System equations development

General

system elements :
 quad 1 - given as system input. x, y coor. θ (will be shown) is not influential
 quad 2 - given as system input. x, y coor. θ (will be shown) is not influential
payload (contrained to quads locations)

system sketch according to 'graphics2D.nb'

General rules

```
Quit[]
```

```
Needs["VariationalMethods`"]
```

```
dispSimp = {devTerm_'[t] → devṪerm, devTerm_''[t] → devT̈erm,
   aTerm_[t] → aTerm, Cos[a_] → c[a], Sin[a_] → s[a], ii_,zz -> Ii};
(* reduce time , trim cos,sin accronyms *)
```

```
QuadsBaseLocations /. dispSimp
```

$\{x_1 \to 0,\ y_1 \to 0,\ x_2 \to 2\, w_p + x_1,\ y_2 \to y_1\}$

Lagrangian

general coordinates are:

```
(q = {{x1[t]}, {y1[t]}, {θ1[t]}, {x2[t]}, {y2[t]}, {θ2[t]}, {xp[t]}, {yp[t]}, {θp[t]}} //
    Flatten) /. dispSimp(*//MatrixForm*)
```

$\{x_1,\ y_1,\ \theta_1,\ x_2,\ y_2,\ \theta_2,\ x_p,\ y_p,\ \theta_p\}$

general kinematics are:

```
(tmp = {
    (Xᵢ = (xᵢ[t]
           yᵢ[t]
           0)),
    (Imatᵢ = (Iᵢ,ₓₓ  0     0
              0     Iᵢ,yy  0
              0     0     Iᵢ,zz)),
    ωᵢ = D[{0, 0, θᵢ[t]}, t],
    (vᵢ = D[Xᵢ, t])
  }) // MatrixForm // TraditionalForm

Table[tmp, {i, {1, 2, p}}] // MatrixForm // TraditionalForm
```

$$\begin{pmatrix} \{x_i(t)\} & \{y_i(t)\} & \{0\} \\ \{\mathbb{i}_{i,\text{xx}}, 0, 0\} & \{0, \mathbb{i}_{i,\text{yy}}, 0\} & \{0, 0, \mathbb{i}_{i,\text{zz}}\} \\ 0 & 0 & \theta_i'(t) \\ \{x_i'(t)\} & \{y_i'(t)\} & \{0\} \end{pmatrix}$$

$$\begin{pmatrix} \begin{pmatrix} \{x_1(t)\} \\ \{y_1(t)\} \\ \{0\} \end{pmatrix} & \begin{pmatrix} \{\mathbb{i}_{1,\text{xx}}, 0, 0\} \\ \{0, \mathbb{i}_{1,\text{yy}}, 0\} \\ \{0, 0, \mathbb{i}_{1,\text{zz}}\} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ \theta_1'(t) \end{pmatrix} & \begin{pmatrix} \{x_1'(t)\} \\ \{y_1'(t)\} \\ \{0\} \end{pmatrix} \\ \begin{pmatrix} \{x_2(t)\} \\ \{y_2(t)\} \\ \{0\} \end{pmatrix} & \begin{pmatrix} \{\mathbb{i}_{2,\text{xx}}, 0, 0\} \\ \{0, \mathbb{i}_{2,\text{yy}}, 0\} \\ \{0, 0, \mathbb{i}_{2,\text{zz}}\} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ \theta_2'(t) \end{pmatrix} & \begin{pmatrix} \{x_2'(t)\} \\ \{y_2'(t)\} \\ \{0\} \end{pmatrix} \\ \begin{pmatrix} \{x_p(t)\} \\ \{y_p(t)\} \\ \{0\} \end{pmatrix} & \begin{pmatrix} \{\mathbb{i}_{p,\text{xx}}, 0, 0\} \\ \{0, \mathbb{i}_{p,\text{yy}}, 0\} \\ \{0, 0, \mathbb{i}_{p,\text{zz}}\} \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ \theta_p'(t) \end{pmatrix} & \begin{pmatrix} \{x_p'(t)\} \\ \{y_p'(t)\} \\ \{0\} \end{pmatrix} \end{pmatrix}$$

```
{(Rp2I = RotationMatrix[θp[t]]) // MatrixForm,
  x1dotSqr = (Transpose[vi].vi)[[1, 1]] /. i → 1,
x2dotSqr = (Transpose[vi].vi)[[1, 1]] /. i → 2,
  IωSqr1 = ωi.Imati .ωi /. i → 1,
  IωSqr2 = ωi.Imati .ωi /. i → 2,
  xpdotSqr = (Transpose[vi].vi)[[1, 1]] /. i → p,
  IωSqrp = ωi.Imati .ωi /. i → p,
  r1[t] = (x1[t]; y1[t]) - ((xp[t]; yp[t]) + Rp2I.{-wp, hp}),
  (* vector of cable length from quad1 to hangPoint1 *)
  r2[t] = (x2[t]; y2[t]) - ((xp[t]; yp[t]) + Rp2I.{wp, hp}),
  (* vector of cable length from quad2 to hangPoint2 *)
  l1 = Sqrt[(r1[t][[1]])^2 + (r1[t][[2]])^2],
  l2 = Sqrt[(r2[t][[1]])^2 + (r2[t][[2]])^2],
  Δ1 = l1 - L01,
  Δ2 = l2 - L02};
(T = 1/2 m1 x1dotSqr + 1/2 IωSqr1 + 1/2 m2 x2dotSqr + 1/2 IωSqr2 + 1/2 mp xpdotSqr + 1/2 IωSqrp);
(*ri=li+Δl*)
V = m1 g (Xi[[2]] /. i → 1) +
   m2 g (Xi[[2]] /. i → 2) + mp g (Xi[[2]] /. i → p) + 1/2 k1 Δ1^2 + 1/2 k2 Δ2^2;
L = (T - V)[[1]] (*Tquad#1+Tquad#2+Tpayload - (Vquad#1+Vquad#2+Vpayload+Vspring#1+Vspring#2)*)
```

$$
\begin{aligned}
&-g\, m_1\, y_1[t] - g\, m_2\, y_2[t] - \frac{1}{2} k_1 \Big(-L0_1 + \sqrt{\big((\mathrm{Sin}[\theta_p[t]]\, h_p + \mathrm{Cos}[\theta_p[t]]\, w_p + x_1[t] - x_p[t])^2 +} \\
&\qquad \overline{(-\mathrm{Cos}[\theta_p[t]]\, h_p + \mathrm{Sin}[\theta_p[t]]\, w_p + y_1[t] - y_p[t])^2\big)}\Big)^2 - \\
&\frac{1}{2} k_2 \Big(-L0_2 + \sqrt{\big((\mathrm{Sin}[\theta_p[t]]\, h_p - \mathrm{Cos}[\theta_p[t]]\, w_p + x_2[t] - x_p[t])^2 +} \\
&\qquad \overline{(-\mathrm{Cos}[\theta_p[t]]\, h_p - \mathrm{Sin}[\theta_p[t]]\, w_p + y_2[t] - y_p[t])^2\big)}\Big)^2 - \\
&g\, m_p\, y_p[t] + \frac{1}{2} m_1 \left(x_1'[t]^2 + y_1'[t]^2\right) + \frac{1}{2} m_2 \left(x_2'[t]^2 + y_2'[t]^2\right) + \\
&\frac{1}{2} \\
&m_p \\
&\left(x_p'[t]^2 + y_p'[t]^2\right) + \\
&\frac{1}{2} \mathbb{i}_{1,zz}\, \theta_1'[t]^2 + \frac{1}{2} \mathbb{i}_{2,zz}\, \theta_2'[t]^2 + \\
&\frac{1}{2} \mathbb{i}_{p,zz}\, \theta_p'[t]^2
\end{aligned}
$$

```
L //. dispSimp // TraditionalForm
```

$$-\frac{1}{2} k_1 \left(\sqrt{\left((w_p\, c(\theta_p) + h_p\, s(\theta_p) - x_p + x_1)^2 + (h_p\,(-c(\theta_p)) + w_p\, s(\theta_p) - y_p + y_1)^2\right)} - \mathrm{L0}_1\right)^2 - \frac{1}{2} k_2 \left(\sqrt{\left((-w_p\, c(\theta_p) + h_p\, s(\theta_p) - x_p + x_2)^2 + (h_p\,(-c(\theta_p)) - w_p\, s(\theta_p) - y_p + y_2)^2\right)} - \mathrm{L0}_2\right)^2 - g\, m_p\, y_p - g\, m_1\, y_1 - g\, m_2\, y_2 + \frac{1}{2} i_1\, \dot{\theta}_1^2 + \frac{1}{2} i_2\, \dot{\theta}_2^2 + \frac{1}{2} m_p \left(\dot{x}_p^2 + \dot{y}_p^2\right) + \frac{1}{2} m_1 \left(\dot{x}_1^2 + \dot{y}_1^2\right) + \frac{1}{2} m_2 \left(\dot{x}_2^2 + \dot{y}_2^2\right) + \frac{1}{2} i_p\, \dot{\theta}_p^2$$

