```
required : system of 2 quads and 1 payload
```

system elements :

quad 1 - given as system input. x,y coor. θ is not influential

quad 2 - given as system input. x,y coor. θ is not influential

payload (contrained to quads locations)

```
In[128]:= Quit[]

In[1]:= Needs["VariationalMethods`"]
```

kinematics :

```
In[2]:= prop2D = {(Xi = ({{xi[t]}, {yi[t]}, {0}})) // MatrixForm // TraditionalForm,
          (Imati = ({{Ii,xx, 0, 0}, {0, Ii,yy, 0}, {0, 0, Ii,zz}})) // MatrixForm // TraditionalForm,
          ωi = D[{0, 0, θi[t]}, t]}
        prop2D /. i → 1
        prop2D /. i → 2
        prop2D /. i → p
```

Out[2]= $\left\{ \begin{pmatrix} x_i(t) \\ y_i(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbb{i}_{i,xx} & 0 & 0 \\ 0 & \mathbb{i}_{i,yy} & 0 \\ 0 & 0 & \mathbb{i}_{i,zz} \end{pmatrix}, \{0, 0, \theta_i'[t]\} \right\}$

Out[3]= $\left\{ \begin{pmatrix} x_1(t) \\ y_1(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbb{i}_{1,xx} & 0 & 0 \\ 0 & \mathbb{i}_{1,yy} & 0 \\ 0 & 0 & \mathbb{i}_{1,zz} \end{pmatrix}, \{0, 0, \theta_1'[t]\} \right\}$

Out[4]= $\left\{ \begin{pmatrix} x_2(t) \\ y_2(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbb{i}_{2,xx} & 0 & 0 \\ 0 & \mathbb{i}_{2,yy} & 0 \\ 0 & 0 & \mathbb{i}_{2,zz} \end{pmatrix}, \{0, 0, \theta_2'[t]\} \right\}$

Out[5]= $\left\{ \begin{pmatrix} x_p(t) \\ y_p(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbb{i}_{p,xx} & 0 & 0 \\ 0 & \mathbb{i}_{p,yy} & 0 \\ 0 & 0 & \mathbb{i}_{p,zz} \end{pmatrix}, \{0, 0, \theta_p'[t]\} \right\}$

```
In[6]:= (vi = D[Xi, t]) // MatrixForm // TraditionalForm
        vi /. i → 1
        vi /. i → 2
        vi /. i → p
```

Out[6]//TraditionalForm=

$$\begin{pmatrix} x_i'(t) \\ y_i'(t) \\ 0 \end{pmatrix}$$

Out[7]= $\{\{x_1'[t]\}, \{y_1'[t]\}, \{0\}\}$

Out[8]= $\{\{x_2'[t]\}, \{y_2'[t]\}, \{0\}\}$

Out[9]= $\{\{x_p'[t]\}, \{y_p'[t]\}, \{0\}\}$

rotations :

```
(*(Rp2I=(RotationMatrix[θp]))//MatrixForm;
HangPoint1=PayloadCenterPos-Rp2I.{lp/2,-hp/2}
    HangPoint2=PayloadCenterPos+Rp2I.{lp/2,hp/2}

     Quad1CenterPos = {xi,zi} /. i→1
       Quad2CenterPos = {xi,zi} /. i→2
        PayloadCenterPos = {xi,zi} /. i→p
         Δ1=(x1
             y1)-(xp
                  yp)-Rp2I.{lp/2,-hp/2}*)
```

```
{3.98866, 4.52335}

{5.22548, 6.09506}

{0, 10}

{10, 10}

{5, 5}

{{-6.01134}, {-0.476653 + y1 - yp}}
```

enrgies :

```
Imat
Imati
Imati /. i → 1
```

```
Imat

{{ii,xx, 0, 0}, {0, ii,yy, 0}, {0, 0, ii,zz}}

{{i1,xx, 0, 0}, {0, i1,yy, 0}, {0, 0, i1,zz}}
```

```
In[10]:= a[e]
         a[e] /. a[a_] → Cos[a]
         a[e] /. a → Cos
```

```
Out[10]= a[e]

Out[11]= Cos[e]

Out[12]= Cos[e]
```

```
In[18]:= dispSimp = {a_[t] → a, Cos[a_] → c[a], Sin[a_] → s[a]};
```

