Q1. Answer.

$$\exists x \forall t\ F(x,t) \land \forall x \exists x\ F(x,t) \land \underbrace{\neg \forall x \forall t\ F(x,t)}_{\exists x \exists t \neg F(x,t)} \Leftarrow$$

#16 .

$$\neg \forall x \exists y\ T(x,y)$$

$$\neg \exists x \forall y\ T(x,y)$$

$$\boxed{\forall x \exists y \neg T(x,y)} \Leftarrow$$

#10. $[\neg\underbrace{(\neg p \land q)} \land \neg\underbrace{(\neg p \land \neg q)}] \lor (p \land r)$

$\overset{DM}{\equiv} [(p \lor \neg q) \land (p \lor q)] \lor (p \land r)$

$\overset{Dis}{\equiv} [p \lor \underbrace{(\neg q \land q)}_{F}] \lor (p \land r) \Leftarrow$

$\overset{Neg}{\equiv} [p \lor F] \lor (p \land r)$

$\overset{Identity}{\equiv} p \lor (p \land r)$

$\overset{Absorption}{\equiv} p$

Absorption law

$\underbrace{p \lor (p \land q)}_{\equiv p}$ $\quad$ $p \land (p \lor q) \equiv p$

#12. pf by Contradiction.

Assume n is odd.

$n = 2k+1 \quad k \in \mathbb{Z}$

$n^3 + 5 = (2k+1)^3 + 5$

$= (2k)^3 + 3(2k)^2(1) + 3(2k)(1)^2 + 1^3 + 5$

1
1 1
1 2 1
1 3 3 1 $\Leftarrow$

$= 8k^3 + 12k^2 + 6k + 6$

$= 2\underbrace{(4k^3 + 6k^2 + 3k + 3)}$

$k \in \mathbb{Z},\ 4k^3 + 6k^2 + 3k + 3 = M,\ M \in \mathbb{Z}$

$n^3 + 5 = 2M$ Even! It reaches the Contradiction.

Therefore, if n is an integer and n^3+5 is odd, then n is even. ▨

#19. if $x \neq 0$, then $x^2 + \frac{1}{x^2} \geqslant 2$. ✓

Backward reasoning: $x^2 + \frac{1}{x^2} \geqslant 2$ ④

$-2 \qquad -2$

$x^2 + \frac{1}{x^2} - 2 \geqslant 0$ ③

$\Rightarrow (x - \frac{1}{x})^2 \geqslant 0$ ②

$x \neq 0$, $\frac{1}{x}$ is defined. ①

$(a-b)^2 = a^2 - 2ab + b^2$

$x^2 \; \boxed{-2} + \frac{1}{x^2}$

verify

$a = x \qquad -2ab = -2 \cdot x \cdot \frac{1}{x}$

$b = \frac{1}{x} \qquad = -2$

Pf since $x \neq 0$, $x^2 \neq 0$, $\frac{1}{x^2} \neq 0$ $\frac{1}{x^2}$ is defined.

$(x - \frac{1}{x})^{\boxed{2}} \geqslant 0$

$x^2 - 2 + \frac{1}{x^2} \geqslant 0$

$x^2 + \frac{1}{x^2} \geqslant 2$ ▨

#18. a. $\exists x \in \mathbb{Q}, 3x^2 - 27 = 0 \qquad x^2 = 9 \qquad x = \pm 3 \in \mathbb{Q}$ T

b. $\forall n \in \mathbb{R}^+ \; \exists m \in \mathbb{R} \; (n = \sqrt{m})$ For every positive real # n, you can always find a corresponding real # m.

$m = n^2$

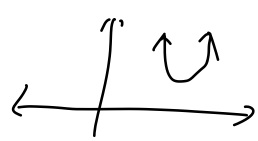
c. $\exists n \in \mathbb{R}, \forall m \in \mathbb{R}^+ \; (n = \sqrt{m})$ False

No one value of n makes $n = \sqrt{m}$ for all positive real # m.

d) $\forall x \in \mathbb{Z}^+ \; [x \neq 0 \rightarrow \exists y \in \mathbb{R}^+ \; (xy = 2)]$ True

$y = \frac{2}{x}$

e). $\forall x \in \mathbb{R} \; (x^2 + 5x + 7 > 0)$

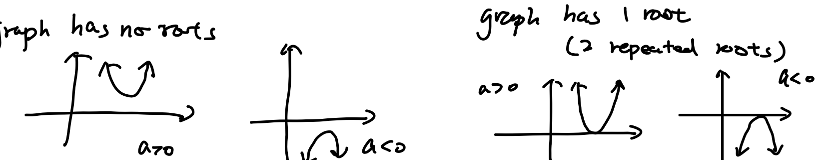


$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Delta = b^2 - 4ac \qquad \Delta < 0 \qquad \Delta = 0$

graph has no roots

graph has 1 root
(2 repeated roots)



x^2+5x+7

$\boxed{\Delta = b^2-4ac}$

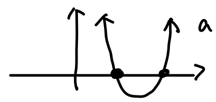
$= 5^2-4(1)(7)$

$= 25-28$

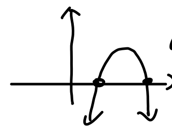
$= -3 < 0$ No root $a=1$

$\Delta > 0$

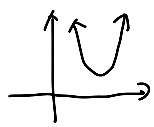
graph has two roots



$a>0$



$a<0$



$\Leftarrow$ $\boxed{\forall x}$ $x^2+5x+7>0$ True.