

required : system of 2 quads and 1 payload

system elements :

quad 1 - given as system input. x,y coor. θ is not influential

quad 2 - given as system input. x,y coor. θ is not influential

payload (constrained to quads locations)

In[29]:= **Quit[]**

Needs["VariationalMethods`"]

kinematics :

In[15]:= **prop2D** = $\left\{ \mathbf{x}_i = \begin{pmatrix} \mathbf{x}_i[t] \\ \mathbf{y}_i[t] \\ 0 \end{pmatrix} \right\}$ // **MatrixForm** // **TraditionalForm**,

$\left(\mathbf{Imat}_i = \begin{pmatrix} \mathbf{I}_{i,xx} & 0 & 0 \\ 0 & \mathbf{I}_{i,yy} & 0 \\ 0 & 0 & \mathbf{I}_{i,zz} \end{pmatrix} \right)$ // **MatrixForm** // **TraditionalForm**,

$\omega_i = \mathbf{D}[\{0, 0, \theta_i[t]\}, t]$

prop2D /. **i** → 1

prop2D /. **i** → 2

prop2D /. **i** → **p**

$(\mathbf{v}_i = \mathbf{D}[\mathbf{x}_i, t])$ // **MatrixForm** // **TraditionalForm**

v_i /. **i** → 1

v_i /. **i** → 2

v_i /. **i** → **p**

Out[15]= $\left\{ \begin{pmatrix} x_i(t) \\ y_i(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \dot{\mathbf{i}}_{i,xx} & 0 & 0 \\ 0 & \dot{\mathbf{i}}_{i,yy} & 0 \\ 0 & 0 & \dot{\mathbf{i}}_{i,zz} \end{pmatrix}, \{0, 0, \theta_i'[t]\} \right\}$

Out[16]= $\left\{ \begin{pmatrix} x_1(t) \\ y_1(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \dot{\mathbf{i}}_{1,xx} & 0 & 0 \\ 0 & \dot{\mathbf{i}}_{1,yy} & 0 \\ 0 & 0 & \dot{\mathbf{i}}_{1,zz} \end{pmatrix}, \{0, 0, \theta_1'[t]\} \right\}$

Out[17]= $\left\{ \begin{pmatrix} x_2(t) \\ y_2(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \dot{\mathbf{i}}_{2,xx} & 0 & 0 \\ 0 & \dot{\mathbf{i}}_{2,yy} & 0 \\ 0 & 0 & \dot{\mathbf{i}}_{2,zz} \end{pmatrix}, \{0, 0, \theta_2'[t]\} \right\}$

Out[18]= $\left\{ \begin{pmatrix} x_p(t) \\ y_p(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \dot{\mathbf{i}}_{p,xx} & 0 & 0 \\ 0 & \dot{\mathbf{i}}_{p,yy} & 0 \\ 0 & 0 & \dot{\mathbf{i}}_{p,zz} \end{pmatrix}, \{0, 0, \theta_p'[t]\} \right\}$

Out[19]//**TraditionalForm**=

$\begin{pmatrix} x_i'(t) \\ y_i'(t) \\ 0 \end{pmatrix}$

Out[20]= $\{\{x_1'[t]\}, \{y_1'[t]\}, \{0\}\}$

Out[21]= $\{\{x_2'[t]\}, \{y_2'[t]\}, \{0\}\}$

Out[22]= $\{\{x_p'[t]\}, \{y_p'[t]\}, \{0\}\}$

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In[23]:= dispSimp = {a_[t] → a, Cos[a_] → c[a], Sin[a_] → s[a], ii,zz → Ii};
{ (Rp2I = RotationMatrix[θp[t]]) // MatrixForm,
  x1dotSqr = (Transpose[vi].vi) [[1, 1]] /. i → 1,
  x2dotSqr = (Transpose[vi].vi) [[1, 1]] /. i → 2,
  IωSqr1 = ωi.Imati.ωi /. i → 1,
  IωSqr2 = ωi.Imati.ωi /. i → 2,
  xpdotSqr = (Transpose[vi].vi) [[1, 1]] /. i → p,
  IωSqrp = ωi.Imati.ωi /. i → p,
  r1[t] =  $\begin{pmatrix} x_1[t] \\ y_1[t] \end{pmatrix} - \left( \begin{pmatrix} x_p[t] \\ y_p[t] \end{pmatrix} + Rp2I \cdot \left\{ -\frac{l_p}{2}, \frac{h_p}{2} \right\} \right),$ 
  r2[t] =  $\begin{pmatrix} x_2[t] \\ y_2[t] \end{pmatrix} - \left( \begin{pmatrix} x_p[t] \\ y_p[t] \end{pmatrix} + Rp2I \cdot \left\{ \frac{l_p}{2}, \frac{h_p}{2} \right\} \right),$ 
  Δ1 =  $\sqrt{(r_1[t] [[1]])^2 + (r_1[t] [[2]])^2 - L0_1},$ 