In[10]:=
```
{(Rp2I = RotationMatrix[θp[t]]) // MatrixForm,
  x1dotSqr = (Transpose[vi].vi)[[1, 1]] /. i → 1,
x2dotSqr = (Transpose[vi].vi)[[1, 1]] /. i → 2,
  IωSqr1 = ωi.Imati .ωi /. i → 1,
  IωSqr2 = ωi.Imati .ωi /. i → 2,
  xpdotSqr = (Transpose[vi].vi)[[1, 1]] /. i → p,
  IωSqrp = ωi.Imati .ωi /. i → p,
```

$$r_1[t] = \begin{pmatrix} x_1[t] \\ y_1[t] \end{pmatrix} - \left(\begin{pmatrix} x_p[t] \\ y_p[t] \end{pmatrix} - \text{Rp2I}.\left\{\frac{l_p}{2}, -\frac{h_p}{2}\right\}\right),$$

$$r_2[t] = \begin{pmatrix} x_2[t] \\ y_2[t] \end{pmatrix} - \left(\begin{pmatrix} x_p[t] \\ y_p[t] \end{pmatrix} + \text{Rp2I}.\left\{\frac{l_p}{2}, \frac{h_p}{2}\right\}\right),$$

$$\Delta_1 = \sqrt{(r_1[t][[1]])^2 + (r_1[t][[2]])^2} - L0_1,$$

$$\Delta_2 = \sqrt{(r_2[t][[1]])^2 + (r_2[t][[2]])^2} - L0_2\};$$

$$\left(T = \frac{1}{2} m_1\, \text{x1dotSqr} + \frac{1}{2} \text{I}\omega\text{Sqr1} + \frac{1}{2} m_2\, \text{x2dotSqr} + \frac{1}{2} \text{I}\omega\text{Sqr2} + \frac{1}{2} m_p\, \text{xpdotSqr} + \frac{1}{2} \text{I}\omega\text{Sqrp}\right);$$

```
(*ri=li+Δl*)
```

$$V = m_1\, g\, (X_i[[2]] \text{ /. } i \to 1) + m_2\, g\, (X_i[[2]] \text{ /. } i \to 2) + m_p\, g\, (X_i[[2]] \text{ /. } i \to p) + \frac{1}{2} k_1 \Delta_1^2 + \frac{1}{2} k_2 \Delta_2^2;$$

```
L = (T - V);
L = (T - V)[[1]]
```

Out[14]=

$$-g\, m_1\, y_1[t] - g\, m_2\, y_2[t] - \frac{1}{2} k_1 \left(-L0_1 + \sqrt{\left(\left(\frac{1}{2} \text{Sin}[\theta_p[t]]\, h_p + \frac{1}{2} \text{Cos}[\theta_p[t]]\, l_p + x_1[t] - x_p[t]\right)^2 + \left(-\frac{1}{2} \text{Cos}[\theta_p[t]]\, h_p + \frac{1}{2} \text{Sin}[\theta_p[t]]\, l_p + y_1[t] - y_p[t]\right)^2\right)}\right)^2 -$$
$$\frac{1}{2} k_2 \left(-L0_2 + \sqrt{\left(\left(\frac{1}{2} \text{Sin}[\theta_p[t]]\, h_p - \frac{1}{2} \text{Cos}[\theta_p[t]]\, l_p + x_2[t] - x_p[t]\right)^2 + \left(-\frac{1}{2} \text{Cos}[\theta_p[t]]\, h_p - \frac{1}{2} \text{Sin}[\theta_p[t]]\, l_p + y_2[t] - y_p[t]\right)^2\right)}\right)^2 -$$
$$g\, m_p\, y_p[t] + \frac{1}{2} m_1 \left(x_1'[t]^2 + y_1'[t]^2\right) + \frac{1}{2} m_2 \left(x_2'[t]^2 + y_2'[t]^2\right) +$$
$$\frac{1}{2}$$
$$m_p \left(x_p'[t]^2 + y_p'[t]^2\right) +$$
$$\frac{1}{2} \mathbb{i}_{1,zz}\, \theta_1'[t]^2 + \frac{1}{2} \mathbb{i}_{2,zz}\, \theta_2'[t]^2 +$$
$$\frac{1}{2} \mathbb{i}_{p,zz}\, \theta_p'[t]^2$$

