```
required : system of 2 quads and 1 payload
```

system elements :
quad 1 - given as system input. x,y coor. θ is not influential
quad 2 - given as system input. x,y coor. θ is not influential
payload (contrained to quads locations)

```
Quit[]

Needs["VariationalMethods`"]
```

kinematics :

$$\text{prop2D} = \left\{\left(X_i = \begin{pmatrix} x_i[t] \\ y_i[t] \\ 0 \end{pmatrix}\right) \text{ // MatrixForm // TraditionalForm}, \left(\text{Imat}_i = \begin{pmatrix} I_{i,xx} & 0 & 0 \\ 0 & I_{i,yy} & 0 \\ 0 & 0 & I_{i,zz} \end{pmatrix}\right) \text{ // MatrixForm // TraditionalForm}, \omega_i = D[\{0, 0, \theta_i[t]\}, t]\right\}$$

```
prop2D /. i → 1
prop2D /. i → 2
prop2D /. i → p
```

$(v_i = D[X_i, t])$ `// MatrixForm // TraditionalForm`

v_i `/. i → 1`

v_i `/. i → 2`

v_i `/. i → p`

$$\left\{\begin{pmatrix} x_i(t) \\ y_i(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbb{i}_{i,xx} & 0 & 0 \\ 0 & \mathbb{i}_{i,yy} & 0 \\ 0 & 0 & \mathbb{i}_{i,zz} \end{pmatrix}, \{0, 0, \theta_i'[t]\}\right\}$$

$$\left\{\begin{pmatrix} x_1(t) \\ y_1(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbb{i}_{1,xx} & 0 & 0 \\ 0 & \mathbb{i}_{1,yy} & 0 \\ 0 & 0 & \mathbb{i}_{1,zz} \end{pmatrix}, \{0, 0, \theta_1'[t]\}\right\}$$

$$\left\{\begin{pmatrix} x_2(t) \\ y_2(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbb{i}_{2,xx} & 0 & 0 \\ 0 & \mathbb{i}_{2,yy} & 0 \\ 0 & 0 & \mathbb{i}_{2,zz} \end{pmatrix}, \{0, 0, \theta_2'[t]\}\right\}$$

$$\left\{\begin{pmatrix} x_p(t) \\ y_p(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbb{i}_{p,xx} & 0 & 0 \\ 0 & \mathbb{i}_{p,yy} & 0 \\ 0 & 0 & \mathbb{i}_{p,zz} \end{pmatrix}, \{0, 0, \theta_p'[t]\}\right\}$$

$$\begin{pmatrix} x_i'(t) \\ y_i'(t) \\ 0 \end{pmatrix}$$

$\{\{x_1'[t]\}, \{y_1'[t]\}, \{0\}\}$

$\{\{x_2'[t]\}, \{y_2'[t]\}, \{0\}\}$

$\{\{x_p'[t]\}, \{y_p'[t]\}, \{0\}\}$

```
dispSimp = {a_[t] → a, Cos[a_] → c[a], Sin[a_] → s[a], ii_,zz -> Ii};
{(Rp2I = RotationMatrix[θp[t]]) // MatrixForm,
  x1dotSqr = (Transpose[vi].vi)[[1, 1]] /. i → 1,
x2dotSqr = (Transpose[vi].vi)[[1, 1]] /. i → 2,
  IωSqr1 = ωi.Imati .ωi /. i → 1,
  IωSqr2 = ωi.Imati .ωi /. i → 2,
  xpdotSqr = (Transpose[vi].vi)[[1, 1]] /. i → p,
  IωSqrp = ωi.Imati .ωi /. i → p,
```

$$r_1[t] = \begin{pmatrix} x_1[t] \\ y_1[t] \end{pmatrix} - \left(\begin{pmatrix} x_p[t] \\ y_p[t] \end{pmatrix} + \text{Rp2I}.\left\{-\frac{l_p}{2}, \frac{h_p}{2}\right\}\right),$$

$$r_2[t] = \begin{pmatrix} x_2[t] \\ y_2[t] \end{pmatrix} - \left(\begin{pmatrix} x_p[t] \\ y_p[t] \end{pmatrix} + \text{Rp2I}.\left\{\frac{l_p}{2}, \frac{h_p}{2}\right\}\right),$$

$$\Delta_1 = \sqrt{(r_1[t][[1]])^2 + (r_1[t][[2]])^2} - L0_1,$$

$$\Delta_2 = \sqrt{(r_2[t][[1]])^2 + (r_2[t][[2]])^2} - L0_2\};$$

$$\left(T = \frac{1}{2} m_1\, \text{x1dotSqr} + \frac{1}{2}\, \text{I}\omega\text{Sqr1} + \frac{1}{2} m_2\, \text{x2dotSqr} + \frac{1}{2}\, \text{I}\omega\text{Sqr2} + \frac{1}{2} m_p\, \text{xpdotSqr} + \frac{1}{2}\, \text{I}\omega\text{Sqrp}\right);$$

(*$r_i = l_i + \Delta l$*)

$$V = m_1\, g\, (X_i[[2]] \text{ /. } i \to 1) + m_2\, g\, (X_i[[2]] \text{ /. } i \to 2) + m_p\, g\, (X_i[[2]] \text{ /. } i \to p) + \frac{1}{2} k_1\, \Delta_1^2 + \frac{1}{2} k_2\, \Delta_2^2;$$

L = (T - V)[[1]] (*$T_{quad\#1} + T_{quad\#2} + T_{payload} - (V_{quad\#1} + V_{quad\#2} + V_{payload} + V_{spring\#1} + V_{spring\#2})$*)

$$-g\, m_1\, y_1[t] - g\, m_2\, y_2[t] - \frac{1}{2} k_1 \left(-L0_1 + \sqrt{\left(\left(\frac{1}{2}\text{Sin}[\theta_p[t]]\, h_p + \frac{1}{2}\text{Cos}[\theta_p[t]]\, l_p + x_1[t] - x_p[t]\right)^2 + \left(-\frac{1}{2}\text{Cos}[\theta_p[t]]\, h_p + \frac{1}{2}\text{Sin}[\theta_p[t]]\, l_p + y_1[t] - y_p[t]\right)^2\right)}\right)^2 -$$

$$\frac{1}{2} k_2 \left(-L0_2 + \sqrt{\left(\left(\frac{1}{2}\text{Sin}[\theta_p[t]]\, h_p - \frac{1}{2}\text{Cos}[\theta_p[t]]\, l_p + x_2[t] - x_p[t]\right)^2 + \left(-\frac{1}{2}\text{Cos}[\theta_p[t]]\, h_p - \frac{1}{2}\text{Sin}[\theta_p[t]]\, l_p + y_2[t] - y_p[t]\right)^2\right)}\right)^2 -$$

$$g\, m_p\, y_p[t] + \frac{1}{2} m_1 \left(x_1'[t]^2 + y_1'[t]^2\right) + \frac{1}{2} m_2 \left(x_2'[t]^2 + y_2'[t]^2\right) +$$

$$\frac{1}{2} m_p \left(x_p'[t]^2 + y_p'[t]^2\right) +$$

$$\frac{1}{2}\, \mathbb{i}_{1,zz}\, \theta_1'[t]^2 + \frac{1}{2}\, \mathbb{i}_{2,zz}\, \theta_2'[t]^2 +$$

$$\frac{1}{2}\, \mathbb{i}_{p,zz}\, \theta_p'[t]^2$$

```
L //. dispSimp // TraditionalForm
```

$$-\frac{1}{2}k_1\left(\sqrt{\left(\left(\frac{1}{2}l_p\,c(\theta_p)+\frac{1}{2}h_p\,s(\theta_p)-x_p+x_1\right)^2+\left(-\frac{1}{2}h_p\,c(\theta_p)+\frac{1}{2}l_p\,s(\theta_p)-y_p+y_1\right)^2\right)}-\text{L0}_1\right)^2-$$
$$\frac{1}{2}k_2\left(\sqrt{\left(\left(-\frac{1}{2}l_p\,c(\theta_p)+\frac{1}{2}h_p\,s(\theta_p)-x_p+x_2\right)^2+\left(-\frac{1}{2}h_p\,c(\theta_p)-\frac{1}{2}l_p\,s(\theta_p)-y_p+y_2\right)^2\right)}-\text{L0}_2\right)^2-g\,m_p\,y_p-g\,m_1\,y_1-$$
$$g\,m_2\,y_2+\frac{1}{2}i_1\,(\theta_1')^2+\frac{1}{2}i_2\,(\theta_2')^2+\frac{1}{2}m_p\left((x_p')^2+(y_p')^2\right)+\frac{1}{2}m_1\left((x_1')^2+(y_1')^2\right)+\frac{1}{2}m_2\left((x_2')^2+(y_2')^2\right)+\frac{1}{2}i_p\,(\theta_p')^2$$

```
(quadEqNominal = EulerEquations[L,
    {(*x1[t],y1[t],θ1[t],x2[t],y2[t],θ2[t],*)xp[t], yp[t], θp[t]}, t]
    (*[[All,1]]*)(*==Q*) // Simplify) // MatrixForm // TraditionalForm
```

