

## § 1.3. Propositional Equivalences

laws

1. De Morgan's law ( $\wedge$ ,  $\vee$ )

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

2. Conditional eq ( $\rightarrow$ ,  $\vee$ )

$$p \rightarrow q \equiv \neg p \vee q$$

3. Identity law

$$p \wedge T \equiv p \quad p \vee F \equiv p$$

4. Domination law

$$p \vee T \equiv T \quad p \wedge F \equiv F$$

5. Idempotent law

$$p \vee p \equiv p \quad p \wedge p \equiv p$$

6. Double Negation. law

$$\neg(\neg p) \equiv p$$

7. Commutative law ( $\vee$ ,  $\wedge$ )

$$p \vee q \equiv q \vee p, \quad p \wedge q \equiv q \wedge p$$

8. Associative Law  $(1+2)+3 = 1+(2+3)$

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

9. Absorption law

$$p \vee (p \wedge q) \equiv p$$

$$p \wedge (p \vee q) \equiv p$$

10. Distributive eq.

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$



$$1 + (2+3) = (1+2) + 3$$

Asso. & Comm.

$$(2+1) + 3$$

Difference: logical operator

#9  
(2 prop)

#10  
(3 prop)

$$3(a+b) = 3a+3b$$

## 11. Negation law

$$P \vee \neg P \equiv T$$

$$P \wedge \neg P \equiv F$$

Ex. Constructing New logical Equivalence.

$$\textcircled{1} \neg [P \vee (\neg P \wedge Q)] \equiv \boxed{\neg P \wedge \neg Q}$$

\*\* Start from more difficult side  
'complicated'

$$\begin{aligned} \text{left: } \neg [P \vee (\neg P \wedge Q)] &\stackrel{\text{DM}}{=} \neg P \wedge \neg (\neg P \wedge Q) \\ &\stackrel{\text{DM}}{=} \neg P \wedge (P \vee \neg Q) \\ &\stackrel{\text{Distrib.}}{=} (\neg P \wedge P) \vee (\neg P \wedge \neg Q) \\ &\stackrel{\text{Neg.}}{=} F \vee (\neg P \wedge \neg Q) \\ &\stackrel{\text{Identity}}{=} \boxed{\neg P \wedge \neg Q} \end{aligned}$$

$$\begin{aligned} \text{Ex. } \neg p \rightarrow (q \rightarrow r) &\equiv \boxed{q \Rightarrow (p \vee r)} \\ &\stackrel{\text{cond.}}{\equiv} \neg p \rightarrow (\neg q \vee r) \\ &\stackrel{\text{cond.}}{\equiv} p \vee (\neg q \vee r) \end{aligned}$$

$$\begin{aligned} \text{Computer } \left[ \begin{array}{l} \stackrel{\text{Asso.}}{\equiv} (p \vee \neg q) \vee r \\ \stackrel{\text{Comm}}{\equiv} (\neg q \vee p) \vee r \\ \stackrel{\text{Asso.}}{\equiv} \neg q \vee (p \vee r) \\ \stackrel{\text{Neg.}}{\equiv} \neg q \vee (p \vee r) \\ \stackrel{\text{Cond.}}{\equiv} q \rightarrow (p \vee r) \end{array} \right] &\stackrel{\text{Asso. \& Commutative}}{\Rightarrow} \equiv \boxed{\neg q \vee (p \vee r)} \\ &\text{human brains} \end{aligned}$$

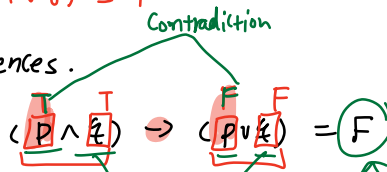
Ex. Show  $(p \wedge q) \rightarrow (p \vee q)$  is a tautology.

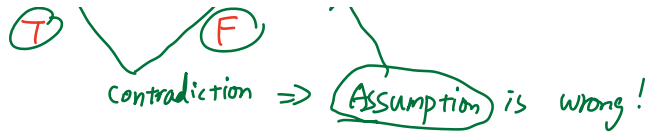
$$\text{Translate: } (p \wedge q) \rightarrow (p \vee q) \equiv T$$

\*\* Method 1

After-class: do the equivalences.

Method 2: Assume  
(Not always works!)





$$(p \wedge q) \rightarrow (p \vee q) \equiv \text{True!}$$

tautology!

Ex. show that  $[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$  is a tautology.

Method 2: Contradiction.

Assume  $[(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r) = \text{False}$

$\rightarrow F = F$

$p = T, \neg p = T$   
 Contradiction!  $\Rightarrow [(p \vee q) \wedge (\neg p \vee r)] \rightarrow (q \vee r)$   
 tautology

Method 1: try after the class