In[19]:=
```
L //. dispSimp // TraditionalForm
```

Out[19]//TraditionalForm=

$$\frac{1}{2}\,\imath_{p,\mathrm{zz}}\,(\theta_p')^2+\frac{1}{2}\,\imath_{1,\mathrm{zz}}\,(\theta_1')^2+\frac{1}{2}\,\imath_{2,\mathrm{zz}}\,(\theta_2')^2-$$
$$\frac{1}{2}\,k_1\left(\sqrt{\left(\left(\frac{1}{2}\,l_p\,c(\theta_p)+\frac{1}{2}\,h_p\,s(\theta_p)-x_p+x_1\right)^2+\left(-\frac{1}{2}\,h_p\,c(\theta_p)+\frac{1}{2}\,l_p\,s(\theta_p)-y_p+y_1\right)^2\right)}-\mathrm{L0}_1\right)^2-$$
$$\frac{1}{2}\,k_2\left(\sqrt{\left(\left(-\frac{1}{2}\,l_p\,c(\theta_p)+\frac{1}{2}\,h_p\,s(\theta_p)-x_p+x_2\right)^2+\left(-\frac{1}{2}\,h_p\,c(\theta_p)-\frac{1}{2}\,l_p\,s(\theta_p)-y_p+y_2\right)^2\right)}-\mathrm{L0}_2\right)^2-$$
$$g\,m_p\,y_p-g\,m_1\,y_1-g\,m_2\,y_2+\frac{1}{2}\,m_p\left((x_p')^2+(y_p')^2\right)+\frac{1}{2}\,m_1\left((x_1')^2+(y_1')^2\right)+\frac{1}{2}\,m_2\left((x_2')^2+(y_2')^2\right)$$

$$(*q=\begin{pmatrix}x_1\\ y_1\\ \theta_1\\ x_2\\ y_2\\ \theta_2\\ x_p\\ y_p\\ \theta_p\end{pmatrix}[t]*)$$

```
{{x1}, {y1}, {θ1}, {x2}, {y2}, {θ2}, {xp}, {yp}, {θp}}[t]
```

after setting L calculate the lagrangian derivatives and equations:

```
(quadEqNominal =
    EulerEquations[L, {x1[t], y1[t], θ1[t], x2[t], y2[t], θ2[t], xp[t], yp[t], θp[t]},
      t](*[[All,1]]*)(*==Q*) // Simplify) // MatrixForm // TraditionalForm
```

In[18]:=
```
(quadEqNominal = EulerEquations[L,
      {(*x1[t],y1[t],θ1[t],x2[t],y2[t],θ2[t],*)xp[t], yp[t], θp[t]}, t]
      (*[[All,1]]*)(*==Q*) // Simplify) // MatrixForm // TraditionalForm
```

Out[18]//TraditionalForm=

$$\frac{k_1\,(h_p\sin(\theta_p(t))+l_p\cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))}{2\sqrt{\frac{1}{4}\,(h_p\sin(\theta_p(t))}}$$

$$\frac{k_1\left(-\frac{1}{2}\,h_p\cos(\theta_p(t))+\frac{1}{2}\,l_p\sin(\theta_p(t))-y_p(t)+y_1(t)\right)\left(\sqrt{\right.}}{\sqrt{\frac{1}{4}\,(h_p\sin(\theta_p(t))+l_p\,\mathrm{co}}}$$

$$\imath_{p,\mathrm{zz}}\,\theta_p''(t)+\frac{k_1\,(h_p\,(x_1(t)\cos(\theta_p(t))-x_p(t)\cos(\theta_p(t))+(y_1(t)-y_p(t))\sin(\theta_p(t)))+l_p\,(x_1(t)\,(-\sin(\theta_p(t)))+x_p(t)\sin(\theta_p(t))+(y_1(t)-}{2\sqrt{\frac{1}{4}\,(h_p\sin(\theta_p(t))+l_p\cos(\theta_p(t))-2\,x_p(t)+}}$$

