Exam. (120 pts)

1. Quantifiers Translation $\forall x\ (M(x) \to C(x))$
   Negation (translate in sentence)

2. Nested quantifier: T or F, provide the reason.

   $\forall x\ \exists y\ (x = y^2)$ F $\quad x = -1 \quad -1 = y^2 \quad y = i^2 \notin R$

3. Truth Table
   ①

P	Q	?
T	T	F
T	F	T
F	T	T
F	F	F

   ? $P \oplus Q$ or $\neg(P \leftrightarrow Q)$

   ② $P \wedge Q$ is true, what is $P \vee K$?

   $P \wedge Q = T$ $\quad P \vee K = \boxed{T} \vee K = T$

   $\neg\ \ T$

4. Set (HW #9, #11 §2.1)

   $\{1, 2, 3\}$

   $1 \in \{1, 2, 3\}$ T

   "$\in$, $\subseteq$, $\subset$, $\emptyset$, $P(A)$"

5. Proofs. (6 Questions: choose 4 to answer)
   rest 2 will be extra credit.

   - Direct
   - Indirect < Contraposition, Contradiction
   - Pf by cases
   - backward reasoning.

6. Equivalence, show the names of equivalence

   $P \to q \overset{\text{Con. eq}}{\equiv} \neg P \vee q$

   $\to \neg(P \vee Q) \to \neg(\neg P \to Q)$

P	Q	$\neg P \wedge \neg Q$
T	T	F
T	F	F
F	T	F
F	F	T

$\forall x\ \exists y$ / $\exists x\ \forall y$ — order < Domain., statement

Pf : $\boxed{\text{n is even}}$ if and only if n^2 is even

$\rightarrow$ if n is even, then n^2 is even
Direct pf

$\leftarrow$ if n^2 is even, then n is even,
Indirect Pf < Pf by Contradiction
Pf by Contraposition. if n is odd, then n^2 is odd.

$\boxed{\text{premise}}$: $\neg p \leftrightarrow q$ = True

§1.3 #15.

e) $\neg(p \to q) \to p$

logical equivalence (Always work)

Neg change keep

$\underbrace{\neg(p \to q)}\ \to\ p$

Con. eq $\equiv (p \to q) \vee p$

Con. eq $\equiv (\neg p \vee q) \vee p$

Commutative & Associative $\equiv (\neg p \vee p) \vee q$

Negation $\equiv T \vee q$

Domination $\equiv \boxed{T}$ tautology

Method 2 (Not always work) only work for $T \to F$ F

by Contradiction. Assumption. $F \vee F$ F

$\underbrace{\neg(p \to q)}_{T} \to \underbrace{p}_{F} = F$ $p = F$

$\neg(p \to q) = T$

$(\underbrace{p}_{T} \to \underbrace{q}_{F}) = F$

p can not be true and false at the same time. reach the Contradiction

$\neg(p \to q) \to p = T$ (tautology)

#29. $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

left: $(p \to r) \vee (q \to r)$ $\overset{\text{Con eq}}{\equiv} (\neg p \vee r) \vee (\neg q \vee r)$

Associative Commutative $\equiv (\neg p \vee \neg q) \vee (r \vee r)$

Idempotent $\equiv \boxed{(\neg p \vee \neg q)} \vee r \to \neg(\neg p \vee \neg q) \to r$

Neg change keep

DM. $\equiv \neg(p \wedge q) \vee r \to (p \wedge q) \to r$

Cond. $\equiv (p \wedge q) \to r$