#19 $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $ad - bc \neq 0$

$A^{-1} = \begin{bmatrix} x & y \\ z & w \end{bmatrix}$ solve for x, y, z, w

$A \cdot A^{-1} = I$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$d\,(ax + bz = 1)$ ①
$-b\,(cx + dz = 0)$ ②

$\begin{cases} ay + bw = 0 \\ cy + dw = 1 \end{cases}$

Elimination Method

$$\begin{array}{r} adx + \cancel{bdz} = d \\ + \quad -bcx - \cancel{bdz} = 0 \\ \hline adx - bcx = d \\ (ad - bc)\,x = d \end{array}$$

$x = \frac{d}{ad-bc}$

② $cx + dz = 0$

$c \cdot \left(\frac{d}{ad-bc}\right) + dz = 0$

$\cancel{d}z = \frac{-c\cancel{d}}{ad-bc}$

$z = \frac{-c}{ad-bc}$

$c\,(ay + bw = 0)$
$-a\,(cy + dw = 1)$

$$\begin{array}{r} a\cancel{cy} + bcw = 0 \\ + \quad -a\cancel{cy} - adw = -a \\ \hline (bc - ad)\,w = -a \end{array}$$

$w = \frac{-a}{bc-ad} = \frac{a}{ad-bc}$

$ay + b \cdot \left(\frac{a}{ad-bc}\right) = 0$

$\frac{\cancel{a}y}{\cancel{a}} = \frac{-\cancel{a}b}{ad-bc} \Big/ \cancel{a}$

$y = \frac{-b}{ad-bc}$

$$A^{-1} = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}$$ $\blacksquare$

§ 2.4 # 16 g. $a_n = -\boxed{a_{n-1}} + n - 1$, $a_0 = 7$

$a_2 = 8$
$a_3 = -a_2 + 3 - 1$
$= -8 + 3 - 1 = -6$

Backward Substitution $= -\left(\boxed{-a_{n-2} + (n-1) - 1}\right) + n - 1$
$= a_{n-2} - (n-1) + 1 + n - 1$
$= a_{n-2} + [-(n-1) + n] + [1 - 1]$
$= [-a_{n-3} + n - 2 - 1] + [-(n-1) + n] + [1 - 1]$

$= -a_{n-3} + (n-2) - 1 + [-(n-1)+n] + [1-1]$

$= -a_{n-3} + [(n-2) - (n-1) + n] + [-1+1-1]$

Forward substitution:

$a_0 = 7$

$a_1 = -a_0 + 1 - 1 = -7$

$a_2 = -a_1 + 2 - 1 = -(-a_0 + 1 - 1) + 2 - 1$

$= 8 \qquad = a_0 - 1 + 1 + 2 - 1$

$= a_0 + (-1+2) + (1-1)$

$a_3 = -a_2 + 3 - 1 = -(a_0 + (-1+2) + (1-1)) + 3 - 1$

$= -a_0 - (-1+2) - (1-1) + 3 - 1$

$= -a_0 - (-1+2-3) - (1-1+1)$

$a_4 = a_0 + (-1+2-3+4) + (1-1+1-1)$

$\vdots$

a_n ① n even $= a_0 + \underbrace{(-1+2-3+4-\cdots+n)} + (\overset{0}{1-1} + \overset{0}{1-1} + \cdots \overset{0}{1-1})$

$= 7 + \sum_{k=1}^{n} (-1)^k k$

$a_2 = 7 + \sum_{k=1}^{2} (-1)^k \cdot k$

$= 7 + (-1)^1 1 + (-1)^2 \cdot 2$

$= 7 - 1 + 2$

$= 8$

② n odd $= -a_0 - (-1+2-3+\cdots-n) - (\overset{0}{1-1} + \overset{0}{1-1} + \overset{0}{1-1} + \cdots \overset{0}{} + 1)$

$= -7 - \sum_{k=1}^{n} (-1)^k k - 1$

$a_3 = -8 - \sum_{k=1}^{3} (-1)^k (k) \implies = -8 - \sum_{k=1}^{n} (-1)^k \cdot k$

$= -8 - [(-1)(1) + (-1)^2(2) + (-1)^3(3)]$

$= -8 - [-1 + 2 - 3]$

$= -8 + 2 = -6$

§ 2.1

#12. a). $\emptyset \; \boxed{\in} \; \{\emptyset\}$ T — element, set, symbol $\qquad \emptyset \subseteq \{\emptyset\}$ T — Null set $\qquad$ Quiz #3, 5

$\heartsuit \in \{\heartsuit\}$ — elem, set $\qquad \boxed{\emptyset \subseteq \emptyset}$ — set, set

