```
required : system of 2 quads and 1 payload
```

system elements :

quad 1 - given as system input. x,y coor. θ is not influential

quad 2 - given as system input. x,y coor. θ is not influential

payload (contrained to quads locations)

In[5]:=
```
Quit[]
```

In[1]:=
```
Needs["VariationalMethods`"]
```

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kinematics :

In[57]:=

$$\text{prop2D} = \left\{\left(X_i = \begin{pmatrix} x_i[t] \\ y_i[t] \\ 0 \end{pmatrix}\right) \text{// MatrixForm // TraditionalForm}, \left(\text{Imat}_i = \begin{pmatrix} I_{i,xx} & 0 & 0 \\ 0 & I_{i,yy} & 0 \\ 0 & 0 & I_{i,zz} \end{pmatrix}\right) \text{// MatrixForm // TraditionalForm}, \omega_i = D[\{0, 0, \theta_i[t]\}, t]\right\}$$

```
prop2D /. i → 1
prop2D /. i → 2
prop2D /. i → p
```

Out[57]=
$$\left\{\begin{pmatrix} x_i(t) \\ y_i(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbb{i}_{i,xx} & 0 & 0 \\ 0 & \mathbb{i}_{i,yy} & 0 \\ 0 & 0 & \mathbb{i}_{i,zz} \end{pmatrix}, \{0, 0, \theta_i'[t]\}\right\}$$

Out[58]=
$$\left\{\begin{pmatrix} x_1(t) \\ y_1(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbb{i}_{1,xx} & 0 & 0 \\ 0 & \mathbb{i}_{1,yy} & 0 \\ 0 & 0 & \mathbb{i}_{1,zz} \end{pmatrix}, \{0, 0, \theta_1'[t]\}\right\}$$

Out[59]=
$$\left\{\begin{pmatrix} x_2(t) \\ y_2(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbb{i}_{2,xx} & 0 & 0 \\ 0 & \mathbb{i}_{2,yy} & 0 \\ 0 & 0 & \mathbb{i}_{2,zz} \end{pmatrix}, \{0, 0, \theta_2'[t]\}\right\}$$

Out[60]=
$$\left\{\begin{pmatrix} x_p(t) \\ y_p(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbb{i}_{p,xx} & 0 & 0 \\ 0 & \mathbb{i}_{p,yy} & 0 \\ 0 & 0 & \mathbb{i}_{p,zz} \end{pmatrix}, \{0, 0, \theta_p'[t]\}\right\}$$

In[6]:=

$$(v_i = D[X_i, t]) \text{ // MatrixForm // TraditionalForm}$$

```
vi /. i → 1
vi /. i → 2
vi /. i → p
```

Out[6]//TraditionalForm=

$$\begin{pmatrix} x_i'(t) \\ y_i'(t) \\ 0 \end{pmatrix}$$

Out[7]= $\{\{x_1'[t]\}, \{y_1'[t]\}, \{0\}\}$

Out[8]= $\{\{x_2'[t]\}, \{y_2'[t]\}, \{0\}\}$

Out[9]= $\{\{x_p'[t]\}, \{y_p'[t]\}, \{0\}\}$

rotations :

```
(*(Rp2I=(RotationMatrix[θp]))//MatrixForm;
HangPoint1=PayloadCenterPos-Rp2I.{lp/2,-hp/2}
    HangPoint2=PayloadCenterPos+Rp2I.{lp/2,hp/2}

      Quad1CenterPos = {xi,zi} /. i→1
        Quad2CenterPos = {xi,zi} /. i→2
         PayloadCenterPos = {xi,zi} /. i→p
          Δ1=(x1;y1)-(xp;yp)-Rp2I.{lp/2,-hp/2}*)
```

Out[1037]= {3.98866, 4.52335}

Out[1038]= {5.22548, 6.09506}

Out[1039]= {0, 10}

Out[1040]= {10, 10}

Out[1041]= {5, 5}

Out[1042]= $\{\{-6.01134\}, \{-0.476653 + y_1 - y_p\}\}$

enrgies :

In[115]:=
```
Imat
Imati
Imati /. i → 1
```

Out[115]= Imat

Out[116]= $\{\{\mathbb{i}_{i,xx}, 0, 0\}, \{0, \mathbb{i}_{i,yy}, 0\}, \{0, 0, \mathbb{i}_{i,zz}\}\}$