In[49]:=
```
quadEqNominal // MatrixForm
```

Out[49]//MatrixForm=
```
k1 (Sin[θp[t]] hp+Cos[θp[t]] lp+2
―――――――――――――――――――――――――――――――――――
2 Sqrt[1/4 (

k1 (-1/2 Cos[θp[t]] hp+1/2 Sin[θp[t]] lp+y1
―――――――――――――――――――――――――――――――――――
Sqrt[1/4 (Sin[

k1 (lp (-Sin[θp[t]] x1[t]+Sin[θp[t]] xp[t]+Cos[θp[t]] (y1[t]-yp[t]))+hp (Cos[θp[t]] x1[t]-Cos[θp[t]] xp[t]+Sin[θp[t]] (y1[t
―――――――――――――――――――――――――――――――――――
2 Sqrt[1/4 (Sin[θp[t]] hp+Cos[θp[t]] lp+2 x1[t]-2 xp[t])^2+(
```

```
terms = {
   (*Sqrt[(1/4 (Sin[θp[t]] hp+Cos[θp[t]] lp+2 x1[t]-2 xp[t])^2+
       (1/2 Cos[θp[t]] hp-1/2 Sin[θp[t]] lp-y1[t]+yp[t])^2)]→dom1,*)
   1/4 (Sin[θp[t]] hp + Cos[θp[t]] lp + 2 x1[t] - 2 xp[t])^2 +
     (1/2 Cos[θp[t]] hp - 1/2 Sin[θp[t]] lp - y1[t] + yp[t])^2 → dom11,
   (*Sqrt[((1/2 Sin[θp[t]] hp-1/2 Cos[θp[t]] lp+x2[t]-xp[t])^2+
       1/4 (Cos[θp[t]] hp+Sin[θp[t]] lp-2 y2[t]+2 yp[t])^2)]→dom2,*)
   (1/2 Sin[θp[t]] hp - 1/2 Cos[θp[t]] lp + x2[t] - xp[t])^2 +
     1/4 (Cos[θp[t]] hp + Sin[θp[t]] lp - 2 y2[t] + 2 yp[t])^2 → dom22
  };
```

In[110]:=
```
(simpStep1 = quadEqNominal /. terms)
  (*//.dispSimp*)(*//Simplify*) //
 MatrixForm(*//TraditionalForm*)
```

Out[110]//MatrixForm=
```
k1 (Sqrt[dom11] -L01) (lp (-Sin[θp[t]] x1[t]+Sin[θp[t]] xp[t]+Cos[θp[t]
―――――――――――――――――――――――――――――――――――
```

In[111]:=
```
simpStep1 //. dispSimp // MatrixForm //
  TraditionalForm
```

Out[111]//TraditionalForm=

$$\left(\begin{array}{c}
\frac{k_1\left(\sqrt{\text{dom11}}-\text{L0}_1\right)\left(l_p\,c(\theta_p)+h_p\,s(\theta_p)-2\,x_p+2\,x_1\right)}{2\sqrt{\text{dom11}}}+\frac{k_2\left(\sqrt{\text{dom22}}-\text{L0}_2\right)\left(-\frac{1}{2}\,l_p\,c(\theta_p)+\frac{1}{2}\,h_p\,s(\theta_p)-x_p+x_2\right)}{\sqrt{\text{dom22}}}=m_p\,x_p'' \cdots \\
\frac{k_1\left(\sqrt{\text{dom11}}-\text{L0}_1\right)\left(-\frac{1}{2}\,h_p\,c(\theta_p)+\frac{1}{2}\,l_p\,s(\theta_p)-y_p+y_1\right)}{\sqrt{\text{dom11}}}+\frac{k_2\left(\sqrt{\text{dom22}}-\text{L0}_2\right)\left(-\frac{1}{2}\,h_p\,c(\theta_p)-\frac{1}{2}\,l_p\,s(\theta_p)-y_p+y_2\right)}{\sqrt{\text{dom22}}}=m_p\,(g+\cdots \\
i_{p,zz}\,\theta_p''+\frac{k_1\left(\sqrt{\text{dom11}}-\text{L0}_1\right)\left(h_p\,(x_1\,c(\theta_p)-x_p\,c(\theta_p)+(y_1-y_p)\,s(\theta_p))+l_p\,((y_1-y_p)\,c(\theta_p)+x_1\,(-s(\theta_p))+x_p\,s(\theta_p))\right)}{2\sqrt{\text{dom11}}}+\frac{k_2\left(\sqrt{\text{dom22}}-\text{L0}_2\right)\left(h_p\,(x_2\,c(\theta_p)-x_p\,c(\theta_p)+(y_2-y_p)\cdots\right.}{2\sqrt{\text{dom22}}}
\end{array}\right.$$