$\emptyset \in \{\ \}$ F

b). $\emptyset \in \{\emptyset, \{\emptyset\}\}$ T

c). $\{\emptyset\} \subset \{\emptyset\}$ F $\qquad \{\emptyset\} \in \{\{\emptyset\}\} = $ T

c) $\{\phi\} \in \{\phi\}$

elem set

d). $\{\phi\} \in \{\{\phi\}\}$ T

e). $\{\phi\} \subset \{\phi, \{\phi\}\}$ T $\{♡\} \subset \{♡, \{\phi\}\}$

f). $\{\{\phi\}\} \subset \{\phi, \{\phi\}\}$ T

g) $\boxed{\{\{\phi\}\}} \not\subset \{\{\phi\}, \{\phi\}\}$ F $\{1, 1\}$ $= \{1\}$ $\{\{\phi\}\} \subseteq \{\{\phi\}, \{\phi\}\}$

$= \boxed{\{\{\phi\}\}}$ T

Proper subset $A \subset B$ $A \neq B$

N

Z $R - Q$

Q

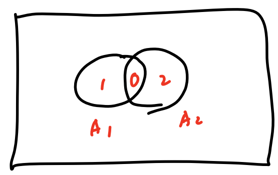
Quiz

Q3 $\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup A_3 \cup \cdots$

$= \{0,1\} \cup \{0,2\} \cup \{0,3\} \cup \cdots$

$= \{0, 1, 2, 3, \ldots\}$

$= N$



$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap A_3 \cdots$

$= \{0,1\} \cap \{0,2\} \cap \{0,3\} \cdots$

$= \{0\}$

$Z^+ \cup \{0\}$

Q5 $a_n = \boxed{\frac{n+2}{n}}$ $n \neq 0$, $n \geq 1$

$a_1 = \frac{1+2}{1} = 3$

$a_2 = \frac{2+2}{2} = \frac{4}{2} = 2$

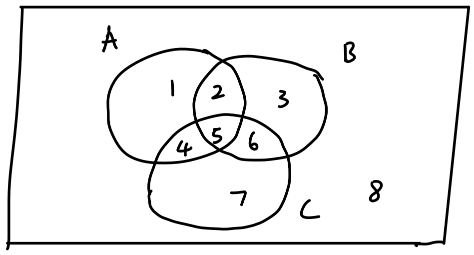
$a_3 = \frac{3+2}{3} = \frac{5}{3}$

$a_4 = \frac{4+2}{4} = \frac{6}{4} = \frac{3}{2}$

$a_5 = \frac{5+2}{5} = \frac{7}{5}$

#18. $(A-B)\cap(C-B) = (A\cap C) - B$

1)



left

$A-B = 1,4$

$C-B = 4,7$

$(A-B)\cap(C-B) = 4 = 4$

right:

$(A\cap C) - B$

$(4,5) - (2,3,5,6)$

4

2) Laws $A-B = A\cap\bar{B}$

$C-B = C\cap\bar{B}$

left: set difference $(A\cap\bar{B})\cap(C\cap\bar{B})$

Associative/commutative laws $= (A\cap C)\cap(\bar{B}\cap\bar{B})$

Idempotent law $= (A\cap C)\cap\bar{B}$

Set difference $= (A\cap C) - B$

3). $(A-B)\cap(C-B)$

by def. difference $= \{x \mid (x\in A \wedge x\notin B)\cap(x\in C \wedge x\notin B)\}$

by def of Intersection $= \{x \mid (x\in A\wedge x\notin B)\wedge(x\in C\wedge x\notin B)\}$

Associative/Commutative eq. $= \{x \mid [(x\in A)\wedge(x\in C)]\wedge[(x\notin B)\wedge(x\notin B)]\}$

Idempotent eq $= \{x \mid [(x\in A)\wedge(x\in C)]\wedge[x\notin B]\}$

by def of Intersection Complement $= \{x \mid (A\cap C)\wedge\bar{B}\}$

by def of set difference $= \{x \mid (A\cap C) - B\}$

Worksheet for the sets

30 IPA

38 CAT

56 NAV

~~IPA only = 0~~ ✓

21 $IPA\wedge CAT$ ✓

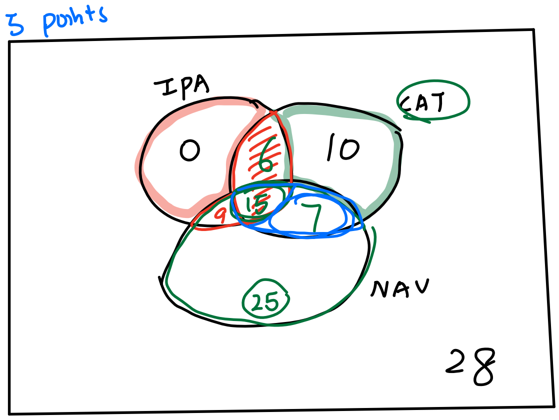
24 $IPA\wedge NAV$ ✓

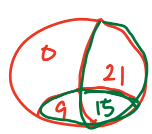
~~10 CAT only~~ ✓

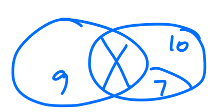
~~28 None~~

5 points

**







a) 15

b) 25

c) $56+6+10+28 = 100$

d) 7

e) $25+28 = 53$

f) $9+17=26$