$$\left(\begin{array}{l}
\dfrac{k_1\,(h_p\sin(\theta_p(t))+l_p\cos(\theta_p(t))-2\,x_p(t)+2\,x_1}{2\sqrt{\frac{1}{4}\,(h_p\sin(\theta_p}} \\[2ex]
\dfrac{k_1\left(-\frac{1}{2}h_p\cos(\theta_p(t))+\frac{1}{2}l_p\sin(\theta_p(t))-y_p(t)+y_1(t)\right)}{\sqrt{\frac{1}{4}\,(h_p\sin(\theta_p(t))+l_{}}} \\[2ex]
i_{p,\text{zz}}\,\theta_p''(t)+\dfrac{k_1\,(h_p\,(x_1(t)\cos(\theta_p(t))-x_p(t)\cos(\theta_p(t))+(y_1(t)-y_p(t))\sin(\theta_p(t)))+l_p\,(x_1(t)\,(-\sin(\theta_p(t)))+x_p(t)\sin(\theta_p(t))+(y_1(}{2\sqrt{\frac{1}{4}\,(h_p\sin(\theta_p(t))+l_p\cos(\theta_p(t))-2\,x_p(}}
\end{array}\right.$$

```
terms2 = {
   (Sin[θp[t]] hp + Cos[θp[t]] lp + 2 x1[t] - 2 xp[t])^1 → (2 r1x),
   (-1/2 Cos[θp[t]] hp + 1/2 Sin[θp[t]] lp + y1[t] - yp[t])^1 → r1y,
   1/2 Cos[θp[t]] hp - 1/2 Sin[θp[t]] lp - y1[t] + yp[t] → -r1y,
   (1/2 Sin[θp[t]] hp - 1/2 Cos[θp[t]] lp + x2[t] - xp[t])^1 → r2x,
   (-1/2 Cos[θp[t]] hp - 1/2 Sin[θp[t]] lp + y2[t] - yp[t])^1 → r2y,
   Cos[θp[t]] hp + Sin[θp[t]] lp - 2 y2[t] + 2 yp[t] → (-2 r2y),
   lp (-Sin[θp[t]] x1[t] + Sin[θp[t]] xp[t] + Cos[θp[t]] (y1[t] - yp[t])) → dr1,
   hp (Cos[θp[t]] x1[t] - Cos[θp[t]] xp[t] + Sin[θp[t]] (y1[t] - yp[t])) → dr2,
   hp (Cos[θp[t]] x2[t] - Cos[θp[t]] xp[t] + Sin[θp[t]] (y2[t] - yp[t])) → dr4,
   lp (Sin[θp[t]] x2[t] - Sin[θp[t]] xp[t] + Cos[θp[t]] (-y2[t] + yp[t])) → dr3
  };
```

```
(simpStep1 =
    (quadEqNominal(*//Simplify*)) /. terms2)
   (*//.dispSimp*)(*//Simplify*) //
  MatrixForm(*//TraditionalForm*)
```

$$\left(\begin{array}{l}
\frac{r1x\,k_1\left(\sqrt{r1x^2+r1y^2}-L0_1\right)}{\sqrt{r1x^2+r1y^2}}+\frac{r2x\,k_2\left(\sqrt{r2x^2+r2y^2}-L0_2\right)}{\sqrt{r2x^2+r2y^2}} == \mathrm{I} \\
\frac{r1y\,k_1\left(\sqrt{r1x^2+r1y^2}-L0_1\right)}{\sqrt{r1x^2+r1y^2}}+\frac{r2y\,k_2\left(\sqrt{r2x^2+r2y^2}-L0_2\right)}{\sqrt{r2x^2+r2y^2}} == m_p \\
\frac{(dr1+dr2)\,k_1\left(\sqrt{r1x^2+r1y^2}-L0_1\right)}{2\sqrt{r1x^2+r1y^2}}+\frac{(dr3+dr4)\,k_2\left(\sqrt{r2x^2+r2y^2}-L0_2\right)}{2\sqrt{r2x^2+r2y^2}} \;-
\end{array}\right.$$

```
terms3 = {
  Sqrt[r1x^2 + r1y^2] -> a,
  Sqrt[r2x^2 + r2y^2] -> b,
  (dr1 + dr2) -> (2 c1),
  (dr3 + dr4) -> (2 c2),
  r1x^2 + r1y^2 -> a^2, r2x^2 + r2y^2 -> b^2,
  Sqrt[a^2] -> a, Sqrt[b^2] -> b}
```

$$\left\{\sqrt{r1x^2+r1y^2}\to a,\ \sqrt{r2x^2+r2y^2}\to b,\ dr1+dr2\to 2\,c1,\ dr3+dr4\to 2\,c2,\ r1x^2+r1y^2\to a^2,\ r2x^2+r2y^2\to b^2,\ \sqrt{a^2}\to a,\ \sqrt{b^2}\to b\right\}$$

```
(*simpStep1//InputForm*)

(*simpStep1//TreeForm*)

(simpStep2 =
    (simpStep1 //. terms3) // Simplify) //
  MatrixForm(*//TraditionalForm*)
```

$$\begin{pmatrix} \frac{r1x\,k_1\,(a-L0_1)}{\sqrt{a^2}}+\frac{r2x\,k_2\,(b-L0_2)}{\sqrt{b^2}} == m_p\,x_p''[t] \\ \frac{r1y\,k_1\,(a-L0_1)}{\sqrt{a^2}}+\frac{r2y\,k_2\,(b-L0_2)}{\sqrt{b^2}} == m_p\,(g+y_p''[t]) \\ \frac{c1\,k_1\,(a-L0_1)}{\sqrt{a^2}}+\frac{c2\,k_2\,(b-L0_2)}{\sqrt{b^2}}+\mathbb{i}_{p,zz}\,\theta_p''[t] == 0 \end{pmatrix}$$

```
(simpStep3 =
   Map[Map[Times[#, a b ] &, #] &, simpStep2] // Expand //
    Simplify) // MatrixForm
```

$$\begin{pmatrix} \frac{\sqrt{a^2}\,b^2\,r1x\,k_1\,(a-L0_1)+a^2\,\sqrt{b^2}\,r2x\,k_2\,(b-L0_2)}{a\,b} == a\,b\,m_p\,x_p''[t] \\ \frac{\sqrt{a^2}\,b^2\,r1y\,k_1\,(a-L0_1)+a^2\,\sqrt{b^2}\,r2y\,k_2\,(b-L0_2)}{a\,b} == a\,b\,m_p\,(g+y_p''[t]) \\ \frac{\sqrt{a^2}\,b^2\,c1\,k_1\,(a-L0_1)+a^2\left(\sqrt{b^2}\,c2\,k_2\,(b-L0_2)+b^2\,\mathbb{i}_{p,zz}\,\theta_p''[t]\right)}{a\,b} == 0 \end{pmatrix}$$

```
simpStep3 //. dispSimp //
   Expand // MatrixForm //
  TraditionalForm
```

$$\begin{pmatrix} -\frac{\sqrt{a^2}\, b\, k_1\, \mathrm{L0}_1\, \mathrm{r1x}}{a} + \sqrt{a^2}\, b\, k_1\, \mathrm{r1x} - \frac{a\sqrt{b^2} \\ -\frac{\sqrt{a^2}\, b\, k_1\, \mathrm{L0}_1\, \mathrm{r1y}}{a} + \sqrt{a^2}\, b\, k_1\, \mathrm{r1y} - \frac{a\sqrt{b^2}\, k_2\, \mathrm{L0}_2\, \mathrm{r}}{b} \\ -\frac{\sqrt{a^2}\, b\, \mathrm{c1}\, k_1\, \mathrm{L0}_1}{a} + \sqrt{a^2}\, b\, \mathrm{c1}\, k_1 - \frac{a\sqrt{b^2}\, \mathrm{c}}{b} \end{pmatrix}$$

$$\left(\mathrm{simpStep4} = k_1\, b\, (a - \mathrm{L0}_1) \begin{pmatrix} \mathrm{r1x} \\ \mathrm{r1y} \\ \mathrm{c1} \end{pmatrix} + k_2\, a\, (b - \mathrm{L0}_1) \begin{pmatrix} \mathrm{r2x} \\ \mathrm{r2y} \\ \mathrm{c2} \end{pmatrix} + \begin{pmatrix} 0 \\ -m_p\, g\, a\, b \\ 0 \end{pmatrix} - a\, b \begin{pmatrix} m_p & 0 & 0 \\ 0 & m_p & 0 \\ 0 & 0 & -I_p \end{pmatrix} . \begin{pmatrix} x_p''[t] \\ y_p''[t] \\ \theta_p''[t] \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}\right) \;//\; \mathrm{MatrixForm}$$

