1. Multiple Choice: How many 8-bit strings begin with "11...

Question	How many 8-bit strings begin with "11" or "010"?
Answer	$2^8 - 2^2 - 2^3$
	$2^6 + 2^5 - 2^3$
	✅ $2^6 + 2^5$
	$\dfrac{2^6 \cdot 2^5}{2^3}$
	2^3

2. Multiple Choice: A group consists of 10 kids and 2 adu...

Question	
	A group consists of 10 kids and 2 adults. On a hike, they must form a line with an adult at the front and an adult at the back. How many ways are there to form the line?
Answer	$2 \cdot 11!$
	$11!$
	✅ $2 \cdot 10!$
	$\dfrac{12!}{2}$
	$\dfrac{12!}{2! \cdot 10!}$

3. Multiple Choice: 10 different prizes are distributed t...

Question	10 different prizes are distributed to 30 students, each student can have at most 1 prize. How many ways can the 10 prizes be distributed?
Answer	$\dfrac{30!}{20! \cdot 10!}$
	✓ $\dfrac{30!}{20!}$
	$\dfrac{30!}{10!}$
	$10!$
	30^{10}

4. Multiple Choice: How many binary strings of length 8 h...

Question	How many binary strings of length 8 have exactly 4 1's or begin with a 1?
Answer	✓ $\dfrac{8!}{4! \cdot 4!} + 2^7 - \dfrac{7!}{4! \cdot 3!}$
	$\dfrac{8!}{4! \cdot 4!} + 2^7 - 1$
	$\dfrac{8!}{4! \cdot 4!} + 2^7$
	$\dfrac{7!}{4! \cdot 3!}$
	$\dfrac{8!}{4!} + 2^7 - \dfrac{7!}{3!}$

5. Multiple Choice: A basket holds a set of balls. Each b...

Question	A basket holds a set of balls. Each ball is either red, green, or blue. How many balls must there be in the basket in order to guarantee that there are at least 5 balls of the same color?
Answer	✅ 13
	14
	15
	16
	17

6. Multiple Choice: A red die and a blue die are thrown. ...

Question	A red die and a blue die are thrown. What is the probability that the red die comes up 6 or the blue die comes up 6?
Answer	$\frac{1}{6} + \frac{5}{36} - \frac{1}{36}$
	$\frac{1}{6} + \frac{5}{36}$
	✅ $\frac{1}{6} + \frac{1}{6} - \frac{1}{36}$
	$\frac{1}{6} + \frac{1}{6}$
	$\frac{1}{6} + \frac{1}{12} - \frac{1}{36}$

7. Multiple Choice: A fair coin is flipped four times. Wh...

Question	A fair coin is flipped four times. What is the probability that the last flip comes up heads, given there are at least three consecutive tails somewhere in the sequence?
Answer	$\dfrac{1}{4}$
	✅ $\dfrac{1}{3}$
	$\dfrac{3}{8}$
	$\dfrac{1}{8}$
	$\dfrac{1}{2}$

8. Multiple Choice: A fair coin is flipped 10 times. What...

Question	A fair coin is flipped 10 times. What is the probability that exactly half flips come up heads?
Answer	✅ $\dfrac{C(10,5)}{2^{10}}$
	$\dfrac{C(10,5)}{10!}$
	$\dfrac{1}{2}$
	$\dfrac{P(10,5)}{2^{10}}$
	$\dfrac{P(10,5)}{10!}$

9. Multiple Answer: Which of the following equations are ...

Question	Which of the following equations are true?
	Select all the correct choices.

Answer	$P(10,7) = P(10,3)$
	✅ $C(10,7) = C(10,3)$
	$P(10,3) \cdot P(10,2) = P(10,5)$
	$C(10,3) \cdot C(10,2) = C(10,5)$
	$P(10,7) \cdot (7!) = C(10,7)$
	✅ $C(10,7) \cdot (7!) = P(10,7)$
	$\dfrac{C(10,3)}{3!} = \dfrac{C(10,7)}{7!}$
	✅ $\dfrac{P(10,3)}{3!} = \dfrac{P(10,7)}{7!}$

☐ **10. Essay: A bank PIN is a string of four digits...**

Question	A bank PIN is a string of four digits, each digit 0-9. How many choices are there for a PIN if the first digit cannot be a 0, also, even digit and odd digit cannot be next to each other? (Repeat digit is allowed.)
	Explain how you arrived at your answer – and DO NOT simplify your answer.
	Answer without explanation will not receive full credit.
Answer	The first digits cannot be 0, there are 9 choices for it.
	The second digits cannot be the same parity as the first digit, so there are 5 choices.
	The third digit cannot be the same parity as the second digit, so there are 5 choices.
	The fourth digit cannot be the same parity as the third digit, so there are 5 choices.
	Therefore, the answer is 9*5*5*5.

☐ **11. Essay: How many bit strings of length 6 begi...**

Question	How many bit strings of length 6 begin with '111' or end with '000'?
	Explain how you arrived at your answer – and simply your answer to an integer.
	Answer without explanation will not receive full credit.
Answer	There are 2^3 bit strings of length 6 that begin with '111' and 2^3 end with '000'.
	There are 1 bit string that begins with '111' and ends with '000', 111000.
	Therefore the answer is 2^3 + 2^3 - 1 = 8 + 8 - 1 = 15.

12. Essay: How many ways are there to permute th...

Question	How many ways are there to permute the letters in SLEEPLESSNESS?
	Explain how you arrived at your answer – and you can leave your answer in fraction form, you don't need to simplify it.
	Answer without explanation will not receive full credit.
Answer	There are 5S, 2L, 4E, 1P, and 1N, a total of 13 letters.
	Therefore, the answer is 13! / (5! 2! 4!).