Equations of Motion

derivating the 9 DOF equations:

```
(quad9EOM =
   EulerEquations[L, {x1[t], y1[t], θ1[t], x2[t], y2[t], θ2[t], xp[t], yp[t], θp[t]},
     t](*[[All,1]]*)(*==Q*) // Simplify) // MatrixForm // TraditionalForm
```

```
$Aborted
```

focusing only on the 3DOF of the payload itself:

```
(quadEqNominal = EulerEquations[L,
      {(*x1[t],y1[t],θ1[t],x2[t],y2[t],θ2[t],*)xp[t], yp[t], θp[t]}, t]
      (*[[All,1]]*)(*==Q*) // Simplify) // MatrixForm // TraditionalForm
```

$$\frac{k_1\,(h_p \sin(\theta_p(t))+w_p \cos(\theta_p(t))-x_p(t)+x_1(t))\,\Big(\sqrt{(h_p \sin(\theta_p(t))+w_p \cos(\theta_p(t))-x_p(t)+x_1(t))^2+(h_p}}{\sqrt{(h_p \sin(\theta_p(t))+w_p \cos(\theta_p(t))-x_p(t)+x_1(t))^2+(h_p \cos(\theta_p(t))-w_p \sin(}}$$

$$\frac{k_1\,(h_p\,(-\cos(\theta_p(t)))+w_p \sin(\theta_p(t))-y_p(t)+y_1(t))\,\Big(\sqrt{(h_p \sin(\theta_p(t))+w_p \cos(\theta_p(t))-x_p(t)+x_1(t))^2+(h_p \cos}}{\sqrt{(h_p \sin(\theta_p(t))+w_p \cos(\theta_p(t))-x_p(t)+x_1(t))^2+(h_p \cos(\theta_p(t))-w_p \sin(\theta_p(t))}}$$

$$\frac{k_1\,(h_p\,(x_1(t)\,(-\cos(\theta_p(t)))+x_p(t) \cos(\theta_p(t))+(y_p(t)-y_1(t)) \sin(\theta_p(t)))+w_p\,(x_1(t) \sin(\theta_p(t))-x_p(t) \sin(\theta_p(t))+(y_p(t)-y_1(t)) \cos(\theta_p(t))))\,\Big(\sqrt{(h_p \sin(\theta_p(t))+w_p \cos(\theta_p(t))-x_p(t)+x_1(t))^2+(h}}{\sqrt{(h_p \sin(\theta_p(t))+w_p \cos(\theta_p(t))-x_p(t)+x_1(t))^2+(h_p \cos(\theta_p(t))-w_p \sin(\theta_p(t))+y_p(t)-y_1(t))^2}}$$

■ the general forces Q_i:

```
(* structural dumping contribution *)
```

$$\text{Cmat} = \begin{pmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & c_\theta \end{pmatrix};$$

```
(*(Qc=(-(Cmat.D[Xi,{t,1}]+Cmat.ωi)/.i→p ))/.dispSimp//MatrixForm//
 TraditionalForm*)
```

$$\text{(*TODO (to consider) : Cmat}_i = \begin{pmatrix} c_x & 0 & 0 \\ 0 & c_y & 0 \\ 0 & 0 & c_\theta \end{pmatrix}; \text{ i=1,2*)}$$

```
(Qc =
    -((Cmat.ωi /. i → p) + (Cmat.((D[Xi, {t, 1}] /. i → p) - (D[Xi, {t, 1}] /. i → 1))) +
        (Cmat.((D[Xi, {t, 1}] /. i → p) - (D[Xi, {t, 1}] /. i → 2)))) //
     Flatten) /. dispSimp // MatrixForm // TraditionalForm
```

$$\begin{pmatrix} c_x\,(-(\dot{x_p}-\dot{x_1})) - c_x\,(\dot{x_p}-\dot{x_2}) \\ c_y\,(-(\dot{y_p}-\dot{y_1})) - c_y\,(\dot{y_p}-\dot{y_2}) \\ c_\theta\left(-\dot{\theta}_p\right) \end{pmatrix}$$

```
(* u,v are the air global velocity in x,y directions *)
```

$$F_x\,\left(*=\tfrac{1}{2}\ \rho\ \theta_p\ C_{F_x\alpha}\ \text{D}[x_p[t],\{t,1\}]^2\,(2\ h_p)*\right) = \frac{1}{2}\rho\, C_D\, \left(\text{D}[x_p[t], \{t, 1\}] - u\right)^2 \left(2\, h_p\right)$$

$$F_y = \frac{1}{2}\rho\, C_D\, \left(\text{D}[y_p[t], \{t, 1\}] - v\right)^2 \left(2\, w_p\right)$$

$$Q_{Aero} = \begin{pmatrix} -F_x \\ -F_y \\ 0 \end{pmatrix} \text{ // Flatten}$$

$\rho\, C_D\, h_p\ (-u + x_p'[t])^2$

$\rho\, C_D\, w_p\ (-v + y_p'[t])^2$

$\{-\rho\, C_D\, h_p\ (-u + x_p'[t])^2,\ -\rho\, C_D\, w_p\ (-v + y_p'[t])^2,\ 0\}$

display manipulations

```
bigTermsToShort = {(*(Sin[θp[t]] hp+Cos[θp[t]] wp+x1[t]-xp[t])^2+
     (Cos[θp[t]] hp-Sin[θp[t]] wp-y1[t]+yp[t])^2→l1,
   (Sin[θp[t]] hp-Cos[θp[t]] wp+x2[t]-xp[t])^2+
     (Cos[θp[t]] hp+Sin[θp[t]] wp-y2[t]+yp[t])^2→l2,*)
   Sin[θp[t]] hp + Cos[θp[t]] wp + x1[t] - xp[t] → dx1,
   (Cos[θp[t]] hp - Sin[θp[t]] wp - y1[t] + yp[t]) → dy1,
   Sin[θp[t]] hp - Cos[θp[t]] wp + x2[t] - xp[t] → dx2,
   Cos[θp[t]] hp + Sin[θp[t]] wp - y2[t] + yp[t] → dy2,
   wp (Sin[θp[t]] x1[t] - Sin[θp[t]] xp[t] + Cos[θp[t]] (-y1[t] + yp[t])) +
     hp (-Cos[θp[t]] x1[t] + Cos[θp[t]] xp[t] + Sin[θp[t]] (-y1[t] + yp[t])) → term1,
   hp (Cos[θp[t]] x2[t] - Cos[θp[t]] xp[t] + Sin[θp[t]] (y2[t] - yp[t])) +
     wp (Sin[θp[t]] x2[t] - Sin[θp[t]] xp[t] + Cos[θp[t]] (-y2[t] + yp[t])) → term2,
   - (Cos[θp[t]] hp - Sin[θp[t]] wp - y1[t] + yp[t]) → -dy1
  }
```

```
{Sin[θp[t]] hp + Cos[θp[t]] wp + x1[t] - xp[t] → dx1,
 Cos[θp[t]] hp - Sin[θp[t]] wp - y1[t] + yp[t] → dy1,
 Sin[θp[t]] hp - Cos[θp[t]] wp + x2[t] - xp[t] → dx2,
 Cos[θp[t]] hp + Sin[θp[t]] wp - y2[t] + yp[t] → dy2,
 wp (Sin[θp[t]] x1[t] - Sin[θp[t]] xp[t] + Cos[θp[t]] (-y1[t] + yp[t])) +
   hp (-Cos[θp[t]] x1[t] + Cos[θp[t]] xp[t] + Sin[θp[t]] (-y1[t] + yp[t])) → term1,
 hp (Cos[θp[t]] x2[t] - Cos[θp[t]] xp[t] + Sin[θp[t]] (y2[t] - yp[t])) +
   wp (Sin[θp[t]] x2[t] - Sin[θp[t]] xp[t] + Cos[θp[t]] (-y2[t] + yp[t])) → term2,
 -Cos[θp[t]] hp + Sin[θp[t]] wp + y1[t] - yp[t] → -dy1}
```

```
(bigTermsToShort[[All, 1]]) //. dispSimp // MatrixForm // TraditionalForm
(bigTermsToShort[[All, 2]]) //. dispSimp // MatrixForm // TraditionalForm
```

$$\begin{pmatrix} w_p\,c(\theta_p) + h_p\,s(\theta_p) - x_p + x_1 \\ h_p\,c(\theta_p) - w_p\,s(\theta_p) + y_p - y_1 \\ -w_p\,c(\theta_p) + h_p\,s(\theta_p) - x_p + x_2 \\ h_p\,c(\theta_p) + w_p\,s(\theta_p) + y_p - y_2 \\ h_p\,(x_1\,(-c(\theta_p)) + x_p\,c(\theta_p) + (y_p - y_1)\,s(\theta_p)) + w_p\,((y_p - y_1)\,c(\theta_p) + x_1\,s(\theta_p) - x_p\,s(\theta_p)) \\ h_p\,(x_2\,c(\theta_p) - x_p\,c(\theta_p) + (y_2 - y_p)\,s(\theta_p)) + w_p\,((y_p - y_2)\,c(\theta_p) + x_2\,s(\theta_p) - x_p\,s(\theta_p)) \\ h_p\,(-c(\theta_p)) + w_p\,s(\theta_p) - y_p + y_1 \end{pmatrix}$$

$$\begin{pmatrix} \mathrm{dx}_1 \\ \mathrm{dy}_1 \\ \mathrm{dx}_2 \\ \mathrm{dy}_2 \\ \mathrm{term1} \\ \mathrm{term2} \\ -\mathrm{dy}_1 \end{pmatrix}$$

9DOF case:

```
quad9EOM /. bigTermsToShort
```

```
quad9EOM /. bigTermsToShort /. dispSimp // MatrixForm // TraditionalForm
```

