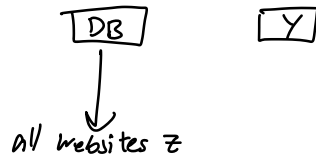


$$\S 1.5 \quad \#5. \quad \exists y \forall z \left( \underbrace{y \neq (DB)}_{\text{student}} \wedge \underbrace{(W(DB, z) \rightarrow W(y, z))}_{\text{student}} \right)$$



$$\#5 f). \quad \exists x \exists y \forall z \left( \underbrace{(x \neq y)}_{\text{student}} \wedge (W(x, z) \leftrightarrow W(y, z)) \right)$$

$$\#39. \quad \forall x \exists y (y^2 = x)$$

$$1, y = \pm 1$$

$$x = 2, y^2 = 2 \Rightarrow y = \pm\sqrt{2} \notin \mathbb{Z}$$

$\S 1.4 \quad \#33$

$$a) \quad \exists x N(x) \quad \neg \exists x N(x) \equiv \forall x \neg N(x)$$

$x \in \text{old dogs}$

All of the old dogs can't learn the new tricks.

$$b) \quad \forall x F(x) \quad \neg \forall x F(x) = \exists x \neg F(x)$$

$x \in \text{birds}$

Some birds cannot fly.

$\S 1.5 \text{ Nested. Quantifiers. (cont.)}$

1. Truth value (last class)

2. Translating

Ex. The sum of two positive integers is always positive.

① Variables:  $x, y$

② predicate:  $x + y > 0$

③ quantifiers:  $\forall x \forall y$

④ Domain:  $x \in \mathbb{Z}^+, y \in \mathbb{Z}^+$

$x, y \in \mathbb{Z}^+$

$$\forall x \in \mathbb{Z}^+ \forall y \in \mathbb{Z}^+ (x + y > 0) \quad \text{or} \quad \forall x \forall y (x + y > 0)$$

Ex. "Ev<sup>y</sup> real number except 0 has a multiplicative inverse."  $x \cdot y = 1$

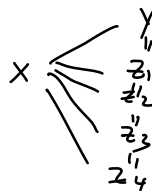
$$\forall x \exists y \left( \begin{array}{l} x \in \mathbb{R} - \{0\} \\ y \in \mathbb{R} \end{array} \quad xy = 1 \right)$$

$$\boxed{x, y \in \mathbb{R}} \quad \forall x \exists y [(x \neq 0) \rightarrow (xy = 1)] = \forall x [(x \neq 0) \rightarrow \exists y (xy = 1)]$$

Ex. Everyone has exactly one best friend.

$B(x, y)$  :  $y$  is  $x$ 's best friend.

$$\forall x \exists y (B(x, y) \wedge \forall z [B(x, z) \rightarrow (y = z)]) \quad \text{Hint: } A \rightarrow \boxed{B} \vee \boxed{C}$$



Contrapositive

$$\forall x \exists y (B(x, y) \wedge \forall z [(y \neq z) \rightarrow \neg B(x, z)])$$

3. Negation.

$$\neg \forall x \exists y (xy = 1) \Rightarrow \exists x \forall y \neg (xy = 1) \\ \Rightarrow \exists x \forall y (xy \neq 1)$$