CSCI 220 | Spring 2022

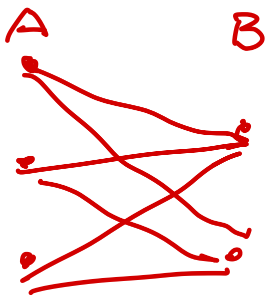
Discrete Structure

Counting

Discrete Mathematics and its Application

Section 6.1, 6.2

Counting Exercise



- Find number of ways to
  - choose apple, blueberry, or pie with juice or milk.

A B

3 * 2 = 6

  - choose a small, medium, large, or extra-large shirt in black or white.

4 sizes

2 colors

4 * 2 = 8

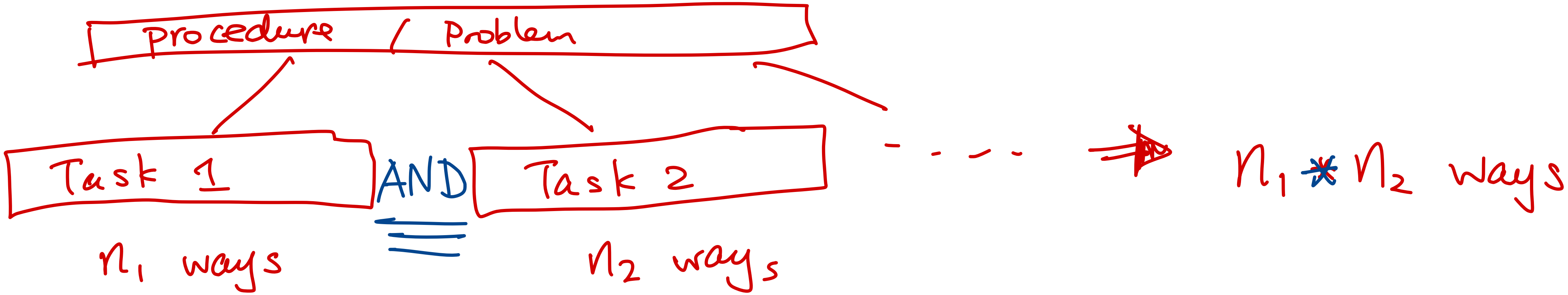
  - answer 2 true or false questions and 2 multiple choices question with 5 choices.

1.T/F 2.T/F 3.MC 4.MC.

2 * 2 * 5 * 5 = 100

The Product Rule

- Suppose that a procedure can be broken down into a sequence of two tasks. If there are n_1 ways to do the first task and for each of these ways of doing the first task, there are n_2 ways to do the second task, then there are $n_1 n_2$ ways to do the procedure.



Counting Exercise

- There are 5 action movies, 7 comedies, and 16 dramas playing at the local movie theater. If you go to the theater to watch a single movie, how many choices do you have for which movie to watch?

$$5 + 7 + 16 = 28$$

Counting Exercise

- At the dealership, there are 10 red trucks, 5 blue trucks, 3 red cars, and 2 blue cars for sale. If you are going to buy exactly one red vehicle, how many choices do you have?

$10 + 3 = 13$

# of way to buy a truck and a car.

$\boxed{\text{a truck}}$ AND $\boxed{\text{a car}}$

$(10+5) * (3+2) = 15 * 5$

Counting Exercise

- Supposed you are playing a card game and the cards in your hand include three 5's, two Jacks, two Aces, one 9, and one King. If you are choosing one card to play in the next round, how many choices do you have for a card to play?

$3+2+2+1+1 = 9$

The Sum Rule

- If a task can be done either in one of n_1 ways or in one of n_2 ways, where none of the set of n_1 ways is the same as any of the set of n_2 ways, then there are $n_1 + n_2$ ways to do the task.

$\boxed{\text{Task}}$

$\boxed{\text{subtask 1}}$ OR $\boxed{\text{subtask2}}$ $\Longrightarrow$ $n_1 + n_2$ ways

n_1 ways n_2 ways

They don't overlap. They don't share same elements.

Counting Exercise

$10 \cdot 36^5$

- Each user on a computer system has a password with six characters, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there?

