Constraining anisotropic universe under $f(R,T)$ theory of gravity

Lokesh Kumar Sharma[a], Suresh Parekh[c,*], Kalyani C.K. Mehta[d,*], Saibal Ray[e], Anil Kumar Yadav[1]

[a] *Department of Physics, GLA University, Mathura 281406, Uttar Pradesh, India*
[b] *Department of Physics, SP Pune University, Pune 411007, Maharastra, India*
[c] *Department of Physics, Eberhard Karls University of Tubingen, Germany*
[d] *Centre for Cosmology, Astrophysics and Space Science (CCASS), GLA University, Mathura 281406, Uttar Pradesh, India*
[e] *Department of Physics, United College of Engineering and Research, Greater Noida 201 306, India*

Abstract

We try to find the possibility of a Bianchi V universe in the modified gravitational field theory of $f(R,T)$. We have considered a Lagrangian model in connection between the trace of the energy-momentum tensor T and the Ricci scalar R. In order to solve the field equations a power law for the scaling factor was also considered. To make a comparison of the model parameters with the observational data we put constraint on the model under the datasets of the Hubble parameter, Baryon Acoustic Oscillations, Pantheon, joint datasets of Hubble parameter + Pantheon and collective datasets of the Hubble parameter + Baryon Acoustic Oscillations + Pantheon. The outcomes for the Hubble parameter in the present epoch are reasonably acceptable, especially our estimation of this H_0 is remarkably consistent with various recent Planck Collaboration studies that utilize the Λ-CDM model.

Keywords: Cosmological Parameters, Observational constraints, Om Diagnostics, Cosmography, MCMC Model

1. Introduction

In 1998, the discovery of a late-time accelerating cosmos and the subsequent development of dark energy (DE) with repulsive pressure led to a significant advancement in cosmology [1, 2, 3, 4, 5, 6]. Numerous cosmological models have been examined to comprehend the characteristics of DE [7, 8, 9, 10, 11]. The cosmological constant Λ is the most viable choice for dark energy, yet it has certain issues with its theoretical implementation [12, 13, 14, 24].Our understanding of dark energy is limited to its phenomenological characteristics, which remain enigmatic and unclear: (i) DE is a cosmic fluid that violates the strong energy condition and has an equation of state parameter. (ii) DE shows a lesser clustering property than dark matter (DM) which is uniformly distributed

*Saibal Ray

Email addresses: lokesh.sharma@gla.ac.in (Lokesh Kumar Sharma), sunil@unizwa.edu.om (Suresh Parekh), sunil@unizwa.edu.om (Kalyani C.K. Mehta), saibal.ray@gla.ac.in (Saibal Ray), abanilyadav@yahoo.co.in (Anil Kumar Yadav)

over the universe on a large cosmological scale. Apart from the Λ-CDM model, there is also the X-CDM model where the dynamical dark energy is parametrized along the spatial direction and the ϕ-CDM model where dynamical dark energy is represented by a scalar field ϕ [16, 17]. We note that the X-CDM model is confirmed for a considerably larger set of cosmological data, such as growth factor information, BAO distance measurements, and type Ia supernova (SN Ia) apparent magnitude observations. This encourages us to build a Bianchi type I space-time model of the accelerating cosmos.

The $f(R,T)$ theory of gravity, which was put forth by Harko et al. [18] in 2011, expresses the Lagrangian in terms of the trace of the stress-energy tensor T and the Ricci scalar R. It is evident that $f(T)$ and matter function as an effective cosmological constant. Therefore, adding a DE component to the $f(R,T)$ theory of gravity may help explain the Universe's late time acceleration. Harko [19] has since looked upon a generalized gravity model where the thermodynamical consequences on the matter-geometry coupling will have a definite imprint. The following works are involved with some significant uses of the $f(R,T)$ gravity theory [20, 21, 22, 23]. Nojiri and Odintsov [24] have studied a cosmological model where $f(R)$ is used in place of the action. Carroll et al. [25] observed that the late-time acceleration of our Universe could be represented satisfactorily by cosmological models motivated by $f(R)$ gravity. The authors have looked at feasible cosmological models that meet the requirements of the Solar system test for the $f(R)$ theory of gravity [26]. We study an anisotropic singular universe model in $f(R,T)$ gravity with nonminimal matter geometry coupling. The field equations have been precisely solved by accounting for the scale factor's power law change, which drives Λ-CDM cosmological scenario. It is important to note that the $f(R,T)$ gravity theory plays an important role in evoking a comprehensive theoretical explanation of the late-time accelerating universe without the assistance of unusual exotic matter or energy.

With the help of the presence of dark energy and dark matter, the $f(R,T)$ theory of gravitation contributes significantly to the explanation of the Universe's current acceleration. Different models regarding different features of the functional form of this theory have been proposed by several scientists [20, 27, 28]. According to this hypothesis, the way that various matter components interact with space-time curvature is a cosmological outcome.Nevertheless, a significant contribution of EMC violation causes rapid growth in the cosmic gravity models of $f(R,T)$. The impact of EMC violation in this theory has not yet been thoroughly investigated. However, a few first investigations on celestial objects by employing this theory have been proposed in the following works [29, 30, 31, 32, 33]. Inspired by the previous conversations, we now set out to study a cosmological model for the following specifications: (i) there will be a connection between nonminimal matter and geometry, (ii) it will be associated with the space-time of Bianchi V and (iii) here gravity will be defined as $f(R,T) = f_1(R) + f_2(R)f_3(T)$. It is noteworthy that the cosmological implications of a nonminimal coupling under the framework of $f(R,T)$ theory are explored by means of a product between R and T or functions of them. We shall use the simplest coupling $f_1(R) = f_2(R) = R$ and $f(T) = \zeta T$, with ζ a constant. Within the $f(R,T)$ gravity formalism, the function $f(R,T)$ has a nontrivial functional form that involves in the nonminimal matter-geometry coupling. Additionally, it benefits from the fact that $\zeta = 0$ is the retrieval of general relativity. A few practical uses for $f(R,T) = f_1(R) + f_2(R)f_3(T)$ are available in various physical situations [20, 34].

The paper is structured as follows: the model and its basic formalism is described in Section 2. The method of observation analysis and observational data sets are given in Section 3. Section 4 deals with the energy conditions and cosmological parameters. Finally, the findings of this paper are summarized in Section 5.

2. Mathematical background of $f(R,T)$ gravity theory

The space-time for Bianchi V can be provided as

$$ds^2 = -c^2dt^2 + (C^2dz^2 + B^2dy^2)e^{2\alpha x} + A(t)^2dx^2, \tag{1}$$

where scale factors along the x, y, and z axes are denoted by $A(t)$, $B(t)$, and $C(t)$, and a constant is represented by α.

The action under $f(R,T)$ theory can be provided as

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}f(R,T) + \int d^4x\sqrt{-g}L_m, \tag{2}$$

where g is the metric determinant and L_m is the matter Lagrangian density.