$$\begin{pmatrix} b\, \mathrm{r1x}\, k_1\, (a - \mathrm{L0}_1) + a\, \mathrm{r2x}\, k_2\, (b - \mathrm{L0}_1) - a\, b\, m_p\, x_p''[t] \\ b\, \mathrm{r1y}\, k_1\, (a - \mathrm{L0}_1) + a\, \mathrm{r2y}\, k_2\, (b - \mathrm{L0}_1) - a\, b\, g\, m_p - a\, b\, m_p\, y_p''[t] \\ b\, \mathrm{c1}\, k_1\, (a - \mathrm{L0}_1) + a\, \mathrm{c2}\, k_2\, (b - \mathrm{L0}_1) + a\, b\, i_p\, \theta_p''[t] \end{pmatrix}$$

```
terms2 //. dispSimp // MatrixForm // TraditionalForm
terms3 //. dispSimp // MatrixForm // TraditionalForm
```

$$\begin{pmatrix} l_p\, c(\theta_p) + h_p\, s(\theta_p) - 2\, x_p + 2\, x_1 \to 2\, \mathrm{r1x} \\ -\frac{1}{2}\, h_p\, c(\theta_p) + \frac{1}{2}\, l_p\, s(\theta_p) - y_p + y_1 \to \mathrm{r1y} \\ \frac{1}{2}\, h_p\, c(\theta_p) - \frac{1}{2}\, l_p\, s(\theta_p) + y_p - y_1 \to -\mathrm{r1y} \\ -\frac{1}{2}\, l_p\, c(\theta_p) + \frac{1}{2}\, h_p\, s(\theta_p) - x_p + x_2 \to \mathrm{r2x} \\ -\frac{1}{2}\, h_p\, c(\theta_p) - \frac{1}{2}\, l_p\, s(\theta_p) - y_p + y_2 \to \mathrm{r2y} \\ h_p\, c(\theta_p) + l_p\, s(\theta_p) + 2\, y_p - 2\, y_2 \to -2\, \mathrm{r2y} \\ l_p\, ((y_1 - y_p)\, c(\theta_p) + x_1\, (-s(\theta_p)) + x_p\, s(\theta_p)) \to \mathrm{dr1} \\ h_p\, (x_1\, c(\theta_p) - x_p\, c(\theta_p) + (y_1 - y_p)\, s(\theta_p)) \to \mathrm{dr2} \\ h_p\, (x_2\, c(\theta_p) - x_p\, c(\theta_p) + (y_2 - y_p)\, s(\theta_p)) \to \mathrm{dr4} \\ l_p\, ((y_p - y_2)\, c(\theta_p) + x_2\, s(\theta_p) - x_p\, s(\theta_p)) \to \mathrm{dr3} \end{pmatrix}$$

$$\begin{pmatrix} \sqrt{\mathrm{r1x}^2 + \mathrm{r1y}^2} \to a \\ \sqrt{\mathrm{r2x}^2 + \mathrm{r2y}^2} \to b \\ \mathrm{dr1} + \mathrm{dr2} \to 2\, \mathrm{c1} \\ \mathrm{dr3} + \mathrm{dr4} \to 2\, \mathrm{c2} \\ \mathrm{r1x}^2 + \mathrm{r1y}^2 \to a^2 \\ \mathrm{r2x}^2 + \mathrm{r2y}^2 \to b^2 \\ \sqrt{a^2} \to a \\ \sqrt{b^2} \to b \end{pmatrix}$$

(* x_p, y_p, θ_p = f (x_1, y_1, x_2, y_2, k_1, k_2, l_p, h_p) *)

---

non - conver forces :

aerodynamic = f ($\dot{x}_p$, $\dot{y}_p$, θ_p, w_x, w_y) ,

w for wind components. = f (relV_x, relV_y) , relV is relative to air

dumping = f ($\dot{l}_i$) = f ($\dot{x}_i$, $\dot{y}_i$, $\dot{x}_p$, $\dot{y}_p$)

---

non dim the full equations

```
quadEqNominal /. terms2
 (*/.terms3*) // MatrixForm
```

$$\left(\begin{array}{l} \frac{\text{r1x}\,k_1\left(\sqrt{\text{r1x}^2+\text{r1y}^2}-\text{L0}_1\right)}{\sqrt{\text{r1x}^2+\text{r1y}^2}}+\frac{\text{r2x}\,k_2\left(\sqrt{\text{r2x}^2+\text{r2y}^2}-\text{L0}_2\right)}{\sqrt{\text{r2x}^2+\text{r2y}^2}} == \text{I} \\ \frac{\text{r1y}\,k_1\left(\sqrt{\text{r1x}^2+\text{r1y}^2}-\text{L0}_1\right)}{\sqrt{\text{r1x}^2+\text{r1y}^2}}+\frac{\text{r2y}\,k_2\left(\sqrt{\text{r2x}^2+\text{r2y}^2}-\text{L0}_2\right)}{\sqrt{\text{r2x}^2+\text{r2y}^2}} == m_p \\ \frac{(\text{dr1}+\text{dr2})\,k_1\left(\sqrt{\text{r1x}^2+\text{r1y}^2}-\text{L0}_1\right)}{2\sqrt{\text{r1x}^2+\text{r1y}^2}}+\frac{(\text{dr3}+\text{dr4})\,k_2\left(\sqrt{\text{r2x}^2+\text{r2y}^2}-\text{L0}_2\right)}{2\sqrt{\text{r2x}^2+\text{r2y}^2}} \end{array}\right.$$

```
(smallEqs =
    quadEqNominal /. terms2 /.
     terms3) // MatrixForm
```

$$\left(\begin{array}{l} \frac{\text{r1x}\,k_1\,(a-\text{L0}_1)}{\sqrt{a^2}}+\frac{\text{r2x}\,k_2\,(b-\text{L0}_2)}{\sqrt{b^2}} == m_p\ \text{X} \\ \frac{\text{r1y}\,k_1\,(a-\text{L0}_1)}{\sqrt{a^2}}+\frac{\text{r2y}\,k_2\,(b-\text{L0}_2)}{\sqrt{b^2}} == m_p\ (g + \\ \frac{\text{c1}\,k_1\,(a-\text{L0}_1)}{\sqrt{a^2}}+\frac{\text{c2}\,k_2\,(b-\text{L0}_2)}{\sqrt{b^2}}+\mathbb{I}_{p,zz}\,\theta_p{}' \end{array}\right.$$

```
(*(NonDimEq=Map[Map[Times[#, 1/(mp ωs^2 L01)]&,#]&,
     (*simpStep1*)smallEqs](*//Expand*)//
    FullSimplify)//MatrixForm*)
```

$$\text{(*}\left(\text{NonDimEq=Map}\left[\text{Map}\left[\text{Times}\left[\#,\frac{1}{m_p\omega_s{}^2\text{L0}_1}\right]\&,\#\right]\&,\ \text{(*simpStep1*)smallEqs}\right]\text{(*//Expand*)//FullSimplify}\right)\text{//MatrixForm*)}$$

NonDimEq manually settings the terms:

$\tilde{y_p}[t] = y_p[t] \,/ L0_1$ or any other of the lengths varialbes $(x_p, r1x, r1y, r2x, r2y, h_p, l_p)$

$t = \tau / \omega_s$

$\omega_s{}^2 = \frac{k_1}{m_p}\left[\frac{g}{l} = \frac{1}{s^2}\right]$

A is non - dimentional form of ' a '