In[112]:=
```
simpStep1 /. θp[t] → 0 /. dispSimp //
  MatrixForm // TraditionalForm
```

Out[112]//TraditionalForm=

$$\left(\begin{array}{c}
\frac{k_1\left(\sqrt{\text{dom11}}-\text{L0}_1\right)\left(l_p-2\,x_p+2\,x_1\right)}{2\sqrt{\text{dom11}}}+\frac{k_2\left(\sqrt{\text{dom22}}-\text{L0}_2\right)\left(-\frac{l_p}{2}-x_p+x_2\right)}{\sqrt{\text{dom22}}}=m_p \cdots \\
\frac{k_1\left(\sqrt{\text{dom11}}-\text{L0}_1\right)\left(-\frac{h_p}{2}-y_p+y_1\right)}{\sqrt{\text{dom11}}}+\frac{k_2\left(\sqrt{\text{dom22}}-\text{L0}_2\right)\left(-\frac{h_p}{2}-y_p+y_2\right)}{\sqrt{\text{dom22}}}=m_p\,(g \cdots \\
i_{p,zz}\,\theta_p''+\frac{k_1\left(\sqrt{\text{dom11}}-\text{L0}_1\right)\left(h_p\,(x_1-x_p)+l_p\,(y_1-y_p)\right)}{2\sqrt{\text{dom11}}}+\frac{k_2\left(\sqrt{\text{dom22}}-\text{L0}_2\right)\left(h_p\,(x_2-x_p)\cdots\right.}{2\sqrt{\text{dom22}}}
\end{array}\right.$$

```
(* planar mass with springs *)
```

In[96]:=
```
(trimmedEq = quadEqNominal /.
    {x1'[t] → 0, x2'[t] → 0, x1''[t] → 0, x2''[t] → 0, θ1′[t] → 0, θ2′[t] → 0,
     θ1''[t] → 0, θ2''[t] → 0, θ1[t] → 0, θ2[t] → 0}) // MatrixForm // TraditionalForm
```

Out[96]//TraditionalForm=

$$\left(\begin{array}{c}
\frac{k_1\,(h_p\sin(\theta_p(t))+l_p\cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))\left(\sqrt{\frac{1}{4}\,(h_p\sin(\theta_p(t))+l_p\cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t)\cdots}\right.}{2\sqrt{\frac{1}{4}\,(h_p\sin(\theta_p(t))+l_p\cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))^2+\left(\frac{1}{2}\,h_p\cos(\theta_p(t))-\cdots\right.}} \\
\frac{k_1\left(-\frac{1}{2}\,h_p\cos(\theta_p(t))+\frac{1}{2}\,l_p\sin(\theta_p(t))-y_p(t)+y_1(t)\right)\left(\sqrt{\frac{1}{4}\,(h_p\sin(\theta_p(t))+l_p\cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))^2+\cdots}\right.}{\sqrt{\frac{1}{4}\,(h_p\sin(\theta_p(t))+l_p\cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))^2+\left(\frac{1}{2}\,h_p\cos(\theta_p(t))-\frac{1}{2}\,l_p\cdots\right.}} \\
i_{p,zz}\,\theta_p''(t)+\frac{k_1\,(h_p\,(x_1(t)\cos(\theta_p(t))-x_p(t)\cos(\theta_p(t))+(y_1(t)-y_p(t))\sin(\theta_p(t)))+l_p\,(x_1(t)\,(-\sin(\theta_p(t)))+x_p(t)\sin(\theta_p(t))+(y_1(t)-y_p(t))\cos(\theta_p(t))))\left(\sqrt{\frac{1}{4}\,(h_p\sin(\theta_p(t))+l_p\cos(\theta_p(t)\cdots}\right.}{2\sqrt{\frac{1}{4}\,(h_p\sin(\theta_p(t))+l_p\cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))^2+\left(\frac{1}{2}\,h_p\cos(\theta_p(t))-\frac{1}{2}\,l_p\sin(\theta_p(t))+y_p(t)-y\cdots\right.}}
\end{array}\right.$$

