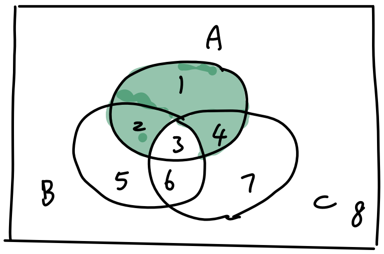
§2.2. HW

\# 29. c)



A = Region 1, 2, 3, 4

B : 2, 3, 5, 6

C : 3, 4, 6, 7

U = 1 − 8

$(A \cap \bar{B}) \cup (A \cap \bar{C})$

$\bar{B}$: 1, 4, 7, 8 $\quad$ $\bar{C}$ = 1, 2, 5, 8

$A \cap \bar{B}$: 1, 2, 3, 4 $\cap$ 1, 4, 7, 8 = 1, 4

$A \cap \bar{C}$: 1, 2, 3, 4 $\cap$ 1, 2, 5, 8 = 1, 2

(1, 4) $\cup$ (1, 2) = 1, 2, 4

§2.2 sets operation.

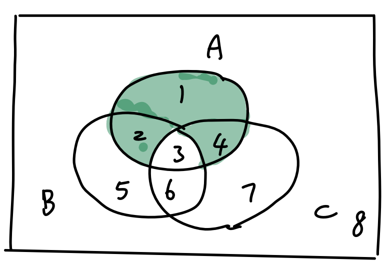
1. Set Identity

1) Distributive law : $\boxed{A \cup} (B \cap C) \equiv (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Ex. $(A \cap \bar{B}) \cup (A \cap \bar{C}) \equiv \boxed{A \cap (\bar{B} \cup \bar{C})}$ $\quad$ HW # 29 (c)

✓



A = Region 1, 2, 3, 4

B : 2, 3, 5, 6 $\quad$ $\bar{B}$: 1, 4, 7, 8

C : 3, 4, 6, 7 $\quad$ $\bar{C}$: 1, 2, 5, 8

U = 1 − 8

$A \cap (\underbrace{\bar{B} \cup \bar{C}})$

(1, 2, 3, 4) $\cap$ (1, 2, 4, 5, 7, 8) = (1, 2, 4)

2. De Morgan's law.

$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$

$\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

Ex. Use the set builders and logical equivalence to establish the DM law $\overline{A \cap B} = \bar{A} \cup \bar{B}$

$$\boxed{\overline{A \cap B}} \overset{\text{define}}{=} \{x \mid x \notin A \cap B\} \quad \text{By def of complement.}$$

$$\overset{\text{eq.}}{=} \{x \mid \neg(x \in A \cap B)\}$$

$$= \{x \mid \neg(\underbrace{x \in A}_{P} \wedge \underbrace{x \in B}_{Q})\} \quad \text{by def of intersection.}$$

$\neg(P \wedge Q)$
$\equiv \neg P \vee \neg Q.$

$$= \{x \mid \neg(x \in A) \vee \neg(x \in B)\} \quad \text{DM law of logical equivalence}$$

$$= \{x \mid (x \notin A) \vee (x \notin B)\}$$

$$= \{x \mid (x \in \bar{A}) \vee (x \in \bar{B})\} \quad \text{by def of complement.}$$

$$= \{x \mid x \in (\bar{A} \cup \bar{B})\} \quad \text{By def of union.}$$

$$\overset{\text{eq.}}{=} \boxed{\bar{A} \cup \bar{B}}$$

$\overline{A \cap B} \equiv \bar{A} \cup \bar{B}$

$==$

$=$

$\equiv$

equivalent

$\boxed{\equiv}$

$a = 3$
↑
defined
Assign

$x+y \equiv y+x$
↑
is defined.
equivalent

$A = \{x \mid x \in \mathbb{Z}\}$

$X :=$
↑
is defined as
in CS.

Set $\xrightarrow[\text{def.}]{}$ logic statement $\xrightarrow[\text{logical equivalences and def}]{}$ Set

Txtboot Page 136 Table 1.

