Test Canvas: Mock Exam 2

The Test Canvas lets you add, edit, and reorder questions, as well as review a test. More Help

1. Multiple Choice: There are 21 balls that need to be di...

Question	There are 21 balls that need to be distributed to 5 boxes. Which of the following statements is true?
Answer	At most one of the boxes will contain at most five balls.
	✔ At least one of the boxes will contain at least five balls.
	At most one of the boxes will contain at least five balls.
	Every box will have at least a ball.
	Every box will have at most five balls.
	One of the boxes will contain more balls than all other boxes.

☐ **2. Multiple Choice: How many different bit strings of len...**

Question	How many different bit strings of length 12 have equal numbers 0's and 1's?
Answer	2^12 / 2^6 2^12 - 2^6 (2^6)(2^6) ✔ 12! / (6! * 6!) 6! * 6! 6! + 6!

☐ **3. Multiple Choice: In a class of 120 students, 60 are ma...**

Question	In a class of 120 students, 60 are majoring in Computer Science. In the same class, 40 students are majoring in Mathematics. The girl in the front row is double majored in Computer Science and Mathematics. The number of students in the class who majored in Computer Science or Mathematics could be
Answer	0. 20. 50. ✔ 80. 100. 120.

☐ **4. Multiple Choice: A super-arrangement of the set {1, 2,...**

Question	

A super-arrangement of the set {1, 2, 3, …, n} is defined to be a permutation/arrangement of the set such that no element remains in the original position.

An example of a super-arrangement of {1, 2, 3} is {2, 3, 1}. {3, 2, 1} will not be a super-arrangement of {1, 2, 3}, since 2 did not change position.

Given the number of super-arrangement of the set {1, 2, …, n} is approximately equal to n!/e. The probability that a permutation/arrangement of the set {1, 2, …, n} is a super-arrangement is approximately equal to

Answer

- 1
- 1/n
- ✓ 1/e
- e/n
- n/e
- 1 - e/n

5. Multiple Choice: There are 26 letters in the English a...

Question

There are 26 letters in the English alphabet and 5 of them are vowels and 21 are consonants. The alphabet is ordered from A to Z. In how many ways can the letters of the alphabet be permuted/arranged if none of the consonants change position?

Answer

- ✓ 5!
- 21!
- 26! - 5!
- 26! - 21!
- 21! - 5!
- 21! + 5!

6. Multiple Choice: A student has four hats, five jackets...

Question	A student has four hats, five jackets and three pairs of shoes. One of the hats is blue, two of the jackets are blue and one pair of shoes are blue. In how many different ways can the student dress up (wearing a hat, a jacket, and shoes) such that they are NOT wearing ALL blue?
Answer	8 10 24 56 ✔ 58 60

☐ 7. Multiple Choice: Two joker cards, one red and one blac...

Question	Two joker cards, one red and one black are added to a standard deck of 52 cards, now you have a deck of 54 cards. Two cards are randomly selected from this deck. What's the probability that you selected both jokers?
Answer	1/27 1/53 1/27 + 1/53 1/54 + 1/53 ✔ 1 / (27*53) 1 / (54*53)

☐ 8. Multiple Choice: An investor has purchased three disjo...

Question	

An investor has purchased three disjoint (non-overlapping) pieces of land. Each piece of land has an independent chance of having oil: one has a 1/2 probability and one has a 1/3 probability and one has a 1/4 probability of having oil. What’s the probability that at least one of the properties/lands that the investor purchased has oil?

Answer

- 1/2 + 1/3 + 1/4 - 1/12
- 1/24
- 1/2
- 2/3
- ✔ 3/4
- 1/2 - 1/3 + 1/4

☐ **9. Multiple Choice: In an election, three people ar...**

Question

In an election, three people are voting for either A or B. Each person votes independently and votes for A with probability 2/5 and B with probability 3/5. The probability that all three votes are not cast for the same candidate is

Answer

- 8/125.
- 27/125.
- 15/125.
- ✔ 90/125.
- 98/125.
- 117/124.