3DOF case:

```
quadEqNominal
```

```
quadEqNominal /. bigTermsToShort /. dispSimp // MatrixForm // TraditionalForm
```

$$\begin{pmatrix} \dfrac{\mathrm{dx}_1\, k_1 \left(\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2} - \mathrm{L0}_1\right)}{\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2}} + \dfrac{\mathrm{dx}_2\, k_2 \left(\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2} - \mathrm{L0}_2\right)}{\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2}} = m_p\, \ddot{x}_p \\ -\dfrac{\mathrm{dy}_1\, k_1 \left(\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2} - \mathrm{L0}_1\right)}{\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2}} = \dfrac{\mathrm{dy}_2\, k_2 \left(\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2} - \mathrm{L0}_2\right)}{\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2}} + g\, m_p + m_p\, \ddot{y}_p \\ \dfrac{k_1\, \mathrm{term1} \left(\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2} - \mathrm{L0}_1\right)}{\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2}} = \dfrac{k_2\, \mathrm{term2} \left(\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2} - \mathrm{L0}_2\right)}{\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2}} + i_p\, \ddot{\theta}_p \end{pmatrix}$$

continue with 3DOF EOM

```
bigTermsToShort /. dispSimp // MatrixForm // TraditionalForm
quadEqNominal /. bigTermsToShort /. dispSimp // MatrixForm // TraditionalForm
QAero /. bigTermsToShort /. dispSimp // MatrixForm // TraditionalForm
Qc /. bigTermsToShort /. dispSimp // MatrixForm // TraditionalForm
```

$$\begin{pmatrix} w_p\, c(\theta_p(t)) + h_p\, s(\theta_p(t)) - x_p + x_1 \to \mathrm{dx}_1 \\ h_p\, c(\theta_p(t)) - w_p\, s(\theta_p(t)) + y_p - y_1 \to \mathrm{dy}_1 \\ -w_p\, c(\theta_p(t)) + h_p\, s(\theta_p(t)) - x_p + x_2 \to \mathrm{dx}_2 \\ h_p\, c(\theta_p(t)) + w_p\, s(\theta_p(t)) + y_p - y_2 \to \mathrm{dy}_2 \\ h_p\,(x_1\,(-c(\theta_p(t))) + x_p\, c(\theta_p(t)) + (y_p - y_1)\, s(\theta_p(t))) + w_p\,((y_p - y_1)\, c(\theta_p(t)) + x_1\, s(\theta_p(t)) - x_p\, s(\theta_p(t))) \to \mathrm{term1} \\ h_p\,(x_2\, c(\theta_p(t)) - x_p\, c(\theta_p(t)) + (y_2 - y_p)\, s(\theta_p(t))) + w_p\,((y_p - y_2)\, c(\theta_p(t)) + x_2\, s(\theta_p(t)) - x_p\, s(\theta_p(t))) \to \mathrm{term2} \\ h_p\,(-c(\theta_p(t))) + w_p\, s(\theta_p(t)) - y_p + y_1 \to -\mathrm{dy}_1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{\mathrm{dx}_1\, k_1 \left(\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2} - \mathrm{L0}_1\right)}{\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2}} + \frac{\mathrm{dx}_2\, k_2 \left(\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2} - \mathrm{L0}_2\right)}{\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2}} = m_p\, \ddot{x}_p \\ -\frac{\mathrm{dy}_1\, k_1 \left(\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2} - \mathrm{L0}_1\right)}{\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2}} = \frac{\mathrm{dy}_2\, k_2 \left(\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2} - \mathrm{L0}_2\right)}{\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2}} + g\, m_p + m_p\, \ddot{y}_p \\ \frac{k_1\, \mathrm{term1} \left(\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2} - \mathrm{L0}_1\right)}{\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2}} = \frac{k_2\, \mathrm{term2} \left(\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2} - \mathrm{L0}_2\right)}{\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2}} + i_p\, \ddot{\theta}_p \end{pmatrix}$$

$$\begin{pmatrix} -\rho\, C_D\, h_p\, (\dot{x}_p - u)^2 \\ -\rho\, C_D\, w_p\, (\dot{y}_p - v)^2 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} c_x\,(-(\dot{x}_p - \dot{x}_1)) - c_x\,(\dot{x}_p - \dot{x}_2) \\ c_y\,(-(\dot{y}_p - \dot{y}_1)) - c_y\,(\dot{y}_p - \dot{y}_2) \\ c_\theta \left(-\dot{\theta}_p\right) \end{pmatrix}$$

■ 3 EOM including general forces :

```
(EOM3D = MapThread[Equal,
      {(quadEqNominal[[All, 1]] - quadEqNominal[[All, 2]]), {0, 0, 0} - QAero - Qc}]) /.
   bigTermsToShort /. dispSimp // MatrixForm // TraditionalForm
```

$$\begin{pmatrix} \frac{\mathrm{dx}_1\, k_1 \left(\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2} - \mathrm{L0}_1\right)}{\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2}} + \frac{\mathrm{dx}_2\, k_2 \left(\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2} - \mathrm{L0}_2\right)}{\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2}} - m_p\, \ddot{x}_p = c_x\,(\dot{x}_p - \dot{x}_1) + c_x\,(\dot{x}_p - \dot{x}_2) + \rho\, C_D\, h_p\,(\dot{x}_p - u)^2 \\ -\frac{\mathrm{dy}_1\, k_1 \left(\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2} - \mathrm{L0}_1\right)}{\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2}} - \frac{\mathrm{dy}_2\, k_2 \left(\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2} - \mathrm{L0}_2\right)}{\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2}} - g\, m_p - m_p\, \ddot{y}_p = c_y\,(\dot{y}_p - \dot{y}_1) + c_y\,(\dot{y}_p - \dot{y}_2) + \rho\, C_D\, w_p\,(\dot{y}_p - v)^2 \\ \frac{k_1\, \mathrm{term1} \left(\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2} - \mathrm{L0}_1\right)}{\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2}} - \frac{k_2\, \mathrm{term2} \left(\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2} - \mathrm{L0}_2\right)}{\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2}} + i_p \left(-\ddot{\theta}_p\right) = c_\theta\, \dot{\theta}_p \end{pmatrix}$$

```
(*Collect[Expand[EOM3D],{D[xp[t],{t,2}],D[yp[t],{t,2}],D[θp[t],{t,2}],k1}]/.
   bigTermsToShort/.dispSimp//MatrixForm//TraditionalForm*)
```

scaling the dimensional variables :

scaling varialbes (length and time related)

```
{x̃p[t] == xp[t] / LO1  /. dispSimp,
 ỹp[t] == yp[t] / LO1  /. dispSimp,
 τ == t ωs /. dispSimp,
 ωs^2 == k1/mp (*[g/1 = 1/s^2]*) /. dispSimp,
 h̃p[t] == hp[t] / LO1  /. dispSimp,
 w̃p[t] == wp[t] / LO1  /. dispSimp}
"as result also:"
{dx̃i[t] == dxi[t] / LO1  /. dispSimp,
 dỹi[t] == dyi[t] / LO1  /. dispSimp,
 term̃i[t] == termi[t] / LO1 / LO1  /. dispSimp,
 F̃i[t] == Fi[t] / LO1 / (LO1^2 * ωs^2)  /. dispSimp,
 ci x̃i'[t] == ci xi'[t] / (LO1 * ωs)  /. dispSimp
}
```

$$\left\{\tilde{x}_p = \frac{x_p}{L0_1},\ \tilde{y}_p = \frac{y_p}{L0_1},\ \tau = t\,\omega_s,\ \omega_s^2 = \frac{k_1}{m_p},\ \tilde{h}_p = \frac{h_p}{L0_1},\ \tilde{w}_p = \frac{w_p}{L0_1}\right\}$$

as result also:

$$\left\{d\tilde{x}_i = \frac{dx_i}{L0_1},\ d\tilde{y}_i = \frac{dy_i}{L0_1},\ \tilde{term}_i = \frac{term_i}{L0_1^2},\ \tilde{F}_i = \frac{F_i}{L0_1^3\,\omega_s^2},\ \dot{x}_i\tilde{c}_i = \frac{\dot{x}_i\,c_i}{L0_1\,\omega_s}\right\}$$

non-dim equations

```
EOM3DshortTerms = EOM3D /. bigTermsToShort
```

$$\Bigg\{\frac{dx_1\,k_1\left(\sqrt{dx_1^2+dy_1^2}-L0_1\right)}{\sqrt{dx_1^2+dy_1^2}}+\frac{dx_2\,k_2\left(\sqrt{dx_2^2+dy_2^2}-L0_2\right)}{\sqrt{dx_2^2+dy_2^2}}-m_p\,x_p''[t]==$$
$$\rho\,C_D\,h_p\,(-u+x_p'[t])^2+c_x\,(-x_1'[t]+x_p'[t])+c_x\,(-x_2'[t]+x_p'[t]),$$
$$-\frac{dy_1\,k_1\left(\sqrt{dx_1^2+dy_1^2}-L0_1\right)}{\sqrt{dx_1^2+dy_1^2}}-\frac{dy_2\,k_2\left(\sqrt{dx_2^2+dy_2^2}-L0_2\right)}{\sqrt{dx_2^2+dy_2^2}}-g\,m_p-m_p\,y_p''[t]==$$
$$\rho\,C_D\,w_p\,(-v+y_p'[t])^2+c_y\,(-y_1'[t]+y_p'[t])+c_y\,(-y_2'[t]+y_p'[t]),$$
$$\frac{term1\,k_1\left(\sqrt{dx_1^2+dy_1^2}-L0_1\right)}{\sqrt{dx_1^2+dy_1^2}}-\frac{term2\,k_2\left(\sqrt{dx_2^2+dy_2^2}-L0_2\right)}{\sqrt{dx_2^2+dy_2^2}}-i_{p,zz}\,\theta_p''[t]==c_\theta\,\theta_p'[t]\Bigg\}$$

