§2.2 Set Operations

Recall: $\{\ \}$, $\subseteq$, $\subset$, $\not\subset$

$\{1,2,3\}$

$1 \in \{1,2,3\}$ $\quad \{1\} \subseteq \{1,2,3\}$ $\quad \{1\} \subset \{1,2,3\}$

3 elements: $2^3 = 8$ $\qquad$ proper subset: $2^3 - 1 = 7$

Subset: $\emptyset$ $\qquad$ proper subset: $\emptyset$

$\{1\}, \{2\}, \{3\}$ $\qquad$ $\{1\}, \{2\}, \{3\}$

$\{1,2\}, \{1,3\}, \{2,3\}$ $\qquad$ $\{1,2\}, \{1,3\}, \{2,3\}$

$\{1,2,3\}$

Power set : $\{$subsets$\}$

$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2,3\}, \{1,2\}, \{1,3\}, \{2,3\}\}$

element

$\emptyset \in P(A)$ $\qquad$ $\emptyset \subseteq P(A)$ (different: set)

subsets of $P(A)$: $2^8 = 256$

$\emptyset$: 0 element $\{\ \}$

1 element → $\{\emptyset\}$, $\{\{1\}\}$, ...

$\{\emptyset, \{1\}\}$, $\{\emptyset, \{2,3\}\}$

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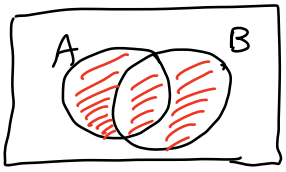
1. Union : $\cup$

Def. let A and B be sets, The union of sets A and B denoted by $A \cup B$, is the set that contains those elements that are either in A or in B, or in both.

$A \cup B = \{x \mid x \in A \vee x \in B\}$

Ex. $A = \{1,3,5\}$, $B = \{1,2,3\}$

$A \cup B = \{1,2,3,5\}$

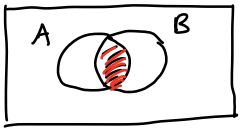


$A \cup B$

2. Intersection : $\cap$ (common)

Def. let A and B be sets, the intersection of A and B, denoted by $A \cap B$. is the set containing those elements in both A and B.

$A \cap B = \{x \mid x \in A \wedge x \in B\}$



Ex. $A = \{1,3,5\}$ $\quad B = \{1,2,3\}$

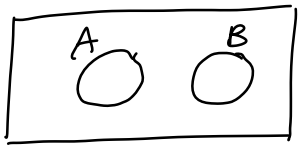
$A \cap B = \{1,3\}$

3. disjoint sets

$A \cap B = \emptyset$

$A = \{1, 3, 5\}$ $B = \{2, 4, 6\}$

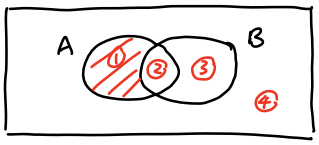
$A \cap B = \emptyset$



4. Difference of sets (order matters)

Def. The Difference of A and B, denoted by $A - B$ is the set containing those elements that are in A but not in B.

$A - B = \{x \mid x \in A \wedge x \notin B\}$



$A - B$

Ex. $A = \{1, 3, 5\}$, $B = \{1, 2, 3\}$

$A - B = \{5\}$

$B - A = \{x \mid x \in B \wedge x \notin A\}$

$B - A$

Ex $A = \{1, 3, 5\}$, $B = \{1, 2, 3\}$

$B - A = \{2\}$

Ex. $A = \{1, 3, 3\}$, $B = \{2, 4, 6\}$

$A - B = \{1, 3, 3\} = A$

Ex. $A = \{1, 3, 5\}$, $B = \{1, 2, 3, 5\}$

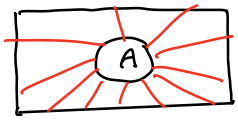
$A - B = \emptyset$

$B - A = \{2\}$

Note: $A - B \neq B - A$

5. Complement set

Def. U = universal set, The complement of set A: $\bar{A}$ or A', is the complement of A with respect to u.



$\bar{A}$

Ex: $u = \{1, 2, 3, 4, 5\}$

$A = \{1, 3\}$

$\bar{A} = \{2, 4, 5\}$

Aside of

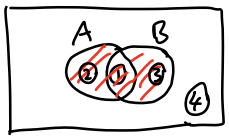
Ex. $p \to q \equiv \neg p \vee q$

$\bar{p} \cup q$

Membership table

	A	B	A∪B
①	∈	∈	∈
	∈	∉	∈
	∉	∈	∈
	∉	∉	∉

A	B	A∨B
T	T	T
T	F	T
F	T	T
F	F	F



or

A	B	A∪B
1	1	1
1	0	1
0	1	1
0	0	0

Ex. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{2, 3, 6\}$

$B = \{1, 4, 5, 7\}$

$C = \{2, 4, 6, 8, 10\}$

① $(A \cup B) \cap C$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$

$(A \cup B) \cap C = \{1, 2, 3, 4, 5, 6, 7\} \cap \{2, 4, 6, 8, 10\}$

$= \{2, 4, 6\}$

② $(\bar{A} \cap C) - B$

③ $(B - \bar{C}) \cup (A - \bar{C})$