Now the given equation can be expressed as

$$\begin{aligned} &[f_1'(R) + f_2'(R)f_3'(T)]R_{ij} - \frac{1}{2}f_1'(R)g_{ij} \\ &+ (g_{ij}\nabla^i\nabla_j - \nabla_i\nabla_j)[f_1'(R) + f_2'(R)f_3'T] \\ &= [8\pi + f_2'(R)f_3'(T)]T_{ij} + f_2(R)\left[f_3'(T)p + \frac{1}{2}f_3(T)\right]g_{ij}, \end{aligned} \tag{3}$$

where $f(R,T)$ has been expressed earlier. Here a prime denotes derivatives with respect to the arrangement under consideration.

In connection with $f(R,T$ gravity, from Eq. (3), we have

$$G_{ij} = 8\pi\tilde{T}_{ij} = 8\pi\left(T_{ij} + T_{ij}^{(\mathrm{ME})}\right), \tag{4}$$

where $\tilde{T}_{ij}$, T_{ij} and $T_{ij}^{(\mathrm{ME})}$ denote, respectively, the effective energy-momentum tensor, the matter energy-momentum tensor and the additional energy term caused by the trace of the energy-momentum tensor which can be obtained as

$$T_{ij}^{(\mathrm{ME})} = \frac{\zeta R}{8\pi}\left(T_{ij} + \frac{3\rho - 7p}{2}g_{ij}\right). \tag{5}$$

From Eq. (4), after plugging the Bianchi identities, we get the following one

$$\nabla^i T_{\mathrm{ij}} = -\frac{\zeta R}{8\pi}\left[\nabla^i(T_{\mathrm{ij}} + pg_{ij}) + \frac{1}{2}g_{\mathrm{ij}}\nabla^i(\rho - 3p)\right]. \tag{6}$$

For the metric (1), the field equation (4) can be explicitly provided as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = -8\pi\tilde{p}, \tag{7}$$

$$\frac{\ddot{C}}{C} + \frac{\ddot{A}}{A} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} = -8\pi\tilde{p}, \tag{8}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\alpha^2}{A^2}=-8\pi\tilde{p}, \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{C}\dot{A}}{CA}+\frac{3\alpha^2}{A^2}=8\pi\tilde{\rho}. \tag{10}$$

In the above, the abbreviated symbols are as follows: $\tilde{\rho}=\rho+\rho^{(\mathrm{ME})}=p-\frac{3\zeta}{8\pi}\left(\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}\right)(3\rho-7p)$, $\tilde{p}=p+p^{(\mathrm{ME})}=p+\frac{9\zeta}{8\pi}\left(\frac{\ddot{a}}{a}+\frac{\dot{a}^2}{a^2}\right)(\rho-3p)$ and a $=(ABC)^{\frac{1}{3}}$ which is the average scale factor.

After a simple reformulation Eqs. (7)–(10) yield the following expression

$$\frac{(ABC)^{\ddot{}}}{ABC}=12\pi\left(\tilde{\rho}-\tilde{p}\right). \tag{11}$$

Now, the Hubble parameter can be defined as

$$H=\frac{\dot{a}}{a}, \tag{12}$$

whereas the average scale factor can be provided as Sharma et al. [22]

$$a=\alpha t^{\beta} \tag{13}$$

where α and β are two constants which should be non-zero and positive in nature.

Again, from Eqs. (7) – (9) along with Eq. (13), we get

$$A(t)=\alpha t^{\beta}, \tag{14}$$

$$B(t)=\xi(\alpha t^{\beta})\exp\left[\frac{\ell t^{1-3\beta}}{\alpha^3(1-3\beta)}\right], \tag{15}$$

$$C(t)=\xi^{-1}(\alpha t^{\beta})\exp\left[-\frac{\ell t^{1-3\beta}}{\alpha^3(1-3\beta)}\right], \tag{16}$$

where ξ and ℓ incorporated above are two arbitrary constants.

3. Observational Analysis

3.1. Model parameters and observational constraints

In order to determine the relevant model parameters, we use the following datasets to draw observational constraints.

1. OHD: From the cosmic chronometric approach, we have collected 55 $H(z)$ Observed Hubble Data (OHD) points in the interval $0 \le z \le 2.36$. Table II contains all of the data points from 55 $H(z)$ of Ref. [35].

2. BAO: Anisotropic Baryon Acoustic Oscillation (BAO) measurements of $D_M(z)/r_d$ and $D_H(z)/r_d$ are included in the final BAO measurements of the SDSS collaboration. Basically this span to eight different redshift intervals (in which $D_H(z)=c/H(z)$ is the Hubble distance and $D_M(z)$ is the

Table 1: Parametric values (viz. H_0, α, β and the age of Universe (Gyr) t) obtained from different datasets due to MCMC and Bayesian analysis where $H_1 = H(z) + Pantheon$ and $H_2 = H(z) + Pantheon + BAO$.

Parameter	**H(z)**	**BAO**	**Pantheon**	H_1	H_2
H_0	$68.790^{+2.350}_{-2.046}$	$69.499^{+2.561}_{-1.863}$	$69.012^{+2.453}_{-2.072}$	$67.633^{+2.448}_{-2.013}$	$68.381^{+2.219}_{-2.032}$
α	$69.012^{+0.092}_{-0.097}$	$68.992^{+0.095}_{-0.112}$	$68.998^{+0.084}_{-0.078}$	$68.979^{+0.099}_{-0.090}$	$69.005^{+0.110}_{-0.092}$
β	$1.001^{+0.009}_{-0.010}$	$0.998^{+0.008}_{-0.011}$	$1.000^{+0.010}_{-0.011}$	$1.006^{+0.009}_{-0.011}$	$1.003^{+0.009}_{-0.010}$
t	14.3430291	14.3599188	14.4902335	14.8743956	14.6678171

comoving angular diameter distance). Table 3 of Ref. [36] compiles all the BAO-only measurements.

3. Pantheon sample: The Pantheon sample was used to get distance modulus measurements of SN Ia [37]. The redshift range $z \in [0.001, 2.26]$ contains 1701 light curves that correspond to 1550 distinct SN Ia events.

For the present purpose the cosmological scale factor and Hubble parameter can be expressed in the following formats:

$$a = \frac{a_0}{1+z} = \alpha t^{\beta}, \tag{17}$$

$$H(z) = -\frac{1}{1+z}\frac{dz'}{dt}, \tag{18}$$

where a_0 is the present value of the scale factor.

By combining Eqs. (17) – (18) and thereafter via a few straightforward steps we can have the following Hubble parameter:

$$H(z) = \beta \left(\frac{a_0}{\alpha}\right)^{-\frac{1}{\beta}} (1+z)^{\frac{1}{\beta}}. \tag{19}$$

From the above Eq. (19), the present value of the Hubble parameter (which is now obviously a constant) can be obtained as $H_0 = \beta \left(\frac{a_0}{\alpha}\right)^{-\frac{1}{\beta}}$.

Figures 1– 6 depict $1D$ marginalized distribution and $2D$ contour diagrams for the derived model and the estimated value of the parameters extracted from OHD, BAO, Pantheon compilation of SN Ia and joined datasets respectively. The parametric values (viz. H_0, α, β and the age of Universe in Gyr t) obtained from different datasets due to MCMC and Bayesian analysis are tabulated in Table 1. The graphical representation of cosmic age over redshift in exhibited in Fig. 7. Five plots are for different combinations of $H(z)$, BAO, Pantheon datasets and their combinations. The Age of universe at $z = 0$ is mentioned in Table 1.