  Δ2 =  $\sqrt{(r_2[t] [[1]])^2 + (r_2[t] [[2]])^2 - L0_2};$ 
  (T =  $\frac{1}{2} m_1 x1dotSqr + \frac{1}{2} I\omega Sqr1 + \frac{1}{2} m_2 x2dotSqr + \frac{1}{2} I\omega Sqr2 + \frac{1}{2} m_p xpdotSqr + \frac{1}{2} I\omega Sqrp$ );
  (*ri=li+Δ1*)
  V = m1 g (Xi [[2]] /. i → 1) +
    m2 g (Xi [[2]] /. i → 2) + mp g (Xi [[2]] /. i → p) +  $\frac{1}{2} k_1 \Delta_1^2 + \frac{1}{2} k_2 \Delta_2^2;$ 
  L = (T - V) [[1]] (*Tquad#1+Tquad#2+Tpayload - (Vquad#1+Vquad#2+Vpayload+Vspring#1+Vspring#2*)
Out[27]= -g m1 y1[t] - g m2 y2[t] -  $\frac{1}{2} k_1 \left( -L0_1 + \sqrt{\left( \left( \frac{1}{2} \sin[\theta_p[t]] h_p + \frac{1}{2} \cos[\theta_p[t]] l_p + x_1[t] - x_p[t] \right)^2 + \left( -\frac{1}{2} \cos[\theta_p[t]] h_p + \frac{1}{2} \sin[\theta_p[t]] l_p + y_1[t] - y_p[t] \right)^2} \right)^2 -$ 
 $\frac{1}{2} k_2 \left( -L0_2 + \sqrt{\left( \left( \frac{1}{2} \sin[\theta_p[t]] h_p - \frac{1}{2} \cos[\theta_p[t]] l_p + x_2[t] - x_p[t] \right)^2 + \left( -\frac{1}{2} \cos[\theta_p[t]] h_p - \frac{1}{2} \sin[\theta_p[t]] l_p + y_2[t] - y_p[t] \right)^2} \right)^2 -$ 
 $g m_p y_p[t] + \frac{1}{2} m_1 (x_1'[t]^2 + y_1'[t]^2) + \frac{1}{2} m_2 (x_2'[t]^2 + y_2'[t]^2) +$ 
 $\frac{1}{2}$ 
 $m_p (x_p'[t]^2 + y_p'[t]^2) +$ 
 $\frac{1}{2} i_{1,zz} \theta_1'[t]^2 + \frac{1}{2} i_{2,zz} \theta_2'[t]^2 +$ 
 $\frac{1}{2} i_{p,zz} \theta_p'[t]^2$ 

```

In[28]:=

L // . dispSimp // TraditionalForm

Out[28]//TraditionalForm=

$$\begin{aligned}
& -\frac{1}{2} k_1 \left(\sqrt{\left(\left(\frac{1}{2} l_p \cos(\theta_p) + \frac{1}{2} h_p \sin(\theta_p) - x_p + x_1 \right)^2 + \left(-\frac{1}{2} h_p \cos(\theta_p) + \frac{1}{2} l_p \sin(\theta_p) - y_p + y_1 \right)^2 \right) - L0_1} \right)^2 - \\
& \frac{1}{2} k_2 \left(\sqrt{\left(\left(-\frac{1}{2} l_p \cos(\theta_p) + \frac{1}{2} h_p \sin(\theta_p) - x_p + x_2 \right)^2 + \left(-\frac{1}{2} h_p \cos(\theta_p) - \frac{1}{2} l_p \sin(\theta_p) - y_p + y_2 \right)^2 \right) - L0_2} \right)^2 - g m_p y_p - g m_1 y_1 - \\
& g m_2 y_2 + \frac{1}{2} i_1 (\theta_1')^2 + \frac{1}{2} i_2 (\theta_2')^2 + \frac{1}{2} m_p ((x_p')^2 + (y_p')^2) + \frac{1}{2} m_1 ((x_1')^2 + (y_1')^2) + \frac{1}{2} m_2 ((x_2')^2 + (y_2')^2) + \frac{1}{2} i_p (\theta_p')^2
\end{aligned}$$

```

(quadEqNominal = EulerEquations[L,
  {(*x1[t], y1[t], theta1[t], x2[t], y2[t], theta2[t], *) xP[t], yP[t], thetaP[t]}, t]
  (*[All, 1] *) (*==Q*) // Simplify) // MatrixForm // TraditionalForm

```

$$\left(\begin{array}{l}
\frac{k_1 (h_p \sin(\theta_p(t)) + l_p \cos(\theta_p(t)) - 2 x_p(t) + 2 x_1(t))}{2 \sqrt{\frac{1}{4} (h_p \sin(\theta_p(t)) + l_p \cos(\theta_p(t)) - 2 x_p(t) + 2 x_1(t))^2}} \\
\frac{k_1 \left(-\frac{1}{2} h_p \cos(\theta_p(t)) + \frac{1}{2} l_p \sin(\theta_p(t)) - y_p(t) + y_1(t) \right)}{\sqrt{\frac{1}{4} (h_p \sin(\theta_p(t)) + l_p \cos(\theta_p(t)) - 2 x_p(t) + 2 x_1(t))^2}} \\
i_{p,zz} \theta_p''(t) + \frac{k_1 (h_p (x_1(t) \cos(\theta_p(t)) - x_p(t) \cos(\theta_p(t)) + (y_1(t) - y_p(t)) \sin(\theta_p(t))) + l_p (x_1(t) (-\sin(\theta_p(t))) + x_p(t) \sin(\theta_p(t)) + (y_1(t) - y_p(t)) \cos(\theta_p(t)))}{2 \sqrt{\frac{1}{4} (h_p \sin(\theta_p(t)) + l_p \cos(\theta_p(t)) - 2 x_p(t) + 2 x_1(t))^2}}
\end{array} \right)$$

```

In[2]:= terms2 = {
  (Sin[θp[t]] hp + Cos[θp[t]] lp + 2 x1[t] - 2 xp[t])1 → (2 r1x),
  (- 1/2 Cos[θp[t]] hp + 1/2 Sin[θp[t]] lp + y1[t] - yp[t])1 → r1y,
  1/2 Cos[θp[t]] hp - 1/2 Sin[θp[t]] lp - y1[t] + yp[t] → -r1y,
  (1/2 Sin[θp[t]] hp - 1/2 Cos[θp[t]] lp + x2[t] - xp[t])1 → r2x,
  (- 1/2 Cos[θp[t]] hp - 1/2 Sin[θp[t]] lp + y2[t] - yp[t])1 → r2y,
  Cos[θp[t]] hp + Sin[θp[t]] lp - 2 y2[t] + 2 yp[t] → (-2 r2y),
  lp (-Sin[θp[t]] x1[t] + Sin[θp[t]] xp[t] + Cos[θp[t]] (y1[t] - yp[t])) → dr1,
  hp (Cos[θp[t]] x1[t] - Cos[θp[t]] xp[t] + Sin[θp[t]] (y1[t] - yp[t])) → dr2,
  hp (Cos[θp[t]] x2[t] - Cos[θp[t]] xp[t] + Sin[θp[t]] (y2[t] - yp[t])) → dr4,
  lp (Sin[θp[t]] x2[t] - Sin[θp[t]] xp[t] + Cos[θp[t]] (-y2[t] + yp[t])) → dr3
};

```