TYOTOT

from ' EOM3DshortTerms' copy, and manually edit :

$$\Big\{ k_1 \left(1 - \frac{L0_1}{\sqrt{dx_1^2 + dy_1^2}}\right) dx_1 + k_2 \left(1 - \frac{L0_2}{\sqrt{dx_2^2 + dy_2^2}}\right) dx_2 - \rho\, C_D\, h_p\, (-u + x_p'[t])^2 - c_x\, (-x_1'[t] + x_p'[t]) - c_x\, (-x_2'[t] + x_p'[t]) = m_p\, x_p''[t],$$

$$- k_1 \left(1 - \frac{L0_1}{\sqrt{dx_1^2 + dy_1^2}}\right) dy_1 - k_2 \left(1 - \frac{L0_2}{\sqrt{dx_2^2 + dy_2^2}}\right) dy_2 - g\, m_p - m_p\, y_p''[t] == \rho\, C_D\, w_p\, (-v + y_p'[t])^2 + c_y\, (-y_1'[t] + y_p'[t]) + c_y\, (-y_2'[t] + y_p'[t]),$$

$$k_1 \frac{\left(\sqrt{dx_1^2 + dy_1^2} - L0_1\right)}{\sqrt{dx_1^2 + dy_1^2}}\, term1 - \frac{k_2 \left(\sqrt{dx_2^2 + dy_2^2} - L0_2\right)}{\sqrt{dx_2^2 + dy_2^2}}\, term2 - I_{p,zz}\, \theta_p''[t] == c_\theta\, \theta_p'[t] \Big\}$$

non dim

$$\Big\{ \frac{k_1}{m_p} \left(1 - \frac{1}{\sqrt{dx_1^2 + dy_1^2}}\right) dx_1\, L0_1 + \frac{k_1}{m_p}\, \frac{k_2}{k_1} \left(1 - \frac{1}{\sqrt{dx_2^2 + dy_2^2}}\, \frac{L0_2}{L0_1}\right) dx_2\, L0_1 - \rho\, C_D\, h_p\, L0_1\, (-u + x_p'[t])^2 \left(L0_1\, \omega_s\right)^2 \frac{1}{m_p} - c_x\, (-x_1'[t] + x_p'[t])\, L0_1\, \omega_s\, \frac{1}{m_p} - c_x\, (-x_2'[t] + x_p'[t])\, L0_1\, \omega_s\, \frac{1}{m_p} = x_p''[t]\, L0_1\, {\omega_s}^2,$$

$$- k_1 \frac{1}{m_p} \left(1 - \frac{1}{\sqrt{dx_1^2 + dy_1^2}}\right) dy_1\, L0_1 - \frac{k_1}{m_p}\, \frac{k_2}{k_1} \left(1 - \frac{1}{\sqrt{dx_2^2 + dy_2^2}}\, \frac{L0_2}{L0_1}\right) dy_2\, L0_1 - g - y_p''[t]\, L0_1\, {\omega_s}^2 == \rho\, C_D\, w_p\, (-v + y_p'[t])^2 \left(L0_1\, \omega_s\right)^2 \frac{1}{m_p} + c_y\, (-y_1'[t] + y_p'[t]) \left(L0_1\, \omega_s\right) \frac{1}{m_p} + c_y\, (-y_2'[t] + y_p'[t]) \left(L0_1\, \omega_s\right) \frac{1}{m_p},$$

$$k_1 \frac{1}{I_{p,zz}} \left(1 - \frac{1}{\sqrt{dx_1^2 + dy_1^2}}\right) term1\, {L0_1}^2 - \frac{k_1}{1}\, \frac{k_2}{k_1}\, \frac{1}{I_{p,zz}} \left(1 - \frac{1}{\sqrt{dx_2^2 + dy_2^2}}\, \frac{L0_2}{L0_1}\right) term2\, {L0_1}^2 - \theta_p''[t]\, {\omega_s}^2 == \frac{1}{I_{p,zz}}\, c_\theta\, \theta_p'[t]\, (\omega_s) \Big\}$$

$$\frac{1}{\omega_s}\frac{1}{m_p} = \frac{1}{k_1}$$

$$\left\{\left(1-\frac{1}{\sqrt{dx_1^2+dy_1^2}}\right)dx_1+\frac{k_2}{k_1}\left(1-\frac{1}{\sqrt{dx_2^2+dy_2^2}}\frac{L0_2}{L0_1}\right)dx_2-\rho\,C_D\,h_p\,(-u+x_p'[t])^2\,(L0_1)^2\frac{1}{m_p}-\right.$$
$$c_x\,(-x_1'[t]+x_p'[t])\frac{1}{k_1}-c_x\,(-x_2'[t]+x_p'[t])\frac{1}{k_1} = x_p''[t],$$
$$-\left(1-\frac{1}{\sqrt{dx_1^2+dy_1^2}}\right)dy_1-\frac{k_2}{k_1}\left(1-\frac{1}{\sqrt{dx_2^2+dy_2^2}}\frac{L0_2}{L0_1}\right)dy_2-\frac{g}{L0_1\,\omega_s{}^2}-y_p''[t]==$$
$$\rho\,C_D\,w_p\,(-v+y_p'[t])^2\,(L0_1)^2\frac{1}{m_p}+c_y\,(-y_1'[t]+y_p'[t])\frac{1}{k_1}+c_y\,(-y_2'[t]+y_p'[t])\frac{1}{k_1},$$
$$\frac{k_1}{\omega_s{}^2}\frac{L0_1{}^2}{I_{p,zz}}\left(1-\frac{1}{\sqrt{dx_1^2+dy_1^2}}\right)\mathrm{term1}-\frac{k_1}{\omega_s{}^2}\frac{k_2}{k_1}\frac{L0_1{}^2}{I_{p,zz}}\left(1-\frac{1}{\sqrt{dx_2^2+dy_2^2}}\frac{L0_2}{L0_1}\right)\mathrm{term2}-\theta_p''[t]==$$
$$\left.\frac{1}{I_{p,zz}}\frac{1}{\omega_s}c_\theta\,\theta_p'[t]\right\}$$

$$\mathcal{X} = \begin{pmatrix} x_p[t] \\ y_p[t] \\ \theta_p[t] \end{pmatrix};$$

```
(*Clear[κ];Clear[ℒ];Clear[u];Clear[v]
  DX1=.;DX2=.;
𝒱1=.;𝒱2=.;*)
```

$$\left(\mathrm{NonDimEOMmatrixForm} = \mathrm{D}[\mathcal{X},\{t,2\}] == \left(\left(\begin{pmatrix}1&0&0\\0&-1&0\\0&0&\alpha\end{pmatrix}\mathrm{DX}_1\right).\mathcal{V}_1+\left(\kappa\begin{pmatrix}1&0&0\\0&-1&0\\0&0&-\alpha\end{pmatrix}\mathrm{DX}_2\right).\mathcal{V}_2-\begin{pmatrix}0\\\gamma\\0\end{pmatrix}-\mathcal{A}-\mathcal{D}\ //\ \mathrm{Flatten}\right)\ /.\ \mathrm{dispSimp}\ //\ \mathrm{MatrixForm}\ //\ \mathrm{TraditionalForm}$$

$$\mathrm{DX}_1 = 1-\frac{1}{\sqrt{dx_1^2+dy_1^2}};$$

$$\mathrm{DX}_2 = \left(*1-\frac{L0_2}{\sqrt{dx_2^2+dy_2^2}}==1-\frac{L0_1}{\sqrt{dx_2^2+dy_2^2}}\frac{L0_2}{L0_1}==*\right)1-\frac{1}{\sqrt{dx_2^2+dy_2^2}}\mathcal{L};$$

$$\left(*\alpha==\frac{L0_1{}^2k_1}{I_p\omega_s{}^2}==\frac{m_pL0_1{}^2}{I_p}\right.$$
$$I_p==\frac{1}{12}m_p\left(\left(2h_p\right)^2+\left(2w_p\right)^2\right)==\frac{1}{3}m_p\left(h_p{}^2+w_p{}^2\right)$$
$$\left.\alpha==\frac{m_pL0_1{}^2}{I_p}==\frac{m_pL0_1{}^2}{\frac{1}{3}m_p\left(h_p{}^2+w_p{}^2\right)}=\frac{3L0_1{}^2}{\left(h_p{}^2+w_p{}^2\right)}=\frac{3}{\left(\left(\frac{h_p}{w_p}\right)^2+1\right)}\left(\frac{L0_1}{w_p}\right)^2*\right)$$

$$\alpha = \frac{3}{\left(\left(\frac{h_p}{w_p}\right)^2 + 1\right)} \left(\frac{1}{w_p}\right)^2$$

(*this is the non-dim version of α. using h_p, w_p which is already normalized *);

$$\mathcal{L} == \frac{L0_2}{L0_1};$$

$$\mathcal{V}_1 = \begin{pmatrix} dx_1 \\ dy_1 \\ term1 \end{pmatrix};$$

$$\mathcal{V}_2 = \begin{pmatrix} dx_2 \\ dy_2 \\ term2 \end{pmatrix};$$

$$\kappa == \frac{k_2}{k_1};$$

$$\gamma == \frac{g\, m_p}{L0_1\, k_1} == \frac{\Omega_1{}^2}{\omega_s{}^2};$$ (*γ>0 because all positivies inside *)

$$\mathcal{A}\,(*= Q_{Aero} * \left(L0_1^3\ \omega_s^2\right) *) = \begin{pmatrix} \rho\, C_D\, h_p\, (-u + x_p'[t])^2 \left(L0_1\right)^2 \frac{1}{m_p} \\ \rho\, C_D\, w_p\, (-v + y_p'[t])^2 \left(L0_1\right)^2 \frac{1}{m_p} \\ 0 \end{pmatrix} \text{ // Flatten}$$

$$\mathcal{D}\,(*= \begin{pmatrix} \left(L0_1\ \omega_s\right) & 0 & 0 \\ 0 & \left(L0_1\ \omega_s\right) & 0 \\ 0 & 0 & (\omega_s) \end{pmatrix} .Q_c *) =$$

$$\begin{pmatrix} c_x\, (-x_1'[t] + x_p'[t])\, \frac{1}{k_1} + c_x\, (-x_2'[t] + x_p'[t])\, \frac{1}{k_1} \\ c_y\, (-y_1'[t] + y_p'[t])\, \frac{1}{k_1} + c_y\, (-y_2'[t] + y_p'[t])\, \frac{1}{k_1} \\ \frac{1}{I_{p,zz}}\, \frac{1}{\omega_s}\, c_\theta\, \theta_p'[t] \end{pmatrix} \text{ // Flatten}$$

