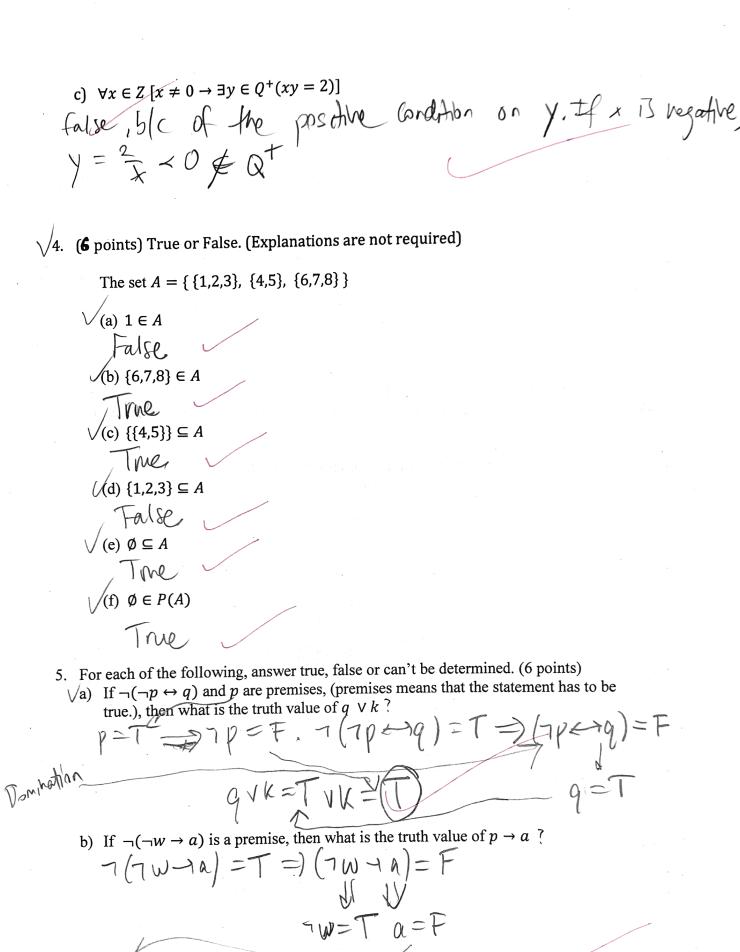
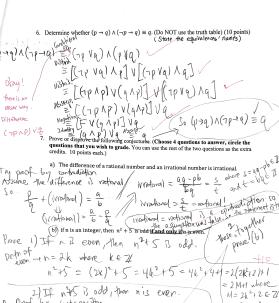
Name: Junesh Thap  MAT 120  EXAM I  Show relevant work, where appropriate, answers without support may receive little
or no credit.  Total: 100 points + 20 points Extra Credits
1. Devise a logical statement that has the truth values shown in the table. (5 pts)  (7 p → 7 q)  P q Statement T T T T T F T F T F T F T F T F T F T F
<ol> <li>Symbolize each of the following quantified statements. Then form the negation, so that no negation appears to the left of a quantifier. Finally, express the negation in simple English. Use the letter appearing in bold to symbolize the embedded simple statement. (18 points)</li> <li>a) Some drivers do not obey the posted speed limits.</li> </ol>
Negation in English: $\frac{AM}{A}$ people one enter not driver or they obey b) All foreign movies are subtitled.
Negation in English: There is a foreign movie that I was subtiled.
c) No one can <b>k</b> eep a secret.
Statement: $\neg \exists x k(x)$ Negation: $\exists x k(x)$
Negation in English: Someone can keep a secret
3. Determine the truth value of the following statements. Justify your answers (i.e. if false, provide a counter-example; if true, show or explain why). (5 points each)
a) $\forall n \in \mathbb{Z} \exists m \in \mathbb{Z} \ (n < m)$ True $b \in \mathbb{Z} \ for only the feel n there is a greater integer m.b \in \mathbb{Z} \ \forall m \in \mathbb{Z} \ (n < m)$
False, ble for an n not every possible integer m softifies. For instance for n=2, m=1 doesn't satisfy.
man - n-1 mstance for n= 1 doesn+ satisfy.



p-1 a = p-1 Fe-This is false of pisme.
This is false of p is false



po proof by contra position its is even.

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So by contraors than it not is and of them in Town

(c) Prove or disprove for every nonnegative integer n that  $2^n + 6^n$  is an even integer.  $2^{n} + 6^{n} = 2^{n} + (2 \cdot 3)^{n} = 2^{n} + 2^{n}(3^{n}) = 2^{n}(1+3^{n})$  $=2[2^{n-1}(1+3^n)]$ = 2M where M = 2h - (1+3") = [ since neZ So 2"+6" is an even integer pf by cases: n=0 (d) Let m and n be integers. If  $m^3 + n^3$  is odd, then m is odd or n is odd. Try proof by contraposition. Need to show if m is even and nit even, then misting is even. m=2k n=21=k,l+21  $m^3 + n^3 = (2k)^3 + (2l)^3 = 8k^3 + 8l^3$ =2(4k3+4l3) =2M where M=4h3+4l3 = Z/Jha K, l=Z/ So by contaposition,

If with 13 is odd, then in is odd or n is odd.

(e) Let x and y be positive real numbers. If  $x \neq y$ , then  $\frac{x}{y} + \frac{y}{x} > 2$ . backwards reasoning X+ + >2 => x2 + x2 >2 => x2+y2>2+y  $x^{2}-2xy+y>0 =)(x-y)^{2}>0$ Proof' Since x and y are distinct positive reals (x-y/50)  $\begin{array}{c} x - y \neq 0 \\ b|c \times \pm y \end{array} \Rightarrow \begin{array}{c} x^2 - 2xy + y^2 > 0 \end{array} \Rightarrow \begin{array}{c} x^2 + y^2 > 2xy \Rightarrow x^2 + y^2 > 2 \end{array}$ valid b/c x‡0 and Since it fails for it m Bodel => m = 2 kH where k = 21 odd, the Statement m2= (2k+1)2=4k2+4k+1 = 8n+1 is false. 462+4k = 8n  $L^2 + 4k = 2a$ K(k+4)=2n Gel: Kisodd Look odd (odd + 4) = odd (odd) = 2k+1+4== bach page odd \$ 2n 2(x+2)++= 2H4/2088 (2k+1 /2l+1) =-4kl+2k+2l+1= 2/2kl+k+l|+1 = 2P+1=0dd

(8) If m is odd, m is odd b/c (244)(2141) = 4k/+2k+2l+1 = 2(2kl+k+l)+1 = 2M+1=old. M== 8n+1 => m2-1 = 8n m²/13 even 1/c m² 13 02d  $m^2=(2k+1)^2$  8n=2(4n)=even by definition can't have (2k+1)(2k+1) So even = even the = original statement Another way to look of A B that  $m^{2'}-1=(m+1)(m-1)=(even)(even)=even$ close. (2A)(2B) = 4AB  $m^2 = (2k+1)^2$ =2(2AB) = 2M = ever =4K2+4K+1 =4K(K+1) +1 K(K+1) consecutive integers then one of them has to be even 50 k(k+1) = 21/2 n  $n^2 = 4 \cdot \frac{2n}{4} + 1 = 8n + 1$