B is non - dimentional form of ' b '

```
(NonDimEq = {k1/mp (1 - 1/A L01/L01) r1x L01 +
      k2/k1 k1/mp (1 - 1/B L02/L01) r2x L01 ==
     L01 ωs^2 xp''[t], k1/mp (1 - 1/A L01/L01) r1y L01 +
      k2/k1 k1/mp (1 - 1/B L02/L01) r2y L01 -
      g == L01 ωs^2 yp''[t],
     k1/(-ip,zz) (1 - 1/A L01/L01) c1 L01^2 + k2/k1 k1/(-ip,zz)
       (1 - 1/B L02/L01) c2 L01^2 == ωs^2 θp''[t]}) //
  Flatten // MatrixForm //
 TraditionalForm
```

$$\begin{pmatrix} \frac{\left(1-\frac{1}{A}\right) k_1 \, \mathrm{L0}_1 \, \mathrm{r1x}}{m_p} + \frac{k_2 \, \mathrm{L0}_1 \, \mathrm{r2x} \left(1-\frac{\mathrm{L0}_2}{B\,\mathrm{L0}_1}\right)}{m_p} = \mathrm{L0}_1 \, \omega_s^2 \, x_p''(t) \\ \frac{\left(1-\frac{1}{A}\right) k_1 \, \mathrm{L0}_1 \, \mathrm{r1y}}{m_p} + \frac{k_2 \, \mathrm{L0}_1 \, \mathrm{r2y} \left(1-\frac{\mathrm{L0}_2}{B\,\mathrm{L0}_1}\right)}{m_p} - g = \mathrm{L0}_1 \, \omega_s^2 \, y_p''(t) \\ -\frac{\left(1-\frac{1}{A}\right) c_1 \, k_1 \, \mathrm{L0}_1^2}{i_{p,zz}} - \frac{c_2 \, k_2 \, \mathrm{L0}_1^2 \left(1-\frac{\mathrm{L0}_2}{B\,\mathrm{L0}_1}\right)}{i_{p,zz}} = \omega_s^2 \, \theta_p''(t) \end{pmatrix}$$

```
(*terms4={(*1- LO_1/a →A,
  1- LO_2/b →B,*)
  k_2/k_1 →k,(*=k_ratio,*)
  g/LO_1 1/ω_s^2 →γ,
  LO_1^2 k_1/(I_p,zz ω_s^2) →E,
  k_1/m_p -> ω_s^2
 }*)

(*Delta_Equilibrium = m_p g/k_i *)

(* g/LO_1 1/ω_s^2 == g 1/LO_1 1/k_1 m_p [ m/s^2 1/m 1/(kg/s^2) kg ]
  LO_1^2 k_1/(I_p,zz ω_s^2) == LO_1^2 k_1/(I_p,zz k_1) m_p [ m^2 kg/(kg m^2) ] *)
```

$$\texttt{greekTerms} = \left\{ \frac{k_2}{k_1} \to \kappa,\ \frac{LO_2}{LO_1} \to \mathcal{L},\ \frac{k_1}{m_p} \text{->} \omega_s{}^2,\ \frac{m_p\, LO_1{}^2}{I_p} \left(= \frac{LO_1{}^2\, k_1}{I_p\, \omega_s{}^2} \right) \to \alpha,\ \frac{g}{LO_1\, \omega_s^2} \left(= \frac{g\, m_p}{LO_1\, k_1} \right) \to \gamma \right\}$$

$$\left(\texttt{NonDimEq} = \left\{ \begin{array}{l} \omega_s^2\, LO_1 \left(1 - \frac{1}{A}\right) \mathrm{r1x} + \kappa\, \omega_s^2\, LO_1 \left(1 - \frac{1}{B} \mathcal{L}\right) \mathrm{r2x} == LO_1\, \omega_s^2\, x_p''[t], \\ \omega_s^2\, LO_1 \left(1 - \frac{1}{A}\right) \mathrm{r1y} + \kappa\, \omega_s^2\, LO_1 \left(1 - \frac{1}{B} \mathcal{L}\right) \mathrm{r2y} - \frac{g}{LO_1\, \omega_s^2} LO_1\, \omega_s^2 == LO_1\, \omega_s^2\, y_p''[t], \\ \frac{k_1}{-i_{p,zz}} \frac{LO_1{}^2}{\omega_s^2} \omega_s^2 \left(1 - \frac{1}{A}\right) c_1 + \kappa \frac{k_1}{-i_{p,zz}} \frac{LO_1{}^2}{\omega_s^2} \omega_s^2 \left(1 - \frac{1}{B} \mathcal{L}\right) c_2 == \omega_s^2\, \theta_p''[t] \end{array} \right\}\right) \texttt{ // Flatten // MatrixForm // TraditionalForm}$$

$$\begin{pmatrix} \left(1 - \frac{1}{A}\right) LO_1\, \mathrm{r1x}\, \omega_s^2 + \kappa\, LO_1\, \mathrm{r2x} \left(1 - \frac{\mathcal{L}}{B}\right) \omega_s^2 = LO_1\, \omega_s^2\, x_p''(t) \\ \left(1 - \frac{1}{A}\right) LO_1\, \mathrm{r1y}\, \omega_s^2 + \kappa\, LO_1\, \mathrm{r2y} \left(1 - \frac{\mathcal{L}}{B}\right) \omega_s^2 - g = LO_1\, \omega_s^2\, y_p''(t) \\ -\frac{\left(1 - \frac{1}{A}\right) c_1\, k_1\, LO_1^2}{i_{p,zz}} - \frac{c_2\, \kappa\, k_1\, LO_1^2 \left(1 - \frac{\mathcal{L}}{B}\right)}{i_{p,zz}} = \omega_s^2\, \theta_p''(t) \end{pmatrix}$$

using ' greekTerms ' list :

```
(NonDimEq = {
    x_p''[t] == (1 - 1/A) r1x + κ (1 - 1/B ℒ) r2x ,
    y_p''[t] == (1 - 1/A) r1y + κ (1 - 1/B ℒ) r2y - γ,
    θ_p''[t] == -α ((1 - 1/A) c1 + κ (1 - 1/B ℒ) c2)
  }) // Flatten // MatrixForm // TraditionalForm
```

$$\begin{pmatrix} x_p''(t) = \left(1 - \frac{1}{A}\right) \mathrm{r1x} + \kappa\, \mathrm{r2x} \left(1 - \frac{\mathcal{L}}{B}\right) \\ y_p''(t) = \left(1 - \frac{1}{A}\right) \mathrm{r1y} + \kappa\, \mathrm{r2y} \left(1 - \frac{\mathcal{L}}{B}\right) - \gamma \\ \theta_p''(t) = -\alpha \left(\left(1 - \frac{1}{A}\right) c_1 + c_2\, \kappa \left(1 - \frac{\mathcal{L}}{B}\right)\right) \end{pmatrix}$$

$$\mathcal{V}_1 = \begin{pmatrix} \mathrm{r1x} \\ \mathrm{r1y} \\ c_1 \end{pmatrix} \quad (x_1, x_p, \theta_p, ..)$$

$$\mathcal{V}_2 = \begin{pmatrix} \mathrm{r2x} \\ \mathrm{r2y} \\ c_2 \end{pmatrix}$$

$$\ddot{\mathcal{X}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\alpha \end{pmatrix} \left(1 - \frac{1}{A}\right) \mathcal{V}_1 + \kappa \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\alpha \end{pmatrix} \left(1 - \frac{1}{B} \mathcal{L}\right) \mathcal{V}_2 - \begin{pmatrix} 0 \\ \gamma \\ 0 \end{pmatrix}$$

```
terms2
terms3
```

$$\{\mathrm{Sin}[\theta_p[t]]\,h_p + \mathrm{Cos}[\theta_p[t]]\,l_p + 2\,x_1[t] - 2\,x_p[t] \to 2\,r1x,$$
$$-\frac{1}{2}\mathrm{Cos}[\theta_p[t]]\,h_p + \frac{1}{2}\mathrm{Sin}[\theta_p[t]]\,l_p + y_1[t] - y_p[t] \to r1y,$$
$$\frac{1}{2}\mathrm{Cos}[\theta_p[t]]\,h_p - \frac{1}{2}\mathrm{Sin}[\theta_p[t]]\,l_p - y_1[t] + y_p[t] \to -r1y,$$
$$\frac{1}{2}\mathrm{Sin}[\theta_p[t]]\,h_p - \frac{1}{2}\mathrm{Cos}[\theta_p[t]]\,l_p + x_2[t] - x_p[t] \to r2x,$$
$$-\frac{1}{2}\mathrm{Cos}[\theta_p[t]]\,h_p - \frac{1}{2}\mathrm{Sin}[\theta_p[t]]\,l_p + y_2[t] - y_p[t] \to r2y,$$
$$\mathrm{Cos}[\theta_p[t]]\,h_p + \mathrm{Sin}[\theta_p[t]]\,l_p - 2\,y_2[t] + 2\,y_p[t] \to -2\,r2y,$$
$$l_p\,(-\mathrm{Sin}[\theta_p[t]]\,x_1[t] + \mathrm{Sin}[\theta_p[t]]\,x_p[t] + \mathrm{Cos}[\theta_p[t]]\,(y_1[t] - y_p[t])) \to dr1,$$
$$h_p\,(\mathrm{Cos}[\theta_p[t]]\,x_1[t] - \mathrm{Cos}[\theta_p[t]]\,x_p[t] + \mathrm{Sin}[\theta_p[t]]\,(y_1[t] - y_p[t])) \to dr2,$$
$$h_p\,(\mathrm{Cos}[\theta_p[t]]\,x_2[t] - \mathrm{Cos}[\theta_p[t]]\,x_p[t] + \mathrm{Sin}[\theta_p[t]]\,(y_2[t] - y_p[t])) \to dr4,$$
$$l_p\,(\mathrm{Sin}[\theta_p[t]]\,x_2[t] - \mathrm{Sin}[\theta_p[t]]\,x_p[t] + \mathrm{Cos}[\theta_p[t]]\,(-y_2[t] + y_p[t])) \to dr3\}$$

$$\{\sqrt{r1x^2 + r1y^2} \to a,\ \sqrt{r2x^2 + r2y^2} \to b,\ dr1 + dr2 \to 2\,c1,$$
$$dr3 + dr4 \to 2\,c2,\ r1x^2 + r1y^2 \to a^2,\ r2x^2 + r2y^2 \to b^2,\ \sqrt{a^2} \to a,\ \sqrt{b^2} \to b\}$$

