Ps 5 Problem 1

Adam Guerin

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Abstract

This short paper will show some plots and a computational solution for calculating the gamma function

1 Introduction

The gamma function is a useful function that appears all over physics especially in the usage of special functions. This is it:

$$\Gamma(a) = \int_0^\infty x^{a-1} e^{-x} dx \tag{1}$$

The function has no closed form solution so you either have to calculate it numerically or look it up in a book or online if you want specific value of it.

2 Methods

By taking the derivative of the integrand and setting it to zero we can see that the max of the function occurs at x = a-1 (you can also see this in figure 1 where I've marked x = a-1 on each graph). Even the smartest computers can't directly integrate an integral without an analytic solution, so we shift the integral so that the previous peak instead falls around x = 1/2 and change the boundaries so that we are integrating from x = 0 to x = 1. Also this form of the gamma function isn't ideal for computers to calculate because over the integrated range x^{a-1} and e^{-x} become very different and computers don't like doing math on number of very different orders, so we rewrite gamma so that we can avoid that issue:

$$\Gamma(a) = \int_0^\infty e^{(a-1)ln(x)-x} dx \tag{2}$$

Taking all of this together the actual integral we are doing end up being this:

$$\Gamma(a) = \int_0^1 \frac{(a-1)}{(1-z)^2} e^{(a-1)ln(\frac{z}{1-z})-\frac{z}{1-z}} dz \tag{3}$$

To actually integrate the value I just used scipy fixed quadrature because I couldn't really be bothered to write a function to shift quadrature roots correctly because either of the types of quadrature i've done already doesn't cover the integral from 0 to infinity.

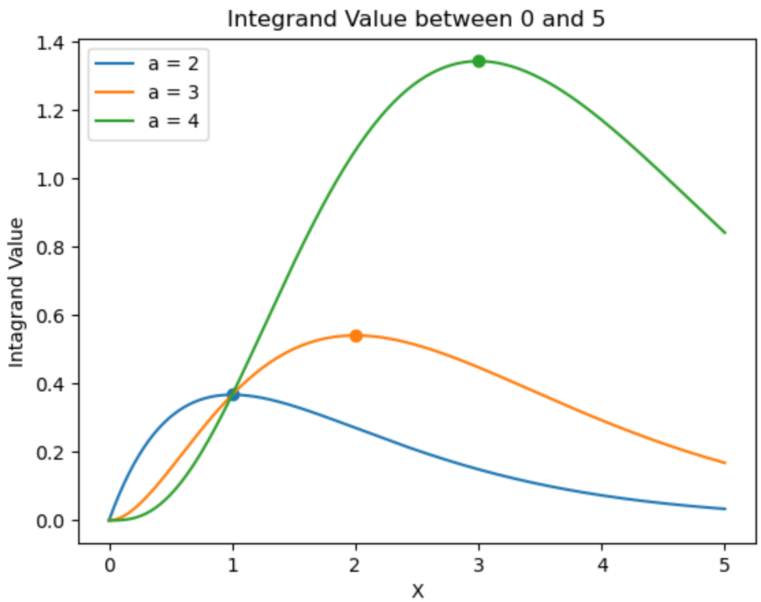

Figure 1: Integrand values for different x and a values. You can see the max at x = a-1

3 Results

For non integer values you can get interesting results for the gamma function, e.g. $\Gamma(\frac{3}{2}) = \frac{\sqrt{\pi}}{2}$ For positive integer, a, $\Gamma(a) =!(a-1)$. You can use the code to see these.

4 Discussion

Like with all of the other homework sets, you have to be a little smart with your code as the most naive method isn't always the best. My calculations were within $10^{(} - 12)$of the actual value.