1. Identity law. $A \cap U = A$
   $A \cup \phi = A$

2. Domination law $A \cup U = U$
   $A \cap \phi = \phi$

3. Idempotent law $A \cup A = A$
   $A \cap A = A$

4. Complementation law $\overline{(\bar{A})} = A$

   ← A
   ← Not A
   ← A

5. Commutative law $A \cup B = B \cup A$
   $A \cap B = B \cap A$

6. Associative law $A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

7. Distributive law $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

8. DM law $\overline{A \cap B} = \boxed{\bar{A} \cup \bar{B}}$

$\overline{A \cup B} = \bar{A} \cap \bar{B}$

9. Absorption law $A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$

10. Complement law $A \cup \bar{A} = U$

$A \cap \bar{A} = \emptyset$ (A and $\bar{A}$ are disjoint)

Ex. $\boxed{\overline{A \cup (B \cap C)}} = (\bar{C} \cup \bar{B}) \cap \bar{A}$

refer textbook page 139 (pf from left to right)

we'll prove from right - left

$$(\bar{C} \cup \bar{B}) \cap \bar{A} = \bar{A} \cap (\bar{B} \cup \bar{C}) \quad \text{by commutative law.}$$

$$= \boxed{\bar{A}} \cap \boxed{\overline{(B \cap C)}} \quad \text{by DM's law}$$

$$= \overline{A \cup (B \cap C)} \quad \text{by DM's law}$$

Prove ① Set builder and logical equivalence $\{x \mid \ldots\}$

refer to Ex. for DM law.

use these two to prove

② Apply existing identities. refer to the Example 14 in txtbook page 139

③ Membership table (similar to Truth table)

$\in$ $\notin$

2. Generalized Union and Intersection.

Example:

4. An auto insurance company has 10,000 policyholders. Each policy holder is classified as
   (i) Young or old;
   (ii) Male or female; and
   (iii) Married or single

   Of these policyholders, 3000 are young, 4600 are male, and 7000 are married. The policyholders can also be classified as 1320 young males, 3010 married males, and 1400 young married persons. Finally, 600 of the policyholders are young married males. Calculate the number of the company's policyholders who are young, female, and single.

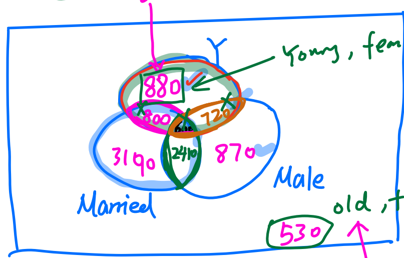
   (a) 280 (b) 423 (c) 486 (d) 880 (e) 896

$3000 - 800 - 600 - 720$

Young = 3000 ← Not a good idea (4 regions)

Male = 4600

Married : 7000



Young ∩ Male = 1320 → 1320 − 600

Married ∩ Male = 3010 → 3010 − 600

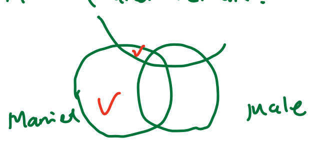
Young ∩ Married = 1400 → 1400 − 600

Married

$10000 - 3190 - 2410 - 800 - 600 - 880 - 720 - 870 = 530$

$\boxed{\text{Young} \cap \text{Married} \cap \text{Male} = 600}$ ← start from here (from 1 region)

# of people who are Married and Female.



$800 + 3190 = 3990$

1/ The union of the collection of sets that contains those elements that are members of all sets in the collection.

$$A_1 \cup A_2 \cup \cdots \cup A_n = \bigcup_{i=1}^{n} A_i$$

2/. The intersection of collection of sets is the set that contains those elements that are members of all the sets in the collection.

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n$$

Ex. $A_i = \{1, 2, 3, \cdots i\}$ for $i = 1, 2, 3, \cdots$

$A_1 = \{1\}$

$A_2 = \{1, 2\}$

$A_{10} = \{1, 2, 3, \cdots 10\}$

$$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup A_3 \cup \cdots = \{1, 2, 3, 4, \ldots\} = Z^+$$

$$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap A_3 \cap \cdots = \{1\}$$