4. Energy conditions and cosmological Parameters

4.1. Energy Conditions

The energy conditions analogously establish cosmic laws that elucidate the distribution of matter and energy throughout the cosmos, which can be derived from Einstein's gravitational equations.

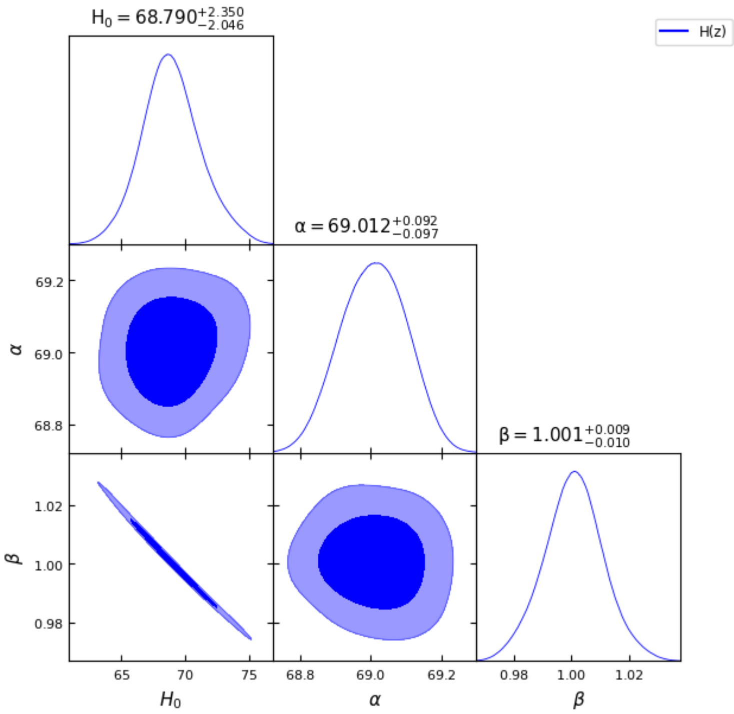

Figure 1: $1D$ marginalized distribution and $2D$ contour diagrams for the present $f(R,T)$ model parameters with the $H(z)$ dataset.

Thus, we obtain certain conditions, viz. Weak Energy Condition (WEC), Null Energy Condition (NEC), Strong Energy Condition (SEC), and Dominant Energy Condition (DEC) reveal the distribution of matter and energy in the cosmic space.

1. WEC: $\rho \geq 0$
   According to the WEC, energy cannot be negative or zero. This requirement keeps the laws of the cosmos impartial and constant. Using the parameters discovered using the Bayesian Analysis, Fig. 14 depicts the WEC's nature for our model.
2. NEC: $\rho - p \geq 0$
   The NEC deals with light. It asserts that energy is a necessary component of all space transit for light and cannot exist in a vacuum. This constraint prevents odd occurrences from occurring and maintains the physics of the cosmos. Figure 11 illustrates the characteristics of NEC for our model utilizing the Bayesian Analysis's parameters.
3. SEC: $\rho + 3p \geq 0$
   Similar to a more rigid counterpart of the NEC, the SEC guarantees that the energy cannot be negative and sets a boundary for how objects respond to gravity. The cosmos benefits from having this rule in place. Figure 13 depicts the characteristics of SEC for our model utilizing the Bayesian analysis.
4. DEC: $\rho + p \geq 0$
   The DEC guarantees that the energy is not just non-negative but also that its distribution cannot be too volatile. It is comparable to asserting that energy cannot go out of control or travel faster than light. Based on the Bayesian Analysis, DEC is shown in Fig. 12.

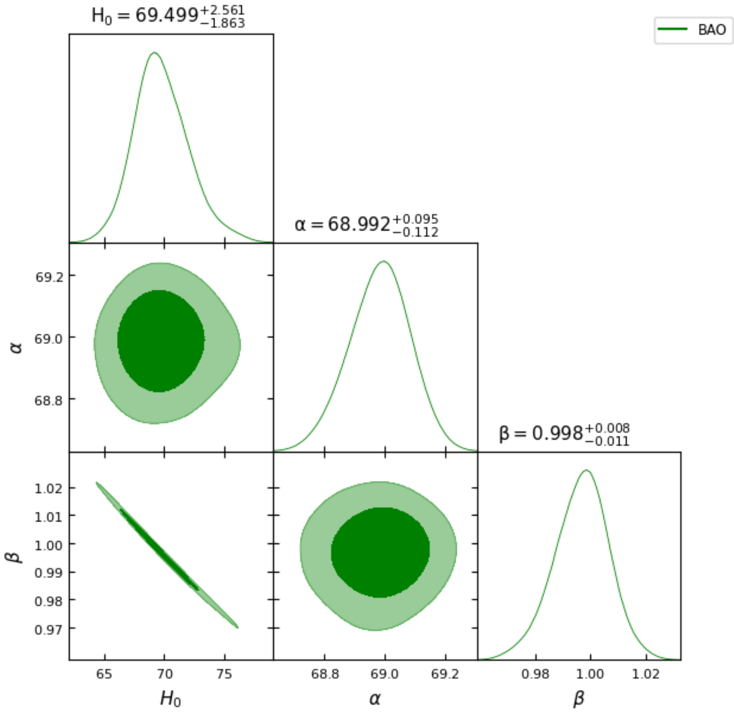

Figure 2: $1D$ marginalized distribution and $2D$ contour diagrams for the present $f(R,T)$ model parameters with the BAO dataset.

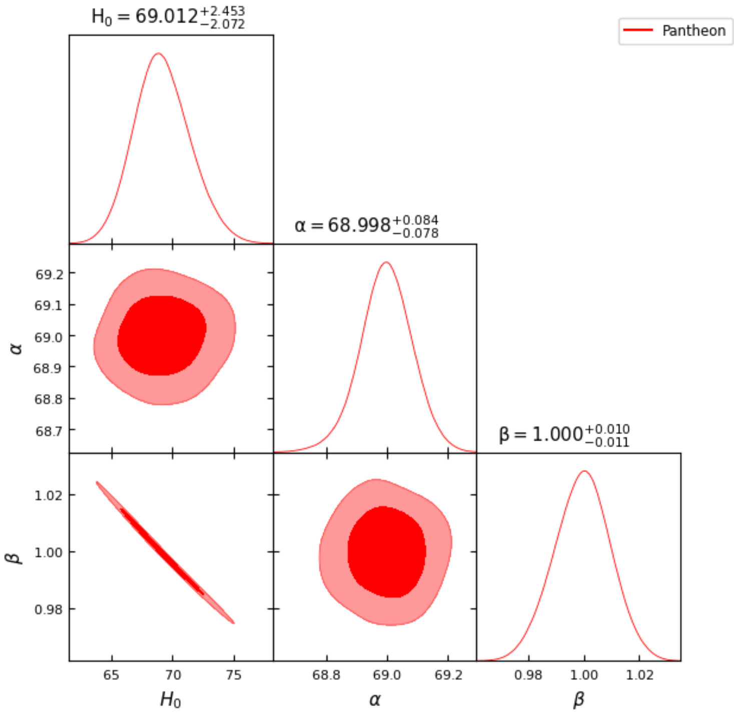

Figure 3: $1D$ marginalized distribution and $2D$ contour diagrams for the present $f(R,T)$ model parameters with the Pantheon dataset.

