# Ps 3 Problem 1

Adam Guerin

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**Abstract**

This document has an outline of a program that numerically calculates derivatives and compares the results to the analytic solution.

## 1 Introduction

The derivative of a function, $f$ at a point $x$ is defined as follows: $\frac{df}{dx} = \lim_{\delta \to 0} \frac{f(x+\delta)-f(x)}{\delta}$ Computers don't like working with very very small or very very large numbers so naively setting delta to smaller and smaller values isn't always the best choice to better calculating derivatives numerically..

## 2 Methods

Let's calculate the derivative of the function $x(x-1)$ by plugging in $\delta$s in decreasing size into a function that acts as the definition of the derivative given above. Analytically the derivative is $2x-1$ so we know the actual answer and can compare the results.

## 3 Results

Error begins to decrease as you take the limit, but then increases again as you being encountering round off errors.

```
the derivative of x(x-1) at x = 1 is 1, with a delta of 10^-2 we get 1.010000000000001 a difference of 10^-2

numerical derivative for delta = 0.01 : 1.010000000000001
error from actual: 0.010000000000000897

numerical derivative for delta = 0.0001 : 1.0000999999998899
error from actual: 9.999999988985486e-05

numerical derivative for delta = 1e-06 : 1.0000009999177333
error from actual: 9.9991773279787e-07

numerical derivative for delta = 1e-08 : 1.0000000039225287
error from actual: 3.922528746258536e-09

numerical derivative for delta = 1e-10 : 1.000000082840371
error from actual: 8.284037100736441e-08

numerical derivative for delta = 1e-12 : 1.000088900583413
error from actual: 8.890058334132256e-05

numerical derivative for delta = 1e-14 : 0.9992007221626509
error from actual: 0.0007992778373491216
```

Figure 1: Output from code. You can see the error decrease then begin to increase.

# 4 Discussion

Computers don't like really small numbers and especially don't like combining really small numbers with much larger numbers. Adding a $\delta$ of power -12 to 1 isn't a good way to calculate anything in a computer.