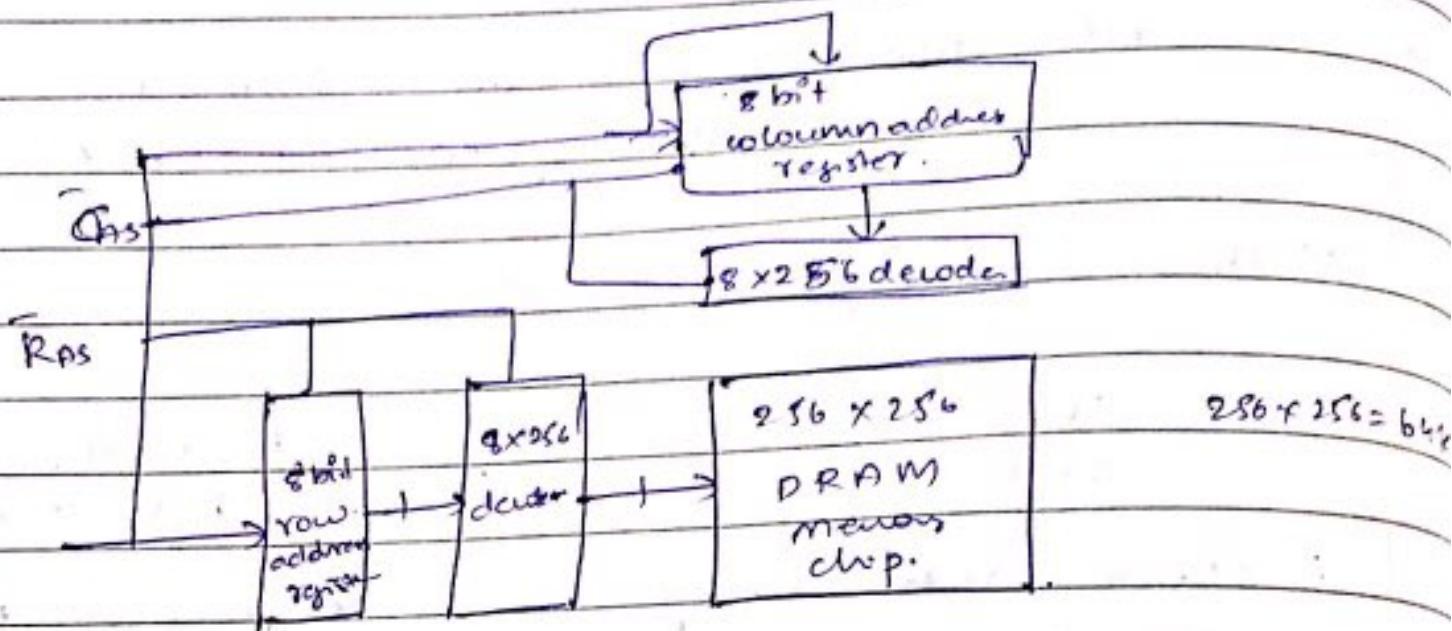
ix) does not need to be refreshed periodically

x) should be refreshed every few seconds

→ RAM is always volatile.

→ ROM, memory disk are non volatile.

Refreshing cycle of DRAM :-



C_{AS} → column address i/p selection

R_{AS} → Row address i/p selection

Error detection and correction code:-

- Detects and corrects code.
- Hamming code.

k-parity bits

n → data bits

Total (n+k) data bits

Hamming Code:-

we have k parity bits and n data bits

Total bits $(k+n)$

parity bits are placed at 2^k position, $k = 0, 1, 2, \dots$
and others are data bits.

→ If we consider 8 bit binary number. (00111000)

$P_1 P_2 D_3 P_4 D_5 D_6 D_7 P_8 D_9 D_{10} D_{11} D_{12}$

(we take 8 data bits)

$P_1 P_2 D_3 P_4 D_5 D_6 D_7 P_8 D_9 D_{10} D_{11} D_{12}$
0 0 1 1 1 0 0 1 0 1 0 0

Calculating parity bits

→ write the position of data bits in binary form.

$D_3 0 0 1 1$

$D_5 0 1 0 1$

$D_6 0 1 1 0$

$D_7 0 1 1 1$

$D_9 1 0 0 1$

$D_{10} 1 0 1 0$

$D_{11} 1 0 1 1$

$D_{12} 1 1 0 0$.

for P_1 , EXOR the data bits whose units position is 1

$P_1 = D_3 \oplus D_5 \oplus D_7 \oplus D_9 \oplus D_{11}$
0 0 1 1 0

→ If there are even no of 1's EXOR = 0

odd no of 1's EXOR = 1

$$\boxed{P_1 = 0}$$

$$P_2 = D_3 \oplus D_6 \oplus D_7 \oplus D_{10} \oplus D_{11}$$

$$= 0 \oplus 1 \oplus 1 \oplus 0 \oplus 0$$

$$\boxed{P_2 = 0}$$

$$P_4 = D_5 \oplus D_6 \oplus D_7 \oplus D_{11}$$

$$= 0 \oplus 1 \oplus 1 \oplus 0$$

$$\boxed{P_4 = 0}$$

$$P_8 = D_9 \oplus D_{10} \oplus D_{11} \oplus D_{12}$$

$$= 1 \oplus 0 \oplus 0 \oplus 0$$

$$\boxed{P_8 = 1}$$

$$\begin{array}{ccccccccc} P_1 & P_2 & D_3 & P_4 & D_5 & D_6 & D_7 & P_8 & D_9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{array} \quad \begin{array}{cccccc} D_{10} & D_{11} & D_{12} \\ 0 & 0 & 0 \end{array}$$

for P_2, P_4, P_8, P_1 there are check bits called C_1, C_2, C_3

$$C_1 = D_3 \oplus D_5 \oplus D_7 \oplus D_9 \oplus D_{11} \oplus P_1$$

$$= 0$$

$$C_2 = 0$$

$$C_3 = 0$$

$$C_8 = 0$$

If all the check bits are zero then there's no error

If $D_S = 1$

$$C_1 = 1$$

$$C_2 = 0$$

$$C_3 = 1$$

$$C_8 = 0$$

$$C_8 \ C_4 \ C_2 \ C_1 \\ 0 \ 1 \ 0 \ 1 \rightarrow D_S$$

→ It cannot detect double errors.

Advanced Hamming code (PLD's)

PLD → Programmable logic devices.

→ Detects 2 errors and corrects 1.

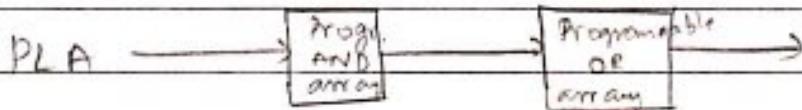
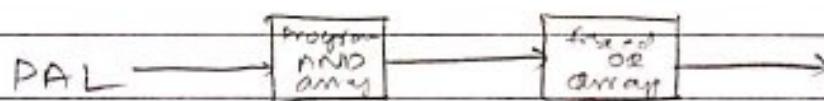
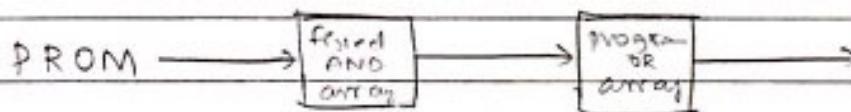
→ There are 3 PLD's.

PROM → programmable read only memory

PAL → programmable array logic

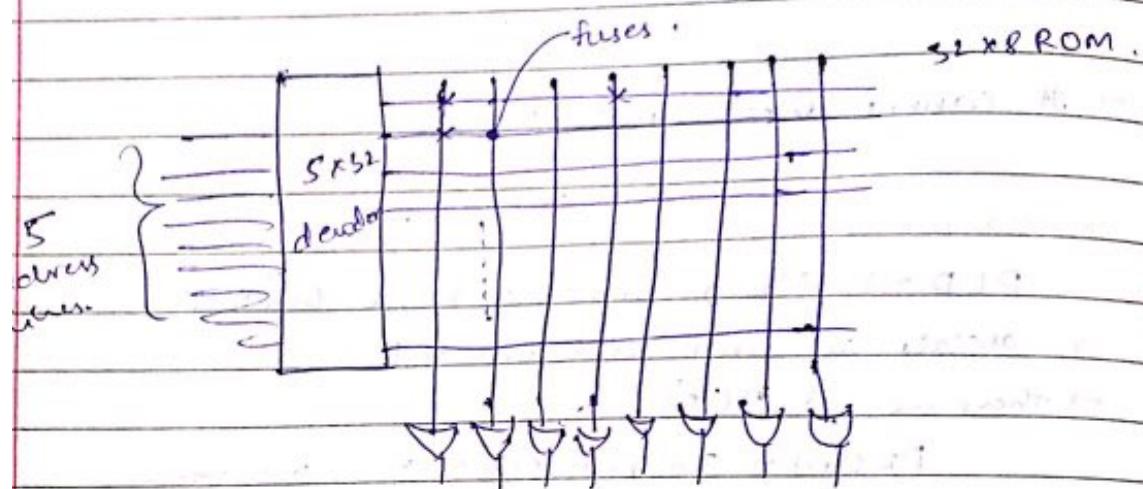
PLA → programmable logic array.

→ There are AND arrays and OR arrays, based on which we decide our PLD.



Read Only memory :- ROM

$$K \rightarrow [2^k \times n] \rightarrow n\text{-bit O/P}$$



→ When there is fuse intact then it is 0.

→ When fuse is blown it is 1.

→ If the power goes off there is no formula lost here
it is non volatile.

