Name: ____________________

MAT 120 **EXAM I**

Show relevant work, where appropriate, answers without support may receive little or no credit.

Total: 100 points + 20 points Extra Credits

1. Devise a logical statement that has the truth values shown in the table. (5 pts)

p	q	Statement
T	T	T
T	F	T
F	T	F
F	F	T

$p \vee \neg q$

2. Symbolize each of the following quantified statements. Then form the negation, so that no negation appears to the left of a quantifier. Finally, express the negation in simple English. Use the letter appearing in bold to symbolize the embedded simple statement. (18 points)

a) Some **d**rivers do not **o**bey the posted speed limits.

Statement: $\exists d \neg O(d)$ Negation: $\forall d\ O(d)$

Negation in English: All drivers obey the posted speed limits.

b) All **f**oreign movies are **s**ubtitled.

Statement: $\forall f\ s(f)$ Negation: $\exists f \neg S(f)$

Negation in English: Some foreign movies are not subtitled

$\neg \exists x\ k(x)$

c) No one can **k**eep a secret.

Statement: $\forall x \neg k(x)$ Negation: $\exists x\ k(x)$

Negation in English: Someone can keep a secret.

3. Determine the truth value of the following statements. Justify your answers (i.e. if false, provide a counter-example; if true, show or explain why). (5 points each)

a) $\forall n \epsilon Z\ \exists m \epsilon Z\ \ (n < m)$

T : $m = n+1$

b) $\exists n \epsilon Z\ \forall m \epsilon Z\ \ (n < m)$

F No integer n is smaller than all of the integers. m

$\cancel{A}=\ m = n-1$

c) $\forall x \in Z\ [x \neq 0 \rightarrow \exists y \in Q^{+}(xy = 2)]$

F $\quad x=-1 \quad y=\frac{2}{x}=-2 \notin Q^{+}$

4. (6 points) True or False. (Explanations are not required)

The set $A = \{\{1,2,3\},\ \{4,5\},\ \{6,7,8\}\}$

(a) $1 \in A$

F

(b) $\{6,7,8\} \in A$

T

(c) $\{\{4,5\}\} \subseteq A$

T

(d) $\{1,2,3\} \subseteq A$

F

(e) $\emptyset \subseteq A$

T

(f) $\emptyset \in P(A)$

T

5. For each of the following, answer true, false or can't be determined. (6 points)

a) If $\neg(\neg p \leftrightarrow q)$ and $\overset{T}{p}$ are premises, (premises means that the statement has to be true.), then what is the truth value of $q \ \vee k$?

$p = T$

$\neg(\neg p \leftrightarrow q) = \neg(\underbrace{F \leftrightarrow q}_{F}) = T$

$q = T$ $\quad T \vee k = \text{True}$

b) If $\neg(\neg w \rightarrow a)$ is a premise, then what is the truth value of $p \rightarrow a$

$\neg(\underbrace{\overset{T}{\neg w} \rightarrow \overset{F}{a}}_{F}) = T$

$a = F$

$p ?$

$? \rightarrow F = \text{CBD}.$

6. Determine whether $(p \to q) \land (\neg p \to q) \equiv q$. (Do NOT use the truth table) (10 points)

$(p \to q) \land (\neg p \to q)$

Conditional eq

$\equiv (\neg p \lor q) \land (p \lor q)$

Distributive

$\equiv (\neg p \land p) \lor q$

Negation

$\equiv F \lor q$

Identity

$\equiv q$

7. Prove or disprove the following conjectures. (**Choose 4 questions to answer, circle the questions that you wish to grade.** You can use the rest of the two questions as the extra credits. 10 points each.)

a) The difference of a rational number and an irrational number is irrational.

