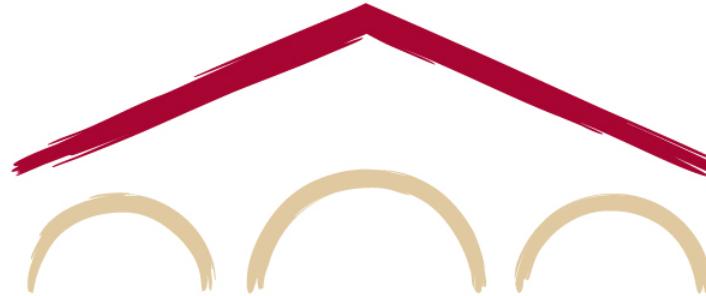


Natural Language Processing with Deep Learning

CS224N/Ling284



Christopher Manning

Lecture 5: Language Models and Recurrent Neural Networks
(oh, and finish neural dependency parsing ☺)

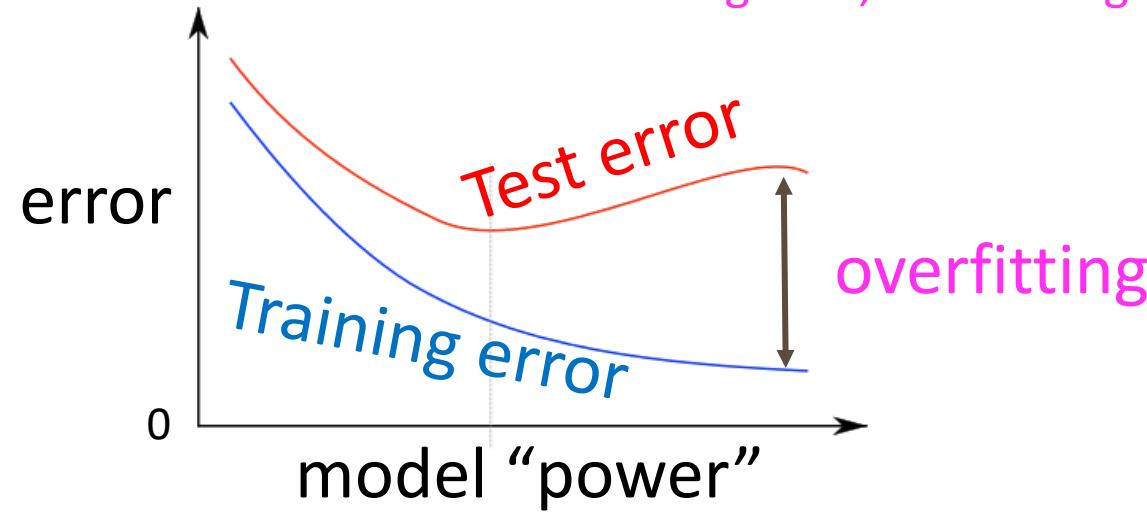
2. A bit more about neural networks

We have models with many parameters! Regularization!

- A full loss function includes **regularization** over all parameters θ , e.g., L2 regularization:

$$J(\theta) = \frac{1}{N} \sum_{i=1}^N -\log \left(\frac{e^{f_{y_i}}}{\sum_{c=1}^C e^{f_c}} \right) + \lambda \sum_k \theta_k^2$$

- Classic view: Regularization works to prevent **overfitting** when we have a lot of features (or later a very powerful/deep model, etc.)
- Now: Regularization **produces** models that generalize well when we have a “big” model
 - We do not care that our models overfit on the training data, even though they are **hugely** overfit



Dropout (Srivastava, Hinton, Krizhevsky, Sutskever, & Salakhutdinov 2012/JMLR 2014)

Preventing Feature Co-adaptation = Good Regularization Method!

- Training time: at each instance of evaluation (in online SGD-training), randomly set 50% of the inputs to each neuron to 0
- Test time: halve the model weights (now twice as many)
- (Except usually only drop first layer inputs a little (~15%) or not at all)
- This prevents feature co-adaptation: A feature cannot only be useful in the presence of particular other features
- In a single layer: A kind of middle-ground between Naïve Bayes (where all feature weights are set independently) and logistic regression models (where weights are set in the context of all others)
- Can be thought of as a form of model bagging (i.e., like an ensemble model)
- Nowadays usually thought of as strong, feature-dependent regularizer
[Wager, Wang, & Liang 2013]

“Vectorization”

- E.g., looping over word vectors versus concatenating them all into one large matrix and then multiplying the softmax weights with that matrix:

```
from numpy import random
N = 500 # number of windows to classify
d = 300 # dimensionality of each window
C = 5 # number of classes
W = random.rand(C,d)
wordvectors_list = [random.rand(d,1) for i in range(N)]
wordvectors_one_matrix = random.rand(d,N)

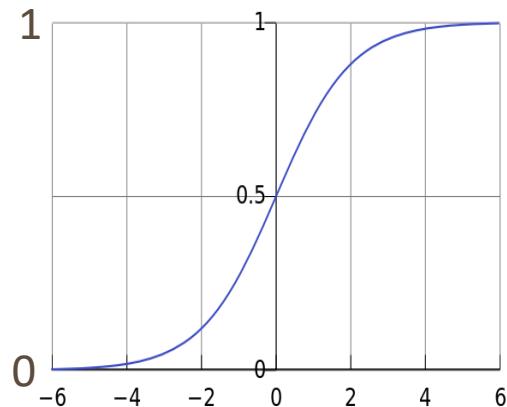
%timeit [W.dot(wordvectors_list[i]) for i in range(N)]
%timeit W.dot(wordvectors_one_matrix)
```

- 1000 loops, best of 3: **639 µs** per loop
- 10000 loops, best of 3: **53.8 µs** per loop ← Now using a single a C x N matrix
- Matrices are awesome!!! Always try to use vectors and matrices rather than for loops!
- The speed gain goes from 1 to 2 orders of magnitude with GPUs!

Non-linearities, old and new

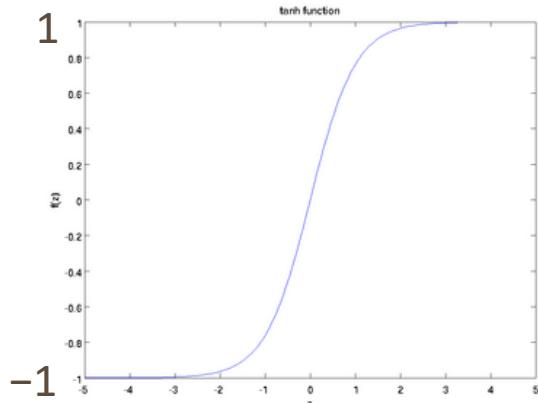
logistic (“sigmoid”)

$$f(z) = \frac{1}{1 + \exp(-z)}.$$



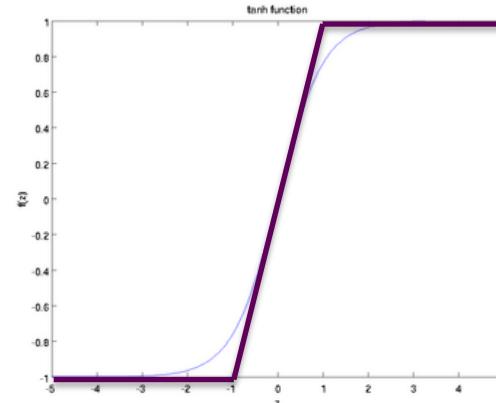
tanh

$$f(z) = \tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}},$$



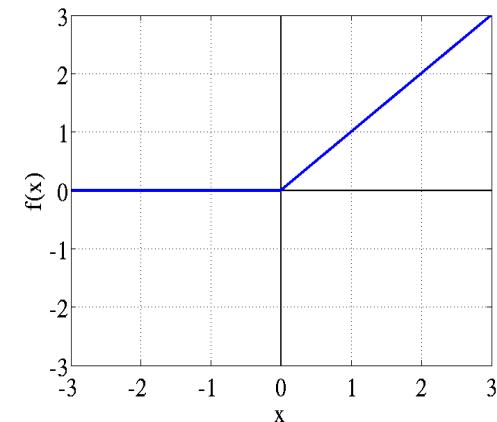
hard tanh

$$\text{HardTanh}(x) = \begin{cases} -1 & \text{if } x < -1 \\ x & \text{if } -1 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$$



ReLU (Rectified Linear Unit)

$$\text{rect}(z) = \max(z, 0)$$

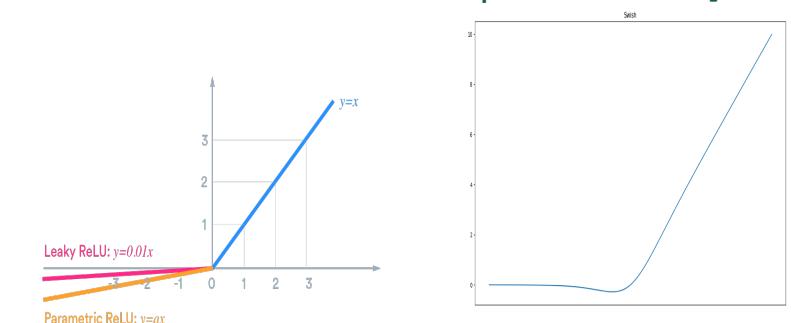


tanh is just a rescaled and shifted sigmoid ($2 \times$ as steep, $[-1,1]$):
 $\tanh(z) = 2\text{logistic}(2z) - 1$

Both logistic and tanh are still used in various places (e.g., to get a probability), but are no longer the defaults for making deep networks

For building a deep network, the first thing you should try is ReLU — it trains quickly and performs well due to good gradient backflow

Leaky ReLU /
Parametric ReLU Swish [Ramachandran,
Zoph & Le 2017]



Parameter Initialization

- You normally must initialize weights to small random values (i.e., not zero matrices!)
 - To avoid symmetries that prevent learning/specialization
- Initialize hidden layer biases to 0 and output (or reconstruction) biases to optimal value if weights were 0 (e.g., mean target or inverse sigmoid of mean target)
- Initialize **all other weights** $\sim \text{Uniform}(-r, r)$, with r chosen so numbers get neither too big or too small [later the need for this is removed with use of layer normalization]
- Xavier initialization has variance inversely proportional to fan-in n_{in} (previous layer size) and fan-out n_{out} (next layer size):

$$\text{Var}(W_i) = \frac{2}{n_{\text{in}} + n_{\text{out}}}$$

Optimizers

- Usually, plain SGD will work just fine!
 - However, getting good results will often require hand-tuning the learning rate
 - See next slide
- For more complex nets and situations, or just to avoid worry, you often do better with one of a family of more sophisticated “adaptive” optimizers that scale the parameter adjustment by an accumulated gradient.
 - These models give differential per-parameter learning rates
 - Adagrad
 - RMSprop
 - Adam ← A fairly good, safe place to begin in many cases
 - SparseAdam
 - ...

Learning Rates

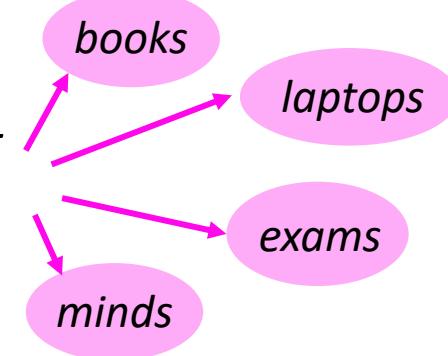
- You can just use a constant learning rate. Start around $lr = 0.001$?
 - It must be order of magnitude right – try powers of 10
 - Too big: model may diverge or not converge
 - Too small: your model may not have trained by the assignment deadline
- Better results can generally be obtained by allowing learning rates to decrease as you train
 - By hand: halve the learning rate every k epochs
 - An epoch = a pass through the data (**shuffled** or sampled – not in same order each time)
 - By a formula: $lr = lr_0 e^{-kt}$, for epoch t
 - There are fancier methods like cyclic learning rates (q.v.)
- Fancier optimizers still use a learning rate but it may be an initial rate that the optimizer shrinks – so you may want to start with a higher learning rate

3. Language Modeling + RNNs

Language Modeling

- **Language Modeling** is the task of predicting what word comes next

the students opened their _____



- More formally: given a sequence of words $x^{(1)}, x^{(2)}, \dots, x^{(t)}$, compute the probability distribution of the next word $x^{(t+1)}$:

$$P(x^{(t+1)} | x^{(t)}, \dots, x^{(1)})$$

where $x^{(t+1)}$ can be any word in the vocabulary $V = \{w_1, \dots, w_{|V|}\}$

- A system that does this is called a **Language Model**

Language Modeling

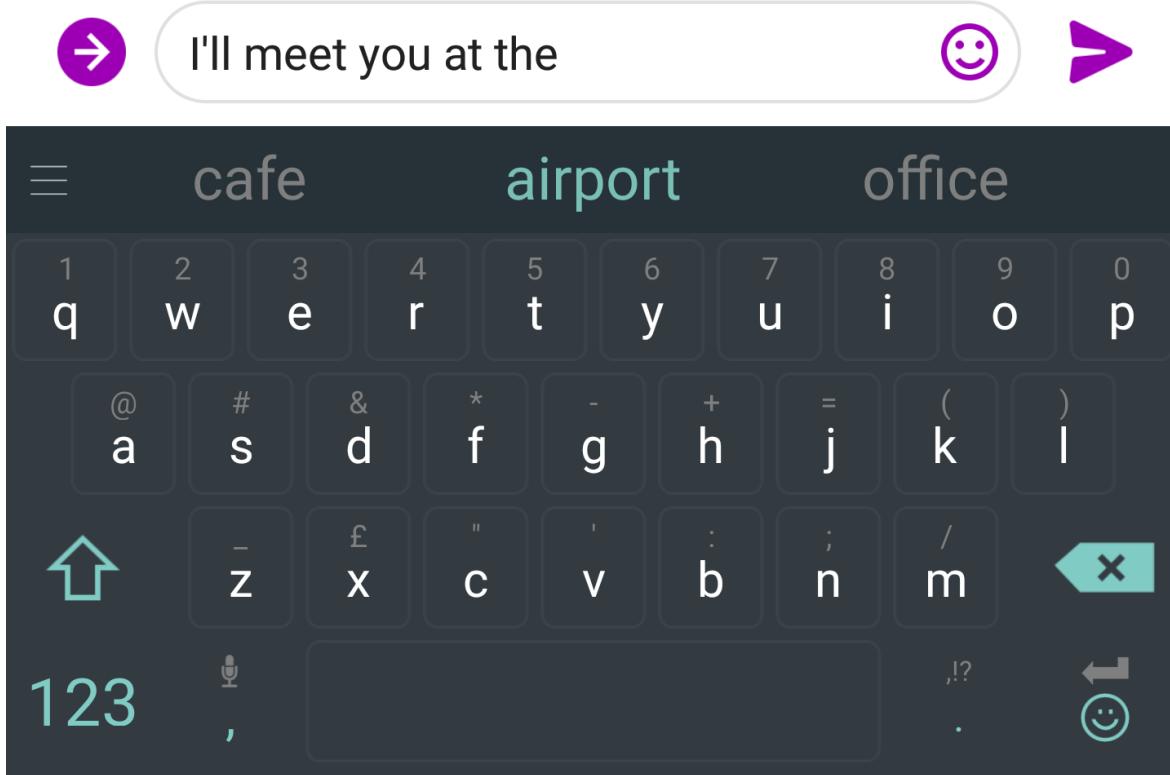
- You can also think of a Language Model as a system that assigns probability to a piece of text
- For example, if we have some text $\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(T)}$, then the probability of this text (according to the Language Model) is:

$$\begin{aligned} P(\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(T)}) &= P(\mathbf{x}^{(1)}) \times P(\mathbf{x}^{(2)} | \mathbf{x}^{(1)}) \times \cdots \times P(\mathbf{x}^{(T)} | \mathbf{x}^{(T-1)}, \dots, \mathbf{x}^{(1)}) \\ &= \prod_{t=1}^T P(\mathbf{x}^{(t)} | \mathbf{x}^{(t-1)}, \dots, \mathbf{x}^{(1)}) \end{aligned}$$



This is what our LM provides

You use Language Models every day!



You use Language Models every day!



A screenshot of a Google search interface. At the top, there is a search bar containing the partial query "what is the |". To the right of the search bar is a microphone icon. Below the search bar, a dropdown menu displays a list of suggested search queries, each preceded by a small blue speech bubble icon. The suggestions are:

- what is the **weather**
- what is the **meaning of life**
- what is the **dark web**
- what is the **xfl**
- what is the **doomsday clock**
- what is the **weather today**
- what is the **keto diet**
- what is the **american dream**
- what is the **speed of light**
- what is the **bill of rights**

At the bottom of the interface are two buttons: "Google Search" and "I'm Feeling Lucky".

n-gram Language Models

the students opened their _____

- **Question:** How to learn a Language Model?
- **Answer** (pre- Deep Learning): learn an *n-gram Language Model!*
- **Definition:** A *n-gram* is a chunk of n consecutive words.
 - **unigrams:** “the”, “students”, “opened”, “their”
 - **bigrams:** “the students”, “students opened”, “opened their”
 - **trigrams:** “the students opened”, “students opened their”
 - **4-grams:** “the students opened their”
- **Idea:** Collect statistics about how frequent different n-grams are and use these to predict next word.

n-gram Language Models

- First we make a **Markov assumption**: $x^{(t+1)}$ depends only on the preceding $n-1$ words

$$P(x^{(t+1)} | x^{(t)}, \dots, x^{(1)}) = P(x^{(t+1)} | \underbrace{x^{(t)}, \dots, x^{(t-n+2)}}_{n-1 \text{ words}}) \quad (\text{assumption})$$

$$\begin{aligned} \text{prob of a n-gram} &\rightarrow P(x^{(t+1)}, x^{(t)}, \dots, x^{(t-n+2)}) \\ \text{prob of a (n-1)-gram} &\rightarrow P(x^{(t)}, \dots, x^{(t-n+2)}) \end{aligned} \quad (\text{definition of conditional prob})$$

- Question:** How do we get these n -gram and $(n-1)$ -gram probabilities?
- Answer:** By **counting** them in some large corpus of text!

$$\approx \frac{\text{count}(x^{(t+1)}, x^{(t)}, \dots, x^{(t-n+2)})}{\text{count}(x^{(t)}, \dots, x^{(t-n+2)})} \quad (\text{statistical approximation})$$

n-gram Language Models: Example

Suppose we are learning a 4-gram Language Model.

~~as the proctor started the clock, the~~ students opened their _____

discard condition on this

$$P(\mathbf{w}|\text{students opened their}) = \frac{\text{count(students opened their } \mathbf{w})}{\text{count(students opened their)}}$$

For example, suppose that in the corpus:

- “students opened their” occurred 1000 times
- “students opened their books” occurred 400 times
 - $\rightarrow P(\text{books} | \text{students opened their}) = 0.4$
- “students opened their exams” occurred 100 times
 - $\rightarrow P(\text{exams} | \text{students opened their}) = 0.1$

Should we have discarded
the “proctor” context?

Sparsity Problems with n-gram Language Models

Sparsity Problem 1

Problem: What if “*students opened their w*” never occurred in data? Then w has probability 0!

(Partial) Solution: Add small δ to the count for every $w \in V$. This is called *smoothing*.

$$P(w|\text{students opened their}) = \frac{\text{count(students opened their } w\text{)}}{\text{count(students opened their)}}$$

Sparsity Problem 2

Problem: What if “*students opened their*” never occurred in data? Then we can’t calculate probability for *any w*!

(Partial) Solution: Just condition on “*opened their*” instead. This is called *backoff*.

Note: Increasing n makes sparsity problems worse. Typically, we can’t have n bigger than 5.

Storage Problems with n-gram Language Models

Storage: Need to store count for all n -grams you saw in the corpus.

$$P(\mathbf{w}|\text{students opened their}) = \frac{\text{count(students opened their } \mathbf{w})}{\text{count(students opened their)}}$$

Increasing n or increasing corpus increases model size!

n-gram Language Models in practice

- You can build a simple trigram Language Model over a 1.7 million word corpus (Reuters) in a few seconds on your laptop*

today the _____

Business and financial news
get probability distribution

company	0.153
bank	0.153
price	0.077
italian	0.039
emirate	0.039
...	

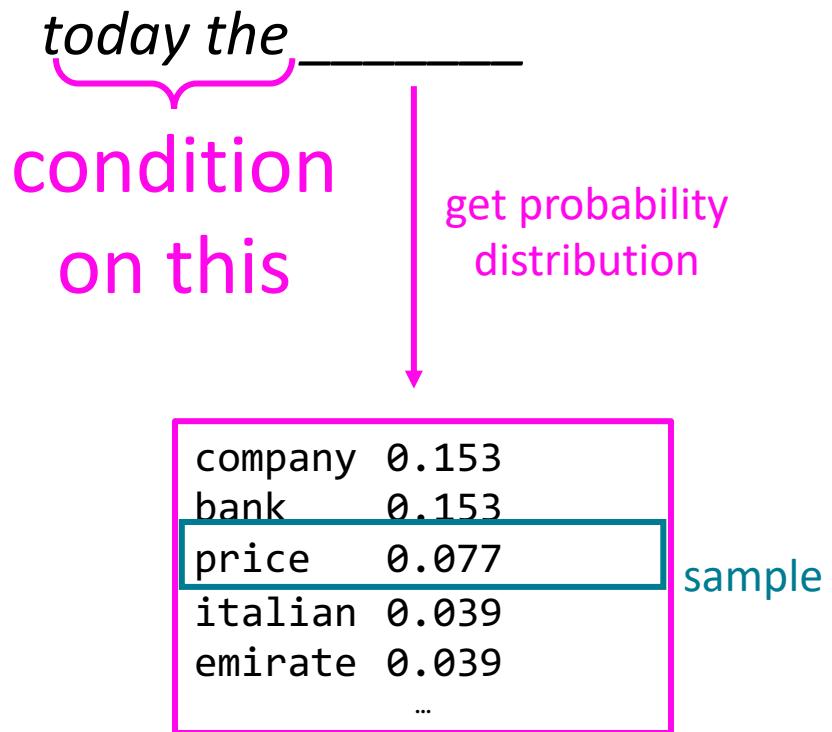
Sparsity problem:
not much granularity
in the probability
distribution

Otherwise, seems reasonable!