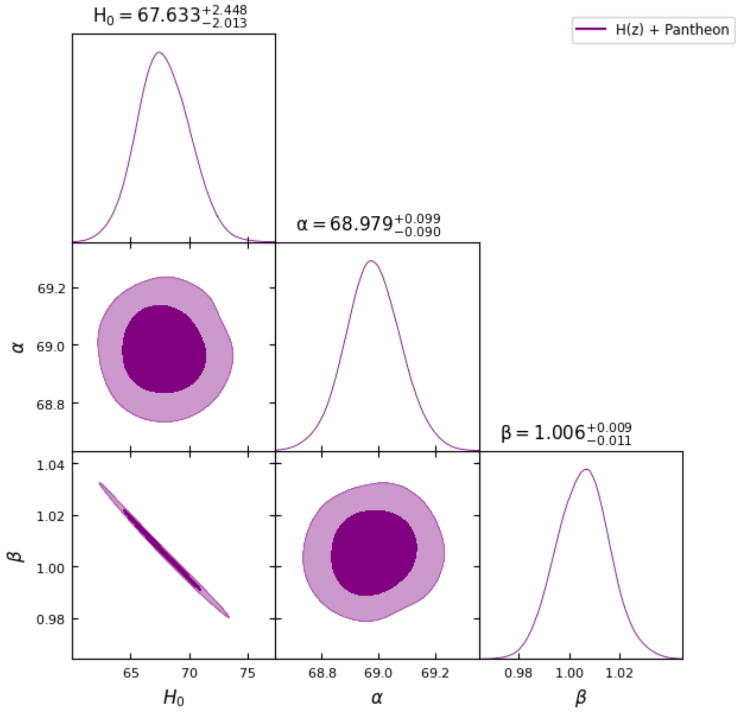

Figure 4: $1D$ marginalized distribution and $2D$ contour diagrams for the present $f(R,T)$ model parameters with the combined $H(z)$ and Pantheon dataset.

All the above-mentioned energy conditions combined are as shown in Fig. 15. Figure 8 displays the distribution of energy density ρ wrt time t, while Fig. 9 displays the distribution of pressure. Our results demonstrate that, except for SEC the others, e.g., NEC, WEC and DEC are fulfilled. The late-rapid expansion of the universe justifies the violation of SEC. The $f(R,T)$ theory of gravity, having a contribution from the trace energy T can suitably explain the late-time accelerated phase of the universe without invoking any need for the cosmological constant or exotic dark energy as the background agent for the phenomenon.

4.2. State Finder Diagnotics

The state finder diagnostics are a kind of pathological testings which can assist to find out the puzzles of dark energy and thus the history of the cosmos. These diagnostics are based on the r and s parameters and we can comprehend the evolution of the cosmos by using these factors in a better way as that provide information on the expansion of the universe and the elements that make it up. These dimensionless parameters help us understand the fundamental dynamics of the cosmos having definitions as follows:

$$r = \frac{\dddot{a}}{aH^3}, \tag{20}$$

$$s = \frac{(-1+r)}{3(-\frac{1}{2}+q)}. \tag{21}$$

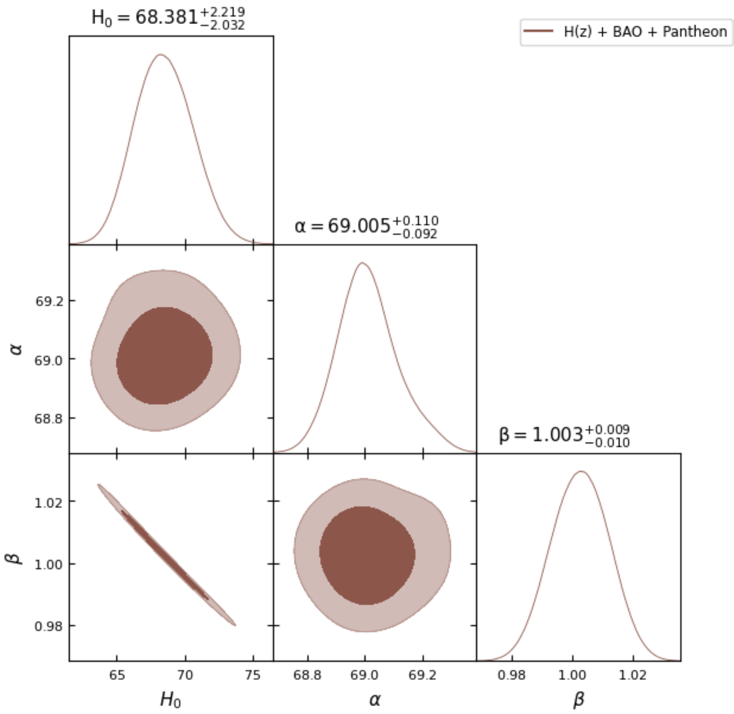

Figure 5: $1D$ marginalized distribution and $2D$ contour diagrams for the present $f(R,T)$ model parameters with the combined $H(z)$, BAO and Pantheon dataset.

The above two equations for the model under consideration take the following forms:

$$r = q(1+2q), \tag{22}$$

$$s = \frac{2}{3}(q+1). \tag{23}$$

The scale factor trajectories in the resulting model may be shown in Fig. 16 to follow a specific set of paths. Interestingly, our strategy is consistent with the results for the cosmic diagnostic pair from power law cosmology.

4.3. Om(z) parameter

The state finder parameters $r-s$ and the $Om(z)$ diagnostic are usually applied to analyse various dark energy ideas. Basically, the $Om(z)$ parameter is deployed when the Hubble parameter and the cosmic redshift merge in a suitable way. The $Om(z)$ parameter in the alternative gravity is read as

$$Om(z) = \frac{[\frac{H(z)}{H_0}]^2 - 1}{(1+z)^3 - 1}. \tag{24}$$

The Hubble parameter is shown by H_0 in this case. Shahalam et al. [38] claim that the negative, zero and positive values of $Om(z)$, respectively, represent the quintessence ($\omega \geq -1$), Λ-CDM and