☐ **10. Multiple Choice: You have a set of 26 alphabet cards, ...**

Question

You have a set of 26 alphabet cards, each card has a different letter of the English alphabet written on one side. The cards are shuffled and you select 7 face down cards and then you order them in a line (without looking at the cards). What’s the probability that when you turn them over, they read, “ILOVENY”?

Answer

- 1 / (26!×7!)
- 7! / 26!
- 1 / (26! - 7!)
- ✔ (26-7)! / 26!
- 1/26! + 1/7!
- 1/(26+7)!

☐ **11. Essay: In how many ways can you create a use...**

Question

In how many ways can you create a username with ten characters where each character is an uppercase letter or a digit (0-9) and your username needs to start with a letter and end with a digit?

Explain how you arrived at your answer – and DO NOT simplify your answer.

Answer

The first character needs to be a letter, so there will be 26 choices.

The last character needs to be a digit, so 10 choices,

The middle 8 characters could be a letter or a digit, so 26+10 = 36 choices.

Therefore, the answer will be 26*10*36^8.

☐ **12. Essay: An exam consists of six multiple-choi...**

Question

An exam consists of six multiple-choice questions, each with five possible answers. If a student randomly selects an answer, he has a 1/5 chance to answer a question correctly. What is the probability this student got exactly half of the questions correct if he guessed on all six?

Explain how you arrived at your answer – and simplify your answer as a single fraction without combination and permutation notation, It's okay to have power or factorial in your fraction.

Answer

The probability of answering a question correctly is 1/5,

P(correct) = 1/5

P(incorrect) = 4/5

We want to find the probability of getting exactly 3 MC questions right, which we will have 3 correct and 3 incorrect.

There are C(6,3) = 6!/(3!3!) = 20 ways to get 3 correct and 3 incorrect. and the probability of each way is (1/5)^3 * (4/5)^3 = 4^3/5^6.

Therefore, the answer is 20 * 4^3/5^6.

☐ **13. Essay: A new lottery awards eight different ...**

Question	A new lottery awards eight different prizes (first prize, second prize, third prize, ..., and eighth prize), and there are only one for each prize. If your numbers are lucky enough, you can win multiple prizes, i.e., you can win only third prize, or both the first and the second prizes, or the third and fifth and eighth prizes, etc. but you can't win two first prizes. In how many ways can you have a winning ticket? (That is, in how many ways can you win at least one prize?) Explain how you arrived at your answer – and DO NOT simplify your answer.
Answer	There are 8 different prizes, you can either win or not win. So there are 2^8 different outcomes to win or not win the prizes. There is only one way not to win any prize. Therefore, the answer is 2^8 - 1.

☐ **14. Essay: What's the probability that a randoml...**

Question	What's the probability that a randomly selected bit string of length 10 begins with at least four consecutive 0's or ends with at least three consecutive 1's? Explain how you arrived at your answer – and leave your answer in fraction form, you don't need to simplify it.
Answer	There are 2^10 10-bits strings. There are 2^6 of them that begin with at least four 0's, 2^7 of them end with at least three 1's, and 2^3 of them begin with at least four 0's and end with at least three 1's. Therefore, the answer is (2^6 + 2^7 - 2^3) / 2^10.

☐ **15. Essay: You have five coins: three fair coins...**

Question	

You have five coins: three fair coins and two two-headed coins (that, naturally, always comes up Heads). One of the five coins is selected at random (meaning each equally likely, from a uniform distribution) and is flipped. What is the probability that the result is Heads?

Explain how you arrived at your answer – and then simplify your answer to a reduced fraction.

Answer

Since we have three fair coins (F) and two bias coins (B), P(F) = 3/5 and P(B) = 2/5.

P(H|F) = 1/2 and P(H|B) = 1.