* Try for yourself: <https://nlpforhackers.io/language-models/>

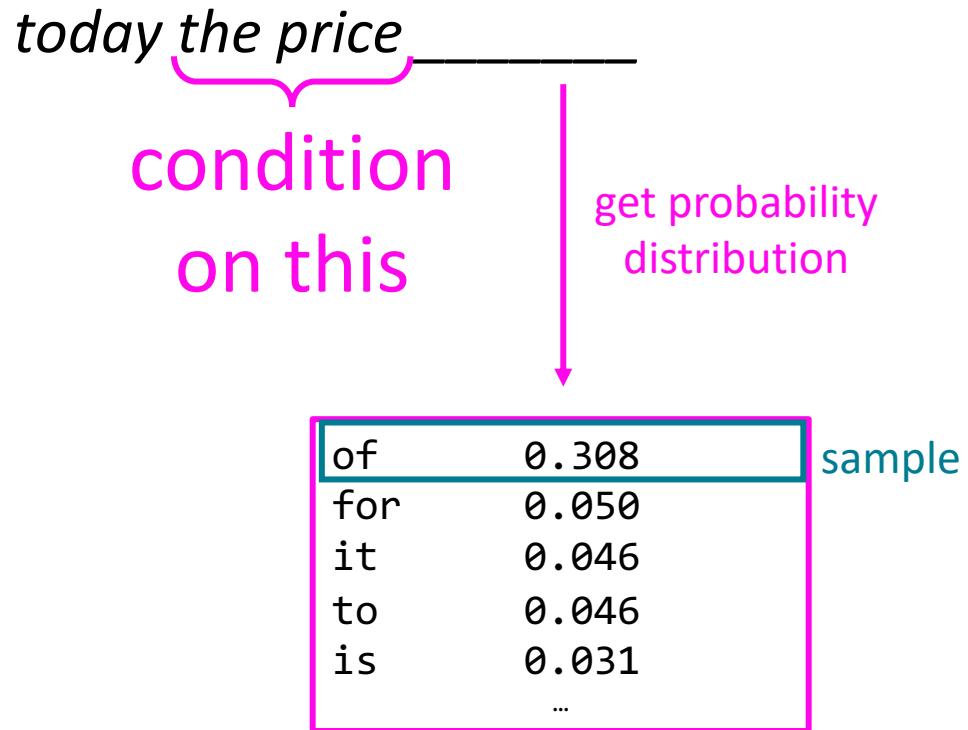
Generating text with a n-gram Language Model

You can also use a Language Model to generate text



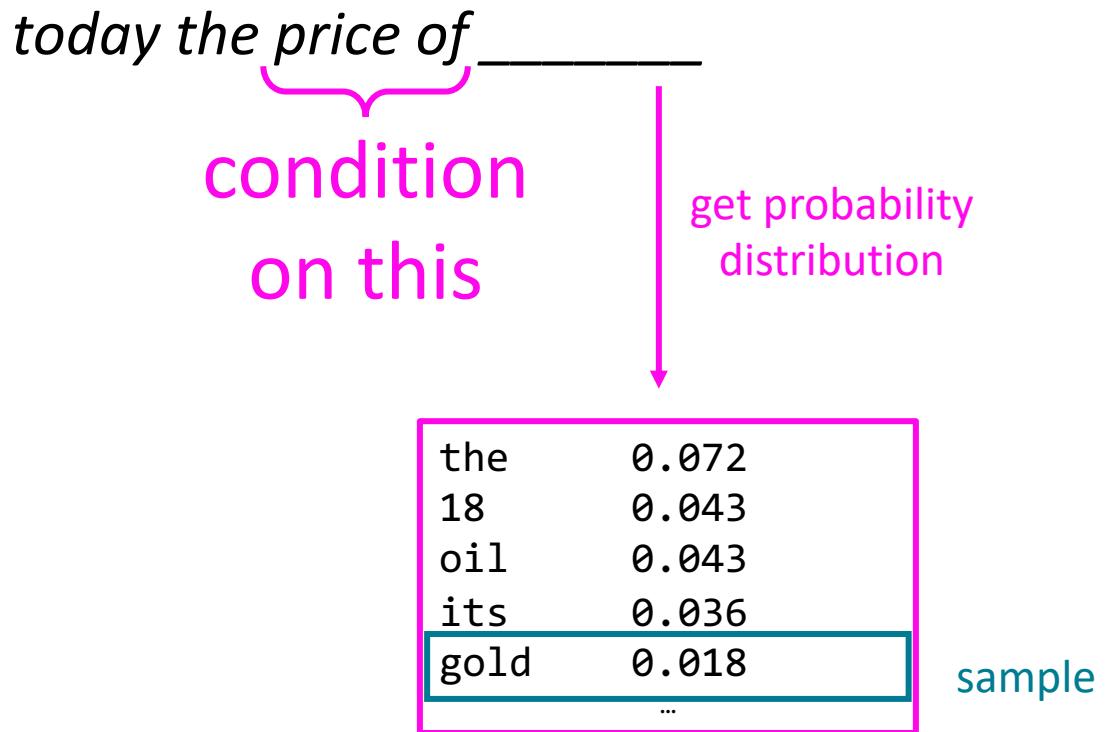
Generating text with a n-gram Language Model

You can also use a Language Model to generate text



Generating text with a n-gram Language Model

You can also use a Language Model to generate text



Generating text with a n-gram Language Model

You can also use a Language Model to generate text

*today the price of gold per ton , while production of shoe
lasts and shoe industry , the bank intervened just after it
considered and rejected an imf demand to rebuild depleted
european stocks , sept 30 end primary 76 cts a share .*

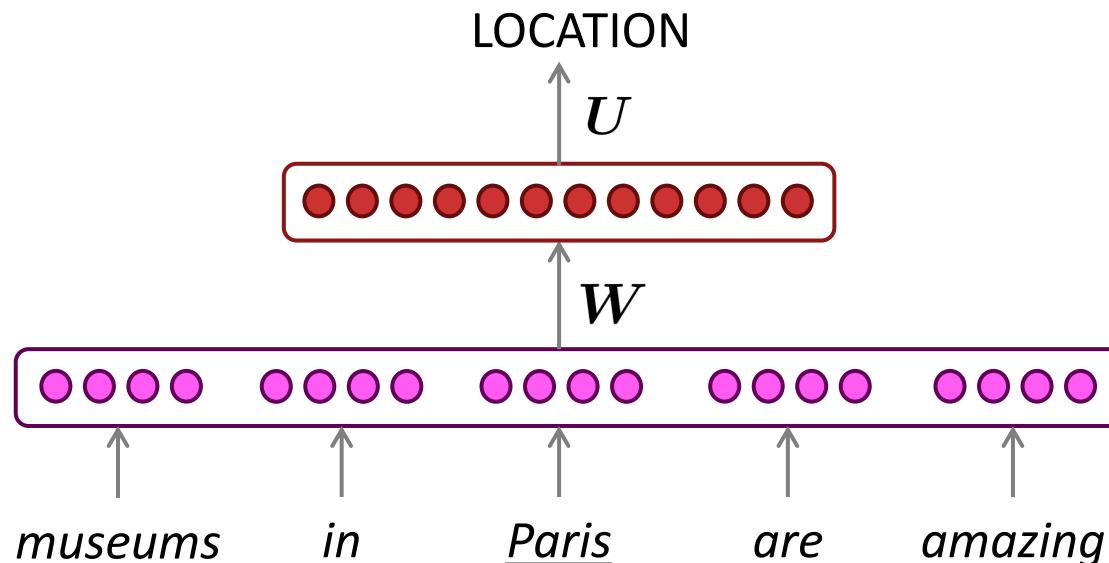
Surprisingly grammatical!

...but **incoherent**. We need to consider more than
three words at a time if we want to model language well.

But increasing n worsens sparsity problem,
and increases model size...

How to build a *neural* Language Model?

- Recall the Language Modeling task:
 - Input: sequence of words $x^{(1)}, x^{(2)}, \dots, x^{(t)}$
 - Output: prob dist of the next word $P(x^{(t+1)} | x^{(t)}, \dots, x^{(1)})$
- How about a *window-based neural model*?
 - We saw this applied to Named Entity Recognition in Lecture 3:



A fixed-window neural Language Model

as the proctor started the clock
discard

the students opened their _____
fixed window

A fixed-window neural Language Model

output distribution

$$\hat{y} = \text{softmax}(U\mathbf{h} + \mathbf{b}_2) \in \mathbb{R}^{|V|}$$

hidden layer

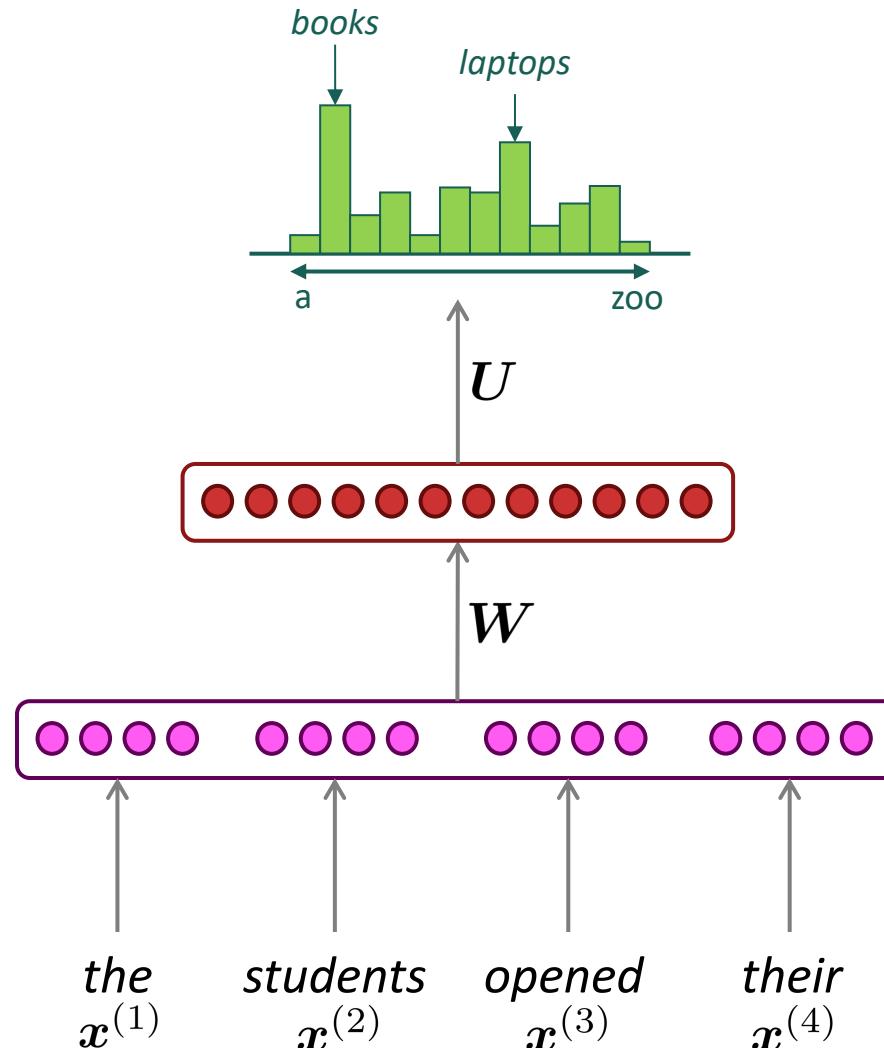
$$\mathbf{h} = f(\mathbf{W}\mathbf{e} + \mathbf{b}_1)$$

concatenated word embeddings

$$\mathbf{e} = [\mathbf{e}^{(1)}; \mathbf{e}^{(2)}; \mathbf{e}^{(3)}; \mathbf{e}^{(4)}]$$

words / one-hot vectors

$$\mathbf{x}^{(1)}, \mathbf{x}^{(2)}, \mathbf{x}^{(3)}, \mathbf{x}^{(4)}$$



A fixed-window neural Language Model

Approximately: Y. Bengio, et al. (2000/2003): A Neural Probabilistic Language Model

Improvements over n -gram LM:

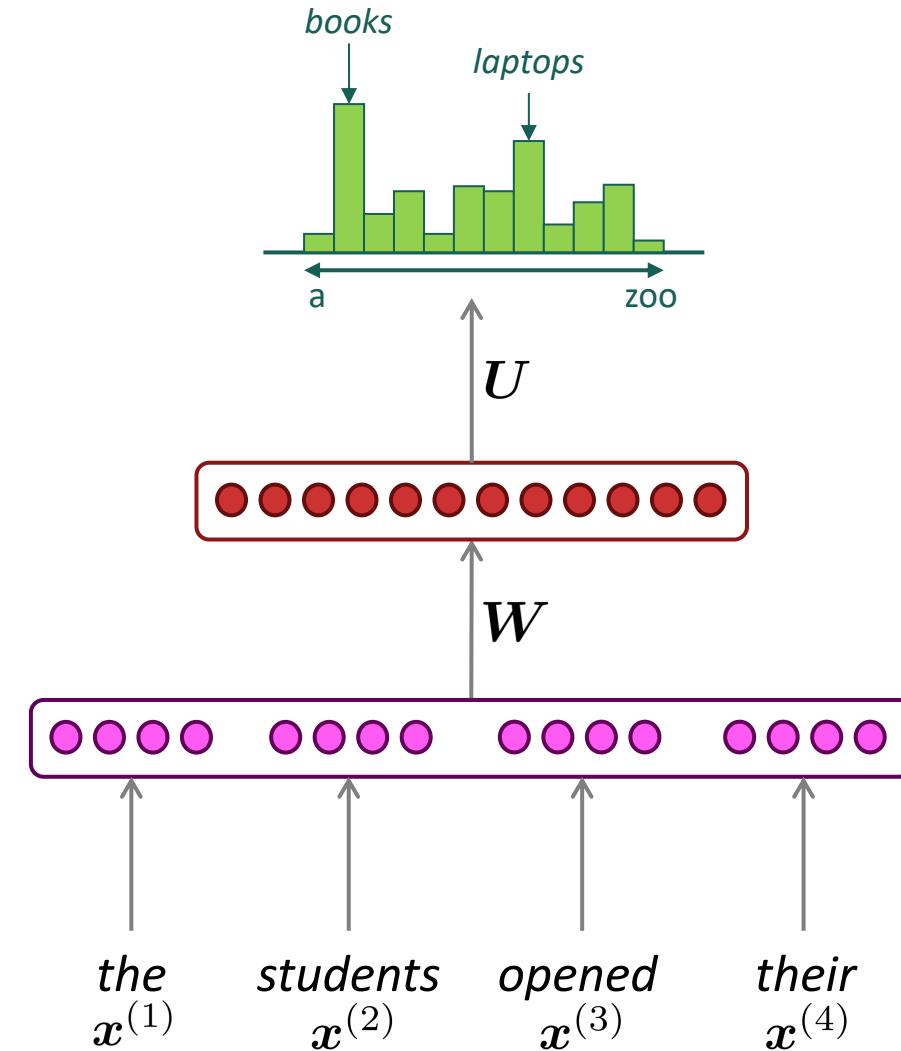
- No sparsity problem
- Don't need to store all observed n -grams

Remaining **problems**:

- Fixed window is **too small**
- Enlarging window enlarges W
- Window can never be large enough!
- $x^{(1)}$ and $x^{(2)}$ are multiplied by completely different weights in W .

No symmetry in how the inputs are processed.

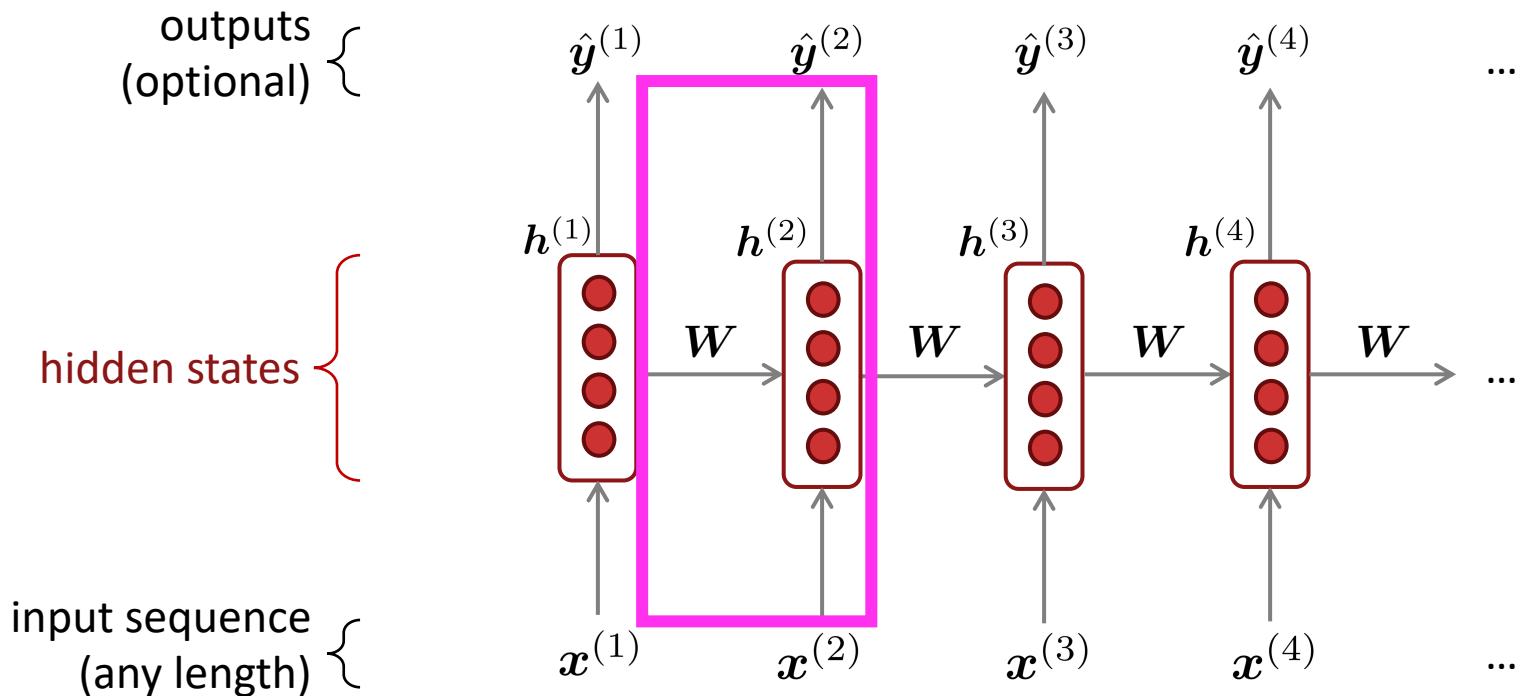
We need a neural architecture
that can process *any length input*



Recurrent Neural Networks (RNN)

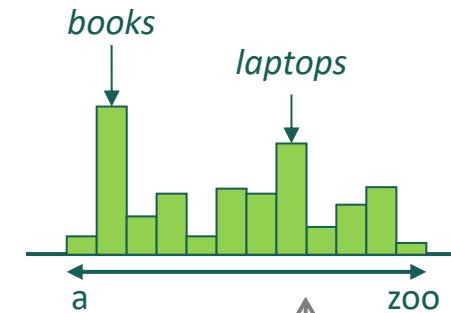
A family of neural architectures

Core idea: Apply
the same weights
 W repeatedly



A Simple RNN Language Model

$$\hat{y}^{(4)} = P(\mathbf{x}^{(5)} | \text{the students opened their})$$



output distribution

$$\hat{y}^{(t)} = \text{softmax}(\mathbf{U}\mathbf{h}^{(t)} + \mathbf{b}_2) \in \mathbb{R}^{|V|}$$

hidden states

$$\mathbf{h}^{(t)} = \sigma(\mathbf{W}_h \mathbf{h}^{(t-1)} + \mathbf{W}_e \mathbf{e}^{(t)} + \mathbf{b}_1)$$

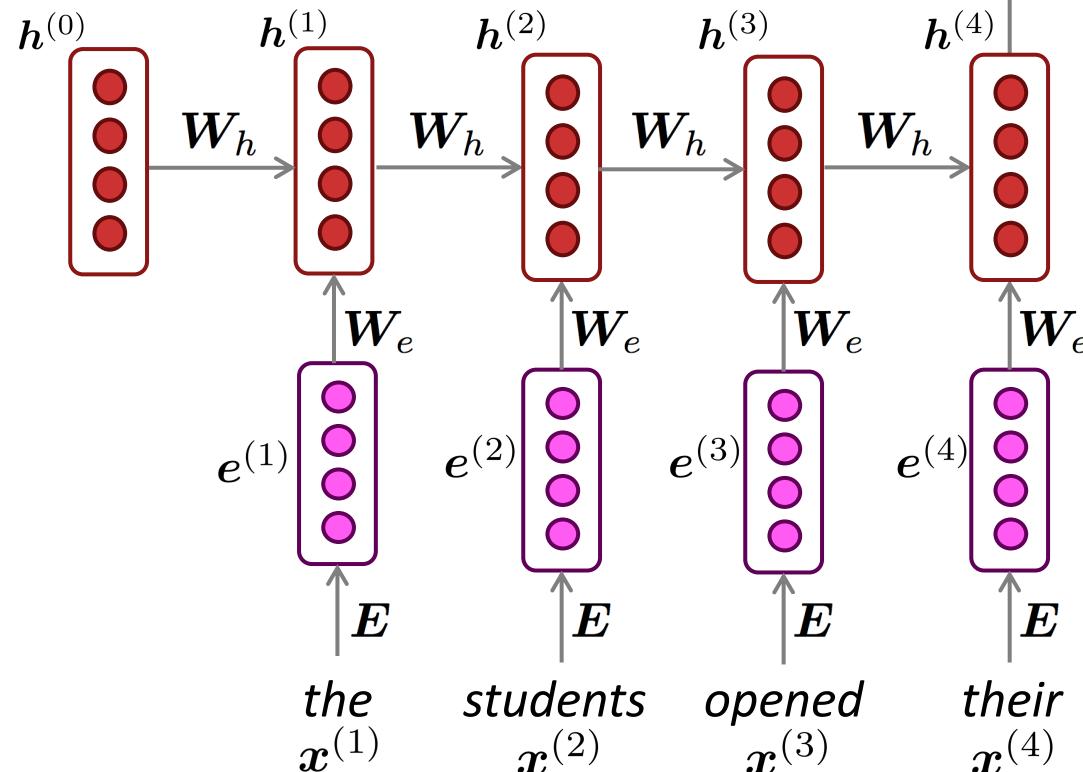
$\mathbf{h}^{(0)}$ is the initial hidden state

word embeddings

$$\mathbf{e}^{(t)} = \mathbf{E}\mathbf{x}^{(t)}$$

words / one-hot vectors

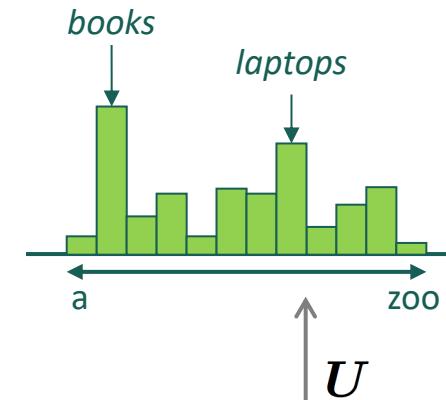
$$\mathbf{x}^{(t)} \in \mathbb{R}^{|V|}$$



Note: this input sequence could be much longer now!