Programmable array logic (PAL):

Find PAL for

$$F_1 = \Sigma(1, 2, 3, 6)$$

$$F_2 = \Sigma(1, 2, 4, 5)$$

$$F_3 = \Sigma(1, 7)$$

	A	B	C	D	F ₁
M ₀	0	1	1	1	0
M ₁	1	0	0	0	1

	A	B	C	D	F ₂
M ₀	0	1	0	1	0
M ₁	1	1	1	0	1

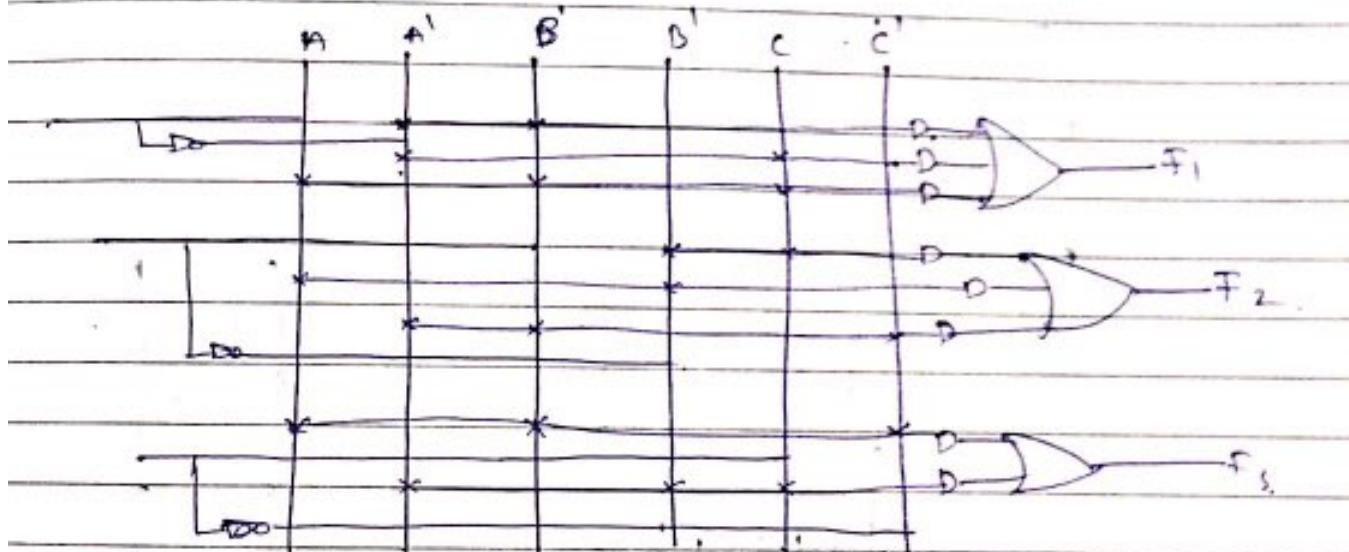
$$F_1 = A'B + A'C + AB'C$$

$$F_2 = B'C + AB + A'BC'$$

F₃

	1		
			1

$$F_3 = A'B'C + ABC'$$



Programmable logic array (PLA):-

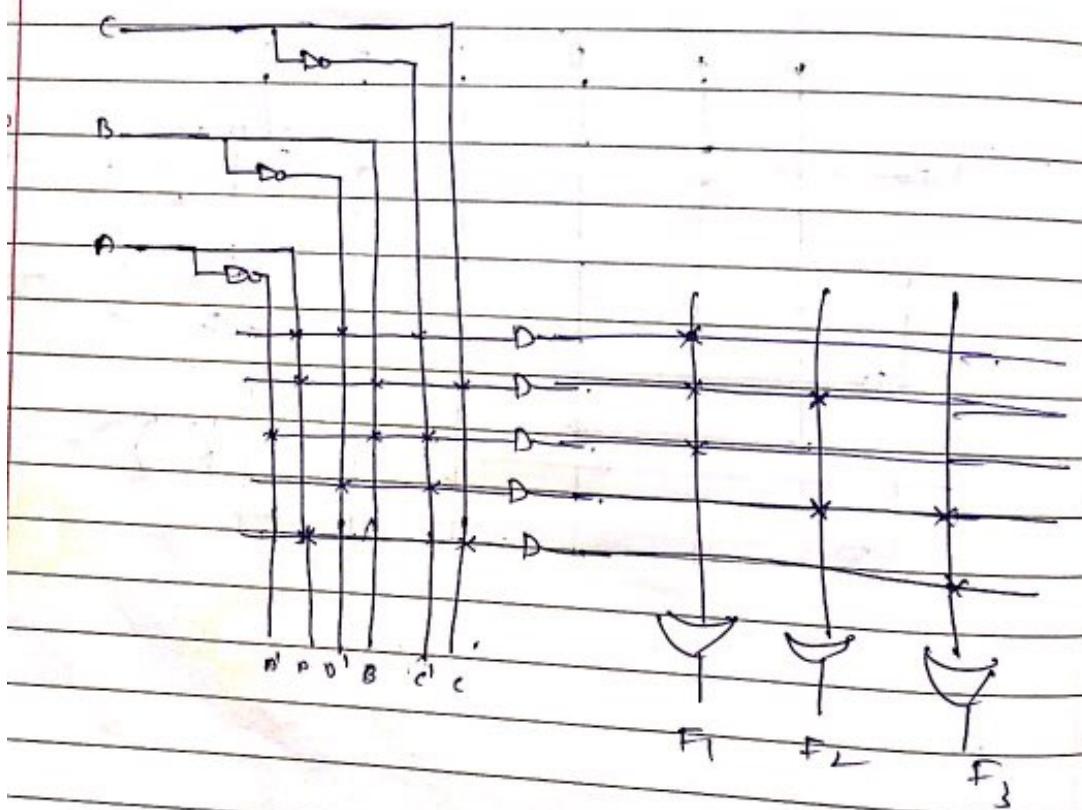
Implemented the function using PLA

$$F_1 = AB'C' + ABC \Rightarrow A'B'C'$$

$$F_2 = ABC + B'C'$$

$$F_3 = AC + B'C'$$

Product	P/A			D/P		
	A	B	C	F ₁	F ₂	F ₃
A'B'C'	1	0	0	1	-	-
ABC	1	1	1	1	1	-
A'B'C'	0	1	0	1	-	-
B'C'	-	0	1	-	1	1
AC	1	-	1	-	-	1



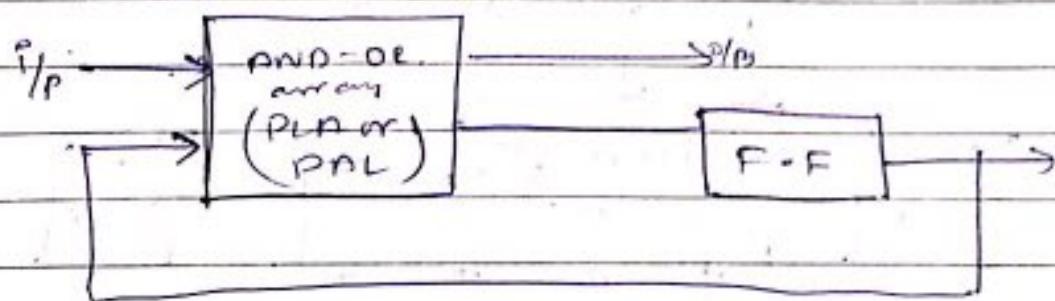
Sequential programmable devices :-

These are designed with flip flops and gates.

There are 3 types:

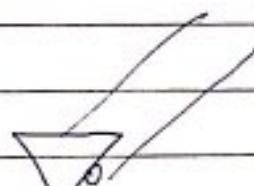
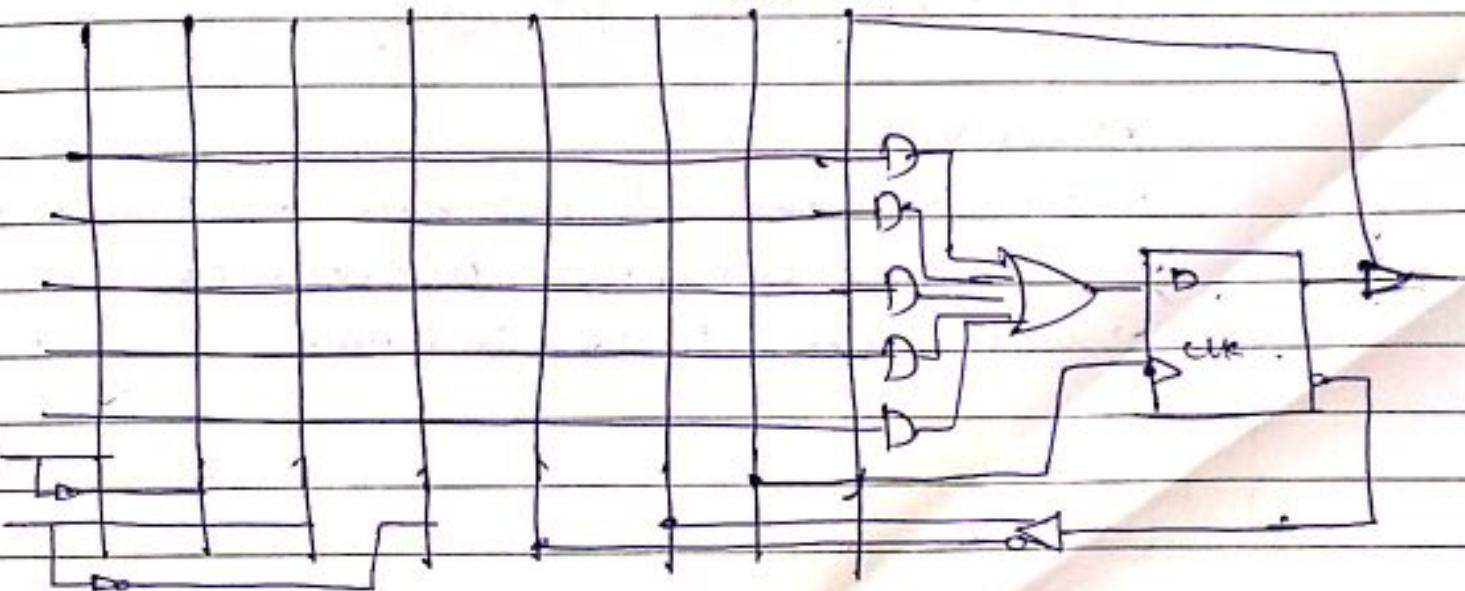
- (SPLD, CPLD, FPGAs)