```

(simpStep1 =

```

```

  (quadEqNominal(*//Simplify*)) /. terms2)

```

```

  (*//.dispSimp*)(*//Simplify*) //

```

```

  MatrixForm(*//TraditionalForm*)

```

$$\left(\begin{array}{l}
 \frac{r_{1x} k_1 \left(\sqrt{r_{1x}^2 + r_{1y}^2} - L_{01} \right)}{\sqrt{r_{1x}^2 + r_{1y}^2}} + \frac{r_{2x} k_2 \left(\sqrt{r_{2x}^2 + r_{2y}^2} - L_{02} \right)}{\sqrt{r_{2x}^2 + r_{2y}^2}} == 1 \\
 \frac{r_{1y} k_1 \left(\sqrt{r_{1x}^2 + r_{1y}^2} - L_{01} \right)}{\sqrt{r_{1x}^2 + r_{1y}^2}} + \frac{r_{2y} k_2 \left(\sqrt{r_{2x}^2 + r_{2y}^2} - L_{02} \right)}{\sqrt{r_{2x}^2 + r_{2y}^2}} == m_p \\
 \frac{(dr_1 + dr_2) k_1 \left(\sqrt{r_{1x}^2 + r_{1y}^2} - L_{01} \right)}{2 \sqrt{r_{1x}^2 + r_{1y}^2}} + \frac{(dr_3 + dr_4) k_2 \left(\sqrt{r_{2x}^2 + r_{2y}^2} - L_{02} \right)}{2 \sqrt{r_{2x}^2 + r_{2y}^2}} .
 \end{array} \right)$$

```

In[3]:= terms3 = {
   $\sqrt{r1x^2 + r1y^2} \rightarrow a,$ 
   $\sqrt{r2x^2 + r2y^2} \rightarrow b,$ 
   $(dr1 + dr2) \rightarrow (2 c1),$ 
   $(dr3 + dr4) \rightarrow (2 c2),$ 
   $r1x^2 + r1y^2 \rightarrow a^2, r2x^2 + r2y^2 \rightarrow b^2,$ 
   $\sqrt{a^2} \rightarrow a, \sqrt{b^2} \rightarrow b$ 
}

Out[3]= { $\sqrt{r1x^2 + r1y^2} \rightarrow a, \sqrt{r2x^2 + r2y^2} \rightarrow b, dr1 + dr2 \rightarrow 2 c1,$ 
   $dr3 + dr4 \rightarrow 2 c2, r1x^2 + r1y^2 \rightarrow a^2, r2x^2 + r2y^2 \rightarrow b^2, \sqrt{a^2} \rightarrow a, \sqrt{b^2} \rightarrow b$ }

(*simpStep1//InputForm*)
(*simpStep1//TreeForm*)

(simpStep2 =
  (simpStep1 //. terms3) // Simplify) //
  MatrixForm(*//TraditionalForm*)

$$\left( \begin{array}{l} \frac{r1x k_1 (a-L0_1)}{\sqrt{a^2}} + \frac{r2x k_2 (b-L0_2)}{\sqrt{b^2}} == m_p x_p''[t] \\ \frac{r1y k_1 (a-L0_1)}{\sqrt{a^2}} + \frac{r2y k_2 (b-L0_2)}{\sqrt{b^2}} == m_p (g + y_p''[t]) \\ \frac{c1 k_1 (a-L0_1)}{\sqrt{a^2}} + \frac{c2 k_2 (b-L0_2)}{\sqrt{b^2}} + i_{p,zz} \Theta_p''[t] == 0 \end{array} \right)$$



---


(simpStep3 =
  Map[Map[Times[#, a b] &, #] &, simpStep2] // Expand //
  Simplify) // MatrixForm

$$\left( \begin{array}{l} \frac{\sqrt{a^2} b^2 r1x k_1 (a-L0_1) + a^2 \sqrt{b^2} r2x k_2 (b-L0_2)}{a b} == a b m_p x_p''[t] \\ \frac{\sqrt{a^2} b^2 r1y k_1 (a-L0_1) + a^2 \sqrt{b^2} r2y k_2 (b-L0_2)}{a b} == a b m_p (g + y_p''[t]) \\ \frac{\sqrt{a^2} b^2 c1 k_1 (a-L0_1) + a^2 (\sqrt{b^2} c2 k_2 (b-L0_2) + b^2 i_{p,zz} \Theta_p''[t])}{a b} == 0 \end{array} \right)$$


```

```

simpStep3 // . dispSimp //
Expand // MatrixForm //
TraditionalForm

```

$$\left(\begin{array}{l} -\frac{\sqrt{a^2} b k_1 L_{01} r_{1x}}{a} + \sqrt{a^2} b k_1 r_{1x} - \frac{a \sqrt{b^2}}{b} \\ -\frac{\sqrt{a^2} b k_1 L_{01} r_{1y}}{a} + \sqrt{a^2} b k_1 r_{1y} - \frac{a \sqrt{b^2} k_2 L_{02} r_{2y}}{b} \\ -\frac{\sqrt{a^2} b c_1 k_1 L_{01}}{a} + \sqrt{a^2} b c_1 k_1 - \frac{a \sqrt{b^2} c_2}{b} \end{array} \right)$$

$$\left(\text{simpStep4} = k_1 b (a - L_{01}) \begin{pmatrix} r_{1x} \\ r_{1y} \\ c_1 \end{pmatrix} + k_2 a (b - L_{01}) \begin{pmatrix} r_{2x} \\ r_{2y} \\ c_2 \end{pmatrix} + \right.$$

$$\left. \begin{pmatrix} 0 \\ -m_p g a b \\ 0 \end{pmatrix} - a b \begin{pmatrix} m_p & 0 & 0 \\ 0 & m_p & 0 \\ 0 & 0 & -I_p \end{pmatrix} \cdot \begin{pmatrix} x_p''[t] \\ y_p''[t] \\ \theta_p''[t] \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right) // \text{MatrixForm}$$

$$\left(\begin{array}{l} b r_{1x} k_1 (a - L_{01}) + a r_{2x} k_2 (b - L_{01}) - a b m_p x_p''[t] \\ b r_{1y} k_1 (a - L_{01}) + a r_{2y} k_2 (b - L_{01}) - a b g m_p - a b m_p y_p''[t] \\ b c_1 k_1 (a - L_{01}) + a c_2 k_2 (b - L_{01}) + a b I_p \theta_p''[t] \end{array} \right)$$