RNN Language Models

$$\hat{y}^{(4)} = P(\mathbf{x}^{(5)} | \text{the students opened their})$$

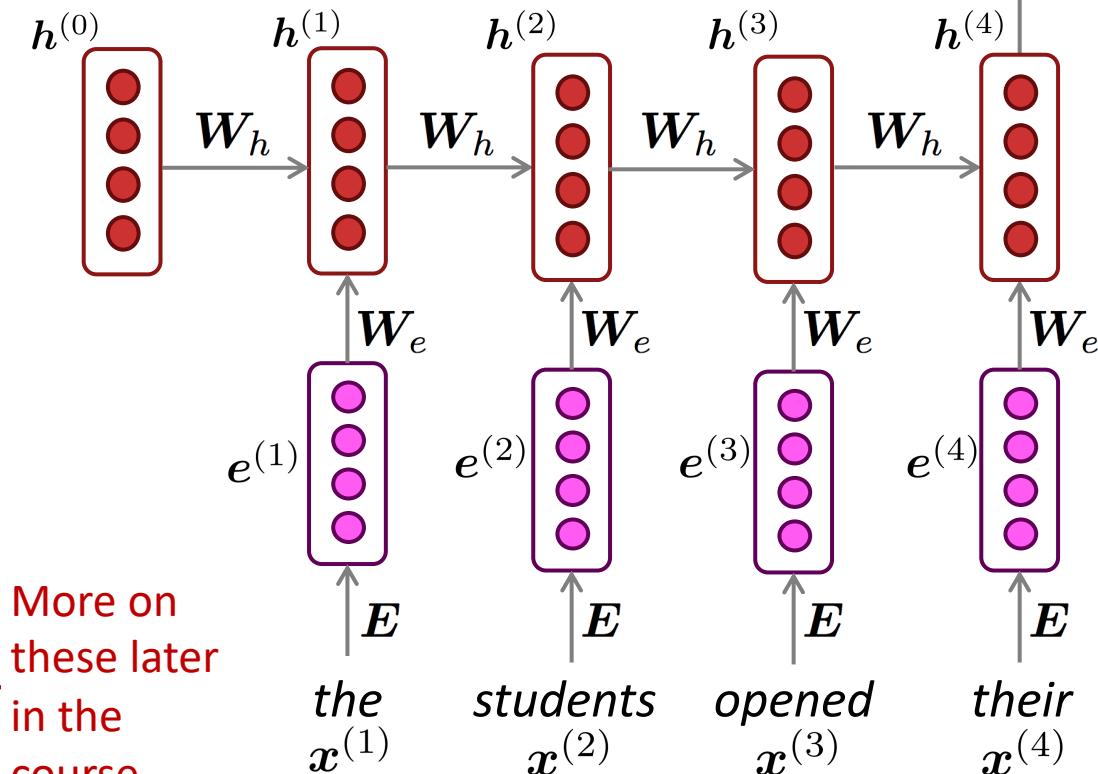


RNN Advantages:

- Can process **any length** input
- Computation for step t can (in theory) use information from **many steps back**
- **Model size doesn't increase** for longer input context
- Same weights applied on every timestep, so there is **symmetry** in how inputs are processed.

RNN Disadvantages:

- Recurrent computation is **slow**
- In practice, difficult to access information from **many steps back**



Training an RNN Language Model

- Get a **big corpus of text** which is a sequence of words $\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(T)}$
- Feed into RNN-LM; compute output distribution $\hat{\mathbf{y}}^{(t)}$ **for every step t .**
 - i.e. predict probability dist of *every word*, given words so far
- **Loss function** on step t is **cross-entropy** between predicted probability distribution $\hat{\mathbf{y}}^{(t)}$, and the true next word $\mathbf{y}^{(t)}$ (one-hot for $\mathbf{x}^{(t+1)}$):

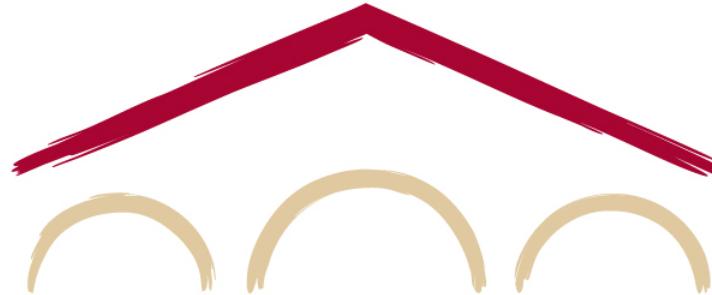
$$J^{(t)}(\theta) = CE(\mathbf{y}^{(t)}, \hat{\mathbf{y}}^{(t)}) = - \sum_{w \in V} \mathbf{y}_w^{(t)} \log \hat{\mathbf{y}}_w^{(t)} = - \log \hat{\mathbf{y}}_{\mathbf{x}_{t+1}}^{(t)}$$

- Average this to get **overall loss** for entire training set:

$$J(\theta) = \frac{1}{T} \sum_{t=1}^T J^{(t)}(\theta) = \frac{1}{T} \sum_{t=1}^T - \log \hat{\mathbf{y}}_{\mathbf{x}_{t+1}}^{(t)}$$

Natural Language Processing with Deep Learning

CS224N/Ling284



Christopher Manning

Lecture 6: Simple and LSTM Recurrent Neural Networks

Lecture Plan

1. RNN Language Models (25 mins)
2. Other uses of RNNs (8 mins)
3. Exploding and vanishing gradients (15 mins)
4. LSTMs (20 mins)
5. Bidirectional and multi-layer RNNs (12 mins)



Overview

- Last lecture we learned:
 - Language models, n-gram language models, and Recurrent Neural Networks (RNNs)
- Today we'll learn how to get RNNs to work for you
 - Training RNNs
 - Uses of RNNs
 - Problems with RNNs (exploding and vanishing gradients) and how to fix them
 - These problems motivate a more sophisticated RNN architecture: LSTMs
 - And other more complex RNN options: bidirectional RNNs and multi-layer RNNs
- Next lecture we'll learn:
 - How we can do Neural Machine Translation (NMT) using an RNN-based architecture called sequence-to-sequence with attention

1. The Simple RNN Language Model

output distribution

$$\hat{y}^{(t)} = \text{softmax}(\mathbf{U}\mathbf{h}^{(t)} + \mathbf{b}_2) \in \mathbb{R}^{|V|}$$

hidden states

$$\mathbf{h}^{(t)} = \sigma(\mathbf{W}_h \mathbf{h}^{(t-1)} + \mathbf{W}_e \mathbf{e}^{(t)} + \mathbf{b}_1)$$

$\mathbf{h}^{(0)}$ is the initial hidden state

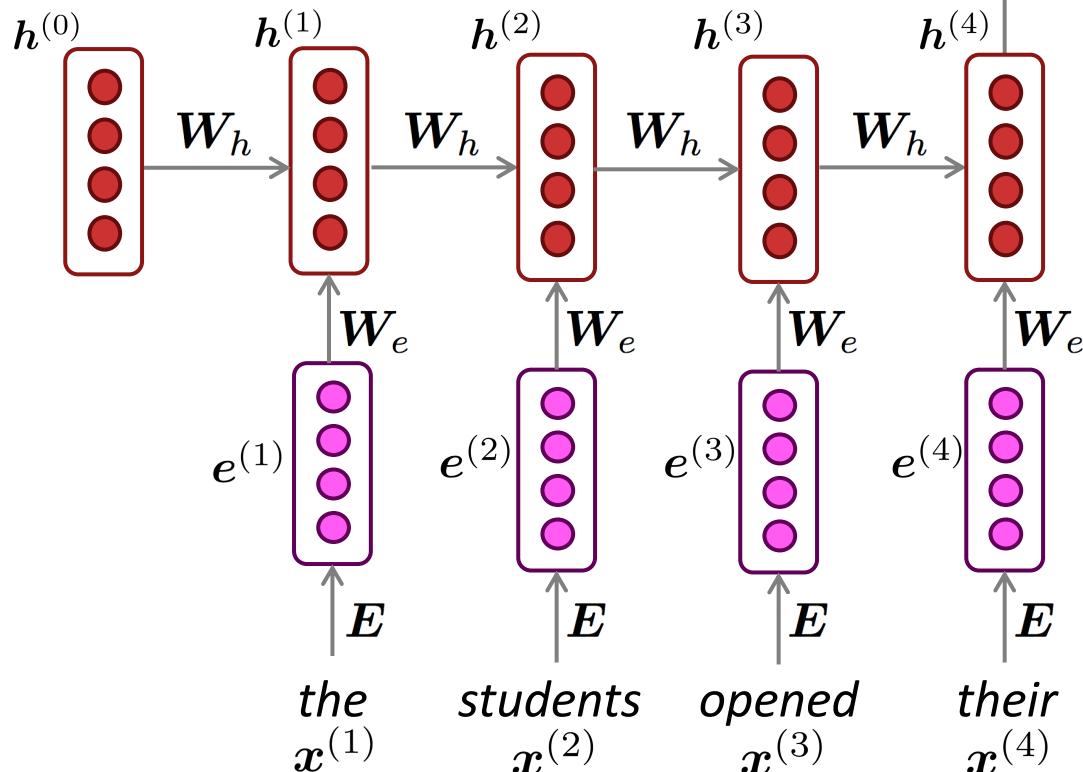
word embeddings

$$\mathbf{e}^{(t)} = \mathbf{E}\mathbf{x}^{(t)}$$

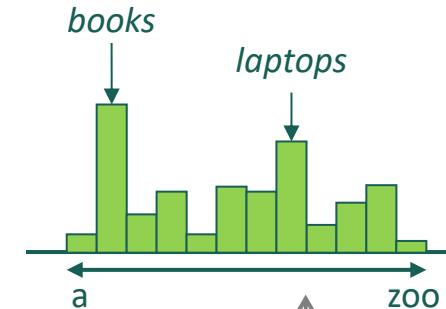
words / one-hot vectors

$$\mathbf{x}^{(t)} \in \mathbb{R}^{|V|}$$

Note: this input sequence could be much longer now!



$$\hat{\mathbf{y}}^{(4)} = P(\mathbf{x}^{(5)} | \text{the students opened their})$$



Training an RNN Language Model

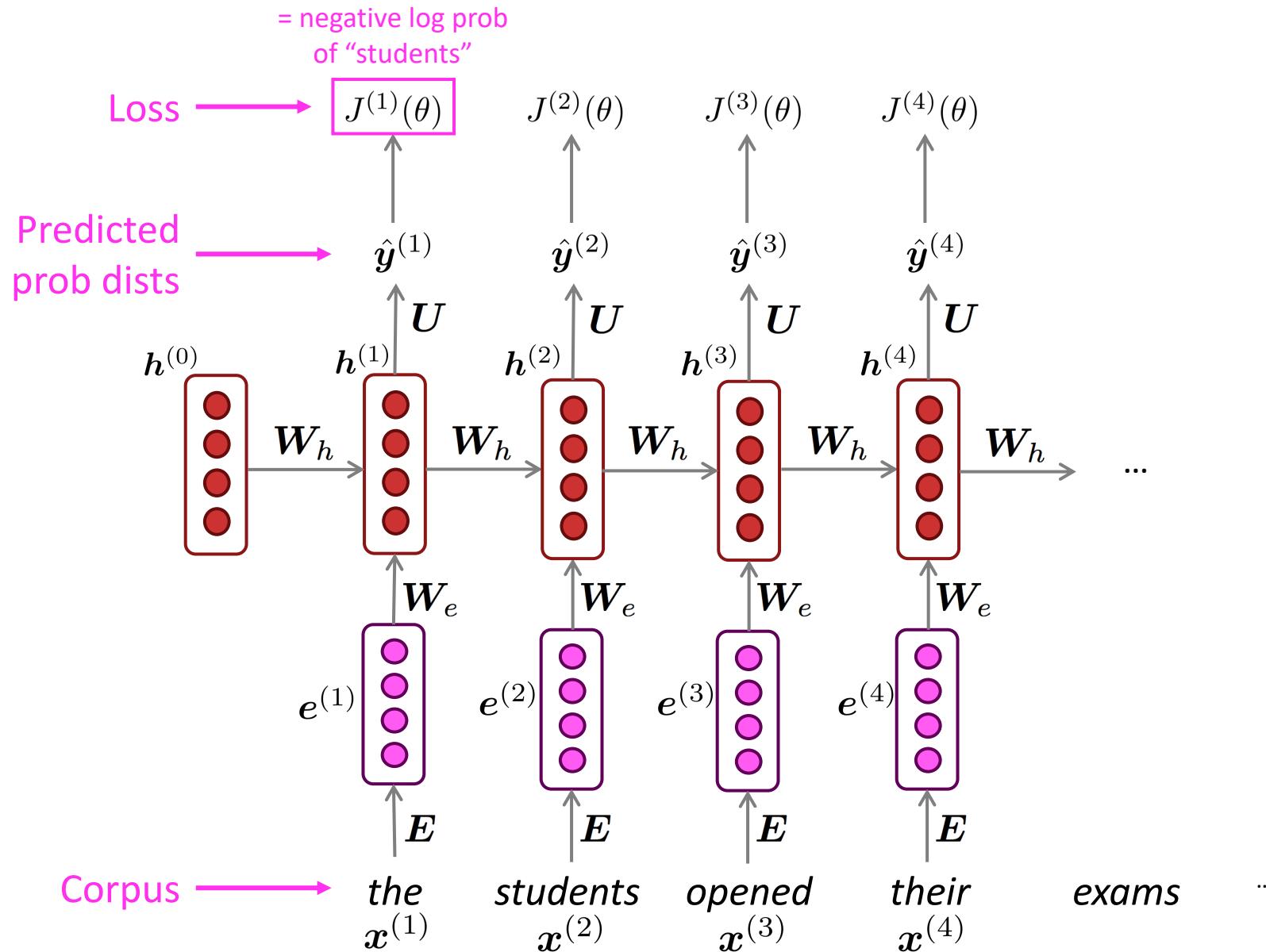
- Get a **big corpus of text** which is a sequence of words $\mathbf{x}^{(1)}, \dots, \mathbf{x}^{(T)}$
- Feed into RNN-LM; compute output distribution $\hat{\mathbf{y}}^{(t)}$ **for every step t .**
 - i.e., predict probability dist of *every word*, given words so far
- **Loss function** on step t is **cross-entropy** between predicted probability distribution $\hat{\mathbf{y}}^{(t)}$, and the true next word $\mathbf{y}^{(t)}$ (one-hot for $\mathbf{x}^{(t+1)}$):

$$J^{(t)}(\theta) = CE(\mathbf{y}^{(t)}, \hat{\mathbf{y}}^{(t)}) = - \sum_{w \in V} \mathbf{y}_w^{(t)} \log \hat{\mathbf{y}}_w^{(t)} = - \log \hat{\mathbf{y}}_{\mathbf{x}_{t+1}}^{(t)}$$

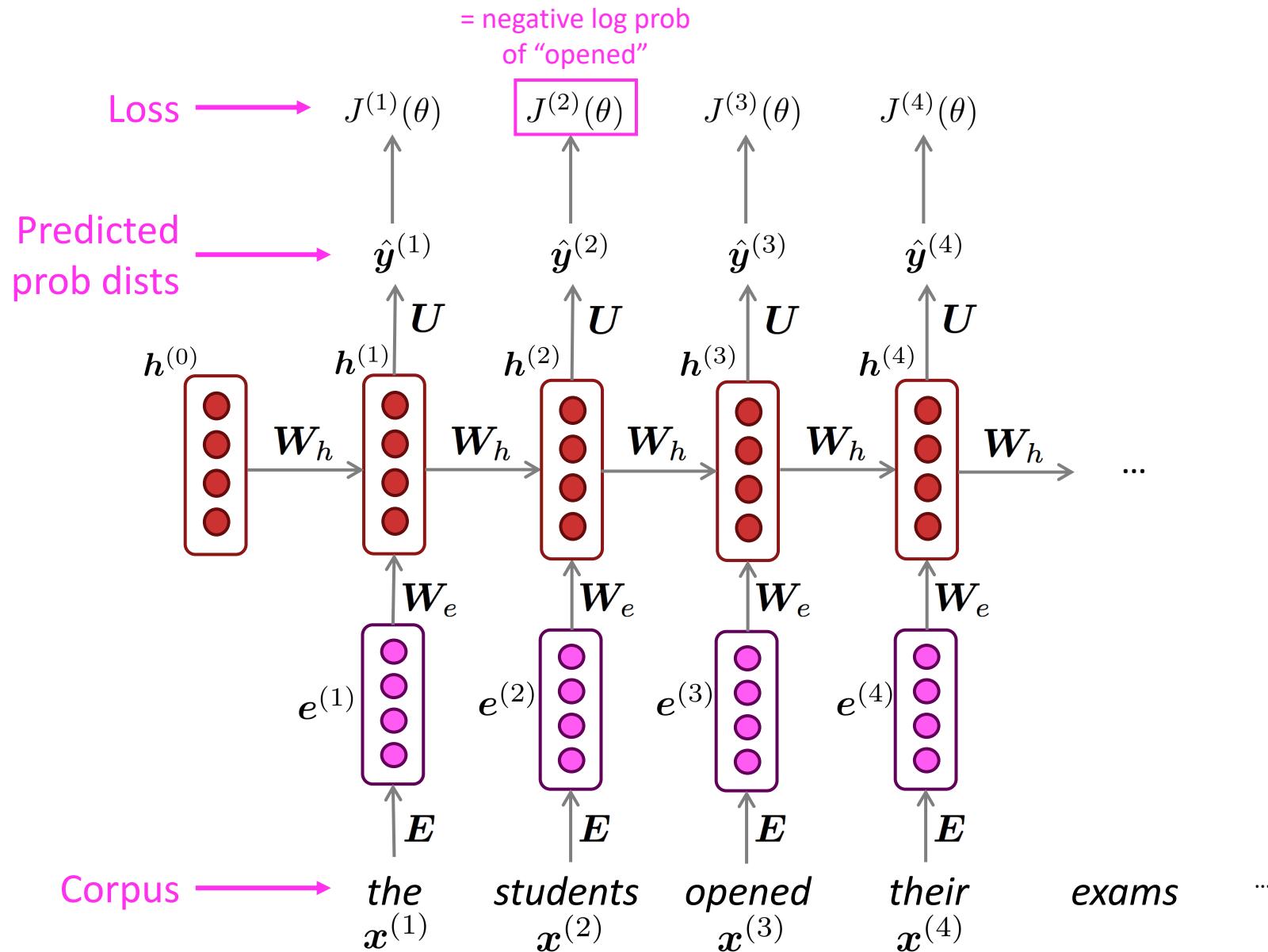
- Average this to get **overall loss** for entire training set:

$$J(\theta) = \frac{1}{T} \sum_{t=1}^T J^{(t)}(\theta) = \frac{1}{T} \sum_{t=1}^T - \log \hat{\mathbf{y}}_{\mathbf{x}_{t+1}}^{(t)}$$