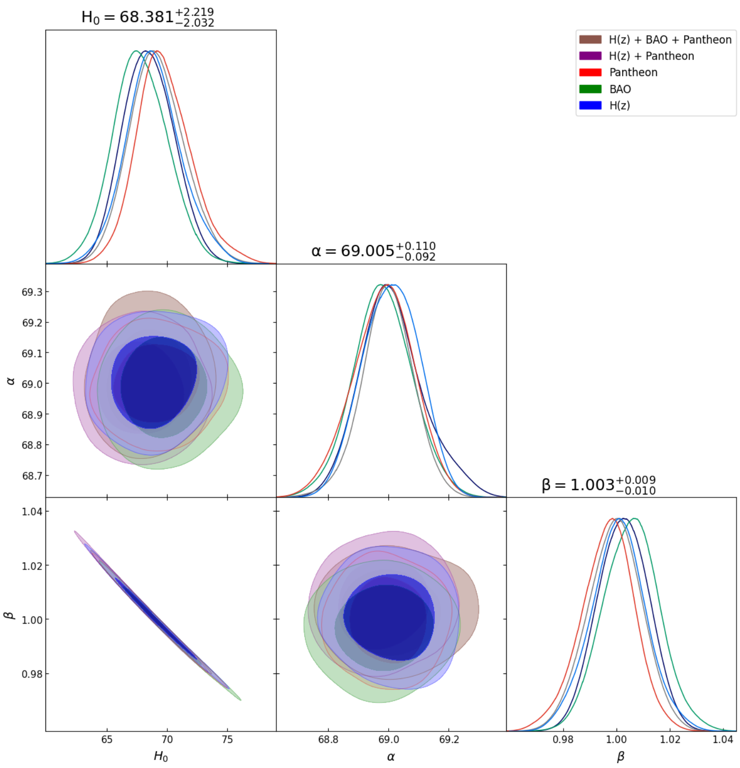

Figure 6: $1D$ marginalized distribution and $2D$ contour diagrams for the present $f(R,T)$ model parameters with the combined variability across all datasets.

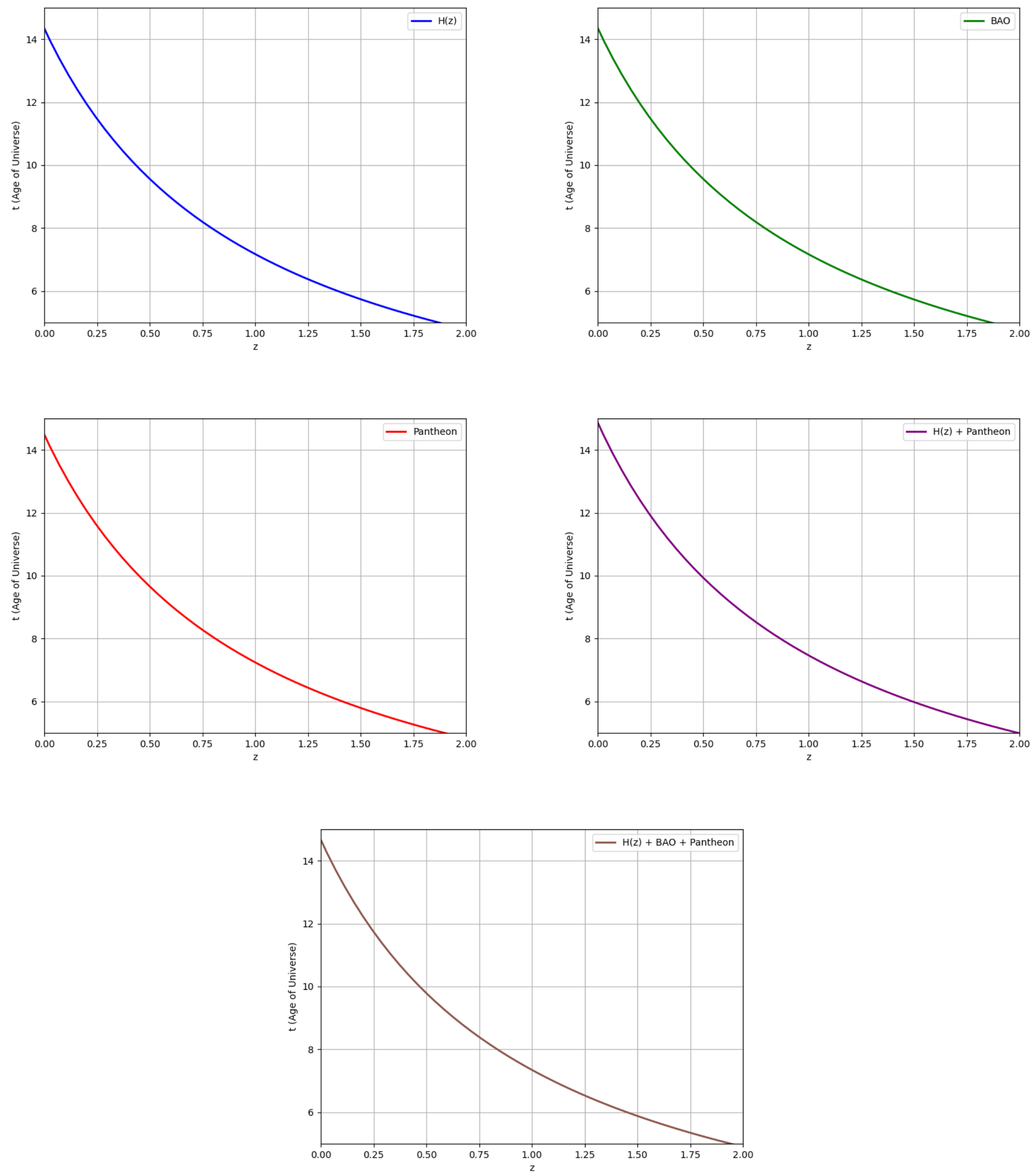

Figure 7: Visual depiction showing the age of the universe as a function of redshift. Validating cosmological models and comprehending cosmic evolution depend on it. We see five graphs here for various permutations of the datasets among $H(z)$, BAO and Pantheon.

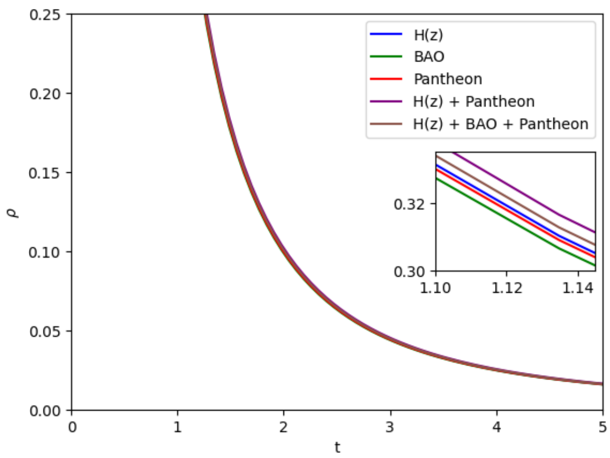

Figure 8: Illustration of the dynamic variation of the energy density over time under various parametric conditions derived from the combined H(z), BAO and Pantheon datasets.

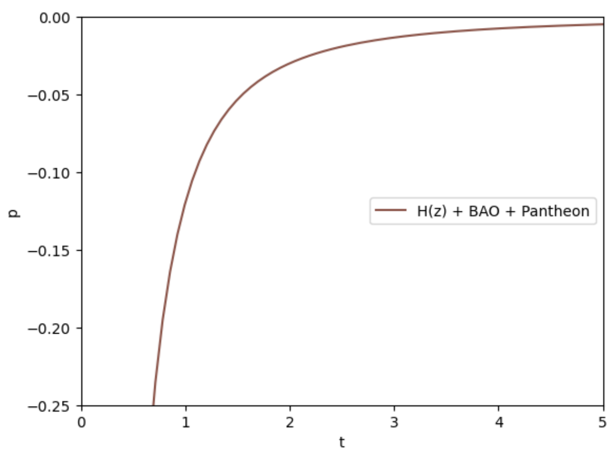

Figure 9: Using the combined H(z), BAO, and Pantheon datasets, this figure illustrates the dynamic pressure variation over time under different parameter circumstances.