NonDimEOMmatrixForm // TraditionalForm

$$\begin{pmatrix} \ddot{x}_p \\ \ddot{y}_p \\ \ddot{\theta}_p \end{pmatrix} = \begin{pmatrix} -\frac{\rho\, \dot{x}_p{}^2\, C_D\, h_p\, L0_1^2}{m_p} + dx_1\left(1 - \frac{1}{\sqrt{dx_1^2 + dy_1^2}}\right) + dx_2\left(1 - \frac{1}{\sqrt{dx_2^2 + dy_2^2}}\right) - \frac{(\dot{x}_p - \dot{x}_1)\, c_x}{k_1} - \frac{(\dot{x}_p - \dot{x}_2)\, c_x}{k_1} \\ -\frac{\rho\, \dot{y}_p{}^2\, C_D\, w_p\, L0_1^2}{m_p} - \gamma + dy_1\left(\frac{1}{\sqrt{dx_1^2 + dy_1^2}} - 1\right) + dy_2\left(\frac{1}{\sqrt{dx_2^2 + dy_2^2}} - 1\right) - \frac{(\dot{y}_p - \dot{y}_1)\, c_y}{k_1} - \frac{(\dot{y}_p - \dot{y}_2)\, c_y}{k_1} \\ -\frac{\dot{\theta}_p\, c_\theta}{i_p\, \omega_s} + \frac{3\, \text{term1}\left(1 - \frac{1}{\sqrt{dx_1^2 + dy_1^2}}\right)}{\left(\frac{h_p^2}{w_p^2} + 1\right) w_p^2} - \frac{3\, \text{term2}\left(1 - \frac{1}{\sqrt{dx_2^2 + dy_2^2}}\right)}{\left(\frac{h_p^2}{w_p^2} + 1\right) w_p^2} \end{pmatrix}$$

$$\left\{\frac{\rho\, C_D\, h_p\, L0_1^2\, x_p'[t]^2}{m_p},\ \frac{\rho\, C_D\, L0_1^2\, w_p\, y_p'[t]^2}{m_p},\ 0\right\}$$

$$\left\{\frac{c_x\, (-x_1'[t] + x_p'[t])}{k_1} + \frac{c_x\, (-x_2'[t] + x_p'[t])}{k_1},\ \frac{c_y\, (-y_1'[t] + y_p'[t])}{k_1} + \frac{c_y\, (-y_2'[t] + y_p'[t])}{k_1},\ \frac{c_\theta\, \theta_p'[t]}{\omega_s\, \mathbb{i}_{p,zz}}\right\}$$

$$\begin{pmatrix} x_p''(t) \\ y_p''(t) \\ \theta_p''(t) \end{pmatrix} = \begin{pmatrix} -\frac{\rho\, C_D\, h_p\, \mathrm{L0}_1^2\, x_p'(t)^2}{m_p} + \mathrm{dx}_1\left(1 - \frac{1}{\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2}}\right) + \mathrm{dx}_2\left(1 - \frac{1}{\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2}}\right) - \frac{c_x\,(x_p'(t)-x_1'(t))}{k_1} - \frac{c_x\,(x_p'(t)-x_2'(t))}{k_1} \\ -\frac{\rho\, C_D\, \mathrm{L0}_1^2\, w_p\, y_p'(t)^2}{m_p} - \gamma + \mathrm{dy}_1\left(\frac{1}{\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2}} - 1\right) + \mathrm{dy}_2\left(\frac{1}{\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2}} - 1\right) - \frac{c_y\,(y_p'(t)-y_1'(t))}{k_1} - \frac{c_y\,(y_p'(t)-y_2'(t))}{k_1} \\ \frac{3\,\mathrm{term1}\left(1-\frac{1}{\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2}}\right)}{\left(\frac{h_p^2}{w_p^2}+1\right)w_p^2} - \frac{c_\theta\,\theta_p'(t)}{\omega_s\, i_{p,zz}} - \frac{3\,\mathrm{term2}\left(1-\frac{1}{\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2}}\right)}{\left(\frac{h_p^2}{w_p^2}+1\right)w_p^2} \end{pmatrix}$$

setting conditions for the rest of the work

```
"symmetric case ."
κ = 1; ℒ = 1;
"no wind. air is static ."
u = 0; v = 0 ;
"quadrotors locations are fixed ."
QuadsBaseLocations = {x1[t] → 0, y1[t] → 0, x2[t] → x1[t] + 2 wp, y2[t] → y1[t]}
```

```
symmetric case .

no wind. air is static .

quadrotors locations are fixed .

{x1[t] → 0, y1[t] → 0, x2[t] → 2 wp + x1[t], y2[t] → y1[t]}
```

equilibrium points

finding the equilibrium points by setting all derivatives to zero

```
(equibZeroDerivatives = {
    Map[Rule[#, 0 ] &, D[q // Flatten, {t, 1}]],
    Map[Rule[#, 0 ] &, D[q // Flatten, {t, 2}]]
   }) (*/.dispSimp*) // MatrixForm // TraditionalForm
equibZeroDerivatives = equibZeroDerivatives // Flatten;
```

$$\begin{pmatrix} x_1'(t)\to 0 & y_1'(t)\to 0 & \theta_1'(t)\to 0 & x_2'(t)\to 0 & y_2'(t)\to 0 & \theta_2'(t)\to 0 & x_p'(t)\to 0 & y_p'(t)\to 0 & \theta_p'(t)\to 0 \\ x_1''(t)\to 0 & y_1''(t)\to 0 & \theta_1''(t)\to 0 & x_2''(t)\to 0 & y_2''(t)\to 0 & \theta_2''(t)\to 0 & x_p''(t)\to 0 & y_p''(t)\to 0 & \theta_p''(t)\to 0 \end{pmatrix}$$

must also assume the inputs of the quadrotors locations:
setting static base locations and with the width of the payload apart (2 w_p)

```
EquibθzeroCondition = {θp[t] → 0}
```

```
{x1[t] → 0, y1[t] → 0, x2[t] → 2 wp + x1[t], y2[t] → y1[t]}

{θp[t] → 0}
```

```
(equibEquations = NonDimEOMmatrixForm /. QuadsBaseLocations /.
      equibZeroDerivatives) /. dispSimp // MatrixForm // TraditionalForm
```

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \mathrm{dx}_1\left(1-\frac{1}{\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2}}\right)+\mathrm{dx}_2\left(1-\frac{1}{\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2}}\right) \\ -\gamma+\mathrm{dy}_1\left(\frac{1}{\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2}}-1\right)-\mathrm{dy}_2\left(1-\frac{1}{\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2}}\right) \\ \frac{3\,\mathrm{term1}\left(1-\frac{1}{\sqrt{\mathrm{dx}_1^2+\mathrm{dy}_1^2}}\right)}{\left(\frac{h_p^2}{w_p^2}+1\right)w_p^2}-\frac{3\,\mathrm{term2}\left(1-\frac{1}{\sqrt{\mathrm{dx}_2^2+\mathrm{dy}_2^2}}\right)}{\left(\frac{h_p^2}{w_p^2}+1\right)w_p^2} \end{pmatrix}$$

```
(dxdytermDetails = Table[Rule[bigTermsToShort[[i, 2]] , bigTermsToShort[[i, 1]]],
      {i, 1, Length[bigTermsToShort]}]) //. dispSimp // MatrixForm // TraditionalForm
```

$$\begin{pmatrix} \mathrm{dx}_1 \to w_p\, c(\theta_p)+h_p\, s(\theta_p)-x_p+x_1 \\ \mathrm{dy}_1 \to h_p\, c(\theta_p)-w_p\, s(\theta_p)+y_p-y_1 \\ \mathrm{dx}_2 \to -w_p\, c(\theta_p)+h_p\, s(\theta_p)-x_p+x_2 \\ \mathrm{dy}_2 \to h_p\, c(\theta_p)+w_p\, s(\theta_p)+y_p-y_2 \\ \mathrm{term1} \to h_p\,(x_1\,(-c(\theta_p))+x_p\, c(\theta_p)+(y_p-y_1)\, s(\theta_p))+w_p\,((y_p-y_1)\, c(\theta_p)+x_1\, s(\theta_p)-x_p\, s(\theta_p)) \\ \mathrm{term2} \to h_p\,(x_2\, c(\theta_p)-x_p\, c(\theta_p)+(y_2-y_p)\, s(\theta_p))+w_p\,((y_p-y_2)\, c(\theta_p)+x_2\, s(\theta_p)-x_p\, s(\theta_p)) \\ -\mathrm{dy}_1 \to h_p\,(-c(\theta_p))+w_p\, s(\theta_p)-y_p+y_1 \end{pmatrix}$$

```
(*equibEquations/.dxdytermDetails//.dispSimp*)
(SymetricEquibWithAssumption =
    (equibEquations /. dxdytermDetails) //. QuadsBaseLocations //.
     EquibθzeroCondition) //. dispSimp
```

$$\{\{0\},\{0\},\{0\}\} == \Big\{\Big\{2\,(w_p-x_p)\left(1-\frac{1}{\sqrt{(w_p-x_p)^2+(h_p+y_p)^2}}\right)\Big\},$$
$$\Big\{-\gamma-(h_p+y_p)\left(1-\frac{1}{\sqrt{(w_p-x_p)^2+(h_p+y_p)^2}}\right)+(h_p+y_p)\left(-1+\frac{1}{\sqrt{(w_p-x_p)^2+(h_p+y_p)^2}}\right)\Big\},$$
$$\Big\{-\frac{3\,(h_p\,(2\,w_p-x_p)+w_p\,y_p)\left(1-\frac{1}{\sqrt{(w_p-x_p)^2+(h_p+y_p)^2}}\right)}{\left(1+\frac{h_p^2}{w_p^2}\right)w_p^2}+\frac{3\,(h_p\,x_p+w_p\,y_p)\left(1-\frac{1}{\sqrt{(w_p-x_p)^2+(h_p+y_p)^2}}\right)}{\left(1+\frac{h_p^2}{w_p^2}\right)w_p^2}\Big\}\Big\}$$

```
(simpleEquibXYSolution = Solve[SymetricEquibWithAssumption, {xp[t], yp[t]}]) //
  MatrixForm // TraditionalForm
```

$$\begin{pmatrix} x_p(t)\to w_p & y_p(t)\to \frac{1}{2}(-\gamma-2\,h_p-2) \\ x_p(t)\to w_p & y_p(t)\to \frac{1}{2}(-\gamma-2\,h_p+2) \end{pmatrix}$$

linearization near equilibrium points

```
simpleEquibXYSolution[[1]]
```

$\left\{x_p[t] \to w_p,\ y_p[t] \to \frac{1}{2}\left(-2-\gamma-2\,h_p\right)\right\}$

```
(*Table[Rule[vec0[[i]] ,simpleEquibXYSolution[[i,2]]],
  {i,1,Length[simpleEquibXYSolution]}]//MatrixForm*)
EquilibiumPointRule = {xp0 -> simpleEquibXYSolution[[1, 1, 2]],
  yp0 -> simpleEquibXYSolution[[1, 2, 2]], θp0 → 0}
```

