Ps 8 Problem 1 and 2

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November 28, 2023

Abstract

This short paper will show some plots and discuss implementing a fourier transform to look at and compare the wave forms and frequencies of a piano and a trumpet. It will also solve a system of differential equations as an example of deterministic chaos.

1 Introduction

Data is often messy and hard to understand at first glance, so processing it into a way that is easier to visualize can be helpful. By Fourier transforming a signal we can break it up into its frequencies, and, if we're lucky or if we're looking at the correct problem, there should be only a few key frequencies that make up the signal. Instruments, when played correctly, will be a primary note and its harmonics. I analyze this for a trombone and a piano signal.

A chaotic system is one in which minuscule differences in initial conditions lead to large differences. Chaos was initially discovered while studying climate patterns. I use a built in scipy integrator to solve an initial value problem for a system of differential equations and show the unpredictable nature it.

2 Methods

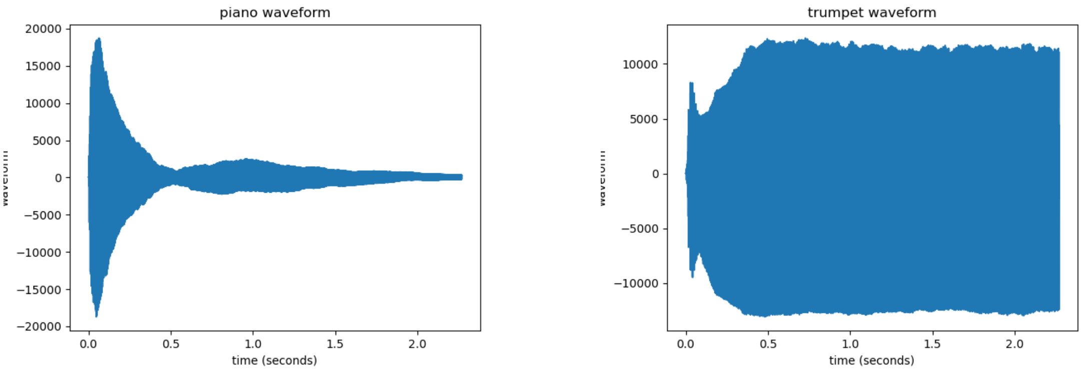
Methods are uninteresting in this problem. For the first part on fourier transforms, I used the built in numpy fast fourier transform functions. Similarly I used a built in scipy integrator with an explicit Runge-Kutta method of order 8. It gave me the expected result on the first try so I did not try other methods.

3 Results

The piano a note of 525 Hz while the trumpet is playing a note of 1043 Hz. I wasn't asked to calculate anything specific for the differential equation portion, so I just include the images with captions discussing them.

4 Discussion

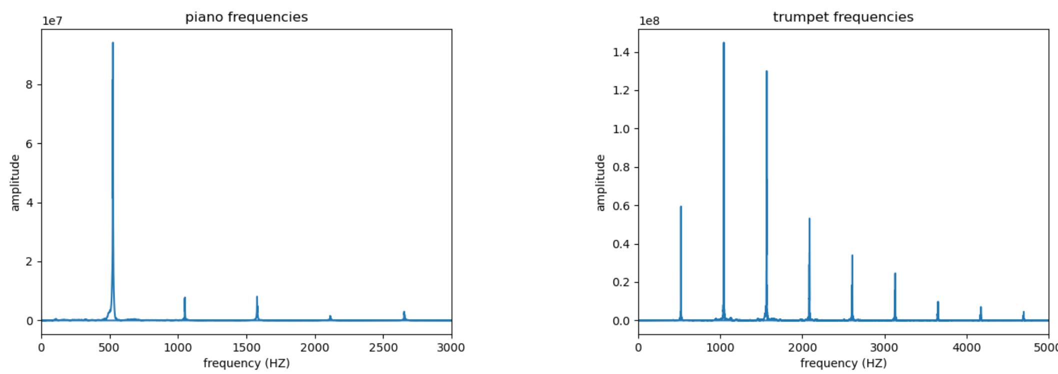
From looking at the waveforms we can immediately tell that the piano is producing a much "cleaner" sound with fewer harmonics and for a shorter period of time. The trumpet is less



(a) Piano waveform

(b) Trumpet waveform

Figure 1: These are both very messy, reducing line width doesn't help as there are many near overlapping but not fully distinct features.

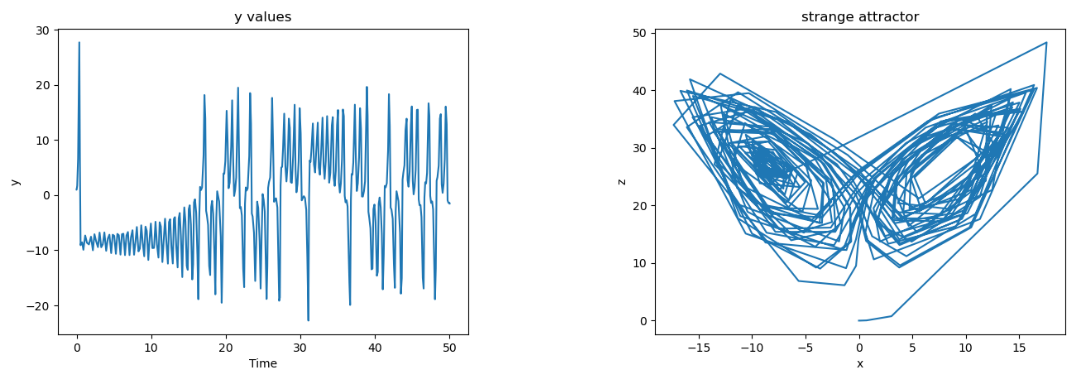


(a) Piano frequencies

(b) Trumpet waveform

Figure 2: This is much better. Now we can see the principle components that go onto making the waveforms and even just by eye pick them out.

clean and the player played the note for a while. Looking at the frequencies you can also see this. Both are playing C, but the piano is playing mostly a normal C of about 525 Hz while the trumpet is playing primarily a C1 of about 1043 Hz from my calculations in the code. The trumpet has far more resonances that the piano because of how brass instruments work, the player is making a C note, but as the note exits it reflects off the air and reverberates back into the instrument cause more harmonics to sounds. All of the frequencies are rational multiples of the same middle c note.



(a) Y values of the differential equation. They are unpredictable and jump all over the place quickly moving from high values to low values are not changing much at all.

(b) The strange attractor shows pseudo-chaotic behavior. The allow values are limited to a range within the attractor, but it never repeats within it.

Figure 3: The both show hall marks of chaotic systems. I only plotted and calculated from t = 0 to t = 50, but these behaviors continue beyond that time frame.