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MAT 120 **EXAM I**

Show relevant work, where appropriate, answers without support may receive little or no credit.

Total: 100 points + 20 points Extra Credits

1. Devise a logical statement that has the truth values shown in the table. (5 pts)

p	q	Statement
T	T	T
T	F	T
F	T	F
F	F	T

$(\neg p \to \neg q)$

2. Symbolize each of the following quantified statements. Then form the negation, so that no negation appears to the left of a quantifier. Finally, express the negation in simple English. Use the letter appearing in bold to symbolize the embedded simple statement. (18 points)

a) Some **d**rivers do not **o**bey the posted speed limits.

Statement: $\exists x(D(x) \wedge \neg O(x))$ Negation: $\forall x(\neg D(x) \vee O(x))$

Negation in English: All people are either not drivers or they obey the posted speed limits.

b) All **f**oreign movies are **s**ubtitled.

Statement: $\forall x(F(x) \to S(x))$ Negation: $\exists x(F(x) \wedge \neg S(x))$

Negation in English: There is a foreign movie that is not subtitled.

c) No one can **k**eep a secret.

Statement: $\neg \exists x K(x)$ Negation: $\exists x\, K(x)$

Negation in English: Someone can keep a secret

3. Determine the truth value of the following statements. Justify your answers (i.e. if false, provide a counter-example; if true, show or explain why). (5 points each)

a) $\forall n \epsilon Z\ \exists m \epsilon Z\ (n < m)$

True, b/c for any integer n there is a greater integer m.

$m = n + 1$ -1

b) $\exists n \epsilon Z\ \forall m \epsilon Z\ (n < m)$

False, b/c for an n not every possible integer m satisfies. For instance for $n = 2$, $m = 1$ doesn't satisfy.

$m = n - 1$.

c) $\forall x \in Z\,[x \neq 0 \to \exists y \in Q^+(xy = 2)]$

false, b/c of the positive condition on y. If x is negative, $y = \frac{2}{x} < 0 \notin Q^+$

√4. (6 points) True or False. (Explanations are not required)

The set $A = \{\{1,2,3\}, \{4,5\}, \{6,7,8\}\}$

√(a) $1 \in A$

False

√(b) $\{6,7,8\} \in A$

True

√(c) $\{\{4,5\}\} \subseteq A$

True

√(d) $\{1,2,3\} \subseteq A$

False

√(e) $\emptyset \subseteq A$

True

√(f) $\emptyset \in P(A)$

True

5. For each of the following, answer true, false or can't be determined. (6 points)

√a) If $\neg(\neg p \leftrightarrow q)$ and p are premises, (premises means that the statement has to be true.), then what is the truth value of $q \vee k$?

$p = T \Rightarrow \neg p = F$. $\neg(\neg p \leftrightarrow q) = T \Rightarrow (\neg p \leftrightarrow q) = F$

$q = T$

Domination

$q \vee k = T \vee k \stackrel{\vee}{=} T$

b) If $\neg(\neg w \to a)$ is a premise, then what is the truth value of $p \to a$?

$\neg(\neg w \to a) = T \Rightarrow (\neg w \to a) = F$

$\Downarrow \quad \Downarrow$

$\neg w = T \quad a = F$

$p \to a \equiv p \to F \leftarrow$ This is false if p is true.
This is true if p is false

6. Determine whether $(p \to q) \wedge (\neg p \to q) \equiv q$. (Do NOT use the truth table) (10 points)
(State the equivalences' names)

$(p\to q)\wedge(\neg p\to q) \overset{\text{Conditional Eq.}}{\equiv} (\neg p \vee q)\wedge(p\vee q)$

Distrib. $\equiv [(\neg p \vee q)\wedge p] \vee [(\neg p \vee q)\wedge q]$

Distrib. $\equiv [(\neg p\wedge p)\vee(q\wedge p)] \vee [(\neg p\vee q)\wedge q]$

Absorp. $\equiv [(\neg p\wedge p)\vee(q\wedge p)]\vee q$

Negation $\equiv [F \vee (q\wedge p)]\vee q$

Identity $\equiv (q\wedge p)\vee q \overset{\text{Absorp.}}{\equiv} q$

So $(p\to q)\wedge(\neg p\to q)\equiv q$

okay! there is an easier way. Distribute $(\neg p \wedge p)\vee q$

2

7. Prove or disprove the following conjectures. (**Choose 4 questions to answer, circle the questions that you wish to grade.** You can use the rest of the two questions as the extra credits. 10 points each.)

a) The difference of a rational number and an irrational number is irrational.

Try proof by contradiction
Assume the difference is rational

So $\frac{p}{q} + (\text{irrational}) = \frac{a}{b}$

$\Downarrow$

$(\text{irrational}) = \frac{a}{b} - \frac{p}{q}$

$\text{irrational} = \frac{aq-pb}{bq} = \frac{s}{t}$ where $s = aq-pb \in \mathbb{Z}$ and $t = bq \in \mathbb{Z}$

$\text{irrational} = \frac{s}{t} = \text{rational}!$

$\text{irrational} = \text{rational}$ is a contradiction so the assumption was false. So the statement is true.

b) if n is an integer, then $n^2 + 5$ ~~is odd~~ **if and only if** ~~n is even~~.

Prove 1) If n is even, then n^2+5 is odd.