$\left\{x_{p0} \to w_p,\ y_{p0} \to \frac{1}{2}\left(-2-\gamma-2\,h_p\right),\ \theta_{p0} \to 0\right\}$

```
(*EquilibiumPoinit={θp0→0, xp0→wp,yp0→-(1/2γ+hp+1)}*)
EquilibiumPointRule
(*GivenEquibPoints={x1[t]→0,y1[t]→0,y2[t]→0,x2[t]→2 wp}*)
QuadsBaseLocations
perturb = {
   θp[t] → θp0 + δθ[t],
   xp[t] → xp0 + δx[t],
   yp[t] → yp0 + δy[t]
  }
perturbD1 = {
   D[θp[t], {t, 1}] → D[δθ[t], {t, 1}],
   D[xp[t], {t, 1}] → D[δx[t], {t, 1}],
   D[yp[t], {t, 1}] → D[δy[t], {t, 1}]
  }
perturbD2 = {
   D[θp[t], {t, 2}] → D[δθ[t], {t, 2}],
   D[xp[t], {t, 2}] → D[δx[t], {t, 2}],
   D[yp[t], {t, 2}] → D[δy[t], {t, 2}]
  }
perturbationsRules = {perturb, perturbD1, perturbD2} // Flatten
smallδθAngleRule = {Cos[δθ[t]] → 1, Sin[δθ[t]] -> δθ[t]}
(* like Taylor for 1st order only . around 0 degrees *)
ruleForNeglectingCombinations =
  {(*δy[t] δx''[t]→0, δy[t] δy''[t]→0,*)a_[t]^2 → 0, a_[t]^3 → 0, a_[t] b_[t] → 0}
```

$\{x_{p0} \to w_p,\ y_{p0} \to \frac{1}{2}\,(-2-\gamma-2\,h_p),\ \theta_{p0} \to 0\}$

$\{x_1[t] \to 0,\ y_1[t] \to 0,\ x_2[t] \to 2\,w_p + x_1[t],\ y_2[t] \to y_1[t]\}$

$\{\theta_p[t] \to \theta_{p0} + \delta\theta[t],\ x_p[t] \to x_{p0} + \delta x[t],\ y_p[t] \to y_{p0} + \delta y[t]\}$

$\{\theta_p'[t] \to \delta\theta'[t],\ x_p'[t] \to \delta x'[t],\ y_p'[t] \to \delta y'[t]\}$

$\{\theta_p''[t] \to \delta\theta''[t],\ x_p''[t] \to \delta x''[t],\ y_p''[t] \to \delta y''[t]\}$

$\{\theta_p[t] \to \theta_{p0} + \delta\theta[t],\ x_p[t] \to x_{p0} + \delta x[t],\ y_p[t] \to y_{p0} + \delta y[t],\ \theta_p'[t] \to \delta\theta'[t],$
$x_p'[t] \to \delta x'[t],\ y_p'[t] \to \delta y'[t],\ \theta_p''[t] \to \delta\theta''[t],\ x_p''[t] \to \delta x''[t],\ y_p''[t] \to \delta y''[t]\}$

$\{\mathrm{Cos}[\delta\theta[t]] \to 1,\ \mathrm{Sin}[\delta\theta[t]] \to \delta\theta[t]\}$

$\{a\_[t]^2 \to 0,\ a\_[t]^3 \to 0,\ a\_[t]\,b\_[t] \to 0\}$

```
NonDimEOMmatrixForm // TraditionalForm
```

$$\begin{pmatrix} x_p''(t) \\ y_p''(t) \\ \theta_p''(t) \end{pmatrix} = \begin{pmatrix} -\frac{\rho\, C_D\, h_p\, \mathrm{L0}_1^2\, x_p'(t)^2}{m_p} + \mathrm{dx}_1 \left(1 - \frac{1}{\sqrt{\mathrm{dx}_1^2 + \mathrm{dy}_1^2}}\right) + \mathrm{dx}_2 \left(1 - \frac{1}{\sqrt{\mathrm{dx}_2^2 + \mathrm{dy}_2^2}}\right) - \frac{c_x\, (x_p'(t) - x_1'(t))}{k_1} - \frac{c_x\, (x_p'(t) - x_2'(t))}{k_1} \\ -\frac{\rho\, C_D\, \mathrm{L0}_1^2\, w_p\, y_p'(t)^2}{m_p} - \gamma + \mathrm{dy}_1 \left(\frac{1}{\sqrt{\mathrm{dx}_1^2 + \mathrm{dy}_1^2}} - 1\right) - \mathrm{dy}_2 \left(1 - \frac{1}{\sqrt{\mathrm{dx}_2^2 + \mathrm{dy}_2^2}}\right) - \frac{c_y\, (y_p'(t) - y_1'(t))}{k_1} - \frac{c_y\, (y_p'(t) - y_2'(t))}{k_1} \\ \frac{3\, \mathrm{term1} \left(1 - \frac{1}{\sqrt{\mathrm{dx}_1^2 + \mathrm{dy}_1^2}}\right)}{\left(\frac{h_p^2}{w_p^2} + 1\right) w_p^2} - \frac{c_\theta\, \theta_p'(t)}{\omega_s\, i_{p,zz}} - \frac{3\, \mathrm{term2} \left(1 - \frac{1}{\sqrt{\mathrm{dx}_2^2 + \mathrm{dy}_2^2}}\right)}{\left(\frac{h_p^2}{w_p^2} + 1\right) w_p^2} \end{pmatrix}$$

```
NonDimEOMmatrixForm /. dxdytermDetails // TraditionalForm
```

$$\begin{pmatrix} x_p''(t) \\ y_p''(t) \\ \theta_p''(t) \end{pmatrix} = \begin{pmatrix} -\frac{\rho\, C_D\, h_p\, \mathrm{L0}_1^2\, x_p'(t)^2}{m_p} + (\sin(\theta_p(t))\, h_p + \cos(\theta_p(t))\, w_p + x_1(t) - x_p(t)) \left(1 - \frac{1}{\sqrt{(\sin(\theta_p(t))\, h_p + \cos(\theta_p(t))\, w_p + x_1(t) - x_p(t))^2 + (\cos(\theta_p(t)) \cdots}} \cdots \right. \\ -\frac{\rho\, C_D\, \mathrm{L0}_1^2\, w_p\, y_p'(t)^2}{m_p} - \gamma + (\cos(\theta_p(t))\, h_p - \sin(\theta_p(t))\, w_p - y_1(t) + y_p(t)) \left(\frac{1}{\sqrt{(\sin(\theta_p(t))\, h_p + \cos(\theta_p(t))\, w_p + x_1(t) - x_p(t))^2 + (\cos(\theta_p(t))\, h_p \cdots}} \cdots \right. \\ \frac{3\, (w_p\, (\sin(\theta_p(t))\, x_1(t) - \sin(\theta_p(t))\, x_p(t) + \cos(\theta_p(t))\, (y_p(t) - y_1(t))) + h_p\, (-\cos(\theta_p(t))\, x_1(t) + \cos(\theta_p(t))\, x_p(t) + \sin(\theta_p(t))\, (y_p(t) - y_1(t)))) \left(1 - \frac{\cdots}{\sqrt{(\sin(\theta_p(t))\, h_p + \cos(\theta_p(t))\, w_p + x_1(t) - x_p(t \cdots}} \right. \cdots}{\left(\frac{h_p^2}{w_p^2} + 1\right) w_p^2} \end{pmatrix}$$

```
(*(𝒱1/.dxdytermDetails)//.dispSimp//TraditionalForm
      (𝒱2/.dxdytermDetails)//.dispSimp//TraditionalForm
     (DX1/.dxdytermDetails)//.dispSimp//TraditionalForm
    (DX2/.dxdytermDetails)//.dispSimp//TraditionalForm
*)"with setting of base locations :"
(v1 = 𝒱1 /. dxdytermDetails //. QuadsBaseLocations) //. dispSimp // TraditionalForm
(v2 = 𝒱2 /. dxdytermDetails //. QuadsBaseLocations) //. dispSimp // TraditionalForm
(d1 = DX1 /. dxdytermDetails //. QuadsBaseLocations) //. dispSimp // TraditionalForm
(d2 = DX2 /. dxdytermDetails //. QuadsBaseLocations) //. dispSimp // TraditionalForm
```

```
with setting of base locations :
```

$$\begin{pmatrix} s(\theta_p)\, h_p + c(\theta_p)\, w_p - x_p \\ c(\theta_p)\, h_p - s(\theta_p)\, w_p + y_p \\ w_p\, (c(\theta_p)\, y_p - s(\theta_p)\, x_p) + h_p\, (c(\theta_p)\, x_p + s(\theta_p)\, y_p) \end{pmatrix}$$

$$\begin{pmatrix} s(\theta_p)\, h_p - c(\theta_p)\, w_p + 2\, w_p - x_p \\ c(\theta_p)\, h_p + s(\theta_p)\, w_p + y_p \\ w_p\, (2\, s(\theta_p)\, w_p - s(\theta_p)\, x_p + c(\theta_p)\, y_p) + h_p\, (2\, c(\theta_p)\, w_p - c(\theta_p)\, x_p - s(\theta_p)\, y_p) \end{pmatrix}$$

$$1 - \frac{1}{\sqrt{(w_p\, c(\theta_p) + h_p\, s(\theta_p) - x_p)^2 + (h_p\, c(\theta_p) - w_p\, s(\theta_p) + y_p)^2}}$$

$$1 - \frac{1}{\sqrt{(-w_p\, c(\theta_p) + h_p\, s(\theta_p) + 2\, w_p - x_p)^2 + (h_p\, c(\theta_p) + w_p\, s(\theta_p) + y_p)^2}}$$

```
n = 1;
tmp = Series[d1, {xp[t], xp0, n}, {yp[t], yp0, n}, {θp[t], θp0, n}] /. θp0 → 0;
"1:";
tmp = Collect[tmp // Normal, {xp[t], yp[t], θp[t]}, Simplify];
"2:";
tmp = ((tmp //. perturb) //. EquilibiumPointRule);
"3:"
DX1taylored = Collect[Refine[tmp, γ > 0], {δx[t], δy[t], δθ[t]}, Simplify]
DX1taylored = Refine[DX1taylored, γ > 0]
"4:"
(*DX1taylored=*) (DX1taylored // Expand)
DX1taylored = (DX1taylored // Expand) /. ruleForNeglectingCombinations
DX1taylored = Collect[DX1taylored, {δx[t], δy[t], δθ[t]}, Simplify]
```