(i) Sequential programmable logic device :-



Macro cell

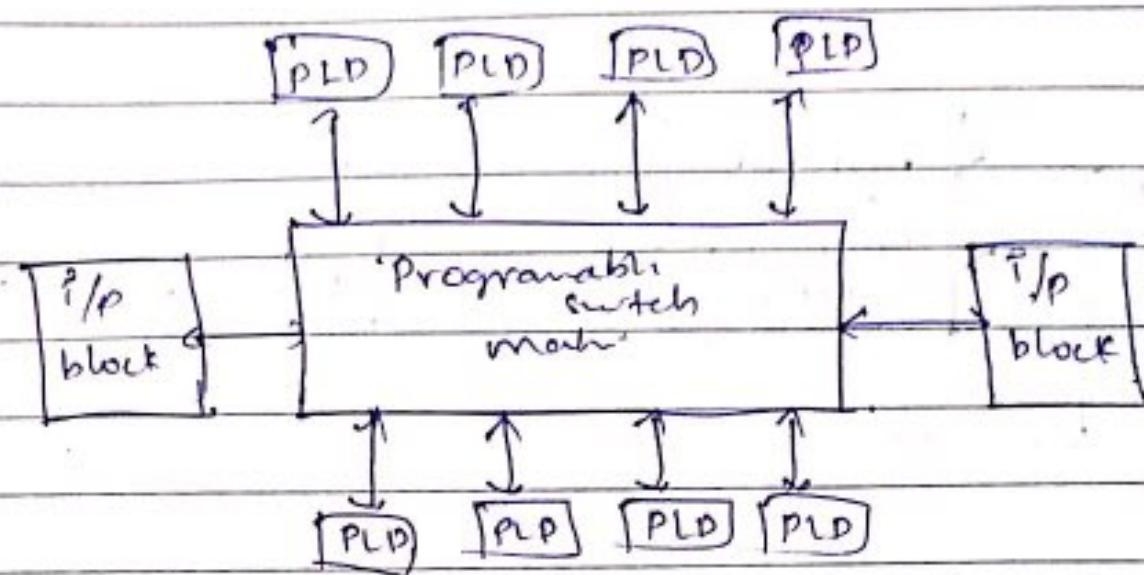
(Optional)
clk. OG



(iii) Complex Programmable Logic device (CPLD):-

→ Connection of several individual PLD's.
(PAL + PLA)

→ The PLD's are interconnected through programmable switch matrix.



(iii) Field programmable gate array:-

→ uses VLSI.

→ Elementary element is logic block and is in 100's or 1000's.

→ It consists of look up table, MUX, gates, FF, etc.

→ Look up table is stored in SRAM.

5. Digital Integrated Circuits

the gates (logic gates/ universal gates) are made by the below technologies.

RTL → Resistor Transistor logic

DTL → Diode Transistor logic

TTL → Transistor Transistor logic

MOS → metal oxide semiconductor ^{n MOS} _{p MOS}

CMOS → complementary metal oxide semiconductor

ECL → Emitter coupled logic.

→ we use BJT's and MOSFET's in these circuits.

→ we commonly use TTL, ECL, CMOS nowadays.

Basic characteristics :-

Fanout → allows the gate to connect connection of other gates → As HIGH as possible.

Power dissipation → Power consumed → LOW

Propagation delay → the time taken to reflect the change → LOW

Noise Margin → max noise that can be allowed.

Fanout :- It is a no of standard nodes that can be

connected to the output of the gate without degrading its normal operation.

(ratio of O/P to I/P current)

Power dissipation :-

$$P_{\text{avg}} = I_{\text{avg}} \cdot V_{\text{cc}}$$

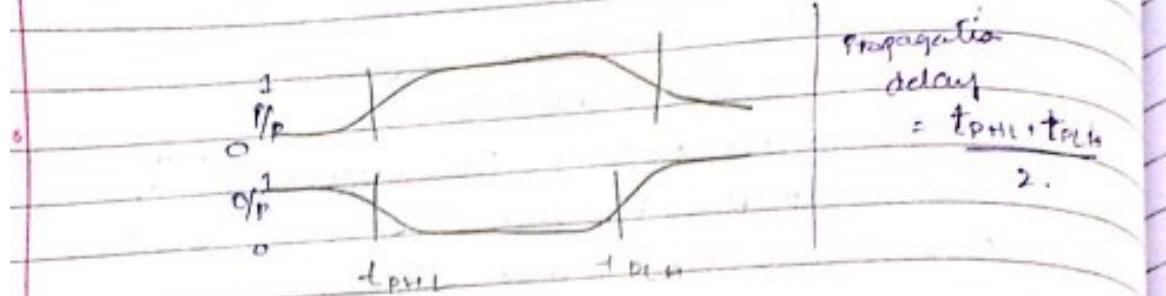
$$I_{\text{avg}} = \frac{I_{\text{CH}}^{\text{high}} + I_{\text{CL}}^{\text{low}}}{2}$$

The current drawn from power supply when output of gate is in high voltage level is: I_{CH} . If it is in low voltage level is I_{CL} .

The power dissipation represents the amount of power needed by the gate.

It is in (mW).

Propagation delay :-



t_{PHL} → The rising transition delay time for the signal to propagate from $I/p \rightarrow O/p$ when the binary I/p signal changes its value.)

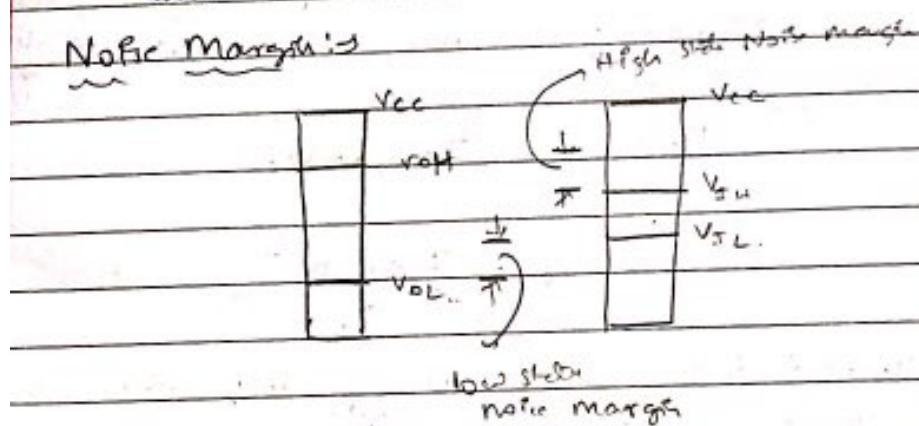
t_{PDL} :-

= the propagation delay from $I/p \rightarrow O/p$ when the change from High to Low

t_{PLH} :-

The propagation delay from $O/p \rightarrow I/p$ when O/p changes from low to high.

Noise Margin



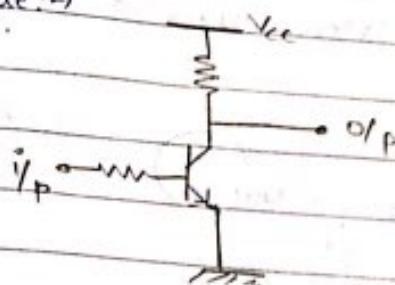
$$\text{noise margin} = V_{OH} - V_{TH} \quad \left. \begin{matrix} \\ \end{matrix} \right\} \text{ whichever is smaller}$$

$$V_{OL} - V_{OL}$$

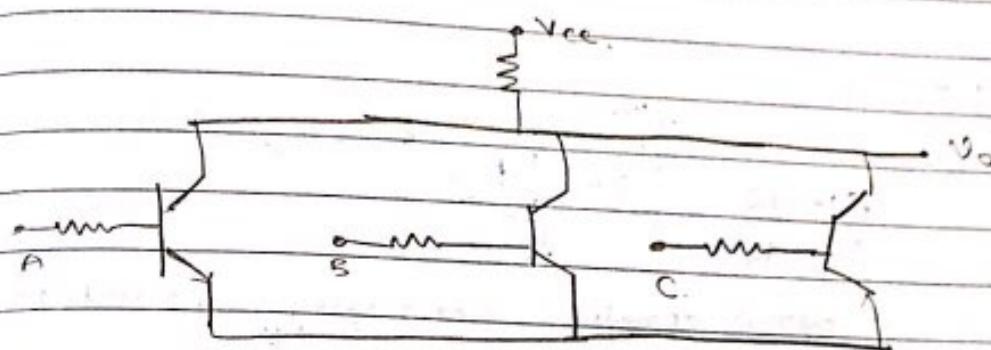
→ $\frac{1}{T}$ is the maximum noise voltage that can be added into the signal that does not cause any undesirable change in the output.