```
terms2 //. dispSimp // MatrixForm // TraditionalForm
terms3 //. dispSimp // MatrixForm // TraditionalForm
```

$$\left(\begin{array}{l} l_p c(\theta_p) + h_p s(\theta_p) - 2 x_p + 2 x_1 \rightarrow 2 r1x \\ -\frac{1}{2} h_p c(\theta_p) + \frac{1}{2} l_p s(\theta_p) - y_p + y_1 \rightarrow r1y \\ \frac{1}{2} h_p c(\theta_p) - \frac{1}{2} l_p s(\theta_p) + y_p - y_1 \rightarrow -r1y \\ -\frac{1}{2} l_p c(\theta_p) + \frac{1}{2} h_p s(\theta_p) - x_p + x_2 \rightarrow r2x \\ -\frac{1}{2} h_p c(\theta_p) - \frac{1}{2} l_p s(\theta_p) - y_p + y_2 \rightarrow r2y \\ h_p c(\theta_p) + l_p s(\theta_p) + 2 y_p - 2 y_2 \rightarrow -2 r2y \\ l_p ((y_1 - y_p) c(\theta_p) + x_1 (-s(\theta_p)) + x_p s(\theta_p)) \rightarrow dr1 \\ h_p (x_1 c(\theta_p) - x_p c(\theta_p) + (y_1 - y_p) s(\theta_p)) \rightarrow dr2 \\ h_p (x_2 c(\theta_p) - x_p c(\theta_p) + (y_2 - y_p) s(\theta_p)) \rightarrow dr4 \\ l_p ((y_p - y_2) c(\theta_p) + x_2 s(\theta_p) - x_p s(\theta_p)) \rightarrow dr3 \end{array} \right)$$

$$\left(\begin{array}{l} \sqrt{r1x^2 + r1y^2} \rightarrow a \\ \sqrt{r2x^2 + r2y^2} \rightarrow b \\ dr1 + dr2 \rightarrow 2 c1 \\ dr3 + dr4 \rightarrow 2 c2 \\ r1x^2 + r1y^2 \rightarrow a^2 \\ r2x^2 + r2y^2 \rightarrow b^2 \\ \sqrt{a^2} \rightarrow a \\ \sqrt{b^2} \rightarrow b \end{array} \right)$$

```
(*xp, yp, θp = f(x1, y1, x2, y2, k1, k2, lp, hp) *)
```

```
non - conver forces :
```

```
aerodynamic = f(xp, yp, θp, wx, wy) ,
```

```
w for wind components. = f(relVx, relVy) , relV is relative to air
```

```
damping = f(li) = f(xi, yi, xp, yp)
```

```
non dim the full equations
```

```
quadEqNominal /. terms2
```

```
(* /. terms3 *) // MatrixForm
```

$$\left(\begin{array}{l} \frac{r1x k_1 \left(\sqrt{r1x^2 + r1y^2} - L0_1 \right)}{\sqrt{r1x^2 + r1y^2}} + \frac{r2x k_2 \left(\sqrt{r2x^2 + r2y^2} - L0_2 \right)}{\sqrt{r2x^2 + r2y^2}} == 1 \\ \frac{r1y k_1 \left(\sqrt{r1x^2 + r1y^2} - L0_1 \right)}{\sqrt{r1x^2 + r1y^2}} + \frac{r2y k_2 \left(\sqrt{r2x^2 + r2y^2} - L0_2 \right)}{\sqrt{r2x^2 + r2y^2}} == m_p \\ \frac{(dr1+dr2) k_1 \left(\sqrt{r1x^2 + r1y^2} - L0_1 \right)}{2 \sqrt{r1x^2 + r1y^2}} + \frac{(dr3+dr4) k_2 \left(\sqrt{r2x^2 + r2y^2} - L0_2 \right)}{2 \sqrt{r2x^2 + r2y^2}} \end{array} \right).$$

```
(smallEqs =
```

```
quadEqNominal /. terms2 /. terms3) // MatrixForm
```

$$\left(\begin{array}{l} \frac{r1x k_1 (a-L0_1)}{\sqrt{a^2}} + \frac{r2x k_2 (b-L0_2)}{\sqrt{b^2}} == m_p z \\ \frac{r1y k_1 (a-L0_1)}{\sqrt{a^2}} + \frac{r2y k_2 (b-L0_2)}{\sqrt{b^2}} == m_p (g + \\ \frac{c1 k_1 (a-L0_1)}{\sqrt{a^2}} + \frac{c2 k_2 (b-L0_2)}{\sqrt{b^2}} + \dot{l}_{p,zz} \Theta_p' \end{array} \right)$$

```
(* (NonDimEq=Map[Map[Times[#, 1/(m_p*omega_s^2*L0_1)] &, #] &,
  (*simpStep1*) smallEqs] (*//Expand*) //
  FullSimplify) // MatrixForm*)
```

NonDimEq manually settings the terms:

$\tilde{y}_p[t] = y_p[t] / L0_1$ or any other of the lengths variables ($x_p, r1x, r1y, r2x, r2y, h_p, l_p$)

$t = \tau / \omega_s$

$$\omega_s^2 = \frac{k_1}{m_p} \left[\frac{g}{1} = \frac{1}{s^2} \right]$$

A is non-dimensional form of 'a'

B is non-dimensional form of 'b'

```
(NonDimEq = {  $\frac{k_1}{m_p} \left( 1 - \frac{1}{A} \frac{L0_1}{L0_1} \right) r1x L0_1 +$   

 $\frac{k_2}{k_1} \frac{k_1}{m_p} \left( 1 - \frac{1}{B} \frac{L0_2}{L0_1} \right) r2x L0_1 ==$   