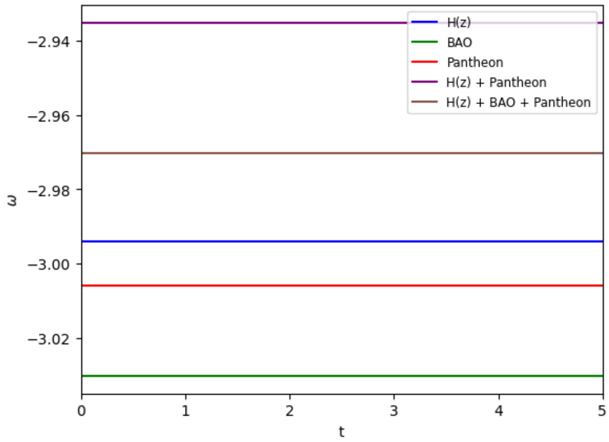

Figure 10: Graphical presentation of the equation of state parameter vs time.

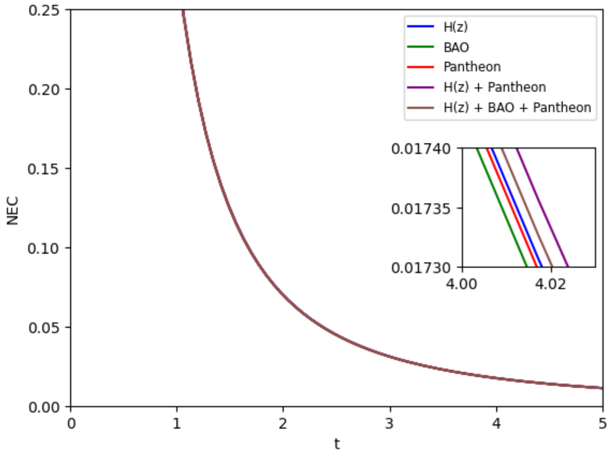

Figure 11: Graphical presentation of NEC vs time for all dataset combinations.

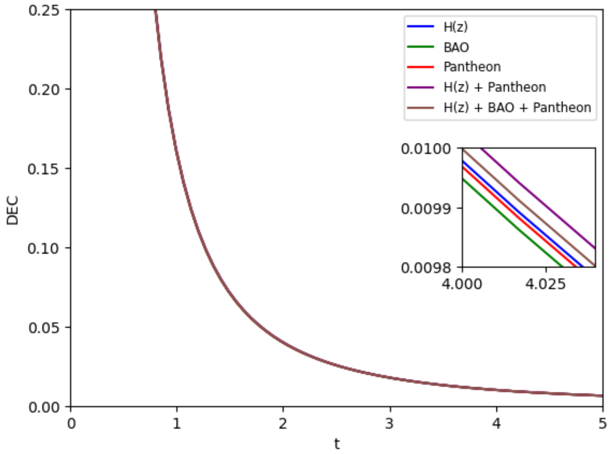

Figure 12: Graphical presentation of DEC vs time for all dataset combinations.

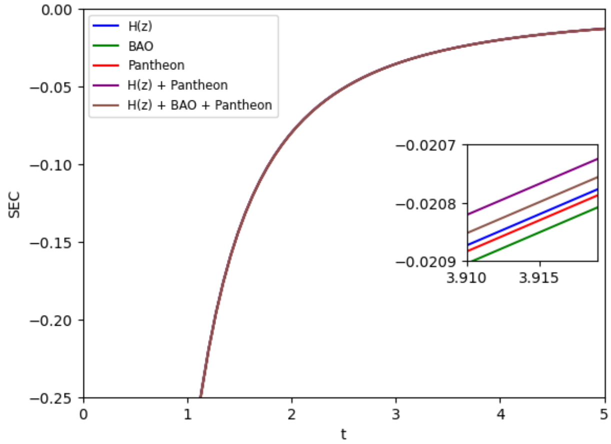

Figure 13: Graphical presentation of SEC vs time for all dataset combinations.

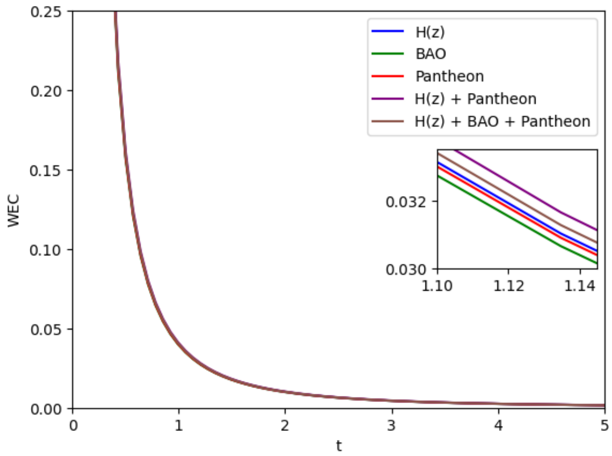

Figure 14: Graphical presentation of WEC vs time for all dataset combinations.

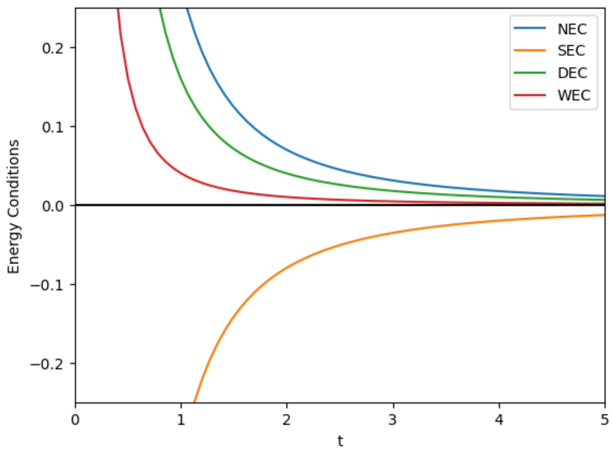

Figure 15: Graphical presentation of all the energy conditions vs time.

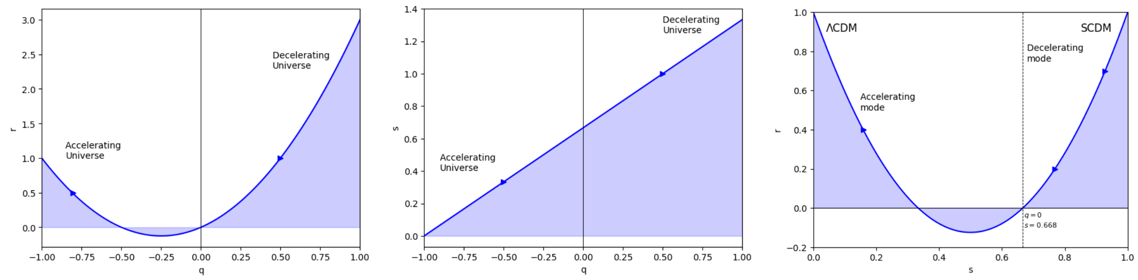

Figure 16: Graphical presentations of the state finder parameters $r-q$, $s-q$ and $r-q$.