13. Essay: A store sells 7 different flavors of ...

Question	A store sells 7 different flavors of candies. How many ways are there to buy 14 candies so that you have at least one of each flavor?
	Explain how you arrived at your answer – and simplify your answer as an expression without combination and permutation notation, It's okay to have fraction, power, or factorial.
	Answer without explanation will not receive full credit.
Answer	Since we want at least one of each flavor, we will get one of each flavor, then select 14 - 7 = 7 more candies.
	To select 7 candies with 7 different flavors, we will have 7 stars to represent the candies, and 6 bars to divide them into 7 different flavors. Then there will be 13! / (7! 6!) ways to arrange the stars and bars.
	Therefore the answer is 13! / (7! 6!).

14. Essay: What is the coefficient of the term ...

Question	What is the coefficient of the x^3y^5 term in $(x - 2y + 3)^{10}$?
	Explain how you arrived at your answer – and simplify your answer as an expression without combination and permutation notation, It's okay to have fraction, power, or factorial.
	Answer without explanation will not receive full credit.
Answer	For (x-2y+3)^10, we need to make 10 selections.
	To have the term x^3 y^5, 3 out of the 10 selections need to be x term, 5 out of the remaining 7 selections need to be -2y, and the rest of the 2 selections need to be 3.
	So we will have C(10,3) x^3 * C(7,5) (-2y)^5 * 3^2 = 10! / (3!7!) * 7! / (5! 2!) * (-2)^5 * 3^2.
	Therefore, the coefficient of x^3y^5 is 10! * (-2)^5 * 3^2 / (3! 5! 2!).

15. Essay: A quiz has 6 multiple-choice question...

Question	A quiz has 6 multiple-choice questions. Each question has four possible choices. If a student makes a random guess on each question, what is the probability that the student answer at least 2 questions correctly?
	Explain how you arrived at your answer – and simplify your answer as an expression without combination and permutation notation, It's okay to have fraction, power, or factorial.
	Answer without explanation will not receive full credit.
Answer	

Since each question has 4 choices, the probability of getting the question correct is 1/4.

So P(correct) = 1/4, P(incorrect) = 3/4.

P(getting at least 2 questions correctly) = 1 - P(getting no question correctly) - P(getting 1 question correctly)

$$= 1 - C(6,0)(3/4)^6 - C(6,1)(1/4)^1 (3/4)^5$$

$$= 1 - 3^6 / 4^6 - 6*3^5 / 4^6$$

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16. Essay: Five cards are randomly selected from...

Question	Five cards are randomly selected from a deck of 52 cards. What's the probability that you are selecting at most 3 picture cards? The picture cards are J, Q, and K, there are 12 of them.
	Explain how you arrived at your answer – and do not simplify your answer. It's okay to have combination, permutation, fraction, power, or factorial.
	Answer without explanation will not receive full credit.
Answer	P(at most 3 picture cards) = 1 - P(more than 3 picture cards)
	= 1 - P(4 picture cards) - P(5 picture cards) = 1 - C(12,4) * 40 / C(52,5) - C(12,5) / C(52,5)
	There are C(52,5) ways to have five cards randomly selected from a deck of 52 cards.
	There are C(12,4)*40 ways to have four picture cards from 12 picture cards with a non-picture card.
	There are C(12,5) ways to have five picture cards from 12 picture cards.

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17. Essay: Can you design a dice where the proba...

Question	Can you design a dice where the probability of 2 occurs is twice the probability of 1 occurs,
	the probability of 3 occurs is three times the probability of 1 occurs,
	the probability of 4 occurs is four times the probability of 1 occurs,
	the probability of 5 occurs is five times the probability of 1 occurs,
	and the probability of 6 occurs is six times the probability of 1 occurs?
	If so, how should the probability of each outcome be? if not, explain why it cannot be done.
	Explain how you arrived at your answer. Answer without explanation will not receive full credit.
Answer	We want P(2) = 2P(1), P(3) = 3P(1), P(4) = 4P(1), P(5) = 5P(1), and P(6) = 6P(1).
	We need P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1.
	P(1) + 2P(1) + 3P(1) + 4P(1) + 5P(1) + 6P(1) = 1
	21P(1) = 1
	P(1) = 1/21.
	We can design the dice so that P(1) = 1/21, P(2) = 2/21, P(3) = 3/21, P(4) = 4/21, P(5) = 5/21, and P(6) = 6/21.

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18. Essay: There are three boxes: the first box ...

Question	

There are three boxes: the first box has 3 red balls and 5 blue balls,

the second box has 4 red balls and 6 yellow balls,

and the third box has 2 blue balls and 7 yellow balls.

The probability that a person randomly selects a box and randomly selects a ball from it.

(a) What is the probability that a person will select a blue ball?

(b) What is the probability that a person selected a blue ball from then the third box?

Explain how you arrived at your answer – and simplify your answer as an expression without combination and permutation notation, It's okay to have fraction, power, or factorial.

Answer without explanation will not receive full credit.

Answer	a) a person can select a blue ball from the first or the third box. P(yellow) = P(box 1 and blue) + P(box 3 and blue) = P(box 1)P(blue|box 1) + P(box 3)P(blue|box 3) = 1/3 * 5/8 + 1/3 * 2/9 B) P(box 3 | blue) = P(box 3 and blue) / P(blue) = (1/3 * 2/9) / [1/3 * 5/8 + 1/3 * 2/9]