 $L0_1 \omega_s^2 x_p''[t], \frac{k_1}{m_p} \left( 1 - \frac{1}{A} \frac{L0_1}{L0_1} \right) r1y L0_1 +$   

 $\frac{k_2}{k_1} \frac{k_1}{m_p} \left( 1 - \frac{1}{B} \frac{L0_2}{L0_1} \right) r2y L0_1 -$   

 $g == L0_1 \omega_s^2 y_p''[t],$   

 $\frac{k_1}{-i_{p,zz}} \left( 1 - \frac{1}{A} \frac{L0_1}{L0_1} \right) c_1 L0_1^2 + \frac{k_2}{k_1} \frac{k_1}{-i_{p,zz}}$   

 $\left( 1 - \frac{1}{B} \frac{L0_2}{L0_1} \right) c_2 L0_1^2 == \omega_s^2 \theta_p''[t] \} \} //$   

Flatten // MatrixForm //  

TraditionalForm
```

$$\left(\begin{array}{l} \frac{(1-\frac{1}{A}) k_1 L0_1 r1x}{m_p} + \frac{k_2 L0_1 r2x (1-\frac{L0_2}{B L0_1})}{m_p} = L0_1 \omega_s^2 x_p''(t) \\ \frac{(1-\frac{1}{A}) k_1 L0_1 r1y}{m_p} + \frac{k_2 L0_1 r2y (1-\frac{L0_2}{B L0_1})}{m_p} - g = L0_1 \omega_s^2 y_p''(t) \\ - \frac{(1-\frac{1}{A}) c_1 k_1 L0_1^2}{i_{p,zz}} - \frac{c_2 k_2 L0_1^2 (1-\frac{L0_2}{B L0_1})}{i_{p,zz}} = \omega_s^2 \theta_p''(t) \end{array} \right)$$

```
(*terms4={(*1-  $\frac{L0_1}{a}$ →A,
  1-  $\frac{L0_2}{b}$ →B,*)
   $\frac{k_2}{k_1}$ →k, (*=kratio,*)
   $\frac{g}{L0_1 \omega_s^2}$ →γ,
   $\frac{L0_1^2 k_1}{I_{p,zz} \omega_s^2}$ →E,
   $\frac{k_1}{m_p}$ →ωs2
}*)
```

```
(*DeltaEquilibrium= $\frac{m_p g}{k_1}$ *)
```

```
(*  $\frac{g}{L0_1 \omega_s^2}$ ==g  $\frac{1}{L0_1} \frac{1}{k_1} m_p$  [ $\frac{m}{s^2} \frac{1}{m} \frac{kg}{s^2}$  kg]
   $\frac{L0_1^2 k_1}{I_{p,zz} \omega_s^2}$ ==  $\frac{L0_1^2 k_1}{I_{p,zz} k_1} m_p$  [ $\frac{m^2 kg}{kg m^2}$  ]*)
```

```
greekTerms = {
```

```
   $\frac{k_2}{k_1}$  → κ,
```

```
   $\frac{L0_2}{L0_1}$  → ℒ,
```

```
   $\frac{k_1}{m_p}$  → ωs2,
```

```
   $\frac{m_p L0_1^2}{I_p} \left( = \frac{L0_1^2 k_1}{I_p \omega_s^2} \right)$  → α,
```

```
   $\frac{g}{L0_1 \omega_s^2} \left( = \frac{g m_p}{L0_1 k_1} \right)$  → γ
```

```
}
```

```
(NonDimEq = {
```

```
  ωs2 L01  $\left( 1 - \frac{1}{A} \right)$  r1x + κ ωs2 L01  $\left( 1 - \frac{1}{B} \right)$  r2x == L01 ωs2 xp''[t],
```

```
  ωs2 L01  $\left( 1 - \frac{1}{A} \right)$  r1y + κ ωs2 L01  $\left( 1 - \frac{1}{B} \right)$  r2y -  $\frac{g}{L0_1 \omega_s^2}$  L01 ωs2 == L01 ωs2 yp''[t],
```

```
   $\frac{k_1}{-i_{p,zz}} \frac{L0_1^2}{\omega_s^2} \omega_s^2 \left( 1 - \frac{1}{A} \right) c_1 + \kappa \frac{k_1}{-i_{p,zz}} \frac{L0_1^2}{\omega_s^2} \omega_s^2 \left( 1 - \frac{1}{B} \right) c_2 == \omega_s^2 \theta_p''[t]$ 
```

```
  } // Flatten // MatrixForm // TraditionalForm
```

```
(  $\left( 1 - \frac{1}{A} \right) L0_1 r1x \omega_s^2 + \kappa L0_1 r2x \left( 1 - \frac{\mathcal{L}}{B} \right) \omega_s^2 = L0_1 \omega_s^2 x_p''(t)$ 
   $\left( 1 - \frac{1}{A} \right) L0_1 r1y \omega_s^2 + \kappa L0_1 r2y \left( 1 - \frac{\mathcal{L}}{B} \right) \omega_s^2 - g = L0_1 \omega_s^2 y_p''(t)$ 
   $-\frac{\left( 1 - \frac{1}{A} \right) c_1 k_1 L0_1^2}{i_{p,zz}} - \frac{c_2 \kappa k_1 L0_1^2 \left( 1 - \frac{\mathcal{L}}{B} \right)}{i_{p,zz}} = \omega_s^2 \theta_p''(t)$  )
```