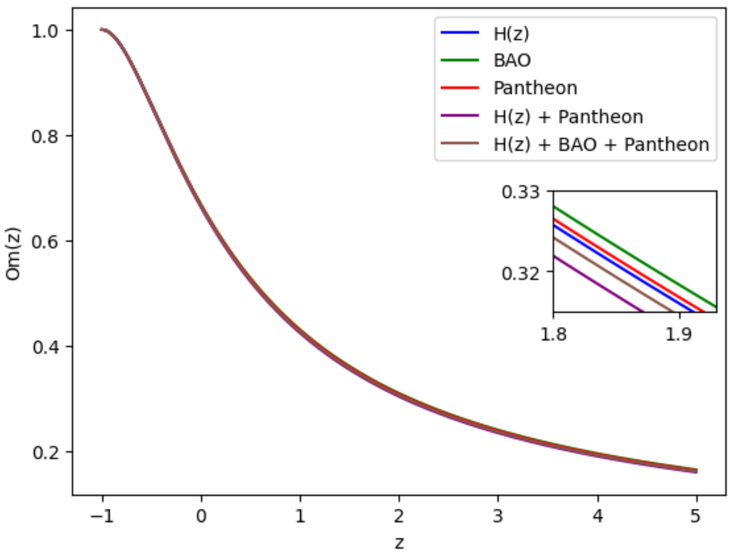

Figure 17: Graphical presentations of $Om(z)$ with z for combined dataset based on the β values obtained from each dataset.

phantom ($\omega \leq -1$) dark energy theories. However, for the present model, we obtain this parameter as follows:

$$Om(z) = \frac{(1+z)^{2/b} - 1}{(1+z)^3 - 1}. \tag{25}$$

4.4. Jerk Parameter

After the Hubble and deceleration parameters, the jerk parameter is one that can provide deeper insights into the universe's history. The universe's acceleration is tracked throughout time by the jerk parameter, indicated by the symbol j. The acceleration is accelerating and speeding up phase when j is positive ($j \geq 0$), whereas the acceleration is said to slow down if j is negative ($j \leq 0$). However, different dark energy theories forecast various jerk parameter values, and thus, scientists can learn more about dark energy by measuring j and comparing it to these predictions. The jerk parameter contributes to the improvement of our models of the cosmos when paired with other cosmic factors. It puts these models to a rigorous examination to ensure they truly reflect what we observe in the physical cosmos. In this context, we note that Fig. 18 depicts the fluctuation of j with respect to redshift z for our model and the derived parameters.

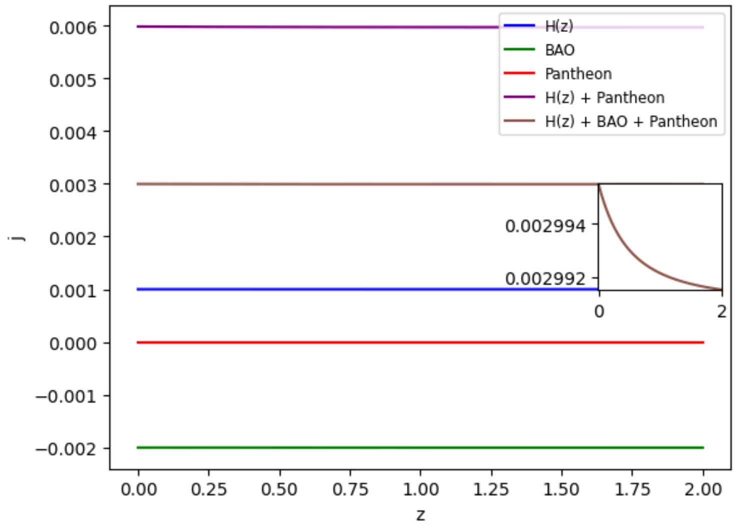

Figure 18: Graphical presentation of the jerk parameter vs redshift. Significant values of J at $z = 0$ are as follows: (i) for H(z) = 0.0009995 s^{-3}, (ii) for BAO = -0.002002 s^{-3}, (iii) for Pantheon = 0.0 s^{-3}, (iv) for $H(z)$ + Pantheon = 0.0059817 s^{-3} and (v) for $H(z)$ + BAO + Pantheon = 0.0029954 s^{-3}

.

5. Conclusion

In the present work our aim was to find the possibility of a Bianchi V type Universe under $f(R,T)$ gravity theory. For this purpose we have proposed for a viable Lagrangian model based on the connection between the trace of the energy-momentum tensor T and the Ricci scalar R. However, in order to solve the related field equations we have considered a power law for the scaling factor.

The investigation provides the following salient features:

(i) To compare the model parameters with the observational data we impose constraint on the model using the datasets from the Hubble parameter $H(z)$, BAO, Pantheon, combined $H(z)$ + Pantheon, and combined $H(z)$ + BAO + Pantheon. The outcomes for H_0 under our $f(R,T)$ gravity model are as follows: $H_0 = 68.790^{+2.250}_{-2.046}$ km s^{-1} Mpc^{-1}, $H_0 = 69.499^{+2.561}_{-1.863}$ km s^{-1} Mpc^{-1}, $H_0 = 69.102^{+2.453}_{-2.072}$ km s^{-1} Mpc^{-1}, $H_0 = 67.633^{+2.448}_{-2.013}$ km s^{-1} Mpc^{-1}, $H_0 = 68.381^{+2.219}_{-2.032}$ km s^{-1} Mpc^{-1}, respectively.

(ii) Our estimation of H_0 is remarkably consistent with various recent Planck Collaboration studies that utilize the Λ-CDM model.

(iii) We have addressed the energy conditions, $Om(z)$ analysis and cosmographical parameters which are equivocally very responsive and informative for physical viability of our model.

(iv) Graphical presentations of different model parameters are shown in several figures which are very informative in their nature. Among all these the Fig. 10, considering the relationship between the state parameter of the equation and the passage of time implies that dark energy plays a role in the accelerated expansion of the universe, while its exact nature varies over time. This finding could have intriguing cosmological implications.

Overall observation and conclusion can be put forward as follows: the obtained results demonstrate that the proposed model mostly agrees with the observational signatures of the cosmological scenario within a certain range of restrictions.

Data Availability

This research did not yield any new data..

Conflict of Interest

The authors declare no conflict of interest.

Acknowledgement

SR would like to express his gratitude to the ICARD facilities at CCASS, GLA University, Mathura, as well as the Visiting Research Associateship Programme at the Inter-University Centre for Astronomy and Astrophysics (IUCAA) in Pune, India.

