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* Newtons optimization *

Let $f(x,y)$ be a convex function. ~~need to~~ be

Task: Find the minimum of $f(x,y)$. ie find (x,y) such that

$f'(x,y) = 0$.

$$\theta = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \quad \text{initial Values}$$

RULE:

$$\boxed{\theta := \theta - (Hf)^{-1} \nabla f}$$

Example: $f(x,y) = x^2 + y^2$ $\quad \theta = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$

$$\nabla f ; \Delta f(10,10) = \begin{bmatrix} 2x \\ 2y \end{bmatrix} = \begin{bmatrix} 20 \\ 20 \end{bmatrix}$$

$$Hf ; Hf(10,10) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\theta := \theta - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 20 \\ 20 \end{bmatrix}$$

Python tutorial for newtons method:

$$X = \begin{bmatrix} 1 & x_0^{(1)} & x_1^{(1)} & \cdots & x_n^{(1)} \\ \vdots & & & & \\ 1 & x_0^{(m)} & x_1^{(m)} & \cdots & x_n^{(m)} \end{bmatrix}_{m \times (n+1)} \qquad y = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(m)} \end{bmatrix}_{(m \times 1)}$$

$$\theta = \begin{bmatrix} \theta_0 \\ \vdots \\ \theta_n \end{bmatrix}_{(n+1) \times 1} \qquad \nabla f = \frac{1}{m} \begin{bmatrix} \sum_{j=1}^{m} (h_\theta(x^{(j)}) - y^{(j)}) x_0^{(j)} \\ \vdots \\ \sum_{j=1}^{m} (h_\theta(x^{(j)}) - y^{(j)}) x_n^{(j)} \end{bmatrix}_{(n+1) \times 1}$$

$$Hf = \begin{bmatrix} \sum_{j=1}^{m} x_0^{(j)} x_0^{(j)} & \sum_{j=1}^{m} x_1^{(j)} x_0^{(j)} & \cdots \\ & \sum_{j=1}^{m} x_1^{(j)} x_1^{(j)} & \\ & & \ddots \\ & & & \sum_{j=1}^{m} x_n^{(j)} x_n^{(j)} \end{bmatrix}_{(n+1) \times (n+1)}$$

$$\nabla f = \left[(X \cdot \theta - y)^T \cdot X \right]^T$$

$$Hf = \underbrace{X^T \cdot X}_{((n+1) \times m) \times (m \times (n+1)) = (n+1) \times (n+1)}$$

update rule:

$$\theta := \theta - (Hf)^{-1} \nabla f$$

$$\boxed{\underbrace{\theta}_{(n+1) \times 1} := \theta - \underbrace{(X^T \cdot X)^{-1}}_{(n+1) \times (n+1)} \underbrace{\left[(X \cdot \theta - y)^T \cdot X \right]^T}_{(n+1) \times 1}}$$