§1.4. Predicates and Quantifiers

Recall: $P(x)$

Quantifiers: ① universal quantifier: $\forall$ "For all", "For every"

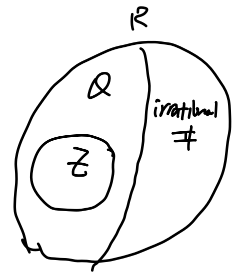
Counter example: Ex. $\forall x \in R\ \ x^2 > 0$ F b/c $x = 0$.

Common Notation. ← set

$R \equiv \{x \mid x \text{ is a Real } \#\}$ Ex. $1, 2, -5, \sqrt{2}, \frac{1}{3}, \pi, e$

(such that; ↑ is defined as)

$R^+ \equiv \{x \mid x \text{ is a positive real } \#\}$



$Q \equiv \{x \mid x \text{ is a rational } \#\}$

def. Rational #: $\frac{a}{b}$ a, b are integers, $b \neq 0$

Ex. $\frac{1}{2}$, $10 \rightarrow \frac{10}{1}$, $-0.\overline{33} = -\frac{1}{3}$, $0 = \frac{0}{10}$

Irrational #: $\pi, e, \sqrt{2}, -\sqrt{5}$

$Z \equiv \{x \mid x \text{ is an integer}\}$ Ex. $-10, 0, 1, 100$

Ex. Non-negative integer: $Z^+ \cup \{0\}$ $x \geq 0$ and x is integer

3. Existential Quantifier: $\exists$: "there exists", "For some", "There is at least one ⋯"

Ex. $\exists x \in Z^+, x^2 = 100$ True $x = 10$

$x = \pm 10$

$\exists x \in Z^+, x^2 = 5$ False $\Rightarrow$ $\exists x \in R, \boxed{x^2 = 5}$

$x = \pm\sqrt{5} \notin Z^+$ $x = \pm\sqrt{5} \in R$ True

Note: As a loop. $\forall x$: loop through all x.

$\exists x$: loop through all x, if at some steps, $p(x)$ is true then $\exists x$ is true and loop terminates

4. Negating quantified expressions.

① $\neg \forall x \; P(x) \; = \; \exists x \; \neg P(x)$

All students have computers.

Neg: Not all students have computers = some students don't have computers.

② $\neg \exists x \; p(x) \; = \; \forall x \; \neg p(x)$

There is an honest politician.

Neg. There is not an honest politician. = All politicians are not honest.

Ex. Negations of the statements.

1) $\forall x \, (x^2 > x)$ 2). $\exists x \, (x^2 = 2)$

Neg: $\exists x \, (x^2 \leq x)$ $\forall x \, (x^2 \neq 2)$

Note: ** quantifiers $\forall$ and $\exists$ have higher precedence than all the logical operators.

Ex show that $\neg \forall x (p(x) \to Q(x)) \equiv \exists x \, (p(x) \wedge \neg Q(x))$

$$= \exists x \; \neg (p(x) \to Q(x))$$

$$\overset{C.E}{=} \exists x \; \neg (\neg p(x) \vee Q(x))$$

$$\overset{DM}{=} \exists x \, . \, (p(x) \wedge \neg Q(x))$$

5. Translating.

① "Every student in this class has studied pre-calc."

$$\forall x \, [C(x) \to P(x)]$$

② Some students in this class has taken calculus.

$$\exists x \, [C(x) \wedge L(x)]$$