# Ps 4 Problem 2

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**Abstract**

This short paper will describe an integration to find the period of an anharmonic oscillator.

## 1 Introduction

Anharmonic oscillators are oscillators that do not have an $x^2$ potential or have additional term in addition to the normal $x^2$ potential. They do not behave in the same ways as harmonic oscillators. One way they differ is in their period: harmonic oscillator period does not depend on the initial amplitude whereas anharmonic oscillators do.

## 2 Methods

The general energy equation for energy is the following:

$$E = \frac{1}{2}(\frac{dx}{dt})^2 + V(x) \tag{1}$$

We can rewrite this as

$$dt = \frac{mdx}{2(E - V(x))} \tag{2}$$

Integrating over a fourth a period, $\frac{T}{4}$, aka from 0 to a, where a is the amplitude, we then get

$$T = \sqrt{8m} \int_0^a \frac{dx}{\sqrt{E - V(x)}} \tag{3}$$

I took V(x) to be $x^4$ so E is $a^4$ and then used scipy quadrature with 20 points to find the period for various points from a = 0 to a = 2.

## 3 Results

The period increases as you move towards 0 and decreases as you move out with a divergence as you approach 0. If you look at the equation we integrated (3 above) this makes sense because as you start smaller and smaller you start dividing by a smaller and smaller number until you divide by zero, and if you move far enough out you end up dividing by a larger and larger number. Thinking about units the period goes as inverse of the amplitude.

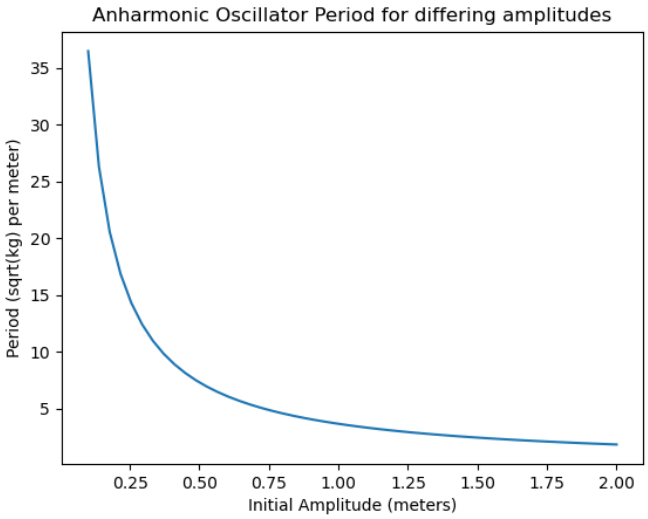


Figure 1: Period of an anharmonic oscillator with potential proportional to $x^4$. The period units are weird because we don't have a constant multiplying the potential.

## 4 Discussion

A lot of integrals are analytic especially when you turn a problem into integral in quadrature. I think thats the name for what we did by solving for dt and then integrating. Scipy makes quick work of them if they aren't very very strange. I was supposed to integrate from a =0 to a = 2, but only did until a = .1 due to the divergence.