```
using ' greekTerms ' list :
```

In[1]:=

```
(NonDimEq = {
  x_p''[t] == (1 - 1/A) r1x + κ (1 - 1/B) L r2x ,
  y_p''[t] == (1 - 1/A) r1y + κ (1 - 1/B) L r2y - γ,
  θ_p''[t] == -α ((1 - 1/A) c1 + κ (1 - 1/B) L c2)
}) // Flatten // MatrixForm // TraditionalForm
```

Out[1]/TraditionalForm=

$$\begin{pmatrix} x_p''(t) = \left(1 - \frac{1}{A}\right) r1x + \kappa r2x \left(1 - \frac{L}{B}\right) \\ y_p''(t) = \left(1 - \frac{1}{A}\right) r1y + \kappa r2y \left(1 - \frac{L}{B}\right) - \gamma \\ \theta_p''(t) = -\alpha \left(\left(1 - \frac{1}{A}\right) c_1 + c_2 \kappa \left(1 - \frac{L}{B}\right)\right) \end{pmatrix}$$

$$\mathcal{V}_1 = \begin{pmatrix} r1x \\ r1y \\ c_1 \end{pmatrix} (x_1, x_p, \theta_p, \dots)$$

$$\mathcal{V}_2 = \begin{pmatrix} r2x \\ r2y \\ c_2 \end{pmatrix}$$

$$\dot{\chi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\alpha \end{pmatrix} \left(1 - \frac{1}{A}\right) \mathcal{V}_1 + \kappa \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\alpha \end{pmatrix} \left(1 - \frac{1}{B} L\right) \mathcal{V}_2 - \begin{pmatrix} 0 \\ \gamma \\ 0 \end{pmatrix}$$

In[8]:= **terms2****terms3**

Out[8]= $\left\{ \begin{aligned} &\sin[\theta_p[t]] h_p + \cos[\theta_p[t]] l_p + 2 x_1[t] - 2 x_p[t] \rightarrow 2 r1x, \\ &-\frac{1}{2} \cos[\theta_p[t]] h_p + \frac{1}{2} \sin[\theta_p[t]] l_p + y_1[t] - y_p[t] \rightarrow r1y, \\ &\frac{1}{2} \cos[\theta_p[t]] h_p - \frac{1}{2} \sin[\theta_p[t]] l_p - y_1[t] + y_p[t] \rightarrow -r1y, \\ &\frac{1}{2} \sin[\theta_p[t]] h_p - \frac{1}{2} \cos[\theta_p[t]] l_p + x_2[t] - x_p[t] \rightarrow r2x, \\ &-\frac{1}{2} \cos[\theta_p[t]] h_p - \frac{1}{2} \sin[\theta_p[t]] l_p + y_2[t] - y_p[t] \rightarrow r2y, \\ &\cos[\theta_p[t]] h_p + \sin[\theta_p[t]] l_p - 2 y_2[t] + 2 y_p[t] \rightarrow -2 r2y, \\ &l_p (-\sin[\theta_p[t]] x_1[t] + \sin[\theta_p[t]] x_p[t] + \cos[\theta_p[t]] (y_1[t] - y_p[t])) \rightarrow dr1, \\ &h_p (\cos[\theta_p[t]] x_1[t] - \cos[\theta_p[t]] x_p[t] + \sin[\theta_p[t]] (y_1[t] - y_p[t])) \rightarrow dr2, \\ &h_p (\cos[\theta_p[t]] x_2[t] - \cos[\theta_p[t]] x_p[t] + \sin[\theta_p[t]] (y_2[t] - y_p[t])) \rightarrow dr4, \\ &l_p (\sin[\theta_p[t]] x_2[t] - \sin[\theta_p[t]] x_p[t] + \cos[\theta_p[t]] (-y_2[t] + y_p[t])) \rightarrow dr3 \end{aligned} \right\}$

Out[9]= $\left\{ \begin{aligned} &\sqrt{r1x^2 + r1y^2} \rightarrow a, \sqrt{r2x^2 + r2y^2} \rightarrow b, dr1 + dr2 \rightarrow 2 c1, \\ &dr3 + dr4 \rightarrow 2 c2, r1x^2 + r1y^2 \rightarrow a^2, r2x^2 + r2y^2 \rightarrow b^2, \sqrt{a^2} \rightarrow a, \sqrt{b^2} \rightarrow b \end{aligned} \right\}$

In[117]:= $\mathcal{X} = \begin{pmatrix} x_p[t] \\ y_p[t] \\ \theta_p[t] \end{pmatrix} (*//\text{Flatten}*)$

greekTermsSymetricCase = {

$(* \frac{k_2}{k_1} \rightarrow *) \kappa \rightarrow 1,$

$(* \frac{L0_2}{L0_1} \rightarrow *) \mathcal{L} \rightarrow 1$

}

greekTermsGeneral = {

$(* \frac{k_2}{k_1} \rightarrow *) \kappa \rightarrow 1,$

$(* \frac{L0_2}{L0_1} \rightarrow *) \mathcal{L} \rightarrow 1,$

$(* \frac{k_1}{m_p} \rightarrow *) \omega_s^2 \rightarrow 1,$

$(* \frac{m_p L0_1^2}{I_p} (= \frac{L0_1^2 k_1}{I_p \omega_s^2}) \rightarrow *) \alpha \rightarrow 1,$

$(* \frac{g}{L0_1 \omega_s^2} (= \frac{g m_p}{L0_1 k_1}) \rightarrow *) \gamma \rightarrow 1$ (* make sure it is not over-determined constant *)

}

(* already here : replacing all former h_p, l_p with new $2h_p, 2l_p$ *)

A(* $\sqrt{r1x^2 + r1y^2}$ *) = $\sqrt{(\sin[\theta_p[t]] h_p + \cos[\theta_p[t]] l_p + (x_1[t] - x_p[t]))^2 + (-\cos[\theta_p[t]] h_p + \sin[\theta_p[t]] l_p + (y_1[t] - y_p[t]))^2}$

B(* $\sqrt{r2x^2 + r2y^2}$ *) = $\sqrt{(\sin[\theta_p[t]] h_p - \cos[\theta_p[t]] l_p + (x_2[t] - x_p[t]))^2 + (-\cos[\theta_p[t]] h_p - \sin[\theta_p[t]] l_p + (y_2[t] - y_p[t]))^2}$