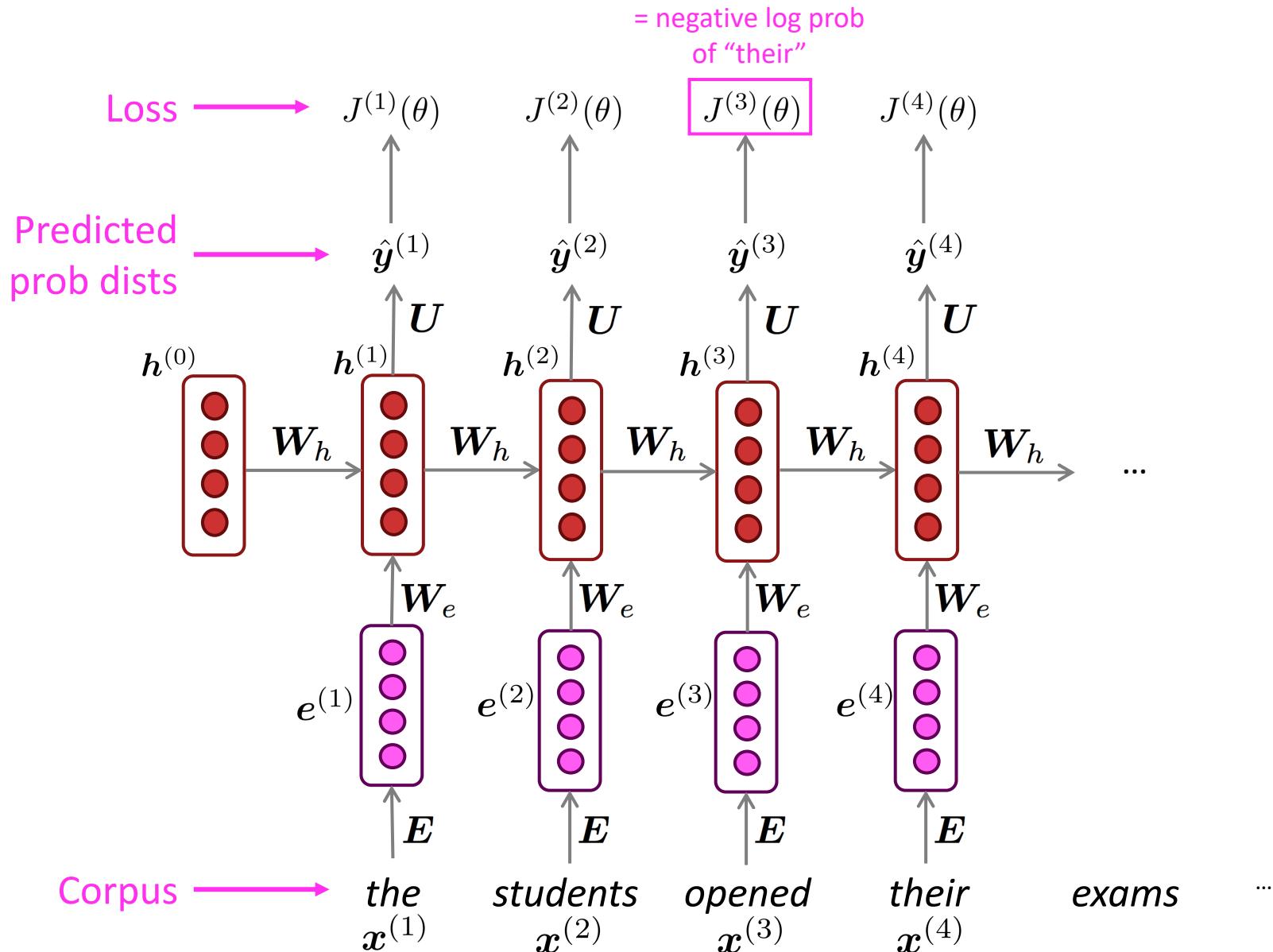
Training an RNN Language Model



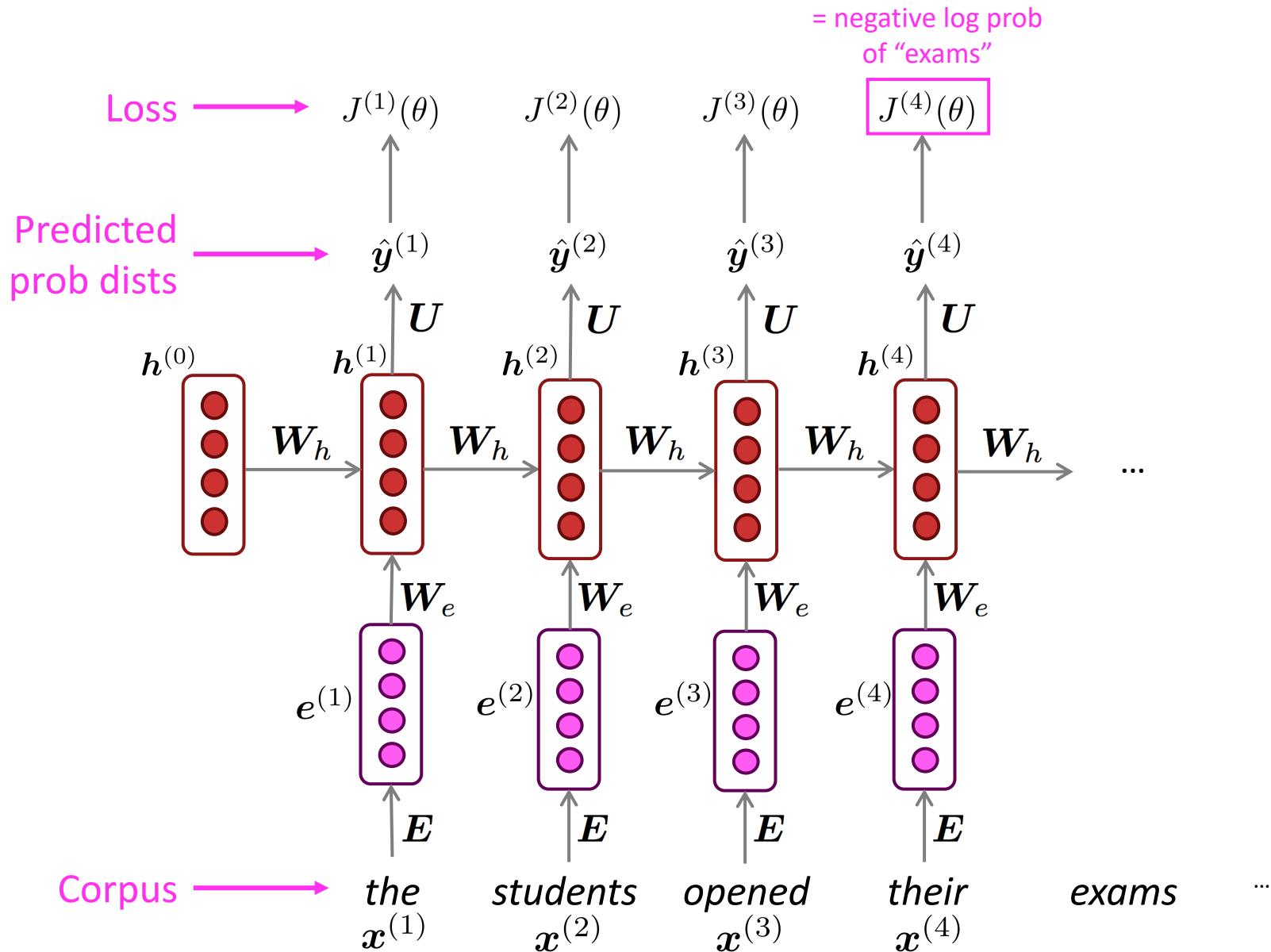
Training an RNN Language Model



Training an RNN Language Model

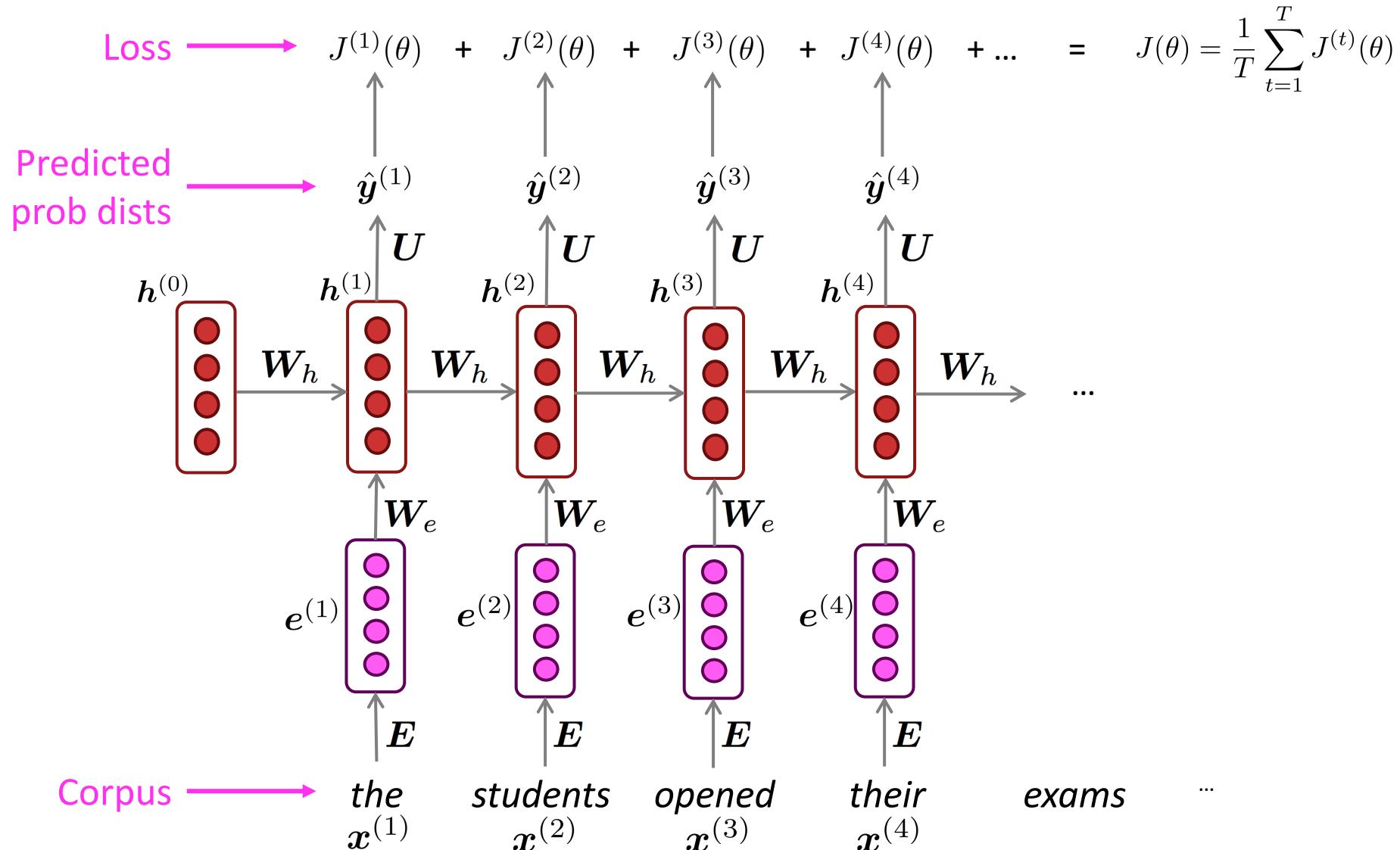


Training an RNN Language Model



Training an RNN Language Model

“Teacher forcing”



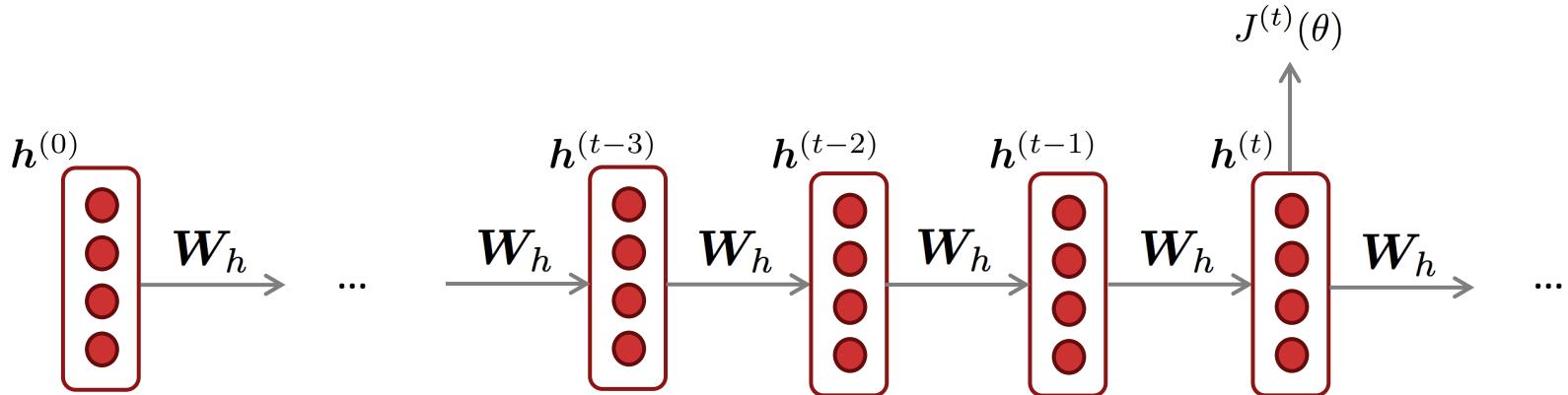
Training a RNN Language Model

- However: Computing loss and gradients across **entire corpus** $x^{(1)}, \dots, x^{(T)}$ is **too expensive!**

$$J(\theta) = \frac{1}{T} \sum_{t=1}^T J^{(t)}(\theta)$$

- In practice, consider $x^{(1)}, \dots, x^{(T)}$ as a **sentence** (or a **document**)
- Recall: **Stochastic Gradient Descent** allows us to compute loss and gradients for small chunk of data, and update.
- Compute loss $J(\theta)$ for a sentence (actually, a batch of sentences), compute gradients and update weights. Repeat.

Training the parameters of RNNs: Backpropagation for RNNs



Question: What's the derivative of $J^{(t)}(\theta)$ w.r.t. the **repeated** weight matrix W_h ?

Answer:
$$\frac{\partial J^{(t)}}{\partial W_h} = \sum_{i=1}^t \frac{\partial J^{(t)}}{\partial W_h} \Big|_{(i)}$$

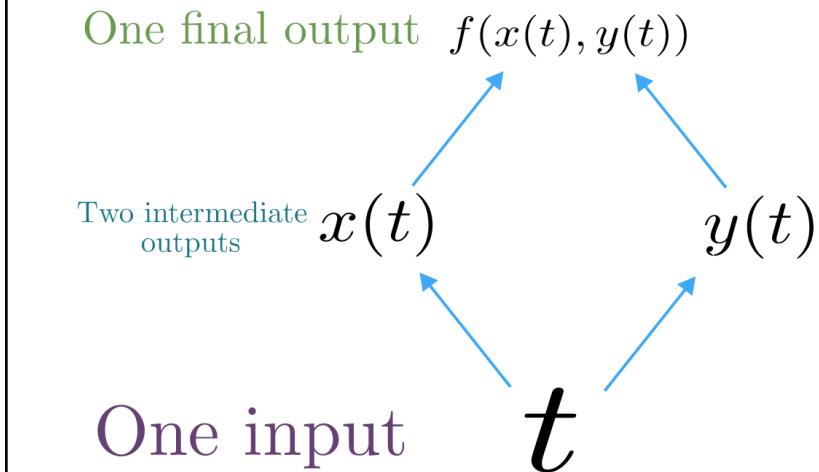
“The gradient w.r.t. a repeated weight
is the sum of the gradient
w.r.t. each time it appears”

Why?

Multivariable Chain Rule

- Given a multivariable function $f(x, y)$, and two single variable functions $x(t)$ and $y(t)$, here's what the multivariable chain rule says:

$$\underbrace{\frac{d}{dt} f(\textcolor{teal}{x}(t), \textcolor{red}{y}(t))}_{\text{Derivative of composition function}} = \frac{\partial f}{\partial \textcolor{teal}{x}} \frac{d\textcolor{teal}{x}}{dt} + \frac{\partial f}{\partial \textcolor{red}{y}} \frac{d\textcolor{red}{y}}{dt}$$



Gradients sum at
outward branches!
(lecture 3)

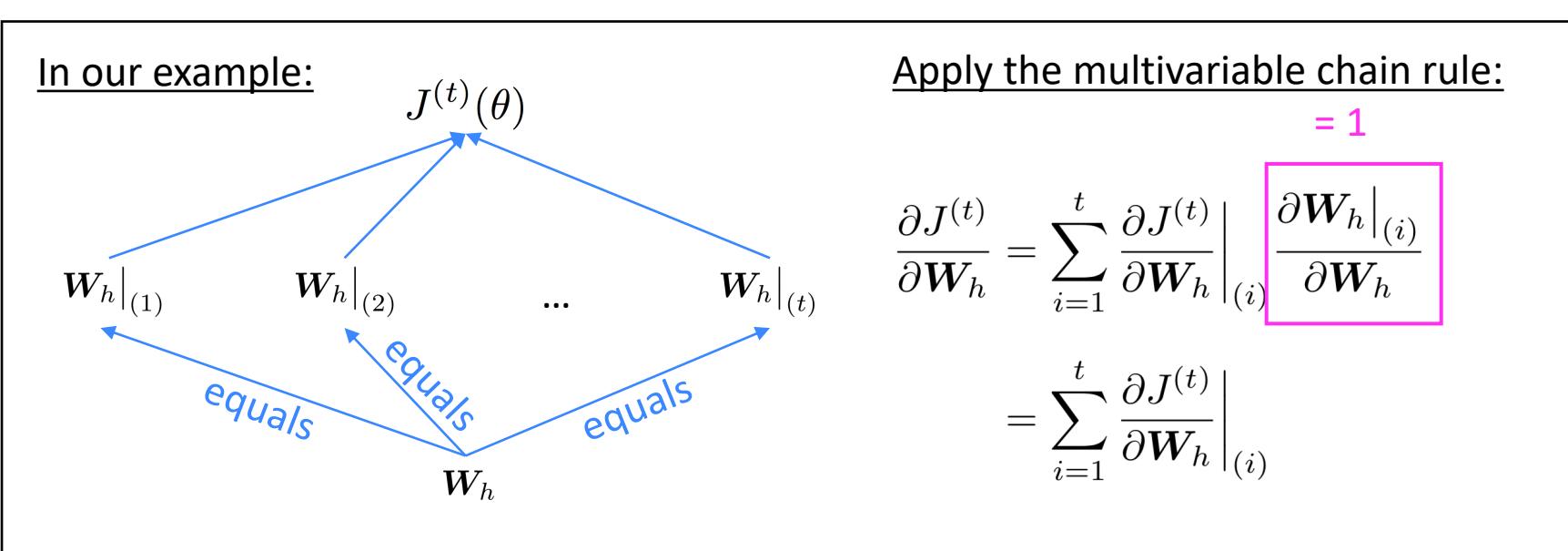
Source:

<https://www.khanacademy.org/math/multivariable-calculus/multivariable-derivatives/differentiating-vector-valued-functions/a/multivariable-chain-rule-simple-version>

Backpropagation for RNNs: Proof sketch

- Given a multivariable function $f(x, y)$, and two single variable functions $x(t)$ and $y(t)$, here's what the multivariable chain rule says:

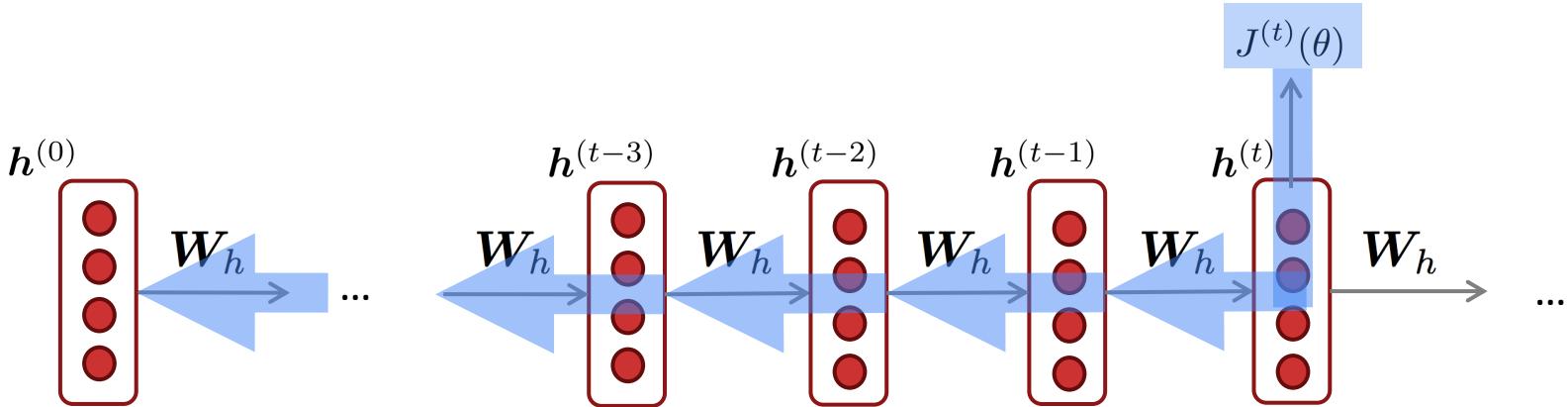
$$\underbrace{\frac{d}{dt} f(\textcolor{teal}{x}(t), \textcolor{red}{y}(t))}_{\text{Derivative of composition function}} = \frac{\partial f}{\partial \textcolor{teal}{x}} \frac{d\textcolor{teal}{x}}{dt} + \frac{\partial f}{\partial \textcolor{red}{y}} \frac{d\textcolor{red}{y}}{dt}$$



Source:

<https://www.khanacademy.org/math/multivariable-calculus/multivariable-derivatives/differentiating-vector-valued-functions/a/multivariable-chain-rule-simple-version>

Backpropagation for RNNs



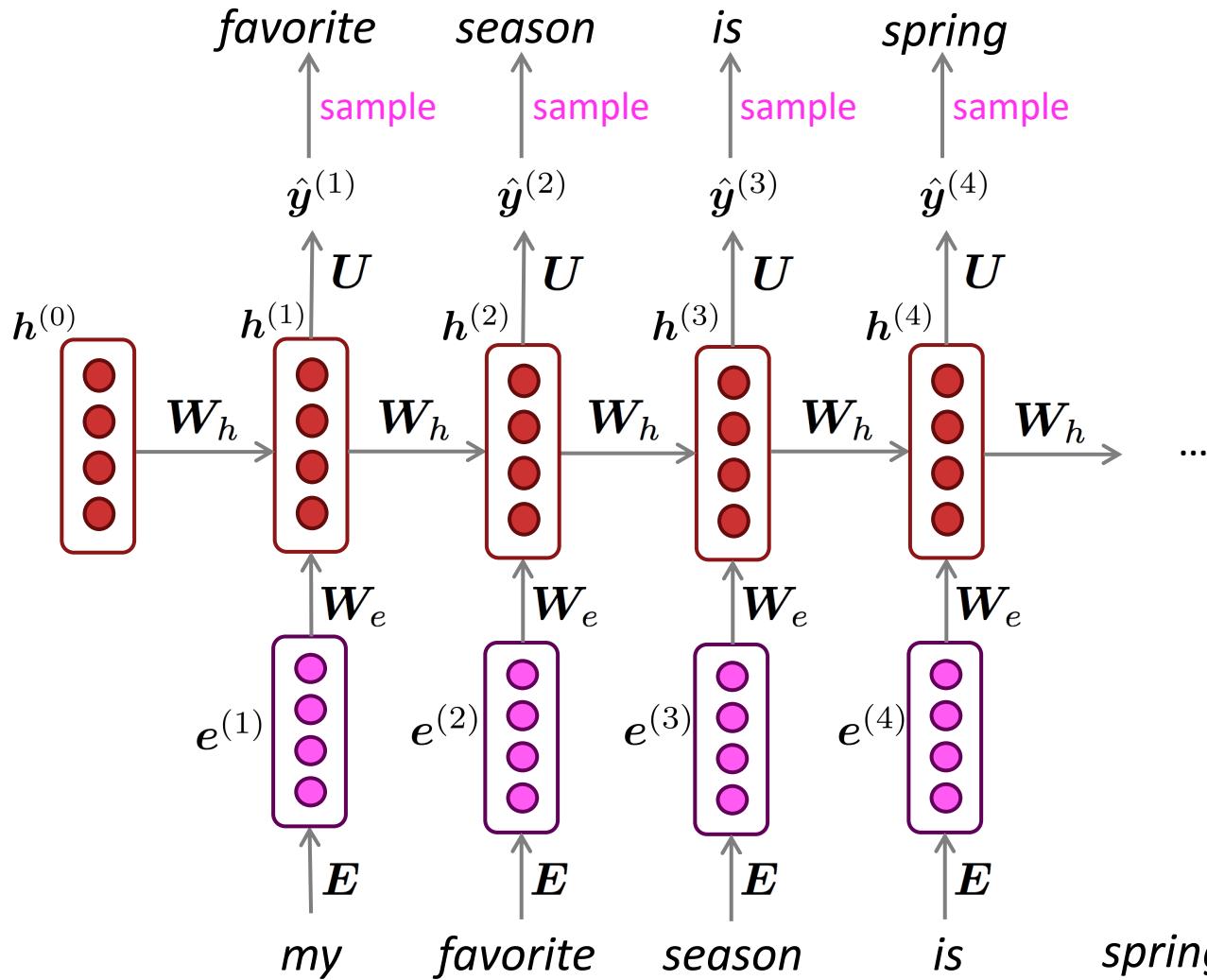
$$\frac{\partial J^{(t)}}{\partial W_h} = \boxed{\sum_{i=1}^t \frac{\partial J^{(t)}}{\partial W_h} \Big|_{(i)}}$$

Question: How do we calculate this?

