

# Chapter 1. The Foundations: Logic and proofs.

## §1.1 propositional logic

### 1. proposition

A proposition is a declarative sentence. that is either True or false but not both.

Ex  $1+1=2$  ✓ (True prop)  
 $(2) \times (4) = 10$  ✓ (False prop)  
What time is it? (Not a proposition, because it is a question)  
 $x+1 = z$  (Not a proposition, b/c we don't its truth value)

Note: Today is Wednesday. = W

We letters to represent the proposition.

### 2 Negation: ("¬" or "¬") !p

Def. let  $p$  = proposition, then the negation  $p$  is  $\neg p$   
(Not p)

The truth value of the negation of  $p$  is the opposite of the truth value of  $p$ .

$p$	$\neg p$
T	F
F	T

Truth Table

≥ 256

Ex. Her phone has at least 256 GB of memory.

Negation: " " " less than 256 GB " " .

### 3. Conjunction ("∧")

Def: let  $p$  and  $Q$  be propositions, the conjunctions of  $p$  and  $Q$ , denoted by  $p \wedge Q$  is the proposition "p and Q".

$p \wedge Q = T$  when both  $p, Q = T$  and is false otherwise.

Ex. CS major (and) GPA ≥ 3.0 to apply for a scholarship.

✓	3.1	✓
Math	3.1	x
CS	2.9	x
Math	2.9	x

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

Note: 2 prop: 4 rows =  $2^2$

3 prop: 8 rows =  $2^3$

n prop:  $2^n$  rows

P	Q	R
T	T	T
T	F	T
F	T	T
F	F	T

Note: In logic, "but" = "and".

#### 4. Disjunction.

① Inclusive OR: " $\vee$ "  $P \vee Q = \text{False}$   $P, Q = \text{False}$

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

② Exclusive OR: " $\oplus$ "

$P \oplus Q = T$  if only one is true.

Ex. Restaurant week:

P	Q	$P \oplus Q$
T	T	F
T	F	T
F	T	T
F	F	F

Appitizer  $\wedge$  Main Course  $\wedge$  Dessert.



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5. Conditional statements:  $P \rightarrow Q$

p implies q

or if p, then q.

or if p, q

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q whenever p

q unless negation p.

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T

ex. If I win the lottery, I will buy you a house.