RTL :- Resistor Transistor logic :-

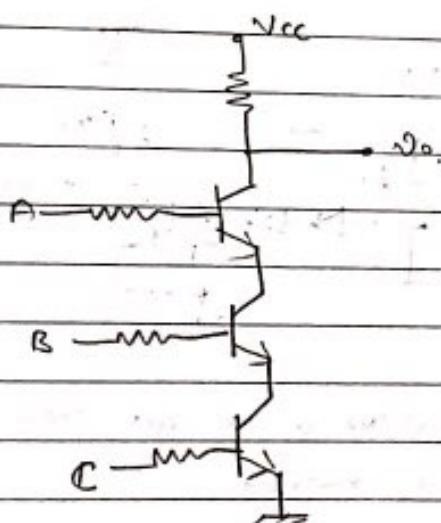
NOT gate :-



NOR gate :-



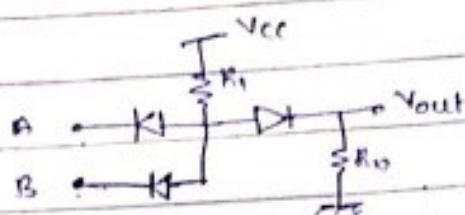
NAND gate :-



DTL → Diode Transistor Logic

First started as DL later converted to DTL.

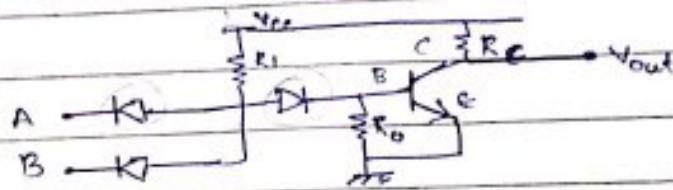
DL →



A	B	out
0	0	0
0	1	0
1	0	0
1	1	1

→ This operation is AND gate

DTL →

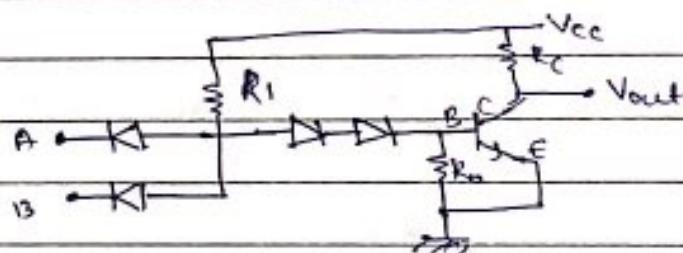


→ This operation is NAND gate (out is inverted)

→ Sometimes due to changes in cut-in voltage of diodes which may effect the o/p.

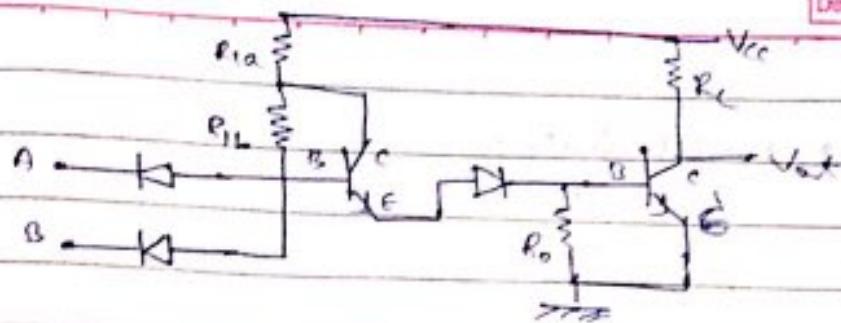
so to correct this they added another diode

DTL - NAND gate



→ we use transistor as we get more current i.e. fanouts are more.

→ Further to increasing current driving capability we made some modifications.



TTL → Transistor Transistor logic

There are 74 Series

HTL → Low power TTL.

SSI → Small scale Integrate

MSI → middle scale Integrate

→ In BJT's earlier it acts as switch

Saturation = ON

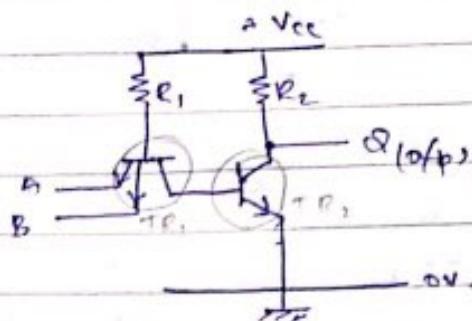
Cutoff = OFF

→ But it's hard to bring the transistor from saturation to cut off

So we use active region for ON of switch

(It's easy to buy one diode for F-D → R-B)
i.e. called Schottky Transistor.

2 input NAND Gate :-



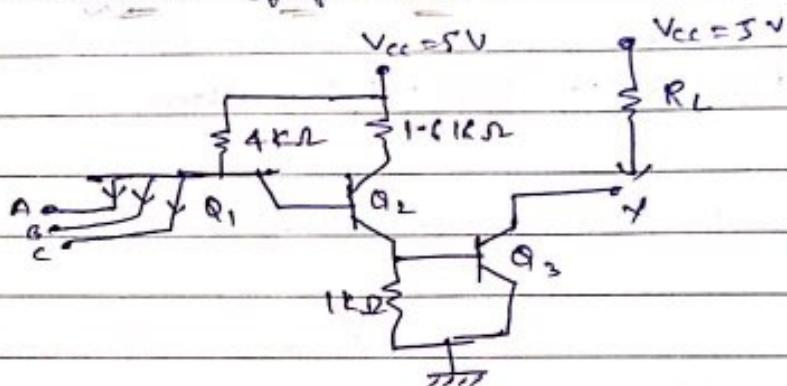
when $A \cap B = 0$ o/p is high.

as TR_1, B R.B

$A \cap B = 1$ o/p is low

TR_1, B F.B (TR_1 drives TR_2 into saturation)

Open collector output gate :-

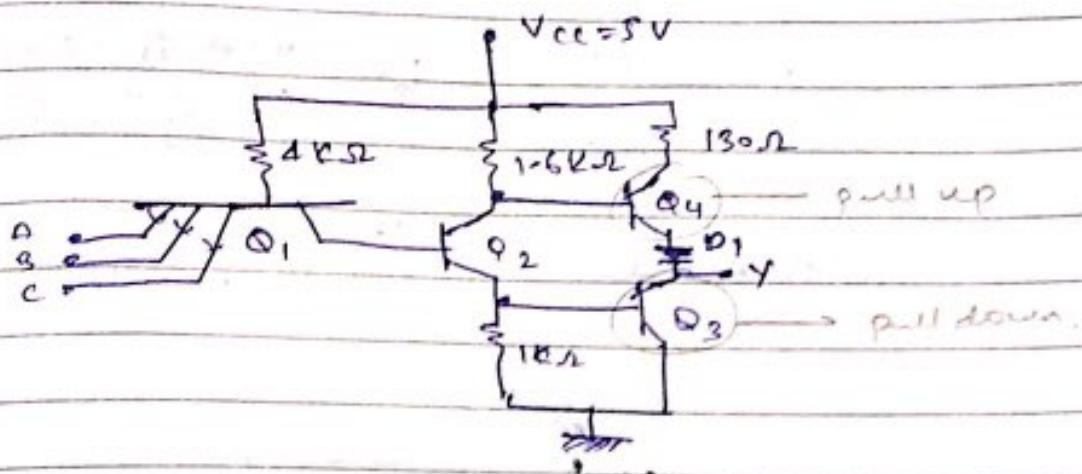


NAND gate

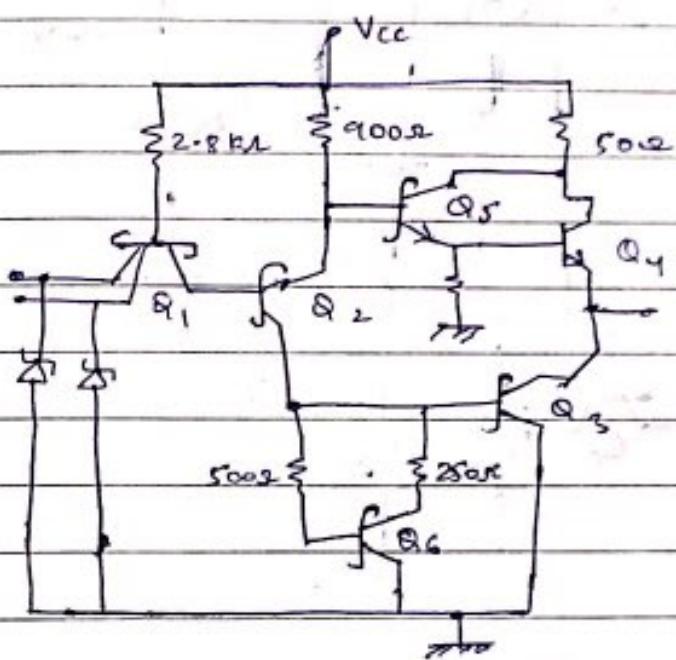
when $A \cap B \cap C = 0$ o/p is high.