Answer: Backpropagate over timesteps $i=t, \dots, 0$, summing gradients as you go.
This algorithm is called “**backpropagation through time**” [Werbos, P.G., 1988, *Neural Networks 1*, and others]

Generating text with a RNN Language Model

Just like a n-gram Language Model, you can use an RNN Language Model to generate text by **repeated sampling**. Sampled output becomes next step's input.



Generating text with an RNN Language Model

Let's have some fun!

- You can train an RNN-LM on any kind of text, then generate text in that style.
- RNN-LM trained on *Harry Potter*:

“Sorry,” Harry shouted, panicking—“I’ll leave those brooms in London, are they?”

“No idea,” said Nearly Headless Nick, casting low close by Cedric, carrying the last bit of treacle Charms, from Harry’s shoulder, and to answer him the common room perched upon it, four arms held a shining knob from when the spider hadn’t felt it seemed. He reached the teams too.

Source: <https://medium.com/deep-writing/harry-potter-written-by-artificial-intelligence-8a9431803da6>

Generating text with an RNN Language Model

Let's have some fun!

- You can train an RNN-LM on any kind of text, then generate text in that style.
- RNN-LM trained on **recipes**:

Title: CHOCOLATE RANCH BARBECUE

Categories: Game, Casseroles, Cookies, Cookies

Yield: 6 Servings

2 tb Parmesan cheese -- chopped

1 c Coconut milk

3 Eggs, beaten

Place each pasta over layers of lumps. Shape mixture into the moderate oven and simmer until firm. Serve hot in bodied fresh, mustard, orange and cheese.

Combine the cheese and salt together the dough in a large skillet; add the ingredients and stir in the chocolate and pepper.

Source: <https://gist.github.com/nylki/1efbaa36635956d35bcc>

Evaluating Language Models

- The standard **evaluation metric** for Language Models is **perplexity**.

$$\text{perplexity} = \prod_{t=1}^T \left(\frac{1}{P_{\text{LM}}(\mathbf{x}^{(t+1)} | \mathbf{x}^{(t)}, \dots, \mathbf{x}^{(1)})} \right)^{1/T}$$



Normalized by
number of words

- This is equal to the exponential of the cross-entropy loss $J(\theta)$:

$$= \prod_{t=1}^T \left(\frac{1}{\hat{\mathbf{y}}_{\mathbf{x}_{t+1}}^{(t)}} \right)^{1/T} = \exp \left(\frac{1}{T} \sum_{t=1}^T -\log \hat{\mathbf{y}}_{\mathbf{x}_{t+1}}^{(t)} \right) = \exp(J(\theta))$$

Lower perplexity is better!

RNNs have greatly improved perplexity

n-gram model →

Increasingly complex RNNs ↓

Model	Perplexity
Interpolated Kneser-Ney 5-gram (Chelba et al., 2013)	67.6
RNN-1024 + MaxEnt 9-gram (Chelba et al., 2013)	51.3
RNN-2048 + BlackOut sampling (Ji et al., 2015)	68.3
Sparse Non-negative Matrix factorization (Shazeer et al., 2015)	52.9
LSTM-2048 (Jozefowicz et al., 2016)	43.7
2-layer LSTM-8192 (Jozefowicz et al., 2016)	30
Ours small (LSTM-2048)	43.9
Ours large (2-layer LSTM-2048)	39.8

Perplexity improves
(lower is better) ↓

Source: <https://research.fb.com/building-an-efficient-neural-language-model-over-a-billion-words/>

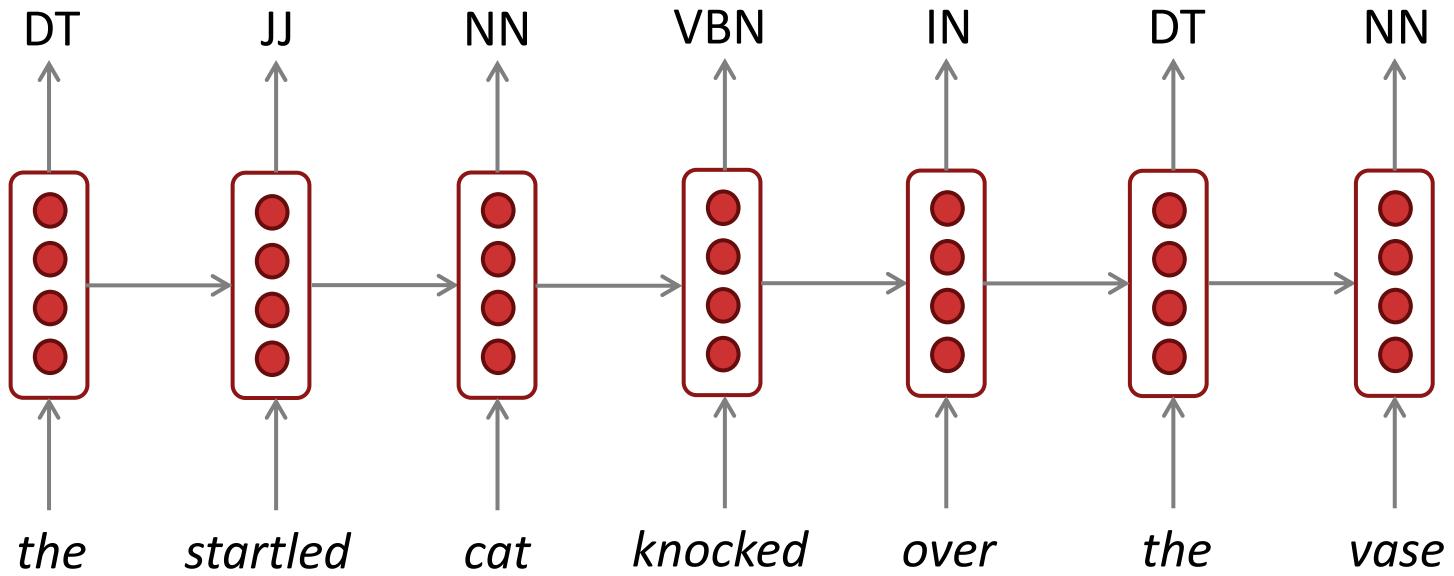
Why should we care about Language Modeling?

- Language Modeling is a **benchmark task** that helps us **measure our progress** on understanding language
- Language Modeling is a **subcomponent** of **many** NLP tasks, especially those involving **generating text** or **estimating the probability of text**:
 - Predictive typing
 - Speech recognition
 - Handwriting recognition
 - Spelling/grammar correction
 - Authorship identification
 - Machine translation
 - Summarization
 - Dialogue
 - etc.

Recap

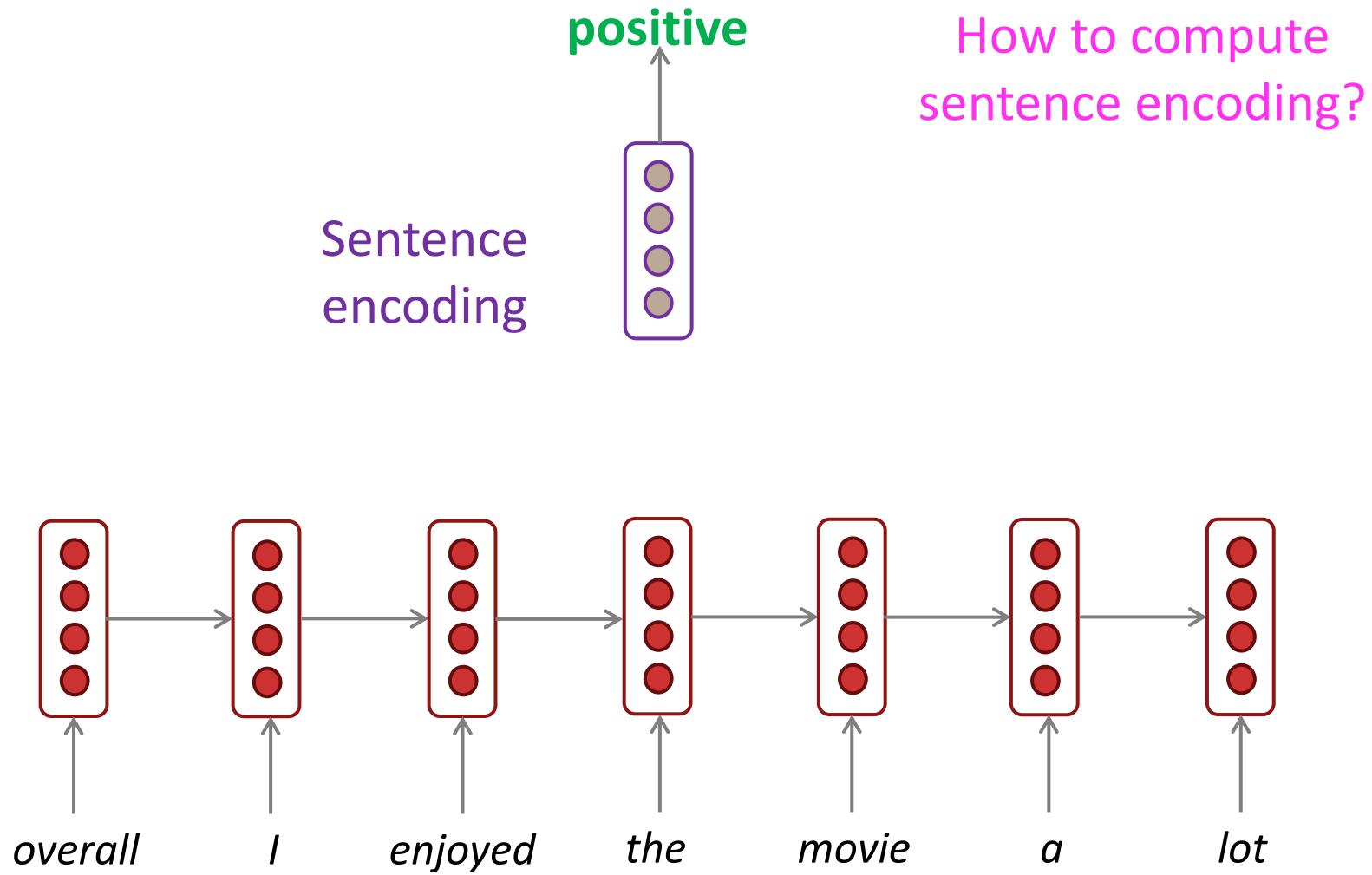
- **Language Model**: A system that predicts the next word
- **Recurrent Neural Network**: A family of neural networks that:
 - Take sequential input of any length
 - Apply the same weights on each step
 - Can optionally produce output on each step
- Recurrent Neural Network \neq Language Model
- We've shown that RNNs are a great way to build a LM
- But RNNs are useful for much more!

2. Other RNN uses: RNNs can be used for sequence tagging e.g., part-of-speech tagging, named entity recognition



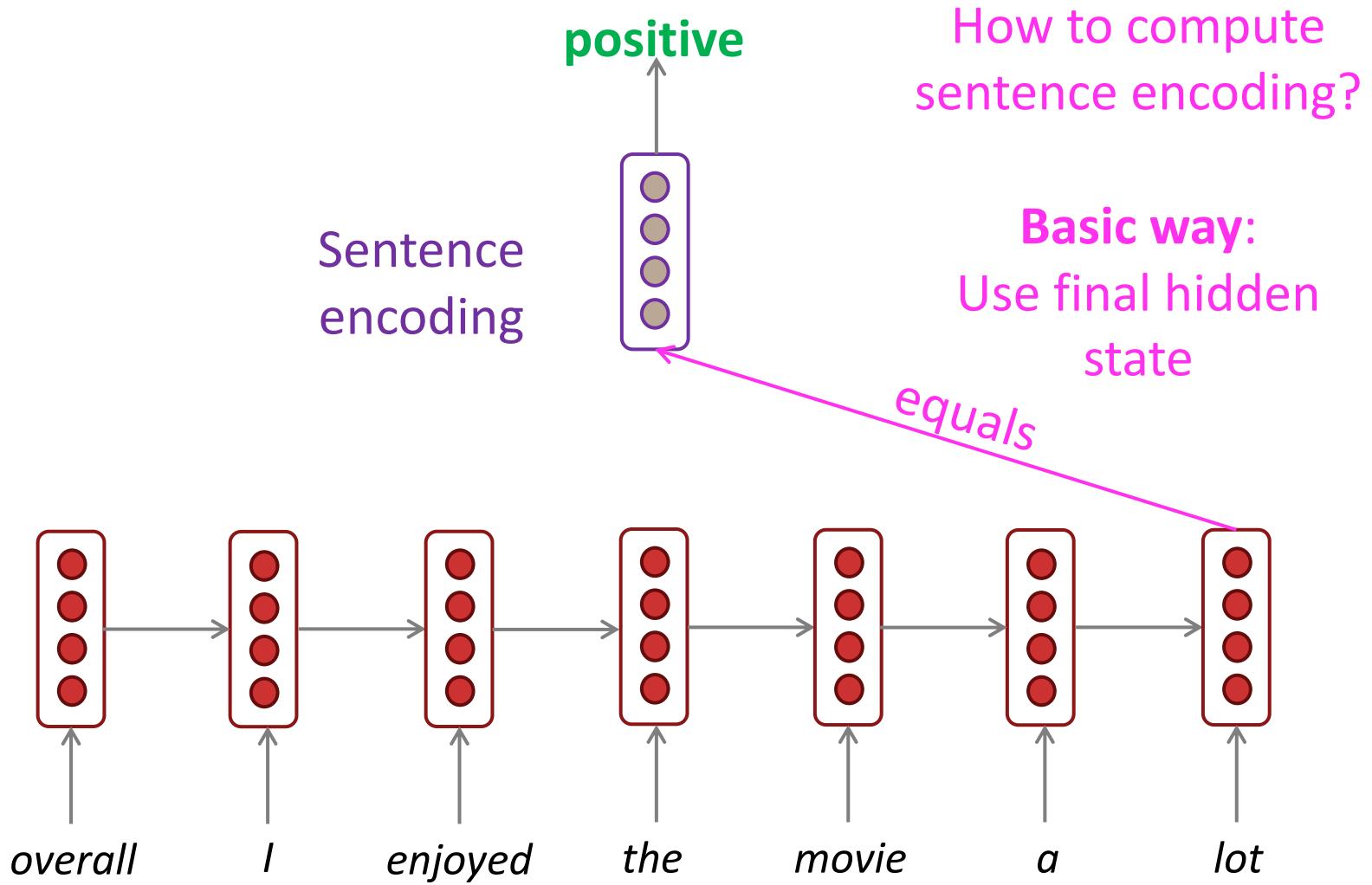
RNNs can be used for sentence classification

e.g., sentiment classification



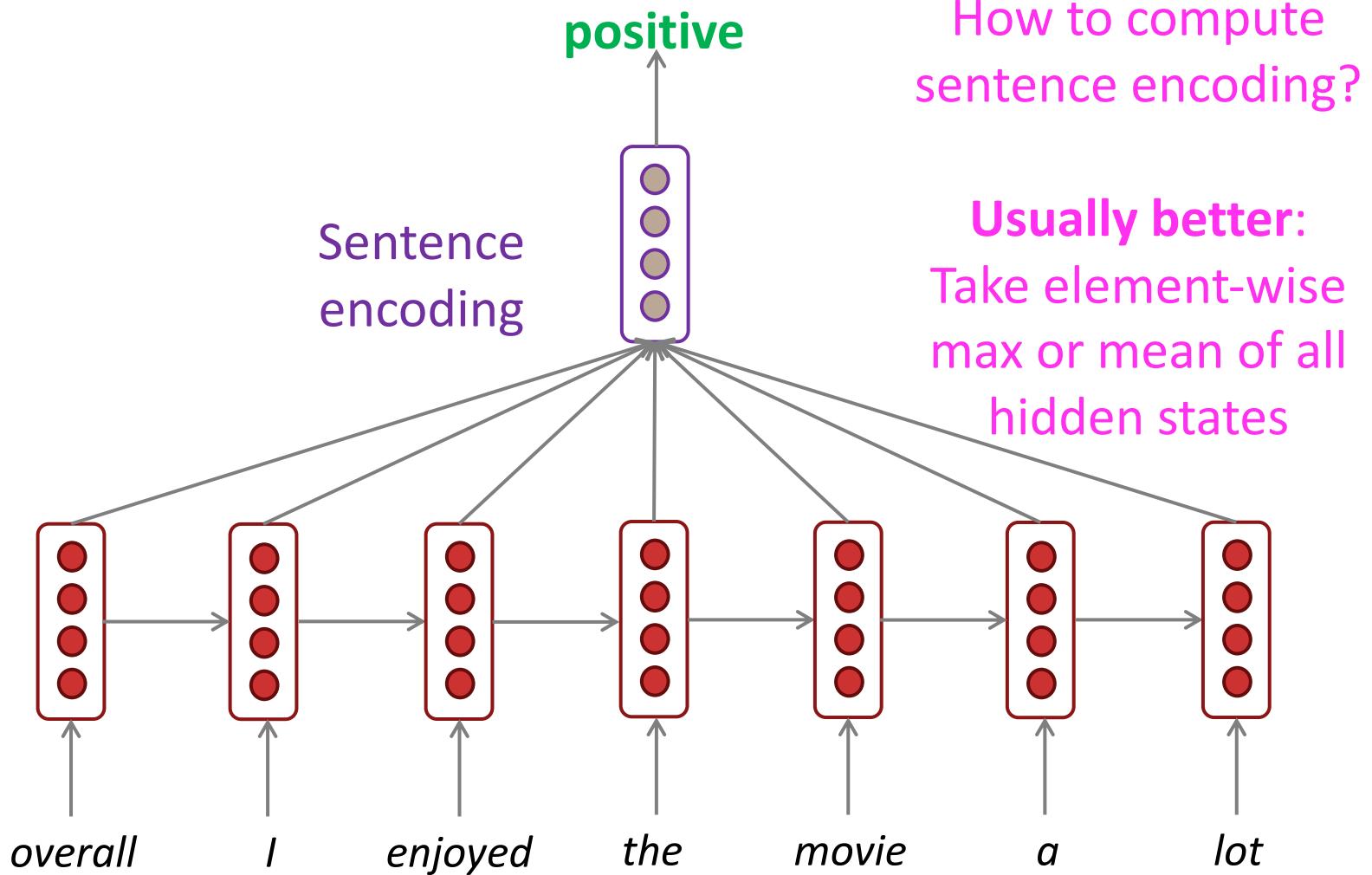
RNNs can be used for sentence classification

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RNNs can be used for sentence classification