In[98]:=
```
eq2D =
  {trimmedEq[[1]]} /. θp[t] → 0 /. y1[t] → 0 /. y2[t] → 0 /. yp[t] → 0 /. lp → 0 /. hp → 0 /.
     dispSimp // Expand // Simplify // TraditionalForm
```

Out[98]//TraditionalForm=

$$\left\{\frac{k_1\left((x_1-x_p)^2-\mathrm{L0}_1\sqrt{(x_1-x_p)^2}\right)}{x_1-x_p}+\frac{k_2\left((x_2-x_p)^2-\mathrm{L0}_2\sqrt{(x_2-x_p)^2}\right)}{x_2-x_p}=m_p\,x_p''\right\}$$

2 D analysis

L

in 2 D case the 2 DOF are x, y_p, looking at lumped mass payload.
X_1, X_2 → 0, k_2 → 0, l, h_p → 0 as well

```
L2D =
  L /. {x1[t] → 0, x1'[t] → 0, x1''[t] → 0, x2[t] → 0, x2'[t] → 0, x2''[t] → 0, θ1'[t] → 0,
       θ1''[t] → 0, θ2'[t] → 0, θ2''[t] → 0, θ1[t] → 0, θ2[t] → 0} /.
     {y1[t] → 0, y1'[t] → 0, y1''[t] → 0, y2[t] → 0, y2'[t] → 0, y2''[t] → 0} /.
    lp → 0 /. hp → 0 /. k2 → 0(*/.θp[t]→0*)
```

$$-g\,m_p\,y_p[t]-\frac{1}{2}k_1\left(-\mathrm{L0}_1+\sqrt{x_p[t]^2+y_p[t]^2}\right)^2+\frac{1}{2}m_p\left(x_p'[t]^2+y_p'[t]^2\right)+\frac{1}{2}\mathrm{i}_{p,zz}\,\theta_p'[t]^2$$

```
(quadEqNominal2D = EulerEquations[L2D, {xp[t], yp[t], θp[t]}, t](*[[All,
     1]]*)(*==Q*) // Expand // Simplify) // MatrixForm // TraditionalForm
```

$$\begin{pmatrix} k_1\,x_p(t)\left(\frac{\mathrm{L0}_1}{\sqrt{x_p(t)^2+y_p(t)^2}}-1\right)=m_p\,x_p''(t) \\ k_1\,y_p(t)\left(\frac{\mathrm{L0}_1}{\sqrt{x_p(t)^2+y_p(t)^2}}-1\right)=m_p\,(g+y_p''(t)) \\ i_{p,zz}\,\theta_p''(t)=0 \end{pmatrix}$$

```
quadEqNominal2D[[1]] // MatrixForm
```

$$k_1\,x_p[t]\left(-1+\frac{\mathrm{L0}_1}{\sqrt{x_p[t]^2+y_p[t]^2}}\right)==m_p\,x_p''[t]$$

```
Series[quadEqNominal2D[[1]], {xp[t], 0, 3}, {yp[t], 0, 3}] // Simplify
```

$$\left(\frac{k_1\,\mathrm{L0}_1}{y_p[t]}-k_1+O[y_p[t]]^4\right)x_p[t]+\left(-\frac{k_1\,\mathrm{L0}_1}{2\,y_p[t]^3}+O[y_p[t]]^4\right)x_p[t]^3+O[x_p[t]]^4==m_p\,x_p''[t]$$

```
Series[quadEqNominal2D[[1]], {yp[t], 0, 3}, {xp[t], 0, 3}] // Simplify
```

