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Math 231 – Midterm Exam 2

Directions. *Read each question on this exam before you start working so you can get the flavor of the questions. Please show all of your work. Unsupported answers will not even be graded. Do not cheat, else you pay with your academic life.*

GRADES	
Problem 1	/10
Problem 2	/10
Problem 3	/10
Problem 4	/10
Problem 5	/10
Problem 6	/10
Problem 7	/10
Problem 8	/10
Problem 9	/10

1. Express

$$A = \begin{bmatrix} 1 & 3 \\ 3 & 6 \end{bmatrix}$$

as a product of elementary matrices.

2. Let

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$$

Find A^{-1} and use it to solve the system $Ax = b$. You must solve for A^{-1} in this problem, otherwise no credit will be given.

3. Find bases for the row space, column space, and null space of

$$A = \begin{bmatrix} 2 & -4 & 5 & 8 & 5 \\ 1 & -2 & 2 & 3 & 1 \\ 4 & -8 & 3 & 2 & 6 \end{bmatrix}.$$

4. Find the standard matrix of the transformation, from $\mathbb{R}^2$ to $\mathbb{R}^2$, given by projection onto the line $y = 2x$.

5. Suppose that $T : \mathbb{R}^n \to \mathbb{R}^n$ is a linear transformation and suppose that v is a vector such that $T(v) \neq 0$ and $T(T(v)) = 0$. Show that v and $T(v)$ are linearly independent.

6. (a) Find the determinant of the matrix

$$A = \begin{bmatrix} 5 & 2 & 6 \\ 1 & 0 & 2 \\ 4 & 2 & 1 \end{bmatrix}$$

(b) Use Cramer's rule to solve the given linear system:

$$2x - y = 5$$
$$x + 3y = -1$$

7. Determine whether the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

is diagonalizable and, if so, find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

8. If

$$A = \begin{bmatrix} -4 & 6 \\ -3 & 5 \end{bmatrix},$$

find A^{50}.

9. (a) If x is an eigenvector of A with eigenvalue $\lambda = 3$, show that x is also an eigenvector of $A^2 - 5A + 2I$. What is the corresponding eigenvalue?

   (b) If A is similar to B with $P^{-1}AP = B$ and x is an eigenvector for A, show that $P^{-1}x$ is an eigenvector for B.