A password of 6 characters

$\boxed{C_1}$ AND $\boxed{C_2}$ AND $\boxed{C_3}$ AND $\boxed{C_4}$ AND $\boxed{C_5}$ AND $\boxed{C_6}$

$C_1 \downarrow$ letter or digit.

$$\boxed{36^6 - 26^6}$$

$$(26+10)(36)(36)(36)(36)(36) = 36^6$$

PW with only letters $(26)(26)\ldots(26) = 26^6$

Pw with at least 1 digit.

C1: 1 digit & 5 letters $10 \cdot 26^5 \cdot 6$

C2: 2 digit & 4 letters $10^2 \cdot 26^4 \cdot \frac{6.5}{2}$

⋮

C6: 6 digit.

Counting Exercise

- Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there?

PW with 6-8 characters

6 chars OR 7 chars OR 8 chars

$$\left(36^6 - 26^6\right) + \left(36^7 - 26^7\right) + \left(36^8 - 26^8\right)$$

Counting Exercise

$1000\,0000 \quad | \quad 0000\,0000$

$1111\,1111$

- How many bit strings of length eight start with a 1 bit or end with the two bits 00?

$1xxx\,xxxx \leftarrow 2^7$

OR

$xxxx\,xx00 \leftarrow 2^6$

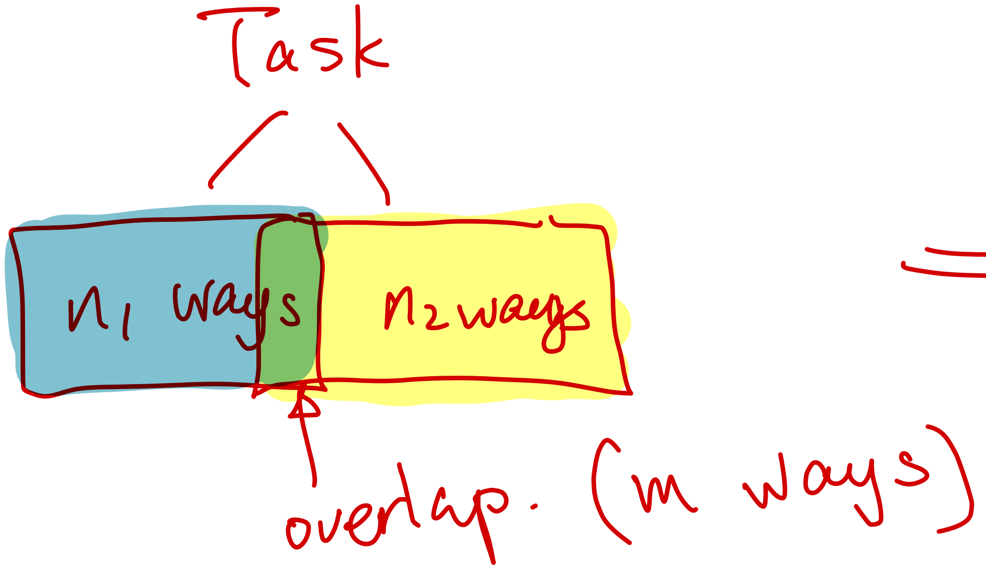
overlap:

$1xxx\,xx00 \leftarrow 2^3$

$$\boxed{2^7 + 2^6 - 2^5}$$

The Subtraction Rule (Inclusion-Exclusion for Two Sets)

- If a task can be done in either n_1 ways or n_2 ways, then the number of ways to do the task is $n_1 + n_2$ minus the number of ways to do the task that are $\underbrace{\text{common}}_{\text{overlap.}}$ to the two different ways.



$$\Longrightarrow n_1 + n_2 - m.$$

Counting Exercise

- A computer company receives 350 applications from college graduates for a job planning a line of new web servers. Suppose that 220 of these applicants majored in computer science, 147 majored in business, and 51 majored both in computer science and in business. How many of these applicants majored neither in computer science nor in business?

$$\begin{array}{lcccccccc} & \text{CS} & & \text{business} & & \text{both} & & \\ \text{CS or business:} & 220 & + & 147 & - & 51 & = & 316 \end{array}$$

$$350 - 316 = \boxed{34}$$

Counting Exercise

- How many total arrangements of the letters in ABC are there?

$\boxed{A}$ $\boxed{B}$ $\boxed{C}$

$\underline{3} \cdot \underline{2} * \underline{1}$

$\boxed{3!}$

Counting Exercise

- How many total arrangements of the letters in MISS are there?