$$\left(k_1\,\mathrm{L0}_1-k_1\,x_p[t]+O[x_p[t]]^4\right)+\left(-\frac{k_1\,\mathrm{L0}_1}{2\,x_p[t]^2}+O[x_p[t]]^4\right)y_p[t]^2+O[y_p[t]]^4==m_p\,x_p''[t]$$

```
Series[L0₁/Sqrt[xp[t]^2 + yp[t]^2], {xp[t], 0, 3}, {yp[t], 0, 3}] // Expand // Simplify
```

$$\left(\frac{L0_1}{y_p[t]} + O[y_p[t]]^4\right) + \left(-\frac{L0_1}{2\, y_p[t]^3} + O[y_p[t]]^4\right) x_p[t]^2 + O[x_p[t]]^4$$

trials :

```
Series[f[x], {x, a, 3}]
```

$$f[a] + f'[a]\,(x-a) + \frac{1}{2} f''[a]\,(x-a)^2 + \frac{1}{6} f^{(3)}[a]\,(x-a)^3 + O[x-a]^4$$

In[115]:=
```
equalibriumTerms = {
   x1'[t] → 0, x1''[t] → 0,
   x2'[t] → 0, x2''[t] → 0,
   θ1′[t] → 0, θ1''[t] → 0,
   θ2′[t] → 0, θ2''[t] → 0,
   y1'[t] → 0, y1''[t] → 0,
   y2'[t] → 0, y2''[t] → 0,
   xp'[t] → 0, xp''[t] → 0,
   yp'[t] → 0, yp''[t] → 0,
   θp′[t] → 0, θp''[t] → 0
  }
```

Out[115]= $\{x_1'[t] \to 0,\ x_1''[t] \to 0,\ x_2'[t] \to 0,\ x_2''[t] \to 0,\ \theta_1'[t] \to 0,\ \theta_1''[t] \to 0,\ \theta_2'[t] \to 0,\ \theta_2''[t] \to 0,\ y_1'[t] \to 0,\ y_1''[t] \to 0,\ y_2'[t] \to 0,\ y_2''[t] \to 0,\ x_p'[t] \to 0,\ x_p''[t] \to 0,\ y_p'[t] \to 0,\ y_p''[t] \to 0,\ \theta_p'[t] \to 0,\ \theta_p''[t] \to 0\}$

In[121]:=
```
simpStep1 /. equalibriumTerms //. dispSimp // MatrixForm //
 TraditionalForm
```

Out[121]//TraditionalForm=

$$\left(\begin{array}{l}
\frac{k_1\left(\sqrt{\text{dom11}}-\text{L0}_1\right)\left(l_p\, c(\theta_p)+h_p\, s(\theta_p)-2\, x_p+2\, x_1\right)}{2\sqrt{\text{dom11}}} + \frac{k_2\left(\sqrt{\text{dom22}}-\text{L0}_2\right)\left(-\frac{1}{2}\, l_p \right.}{\sqrt{\text{dor}}} \\
\frac{k_1\left(\sqrt{\text{dom11}}-\text{L0}_1\right)\left(-\frac{1}{2}\, h_p\, c(\theta_p)+\frac{1}{2}\, l_p\, s(\theta_p)-y_p+y_1\right)}{\sqrt{\text{dom11}}} + \frac{k_2\left(\sqrt{\text{dom22}}-\text{L0}_2\right)\left(-\frac{1}{2}\, h_p \right.}{\sqrt{\text{dom}}} \\
\frac{k_1\left(\sqrt{\text{dom11}}-\text{L0}_1\right)\left(h_p\left(x_1\, c(\theta_p)-x_p\, c(\theta_p)+(y_1-y_p)\, s(\theta_p)\right)+l_p\left((y_1-y_p)\, c(\theta_p)+x_1\,(-s(\theta_p))+x_p\, s(\theta_p)\right)\right)}{2\sqrt{\text{dom11}}} + \frac{k_2\left(\sqrt{\text{dom22}}-\text{L0}_2\right)\left(h_p \right.}{}
\end{array}\right.$$