CSCI 220 | Spring 2022

Discrete Structure

Permutations and Combinations

Discrete Mathematics and its Application

Section 6.3, 6.5

Counting Exercise:

- In how many ways can we select three students from a group of five students to stand in line for a picture? In how many ways can we arrange all five of these students in a line for a picture?

$5 \cdot 4 \cdot 3$ X X

$P(5, 3)$

$$\frac{5!}{(5-3)!} = \frac{5!}{2!}$$

Permutations

- A permutation of a set of distinct objects is an ordered arrangement of these objects.
- An ordered arrangement of r elements of a set is called an r-permutation.
- If n is a positive integer and r is an integer with $1 \leq r \leq n$, then there are

$$(n)_r = {}_nP_r = P(n,r) = n(n-1)(n-2)\cdots\underbrace{(n-r+1)}_{r\text{th}} = \frac{n!}{(n-r)!}$$

r-permutations of a set with n distinct elements.

Counting Exercise:

- How many ways are there to select a first-prize winner, a second-prize winner, and a third-prize winner from 100 different people who have entered a contest?

$$100 \cdot 99 \cdot 98$$

$$P(100, 3) = \frac{100!}{(100-3)!} = \frac{100!}{97!}$$

Counting Exercise:

- Suppose that a salesman has to visit eight different cities. He must begin his trip in a specified city, but he can visit the other seven cities in any order he wishes. How many possible orders can the salesman use when visiting these cities?

$$7! = (8-1)!$$

Counting Exercise:

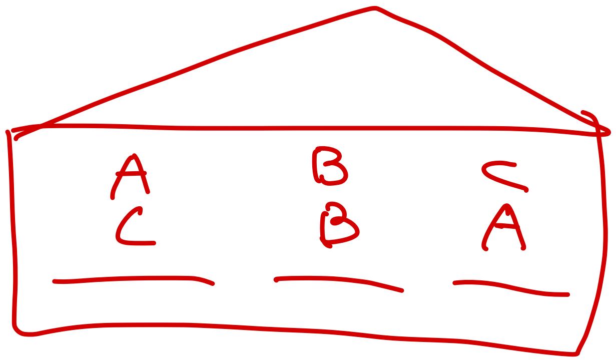
- How many permutations of the letters ABCDEFGH contain the substring “ABC” ?

$\boxed{ABC}\ \boxed{D}\ \boxed{E}\ \boxed{F}\ \boxed{G}\boxed{H}$

$6!$

Counting Exercise:

- How many different committees of three students can be formed from a group of four students?



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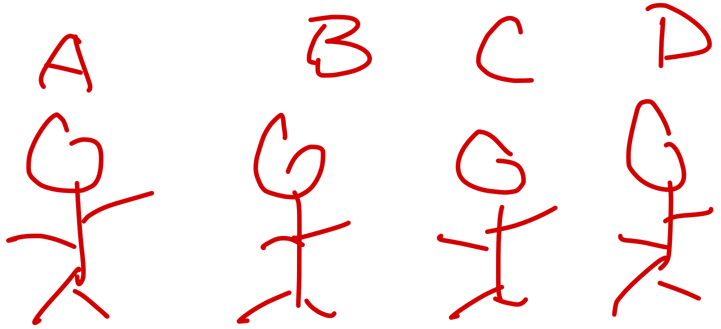

$$\frac{4 \times 3 \times 2}{3!} = 4$$

Combinations

- An r-combination of elements of a set is an unordered selection of r elements from the set.
- The number of r-combinations of a set with n elements, where n is a nonnegative integer and r is an integer with $0 \leq r \leq n$, equals

$$C(n,r) = \frac{n!}{r!\,(n-r)!}.$$

$$C(n,r) = \frac{P(n,r)}{r!}$$

Counting Exercise:

- How many poker hands of five cards can be dealt from a standard deck of 52 cards? Also, how many ways are there to select 47 cards from a standard deck of 52 cards?

$$C(52,5) = \frac{52!}{47!\,5!}$$

$$C(52,47) = \frac{52!}{47!\,5!}$$

the same.