M I S_1 S_2 S_3 M I S S S

M I S_1 S_2 S_3

S_2 S_1 S_3

$4 \cdot 3 \cdot 2 \cdot 1$

$$\boxed{\frac{4!}{2}}$$

$$\frac{5!}{3!}$$

The Division Rule

- How many total arrangements of the letters in MISSISSIPPI are there?

1 $\boxed{M}$ 4 $\boxed{I}$ 4 $\boxed{S}$ 2 $\boxed{P}$ $\Rightarrow$ total of 11 Letters.

$$\boxed{\frac{11!}{4!\,4!\cdot 2}}$$

M I S S I S S I P P I

M I S P

$1 * 4! * 4! * 2$

There are n/d ways to do a task if it can be done using a procedure that can be carried out in n ways, and for each ways w, exactly d of n ways correspond to way w.

Counting Exercise

- How many ways to arrange 7 distinct books on the shelf?

$7!$

Counting Exercise

- Five students walk into a classroom with 10 remaining seats available. How many ways can they be seated?

$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6$

$\boxed{S1}$ $\boxed{S2}$ $\boxed{S3}$ $\boxed{S4}$ $\boxed{S5}$ $\boxed{E}$ $\times 5$

$$\frac{10!}{5!}$$

$$\frac{10!}{5!}$$

Counting Exercise

- How many different license plates can be made if each plate contains a sequence of three uppercase English letters followed by three digits (and no sequences of letters are prohibited, even if they are obscene)?

$26^3 \cdot 10^3$

$$\boxed{\underbrace{26 * 26 * 26}_{\text{letters}} * \underbrace{10 * 10 * 10}_{\text{digits}}}$$

← Repetition allow

$$\boxed{26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8}$$

$\frac{26!}{23!}$

← Repetition does not allow

Counting Exercise

$\lceil \frac{8}{7} \rceil = 2$

- True or False
  - Among any group of 8 people, there must be at least two with the birthday on the same day of the week.

    True

    Sat, Wed, Fri, Mon, Sun, Tue, Thu,

  - Among any group of 21 people, there must be at least four with the birthday on the same day of the week.

    False

    $7 * 3 = 21$

    $\lceil \frac{21}{7} \rceil = 3$

  - Among any group of 25 people, there must be at least three with the birthday on the same month.

    True.

    ≤ 3

    $12 * 2 = 24$

    $\lceil \frac{25}{12} \rceil = 3$

    $> 3 \rightarrow \geq 2$

The Pigeonhole Principle

- If k is a positive integer and $k + 1$ or more objects are placed into k boxes, then there is at least one box containing two or more of the objects.

The Pigeonhole Principle

- If k is a positive integer and $k + 1$ or more objects are placed into k boxes, then there is at least one box containing two or more of the objects.

- Examples:
  - Among any group of 367 people, there must be at least two with the same birthday, because there are only 366 possible birthdays.

  - In any group of 27 English words, there must be at least two that begin with the same letter, because there are 26 letters in the English alphabet.

The Generalized Pigeonhole Principle

- If N objects are placed into k boxes, then there is at least one box containing at least $\left\lceil \frac{N}{k} \right\rceil$ objects.

round up.

Counting Exercise

- How many students must be in a class to guarantee that at least two students receive the same score on the final exam, if the exam is graded on a scale from 0 to 100 points?

$101 + 1 = \boxed{102}$

Counting Exercise

- What is the minimum number of students required in a discrete structure class to be sure that at least six will receive the same grade, if there are five possible grades, A, B, C, D, and F?

$\boxed{26}$?

$5 \times 5 + 1$

$\left\lceil \frac{N}{5} \right\rceil = 6$

$\left\lceil \frac{25}{5} \right\rceil = 5$

$\left\lceil \frac{26}{5} \right\rceil = 6$

Counting Exercise

- A bag contains 3 red balls, 4 blue balls, and 5 yellow balls. You are selecting balls at random without looking at them.
  - How many balls must you take out to be sure that you have at least two balls of the same color?

    $3+1 = \boxed{4}$

  - How many balls must you take out to be sure that you have at least one ball of each color?

    $\underbrace{5}_{\text{Yellow}} + \underbrace{4}_{\text{Blue}} + \underbrace{1}_{\text{Red}} = \boxed{10}$