$A, B, C = 1$ o/p is low. (as Q_1 drives Q_2, Q_3 into saturation)

TTL gate with totem-pole output \Rightarrow



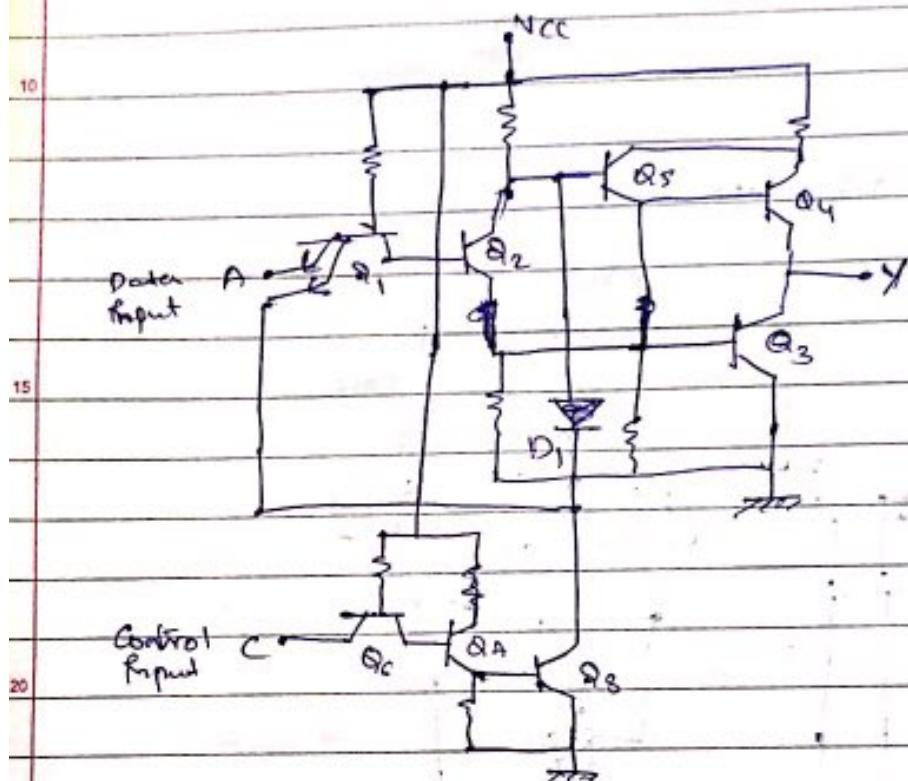
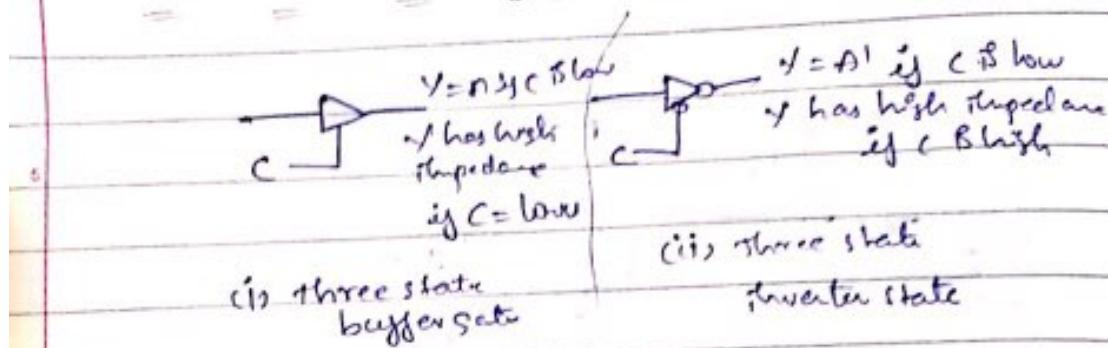
Schottky TTL gate \Rightarrow



\rightarrow has more speed

\rightarrow Because schottky Transistor are used
(switches b/w active and cut off)

Three state TTL gate :-



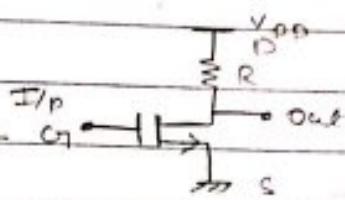
MOS :: Metal Oxide Semiconductor

We prefer NMOS to PMOS because

- the charge carrier moving are e^-
- and mobility of e^- is more than holes
- easy size reduction
- less noise compared to PMOS.

NMOS logic gates :: → These are positive logic gate devices.

S → Low → NMOS
→ High → PMOS.



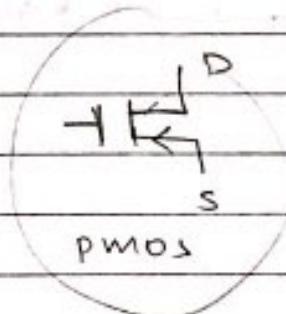
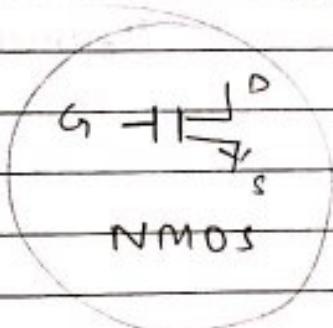
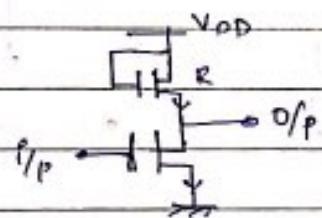
i/p	o/p
0.	1
1	0

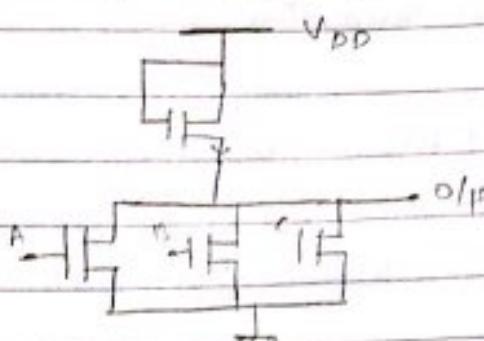
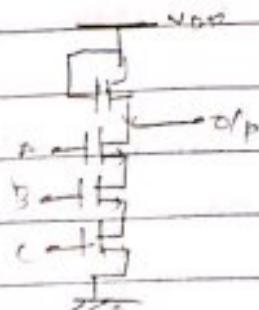
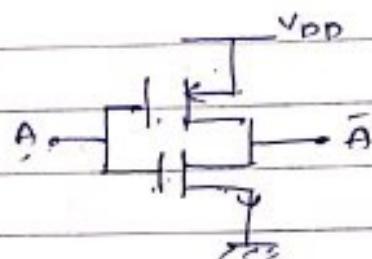
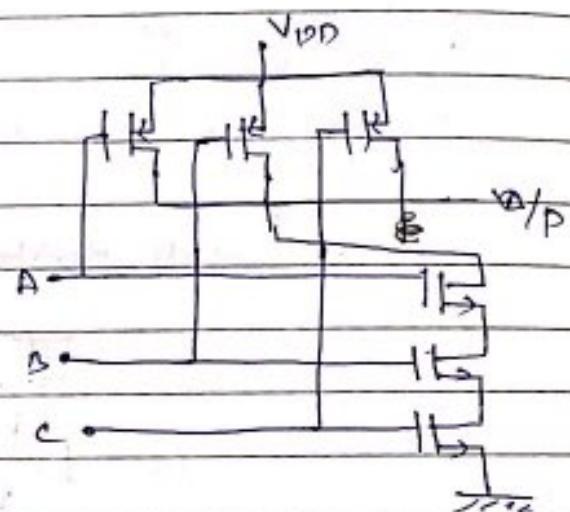
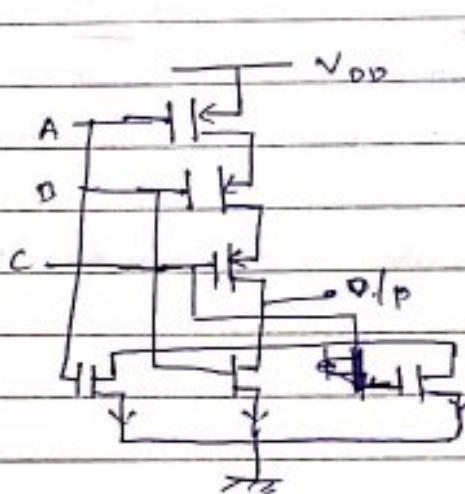
invert

PUN → pull the o/p to V_{DD} ($O/I/P \rightarrow$ supply) (use PMOS)

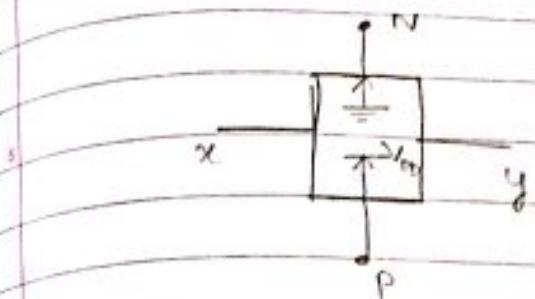
PDN → pull down the o/p to ground ($O/I/P \rightarrow$ ground) (use NMOS)

Instead of Resistor R we use NMOS or PMOS

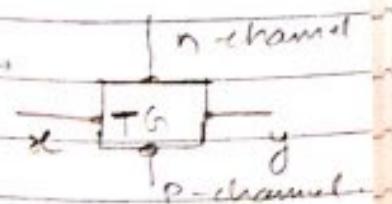


3 i/p NOR gate3 i/p NAND gateCMOS : → complementary metal oxide semiconductorCMOS logic inverter3 i/p CMOS NOR gate(pmos \rightarrow Nel
 $n_{mn} \rightarrow S_{nn}$)3 i/p CMOS NAND gate ::(nmos \rightarrow 11st
pmos \rightarrow (11))

CMOS Transmission gates :-

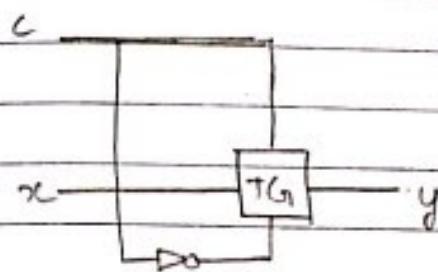


Logic symbol :-



→ It is a bilateral switch.

