§1.3 propositional Equivalences.

1. Tautology vs. Contradiction vs. contigency

1) Tautology : A compound proposition that is always true, No matter what the truth values of the propositional variables that occur in it.

Ex. $p \vee \neg p$ = True ! Tautology.

2). Contradiction : · · · · · · · · · that is always False, · · · · · · · · .

Ex. $p \wedge \neg p$ = False ! Contradiction.

3). Contigency : Not a tautology or Contradiction. Ex.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

2. Def. the compound propositions p and q are logically equivalent if $p \leftrightarrow q$ is a tautology. $p \equiv q$

Ex. $p \to q \equiv \neg q \to \neg p$
- ① Truth Table
- ② $(p \to q) \longleftrightarrow (\neg q \to \neg p) = T$ No matter what values of q and p.

3. Equivalence laws

① De Morgan's law   Ex. $-(a-b) = -a+b$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

② Conditional Equivalence

$p \to q \equiv \neg p \vee q$

Neg change keep   Neg change keep

Ex. show $\neg(p \to q) \equiv p \wedge \neg q$.

$\neg(p \to q) \overset{\text{Conditional eq}}{\equiv} \neg(\neg p \vee q)$

$\overset{DM}{\equiv} p \wedge \neg q$

$p \wedge \neg q \overset{DM}{\equiv} \neg(\neg p \vee q)$ neg change keep

$\overset{\text{Conditional}}{\equiv} \neg(p \to q)$