§ 1.5. Nested Quantifiers.

1. Nested Quantifiers: often necessary to express the meaning of sentences in English as well as important concepts in CS and Math.

Ex. $\forall x \, \forall y \; (x+y = y+x)$

Translate: $x+y = y+x$ for all real # x and y.

(order doesn't matter)

$= \forall y \, \forall x \; (x+y = y+x)$

$\exists x \, \exists y \; (xy = 1)$ (order doesn't matter)

Translate: there is a pair of real # x and y such that $x \cdot y = 1$

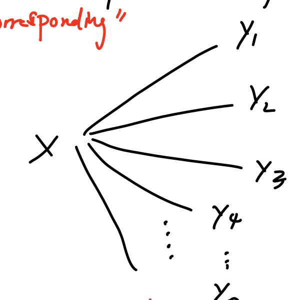
Ex. 1) $\forall x \, \exists y \; (x+y=0)$ $x, y \in R$ True!

For every Real # x, there is a real # y s.t $x+y=0$.

"Corresponding"

$x = 1$ $y = -1$

$x = -1000$, $y = 1000$



2) $\underline{\exists x \, \forall y \; (x+y=0)}$ $x, y \in R$ False

Translate: there is a x, for all real # y. S.t $\underline{x+y=0}$.

"magic"

impossible b/c there is no such x can satisfy for all ys.

Ex. $\exists x \, \forall y \; (xy = 0)$ $x = 0$  Y $0 \cdot y = 0$

True!

Ex. ① $\forall x \in R \; \exists y \in R \; (x+y=1)$

True. $\underline{y = 1-x}$

② $\exists x \in \mathbb{R}\ \forall y \in \mathbb{R}\ (x+y=1)$

F. NO one value x makes $x+y=1$ for all real y.

③ $\exists x \in \mathbb{Z}\ \forall y \in \mathbb{Z}^+\ (x+y=y)$

True $x=0$, $0+y=y$

④ $\exists x \in \mathbb{Z}^+\ \forall y \in \mathbb{Z}\ (x+\boxed{y}=\boxed{y})$

False $x+y > y$

$x=1$ $1+y > y$

Note: * $\forall x \forall y = \forall y \forall x$

$\exists x \exists y = \exists y \exists x$

$\forall x \exists y \neq \exists x \forall y$

$\neq$

or $\exists y \forall x$

textbook: §1.5 page 63.

Statement	True?	False?	
$\forall x \forall y\ p(x,y)$ $\forall y \forall x\ p(x,y)$	$p(x,y)=$ True for every pair (x,y)	There is a pair for which $p(x,y)=F$	Counter-example
$\exists x \exists y\ p(x,y)$ $\exists y \exists x\ p(x,y)$	one pair $(x,y)=T$	Every pair is false	
$\forall x \exists y\ p(x,y)$	For every x, there is a corresponding y	There is an x s.t $p(x,y)=$ False for every y.	
$\exists x \forall y\ p(x,y)$	one x works for all y	For every x, there is a corresponding y for $p(x,y)=F$	

Ex. $\exists x \exists y\ (x^2+y^2=6)$ $x,y \in \mathbb{Z}$ F

$\exists x \exists y\ (x^2+y^2=5)$ $x,y \in \mathbb{Z}$ T $(1,2)$ $(-1,2)$ $(1,-2)$ $(2,1$ $(-1,-2)$