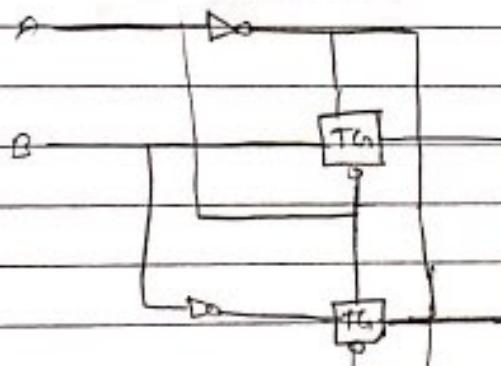
→ It conducts both the sides. ($x \rightarrow y$ or $y \rightarrow x$)



$C = 0$ Switch open (OFF)

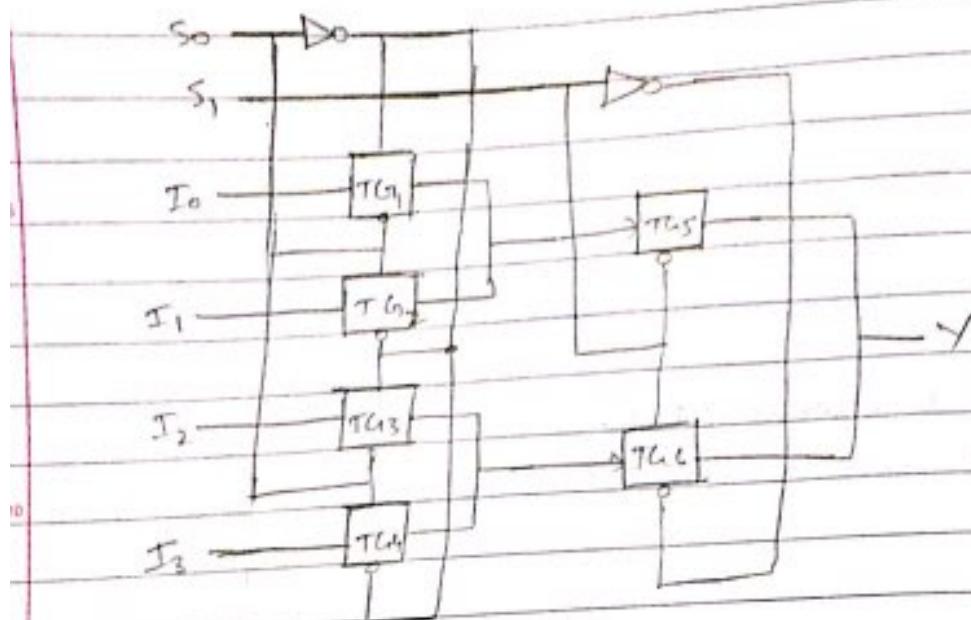
$C = 1$ Switch closed (ON)

B-YOR Gate :-



A	B	V
0	0	0
0	1	1
1	0	1
1	1	0

4x1 MUX using T-G:-



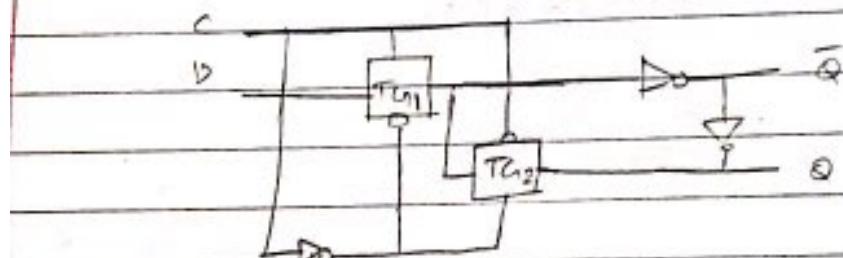
$S_1 \quad S_0 \quad Y$

0 0 I_0

0 1 I_1

1 0 I_2

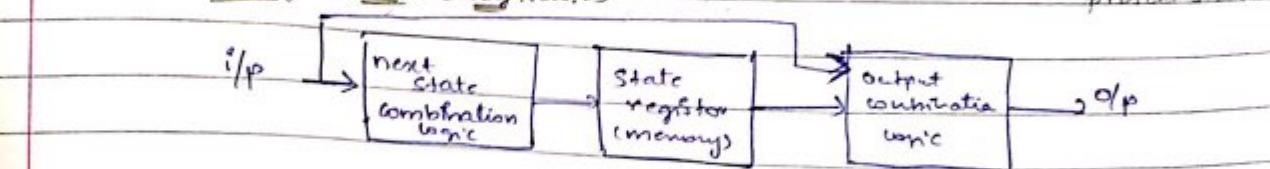
1 1 I_3



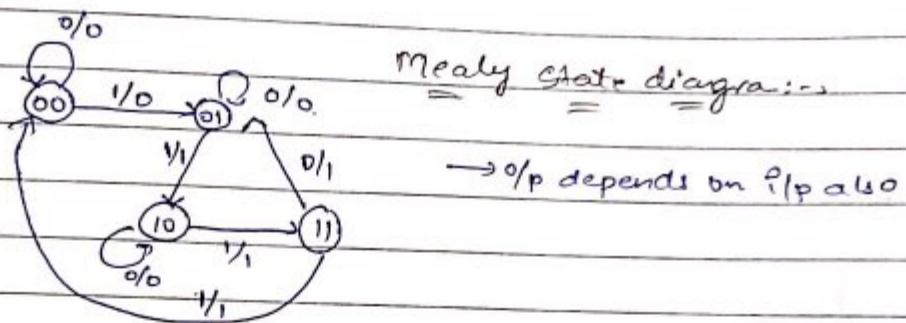
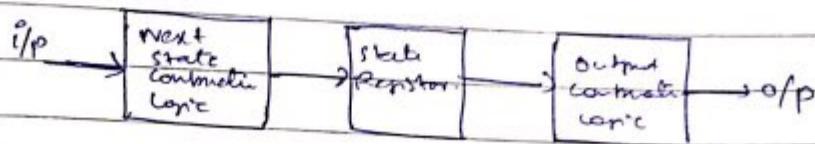
State Machines:-

These are 2 types

Mealy: block diagram :-



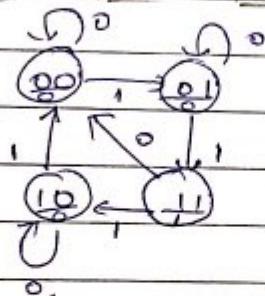
Moore: block diagram



= = = = =
Mealy state diagram :-

→ o/p depends on i/p also

Moore state diagram:-



→ o/p is not influenced by i/p.

→ only influenced by state.

State Equations :-

D, JK and FF are mostly used.

→ The state equations are on next state, \bar{Q}_n and ^{present} Q_n .

for DFF

P.S	\bar{Q}_n	D	Q_{n+1}
0	0	0	0
0	1	1	1
1	0	0	1
1	1	1	0

→ Irrespective of past or present state the output is same as input D value.

$$\boxed{Q_{n+1} = D}$$

for JK FF

\bar{Q}_n	J	K	Q_{n+1}	
0	0	0	0	reset
0	0	1	0	reset
0	1	0	1	set
0	1	1	1	\bar{Q}_n
1	0	0	1	reset
1	1	0	1	set
1	1	1	0	\bar{Q}_n
1	0	1	0	reset

