Non-dimensional Equations of motion are:

$$\ddot{\mathcal{X}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\alpha \end{pmatrix} \left(1 - \tfrac{1}{A}\right) \mathcal{V}_1 + \kappa \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\alpha \end{pmatrix} \left(1 - \tfrac{1}{B}\mathcal{L}\right) \mathcal{V}_2 - \begin{pmatrix} 0 \\ \gamma \\ 0 \end{pmatrix} \qquad (1)$$

Where:

$$\mathcal{X} = \begin{pmatrix} x_p \\ y_p \\ \theta_p \end{pmatrix}$$

$$A = \sqrt{\left(h_p \sin(\theta_p) + l_p \cos(\theta_p) - x_p + x_1\right)^2 + \left(-h_p \cos(\theta_p) + l_p \sin(\theta_p) - y_p + y_1\right)^2}$$

$$B = \sqrt{\left(h_p \sin(\theta_p) - l_p \cos(\theta_p) - x_p + x_2\right)^2 + \left(-h_p \cos(\theta_p) - l_p \sin(\theta_p) - y_p + y_2\right)^2}$$

$$\mathcal{V}_1 = \begin{pmatrix} \sin(\theta_p)\, h_p + \cos(\theta_p)\, l_p + x_1 - x_p \\ -\cos(\theta_p)\, h_p + \sin(\theta_p)\, l_p + y_1 - y_p \\ l_p \left(\cos(\theta_p)\,(y_1 - y_p) - \sin(\theta_p)\,(x_1 - x_p)\right) + h_p \left(\cos(\theta_p)\,(x_1 - x_p) + \sin(\theta_p)\,(y_1 - y_p)\right) \end{pmatrix}$$

$$\mathcal{V}_2 = \begin{pmatrix} \sin(\theta_p)\, h_p - \cos(\theta_p)\, l_p + x_2 - x_p \\ -\cos(\theta_p)\, h_p - \sin(\theta_p)\, l_p + y_2 - y_p \\ h_p \left(\cos(\theta_p)\,(x_2 - x_p) + \sin(\theta_p)\,(y_2 - y_p)\right) + l_p \left(\sin(\theta_p)\,(x_2 - x_p) + \cos(\theta_p)\,(y_p - y_2)\right) \end{pmatrix}$$

$$\alpha = \frac{m_p \mathrm{L0_1}^2}{I_p} = \frac{3\mathrm{L0_1}^2}{\left({w_p}^2 + {h_p}^2\right)} \quad (\mathit{assuming\ Inertia\ of\ a\ rectangular\ payload})$$

$$\kappa = \frac{k_2}{k_1}$$

$$\mathcal{L} = \frac{\mathrm{L0_2}}{\mathrm{L0_1}}$$

$$\gamma = \frac{g}{\mathrm{L0_1}} \frac{m_p}{k_1}$$

All dimensional quantities are originally normalized by :

$$\omega_s^{\ 2} = \frac{k_1}{m_p}$$

A symmetrical case is considered and it is:

$$k_1 \to 200,$$

$$\mathrm{L0}_1 \to 2$$

$$k_2 \to k_1 + 0,$$

$$, \mathrm{L0}_2 \to \mathrm{L0}_1 + 0,$$

$$m_p \to 2,$$

$$h_p \to 0.1,$$

$$w_p \to 1,$$

$$g \to 9.81$$

$$x_1[t] \to 0$$
$$y_1[t] \to 0$$
$$x_2[t] \to 2w_p$$
$$y_2[t] \to y_1[t]$$

Equilibrium point is :

$$\boldsymbol{X} = \begin{pmatrix} \boldsymbol{w_p} \\ -(\frac{\mathbf{1}}{\mathbf{2}}\boldsymbol{\gamma} + \boldsymbol{h_p} + \mathbf{1}) \\ \mathbf{0} \end{pmatrix}$$

That point is known to be stable

Linearizing about that equilibrium point:

$$x_p \to \delta\mathrm{x} + x_{p_0}$$

$$y_p \to \delta\mathrm{y} + y_{p_0}$$

$$\theta_p \to \delta\theta + \theta_{p_0}$$

…

After all calculations we can describe **the linearized equations as:**

$\mathbf{M}\ddot{x} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{F}$

Where :

$$\boldsymbol{M} = \begin{pmatrix} (\gamma+2) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (\gamma+2) \end{pmatrix}$$

$$\boldsymbol{F} = \{\mathbf{0}\}, \boldsymbol{C} = \{\mathbf{0}\}$$

$$\boldsymbol{K} = \begin{pmatrix} 2\gamma & 0 & -2\gamma h_p \\ 0 & 2 & 0 \\ -2\alpha\gamma h_p & 0 & \alpha\left(2\gamma h_p^2 + \gamma(\gamma+2)h_p + 2(\gamma+2)w_p^2\right) \end{pmatrix}$$

The natural frequencies can be calculated from :

$$\mathrm{Det}[K - \omega^2 M] = 0$$

For the y component we get :

$$\omega_y{}^2 = 2$$

The other frequencies can be shown numerically. For example:

$$\{\{\omega \to -0.21\}, \{\omega \to 0.21\}, \{\omega \to -9.80\}, \{\omega \to 9.80\}\}$$

OR:

$$\{\{\omega \to -0.61\}, \{\omega \to 0.61\}, \{\omega \to -10.27\}, \{\omega \to 10.27\}\}$$