CSCI 220 | Spring 2022

Discrete Structure

Expected Value and Variance

Discrete Mathematics and its Application

Section 7.4

Expected Value Exercise:

- How many heads are expected to appear when a fair coin is flipped 100 times?

$$P(H) = \frac{1}{2}$$

$$\frac{1}{2} * 100 = 50$$

Expected Value Exercise:

- If a coin is bias, the probability of flipping a head is 2/5, how many heads are expected to appear when this coin is flipped 100 times?

$$P(H) = \frac{2}{5}$$

$$\frac{2}{5} * 100 = 40$$

Expected Values

- The *expected value*, also called the *expectation* or *mean*, of the random variable X on the sample space S is equal to

$$E(X) = \sum_{s \in S} p(s)X(s) \, .$$

- The *deviation* of X at $s \in S$ is $X(s) - E(X)$, the difference between the value of X and the mean of X.

Expected Value of a Die

- Let X be the number that comes up when a fair die is rolled. What is the expected value of X?

$E(X) =$

$X(s)$		$P(s)$	
1	*	1/6	1/6
2	*	1/6	2/6
3	*	1/6	3/6
4	*	1/6	4/6
5	*	1/6	5/6
6	*	1/6	+ 6/6

$21/6 = 7/2 = \boxed{3.5}$

Exercise on Expected Values

- A fair coin is flipped three times. Let S be the sample space of the eight possible outcomes and let X be the random variable that assigns to an outcome the number of heads in this outcome. What is the expected value of X?

X		P		
0	*	1/8	=	0
1	*	3/8	=	3/8
2	*	3/8	=	6/8
3	*	1/8	=	3/8
				12/8

$= 3/2 = 1.5$

Expected Values

- If X is a random variable and $p(X = r)$ is the probability that $X = r$, so that $p(X = r) = \sum_{s \in S, X(s)=r} p(s)$ then

$$E(X) = \sum_{r \in X(s)} p(X = r)\, r \ .$$

Exercise on Expected Values

- What is the expected value of the sum of the numbers that appear when a pair of fair dice is rolled?

X	P
2	1/36
3	2/36
4	3/36
5	
⋮	⋮
12	1/36

$$E(X) = 2(1/36) + 3(2/36) + 4(3/36) + 5(4/36) + 6(5/36) + 7(6/36) + 8(5/36) + 9(4/36) + 10(3/36) + 11(2/36) + 12(1/36)$$

$$= \frac{2+6+12+20+30+42+40+36+30+22+12}{36} = \frac{252}{36} = 7$$

Variance

- Let X be a random variable on a sample space S. The variance of X, denoted by $V(X)$, is

$$V(X) = E(X^2) - E(X)^2.$$

$$\sum_{s \in S} (X(s) - E(s))^2 P(s)$$

That is, $V(X)$ is the weighted average of the square of the deviation of X. The standard deviation of X, denoted $\sigma(X)$, is defined to be $\sqrt{V(X)}$.

Variance of the Value of a Die

- What is the variance of the random variable X, where X is the number that comes up when a fair die is rolled?

$$V(X) = E(X^2) - E(X)^2 = \frac{91}{6} - \left(\frac{7}{2}\right)^2 = \frac{35}{12}$$

X^2	$P(X)$
$1^2 = 1$	$\frac{1}{6}$
$2^2 = 4$	$\frac{1}{6}$
⋮	⋮
$6^2 = 36$	$\frac{1}{6}$

$1 + 4 + 9 + 16 + 25 + 36 = \frac{91}{6}$

$$\sqrt{x} \times 8 + x * 20\%$$

$$\sqrt{100} \times 8 + 100 * 20\% = 100$$

$$\sqrt{49} \times 8 + 49 * 20\%$$

$$7 \times 8 + 10 = 66$$