Q_{n+1}	$Q_n \bar{T}$	$Q_n \bar{K}'$
0	0	1
0	0	1
1	1	0
1	0	0

$$Q_{n+1} = Q_n \bar{T} + Q_n \bar{K}'$$

$$Q_{n+1} = Q_n \bar{T} + Q_n K'$$

for T FF

$$Q_n \quad T \quad Q_{n+1}$$

$$0 \quad 0, \quad 0 \quad Q_n$$

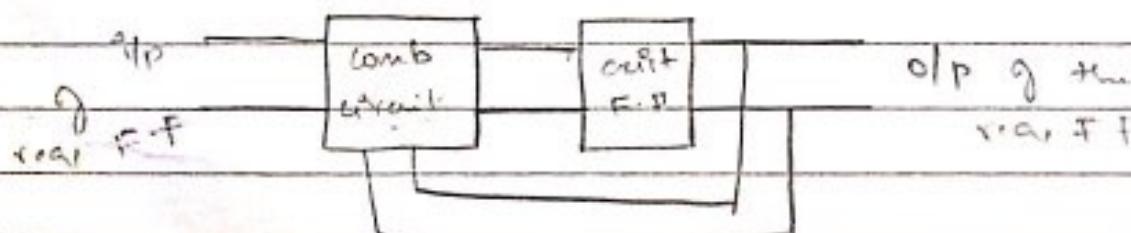
$$0 \quad 1 \quad 1, \quad \bar{Q}_n$$

$$1 \quad 1 \quad 0 \quad \bar{Q}_n$$

$$1 \quad 0 \quad 1 \quad Q_n$$

$$Q_{n+1} = T \oplus Q_n$$

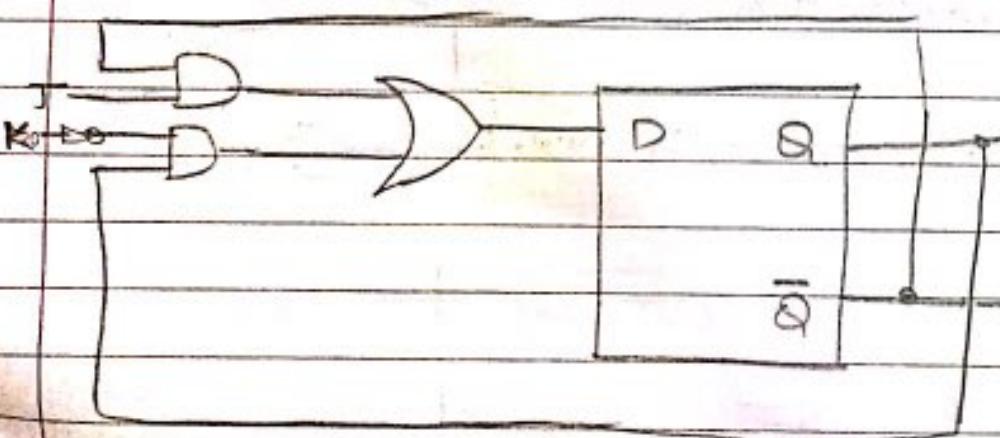
Conversions between flip flops:-



Convert D FF into JK FF

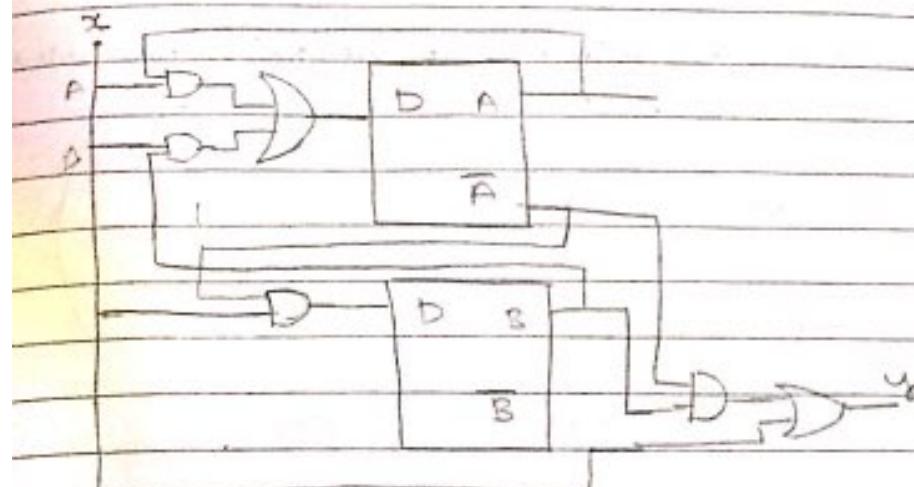
	I/p of new F.F		Present state	Next state	I/p Q. exist FF
	J	K	Q_n	Q_{n+1}	D
0	0	0	0	0	0
0	0	1	1	1	1
10	0	1	0	0	0
0	1	1	0	0	0
1	0	0	1	1	1
1	0	1	1	1	1
11	1	1	0	1	1
15	1	1	1	0	0

$$D = JQ_n + K'Q_n$$



Sequential circuit Analysis Procedure :-

(i) D- Flip Flop :-)



$$A(t+1) = A(t)x(t) + B(t)\bar{x}(t) = Ax + B\bar{x}$$

$$B(t+1) = \bar{A}(t)x(t) = \bar{A}x$$

$$y(t+1) = \bar{A}B + x(t+1)\bar{A}(t)B(t) + x(t)$$

State Table :-

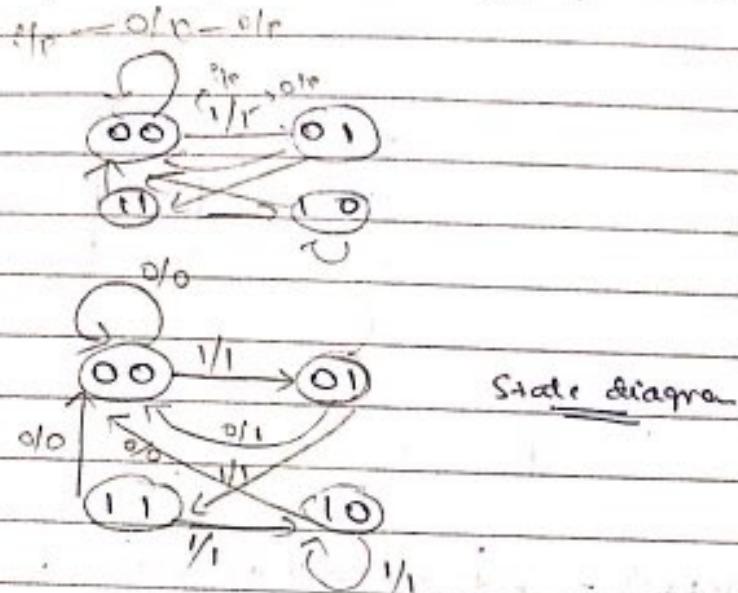
Present State		I/P	next state		O/P
D	B	x	A	B	y
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	0	0	1
0	1	1	1	1	1
1	0	0	0	0	0
1	0	1	1	0	1
1	1	0	0	0	0
1	1	1	1	0	1

→ This circuit may exist 4 states

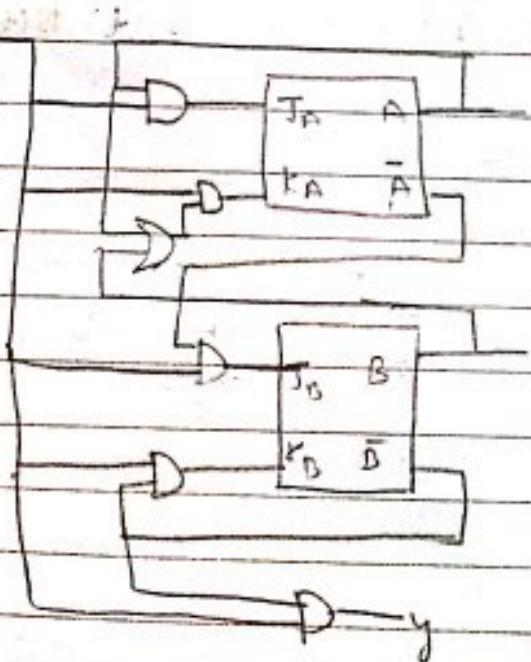
00 01

11 10

→ State diagram represents the i/p , o/p and etc. for state transitions.



(ii) JK - Flip Flop :-



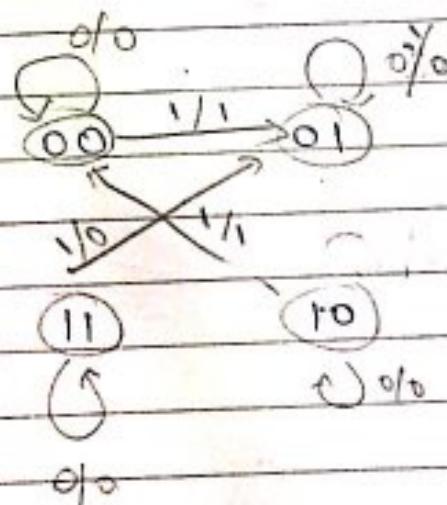
$$F = \bar{P} \cdot Q \cdot R$$

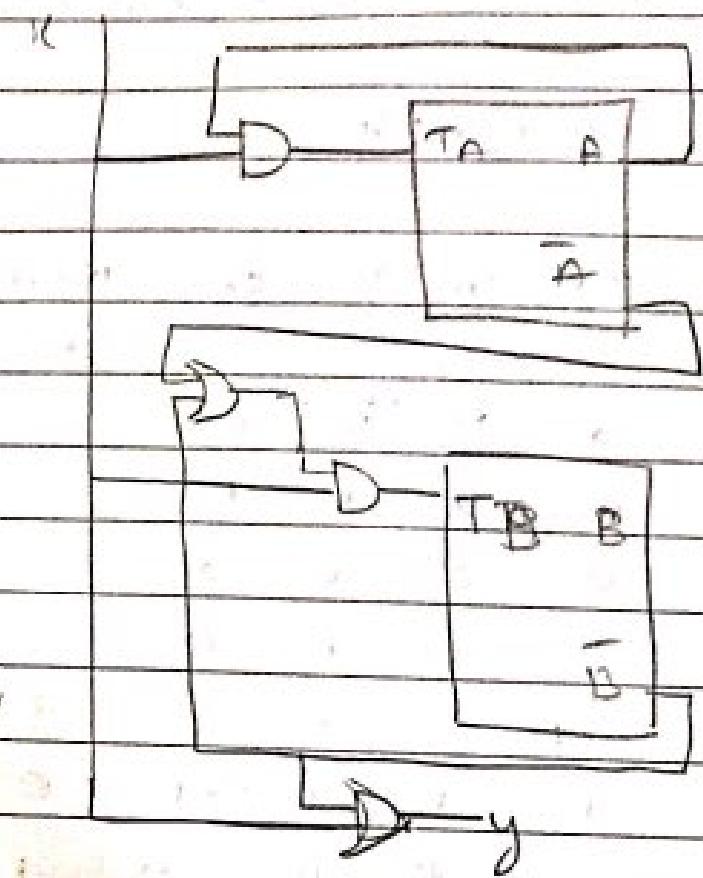
$$T_A = A(t) \cdot x(t) \quad k_A = (A(t) + B(t)) \cdot x(t)$$

$$T_B = \bar{A}(t) \cdot x(t) \quad k_B = x(t) \cdot \bar{B}(t)$$

$$y = \bar{B}(t) \cdot x(t)$$

Present state		\bar{P}	Q_A	R_A	T_A	k_A	Next state		\bar{P}
A	B	x	A	B	y
0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	1	1	0	1	1
0	1	0	0	0	0	0	0	1	0
0	1	1	0	1	1	0	0	1	0
1	0	0	0	0	0	0	1	0	0
1	0	1	1	1	0	1	0	0	1
1	1	0	0	0	0	0	1	1	0
1	1	1	1	1	0	0	0	1	0