```

(*c1(*->dr1+dr2*)=l_p (-Sin[θ_p[t]] (x_1[t]- x_p[t])+Cos[θ_p[t]] (y_1[t]-y_p[t]))+
  h_p (Cos[θ_p[t]] ( x_1[t]- x_p[t])+Sin[θ_p[t]] (y_1[t]-y_p[t]))
  c2(*->dr3+dr4*)=l_p (Sin[θ_p[t]] (x_2[t]- x_p[t])+Cos[θ_p[t]] (-y_2[t]+y_p[t]))+
  h_p (Cos[θ_p[t]] ( x_2[t]- x_p[t])+Sin[θ_p[t]] (y_2[t]-y_p[t]))*)
V1(*= (r1x
      r1y
      c1) *) =
  (
    (Sin[θ_p[t]] h_p + Cos[θ_p[t]] l_p + (x_1[t] - x_p[t]))
    (- Cos[θ_p[t]] h_p + Sin[θ_p[t]] l_p + (y_1[t] - y_p[t]))
    l_p (- Sin[θ_p[t]] (x_1[t] - x_p[t]) + Cos[θ_p[t]] (y_1[t] - y_p[t])) + h_p (Cos[θ_p[t]] (x_1[t] - x_p[
  )
V2(*= (r2x
      r2y
      c2) *) =
  (
    (Sin[θ_p[t]] h_p - Cos[θ_p[t]] l_p + (x_2[t] - x_p[t]))
    (- Cos[θ_p[t]] h_p - Sin[θ_p[t]] l_p + (y_2[t] - y_p[t]))
    l_p (Sin[θ_p[t]] (x_2[t] - x_p[t]) + Cos[θ_p[t]] (-y_2[t] + y_p[t])) + h_p (Cos[θ_p[t]] (x_2[t] - x_p[
  )

```

"equations with no general forces :"

EOM =

$$D[\chi, \{t, 2\}] = \left(\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\alpha \end{pmatrix} \left(1 - \frac{1}{A} \right) \right) \cdot \mathcal{V}_1 + \left(\kappa \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\alpha \end{pmatrix} \left(1 - \frac{1}{B} \right) \right) \cdot \mathcal{V}_2 - \begin{pmatrix} 0 \\ \gamma \\ 0 \end{pmatrix} // \text{Flatten};$$

Out[117]= {{x_p[t]}, {y_p[t]}, {θ_p[t]}}

Out[118]= {κ → 1, ℒ → 1}

Out[119]= {κ → 1, ℒ → 1, ω_s^2 → 1, α → 1, γ → 1}

Out[120]= $\sqrt{\left((\sin[\theta_p[t]] h_p + \cos[\theta_p[t]] l_p + x_1[t] - x_p[t])^2 + (-\cos[\theta_p[t]] h_p + \sin[\theta_p[t]] l_p + y_1[t] - y_p[t])^2 \right)}$

Out[121]= $\sqrt{\left((\sin[\theta_p[t]] h_p - \cos[\theta_p[t]] l_p + x_2[t] - x_p[t])^2 + (-\cos[\theta_p[t]] h_p - \sin[\theta_p[t]] l_p + y_2[t] - y_p[t])^2 \right)}$

Out[122]= {{Sin[θ_p[t]] h_p + Cos[θ_p[t]] l_p + x_1[t] - x_p[t]},
{-Cos[θ_p[t]] h_p + Sin[θ_p[t]] l_p + y_1[t] - y_p[t]},
{l_p (-Sin[θ_p[t]] (x_1[t] - x_p[t]) + Cos[θ_p[t]] (y_1[t] - y_p[t])) +
h_p (Cos[θ_p[t]] (x_1[t] - x_p[t]) + Sin[θ_p[t]] (y_1[t] - y_p[t]))}}

Out[123]= {{Sin[θ_p[t]] h_p - Cos[θ_p[t]] l_p + x_2[t] - x_p[t]},
{-Cos[θ_p[t]] h_p - Sin[θ_p[t]] l_p + y_2[t] - y_p[t]},
{h_p (Cos[θ_p[t]] (x_2[t] - x_p[t]) + Sin[θ_p[t]] (y_2[t] - y_p[t])) +
l_p (Sin[θ_p[t]] (x_2[t] - x_p[t]) + Cos[θ_p[t]] (-y_2[t] + y_p[t]))}}

Out[124]= equations with no general forces :

```
In[128]:= nameChange = {lp → wp} ;
EOM /. nameChange /. greekTermsSymetricCase // Flatten // MatrixForm //
TraditionalForm
```

Out[129]//TraditionalForm=

$$\begin{pmatrix} x_p''(t) \\ y_p''(t) \\ \theta_p''(t) \end{pmatrix} = \begin{pmatrix} (\sin(\theta_p(t)) h_p + \cos(\theta_p(t)) w_p + x_1(t) - x_p(t)) \\ -\gamma + \left(1 - \frac{1}{\sqrt{(\sin(\theta_p(t)) h_p + \cos(\theta_p(t)) w_p + x_1(t) - x_p(t))^2 + (-\cos(\theta_p(t)) h_p - \sin(\theta_p(t)) w_p - y_p(t))^2}} \right) \\ -\alpha (w_p (\cos(\theta_p(t)) (y_1(t) - y_p(t)) - \sin(\theta_p(t)) (x_1(t) - x_p(t))) + h_p (\cos(\theta_p(t)) (x_1(t) - x_p(t)) + \sin(\theta_p(t)) (y_1(t) - y_p(t))) \end{pmatrix}$$

```
In[144]:= "set derivatives to zero: "
(equibTerms = {Map[Rule[#, 0] &, D[X // Flatten, {t, 1}]],
Map[Rule[#, 0] &, D[X // Flatten, {t, 2}]]} // Flatten)
(*//MatrixForm*) // TraditionalForm
```

Out[144]= set derivatives to zero:

Out[145]//TraditionalForm=

$$\{x_p'(t) \rightarrow 0, y_p'(t) \rightarrow 0, \theta_p'(t) \rightarrow 0, x_p''(t) \rightarrow 0, y_p''(t) \rightarrow 0, \theta_p''(t) \rightarrow 0\}$$

$$(*\text{EquibStartConditions}=\{\mathbf{x}_1[0] \rightarrow 0, \mathbf{y}_1[0] \rightarrow 0, \mathbf{x}_2[0] \rightarrow D, \mathbf{y}_2[0] \rightarrow \mathbf{y}_1[0]\}*)$$

```
In[150]:= EquibInputConditions = {x1[t] → 0, y1[t] → 0, x2[t] → 2 wp, y2[t] → y1[t]}
```

Out[150]= {x₁[t] → 0, y₁[t] → 0, x₂[t] → 2 w_p, y₂[t] → y₁[t]}