In[1]:= $\mathcal{X} = \begin{pmatrix} x_p[t] \\ y_p[t] \\ \theta_p[t] \end{pmatrix}$ (*//Flatten*)

greekTermsSymetricCase = {

$(*\frac{k_2}{k_1}\to*)\ \kappa \to 1,$

$(*\frac{L0_2}{L0_1}\to*)\ \mathcal{L} \to 1$

}

greekTermsGeneral = {

$(*\frac{k_2}{k_1}\to*)\ \kappa \to 1,$

$(*\frac{L0_2}{L0_1}\to*)\ \mathcal{L} \to 1,$

$(*\frac{k_1}{m_p}{-}{>}*)\ \omega_s{}^2 \to 1,$

$(*\frac{m_p L0_1{}^2}{I_p}\left(=\frac{L0_1{}^2 k_1}{I_p \omega_s{}^2}\right)\to*)\ \alpha \to 1,$

$(*\frac{q}{L0_1\ \omega_s^2}\left(=\frac{q\ m_p}{L0_1 k_1}\right)\to\ *)\ \gamma \to 1$ (* make sure it is not over-determined constant *)

}

(* already here : replacing all former h_p,l_p with new $2h_p$,$2l_p$*)

$$A\,(*\to\sqrt{r1x^2{+}r1y^2}\ *) = \sqrt{\left((\mathrm{Sin}[\theta_p[t]]\,h_p + \mathrm{Cos}[\theta_p[t]]\,l_p + (x_1[t] - x_p[t]))^2 + (-\,\mathrm{Cos}[\theta_p[t]]\,h_p + \mathrm{Sin}[\theta_p[t]]\,l_p + (y_1[t] - y_p[t]))^2\right)}$$

$$B\,(*\to\sqrt{r2x^2{+}r2y^2}\ *) = \sqrt{\left((\mathrm{Sin}[\theta_p[t]]\,h_p - \mathrm{Cos}[\theta_p[t]]\,l_p + (x_2[t] - x_p[t]))^2 + (-\,\mathrm{Cos}[\theta_p[t]]\,h_p - \mathrm{Sin}[\theta_p[t]]\,l_p + (y_2[t] - y_p[t]))^2\right)}$$

```
(*c1(*→dr1+dr2*)=lp (-Sin[θp[t]] (x1[t]- xp[t])+Cos[θp[t]] (y1[t]-yp[t]))+
   hp (Cos[θp[t]]( x1[t]- xp[t])+Sin[θp[t]] (y1[t]-yp[t]))
    c2(*→dr3+dr4*)=lp (Sin[θp[t]] (x2[t]- xp[t])+Cos[θp[t]] (-y2[t]+yp[t]))+
   hp (Cos[θp[t]]( x2[t]- xp[t])+Sin[θp[t]] (y2[t]-yp[t]))*)
```

$\mathcal{V}_1$ (*= $\begin{pmatrix} r1x \\ r1y \\ c_1 \end{pmatrix}$ *) =

$$\begin{pmatrix} (\mathrm{Sin}[\theta_p[t]]\, h_p + \mathrm{Cos}[\theta_p[t]]\, l_p + (x_1[t] - x_p[t])) \\ (-\mathrm{Cos}[\theta_p[t]]\, h_p + \mathrm{Sin}[\theta_p[t]]\, l_p + (y_1[t] - y_p[t])) \\ l_p\, (-\mathrm{Sin}[\theta_p[t]]\, (x_1[t] - x_p[t]) + \mathrm{Cos}[\theta_p[t]]\, (y_1[t] - y_p[t])) + h_p\, (\mathrm{Cos}[\theta_p[t]]\, (x_1[t] - x_p[\end{pmatrix}$$

$\mathcal{V}_2$ (*= $\begin{pmatrix} r2x \\ r2y \\ c_2 \end{pmatrix}$ *) =

$$\begin{pmatrix} (\mathrm{Sin}[\theta_p[t]]\, h_p - \mathrm{Cos}[\theta_p[t]]\, l_p + (x_2[t] - x_p[t])) \\ (-\mathrm{Cos}[\theta_p[t]]\, h_p - \mathrm{Sin}[\theta_p[t]]\, l_p + (y_2[t] - y_p[t])) \\ l_p\, (\mathrm{Sin}[\theta_p[t]]\, (x_2[t] - x_p[t]) + \mathrm{Cos}[\theta_p[t]]\, (-y_2[t] + y_p[t])) + h_p\, (\mathrm{Cos}[\theta_p[t]]\, (x_2[t] - x_p[\end{pmatrix}$$

"equations with no general forces :"

EOM =

$$D[\mathcal{X}, \{t, 2\}] == \left(\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\alpha \end{pmatrix} \left(1 - \frac{1}{A}\right)\right).\mathcal{V}_1 + \left(\kappa \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\alpha \end{pmatrix} \left(1 - \frac{1}{B}\mathcal{L}\right)\right).\mathcal{V}_2 - \begin{pmatrix} 0 \\ \gamma \\ 0 \end{pmatrix} \text{ // Flatten;}$$

Out[1]= $\{\{x_p[t]\}, \{y_p[t]\}, \{\theta_p[t]\}\}$

Out[2]= $\{\kappa \to 1, \mathcal{L} \to 1\}$

Out[3]= $\{\kappa \to 1, \mathcal{L} \to 1, \omega_s^2 \to 1, \alpha \to 1, \gamma \to 1\}$

Out[4]= $\sqrt{\left((\mathrm{Sin}[\theta_p[t]]\, h_p + \mathrm{Cos}[\theta_p[t]]\, l_p + x_1[t] - x_p[t])^2 + (-\mathrm{Cos}[\theta_p[t]]\, h_p + \mathrm{Sin}[\theta_p[t]]\, l_p + y_1[t] - y_p[t])^2\right)}$

Out[5]= $\sqrt{\left((\mathrm{Sin}[\theta_p[t]]\, h_p - \mathrm{Cos}[\theta_p[t]]\, l_p + x_2[t] - x_p[t])^2 + (-\mathrm{Cos}[\theta_p[t]]\, h_p - \mathrm{Sin}[\theta_p[t]]\, l_p + y_2[t] - y_p[t])^2\right)}$

Out[6]= $\{\{\mathrm{Sin}[\theta_p[t]]\, h_p + \mathrm{Cos}[\theta_p[t]]\, l_p + x_1[t] - x_p[t]\},$
$\{-\mathrm{Cos}[\theta_p[t]]\, h_p + \mathrm{Sin}[\theta_p[t]]\, l_p + y_1[t] - y_p[t]\},$
$\{l_p\, (-\mathrm{Sin}[\theta_p[t]]\, (x_1[t] - x_p[t]) + \mathrm{Cos}[\theta_p[t]]\, (y_1[t] - y_p[t])) +$
$h_p\, (\mathrm{Cos}[\theta_p[t]]\, (x_1[t] - x_p[t]) + \mathrm{Sin}[\theta_p[t]]\, (y_1[t] - y_p[t]))\}\}$

Out[7]= $\{\{\mathrm{Sin}[\theta_p[t]]\, h_p - \mathrm{Cos}[\theta_p[t]]\, l_p + x_2[t] - x_p[t]\},$
$\{-\mathrm{Cos}[\theta_p[t]]\, h_p - \mathrm{Sin}[\theta_p[t]]\, l_p + y_2[t] - y_p[t]\},$
$\{h_p\, (\mathrm{Cos}[\theta_p[t]]\, (x_2[t] - x_p[t]) + \mathrm{Sin}[\theta_p[t]]\, (y_2[t] - y_p[t])) +$
$l_p\, (\mathrm{Sin}[\theta_p[t]]\, (x_2[t] - x_p[t]) + \mathrm{Cos}[\theta_p[t]]\, (-y_2[t] + y_p[t]))\}\}$

Out[8]= equations with no general forces :

