§4.2. Integer Representation and Algorithm.

1. Representations of integers

$541_{10} = 5\times10^2 + 4\times10^1 + 1\times10^0$

$541_8 = 5\times8^2 + 4\times8^1 + 1\times8^0$ (Base 8: digit: 0 - 7)

$\underset{\uparrow\,2^3\ \uparrow\,2^2\ \uparrow\,2^1\ \uparrow\,2^0}{1001_2} = 1\times2^3 + 0\times2^2 + 0\times2^1 + 1\times2^0$ (Base 2: digit: 0 - 1)

$A95_{16} = A\times16^2 + 9\times16^1 + 5\times16^0$ (Base 16: digit: 0 - 9, $\underset{10}{A}, \underset{11}{B}, \underset{12}{C}, \underset{13}{D}, \underset{14}{E}, \underset{15}{F}$)
0 - 15

Ex

Base 21: 0 - 9, A, B, C, D, E, ...
0 - 20

Theorem 1 (Base b expansion of n) let b (← base) be an integer greater than 1, then if n is a positive integer,

it can be expressed uniquelly in the form.

$$n = a_k\cdot b^k + a_{k-1}\cdot b^{k-1} + \cdots + a_1 b^1 + a_0\cdot b^0 \quad \text{where } k\in \mathbb{Z}^+\cup\{0\}$$

$a_0, a_1, \ldots a_k$ are non-negative integers that is less than b and $a_k \neq 0$

Base 8: You'll not have some numbers: $7\underset{=8}{8}\underset{>8}{9}_8$ (impossible)

$112 = 0112$ ← we don't write in this way.

Theorem 1 can help you to convert: Other Bases to Base 10

$571_8 = 5\times8^2 + 7\times8^1 + 1 = 320 + 56 + 1 = 377_{10}$

$$\begin{array}{|c|c|c|}\hline 5 & 7 & 1 \\ \hline \end{array}$$
$\uparrow 8^2 \quad \uparrow 8^1 \quad \uparrow 8^0$

$(101100)_2 = 1\times2^5 + 0 + 1\times2^3 + 1\times2^2 + 0 + 0 = 32 + 8 + 4 = 44$

$$\begin{array}{|c|c|c|c|c|c|}\hline 1 & 0 & 1 & 1 & 0 & 0 \\ \hline \end{array}$$
$\uparrow 2^5 \quad \uparrow 2^4 \quad \uparrow 2^3 \quad \uparrow 2^2 \quad \uparrow 2^1 \quad \uparrow 2^0$

$(2AE0B)_{16} = 2\times16^4 + \overset{10}{A}\times16^3 + \overset{14}{E}\times16^2 + 0 + \overset{11}{B}\times16^0$

$= 175627$

$$\begin{array}{|c|c|c|c|c|} \hline 2 & A & E & 0 & B \\ \hline \end{array}$$
$$\begin{array}{ccccc} \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ 16^4 & 16^3 & 16^2 & 16^1 & 16^0 \end{array}$$

2. Base Conversion. (Base 10 → other Bases)

① $\underset{\text{integer}}{n} = \underset{\text{base}}{b} \cdot \underset{\text{quotient}}{q_0} + \underset{\text{remainder}}{a_0}$ $\quad 0 \le a_0 < b$

Note: $\underline{a_0}$ will be the rightmost digit in the base $b$ expansion of $n$.

② $q_0 = b \cdot q_1 + a_1$

$a_1$ will be the second digit from the right

⋮

③ $q_1 = b \cdot q_2 + a_2$

⋮

This process terminates when $q = 0$

Ex. $(12345)_{10} \to (\ ?\ )_8$ Octal expansion.

$12345 = 8(\underline{1543}) + 1 \leftarrow a_0$ $\Leftarrow$ ① $12345 \div 8 = 1543. \cdots$

$1543 = 8(\underline{192}) + 7$

$192 = 8(\underline{24}) + 0$

$24 = 8(\underline{3}) + 0$

$3 = 8(0) + 3$

stop!

$(12345)_{10} = (30071)_8$

Ex. Convert $(177130)_{10}$ to Base 16 (Hexadecimal expansion)

$177130 = 16 \times (11070) + 10 \Rightarrow A$

$11070 = 16 \times (691) + 14 \to E$

$691 = 16 \times (43) + 3$

$(2B3EA)_{16}$

$43 = 16 \times (2) + \boxed{11} \Rightarrow B$

$2 = 16 \times 0 + 2$

Ex $(\quad)_8 \Rightarrow (\quad)_2$

$(\quad)_8 \Rightarrow (\quad)_{10} \Rightarrow (\quad)_2$

other Bases → Base 10 ⇒ other Bases

3. Integer Operations.

① Addition.

Ex. $a: (1110)_2$ and $b\ (1011)_2$

$$
\begin{array}{r}
1\,1\,1\phantom{0} \\
1\,1\,1\,0_2 \\
+\ 1\,0\,1\,1_2 \\
\hline
1\,1\,0\,0\,1_2
\end{array}
$$

$$
\begin{array}{r}
1 \\
2\,\overline{)\,3} \\
\underline{2} \\
1
\end{array}
\qquad
\begin{array}{r}
1 \\
2\,\overline{)\,2} \\
\underline{2} \\
0
\end{array}
$$

$2_{10} = (10)_2$

$2 = 2 \times 1 + 0$

$1 = 2 \times 0 + 1$

$35 + 46$

vertical form

$$
\begin{array}{r}
1\phantom{5} \\
3\,5 \\
+\ 4\,6 \\
\hline
8\,1
\end{array}
\qquad
\begin{array}{r}
1\phantom{4} \\
1\,4 \\
+\ \ \ 6 \\
\hline
2\,0
\end{array}
$$

$8_{10} = (10)_8$

10