Out[117]= $\{\{\mathbb{i}_{1,xx}, 0, 0\}, \{0, \mathbb{i}_{1,yy}, 0\}, \{0, 0, \mathbb{i}_{1,zz}\}\}$

In[61]:=
```
{(Rp2I = RotationMatrix[θp[t]]) // MatrixForm,
  x1dotSqr = (Transpose[vi].vi)[[1, 1]] /. i → 1,
x2dotSqr = (Transpose[vi].vi)[[1, 1]] /. i → 2,
  IωSqr1 = ωi.Imati .ωi /. i → 1,
  IωSqr2 = ωi.Imati .ωi /. i → 2,
  xpdotSqr = (Transpose[vi].vi)[[1, 1]] /. i → p,
  IωSqrp = ωi.Imati .ωi /. i → p,
```

$$r_1[t] = \begin{pmatrix} x_1[t] \\ y_1[t] \end{pmatrix} - \left(\begin{pmatrix} x_p[t] \\ y_p[t] \end{pmatrix} - \mathrm{Rp2I}.\left\{\frac{l_p}{2}, -\frac{h_p}{2}\right\}\right),$$

$$r_2[t] = \begin{pmatrix} x_2[t] \\ y_2[t] \end{pmatrix} - \left(\begin{pmatrix} x_p[t] \\ y_p[t] \end{pmatrix} + \mathrm{Rp2I}.\left\{\frac{l_p}{2}, \frac{h_p}{2}\right\}\right),$$

$$\Delta_1 = \sqrt{(r_1[t][[1]])^2 + (r_1[t][[2]])^2} - L0_1,$$

$$\Delta_2 = \sqrt{(r_2[t][[1]])^2 + (r_2[t][[2]])^2} - L0_2\};$$

$$\left(T = \frac{1}{2} m_1\, \mathrm{x1dotSqr} + \frac{1}{2} \mathrm{I\omega Sqr1} + \frac{1}{2} m_2\, \mathrm{x2dotSqr} + \frac{1}{2} \mathrm{I\omega Sqr2} + \frac{1}{2} m_p\, \mathrm{xpdotSqr} + \frac{1}{2} \mathrm{I\omega Sqrp}\right);$$

```
(*ri=li+Δl*)
```

$$V = m_1\, g\, (X_i[[2]] \;/.\; i \to 1) + m_2\, g\, (X_i[[2]] \;/.\; i \to 2) + m_p\, g\, (X_i[[2]] \;/.\; i \to p) + \frac{1}{2} k_1 \Delta_1^2 + \frac{1}{2} k_2 \Delta_2^2;$$

```
L = (T - V);
L = (T - V)[[1]]
```

Out[65]=

$$-g\, m_1\, y_1[t] - g\, m_2\, y_2[t] - \frac{1}{2} k_1 \left(-L0_1 + \sqrt{\left(\left(\frac{1}{2} \mathrm{Sin}[\theta_p[t]]\, h_p + \frac{1}{2} \mathrm{Cos}[\theta_p[t]]\, l_p + x_1[t] - x_p[t]\right)^2 + \left(-\frac{1}{2} \mathrm{Cos}[\theta_p[t]]\, h_p + \frac{1}{2} \mathrm{Sin}[\theta_p[t]]\, l_p + y_1[t] - y_p[t]\right)^2\right)}\right)^2 -$$

$$\frac{1}{2} k_2 \left(-L0_2 + \sqrt{\left(\left(\frac{1}{2} \mathrm{Sin}[\theta_p[t]]\, h_p - \frac{1}{2} \mathrm{Cos}[\theta_p[t]]\, l_p + x_2[t] - x_p[t]\right)^2 + \left(-\frac{1}{2} \mathrm{Cos}[\theta_p[t]]\, h_p - \frac{1}{2} \mathrm{Sin}[\theta_p[t]]\, l_p + y_2[t] - y_p[t]\right)^2\right)}\right)^2 -$$

$$g\, m_p\, y_p[t] + \frac{1}{2} m_1 \left(x_1'[t]^2 + y_1'[t]^2\right) + \frac{1}{2} m_2 \left(x_2'[t]^2 + y_2'[t]^2\right) +$$

$$\frac{1}{2} m_p \left(x_p'[t]^2 + y_p'[t]^2\right) +$$

$$\frac{1}{2} \mathbb{i}_{1,zz}\, \theta_1'[t]^2 + \frac{1}{2} \mathbb{i}_{2,zz}\, \theta_2'[t]^2 +$$

$$\frac{1}{2} \mathbb{i}_{p,zz}\, \theta_p'[t]^2$$

In[35]:=
```
q = (x₁ y₁ θ₁ x₂ y₂ θ₂ xp yp θp)[t]
```

$$q = \begin{pmatrix} x_1 \\ y_1 \\ \theta_1 \\ x_2 \\ y_2 \\ \theta_2 \\ x_p \\ y_p \\ \theta_p \end{pmatrix}[t]$$

