Quiz 2. 2.1, 2.2, 2.3, 2.4

$\uparrow$

1-1
onto

§2.6 Matrices.

1. Matrix.

<u>def.</u> A matrix is a rectangular array of numbers. A matrix with m rows and n columns is called $m \times n$ matrix.

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \\ 1 & 3 \end{bmatrix} \quad 3 \times 2 \text{ matrix.}$$

Note: A matrix with the same # of rows and columns is called square matrix.

Ex. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

<u>Def</u> 2. let m and n be positive integer,

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$(i,j)^{th}$ element or entry of A. a_{ij}

the i^{th} row of A is $1 \times n$ matrix $[a_{i1}, a_{i2}, \cdots a_{in}]$

the j^{th} column of A is $m \times 1$ matrix $\begin{bmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{bmatrix}$

Ex. $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ $\quad (3,2)^{th}$ element $= 0$

3^{rd} row $[0 \ 0 \ 1]$ $\quad 3^{rd}$ column. $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$

2. Matrix Arithmetic

1) Addition/Subtraction: A and B (same size) $m \times n$ matrices

Ex. $A: 2\times 3$, $B: 3\times 2$ $\quad A \pm B$ No answer!

$A=[a_{ij}]$ $B=[b_{ij}]$ $m\times n$ matrices

$A+B = m\times n$ matrix that has $a_{ij}+b_{ij}$.

Ex.
$$\underset{3\times 3}{\begin{bmatrix} 1 & 0 & -1 \\ 2 & 2 & -3 \\ 3 & 4 & 0 \end{bmatrix}} + \underset{3\times 3}{\begin{bmatrix} 3 & 4 & -1 \\ 1 & -3 & 0 \\ -1 & 1 & 2 \end{bmatrix}} = \underset{3\times 3}{\begin{bmatrix} 4 & 4 & -2 \\ 3 & -1 & -3 \\ 2 & 5 & 2 \end{bmatrix}}$$

(Add or subtract the corresponding entry)

2). Multiplication

$A = m\times k$ matrix $\quad A\times B$: Column # of A = row # of B

$B = k \times n$ matrix $\quad A\times B = m\times n$ matrix.

$A=[a_{ij}]$ $B=[b_{ij}]$

$AB=[C_{ij}]$ $\quad C_{ij} = a_{i1}b_{1j} + a_{i2}b_{2j} + a_{i3}b_{3j} + \cdots a_{ik}b_{kj}$

Ex. $A = \underset{4\times 3}{\begin{bmatrix} 1 & 0 & 4 \\ 2 & 1 & 1 \\ 3 & 1 & 0 \\ 0 & 2 & 2 \end{bmatrix}}$ $\quad B = \underset{3\times 2}{\begin{bmatrix} 2 & 4 \\ 1 & 1 \\ 3 & 0 \end{bmatrix}}$

$$A\cdot B = \underset{4\times 3}{\begin{bmatrix} 1 & 0 & 4 \\ 2 & 1 & 1 \\ 3 & 1 & 0 \\ 0 & 2 & 2 \end{bmatrix}} \cdot \underset{3\times 2}{\begin{bmatrix} 2 & 4 \\ 1 & 1 \\ 3 & 0 \end{bmatrix}} = \underbrace{\begin{bmatrix} 1\times2+0\times1+4\times3 & 1\times4+0\times1+4\times0 \\ 2\times2+1\times1+1\times3 & 2\times4+1\times1+1\times0 \\ 3\times2+1\times1+0\times3 & 3\times4+1\times1+0\times0 \\ 0\times2+2\times1+2\times3 & 0\times4+2\times1+2\times0 \end{bmatrix}}$$

$$A \cdot B = \begin{bmatrix} 14 & 4 \\ 8 & 9 \\ 7 & 13 \\ 8 & 2 \end{bmatrix}$$

$$\underline{\hspace{3cm}}$$
4×2

Note: $\underset{k \times n}{B} \cdot \underset{m \times k}{A}$ unless $n = m$, otherwise $B \cdot A$ doesn't exist!

**. $A \cdot B$ is not necarily $= B \cdot A$

Ex. $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

$A \cdot B = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$

$B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

$B \cdot A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$

$AB \neq BA$

3. Transposes and Powers of Matrices.

① Identity Matrix of order n ($n \times n$ matrix. $I_n = [\delta_{ij}]$ where $\delta_{ij} = 1$ if $i = j$, and $\delta_{ij} = 0$ if $i \neq j$)

$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(3x3 identity)

Ex. $1 \times 5 = 5$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\underset{m \times n}{A} \cdot I_n = I_m \cdot A = A$$

$A: 3 \times 4$ $A \cdot I_4 = A$

$I_3 \cdot A = A$

$A \cdot A^{-1} = I$ $\qquad\qquad x \cdot \frac{1}{x} = 1$

Ex. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ Find A^{-1}.

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{cases} -3(1 \cdot a + 2c = 1) \\ 3a + 4c = 0 \end{cases} \qquad \begin{cases} 1 \cdot b + 2d = 0 \Rightarrow b = -2d \\ 3b + 4d = 1 \end{cases}$$

Elimination method

$$\begin{array}{r} \cancel{-3a} - 6c = -3 \\ + \quad \cancel{3a} + 4c = 0 \\ \hline -2c = -3 \end{array}$$

$c = \frac{3}{2}$

$3a + 4\left(\frac{3}{2}\right) = 0$

$3a + 6 = 0$

$a = -2$

substitution method

$3(-2d) + 4d = 1$

$-6d + 4d = 1$

$-2d = 1$

$d = -\frac{1}{2}$

$b = -2\left(-\frac{1}{2}\right) = 1$

$$\boxed{A^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}}$$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Textbook page 194 # 19 $\qquad 2 \times 2 \qquad A^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \rightarrow A^{-1} = \frac{1}{1(4) - 2(3)} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \frac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 \cdot \left(-\frac{1}{2}\right) & -2\left(-\frac{1}{2}\right) \\ -3\left(-\frac{1}{2}\right) & 1 \cdot \left(-\frac{1}{2}\right) \end{bmatrix} = \boxed{\begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}}$$