Pf by Contradiction: The difference of a rational number and irrational # is rational

let $x \in \mathbb{Q}$, $x = \frac{a}{b}$, $a, b \in \mathbb{Z}$, $b \neq 0$

$y \notin \mathbb{Q}$, $z \in \mathbb{Q}$, $z = \frac{c}{d}$, $c, d \in \mathbb{Z}$, $d \neq 0$

$x - y = z$

$\frac{a}{b} - y = \frac{c}{d}$

$y = \frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$ $b, d \neq 0$

$a, b, c, d \in \mathbb{Z}$

$ad - bc \in \mathbb{Z}$

$bd \in \mathbb{Z}$

by def y is Rational #. $\blacksquare$

b) if n is an integer, then $n^2 + 5$ is odd **if and only if** n is even.

$\to$ if n^2+5 is odd, then. n is even

Pf by Contrapositive, if n is odd, then n^2+5 is even.

let $n = 2k+1$, $k \in \mathbb{Z}$

$n^2 + 5 = (2k+1)^2 + 5$

$= 4k^2 + 4k + 1 + 5$

$= 4k^2 + 4k + 6 = 2(2k^2 + 2k + 3)$

let $2k^2 + 2k + 3 = M$, $M \in \mathbb{Z}$

$n^2 + 5 = 2M$. is even.

$\leftarrow$ if n is even, then n^2+5 is odd

let $n = 2k$, $k \in \mathbb{Z}$

$n^2 + 5 = (2k)^2 + 5$

$= 4k^2 + 5$

$= 4k^2 + 4 + 1$

$= 2(2k^2 + 2) + 1$

let $2k^2 + 2 = M$, $M \in \mathbb{Z}$

$n^2 + 5 = 2M + 1$, is odd.

$\blacksquare$

c) Prove or disprove for every nonnegative integer n that $2^n + 6^n$ is an even integer.

pf. 2^n+6^n

$= 2^n + 2^n \cdot 3^n$

$= 2^n \cdot (1+3^n)$

$= 2 \cdot 2^{n-1}(1+3^n)$

Case 1 $n=0$

$2^n+6^n = 2$

Case 2 $n \geq 1$

$2^n+6^n = 2 \cdot 2^{n-1}(1+3^n)$

b/c $n \geq 1$

2^{n-1} will be an integer

and $(1+3^n)$ is an integer

we can let $2^{n-1} \cdot (1+3^n) = M \in \mathbb{Z}$

$2^n+6^n = 2M$

even.

d) Let m and n be integers. If $m^3 + n^3$ is odd, then m is odd or n is odd.

pf by contrapositive: if m is even and n is even, then m^3+n^3 is even.

let $m=2k$, $n=2p$

$m^3+n^3 = (2k)^3 + (2p)^3$

$= 8k^3 + 8p^3$

$= 2(4k^3+4p^3)$

let $4k^3+4p^3 = M \in \mathbb{Z}$

m^3+n^3 is even

e) Let x and y be positive real numbers. If $x \neq y$, then $\frac{x}{y}+\frac{y}{x}>2$.

Backward reasoning: $\frac{x}{y}+\frac{y}{x}>2$

$$x^2+y^2>2xy$$

$$x^2-2xy+y^2>0$$

$$(x-y)^2>0$$

$$x \neq y$$

Pf. since $x\neq y$, $(x-y)^2>0$

$$x^2-2xy+y^2>0$$

$$x^2+y^2>2xy$$

$$\frac{x}{y}+\frac{y}{x}>2.$$

$\boxed{}$

f) $\forall m \in \mathbb{Z}\, \exists n \in \mathbb{Z}\left[m \text{ odd} \rightarrow m^2=8n+1\right]$

let $m=2k+1$, $k\in\mathbb{Z}$

$$m^2=(2k+1)^2$$

$$=4k^2+4k+1$$

$$=4k(k+1)+1$$

since $k\in\mathbb{Z}$ then $k(k+1)$ is even because they are consecutive integers

let $k(k+1)=2n$, $n\in\mathbb{Z}$

$$m^2=4\cdot 2n+1=8n+1$$

$\boxed{}$