3:

$$1-\frac{2}{\sqrt{(2+\gamma)^2}}+\frac{4\,w_p\,\delta\theta[t]}{(2+\gamma)\,\sqrt{(2+\gamma)^2}}+\delta y[t]\left(-\frac{4}{(2+\gamma)\,\sqrt{(2+\gamma)^2}}+\frac{16\,w_p\,\delta\theta[t]}{\left((2+\gamma)^2\right)^{3/2}}\right)+\delta x[t]\left(-\frac{8\,h_p\,\delta\theta[t]}{\left((2+\gamma)^2\right)^{3/2}}-\frac{48\,h_p\,\delta y[t]\,\delta\theta[t]}{(2+\gamma)^3\,\sqrt{(2+\gamma)^2}}\right)$$

$$1-\frac{2}{2+\gamma}+\frac{4\,w_p\,\delta\theta[t]}{(2+\gamma)^2}+\delta y[t]\left(-\frac{4}{(2+\gamma)^2}+\frac{16\,w_p\,\delta\theta[t]}{(2+\gamma)^3}\right)+\delta x[t]\left(-\frac{8\,h_p\,\delta\theta[t]}{(2+\gamma)^3}-\frac{48\,h_p\,\delta y[t]\,\delta\theta[t]}{(2+\gamma)^4}\right)$$

4:

$$1-\frac{2}{2+\gamma}-\frac{4\,\delta y[t]}{(2+\gamma)^2}+\frac{4\,w_p\,\delta\theta[t]}{(2+\gamma)^2}-\frac{8\,h_p\,\delta x[t]\,\delta\theta[t]}{(2+\gamma)^3}+\frac{16\,w_p\,\delta y[t]\,\delta\theta[t]}{(2+\gamma)^3}-\frac{48\,h_p\,\delta x[t]\,\delta y[t]\,\delta\theta[t]}{(2+\gamma)^4}$$

$$1-\frac{2}{2+\gamma}-\frac{4\,\delta y[t]}{(2+\gamma)^2}+\frac{4\,w_p\,\delta\theta[t]}{(2+\gamma)^2}$$

$$\frac{\gamma}{2+\gamma}-\frac{4\,\delta y[t]}{(2+\gamma)^2}+\frac{4\,w_p\,\delta\theta[t]}{(2+\gamma)^2}$$

```
n = 1;
tmp = Series[d2, {xp[t], xp0, n}, {yp[t], yp0, n}, {θp[t], θp0, n}] /. θp0 → 0;
"1:";
tmp = Collect[tmp // Normal, {xp[t], yp[t], θp[t]}, Simplify];
"2:";
tmp = ((tmp //. perturb) //. EquilibiumPointRule);
"3:"
varTaylored = Collect[Refine[tmp, γ > 0], {δx[t], δy[t], δθ[t]}, Simplify]
varTaylored = Refine[varTaylored, γ > 0]
"4:"
(*varTaylored=*)(varTaylored // Expand)
varTaylored = (varTaylored // Expand) /. ruleForNeglectingCombinations
DX2taylored = Collect[varTaylored, {δx[t], δy[t], δθ[t]}, Simplify]
```

3:

$$\frac{\gamma}{2+\gamma} - \frac{4\, w_p\, \delta\theta[t]}{(2+\gamma)^2} + \delta y[t]\left(-\frac{4}{(2+\gamma)^2} - \frac{16\, w_p\, \delta\theta[t]}{(2+\gamma)^3}\right) + \delta x[t]\left(-\frac{8\, h_p\, \delta\theta[t]}{(2+\gamma)^3} - \frac{48\, h_p\, \delta y[t]\, \delta\theta[t]}{(2+\gamma)^4}\right)$$

$$\frac{\gamma}{2+\gamma} - \frac{4\, w_p\, \delta\theta[t]}{(2+\gamma)^2} + \delta y[t]\left(-\frac{4}{(2+\gamma)^2} - \frac{16\, w_p\, \delta\theta[t]}{(2+\gamma)^3}\right) + \delta x[t]\left(-\frac{8\, h_p\, \delta\theta[t]}{(2+\gamma)^3} - \frac{48\, h_p\, \delta y[t]\, \delta\theta[t]}{(2+\gamma)^4}\right)$$

4:

$$\frac{\gamma}{2+\gamma} - \frac{4\, \delta y[t]}{(2+\gamma)^2} - \frac{4\, w_p\, \delta\theta[t]}{(2+\gamma)^2} - \frac{8\, h_p\, \delta x[t]\, \delta\theta[t]}{(2+\gamma)^3} - \frac{16\, w_p\, \delta y[t]\, \delta\theta[t]}{(2+\gamma)^3} - \frac{48\, h_p\, \delta x[t]\, \delta y[t]\, \delta\theta[t]}{(2+\gamma)^4}$$

$$\frac{\gamma}{2+\gamma} - \frac{4\, \delta y[t]}{(2+\gamma)^2} - \frac{4\, w_p\, \delta\theta[t]}{(2+\gamma)^2}$$

$$\frac{\gamma}{2+\gamma} - \frac{4\, \delta y[t]}{(2+\gamma)^2} - \frac{4\, w_p\, \delta\theta[t]}{(2+\gamma)^2}$$

```
(*d3=-1/(DX1-1)/.dxdytermDetails//.QuadsBaseLocations
    d3=-1/(DX2-1)/.dxdytermDetails//.QuadsBaseLocations*)
```

$$\sqrt{\left((\mathrm{Sin}[\theta_p[t]]\, h_p + \mathrm{Cos}[\theta_p[t]]\, w_p - x_p[t])^2 + (\mathrm{Cos}[\theta_p[t]]\, h_p - \mathrm{Sin}[\theta_p[t]]\, w_p + y_p[t])^2\right)}$$

$$\sqrt{\left((\mathrm{Sin}[\theta_p[t]]\, h_p + 2\, w_p - \mathrm{Cos}[\theta_p[t]]\, w_p - x_p[t])^2 + (\mathrm{Cos}[\theta_p[t]]\, h_p + \mathrm{Sin}[\theta_p[t]]\, w_p + y_p[t])^2\right)}$$

```
tmpVar = v1 /. perturbationsRules /. EquilibiumPointRule //. smallδθAngleRule;
(*Collect[linearizedv1,{δx[t],δy[t],δθ[t]},Simplify]//MatrixForm*)
(tmpVar // Expand)
tmpVar = (tmpVar // Expand) /. ruleForNeglectingCombinations;
(linearizedv1 = Collect[tmpVar, {δx[t], δy[t], δθ[t]}, Simplify]) // MatrixForm
```

$$\left\{\{-\delta x[t] + h_p\,\delta\theta[t]\},\ \left\{-1 - \frac{\gamma}{2} + \delta y[t] - w_p\,\delta\theta[t]\right\},\ \left\{-w_p - \frac{\gamma\, w_p}{2} + h_p\,\delta x[t] + w_p\,\delta y[t] - h_p\,\delta\theta[t] - \frac{1}{2}\gamma\, h_p\,\delta\theta[t] - h_p^2\,\delta\theta[t] - w_p^2\,\delta\theta[t] - w_p\,\delta x[t]\,\delta\theta[t] + h_p\,\delta y[t]\,\delta\theta[t]\right\}\right\}$$

$$\begin{pmatrix} -\delta x[t] + h_p\,\delta\theta[t] \\ -1 - \frac{\gamma}{2} + \delta y[t] - w_p\,\delta\theta[t] \\ -\frac{1}{2}(2+\gamma)\,w_p + h_p\,\delta x[t] + w_p\,\delta y[t] + \left(-\frac{1}{2}(2+\gamma)\,h_p - h_p^2 - w_p^2\right)\delta\theta[t] \end{pmatrix}$$

```
tmpVar = v2 /. perturbationsRules /. EquilibiumPointRule //. smallδθAngleRule;
(*Collect[linearizedv1,{δx[t],δy[t],δθ[t]},Simplify]//MatrixForm*)
(tmpVar // Expand)
tmpVar = (tmpVar // Expand) /. ruleForNeglectingCombinations;
(linearizedv2 = Collect[tmpVar, {δx[t], δy[t], δθ[t]}, Simplify]) // MatrixForm
```

$$\left\{\{-\delta x[t] + h_p\,\delta\theta[t]\},\ \left\{-1 - \frac{\gamma}{2} + \delta y[t] + w_p\,\delta\theta[t]\right\},\ \left\{-w_p - \frac{\gamma\, w_p}{2} - h_p\,\delta x[t] + w_p\,\delta y[t] + h_p\,\delta\theta[t] + \frac{1}{2}\gamma\, h_p\,\delta\theta[t] + h_p^2\,\delta\theta[t] + w_p^2\,\delta\theta[t] - w_p\,\delta x[t]\,\delta\theta[t] - h_p\,\delta y[t]\,\delta\theta[t]\right\}\right\}$$

$$\begin{pmatrix} -\delta x[t] + h_p\,\delta\theta[t] \\ -1 - \frac{\gamma}{2} + \delta y[t] + w_p\,\delta\theta[t] \\ -\frac{1}{2}(2+\gamma)\,w_p - h_p\,\delta x[t] + w_p\,\delta y[t] + \left(\frac{1}{2}(2+\gamma)\,h_p + h_p^2 + w_p^2\right)\delta\theta[t] \end{pmatrix}$$

```
DX1taylored
DX2taylored
linearizedv1
linearizedv2
```