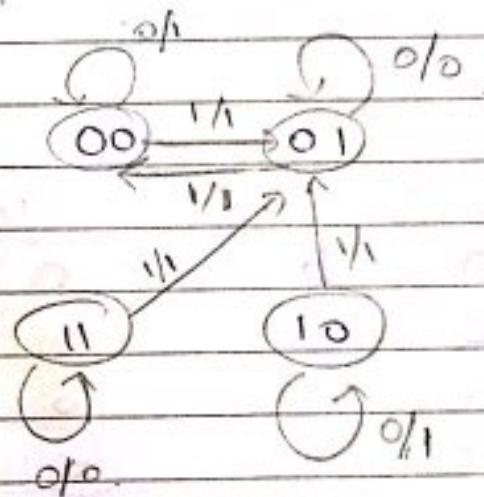
(iii) T-Flip Flop:-

$$T_A = A(+) \cdot x(+)$$

$$T_B = [\bar{A}(+) + \bar{B}(+)] \cdot x(+)$$

$$y = \bar{B}(+) \cdot x$$

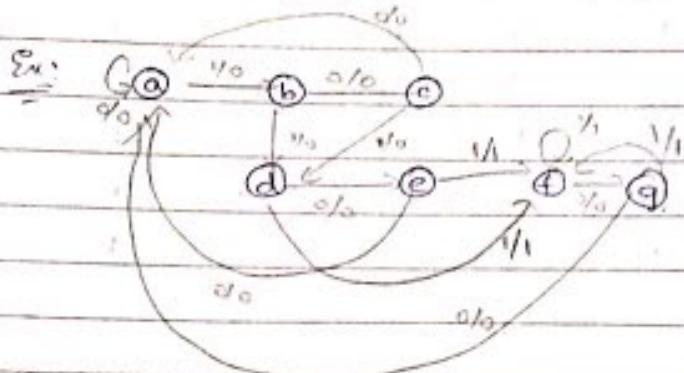
Present state		$\bar{P}lP$	T_A	T_B	Next state		$\bar{o}LP$
A	B	x			A	B	y
0	0	0	0	0	0	0	a -1
0	0	1	0	1	0	1	b -1
0	1	0	0	0	0	1	c 0
0	1	1	0	1	0	0	d 1
1	0	0	0	0	1	0	e 1
1	0	1	1	1	0	1	f -1
1	1	0	0	0	1	1	g 0
1	1	1	1	0	0	1	h -1



Sequential circuit design procedure :-

State Reduction and assignment :-

Choose the states and assignment variable for the



→ Reducing the states i.e. state reduction

State Table :-

Preset State	Next State		
	$X=0$	$X=1$	
a	b	c	$\begin{matrix} 0 & 1 \\ 0 & 1 \end{matrix}$
b	c	d	0 0
c	a	d	0 0
d	e	f	0 1
e	a	f	0 1
f	g	g	0 1
g	a	f	0 1

→ for some combination of I/P, we get same state and same O/P ; then one of combination can be removed.

1st → e f g

2nd → d f f

Reduced state table:-

P.S	N.S $n=0$ $n=1$	S.P $n=0$ $x=1$
a	a b	0 0
b	c d	0 0
c	a d	0 0
d	e d	0 1
e	a d.	0 1

→ The states should be assigned with binary values,
i.e state assignment

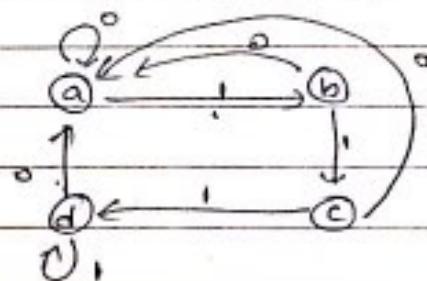
<u>state</u>	<u>binary</u>	<u>gray code</u>	<u>one shot</u>
a	0 0 0	0 0 0	0 0 0 0 1
b	0 0 1	0 0 1	0 0 0 1 0
c	0 1 0	0 1 1	0 0 1 0 0
d	0 1 1	0 1 0	0 1 0 0 0
e	1 0 0	1 1 0	1 0 0 0 0

any of these can be used for designing

Synchronous sequential circuit design :-

- i) From the design objective, the state diagram has to be derived.
- ii) If possible, states are to be reduced.
- iii) For the final states, binary values are to be assigned.
- iv) Then required flip flops are to be chosen.
- v) The flip flop inputs and outputs are to be derived.
- vi) The logic diagram of the required synchronous sequential circuit is to be drawn.

Design a synchronous sequential circuit which detects 3 or more consecutive 1's in the sequence of bits.



P.S.	N.S.	P.S.	I/p	N.S.
a	$x=0$ $x=1$	A' B	A	A B
a	a b	0' 0	0	0 0
b	a c	0 1	1	1 0
c	a d	1 0	0	0 0
d	a a	1 1	0	0 0

The flip flop chosen is 'D'.

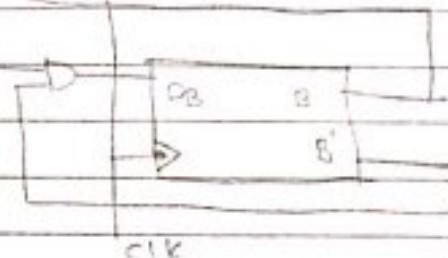
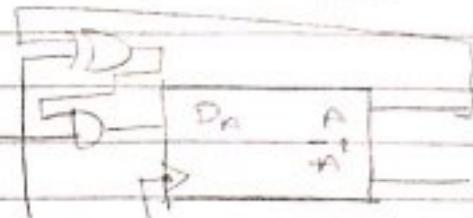
$$D_n = AB'x + A'Bx \\ = x(A \oplus B)$$

	0	1	0	1	0
0	0	0	1	0	0
1	0	1	0	0	0

$$DB = B'x$$

B' 00 01 11 10			
0	0 1	0 0	
1	0 1	0 0	

x



Design a synchronous sequential circuit which acts as a 3-bit binary counter.

P.S.

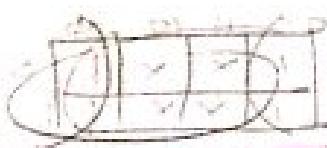
N.S.

A	B	C	A	B	C	J _A	K _A	J _B	K _B	J _C	K _C	T _A	T _B	T _C
0	0	0	0	0	1	0	x	0	x	1	x	0	0	1
0	0	1	0	1	0	0	x	1	x	x	1	0	1	1
0	1	0	0	1	1	0	x	x	0	1	x	0	0	1
0	1	1	1	0	0	1	x	1	x	1	1	1	1	1
1	0	0	1	0	1	x	0	0	x	1	x	0	0	1
1	0	1	1	1	0	x	0	1	x	x	1	0	1	1
1	1	0	1	1	1	x	0	x	0	1	x	0	0	1
1	1	1	0	0	0	x	1	x	1	x	1	1	1	1

A	B	C	J _A	K _A	J _B	K _B	J _C	K _C
0	0	0	1	0	0	x	x	0
1	0	1	x	0	0	x	1	0

$$J_A = BC \quad K_A = BC \quad J_C = AC$$

K_B	0	0	1	0
\bar{K}_B	1	1	0	0
T_B	P	A		
\bar{T}_B	\bar{A}			

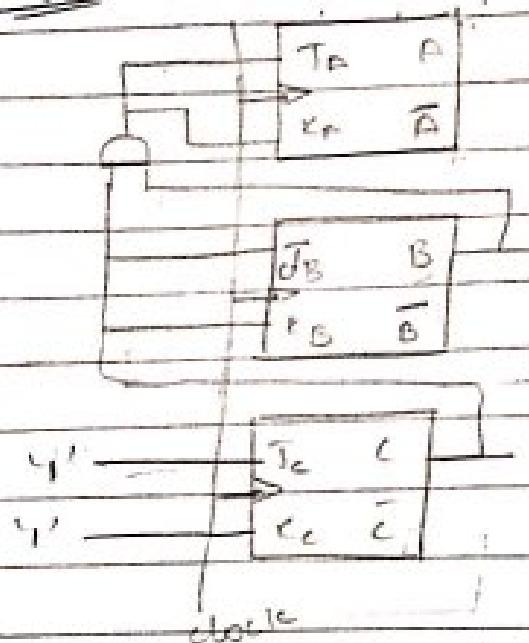


$$K_B = C$$

$$T_C = 1$$

$$K_C = 1'$$

using JK



using T

0	0	1	0
0	0	1	0

$$T_B = BC$$

0	1	1	0
0	1	1	0

$$T_B = C$$

1	1	1	1
1	1	1	1

$$T_C = 1$$