P(H) = P(F and H) + P(B and H) = (3/5)(1/2) + (2/5)*1 = 3/10 + 4/10 = 7/10

☐ **16. Essay: You have five coins: three fair coins...**

Success: Question edited.

Question

You have five coins: three fair coins and two two-headed coins (that, naturally, always comes up Heads). One of the five coins is selected at random (meaning each equally likely, from a uniform distribution) and is flipped. It comes up Heads. What's the probability that a fair coin was used?

Explain how you arrived at your answer – and then simplify your answer to a reduced fraction.

Answer

Since we have three fair coins (F) and two bias coins (B), P(F) = 3/5 and P(B) = 2/5.

P(H|F) = 1/2 and P(H|B) = 1.

P(H) = P(F and H) + P(B and H) = (3/5)(1/2) + (2/5)*1 = 3/10 + 4/10 = 7/10

P(F|H) = P(F and H) / P(H) = (3/5)(1/2) / (7/10) = (3/10) / (7/10) = 3/7

☐ **17. Essay: Ten distinct integers are chosen from...**

Question

Ten distinct integers are chosen from 1 to 18. Briefly explain in a full sentence or two why it must be true that two of the numbers selected will have products with an even number.

(Given a theorem/concept's name is not enough, you need to explain it.)

Answer

We are selecting 10 distinct numbers from 1 to 18.

From 1 to 18, there are 9 even and 9 odd numbers.

no matter how you select the 10 numbers, at least one of them will be even, since there are only 9 odd.

Since you will select at least one even number, take the even number and multiply with any other number, you will get a product of even number.

☐ **18. Essay: There are 50 balls labeled from 1 to ...**

Question

There are 50 balls labeled from 1 to 50. You are asked to distribute the 50 balls into two boxes (call box A and box B), so that none of the boxes is empty. In how many ways can this be done?

Explain how you arrived at your answer – and leave your answer in a 'simple' form (you do NOT need to simplify your answer).

Answer

There are 50 distinct balls, each ball can be placed in box A or box B, so there are 2^50 ways to place the balls.

We don't want the box to be empty, which we can't place all the balls into box A or all in box B, so we have 2 bad cases.

Therefore, the answer is 2^50 - 2.

☐ **19. Essay: How many distinct permutations are th...**

Question

How many distinct permutations are there of the word PEOPLE? That is, how many different six-letter strings can be formed using those letters?

Explain how you arrived at your answer – and then simplify your answer to an integer. (Note that you don't have to make 'real' words, any rearrangement is fine.)

Answer

There are 2 P's, 2 E's, 1 O's, and 1 L's, a total of 6 letters in PEOPLE.

Therefore, the answer are 6! / (2!2!) = 6*5*4*3*2*1 / (2*1*2*1) = 180.

☐ **20. Essay: A student is designing a special six-...**

Question

A student is designing a special six-sided die. Her design requires that when the die is rolled,

(i) a 6 should come up with probability 1/6 and

(ii) a 5 should come up with probability 1/5 and

(iii) a 4 should come up with probability 1/4 and

(iv) a 3 should come up with probability 1/3 and

(v) a 2 should come up with probability 1/2.

The student asks you if her design makes sense. And if it does, what would happen when the die is rolled and a 1 comes up?

Explain what you would tell the student – and why.

Answer

P(2) = 1/2, P(3) = 1/3, P(4) = 1/4, P(5) = 1/5, P(6) = 1/6

We want to find P(1).

We also know that P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1

P(1) + 1/2 + 1/3 + 1/4 + 1/5 + 1/6 = 1

P(1) = 1- (1/2 + 1/3 + 1/4 + 1/5 + 1/6) = 1 - (30/60 + 20/60 + 15/60 + 12/60 + 10/60) = 1 - 87/60 = -27/60 = - 9/20

This design cannot be done, since it will force the probability of getting 1 to be negative and the probability cannot be negative.