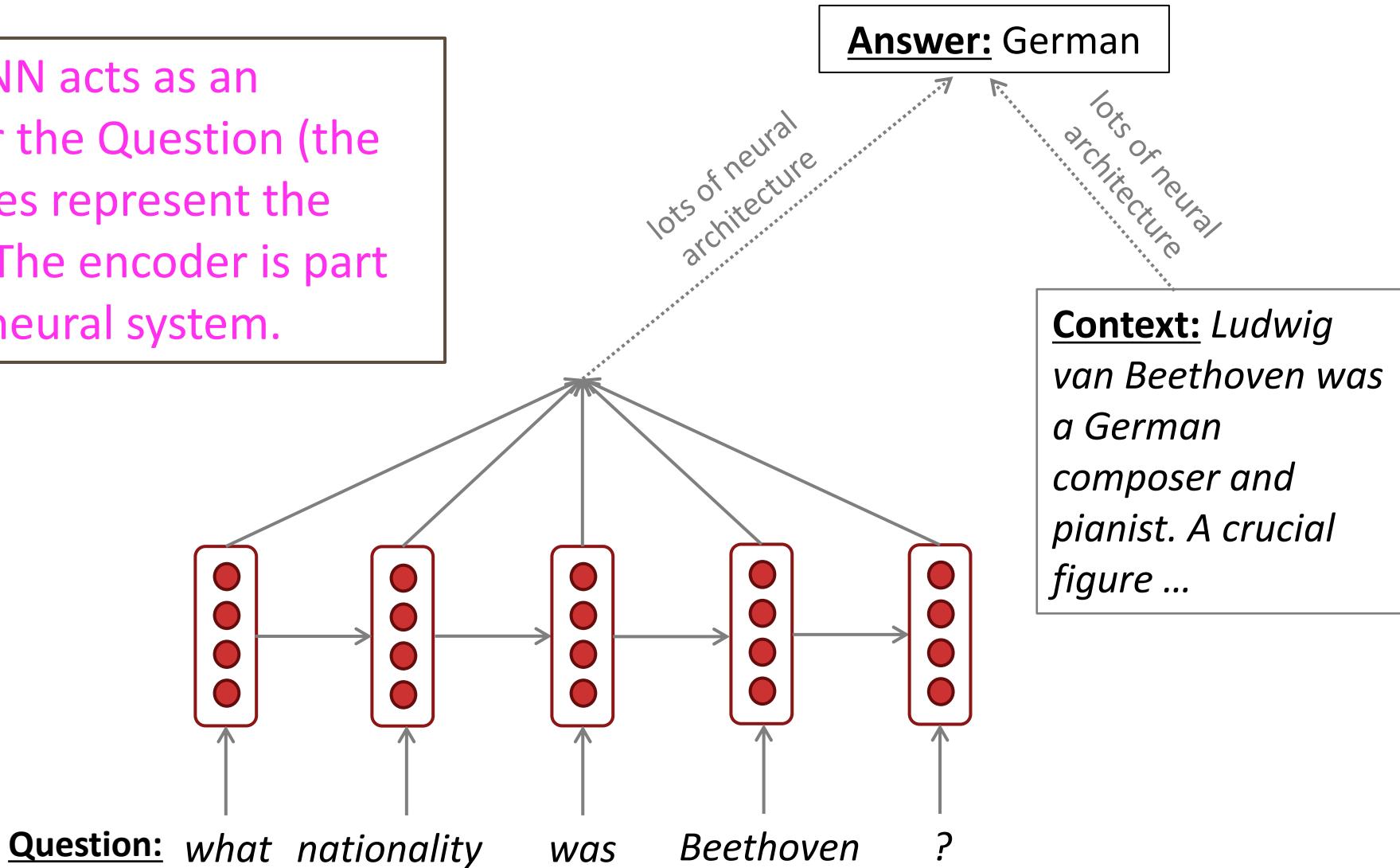
e.g., sentiment classification



RNNs can be used as a language encoder module

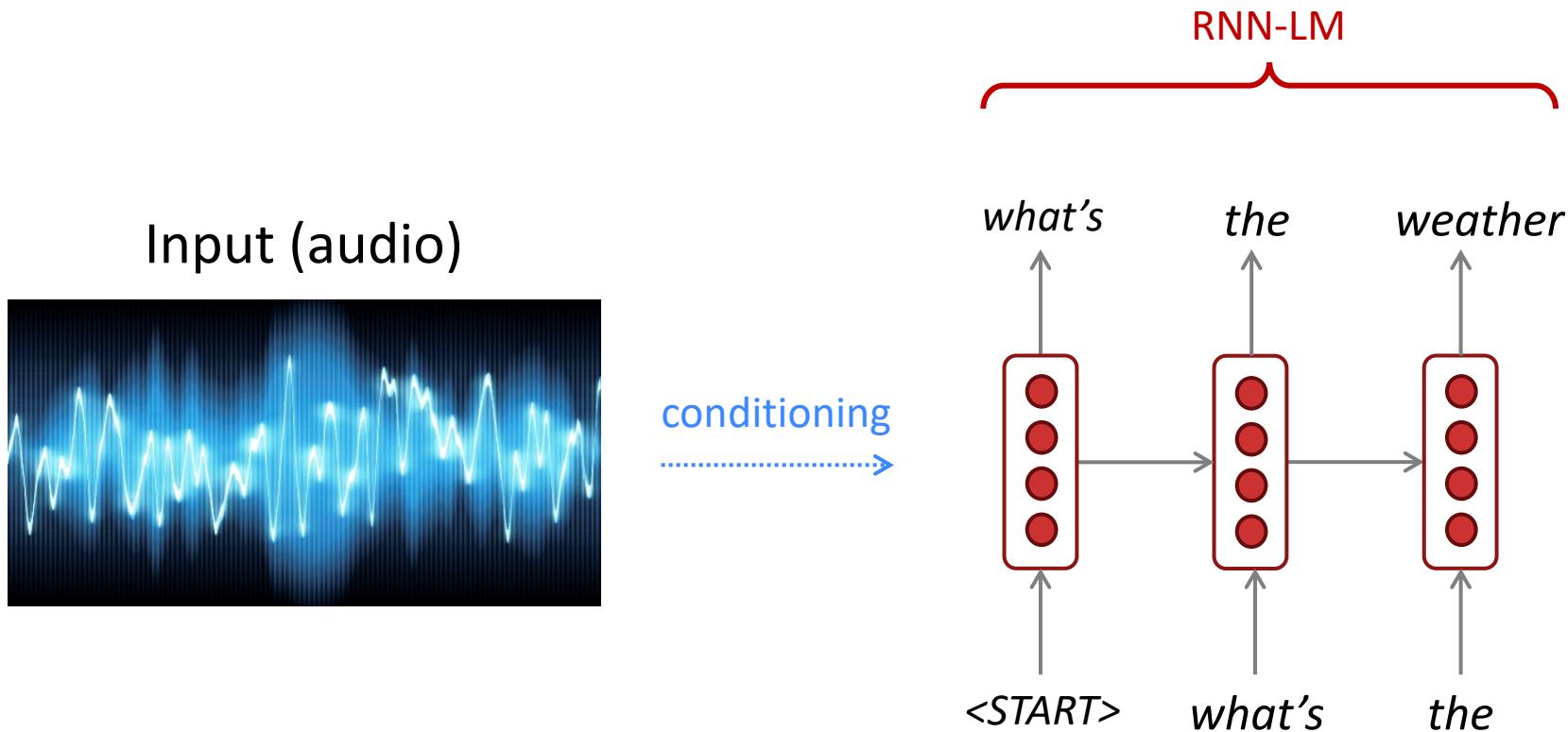
e.g., question answering, machine translation, *many other tasks!*

Here the RNN acts as an **encoder** for the Question (the hidden states represent the Question). The encoder is part of a larger neural system.



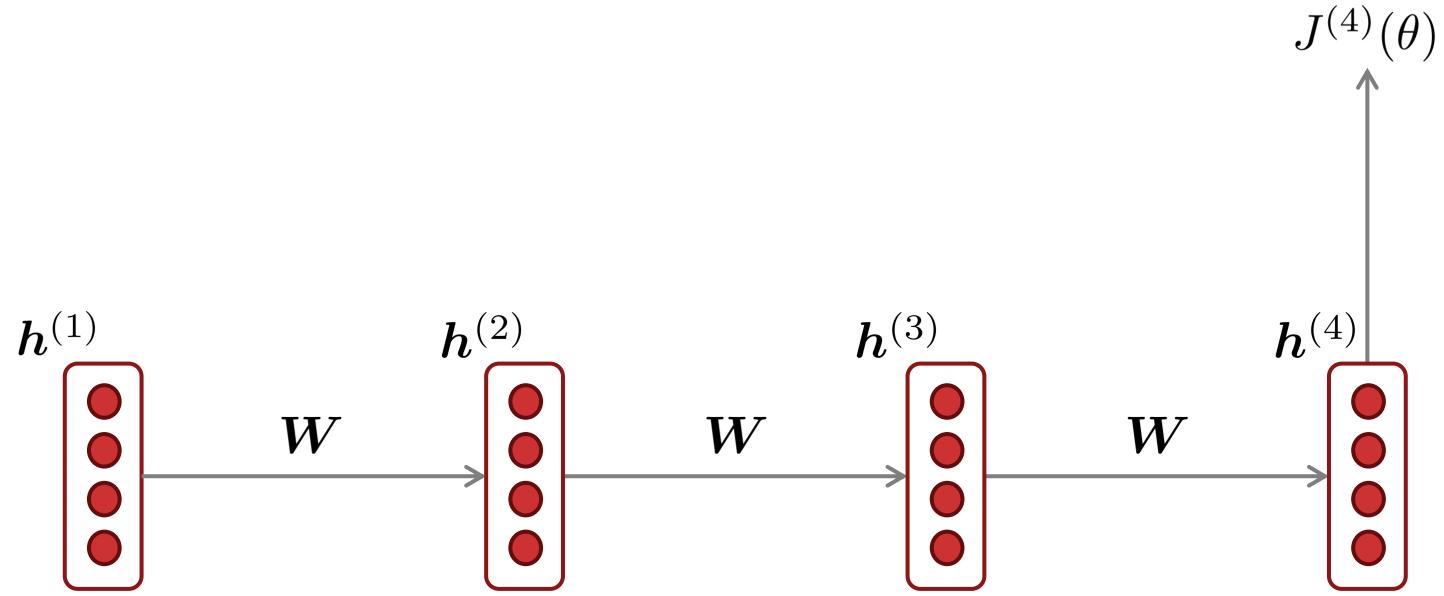
RNN-LMs can be used to generate text

e.g., speech recognition, machine translation, summarization

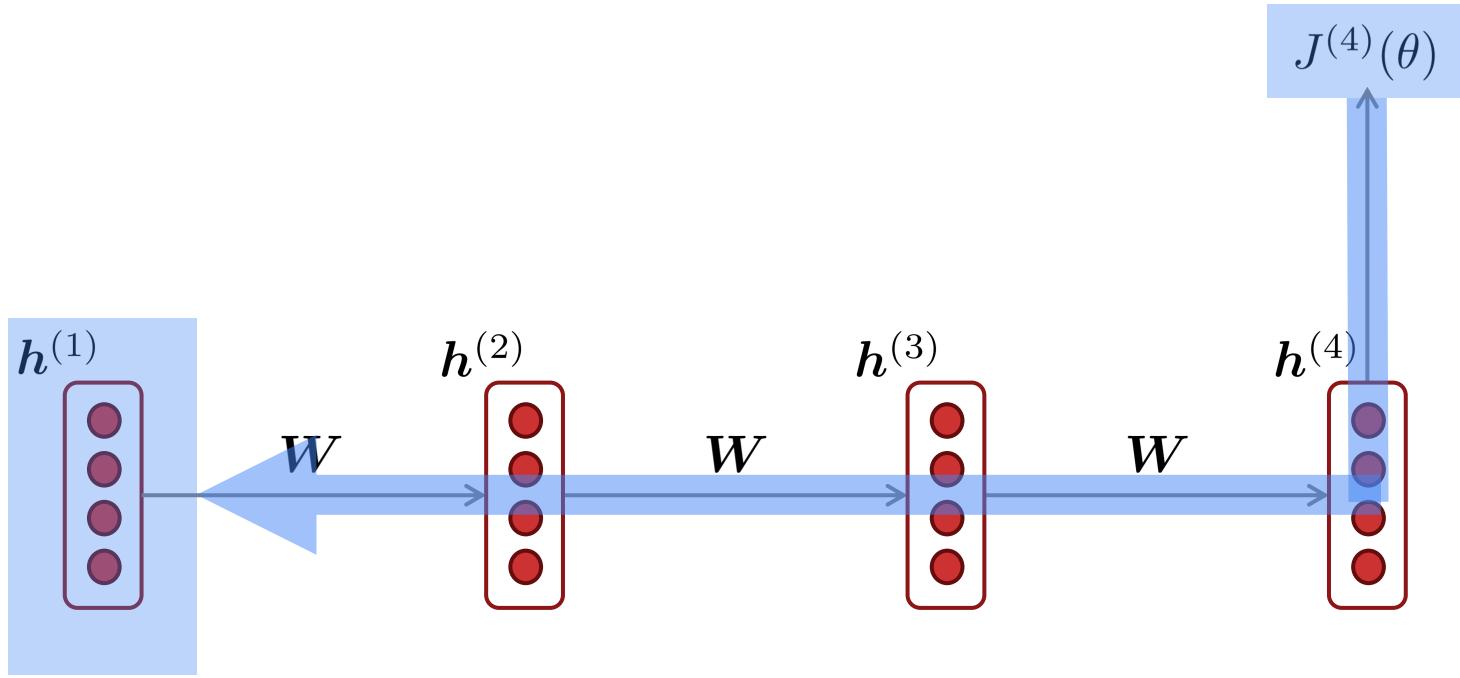


This is an example of a *conditional language model*.
We'll see Machine Translation in much more detail next class.

3. Problems with Vanishing and Exploding Gradients

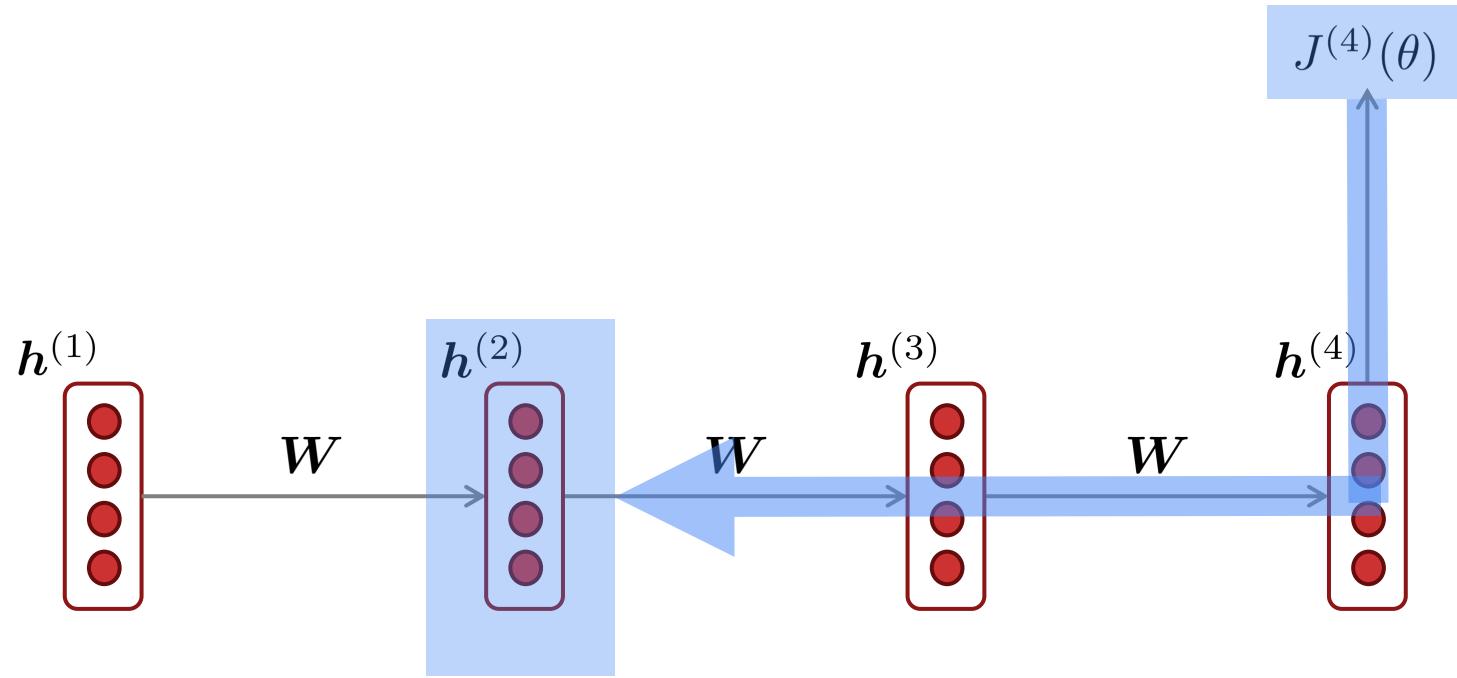


Vanishing gradient intuition



$$\frac{\partial J^{(4)}}{\partial h^{(1)}} = ?$$

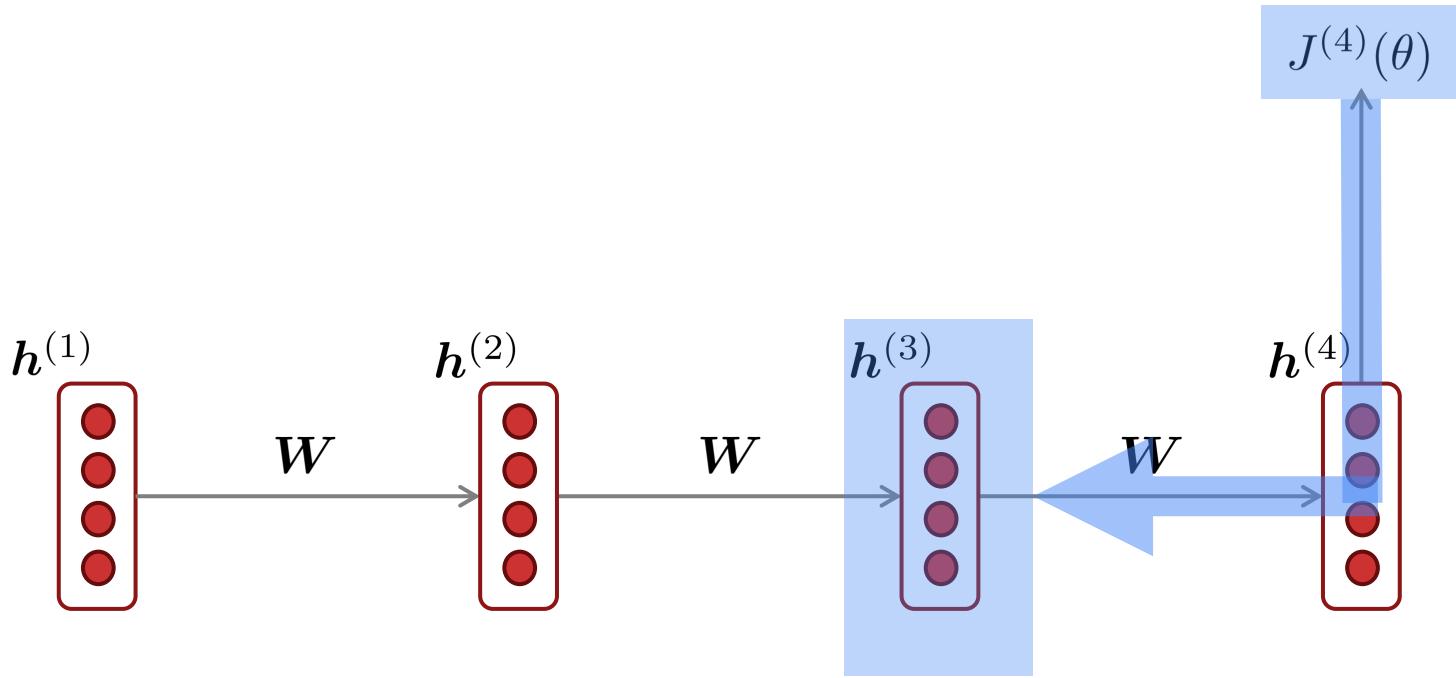
Vanishing gradient intuition



$$\frac{\partial J^{(4)}}{\partial \mathbf{h}^{(1)}} = \frac{\partial \mathbf{h}^{(2)}}{\partial \mathbf{h}^{(1)}} \times \frac{\partial J^{(4)}}{\partial \mathbf{h}^{(2)}}$$

chain rule!

Vanishing gradient intuition

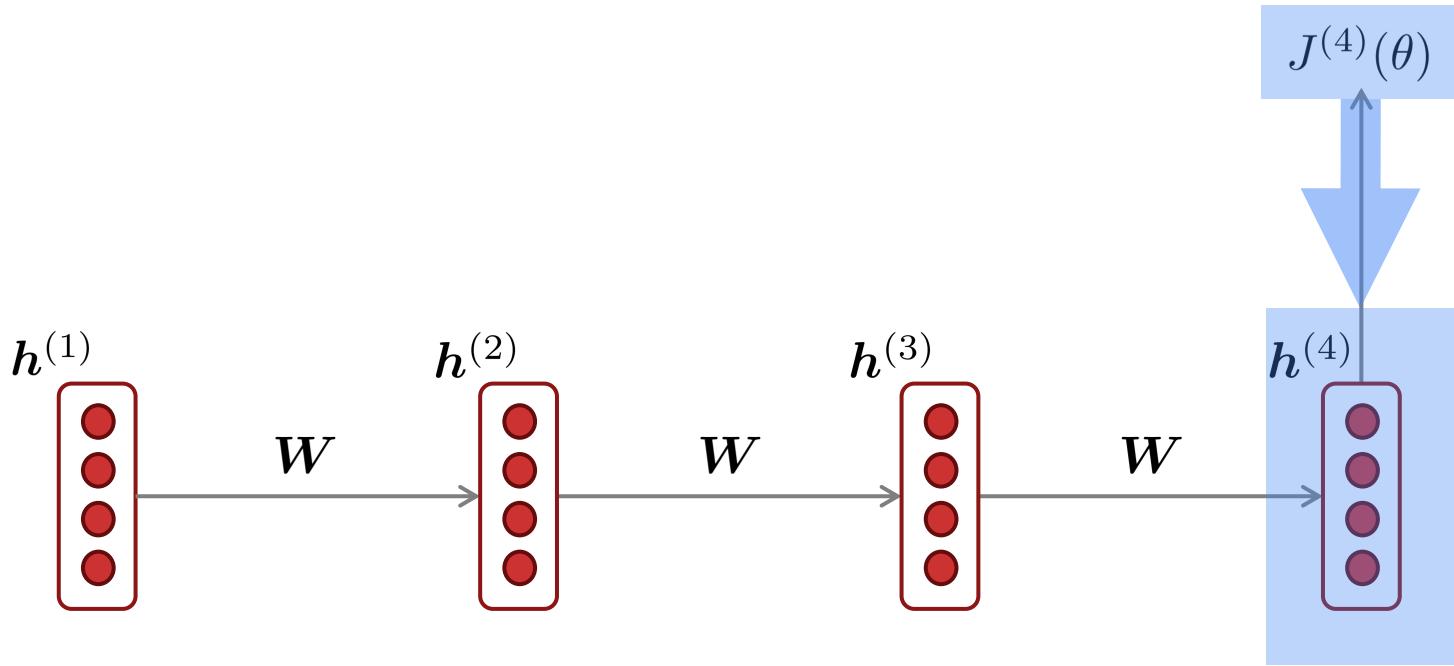


$$\frac{\partial J^{(4)}}{\partial \mathbf{h}^{(1)}} = \frac{\partial \mathbf{h}^{(2)}}{\partial \mathbf{h}^{(1)}} \times$$

$$\frac{\partial \mathbf{h}^{(3)}}{\partial \mathbf{h}^{(2)}} \times \frac{\partial J^{(4)}}{\partial \mathbf{h}^{(3)}}$$

chain rule!