Combinations

- Let n and r be nonnegative integers with $r \leq n$. Then $C(n,r) = C(n,n-r)$.

$$\frac{n!}{r!\,(n-r)!} = \frac{n!}{(n-r)!\,r!}$$

$$(n-(n-r))!$$

Counting Exercise:

- How many ways are there to select five players from a 10-member tennis team to make a trip to a match at another school?

$$_{10}C_5 = \frac{10!}{5!\,5!}$$

Counting Exercise:

m1, m2, ... m9

c1, c2, ... c11.

- Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department?

math

cs

$$\boxed{C(9,3) * C(11,4)}$$

$$C(9,3) + C(11,4)$$

Select 3 members at least 1 from each department.

$$\boxed{C(20,3) - C(9,3) - C(11,3)}$$

C1: 1 m and 2 cs

C2: 2 m and 1 cs

$$\boxed{C(9,1) * C(11,2) + C(9,2) * C(11,1)}$$

Counting Exercise:

- How many strings of length 8 can be formed from the uppercase letters of the English alphabet?

$\overline{26} * \overline{26} * \overline{26} * \overline{26} * \overline{26} * \overline{26} * \overline{26} * \overline{26} = \boxed{26^8}$

Permutations with Repetition

- The number of r-permutations of a set of n objects with repetition allowed is n^r.

$$\underbrace{\underline{\quad}\;\underline{\quad}\;\underline{\quad}\;\underline{\quad}\;\cdots\;\underline{\quad}}_{}$$

$$\underbrace{n \quad n \quad n \quad n \quad \cdots \quad n}_{r \text{ times.}} = n^r.$$

Counting Exercise:

- How many ways are there to select four pieces of fruit from a bowl containing apples, oranges, and pears if the order in which the pieces are selected does not matter, only the type of fruit and not the individual piece matters, and there are at least four pieces of each type of fruit in the bowl?

A	O	P
4	0	0
3	1	0
3	0	1
2	2	0
⋮	⋮	⋮

A O P

****||

***|*|

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$C(6,2) = C(6,4)$

Counting Exercise:

- How many ways are there to select five bills from a cash box containing $1 bills, $2 bills, $5 bills, $10 bills, $20 bills, $50 bills, and $100 bills? Assume that the order in which the bills are chosen does not matter, that the bills of each denomination are indistinguishable, and that there are at least five bills of each type.

1 2 5 10 20 50 100

* | * * | * | * | | |

$C(11,5)$ ← choosing stars.

$= C(11,6)$ ← choosing bars.

Combinations with Repetition

- There are $C(n + r - 1, r) = C(n + r - 1, n - 1)$ r-combinations from a set with n elements when repetition of elements is allowed.

stars * selecting r objects.

C(r+n-1, r)

C(r+n-1, n-1)

n type of objects

n-1 bars | to seperate the objects

Counting Exercise:

c | s | v, | m

- Suppose that a cookie shop has four different kinds of cookies. How many ways can six cookies be chosen? Assume that only the type of cookie, and not the individual cookies or the order in which they are chosen, matters.

$* \; * \mid * \; * \mid * \; * \mid$

$4-1$

6 stars and 3 bars.

$C(9, 6)$

$C(9, 3)$

Summary

- Combinations and Permutations With and Without Repetition

Type	***Repetition Allowed?***	***Formula***
r-permutations	No	$\frac{n!}{(n-r)!}$
r-combinations	No	$\frac{n!}{r!\,(n-r)!}$
r-permutations	Yes	n^r
r-combinations	Yes	$\frac{(n+r-1)!}{r!\,(n-1)!}$

Counting Exercise:

- How many ways are there to distribute hands of 5 cards to each of four players from the standard deck of 52 cards?

$$C(52, 5) * C(47, 5) * C(42, 5) * C(37, 5)$$

for player 1   player 2   player 3   player 4