In[24]:=
```
nameChange = {lp → wp, a_[t] → a};
EOM /. nameChange /. greekTermsSymetricCase // Flatten
(*//MatrixForm*)(*//TraditionalForm*)
EOM /. nameChange /. greekTermsSymetricCase // Flatten
 (*//MatrixForm*) // TraditionalForm
```

Out[25]=
$$\{\{x_p''\},\{y_p''\},\{\theta_p''\}\} == \Big\{\Big\{(\mathrm{Sin}[\theta_p]\,h_p + \mathrm{Cos}[\theta_p]\,w_p + x_1 - x_p)\left(1 - \frac{1}{\sqrt{(\mathrm{Sin}[\theta_p]\,h_p + \mathrm{Cos}[\theta_p]\,w_p + x_1 - x_p)^2 + (-\mathrm{Cos}[\theta_p]\,h_p + \mathrm{Sin}[\theta_p]\,w_p + y_1 - y_p)^2}}\right) + (\mathrm{Sin}[\theta_p]\,h_p - \mathrm{Cos}[\theta_p]\,w_p + x_2 - x_p)\left(1 - \frac{1}{\sqrt{(\mathrm{Sin}[\theta_p]\,h_p - \mathrm{Cos}[\theta_p]\,w_p + x_2 - x_p)^2 + (-\mathrm{Cos}[\theta_p]\,h_p - \mathrm{Sin}[\theta_p]\,w_p + y_2 - y_p)^2}}\right)\Big\},$$
$$\Big\{-\gamma + \left(1 - \frac{1}{\sqrt{(\mathrm{Sin}[\theta_p]\,h_p + \mathrm{Cos}[\theta_p]\,w_p + x_1 - x_p)^2 + (-\mathrm{Cos}[\theta_p]\,h_p + \mathrm{Sin}[\theta_p]\,w_p + y_1 - y_p)^2}}\right)(-\mathrm{Cos}[\theta_p]\,h_p + \mathrm{Sin}[\theta_p]\,w_p + y_1 - y_p) + \left(1 - \frac{1}{\sqrt{(\mathrm{Sin}[\theta_p]\,h_p - \mathrm{Cos}[\theta_p]\,w_p + x_2 - x_p)^2 + (-\mathrm{Cos}[\theta_p]\,h_p - \mathrm{Sin}[\theta_p]\,w_p + y_2 - y_p)^2}}\right)(-\mathrm{Cos}[\theta_p]\,h_p - \mathrm{Sin}[\theta_p]\,w_p + y_2 - y_p)\Big\},$$
$$\Big\{-\alpha\,(w_p\,(-\mathrm{Sin}[\theta_p]\,(x_1 - x_p) + \mathrm{Cos}[\theta_p]\,(y_1 - y_p)) + h_p\,(\mathrm{Cos}[\theta_p]\,(x_1 - x_p) + \mathrm{Sin}[\theta_p]\,(y_1 - y_p)))\left(1 - \frac{1}{\sqrt{(\mathrm{Sin}[\theta_p]\,h_p + \mathrm{Cos}[\theta_p]\,w_p + x_1 - x_p)^2 + (-\mathrm{Cos}[\theta_p]\,h_p + \mathrm{Sin}[\theta_p]\,w_p + y_1 - y_p)^2}}\right) - \alpha\left(1 - \frac{1}{\sqrt{(\mathrm{Sin}[\theta_p]\,h_p - \mathrm{Cos}[\theta_p]\,w_p + x_2 - x_p)^2 + (-\mathrm{Cos}[\theta_p]\,h_p - \mathrm{Sin}[\theta_p]\,w_p + y_2 - y_p)^2}}\right)(h_p\,(\mathrm{Cos}[\theta_p]\,(x_2 - x_p) + \mathrm{Sin}[\theta_p]\,(y_2 - y_p)) + w_p\,(\mathrm{Sin}[\theta_p]\,(x_2 - x_p) + \mathrm{Cos}[\theta_p]\,(-y_2 + y_p)))\Big\}\Big\}$$

Out[26]//TraditionalForm=
$$\begin{pmatrix} x_p'' \\ y_p'' \\ \theta_p'' \end{pmatrix} = \left(\begin{array}{l} (\sin(\theta_p)\,h_p + \cos(\theta_p)\,w_p + x_1 - x_p)\left(1 - \frac{\quad}{\sqrt{(\sin(\theta_p)\,h_p + \cos(\theta_p)\,w_p + x_1 - x_p}}\right. \\ -\gamma + \left(1 - \frac{1}{\sqrt{(\sin(\theta_p)\,h_p + \cos(\theta_p)\,w_p + x_1 - x_p)^2 + (-\cos(\theta_p)\,h_p + \sin(\theta_p)\,w_p + y_1 - y_p)^2}}\right)(-\mathrm{cc} \\ -\alpha\,(w_p\,(\cos(\theta_p)\,(y_1 - y_p) - \sin(\theta_p)\,(x_1 - x_p)) + h_p\,(\cos(\theta_p)\,(x_1 - x_p) + \sin(\theta_p)\,(y_1 - y_p)))\left(1 - \frac{\quad}{\sqrt{(\sin(\theta_p)\,h_p + \cos(\theta_p)\,w_p + x_1 - x}}\right. \end{array} \right.$$

```
"set derivatives to zero: "
(equibTerms = {Map[Rule[#, 0 ] &, D[𝒳 // Flatten, {t, 1}]],
     Map[Rule[#, 0 ] &, D[𝒳 // Flatten, {t, 2}]]} // Flatten)
  (*//MatrixForm*) // TraditionalForm
```

set derivatives to zero:

$$\{x_p'(t)\to 0,\ y_p'(t)\to 0,\ \theta_p'(t)\to 0,\ x_p''(t)\to 0,\ y_p''(t)\to 0,\ \theta_p''(t)\to 0\}$$

```
EquibInputConditions = {x1[t] → 0, y1[t] → 0, x2[t] → 2 wp, y2[t] → y1[t]}
```

$$\{x_1[t]\to 0,\ y_1[t]\to 0,\ x_2[t]\to 2\,w_p,\ y_2[t]\to y_1[t]\}$$

```
(SymetricEquib = EOM /. nameChange /. greekTermsSymetricCase /. equibTerms //.
     EquibInputConditions) // MatrixForm // TraditionalForm
```

$$\begin{pmatrix}0\\0\\0\end{pmatrix}=\left(\begin{array}{r} (\sin(\theta_p(t))\,h_p-\cos(\theta_p(t))\,w_p+2\,w_p-x_p(t))\left(1-\frac{}{\sqrt{(\sin(\theta_p(t))\,h_p-\cos(\theta_p(t))\,\cdots}}\right.\\ -\gamma+\left(1-\frac{1}{\sqrt{(\sin(\theta_p(t))\,h_p-\cos(\theta_p(t))\,w_p+2\,w_p-x_p(t))^2+(-\cos(\theta_p(t))\,h_p-\sin(\theta_p(t))\,w_p-y_p(t))^2}}\right.\\ -\alpha\left(1-\frac{1}{\sqrt{(\sin(\theta_p(t))\,h_p-\cos(\theta_p(t))\,w_p+2\,w_p-x_p(t))^2+(-\cos(\theta_p(t))\,h_p-\sin(\theta_p(t))\,w_p-y_p(t))^2}}\right)(w_p\,(\sin(\theta_p(t))\,(2\,w_p-x_p(t))+\cos(\theta_p(t))\,y_p(t))+\cdots \end{array}\right.$$

```
(*EquibStartConditions={x1[0]→0,y1[0]→0,x2[0]→D,y2[0]→y1[0]}*)
horizontalState = {θp[t] → 0};
SymetricEquibWithAssumption = SymetricEquib /. horizontalState
```

$$\{\{0\},\{0\},\{0\}\} == \left\{\left\{2\,(w_p-x_p[t])\left(1-\frac{1}{\sqrt{(w_p-x_p[t])^2+(-h_p-y_p[t])^2}}\right)\right\},\right.$$
$$\left\{-\gamma+2\left(1-\frac{1}{\sqrt{(w_p-x_p[t])^2+(-h_p-y_p[t])^2}}\right)(-h_p-y_p[t])\right\},$$
$$\left\{-\alpha\left(1-\frac{1}{\sqrt{(w_p-x_p[t])^2+(-h_p-y_p[t])^2}}\right)(-h_p\,x_p[t]-w_p\,y_p[t])-\right.$$
$$\left.\left.\alpha\left(1-\frac{1}{\sqrt{(w_p-x_p[t])^2+(-h_p-y_p[t])^2}}\right)(h_p\,(2\,w_p-x_p[t])+w_p\,y_p[t])\right\}\right\}$$

```
(simpleEquibXYSolution = Solve[SymetricEquibWithAssumption, {xp[t], yp[t]}]) //
  MatrixForm // TraditionalForm
```

$$\begin{pmatrix} x_p(t)\to w_p & y_p(t)\to \frac{1}{2}(-\gamma-2\,h_p-2)\\ x_p(t)\to w_p & y_p(t)\to \frac{1}{2}(-\gamma-2\,h_p+2)\end{pmatrix}$$

```
EquibInputConditions = {x1[t] → 0, y1[t] → 0, x2[t] → (*2*) wp, y2[t] → y1[t]}
```

