§1.3. Propositional Equivalences

laws

1. De Morgan's law ($\wedge$, $\vee$)

$\neg(p \wedge q) \equiv \neg p \vee \neg q$
$\neg(p \vee q) \equiv \neg p \wedge \neg q$

2. Conditional Eq ($\rightarrow$, $\vee$) **

$p \rightarrow q \equiv \neg p \vee q$

3. Identity law

$p \wedge T \equiv p$ $\quad$ $p \vee F \equiv p$

4. Domination law

$p \vee T \equiv T$ $\quad$ $p \wedge F \equiv F$

5. Idempotent law

$p \vee p \equiv p$ $\quad$ $p \wedge p \equiv p$

6. Double Negation. law

$\neg(\neg p) \equiv p$

7. Commutative law ($\vee$, $\wedge$)

$p \vee q \equiv q \vee p$, $p \wedge q \equiv q \wedge p$

$1 + (2+3) = (1+2)+3$
Asso. & Commu.
$= (2+1) + 3$

8. Associative Law $(1+2)+3 = 1+(2+3)$

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

Difference : logical operator

9. Absorption law

#9 (2 prop)

$p \vee (p \wedge q) \equiv p$

$p \wedge (p \vee q) \equiv p$

#10 (3 prop)

10. Distributive eq. $\quad 3(a+b) = 3a+3b$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

11. Negation law

$$p \vee \neg p \equiv T$$

$$p \wedge \neg p \equiv F$$

Ex. Constructing New logical Equivalence.

① $\neg[p \vee (\neg p \wedge q)] \equiv \boxed{\neg p \wedge \neg q}$

** Start from more difficult side "complicated"

left: $\neg[p \vee (\neg p \wedge q)] \overset{DM}{\equiv} \neg p \wedge \neg(\neg p \wedge q)$

$\overset{DM}{\equiv} \neg p \wedge (p \vee \neg q)$

$\overset{Distri}{\equiv} (\neg p \wedge p) \vee \boxed{(\neg p \wedge \neg q)}$

$\overset{Neg.}{\equiv} F \vee (\neg p \wedge \neg q)$

$\overset{Identity}{\equiv} \neg p \wedge \neg q$

Ex. $\neg p \rightarrow (q \rightarrow r) \equiv q \Rightarrow (p \vee r)$

$\neg p \rightarrow (q \rightarrow r) \overset{cond.}{\equiv} \neg p \rightarrow (\neg q \vee r)$

$\overset{Cond.}{\equiv} p \vee (\neg q \vee r)$

Computer

$\overset{Asso.}{\equiv} (p \vee \neg q) \vee r$

$\overset{comm}{\equiv} (\neg q \vee p) \vee r$

$\overset{Asso.}{\equiv} \underset{Neg}{\neg q} \underset{cha.}{\vee} \underset{keep}{(p \vee r)}$

$\overset{Cond.}{\equiv} q \rightarrow (p \vee r)$

Asso. & Commutative $\Rightarrow \equiv \neg q \vee (p \vee r)$

human brains

Ex. Show $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

Translate: $(p \wedge q) \rightarrow (p \vee q) \equiv T$

** Method 1

After-class: do the equivalences.

Method 2: Assume $(\overset{T}{p} \wedge \overset{T}{q}) \rightarrow (\overset{F}{p} \vee \overset{F}{q}) = F$

Contradiction

(Not always working)

T F

contradiction $\Rightarrow$ Assumption is wrong!

$(p\wedge q)\to(p\vee q)\equiv$ True!

tautology!

Ex. show that $[(p\vee q)\wedge(\neg p\vee r)]\to(q\vee r)$ is a tautology.

Method 2: Contradiction.

Assume $[(p\vee q)\wedge(\neg p\vee r)]\to(q\vee r)=$ False

$$\underbrace{[(\overset{T}{\boxed{p}}\vee \overset{F}{q})\overset{T}{\wedge}(\overset{T}{\boxed{\neg p}}\vee \overset{F}{r})]}_{T}\to\underbrace{(\overset{F}{q}\vee \overset{F}{r})}_{F}=F$$

$p=T,\ \neg p=T$

contradiction! $\Rightarrow [(p\vee q)\wedge(\neg p\vee r)]\to(q\vee r)$

tautology

Method 1: try after the class