Out[35]= $\{\{x_1\}, \{y_1\}, \{\theta_1\}, \{x_2\}, \{y_2\}, \{\theta_2\}, \{x_p\}, \{y_p\}, \{\theta_p\}\}[t]$

In[68]:=
```
(quadEqNominal =
    EulerEquations[L, {x₁[t], y₁[t], θ₁[t], x₂[t], y₂[t], θ₂[t], xp[t], yp[t], θp[t]},
     t](*[[All,1]]*)(*==Q*) // Simplify) // MatrixForm // TraditionalForm
```

Out[68]//TraditionalForm=

$$k_1\,(h_p \sin(\theta_p(t))+l_p$$

$$g\,m_1 + \frac{k_1\left(-\frac{1}{2}\,h_p \cos(\theta\right.}{}$$

$$k_2\left(\frac{1}{2}\,h_p \sin(\theta_p(t))-\right.$$

$$g\,m_2 + \frac{k_2\left(-\frac{1}{2}\,h_p \cos(\theta\right.}{}$$

$$\frac{k_1\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))\left(\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t)}\right.}{2\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))^2+\left(\frac{1}{2}\,h_p \cos(\theta_p(t))-\right.}}$$

$$\frac{k_1\left(-\frac{1}{2}\,h_p \cos(\theta_p(t))+\frac{1}{2}\,l_p \sin(\theta_p(t))-y_p(t)+y_1(t)\right)\left(\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))^2+}\right.}{\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))^2+\left(\frac{1}{2}\,h_p \cos(\theta_p(t))-\frac{1}{2}\,l_p\right.}}$$

$$i_{p,zz}\,\theta_p''(t) + \frac{k_1\,(h_p\,(x_1(t)\cos(\theta_p(t))-x_p(t)\cos(\theta_p(t))+(y_1(t)-y_p(t))\sin(\theta_p(t)))+l_p\,(x_1(t)\,(-\sin(\theta_p(t)))+x_p(t)\sin(\theta_p(t))+(y_1(t)-y_p(t))\cos(\theta_p(t))))\left(\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t)}\right.}{2\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))^2+\left(\frac{1}{2}\,h_p \cos(\theta_p(t))-\frac{1}{2}\,l_p \sin(\theta_p(t))+y_p(t)-y\right.}}$$

In[69]:=
```
(quadEqNominal = EulerEquations[L,
    {(*x1[t],y1[t],θ1[t],x2[t],y2[t],θ2[t],*)xp[t], yp[t], θp[t]}, t]
    (*[[All,1]]*)(*==Q*) // Simplify) // MatrixForm // TraditionalForm
```

Out[69]//TraditionalForm=

$$\left(\begin{array}{c} \dfrac{k_1\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))\left(\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t)}\right.}{2\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))^2+\left(\frac{1}{2}\,h_p \cos(\theta_p(t))-\right.}} \\ \dfrac{k_1\left(-\frac{1}{2}\,h_p \cos(\theta_p(t))+\frac{1}{2}\,l_p \sin(\theta_p(t))-y_p(t)+y_1(t)\right)\left(\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))^2+}\right.}{\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))^2+\left(\frac{1}{2}\,h_p \cos(\theta_p(t))-\frac{1}{2}\,l_p\right.}} \\ i_{p,zz}\,\theta_p''(t)+\dfrac{k_1\,(h_p\,(x_1(t)\cos(\theta_p(t))-x_p(t)\cos(\theta_p(t))+(y_1(t)-y_p(t))\sin(\theta_p(t)))+l_p\,(x_1(t)\,(-\sin(\theta_p(t)))+x_p(t)\sin(\theta_p(t))+(y_1(t)-y_p(t))\cos(\theta_p(t))))\left(\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t)}\right.}{2\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))^2+\left(\frac{1}{2}\,h_p \cos(\theta_p(t))-\frac{1}{2}\,l_p \sin(\theta_p(t))+y_p(t)-y\right.}} \end{array}\right.$$

