Lecture 1 Note Tuesday, 02/01/2022

Announcement
- Please download and read the syllabus from Blackboard if you were not in class when we went over it.
- For students who didn't attend the class, make sure you attend at least one of the first five in-person classes to avoid the WN grade and being withdrawn by the registrar.
- The Lecture PowerPoints will not be uploaded, instead, I will upload Lecture notes that include all the PowerPoint content and explanation of what we did in the class.
- The Lecture Recordings will be available on the following YouTube Playlists Link: https://youtube.com/playlist?list=PLZaTmV9UMKliRBuEs0-dL968iQqE0qwpc

Go over the syllabus

Division
- Which of following are representing "a divides b"?
  1) a/b
  2) b/a
  3) $a|b$
  4) $b|a$
  5) a is a multiple of b.
  6) b is a multiple of a.

  - The answer will be 3 and 6. "|" is the notation for divide.
  - We say a divides b when a goes into b evenly, then b will need to be a multiple of a.
  - The difference between divide (|) and divided by (/): divided by is an arithmetic operation, for examples, we can have $2/4 = 1/2 = 0.5$ or $4/2 = 2$. Where for divide when we have $2|4$, we know it's a true statement since 2 go into 4. But, for $4|2$, it will be a false statement since 4 cannot go into 2.
  - We can say that a divides b iff (if and only if) b divided by a is an integer.

Definition of Division
- If a and b are integers with $a \neq 0$, we say that a divides b if there is an integer c such that $b = ac$ (or equivalently, if $\frac{b}{a}$ is an integer).
  When a divides b we say that a is a factor or divisor of b, and that b is a multiple of a. The notation $a|b$ denotes that a divides b.
  We write $a \nmid b$ when a does not divide b.
  - The Definition in Mathematical symbol:

$$a \mid b \leftrightarrow \exists\, c \in \mathbb{Z} : b = a$$

    - $\leftrightarrow$: iff, if and only if symbol, double implication, means the implication will go both directions.
    - $\exists$: "there exist", existential quantifier. You might also see $\forall$: "for all", universal quantifier.
    - $\in$: "belongs to" symbol, referring to an element belongs to a set.
    - $\mathbb{Z}$: the set of all integers. You might also see $\mathbb{R}$ for real numbers and $\mathbb{Q}$ for rational numbers.

Examples of Division

- Determine whether $7|25$.
  $7|25$ is false, since if it's true, we can find an integer c that $25 = 7c$, to solve for c, we have $c = \frac{25}{7}$ and $\frac{25}{7}$ is not an integer.
  Therefore, $7|25$ is false.

- Determine whether $7|35$.
  $7|35$ is true, since we can write 35 as multiple of 7, $35 = 7 \cdot 5$.

Theorem

- Let a, b, and c be integers, where $a \neq 0$. Then
  (i) if $a \mid b$ and $a \mid c$, then $a \mid (b+c)$;
  (ii) if $a \mid b$, then $a \mid bc$ for all integers c;
  (iii) if $a \mid b$ and $b \mid c$, then $a \mid c$.

  - Example for (i): Given $3|12$ and $3|15$, we know that $3|(12+15)$ is true since $12+15=27$ and $27 = 3 \cdot 9$, therefore, $3|(12+15)$.
  - Proof for (i):
    Given that $a \mid b$ and $a \mid c$.
    By definition, we know $\exists\, k \in \mathbb{Z} : b = ak$ and $\exists\, s \in \mathbb{Z} : c = as$.
    We want to show that $a \mid (b+c)$ which $b+c$ is a multiple of a.
    $b + c = ak + as = a(k+s)$, $k+s$ is an integer. Thus, $a|(b+c)$.

  - Example for (ii): Given $3|12$, we know that $3|(12 \cdot 4)$ is also true since $12 \cdot 4 = 48$ and $48 = 3 \cdot 16$, therefore, $3|(12 \cdot 4)$.
  - Proof for (ii):
    Given that $a \mid b$ and c is an integer.
    By definition, we know $\exists\, k \in \mathbb{Z} : b = ak$.
    We want to show that $a \mid bc$ which bc is a multiple of a.
    $bc = (ak)c = a(kc)$, kc is an integer. Thus, $a|bc$.

  - Example for (iii): Given $3|12$ and $12|48$, we can have $3|48$ since $48 = 3 \cdot 16$, therefore, $3|48$.
  - Proof for (iii):
    Given that $a \mid b$ and $b \mid c$.
    By definition, we know $\exists\, k \in \mathbb{Z} : b = ak$ and $\exists\, s \in \mathbb{Z} : c = bs$.
    We want to show that $a \mid c$ which c is a multiple of a.
    $c = bs = (ak)s = a(ks)$, ks is an integer. Thus, $a|c$.

Corollary

- If a, b, and c are integers, where $a \neq 0$, such that $a \mid b$ and $a \mid c$, then $a \mid mb + nc$ whenever m and n are integers.
  - Given $a \mid b$ and $a \mid c$ and, m and n are integers, we know $a \mid mb$ and $a \mid nc$ is true by the theorem (ii) above. Then, we can also have $a \mid mb + nc$ by the theorem (i).
    We have shown that $a \mid mb + nc$ is true.

What to expect or prepare for the next class:

- We will do division algorithm and cover congruences and modulus.

Suggested Problems (You don't need to hand in.)

- Discrete Mathematics and its Application 4.1 # 1-10
- zyBook Additional Exercises #1.1.1, 1.1.2