§ 2.1 (Cont.)

Theorem 1. For every set S

i) $\emptyset \subseteq S$

ii). $S \subseteq S$

Pf. By def. $A \subseteq B \Rightarrow \boxed{\forall x(x \in A \rightarrow x \in B)}$ T

i). $\emptyset \subseteq S$ let S be the set

We want to show that $\forall x\,(x \in \emptyset \rightarrow x \in S)$

Since the empty set contains no element.

$x \in \emptyset = F$

$\underbrace{x \in \emptyset}_{F} \rightarrow x \in S$

$F \rightarrow ?? = \text{True}$

Pf by vacuous.

$\forall x(x \in \emptyset \rightarrow x \in S) = \text{True} \Rightarrow \emptyset \subseteq S$ ∎

ii). $S \subseteq S \rightarrow \forall x\,(x \in S \rightarrow x \in S)$

Case 1. $x \in S = \text{True}$

$T \rightarrow T = T$

Case 2 $x \in S = \text{False}\ (x \notin S)$

$F \rightarrow F = T$

$\forall x\,(x \in S \rightarrow x \in S) = T \Rightarrow S \subseteq S.$ ∎

$\forall x\ (x \notin S) \vee (x \in S) = T$

Negation. $p \vee \neg p = T$

subset

$\subseteq\ (\leq)$

6. proper subset "$\subset$" $<$

Set A is a subset of a set B but $A \neq B$

$\forall x(x \in A \rightarrow x \in B) \wedge \exists y\,(y \in B \wedge y \notin A)$

Ex. $A = \{1, 3, 5\}$

3 elements : $|A| = 3$, 2^3 subsets $= 8$ subsets

$2^3 - 1$ proper subsets $= 7$ proper subsets.

proper subsets: ϕ, $\{1\}$, $\{3\}$, $\{5\}$, $\{1,3\}$, $\{1,5\}$, $\{3,5\}$.

summary: element $\in$ set $A=\{1,3,5\}$

$\uparrow$ belongs to $1 \in \{1,3,5\}$

set $\subseteq$ set $\{1\} \subseteq \{1,3,5\}$

set $\subset$ set

7. power set

Def: Given a set S, the power set of S is the set of all subsets of the set S. $P(S)$

Ex. $A=\{1,3,5\}$

$P(A)=\{\phi, \{1\}, \{3\}, \{5\}, \{1,3\}, \{1,5\}, \{3,5\}, \{1,3,5\}\}$

elem, element, set

$\{1\} \in P(A)$ $\{\{1\}\} \subseteq P(A)$ (set, set) $1 \notin P(A)$

$1 \nsubseteq P(A)$

8. show two sets that are equal: $A=B$

Pf for it: 1) $A \subseteq B$

2) $B \subseteq A$

9. The size of a set: $|\ |$

Def. let S be a set. If there are exactly n distinct elements in S, $n \in \mathbb{Z}^+ \cup \{0\}$.

S is a finite set and that n is the cardinality of S. $|S|$

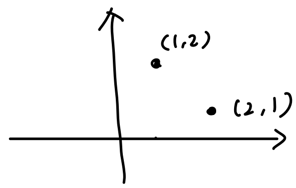
Ex. $|\{1,1,3,5,3\}| = 3$ $|\mathbb{Z}|: \infty$

10. Cartisian Products.

Def. The ordered n-tuple $(a_1, a_2, \dots a_n)$ is the ordered collection that a_1 as its first elements, a_2 is the second element, ...

odered 2-tuples are called ordered pairs. $(a,b) \neq (b,a)$ is not necessary

Ex. $(1,2) \neq (2,1)$ $\{1,2\} = \{2,1\}$



<u>Def</u>. let A and B be sets, The cartisian product of A and B, denoted by $A \times B$ is the set of all ordered pairs (a,b), $a \in A$, $b \in B$

$$A \times B = \{(a,b) \mid a \in A \wedge b \in B\}$$

Ex $A = \{1, 2\}$, $B = \{a, b, c\}$

$$A \times B = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\}$$

$$B \times A = \{(a,1), (a,2), (b,1), (b,2), (C,1), (C,2)\}$$

Note: $A^2 = A \times A$ $A^3 = A \times A \times A$

Ex. $A = \{0, 1, 3\}$

$$A^2 = \{(0,0), \boxed{(0,1)}, (0,3), \boxed{(1,0)}, (1,1), (1,3), (3,0), (3,1), (3,3)\}$$

order matters

$(1,0)$ $(0,1)$

$$(\underline{3}, \times \underline{3}) = 9$$

$$A^3 = \{(0,0,0), (0,0,1), (0,0,3), (0,1,0), (0,1,1), (0,1,3) \dots\}$$

$$(\underline{3}, \times \underline{3} \times, \underline{3}) = 27$$