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Math 231 – Practice Midterm Exam 3

Directions. *Read each question on this exam before you start working so you can get the flavor of the questions. Please show all of your work. Unsupported answers will not even be graded. Do not cheat, else you pay with your academic life.*

GRADES	
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Problem 2	/10
Problem 3	/10
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Problem 9	/10

1. Find the orthogonal decomposition of $v = [4, -2, 3]$ with respect to $W = \text{span}(\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix})$

2. Find an orthogonal basis for $\mathbb{R}^3$ that contains the vector $\begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}$. Hint: recall that first you need to find an ordinary basis containing $\begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}$. To do this, you may find a basis for $W^{\perp}$ where $W = \text{span}(\begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix})$. Then include the basis of $W^{\perp}$ with $\begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}$ to get an ordinary basis for $\mathbb{R}^3$. Finally, apply Gram-Schmidt to the result.

3. Find the QR-factorization of the matrix

$$\begin{bmatrix} 2 & 8 & 2 \\ 1 & 7 & -1 \\ -2 & -2 & 1 \end{bmatrix}$$

4. Orthogonally diagonalize the matrix

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

5. Find a symmetric 2×2 matrix with eigenvalues $\lambda_1 = -1, \lambda_2 = 2$, and corresponding eigenvectors $v_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$.

6. Determine whether W is a subspace of V:

   1)

$$V = M_{22}, \quad W = \{\begin{bmatrix} a & b \\ b & 2a \end{bmatrix}\}$$

   2)

$$V = \mathcal{F}, W = \{f \in \mathcal{F} \text{ such that } f(0) = 1\}$$

7. Is $\mathcal{P}_2$ spanned by $1 + x + 2x^2, 2 + x + 2x^2, -1 + x + 2x^2$?

   Hint: There's an easy way to do this question. You have three vectors here, living in a three-dimensional space $\mathcal{P}_2$. By one of our theorems, if they are linearly independent, then they span. If not, then argue that they cannot span.

8. Extend

$$\left\{\begin{bmatrix}0 & 1\\0 & 1\end{bmatrix}, \begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\right\}$$

to a basis for M_{22}.

9. Let $p(x) = 4 - 2x - x^2$, $\mathcal{B} = \{x, 1 + x^2, x + x^2\}$, $\mathcal{C} = \{1, 1 + x, x^2\}$ in $\mathcal{P}_2$.

(A) Find the coordinate vectors $[x]_{\mathcal{B}}$ and $[x]_{\mathcal{C}}$ of x with respect to the bases $\mathcal{B}$ and $\mathcal{C}$, respectively.

(B) Find the change of basis matrix $P_{\mathcal{C}\leftarrow\mathcal{B}}$ from $\mathcal{B}$ to $\mathcal{C}$.

(C) Specifically use your answer in part (b) to compute $[x]_{\mathcal{C}}$, and compare your answer with the one found in (A).

(D) Find the change of basis matrix $P_{\mathcal{B}\leftarrow\mathcal{C}}$ from $\mathcal{C}$ to $\mathcal{B}$.

(E) Use your answers to part (C) and (D) to compute $[x]_{\mathcal{B}}$, and compare your answer with the one found in (A).