In[158]:=
```
(trimmedEq = quadEqNominal /.
   {x1'[t] → 0, x2'[t] → 0, x1''[t] → 0, x2''[t] → 0, θ1'[t] → 0, θ2'[t] → 0,
    θ1''[t] → 0, θ2''[t] → 0, θ1[t] → 0, θ2[t] → 0}) // MatrixForm // TraditionalForm
```

Out[158]//TraditionalForm=

$$\left(\begin{array}{c}
\dfrac{k_1\,(x_1(t)-x_p(t))\left(\sqrt{(x_1(t)-x_p(t))^2+(y_1(t)-y_p(t))^2}-\mathrm{L0}_1\right)}{\sqrt{(x_1(t)-x_p(t))^2+(y_1(t)-y_p(t))^2}}=0 \\
g\,m_1+\dfrac{k_1\,(y_1(t)-y_p(t))\left(\sqrt{(x_1(t)-x_p(t))^2+(y_1(t)-y_p(t))^2}-\mathrm{L0}_1\right)}{\sqrt{(x_1(t)-x_p(t))^2+(y_1(t)-y_p(t))^2}}+m_1\,y_1''(t)=0 \\
\text{True} \\
\dfrac{k_2\,(x_2(t)-x_p(t))\left(\sqrt{(x_2(t)-x_p(t))^2+(y_2(t)-y_p(t))^2}-\mathrm{L0}_2\right)}{\sqrt{(x_2(t)-x_p(t))^2+(y_2(t)-y_p(t))^2}}=0 \\
g\,m_2+\dfrac{k_2\,(y_2(t)-y_p(t))\left(\sqrt{(x_2(t)-x_p(t))^2+(y_2(t)-y_p(t))^2}-\mathrm{L0}_2\right)}{\sqrt{(x_2(t)-x_p(t))^2+(y_2(t)-y_p(t))^2}}+m_2\,y_2''(t)=0 \\
\text{True} \\
\dfrac{k_1\,(x_1(t)-x_p(t))\left(\sqrt{(x_1(t)-x_p(t))^2+(y_1(t)-y_p(t))^2}-\mathrm{L0}_1\right)}{\sqrt{(x_1(t)-x_p(t))^2+(y_1(t)-y_p(t))^2}}+\dfrac{k_2\,(x_2(t)-x_p(t))\left(\sqrt{(x_2(t)-x_p(t))^2+(y_2(t)-y_p(t))^2}-\mathrm{L0}_2\right)}{\sqrt{(x_2(t)-x_p(t))^2+(y_2(t)-y_p(t))^2}}=m_p\,x_p''(t) \\
\dfrac{k_1\,(y_1(t)-y_p(t))\left(\sqrt{(x_1(t)-x_p(t))^2+(y_1(t)-y_p(t))^2}-\mathrm{L0}_1\right)}{\sqrt{(x_1(t)-x_p(t))^2+(y_1(t)-y_p(t))^2}}+\dfrac{k_2\,(y_2(t)-y_p(t))\left(\sqrt{(x_2(t)-x_p(t))^2+(y_2(t)-y_p(t))^2}-\mathrm{L0}_2\right)}{\sqrt{(x_2(t)-x_p(t))^2+(y_2(t)-y_p(t))^2}}=m_p\,(g+y_p''(t)) \\
i_{p,zz}\,\theta_p''(t)=0
\end{array}\right)$$

In[170]:=
```
eq2D =
  {trimmedEq[[7]], trimmedEq[[8]]} /. y1[t] → 0 /. y2[t] → 0 /. yp[t] → 0 // Expand //
   Simplify // TraditionalForm
```

Out[170]//TraditionalForm=

$$\left\{\frac{k_1\left((x_1(t)-x_p(t))^2-\text{L0}_1\sqrt{(x_1(t)-x_p(t))^2}\right)}{x_1(t)-x_p(t)}+\frac{k_2\left((x_2(t)-x_p(t))^2-\text{L0}_2\sqrt{(x_2(t)-x_p(t))^2}\right)}{x_2(t)-x_p(t)}=m_p\,x_p''(t),\right.$$

$$\left.m_p\,(g+y_p''(t))=0\right\}$$