Chapter 2. Basic structures: Sets, Functions, Sequences and Matrix

§2.1 Sets

1. Def. A set is an unordered collection of objects.
elements, or members.

{ } ← Notation for a set.

upper case letter to represent set

Ex. $\underbrace{Z = \{ x \mid x \text{ is an integer} \}}_{\text{set builder}}$

Another way for Z: $\{ \ldots, -2, -1, 0, 1, 2, 3, \ldots \}$

Notation: $\in$ v $\notin$

$1 \in Z$: 1 is an element of the integer set.
↑ belongs to

$\frac{4}{5} \notin Z$
↑ doesn't belong to

Ex. The set P of odd positive integers less than 10.

$P = \{1, 3, 5, 7, 9\}$

the number of elements in the set. $|P| = 5$

**. $Q = \{1, 3, 3, 5, 7, 9\}$ $|Q| = 5$

2. Equal sets

Def. Two sets are equal if and only if they have the same elements.

Translation: $A = B$ iff $\forall x \, (x \in A \leftrightarrow x \in B)$

Ex. $\{1, 3, 5\} = \{3, 1, 5\}$
$\{1, 3, 5\} = \{1, 3, 3, 5, 5, 5\}$ they have the same elements.

3. The empty set: A special set has no element.

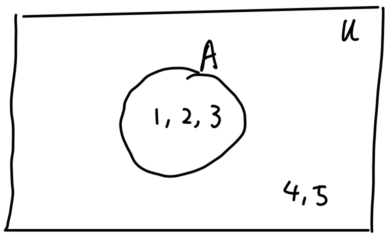
or the null set: $\{ \}$, $\emptyset$

Common errors: $\phi \neq \{\emptyset\}$
{ }

Ex. $A = \{x \mid x \in \mathbb{Z} \wedge x^2 = 10\} = \{\ \}$ or $\emptyset$ $\qquad B = \{x \mid \underline{x \in \mathbb{Z}} \wedge \underline{x^2 = 4}\} = \{-2, 2\}$

$x = \pm\sqrt{10} \notin \mathbb{Z}$

4. Venn Diagram.



→ universal set
(Contains all the objects under consideration)

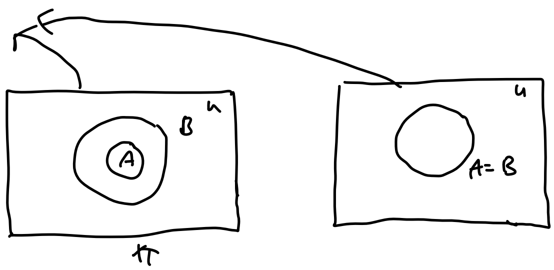
$U = \{1, 2, 3, 4, 5\}$

$A = \{1, 2, 3\}$

5. subsets.

Def. The A is a subset of B iff every element of A is also an element of B.

$A \subseteq B$

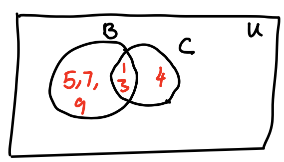


Ex. $U = \{1, 2, 3, \dots, 10\}$

$A = \{1, 3, 5\}$ $\quad B = \{1, 3, 5, 7, 9\}$

$A \subseteq B$: $\quad B \nsubseteq A$

$C = \{1, 3, 4\}$ $\quad C \nsubseteq B$



Translate the def : $A \subseteq B \longleftrightarrow \forall x (x \in A \rightarrow x \in B)$

Ex. $A = \{1, 3, 5\}$, List subsets of A. $\qquad |A| = 3 \quad$ # of subsets $= 2^3 = 8$

$\vdots$

$|A| = n \quad \cdots\cdots = 2^n$

⇒ 0 element : $\emptyset$

1 element : $\{1\}, \{3\}, \{5\}$

2 elements : $\{1, 3\}, \{1, 5\}, \{3, 5\}$ $\quad <$

⇒ 3 elements : $\{1, 3, 5\}$ $\quad (=)$

Theorem 1. For every set S, 1) $\emptyset \subseteq S$ 2) $S \subseteq S$

$\{$Monday, T, W, Th$\}$

$\emptyset$

$\vdots$

$\{M, T, W\ Th\}$

Pf.