References

[1] A. G. Riess *et al.* [Supernova Search Team], Observational evidence from supernovae for an accelerating universe and a cosmological constant, Astron. J. **116**, 1009-1038 (1998)

[2] A. G. Riess *et al.* [Supernova Search Team], Type Ia supernova discoveries at z > 1 from the Hubble Space Telescope: Evidence for past deceleration and constraints on dark energy evolution, Astrophys. J. **607**, 665-687 (2004)

[3] M. Tegmark *et al.* [SDSS], Cosmological parameters from SDSS and WMAP, Phys. Rev. D **69**, 103501 (2004)

[4] E. Calabrese, M. Migliaccio, L. Pagano, G. De Troia, A. Melchiorri and P. Natoli, Cosmological constraints on the matter equation of state, Phys. Rev. D **80**, 063539 (2009)

[5] Y. Wang and M. Dai, Exploring uncertainties in dark energy constraints using current observational data with Planck 2015 distance priors, Phys. Rev. D **94**, no.8, 083521 (2016)

[6] M. M. Zhao, D. Z. He, J. F. Zhang and X. Zhang, Search for sterile neutrinos in holographic dark energy cosmology: Reconciling Planck observation with the local measurement of the Hubble constant, Phys. Rev. D **96**, no.4, 043520 (2017)

[7] B. Wang, E. Abdalla, F. Atrio-Barandela and D. Pavon, Dark Matter and Dark Energy Interactions: Theoretical Challenges, Cosmological Implications and Observational Signatures, Rept. Prog. Phys. **79**, 096901 (2016)

[8] P. J. E. Peebles and B. Ratra, The Cosmological Constant and Dark Energy, Rev. Mod. Phys. **75**, 559-606 (2003)

[9] T. Padmanabhan, Cosmological constant: The Weight of the vacuum, Phys. Rept. **380**, 235-320 (2003)

[10] E. Abdalla and A. Marins, The Dark Sector Cosmology, Int. J. Mod. Phys. D **29**, 2030014 (2020)

[11] M. Li, X. D. Li, S. Wang and Y. Wang, Dark Energy: A Brief Review, Front. Phys. (Beijing) **8**, 828-846 (2013)

[12] V. Sahni, The Cosmological constant problem and quintessence, Class. Quant. Grav. **19**, 3435-3448 (2002)

[13] J. Garriga and A. Vilenkin, Solutions to the cosmological constant problems, Phys. Rev. D **64**, 023517 (2001)

[14] J. Frieman, M. Turner and D. Huterer, Dark Energy and the Accelerating Universe, Ann. Rev. Astron. Astrophys. **46**, 385-432 (2008)

[15] S. Nojiri and S. D. Odintsov, Unified cosmic history in modified gravity: from $F(R)$ theory to Lorentz non-invariant models, Phys. Rept. **505**, 59-144 (2011)

[16] L. Samushia and B. Ratra, Constraining dark energy with gamma-ray bursts, Astrophys. J. **714**, 1347-1354 (2010)

[17] M. Yashar, B. Bozek, A. Abrahamse, A. Albrecht and M. Barnard, Exploring Parameter Constraints on Quintessential Dark Energy: the Inverse Power Law Model, Phys. Rev. D **79**, 103004 (2009)

[18] Harko, Tiberiu, Francisco SN Lobo, Shin'ichi Nojiri, and Sergei D. Odintsov, $f(R,T)$ gravity, Phys. Rev. D **84**, 024020 (2011)

[19] T. Harko, Thermodynamic interpretation of the generalized gravity models with geometry - matter coupling, Phys. Rev. D **90**, no.4, 044067 (2014)

[20] L. K. Sharma, A. K. Yadav, P. K. Sahoo and B. K. Singh, Non-minimal matter-geometry coupling in Bianchi I space-time, Res. Phys. **10**, 738-742 (2018)

[21] A. K. Yadav, L. K. Sharma, B. K. Singh and P. K. Sahoo, Existence of bulk viscous universe in $f(R,T)$ gravity and confrontation with observational data, New Astron. **78**, 101382 (2020)

[22] L. K. Sharma, A. K. Yadav and B. K. Singh, Power-law solution for homogeneous and isotropic universe in $f(R,T)$ gravity, New Astron. **79**, 101396 (2020)

[23] L. K. Sharma, B. K. Singh and A. K. Yadav, Viability of Bianchi type V universe in $f(R,T) = f_1(R) + f_2(R)f_3(T)$ gravity, Int. J. Geom. Meth. Mod. Phys. **17**, 2050111 (2020)

[24] S. Nojiri and S. D. Odintsov, Unified cosmic history in modified gravity: from $F(R)$ theory to Lorentz non-invariant models, Phys. Rept. **505**, 59-144 (2011)

[25] S. M. Carroll, V. Duvvuri, M. Trodden and M. S. Turner, Is cosmic speed-up due to new gravitational physics? Phys. Rev. D **70**, 043528 (2004)

[26] W. Hu and I. Sawicki, Models of $f(R)$ Cosmic Acceleration that Evade Solar-System Tests, Phys. Rev. D **76**, 064004 (2007)

[27] F. G. Alvarenga, A. de la Cruz-Dombriz, M. J. S. Houndjo, M. E. Rodrigues and D. Sáez-Gómez, Dynamics of scalar perturbations in $f(R,T)$ gravity, Phys. Rev. D **87**, 103526 (2013)

[28] P. K. Sahoo, P. Sahoo and B. K. Bishi, Anisotropic cosmological models in $f(R,T)$ gravity with variable deceleration parameter, Int. J. Geom. Meth. Mod. Phys. **14**, 1750097 (2017)

[29] Z. Yousaf, K. Bamba and M. Z. u. H. Bhatti, Causes of Irregular Energy Density in $f(R,T)$ Gravity, Phys. Rev. D **93**, 124048 (2016)

[30] Z. Yousaf, M. Z. u. H. Bhatti and M. Ilyas, Existence of compact structures in $f(R,T)$ gravity, Eur. Phys. J. C **78**, 307 (2018)

[31] Z. Yousaf, K. Bamba and M. Z. u. H. Bhatti, Influence of Modification of Gravity on the Dynamics of Radiating Spherical Fluids, Phys. Rev. D **93**, 064059 (2016)

[32] Z. Yousaf, K. Bamba, M. Z. Bhatti and U. Ghafoor, Charged Gravastars in Modified Gravity, Phys. Rev. D **100**, 024062 (2019)

[33] A. Das, F. Rahaman, B. K. Guha and S. Ray, Compact stars in $f(R,\mathcal{T})$ gravity, Eur. Phys. J. C **76**, 654 (2016)

[34] A. K. Yadav, P. K. Sahoo and V. Bhardwaj, Bulk viscous Bianchi-I embedded cosmological model in $f(R,T) = f1(R) + f2(R)f3(T)$ gravity, Mod. Phys. Lett. A **34**, 1950145 (2019)

[35] S. V. Lohakare, B. Mishra, S. K. Maurya, Ksh. N. Singh, Analyzing the Geometrical and Dynamical Parameters of Modified Teleparallel- Gauss-Bonnet Model, arXiv: 2209.13197.

[36] S. Alam et al. (eBOSS), Completed SDSS-IV extended Baryon Oscillation Spectroscopic Survey: Cosmological implications from two decades of spectroscopic surveys at the Apache Point Observatory, Phys. Rev. D 103, 083533 (2021).

[37] D. M. Scolnic et al., The Complete Light-curve Sample of Spectroscopically Confirmed SNe Ia from Pan-STARRS1 and Cosmological Constraints from the Combined Pantheon Sample, Astrophys. J. 859, 101 (2018).

[38] M. Shahalam, S. Sami and A. Agarwa, Mon. Not. R. Astron. Soc. **448**, 2948 (2015).