Vanishing gradient intuition



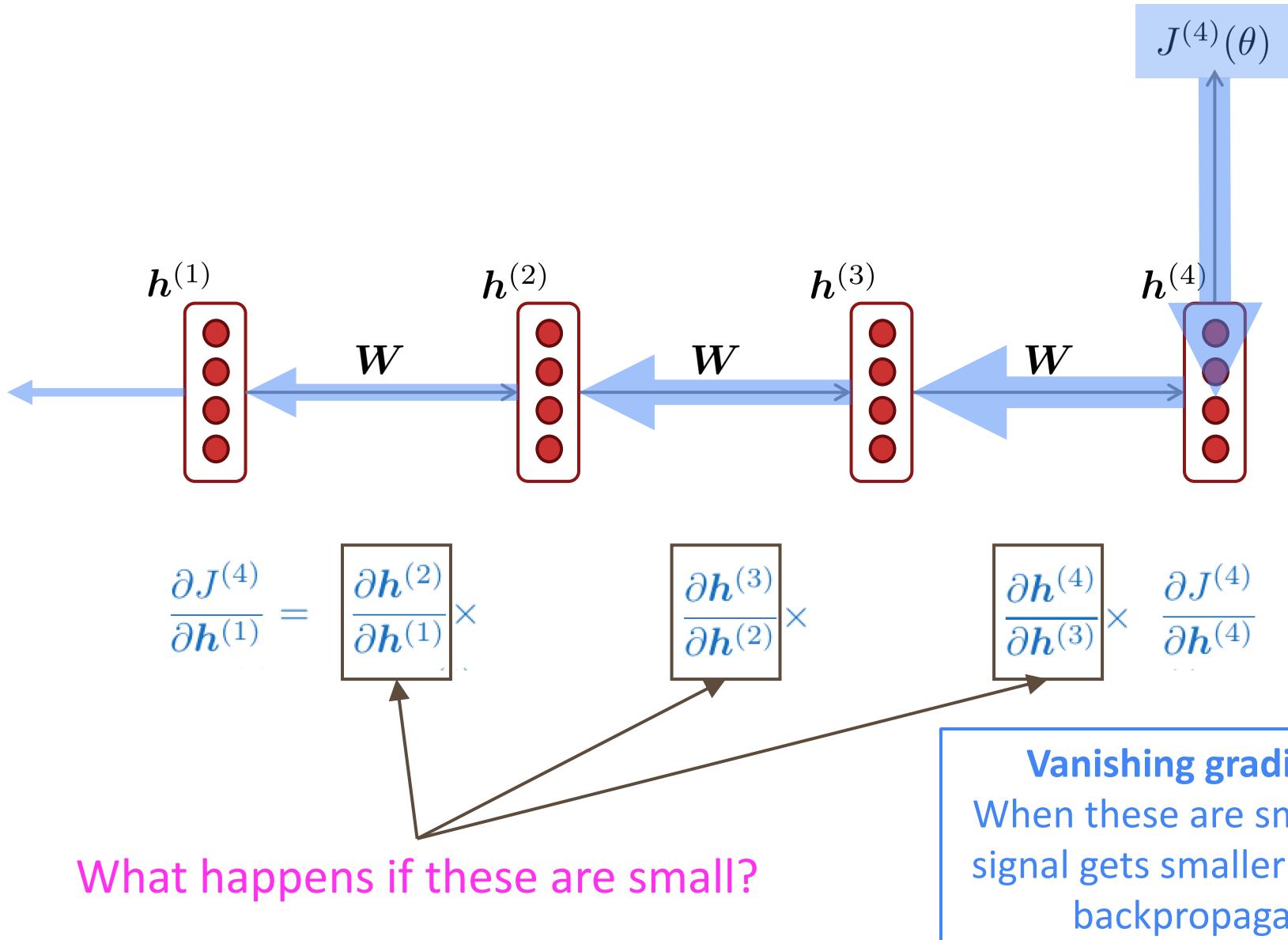
$$\frac{\partial J^{(4)}}{\partial \mathbf{h}^{(1)}} = \frac{\partial \mathbf{h}^{(2)}}{\partial \mathbf{h}^{(1)}} \times$$

$$\frac{\partial \mathbf{h}^{(3)}}{\partial \mathbf{h}^{(2)}} \times$$

$$\frac{\partial \mathbf{h}^{(4)}}{\partial \mathbf{h}^{(3)}} \times \frac{\partial J^{(4)}}{\partial \mathbf{h}^{(4)}}$$

chain rule!

Vanishing gradient intuition



Vanishing gradient proof sketch (linear case)

- Recall:
- What if σ were the identity function, $\sigma(x) = x$?

$$\begin{aligned}\frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{h}^{(t-1)}} &= \text{diag} \left(\sigma' \left(\mathbf{W}_h \mathbf{h}^{(t-1)} + \mathbf{W}_x \mathbf{x}^{(t)} + \mathbf{b}_1 \right) \right) \mathbf{W}_h && \text{(chain rule)} \\ &= \mathbf{I} \quad \mathbf{W}_h = \mathbf{W}_h\end{aligned}$$

- Consider the gradient of the loss $J^{(i)}(\theta)$ on step i , with respect to the hidden state $\mathbf{h}^{(j)}$ on some previous step j . Let $\ell = i - j$

$$\frac{\partial J^{(i)}(\theta)}{\partial \mathbf{h}^{(j)}} = \frac{\partial J^{(i)}(\theta)}{\partial \mathbf{h}^{(i)}} \prod_{j < t \leq i} \frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{h}^{(t-1)}} \quad \text{(chain rule)}$$

$$= \frac{\partial J^{(i)}(\theta)}{\partial \mathbf{h}^{(i)}} \prod_{j < t \leq i} \mathbf{W}_h = \frac{\partial J^{(i)}(\theta)}{\partial \mathbf{h}^{(i)}} \boxed{\mathbf{W}_h^\ell}$$

↑
(value of $\frac{\partial \mathbf{h}^{(t)}}{\partial \mathbf{h}^{(t-1)}}$)

If \mathbf{W}_h is “small”, then this term gets exponentially problematic as ℓ becomes large

Vanishing gradient proof sketch (linear case)

- What's wrong with W_h^ℓ ?
- Consider if the eigenvalues of W_h are all less than 1:
sufficient but
not necessary

$$\lambda_1, \lambda_2, \dots, \lambda_n < 1$$
$$q_1, q_2, \dots, q_n \text{ (eigenvectors)}$$

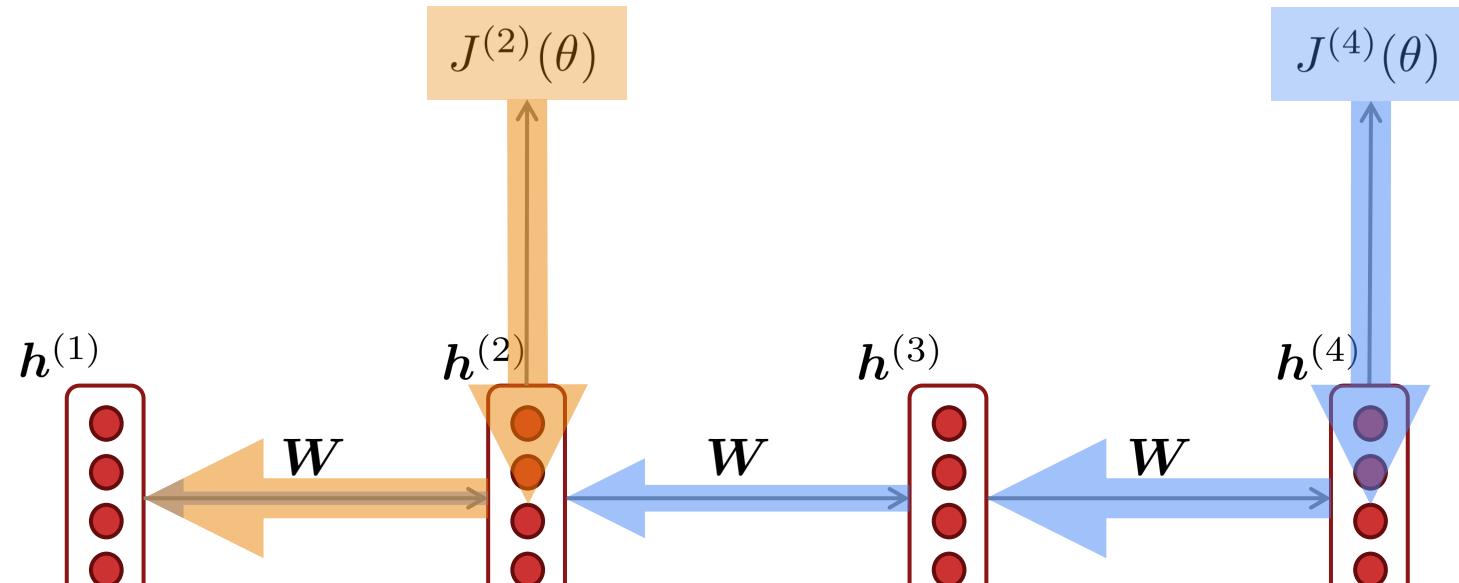
- We can write $\frac{\partial J^{(i)}(\theta)}{\partial h^{(i)}} W_h^\ell$ using the eigenvectors of W_h as a basis:

$$\frac{\partial J^{(i)}(\theta)}{\partial h^{(i)}} W_h^\ell = \sum_{i=1}^n c_i \boxed{\lambda_i^\ell} q_i \approx \mathbf{0} \text{ (for large } \ell\text{)}$$

Approaches 0 as ℓ grows, so gradient vanishes

- What about nonlinear activations σ (i.e., what we use?)
 - Pretty much the same thing, except the proof requires $\lambda_i < \gamma$ for some γ dependent on dimensionality and σ

Why is vanishing gradient a problem?



Gradient signal from far away is lost because it's much smaller than gradient signal from close-by.

So, model weights are updated only with respect to near effects, not long-term effects.

Effect of vanishing gradient on RNN-LM

- **LM task:** *When she tried to print her tickets, she found that the printer was out of toner. She went to the stationery store to buy more toner. It was very overpriced. After installing the toner into the printer, she finally printed her _____*
- To learn from this training example, the RNN-LM needs to **model the dependency** between “*tickets*” on the 7th step and the target word “*tickets*” at the end.
- But if gradient is small, the model **can't learn this dependency**
 - So, the model is **unable to predict similar long-distance dependencies** at test time

Why is exploding gradient a problem?

- If the gradient becomes too big, then the SGD update step becomes too big:

$$\theta^{new} = \theta^{old} - \underbrace{\alpha \nabla_{\theta} J(\theta)}_{\text{gradient}}$$

learning rate

- This can cause **bad updates**: we take too large a step and reach a weird and bad parameter configuration (with large loss)
 - You think you've found a hill to climb, but suddenly you're in Iowa
- In the worst case, this will result in **Inf** or **NaN** in your network
(then you have to restart training from an earlier checkpoint)

Gradient clipping: solution for exploding gradient

- **Gradient clipping**: if the norm of the gradient is greater than some threshold, scale it down before applying SGD update

Algorithm 1 Pseudo-code for norm clipping

```
 $\hat{\mathbf{g}} \leftarrow \frac{\partial \mathcal{E}}{\partial \theta}$ 
if  $\|\hat{\mathbf{g}}\| \geq \text{threshold}$  then
     $\hat{\mathbf{g}} \leftarrow \frac{\text{threshold}}{\|\hat{\mathbf{g}}\|} \hat{\mathbf{g}}$ 
end if
```

- **Intuition**: take a step in the same direction, but a smaller step
- In practice, remembering to clip gradients is important, but exploding gradients are an easy problem to solve

How to fix the vanishing gradient problem?

- The main problem is that *it's too difficult for the RNN to learn to preserve information over many timesteps.*
- In a vanilla RNN, the hidden state is constantly being **rewritten**

$$\mathbf{h}^{(t)} = \sigma \left(\mathbf{W}_h \mathbf{h}^{(t-1)} + \mathbf{W}_x \mathbf{x}^{(t)} + \mathbf{b} \right)$$

- How about a RNN with separate **memory**?

4. Long Short-Term Memory RNNs (LSTMs)

- A type of RNN proposed by Hochreiter and Schmidhuber in 1997 as a solution to the vanishing gradients problem.
 - Everyone cites that paper but really a crucial part of the modern LSTM is from Gers et al. (2000) 
- On step t , there is a hidden state $\mathbf{h}^{(t)}$ and a cell state $\mathbf{c}^{(t)}$
 - Both are vectors length n
 - The cell stores long-term information
 - The LSTM can read, erase, and write information from the cell
 - The cell becomes conceptually rather like RAM in a computer
- The selection of which information is erased/written/read is controlled by three corresponding gates
 - The gates are also vectors length n
 - On each timestep, each element of the gates can be open (1), closed (0), or somewhere in-between
 - The gates are dynamic: their value is computed based on the current context

“Long short-term memory”, Hochreiter and Schmidhuber, 1997. <https://www.bioinf.jku.at/publications/older/2604.pdf>

“Learning to Forget: Continual Prediction with LSTM”, Gers, Schmidhuber, and Cummins, 2000. <https://dl.acm.org/doi/10.1162/089976600300015015>

Long Short-Term Memory (LSTM)

We have a sequence of inputs $x^{(t)}$, and we will compute a sequence of hidden states $h^{(t)}$ and cell states $c^{(t)}$. On timestep t :

Forget gate: controls what is kept vs forgotten, from previous cell state

Input gate: controls what parts of the new cell content are written to cell

Output gate: controls what parts of cell are output to hidden state

Sigmoid function: all gate values are between 0 and 1

$$f^{(t)} = \sigma(W_f h^{(t-1)} + U_f x^{(t)} + b_f)$$

$$i^{(t)} = \sigma(W_i h^{(t-1)} + U_i x^{(t)} + b_i)$$

$$o^{(t)} = \sigma(W_o h^{(t-1)} + U_o x^{(t)} + b_o)$$

New cell content: this is the new content to be written to the cell

Cell state: erase (“forget”) some content from last cell state, and write (“input”) some new cell content

Hidden state: read (“output”) some content from the cell

$$\tilde{c}^{(t)} = \tanh(W_c h^{(t-1)} + U_c x^{(t)} + b_c)$$

$$c^{(t)} = f^{(t)} \circ c^{(t-1)} + i^{(t)} \circ \tilde{c}^{(t)}$$

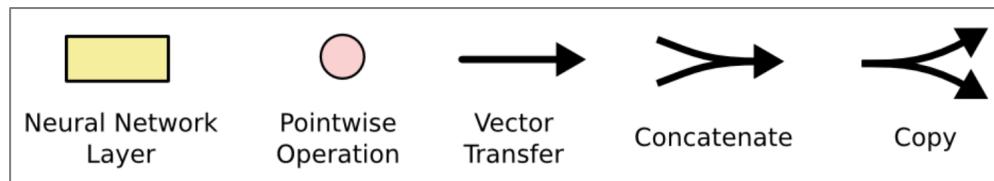
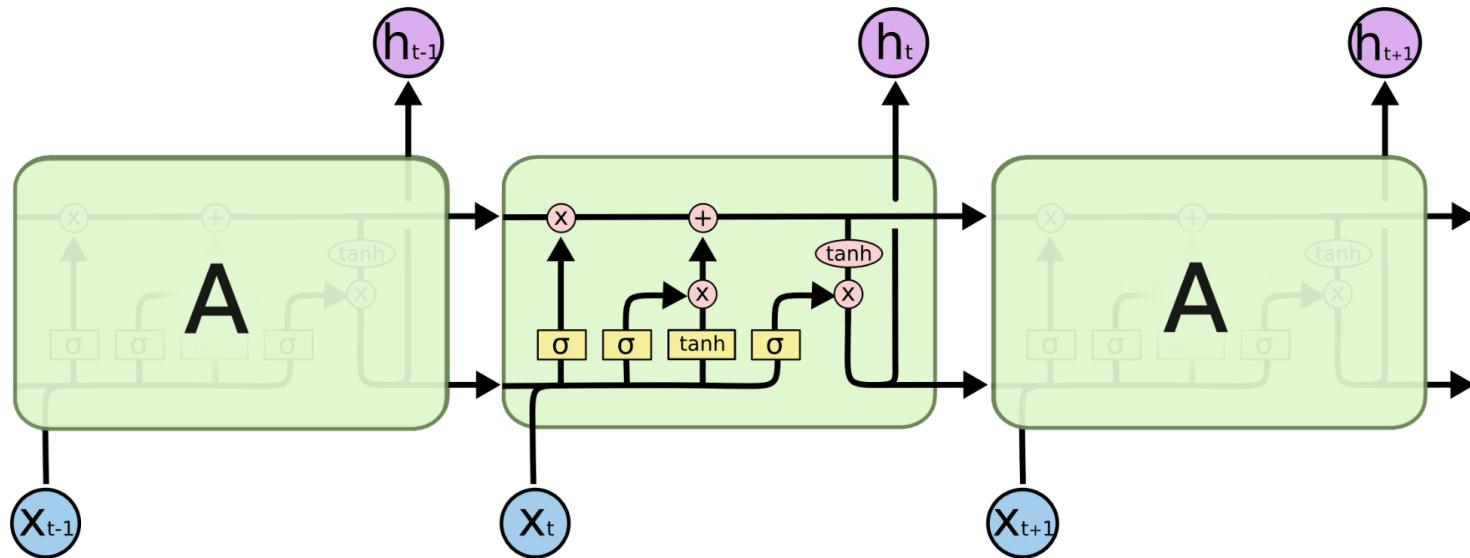
$$h^{(t)} = o^{(t)} \circ \tanh c^{(t)}$$

All these are vectors of same length n

Gates are applied using element-wise (or Hadamard) product: \odot

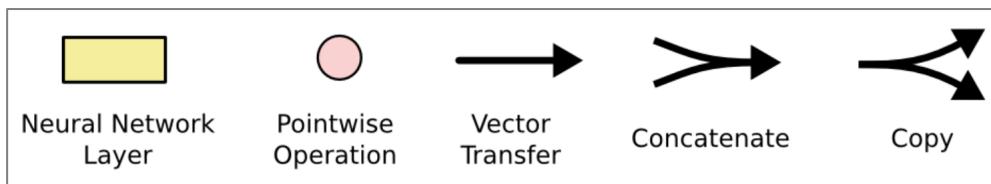
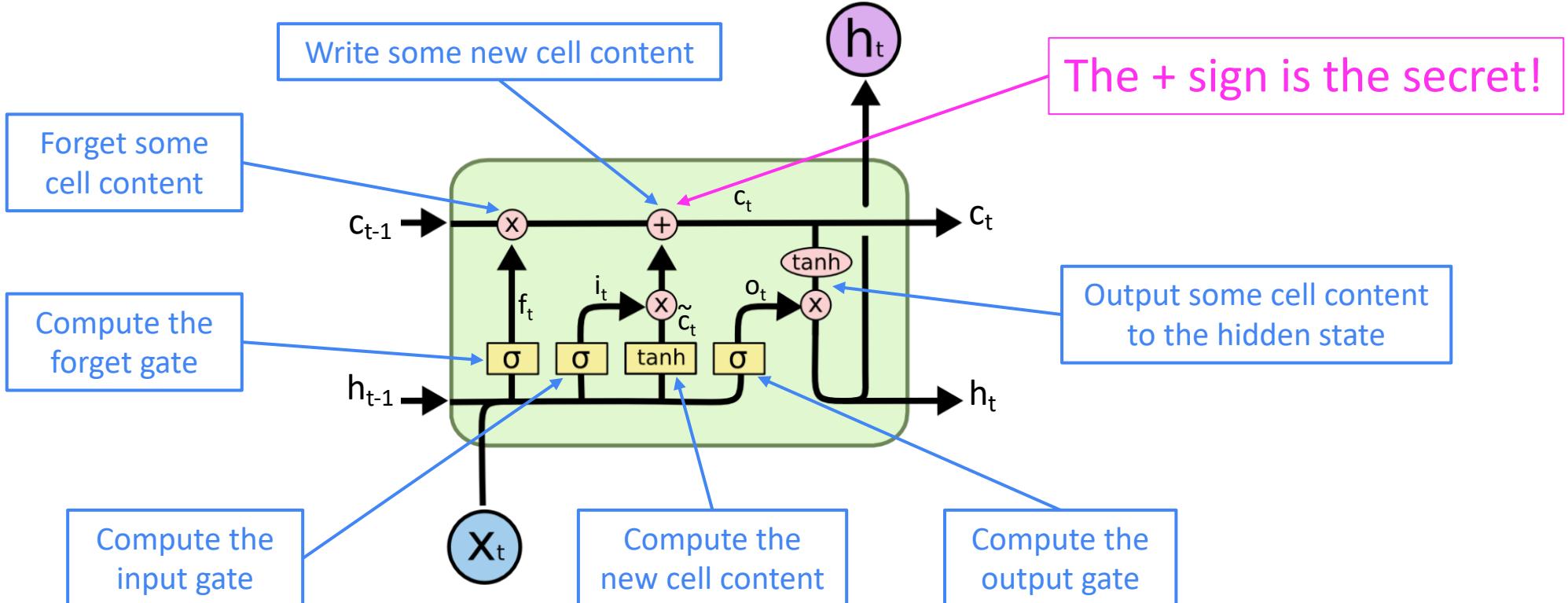
Long Short-Term Memory (LSTM)

You can think of the LSTM equations visually like this:



Long Short-Term Memory (LSTM)

You can think of the LSTM equations visually like this:



Source: <http://colah.github.io/posts/2015-08-Understanding-LSTMs/>

How does LSTM solve vanishing gradients?

- The LSTM architecture makes it easier for the RNN to **preserve information over many timesteps**
 - e.g., if the forget gate is set to 1 for a cell dimension and the input gate set to 0, then the information of that cell is preserved indefinitely.
 - In contrast, it's harder for a vanilla RNN to learn a recurrent weight matrix W_h that preserves info in the hidden state
 - In practice, you get about 100 timesteps rather than about 7
- LSTM doesn't *guarantee* that there is no vanishing/exploding gradient, but it does provide an easier way for the model to learn long-distance dependencies

LSTMs: real-world success

- In 2013–2015, LSTMs started achieving state-of-the-art results
 - Successful tasks include handwriting recognition, speech recognition, machine translation, parsing, and image captioning, as well as language models
 - LSTMs became the dominant approach for most NLP tasks
- Now (2021), other approaches (e.g., Transformers) have become dominant for many tasks
 - For example, in WMT (a Machine Translation conference + competition):
 - In WMT 2016, the summary report contains “RNN” 44 times
 - In WMT 2019: “RNN” 7 times, “Transformer” 105 times

Source: "Findings of the 2016 Conference on Machine Translation (WMT16)", Bojar et al. 2016, <http://www.statmt.org/wmt16/pdf/W16-2301.pdf>

Source: "Findings of the 2018 Conference on Machine Translation (WMT18)", Bojar et al. 2018, <http://www.statmt.org/wmt18/pdf/WMT028.pdf>

Source: "Findings of the 2019Conference on Machine Translation (WMT19)", Barrault et al. 2019, <http://www.statmt.org/wmt18/pdf/WMT028.pdf>

Is vanishing/exploding gradient just a RNN problem?