$\{x_1[t] \to 0, y_1[t] \to 0, x_2[t] \to w_p, y_2[t] \to y_1[t]\}$

```
NumericParametersTest =
  {k1 → 200, k2 → k1 + 0, LO1 → 2, LO2 → LO1 + 0, mp → 2, hp → 0.1, wp → 1, g → 9.81}
greekTermsGeneralForTest = {
```

$$\kappa \to \frac{k_2}{k_1},\quad \mathcal{L} \to \frac{L0_2}{L0_1},\quad \omega_s{}^2 \to \frac{k_1}{m_p},\quad \alpha \to \frac{3\,L0_1{}^2}{\left(w_p{}^2 + h_p{}^2\right)},\quad \gamma \to \left(\frac{g\,m_p}{L0_1\,k_1}\right)$$

```
}
```

$$\frac{g}{L0_1}\ \text{/. NumericParametersTest}$$

```
NumericTestParams = greekTermsGeneralForTest //. NumericParametersTest
```

$\{k_1 \to 200, k_2 \to k_1, L0_1 \to 2, L0_2 \to L0_1, m_p \to 2, h_p \to 0.1, w_p \to 1, g \to 9.81\}$

$$\left\{\kappa \to \frac{k_2}{k_1}, \mathcal{L} \to \frac{L0_2}{L0_1}, \omega_s^2 \to \frac{k_1}{m_p}, \alpha \to \frac{3\,L0_1^2}{h_p^2 + w_p^2}, \gamma \to \frac{g\,m_p}{k_1\,L0_1}\right\}$$

4.905

$\{\kappa \to 1, \mathcal{L} \to 1, \omega_s^2 \to 100, \alpha \to 11.8812, \gamma \to 0.04905\}$

```
(SymetricEquib = EOM /. nameChange /. greekTermsSymetricCase /. equibTerms //.
    EquibInputConditions) // MatrixForm // TraditionalForm
```

$$\begin{pmatrix}0\\0\\0\end{pmatrix} = \left(\begin{array}{r} (\sin(\theta_p(t))\,h_p - \cos(\theta_p(t))\,w_p + w_p - x_p(t))\left(1 - \frac{}{\sqrt{(\sin(\theta_p(t))\,h_p - \cos(\theta_p(t))\,w_p +}}\right. \\ -\gamma + \left(1 - \frac{1}{\sqrt{(\sin(\theta_p(t))\,h_p - \cos(\theta_p(t))\,w_p + w_p - x_p(t))^2 + (-\cos(\theta_p(t))\,h_p - \sin(\theta_p(t))\,w_p - y_p(t))^2}}\right)(\\ -\alpha\left(1 - \frac{1}{\sqrt{(\sin(\theta_p(t))\,h_p - \cos(\theta_p(t))\,w_p + w_p - x_p(t))^2 + (-\cos(\theta_p(t))\,h_p - \sin(\theta_p(t))\,w_p - y_p(t))^2}}\right)(w_p\,(\sin(\theta_p(t))\,(w_p - x_p(t)) + \cos(\theta_p(t))\,y_p(t)) + h_p \end{array}\right.$$

```
"see the remaining equations are only function
  of the payload location and orientation (in 3DOF):"
(NSymetricEquib = SymetricEquib //. NumericTestParams //. NumericParametersTest) //
 TraditionalForm
```

```
see the remaining equations are only
  function of the payload location and orientation (in 3DOF):
```

$$\begin{pmatrix}0\\0\\0\end{pmatrix}=\left(\begin{array}{r}(-\cos(\theta_p(t))+0.1\sin(\theta_p(t))-x_p(t)+1)\left(1-\frac{}{\sqrt{(-\cos(\theta_p(t))+0.1\sin(\theta_p(t))}}\right.\\ \left(1-\frac{1}{\sqrt{(-\cos(\theta_p(t))+0.1\sin(\theta_p(t))-x_p(t)+1)^2+(-0.1\cos(\theta_p(t))-\sin(\theta_p(t))-y_p(t))^2}}\right)(-0.1\ \mathrm{cc}\\ -11.8812\left(1-\frac{1}{\sqrt{(-\cos(\theta_p(t))+0.1\sin(\theta_p(t))-x_p(t)+1)^2+(-0.1\cos(\theta_p(t))-\sin(\theta_p(t))-y_p(t))^2}}\right)(\sin(\theta_p(t))\,(1-x_p(t))+\cos(\theta_p(t))\,y_p(t)+0.1\,(\end{array}\right.$$

```
simpleEquibXYSolution
```

$$\left\{\left\{x_p[t]\to w_p,\ y_p[t]\to\frac{1}{2}(-2-\gamma-2h_p)\right\},\ \left\{x_p[t]\to w_p,\ y_p[t]\to\frac{1}{2}(2-\gamma-2h_p)\right\}\right\}$$

```
(*Solve[SymetricEquib,{xp[t],yp[t],θp[t]}]*)
NSymetricEquib /. {xp[t] → 1(*,yp[t]→1*), θp[t] → 1}
NSymetricEquib /. {xp[t] → 1(*,yp[t]→1*), θp[t] → 0}
```

$$\{\{0\},\{0\},\{0\}\}==\left\{\{0.\},\left\{-0.04905+2\left(1-\frac{1}{\sqrt{0.+(-0.1-y_p[t])^2}}\right)(-0.1-y_p[t])\right\},\right.$$
$$\left.\left\{-11.8812\left(1-\frac{1}{\sqrt{0.+(-0.1-y_p[t])^2}}\right)(-0.1-y_p[t])-11.8812\left(1-\frac{1}{\sqrt{0.+(-0.1-y_p[t])^2}}\right)(0.1+y_p[t])\right\}\right\}$$

```
NSolve[NSymetricEquib /. θp[t] → Pi, {xp[t], yp[t](*,θp[t]*)}, Reals]
```

$\{\{x_p[t]\to 1.,\ y_p[t]\to 0.0509647\}\}$

```
NSolve[NSymetricEquib /. θp[t] → 0.0, {xp[t], yp[t](*,θp[t]*)}, Reals]
```

$\{\{x_p[t]\to 1.,\ y_p[t]\to -1.12452\},\ \{x_p[t]\to 1.,\ y_p[t]\to 0.875475\}\}$

```
NSolve[NSymetricEquib /. θp[t] → Pi/2, {xp[t], yp[t](*,θp[t]*)}, Reals]
```

NSolve::cadpr :

The cylindrical algebraic decomposition algorithm used by NSolve failed due to a too low WorkingPrecision. Increasing the value of WorkingPrecision may allow the algorithm to succeed. >>

NSolve::ratnz : NSolve was unable to solve the system with inexact coefficients.
The answer was obtained by solving a corresponding exact system and numericizing the result. >>

```
{}
```

NSolve::infsolns : Infinite solution set has dimension at least 1.
Returning intersection of solutions with $-\frac{113492\, x_p[t]}{178835}-\frac{121484\, y_p[t]}{178835}+\frac{171802\, \theta_p[t]}{178835}==1.$ >>

```
{{xp[t] → 1., yp[t] → -1.12453, θp[t] → 0.906364},
 {xp[t] → 1., yp[t] → 0.875475, θp[t] → 2.3206}}
```

simple case testings:

(*trajectory:

τ=0: $\ddot{y}=1m/s^2$ until $y_1=y_2=10L0_1$

$\ddot{y}=-1m/s^2$ until $\dot{y}_1=\dot{y}_2=0$

$\ddot{x}_1=\ddot{x}_2=1m/s^2$ until $\dot{x}_1=\dot{x}_2=2m/s$

disterbunce can be input by x_1+=5$L0_1$ over $\frac{1}{100\sqrt{\omega_s}}$*)