$$\frac{\gamma}{2+\gamma} - \frac{4\,\delta y[t]}{(2+\gamma)^2} + \frac{4\,w_p\,\delta\theta[t]}{(2+\gamma)^2}$$

$$\frac{2+\gamma}{2} - \delta y[t] - w_p\,\delta\theta[t]$$

$$\left\{\{-\delta x[t] + h_p\,\delta\theta[t]\},\ \left\{-1-\frac{\gamma}{2}+\delta y[t] - w_p\,\delta\theta[t]\right\},\ \left\{-\frac{1}{2}\,(2+\gamma)\,w_p + h_p\,\delta x[t] + w_p\,\delta y[t] + \left(-\frac{1}{2}\,(2+\gamma)\,h_p - h_p^2 - w_p^2\right)\delta\theta[t]\right\}\right\}$$

$$\left\{\{-\delta x[t] + h_p\,\delta\theta[t]\},\ \left\{-1-\frac{\gamma}{2}+\delta y[t] + w_p\,\delta\theta[t]\right\},\ \left\{-\frac{1}{2}\,(2+\gamma)\,w_p - h_p\,\delta x[t] + w_p\,\delta y[t] + \left(\frac{1}{2}\,(2+\gamma)\,h_p + h_p^2 + w_p^2\right)\delta\theta[t]\right\}\right\}$$

```
tmp = 𝒳 /. perturbationsRules /. EquilibiumPointRule
D[tmp, {t, 2}]
```

$$\left\{\{w_p + \delta x[t]\},\ \left\{\frac{1}{2}\,(-2-\gamma-2\,h_p) + \delta y[t]\right\},\ \{\delta\theta[t]\}\right\}$$

$$\{\{\delta x''[t]\},\ \{\delta y''[t]\},\ \{\delta\theta''[t]\}\}$$

```
"next is based on last setting of 'NonDimEOMmatrixForm'"
```

$$\left(\text{linearizedEOM} = \left(D[\mathcal{X},\{t,2\}] == \left(\left(\begin{pmatrix}1&0&0\\0&-1&0\\0&0&\alpha\end{pmatrix}\text{DX1taylored}\right).\text{linearizedv1} + \left(\kappa\begin{pmatrix}1&0&0\\0&-1&0\\0&0&-\alpha\end{pmatrix}\text{DX2taylored}\right).\text{linearizedv2} - \begin{pmatrix}0\\\gamma\\0\end{pmatrix} - \mathcal{A} - \mathcal{D}\ //\ \text{Flatten}\right)\ /.\ \text{perturbationsRules}\ /.\ \text{EquilibiumPointRule}\right)\ /.\ \text{dispSimp}\ //\ \text{MatrixForm}\ //\ \text{TraditionalForm}\right)$$

$$\begin{pmatrix}\dot{\delta x}\\\dot{\delta y}\\\dot{\delta\theta}\end{pmatrix} = \left(\begin{array}{c} -\frac{\rho\,\dot{\delta x}^2\,C_D\,h_p\,\mathrm{L0}_1^2}{m_p} + (\delta\theta\,h_p - \delta x)\left(\frac{\gamma+2}{2} - \delta y - \delta\theta\,w_p\right) + (\delta\theta\,h_p - \delta x)\left(\frac{\gamma}{\gamma+2} - \frac{4\,\delta y}{(\gamma+2)^2} + \frac{4\,\delta\theta\,w_p}{(\gamma+2)^2}\right) - \frac{(\dot{\delta x} - \dot{x_1})\,c_x}{k_1} - \cdots \\ -\frac{\rho\,\dot{\delta y}^2\,C_D\,w_p\,\mathrm{L0}_1^2}{m_p} - \gamma + \left(\frac{1}{2}(-\gamma-2) + \delta y + \delta\theta\,w_p\right)\left(-\frac{\gamma}{2} + \delta y + \delta\theta\,w_p - 1\right) + \left(-\frac{\gamma}{2} + \delta y - \delta\theta\,w_p - 1\right)\left(-\frac{\gamma}{\gamma+2} + \frac{4\,\delta y}{(\gamma+2)^2} - \frac{4\,\delta\theta\,\cdots}{(\gamma+2\cdots}\right. \\ -\frac{\dot{\delta\theta}\,c_\theta}{i_p\,\omega_s} + \frac{3\left(\frac{\gamma}{\gamma+2} - \frac{4\,\delta y}{(\gamma+2)^2} + \frac{4\,\delta\theta\,w_p}{(\gamma+2)^2}\right)\left(\delta x\,h_p - \frac{1}{2}(\gamma+2)\,w_p + \delta y\,w_p + \delta\theta\left(-h_p^2 - \frac{1}{2}(\gamma+2)\,h_p - w_p^2\right)\right)}{\left(\frac{h_p^2}{w_p^2}+1\right)w_p^2} - \frac{3\left(\frac{\gamma+2}{2} - \delta y - \delta\theta\,w_p\right)\left(-\delta x\,h_p - \frac{1}{2}(\gamma+2)\,w_p + \delta y\,w_p + \delta\theta\left(h_p^2\cdots\right.\right.}{\left(\frac{h_p^2}{w_p^2}+1\right)w_p^2} \end{array}\right.$$

```
(conservativeLinearEOM = linearizedEOM /. CD → 0 /. ci_ → 0) // MatrixForm //
 TraditionalForm
```

$$\begin{pmatrix} \delta x''(t) \\ \delta y''(t) \\ \delta\theta''(t) \end{pmatrix} = \begin{pmatrix} (h_p\,\delta\theta(t) - \delta x(t))\left(\frac{\gamma+2}{2} - \delta y(t) - w_p\,\delta\theta(t)\right) + (h_p\,\delta\theta(t) - \delta x(t))\left(\frac{\gamma}{\gamma+2} - \frac{4\,\delta y(t)}{(\gamma+2)^2} + \frac{4\,w_p\,\delta\theta(t)}{(\gamma+2)^2}\right) \\ -\gamma + \left(\frac{1}{2}(-\gamma-2) + \delta y(t) + w_p\,\delta\theta(t)\right)\left(-\frac{\gamma}{2} + \delta y(t) + w_p\,\delta\theta(t) - 1\right) + \left(-\frac{\gamma}{2} + \delta y(t) - w_p\,\delta\theta(t) - 1\right)\left(-\frac{\gamma}{\gamma+2} + \frac{4\,\delta y(t)}{(\gamma+2)^2} \cdots\right. \\ \frac{3\left(\frac{\gamma}{\gamma+2} - \frac{4\,\delta y(t)}{(\gamma+2)^2} + \frac{4\,w_p\,\delta\theta(t)}{(\gamma+2)^2}\right)\left(-\frac{1}{2}(\gamma+2)\,w_p + \delta y(t)\,w_p + h_p\,\delta x(t) + \left(-h_p^2 - \frac{1}{2}(\gamma+2)\,h_p - w_p^2\right)\delta\theta(t)\right)}{\left(\frac{h_p^2}{w_p^2}+1\right)w_p^2} - \frac{3\left(\frac{\gamma+2}{2} - \delta y(t) - w_p\,\delta\theta(t)\right)\left(-\frac{1}{2}(\gamma+2)\,w_p + \delta y(t)\,w_p - h_p\,\delta x(t) + \left(h_p^2 + \frac{1}{2}(\cdots\right.\right.}{\left(\frac{h_p^2}{w_p^2}+1\right)w_p^2} \end{pmatrix}$$

```
tmpVar = conservativeLinearEOM;
(tmpVar // Expand);
tmpVar = (tmpVar // Expand) /.
   ruleForNeglectingCombinations;
(trimmedEOM = Collect[tmpVar,
     {δx[t], δy[t], δθ[t]}, Simplify]) //
  MatrixForm // TraditionalForm
```

$$\begin{pmatrix} \delta x''(t) \\ \delta y''(t) \\ \delta\theta''(t) \end{pmatrix} = \begin{pmatrix} \frac{(\gamma^2+6\gamma+4)\,h_p\,\delta\theta(t)}{2(\gamma+2)} - \frac{(\gamma^2+\cdots}{} \\ \frac{1}{4}\left(\gamma^2+2\gamma+4\right) + (-\gamma-3)\,\delta y(t\cdots \\ -\frac{3(\gamma+1)\,\delta y(t)\,w_p}{h_p^2+w_p^2} + \frac{3(\gamma^2+2\gamma+4)\,w_p}{4(h_p^2+w_p^2)} + \frac{3(\gamma^2+6\gamma+4)\,h_p\,\delta x(t)}{2(\gamma+2)(h_p^2+w_p^2)} - \frac{3(2(\gamma\cdots}{} \end{pmatrix}$$

```
trimmedEOM // Expand // Simplify // TraditionalForm
"desired form of : M x''+C x'+K x==F "
```

$$M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$K = \begin{pmatrix} \frac{(\gamma^2+6\gamma+4)}{2\gamma+4} & 0 & \frac{-(\gamma^2+6\gamma+4)\,h_p}{2(\gamma+2)} \\ 0 & -(-\gamma-3) & (\gamma+1)\,w_p \\ -\frac{3(\gamma^2+6\gamma+4)\,h_p}{2(\gamma+2)(h_p^2+w_p^2)} & \frac{3(\gamma+1)\,w_p}{h_p^2+w_p^2} & \frac{3\left(2(\gamma^2+6\gamma+4)\,h_p^2 + (\gamma^3+8\gamma^2+16\gamma+8)\,h_p + 4(\gamma^2+5\gamma+6)\,w_p^2\right)}{4(\gamma+2)(h_p^2+w_p^2)} \end{pmatrix}$$

```
Solve[Det[K - ω² M] == 0, ω]
Manipulate[(Solve[Det[K - ω² M] == 0 /. hp → hp /. γ → gamma, ω]),
 {{hp, 1}, 0.1, 10}, {{gamma, 1}, 0.1, 5}]
```