```
In[152]:= (SymetricEquib = EOM /. nameChange /. greekTermsSymetricCase /. equibTerms // .
EquibInputConditions) // MatrixForm // TraditionalForm
```

Out[152]//TraditionalForm=

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} (\sin(\theta_p(t)) h_p - \cos(\theta_p(t)) w_p + 2 w_p - x_p(t)) \left(1 - \frac{1}{\sqrt{(\sin(\theta_p(t)) h_p - \cos(\theta_p(t)) w_p + 2 w_p - x_p(t))^2 + (-\cos(\theta_p(t)) h_p - \sin(\theta_p(t)) w_p - y_p(t))^2}} \right) \\ -\gamma + \left(1 - \frac{1}{\sqrt{(\sin(\theta_p(t)) h_p - \cos(\theta_p(t)) w_p + 2 w_p - x_p(t))^2 + (-\cos(\theta_p(t)) h_p - \sin(\theta_p(t)) w_p - y_p(t))^2}} \right) \\ -\alpha \left(1 - \frac{1}{\sqrt{(\sin(\theta_p(t)) h_p - \cos(\theta_p(t)) w_p + 2 w_p - x_p(t))^2 + (-\cos(\theta_p(t)) h_p - \sin(\theta_p(t)) w_p - y_p(t))^2}} \right) (w_p (\sin(\theta_p(t)) (2 w_p - x_p(t)) + \cos(\theta_p(t)) y_p(t)) + \end{pmatrix}$$

```
In[158]:= horizontalState = {θp[t] → 0}
```

Out[158]= {θ_p[t] → 0}

```
In[160]:= SymetricEquibWithAssumption = SymetricEquib /. horizontalState
```

$$\text{Out[160]} = \left\{ \{0\}, \{0\}, \{0\} \right\} = \left\{ \left\{ 2 (w_p - x_p[t]) \left(1 - \frac{1}{\sqrt{(w_p - x_p[t])^2 + (-h_p - y_p[t])^2}} \right) \right\}, \right. \\ \left. \left\{ -\gamma + 2 \left(1 - \frac{1}{\sqrt{(w_p - x_p[t])^2 + (-h_p - y_p[t])^2}} \right) (-h_p - y_p[t]) \right\}, \right. \\ \left. \left\{ -\alpha \left(1 - \frac{1}{\sqrt{(w_p - x_p[t])^2 + (-h_p - y_p[t])^2}} \right) (-h_p x_p[t] - w_p y_p[t]) - \right. \right. \\ \left. \left. \alpha \left(1 - \frac{1}{\sqrt{(w_p - x_p[t])^2 + (-h_p - y_p[t])^2}} \right) (h_p (2 w_p - x_p[t]) + w_p y_p[t]) \right\} \right\}$$

```
In[164]:= simpleEquibXYSolution =
```

```
Solve[SymetricEquibWithAssumption, {x_p[t], y_p[t]}] // MatrixForm // TraditionalForm
```

```
Out[164]//TraditionalForm=
```

$$\begin{pmatrix} x_p(t) \rightarrow w_p & y_p(t) \rightarrow \frac{1}{2}(-\gamma - 2 h_p - 2) \\ x_p(t) \rightarrow w_p & y_p(t) \rightarrow \frac{1}{2}(-\gamma - 2 h_p + 2) \end{pmatrix}$$

```
In[163]:= Solve[SymetricEquib, {x_p[t], y_p[t], \theta_p[t]}]
```

```
Out[163]= $Aborted
```

simple case testings:

(*y₁=y₂ , k₁=k₂ , L₀₁=L₀₂

$$\theta_p=0, \quad x_p=\frac{x_1+x_2}{2}, \quad \frac{y_p}{L_{01}}=-\left(1+\frac{1}{2}D\right)*$$

(*trajectory:

\tau=0: \ddot{y}=1m/s^2 \quad \text{until } y_1=y_2=10L_{01}

\ddot{y}=-1m/s^2 \quad \text{until } \dot{y}_1=\dot{y}_2=0

\dot{x}_1=\dot{x}_2=1m/s^2 \quad \text{until } x_1=x_2=2m/s

disterbunce can be input by x₁+=5L₀₁ over $\frac{1}{100 \sqrt{\omega_s}} *$)

(*what needs to be done in order to keep horizontal payload? $(\theta_p[t] \rightarrow 0)$:
simpStep1/. $\theta_p[t] \rightarrow 0$ /.dispSimp//MatrixForm//TraditionalForm

$$\left(\begin{array}{l} \frac{k_1 r_{1x} \left(\sqrt{r_{1x}^2 + r_{1y}^2} - L_{01} \right)}{\sqrt{r_{1x}^2 + r_{1y}^2}} + \frac{k_2 r_{2x} \left(\sqrt{r_{2x}^2 + r_{2y}^2} - L_{02} \right)}{\sqrt{r_{2x}^2 + r_{2y}^2}} = m_p \ddot{x}_p \\ \frac{k_1 r_{1y} \left(\sqrt{r_{1x}^2 + r_{1y}^2} - L_{01} \right)}{\sqrt{r_{1x}^2 + r_{1y}^2}} + \frac{k_2 r_{2y} \left(\sqrt{r_{2x}^2 + r_{2y}^2} - L_{02} \right)}{\sqrt{r_{2x}^2 + r_{2y}^2}} = m_p (g + \ddot{y}_p) \\ \frac{k_1 (dr_1 + dr_2) \left(\sqrt{r_{1x}^2 + r_{1y}^2} - L_{01} \right)}{2 \sqrt{r_{1x}^2 + r_{1y}^2}} + \frac{k_2 (dr_3 + dr_4) \left(\sqrt{r_{2x}^2 + r_{2y}^2} - L_{02} \right)}{2 \sqrt{r_{2x}^2 + r_{2y}^2}} + \ddot{\theta}_p = 0 \end{array} \right)$$

what needs to be done in order to keep horizontal payload±ε? $(\theta_p[t] \rightarrow \delta\theta[t])$:*)