Defn of even $\to n = 2k$ where $k\in\mathbb{Z}$

$n^2+5 = (2k)^2+5 = 4k^2+5 = 4k^2+4+1 = 2(2k^2+2)+1$
$= 2M+1$ where $M = 2k^2+2\in\mathbb{Z}$ since $k\in\mathbb{Z}$
$\Rightarrow$ odd by defn.
Proved part 1

2) If n^2+5 is odd, then n is even.

Do Proof by Contraposition
If n is odd, then n^2+5 is even.

Defn of odd $\to n = 2l+1$ where $l\in\mathbb{Z}$

$n^2+5 = (2l+1)^2+5 = 4l^2+4l+1+5 = 4l^2+4l+6$
$= 2(2l^2+2l+3)$
$= 2M$ where $M = 2l^2+2l+3\in\mathbb{Z}$ since $l\in\mathbb{Z}$
$\Rightarrow$ even

So, by contraposition if n^2+5 is odd, then n is even.

these 2 together prove (b)

+10

(c) Prove or disprove for every nonnegative integer n that $2^n + 6^n$ is an even integer.

$2^n + 6^n = 2^n + (2\cdot 3)^n = 2^n + 2^n(3^n) = 2^n(1+3^n)$

$= 2\left[2^{n-1}(1+3^n)\right]$

$= 2M$ where $M = 2^{n-1}(1+3^n) \in \mathbb{Z}$ since $n \in \mathbb{Z}$

$=$ even

So $2^n + 6^n$ is an even integer

You need to be careful with this part ∵ because n is nonnegative
What if $n=0$

-1

pf by cases : case 1 $n=0$
case 2 $n \geq 1$.

(d) Let m and n be integers. If $m^3 + n^3$ is odd, then m is odd or n is odd.

good!

Try proof by contraposition.
Need to show if m is even and n is even, then m^3+n^3 is even.

$m = 2k$, $n = 2l$ $k, l \in \mathbb{Z}$

$m^3 + n^3 = (2k)^3 + (2l)^3 = 8k^3 + 8l^3$

$= 2(4k^3 + 4l^3)$

$= 2M$ where $M = 4k^3 + 4l^3 \in \mathbb{Z}$ since $k, l \in \mathbb{Z}$

$=$ even

So by contraposition,
If m^3+n^3 is odd, then m is odd or n is odd.

e) Let x and y be positive real numbers. If $x \neq y$, then $\frac{x}{y} + \frac{y}{x} > 2$.

backwards reasoning

$$\frac{x}{y} + \frac{y}{x} > 2 \Rightarrow \frac{x^2}{xy} + \frac{y^2}{yx} > 2 \Rightarrow \frac{x^2+y^2}{xy} > 2 \Rightarrow x^2+y^2 > 2xy$$

$$x^2 - 2xy + y^2 > 0 \Rightarrow (x-y)^2 > 0$$

Proof: Since x and y are distinct positive reals, $(x-y)^2 > 0$,

$$x - y \neq 0 \text{ b/c } x \neq y \Rightarrow x^2 - 2xy + y^2 > 0 \Rightarrow x^2 + y^2 > 2xy \Rightarrow \frac{x^2+y^2}{xy} > 2$$

valid b/c $x \neq 0$ and $y \neq 0$

$$\frac{x^2}{xy} + \frac{y^2}{xy} > 2 \Rightarrow \frac{x}{y} + \frac{y}{x} > 2$$

f) $\forall m \in \mathbb{Z}\ \exists n \in \mathbb{Z}\ [m \text{ odd} \rightarrow m^2 = 8n+1]$

m is odd $\Rightarrow m = 2k+1$ where $k \in \mathbb{Z}$

$$m^2 = (2k+1)^2 = 4k^2 + 4k + 1 \stackrel{?}{=} 8n+1$$

$$\Downarrow$$

$$4k^2 + 4k = 8n$$

$$k^2 + 4k = 2n$$

$$k(k+4) = 2n$$

Case 1: k is odd

odd(odd + 4) =

odd(odd) =

odd $\neq$ 2n

$2k+1+4 =$
$2k+4+1 =$
$2(k+2)+1 =$
$2N+1 = \text{odd}$

$(2k+1)(2l+1) =$
$4kl + 2k + 2l + 1 =$
$2(2kl + k + l) + 1 = 2P + 1 = \text{odd}$

Since it fails for the case where k is odd, the statement is false.

Look at back of page for (f)

(f) If m is odd, m^2 is odd b/c $(2k+1)(2l+1) =$ ×
$4kl+2k+2l+1 = 2(2kl+k+l)+1 = 2M+1 = \text{odd}.$

$m^2 \stackrel{?}{=} 8n+1 \Rightarrow m^2-1 = 8n$

m^2-1 is even b/c m^2 is odd
$8n = 2(4n) =$ even by definition
So
even = even true $\Rightarrow$ Original statement is true

$m=2k+1$
$m^2=(2k+1)^2$
can't have $(2k+1)(2l+1)$

Another way to look at it is that
$m^2-1 = (m+1)(m-1) = (\text{even})(\text{even}) = \text{even}$
close.

$(2A)(2B) = 4AB$
$= 2(2AB)$
$= 2M = \text{even}$

$m^2 = (2k+1)^2$
$= \underbrace{4k^2+4k}+1$
$= \underbrace{4k(k+1)}+1$

$k(k+1)$ consecutive integers
then one of them has to be even

+2

So $k(k+1) = \cancel{2M}\,2n$

$m^2 = 4 \cdot \cancel{2M}^{\,2n} + 1 = 8n+1$

$\boxed{\text{///}}$