- No! It can be a problem for all neural architectures (including **feed-forward** and **convolutional**), especially **very deep** ones.
 - Due to chain rule / choice of nonlinearity function, gradient can become vanishingly small as it backpropagates
 - Thus, lower layers are learned very slowly (hard to train)
- Solution: lots of new deep feedforward/convolutional architectures that **add more direct connections** (thus allowing the gradient to flow)

For example:

- **Residual connections** aka “ResNet”
- Also known as **skip-connections**
- The **identity connection** **preserves information** by default
- This makes **deep** networks much **easier to train**

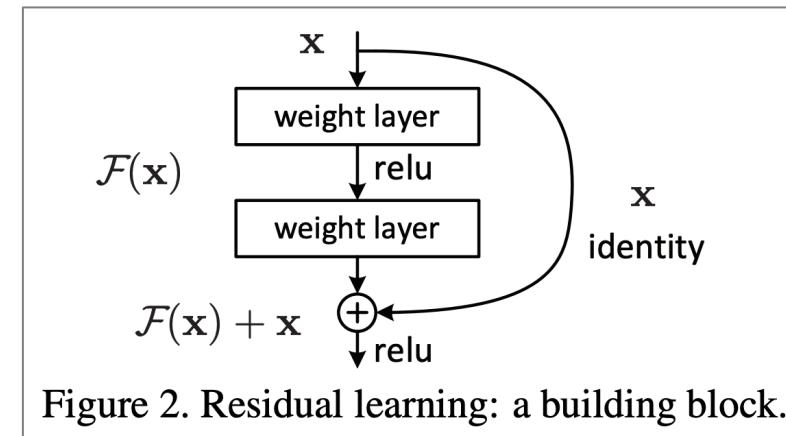


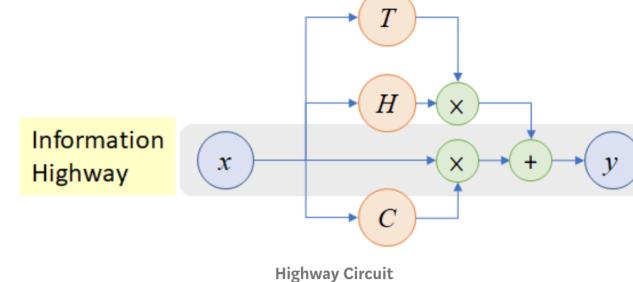
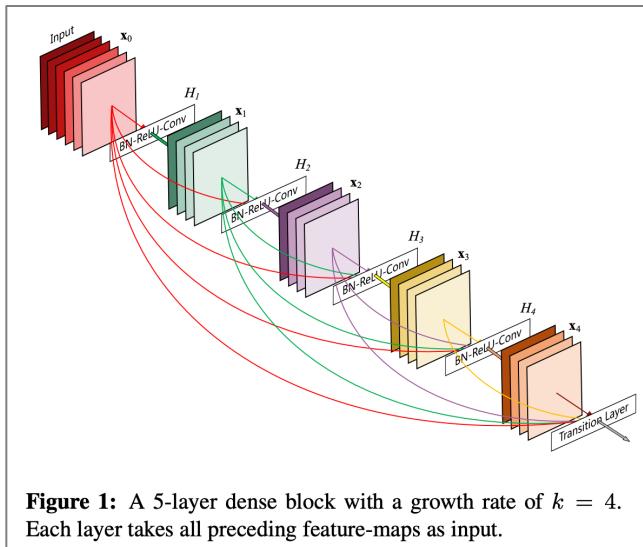
Figure 2. Residual learning: a building block.

Is vanishing/exploding gradient just a RNN problem?

- Solution: lots of new deep feedforward/convolutional architectures that **add more direct connections** (thus allowing the gradient to flow)

Other methods:

- Dense connections** aka “DenseNet”
- Directly connect each layer to all future layers!
- Highway connections** aka “HighwayNet”
- Similar to residual connections, but the identity connection vs the transformation layer is controlled by a **dynamic gate**
- Inspired by LSTMs, but applied to deep feedforward/convolutional networks

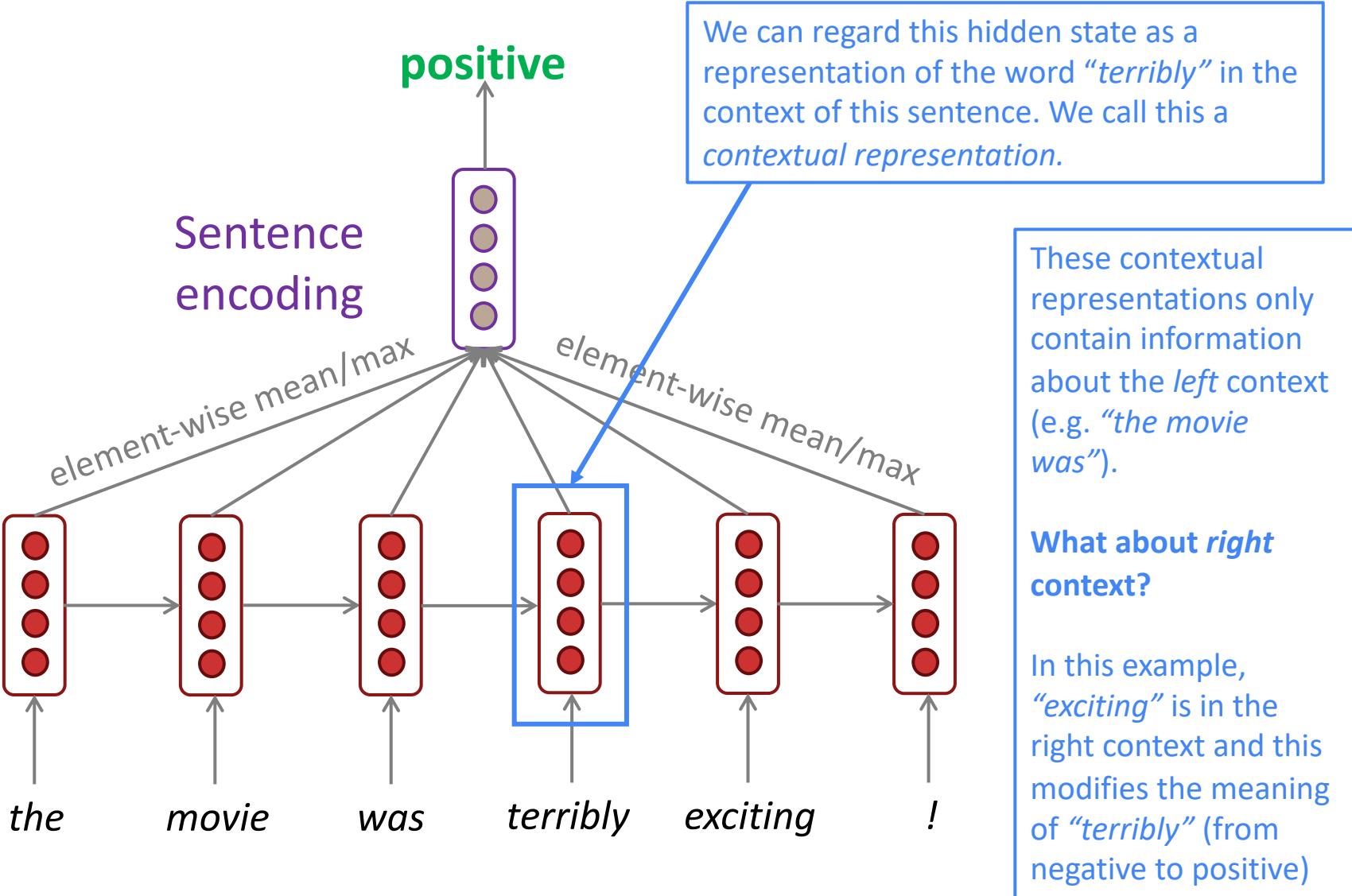


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 - Due to chain rule / choice of nonlinearity function, gradient can become vanishingly small as it backpropagates
 - Thus, lower layers are learned very slowly (hard to train)
- Solution: lots of new deep feedforward/convolutional architectures that **add more direct connections** (thus allowing the gradient to flow)
- **Conclusion:** Though vanishing/exploding gradients are a general problem, **RNNs are particularly unstable** due to the repeated multiplication by the **same** weight matrix [Bengio et al, 1994]

5. Bidirectional and Multi-layer RNNs: motivation

Task: Sentiment Classification



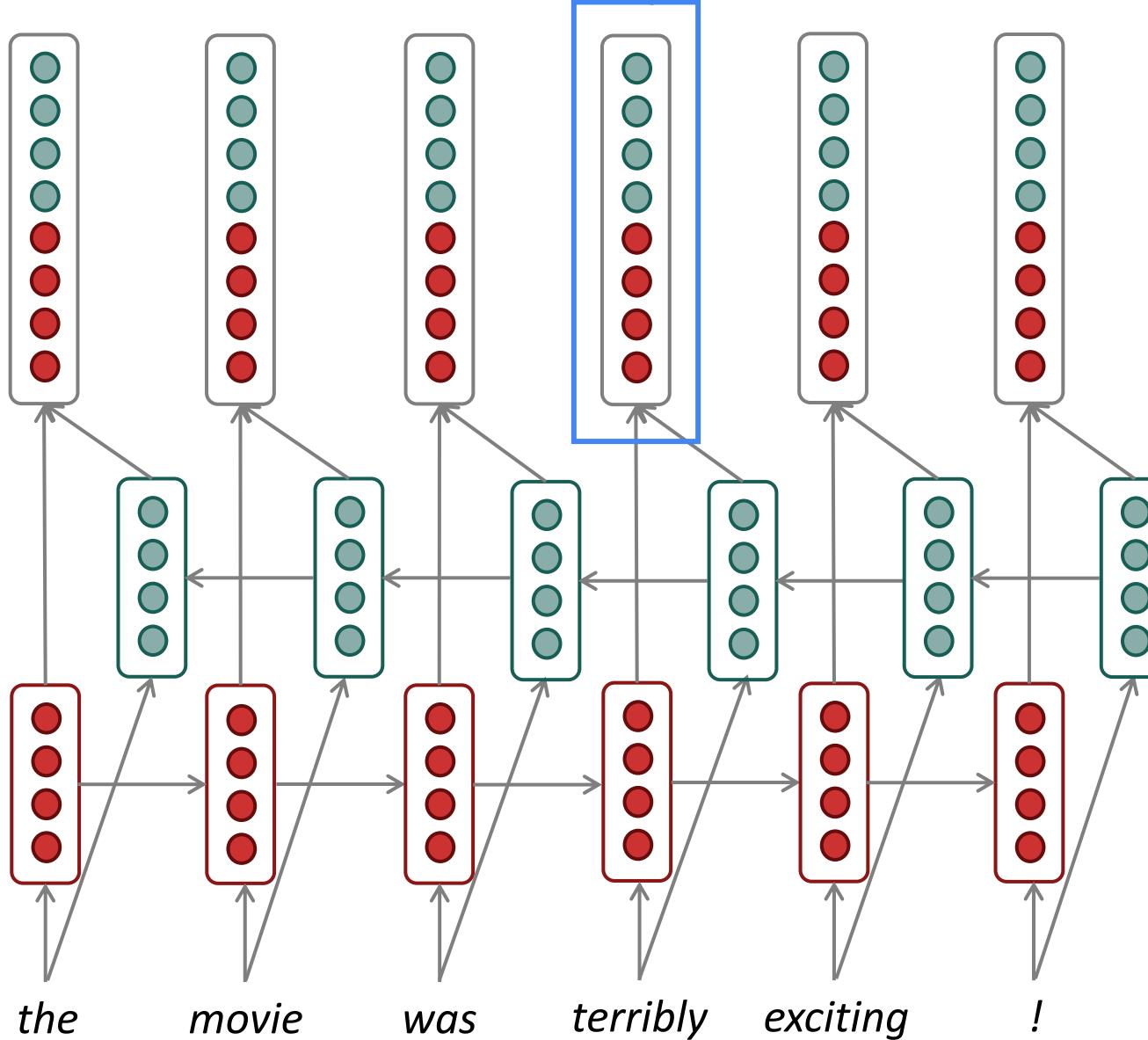
Bidirectional RNNs

Concatenated
hidden states

Backward RNN

Forward RNN

This contextual representation of “terribly”
has both left and right context!



Bidirectional RNNs

On timestep t :

This is a general notation to mean “compute one forward step of the RNN” – it could be a vanilla, LSTM or GRU computation.

Forward RNN $\vec{h}^{(t)} = \text{RNN}_{\text{FW}}(\vec{h}^{(t-1)}, \mathbf{x}^{(t)})$

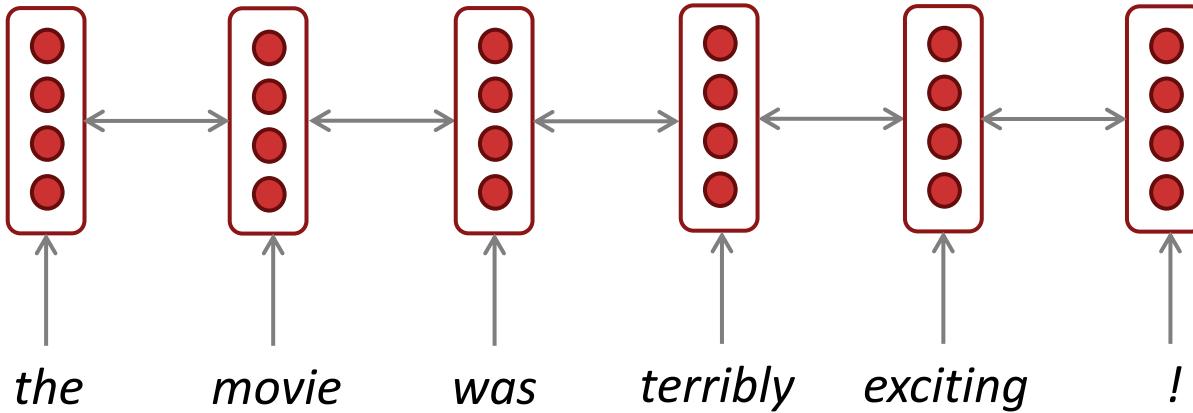
Backward RNN $\overleftarrow{h}^{(t)} = \text{RNN}_{\text{BW}}(\overleftarrow{h}^{(t+1)}, \mathbf{x}^{(t)})$

Concatenated hidden states $\boxed{\mathbf{h}^{(t)}} = [\vec{h}^{(t)}; \overleftarrow{h}^{(t)}]$

Generally, these two RNNs have separate weights

We regard this as “the hidden state” of a bidirectional RNN. This is what we pass on to the next parts of the network.

Bidirectional RNNs: simplified diagram



The two-way arrows indicate bidirectionality and the depicted hidden states are assumed to be the concatenated forwards+backwards states

Bidirectional RNNs

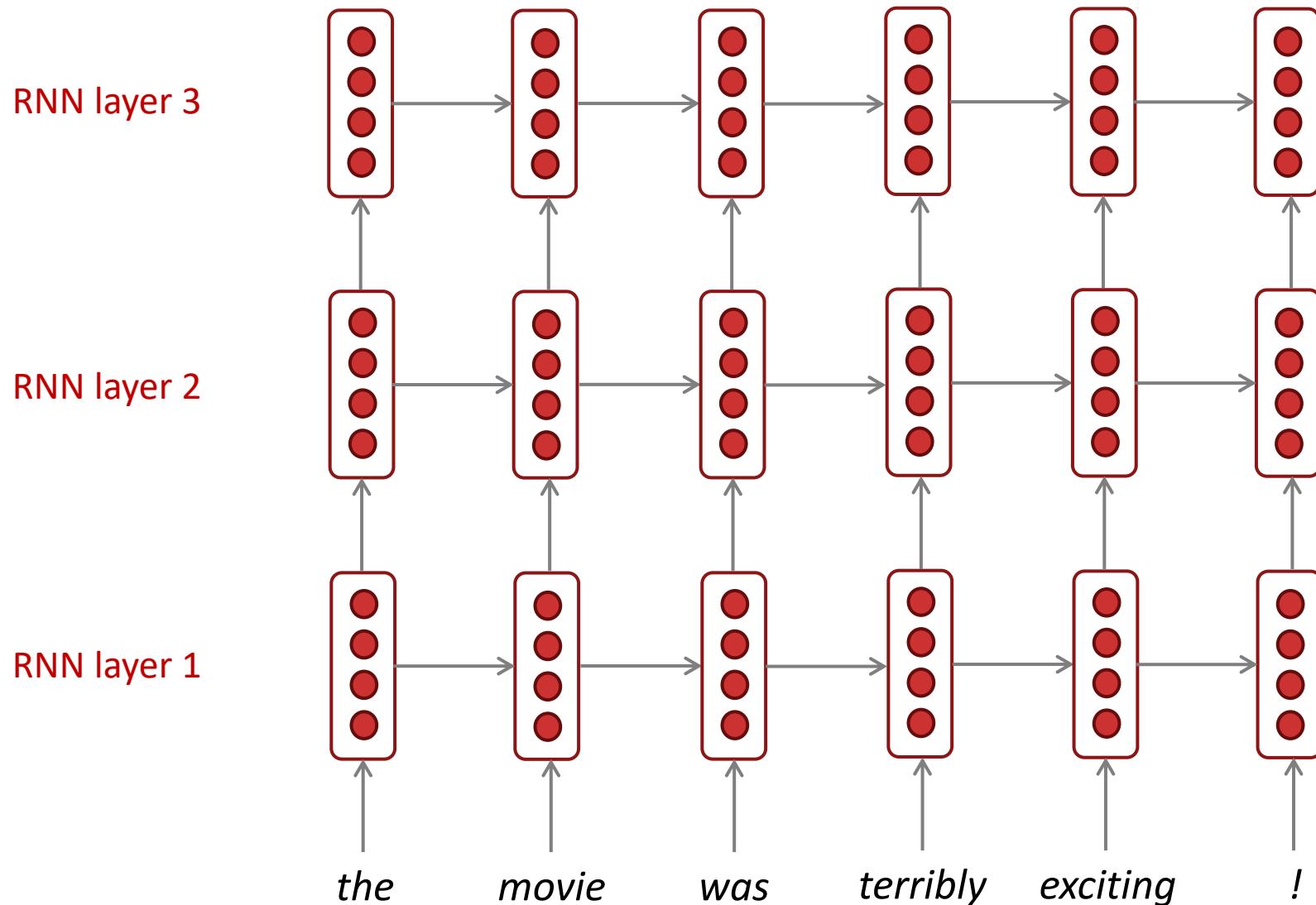
- Note: bidirectional RNNs are only applicable if you have access to the **entire input sequence**
 - They are **not** applicable to Language Modeling, because in LM you *only* have left context available.
- If you do have entire input sequence (e.g., any kind of encoding), **bidirectionality is powerful** (you should use it by default).
- For example, **BERT** (**Bidirectional** Encoder Representations from Transformers) is a powerful pretrained contextual representation system **built on bidirectionality**.
 - You will learn more about **transformers** include BERT in a couple of weeks!

Multi-layer RNNs

- RNNs are already “deep” on one dimension (they unroll over many timesteps)
- We can also make them “deep” in another dimension by applying multiple RNNs – this is a multi-layer RNN.
- This allows the network to compute more complex representations
 - The lower RNNs should compute lower-level features and the higher RNNs should compute higher-level features.
- Multi-layer RNNs are also called *stacked RNNs*.

Multi-layer RNNs

The hidden states from RNN layer i
are the inputs to RNN layer $i+1$

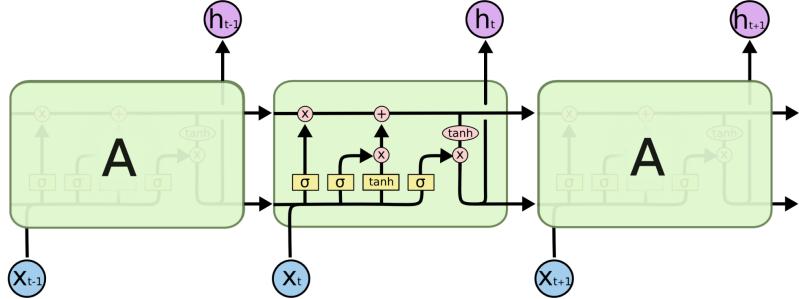


Multi-layer RNNs in practice

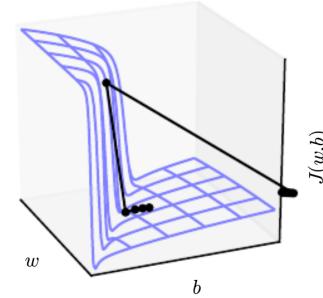
- High-performing RNNs are often multi-layer (but aren't as deep as convolutional or feed-forward networks)
- For example: In a 2017 paper, Britz et al find that for Neural Machine Translation, 2 to 4 layers is best for the encoder RNN, and 4 layers is best for the decoder RNN
 - Usually, skip-connections/dense-connections are needed to train deeper RNNs (e.g., 8 layers)
- Transformer-based networks (e.g., BERT) are usually deeper, like 12 or 24 layers.
 - You will learn about Transformers later; they have a lot of skipping-like connections

In summary

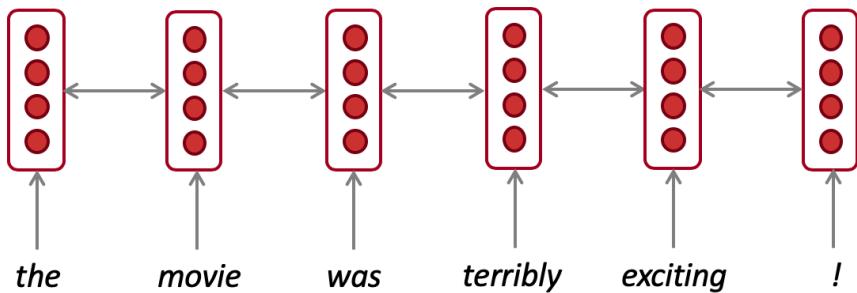
Lots of new information today! What are some of the [practical takeaways](#)?



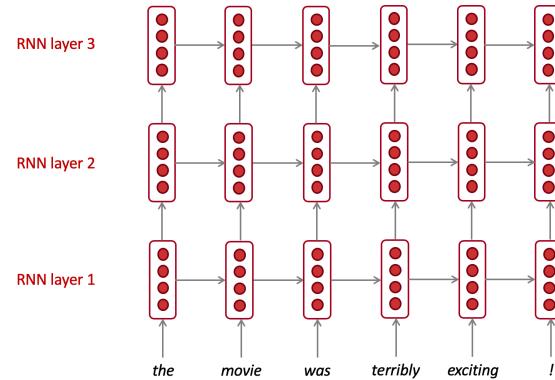
1. LSTMs are powerful



2. Clip your gradients



3. Use bidirectionality when possible



4. Multi-layer RNNs are more powerful, but you might need skip connections if it's deep