(*what needs to be done in order to keep horizontal payload?$(\theta_p[t]\to 0)$:

simpStep1/.θ_p[t]→0/.dispSimp//MatrixForm//TraditionalForm

$$\begin{pmatrix} \frac{k_1\, r1x\left(\sqrt{r1x^2+r1y^2}-L0_1\right)}{\sqrt{r1x^2+r1y^2}}+\frac{k_2\, r2x\left(\sqrt{r2x^2+r2y^2}-L0_2\right)}{\sqrt{r2x^2+r2y^2}}=m_p\, x_p'' \\ \frac{k_1\, r1y\left(\sqrt{r1x^2+r1y^2}-L0_1\right)}{\sqrt{r1x^2+r1y^2}}+\frac{k_2\, r2y\left(\sqrt{r2x^2+r2y^2}-L0_2\right)}{\sqrt{r2x^2+r2y^2}}=m_p\,(g+y_p'') \\ \frac{k_1\,(dr1+dr2)\left(\sqrt{r1x^2+r1y^2}-L0_1\right)}{2\sqrt{r1x^2+r1y^2}}+\frac{k_2\,(dr3+dr4)\left(\sqrt{r2x^2+r2y^2}-L0_2\right)}{2\sqrt{r2x^2+r2y^2}}+i_p\,\theta_p''=0 \end{pmatrix}$$

what needs to be done in order to keep horizontal payload±ϵ?(θ_p[t]→$\delta\theta$[t]) :*)

```
n = 1; Series[√f[x, y, θ] , {x, 0, n}, {y, 0, n}, {θ, 0, n}]
```

$$\begin{aligned}&\left(\left(\sqrt{f[0,0,0]}+\frac{f^{(0,0,1)}[0,0,0]\,\theta}{2\sqrt{f[0,0,0]}}+O[\theta]^2\right)+\right.\\&\quad\left.\left(\frac{f^{(0,1,0)}[0,0,0]}{2\sqrt{f[0,0,0]}}+\left(-\frac{f^{(0,0,1)}[0,0,0]\,f^{(0,1,0)}[0,0,0]}{4\,f[0,0,0]^{3/2}}+\frac{f^{(0,1,1)}[0,0,0]}{2\sqrt{f[0,0,0]}}\right)\theta+O[\theta]^2\right)y+O[y]^2\right)+\\&\left(\left(\frac{f^{(1,0,0)}[0,0,0]}{2\sqrt{f[0,0,0]}}+\left(-\frac{f^{(0,0,1)}[0,0,0]\,f^{(1,0,0)}[0,0,0]}{4\,f[0,0,0]^{3/2}}+\frac{f^{(1,0,1)}[0,0,0]}{2\sqrt{f[0,0,0]}}\right)\theta+O[\theta]^2\right)+\right.\\&\quad\left(\left(-\frac{f^{(0,1,0)}[0,0,0]\,f^{(1,0,0)}[0,0,0]}{4\,f[0,0,0]^{3/2}}+\frac{f^{(1,1,0)}[0,0,0]}{2\sqrt{f[0,0,0]}}\right)+\right.\\&\qquad\left(\left(3\,f^{(0,0,1)}[0,0,0]\,f^{(0,1,0)}[0,0,0]\,f^{(1,0,0)}[0,0,0]\right)\Big/\left(8\,f[0,0,0]^{5/2}\right)-\right.\\&\qquad\quad\frac{f^{(0,1,1)}[0,0,0]\,f^{(1,0,0)}[0,0,0]}{4\,f[0,0,0]^{3/2}}-\\&\qquad\quad\frac{f^{(0,1,0)}[0,0,0]\,f^{(1,0,1)}[0,0,0]}{4\,f[0,0,0]^{3/2}}-\frac{f^{(0,0,1)}[0,0,0]\,f^{(1,1,0)}[0,0,0]}{4\,f[0,0,0]^{3/2}}+\\&\qquad\quad\left.\left.\left.\frac{f^{(1,1,1)}[0,0,0]}{2\sqrt{f[0,0,0]}}\right)\theta+O[\theta]^2\right)y+O[y]^2\right)x+O[x]^2\end{aligned}$$

```
% // Simplify
```

$$\begin{aligned}&\left(\left(\sqrt{f[0,0,0]}+\frac{f^{(0,0,1)}[0,0,0]\,\theta}{2\sqrt{f[0,0,0]}}+O[\theta]^2\right)+\right.\\&\quad\left(\frac{f^{(0,1,0)}[0,0,0]}{2\sqrt{f[0,0,0]}}+\left(\left(-f^{(0,0,1)}[0,0,0]\,f^{(0,1,0)}[0,0,0]+2\,f[0,0,0]\,f^{(0,1,1)}[0,0,0]\right)\right.\right.\\&\qquad\left.\left.\left.\theta\right)\Big/\left(4\,f[0,0,0]^{3/2}\right)+O[\theta]^2\right)y+O[y]^2\right)+\\&\left(\left(\frac{f^{(1,0,0)}[0,0,0]}{2\sqrt{f[0,0,0]}}+\left(\left(-f^{(0,0,1)}[0,0,0]\,f^{(1,0,0)}[0,0,0]+2\,f[0,0,0]\,f^{(1,0,1)}[0,0,0]\right)\right.\right.\right.\\&\qquad\left.\left.\theta\right)\Big/\left(4\,f[0,0,0]^{3/2}\right)+O[\theta]^2\right)+\\&\quad\left(\left(-f^{(0,1,0)}[0,0,0]\,f^{(1,0,0)}[0,0,0]+2\,f[0,0,0]\,f^{(1,1,0)}[0,0,0]\right)\Big/\left(4\,f[0,0,0]^{3/2}\right)+\right.\\&\qquad\frac{1}{8\,f[0,0,0]^{5/2}}\left(f^{(0,0,1)}[0,0,0]\right.\\&\qquad\quad\left(3\,f^{(0,1,0)}[0,0,0]\,f^{(1,0,0)}[0,0,0]-2\,f[0,0,0]\,f^{(1,1,0)}[0,0,0]\right)+\\&\qquad\quad 2\,f[0,0,0]\left(-f^{(0,1,1)}[0,0,0]\,f^{(1,0,0)}[0,0,0]-f^{(0,1,0)}[0,0,0]\,f^{(1,0,1)}[\right.\\&\qquad\qquad\left.\left.\left.\left.0,0,0]+2\,f[0,0,0]\,f^{(1,1,1)}[0,0,0]\right)\right)\theta+O[\theta]^2\right)y+O[y]^2\right)x+O[x]^2\end{aligned}$$

```
EOM // MatrixForm // TraditionalForm
nameChange
greekTermsSymetricCase
equibTerms
EquibInputConditions
```

$$\begin{pmatrix} x_p''(t) \\ y_p''(t) \\ \theta_p''(t) \end{pmatrix} = \begin{pmatrix} (\sin(\theta_p(t))\, h_p + \cos(\theta_p(t))\, l_p + x_1(t) - x_p(t)) \\ -\gamma + \left(1 - \frac{1}{\sqrt{(\sin(\theta_p(t))\, h_p + \cos(\theta_p(t))\, l_p + x_1(t) - x_p(t))^2 + (-\cos}}\right. \\ -\alpha\, (l_p\, (\cos(\theta_p(t))\, (y_1(t) - y_p(t)) - \sin(\theta_p(t))\, (x_1(t) - x_p(t))) + h_p\, (\cos(\theta_p(t))\, (x_1(t) - x_p(t)) + \sin(\theta_p(t))\, (y_1(t) - y_p(t))) \end{pmatrix}$$

```
{lp → wp}
{κ → 1, ℒ → 1}
{xp'[t] → 0, yp'[t] → 0, θp'[t] → 0, xp''[t] → 0, yp''[t] → 0, θp''[t] → 0}
{x1[t] → 0, y1[t] → 0, x2[t] → wp, y2[t] → y1[t]}
```

```
(* greekTermsSymetricCase must be used again
 because this is where the equilibrium point is refering to .
  EquibInputConditions might be turned of later for better investigation *)
(SymetricEOMtoInvestigate = EOM /. nameChange /. greekTermsSymetricCase //.
    EquibInputConditions) // MatrixForm // TraditionalForm
```

$$\begin{pmatrix} x_p''(t) \\ y_p''(t) \\ \theta_p''(t) \end{pmatrix} = \begin{pmatrix} (\sin(\theta_p(t))\, h_p - \cos(\theta_p(t))\, w_p + w_p - x_p(t)) \left(1 - \frac{}{\sqrt{(\sin(\theta_p(t))\, h_p - \cos(\theta_p(t}}\right. \\ -\gamma + \left(1 - \frac{1}{\sqrt{(\sin(\theta_p(t))\, h_p - \cos(\theta_p(t))\, w_p + w_p - x_p(t))^2 + (-\cos(\theta_p(t))\, h_p - \sin(\theta_p(t))\, w_p - y_p(t}}\right. \\ -\alpha \left(1 - \frac{1}{\sqrt{(\sin(\theta_p(t))\, h_p - \cos(\theta_p(t))\, w_p + w_p - x_p(t))^2 + (-\cos(\theta_p(t))\, h_p - \sin(\theta_p(t))\, w_p - y_p(t))^2}}\right) (w_p\, (\sin(\theta_p(t))\, (w_p - x_p(t)) + \cos(\theta_p(t))\, y_p(t)) \end{pmatrix}$$