required : system of 2 quads and 1 payload

system elements :

quad 1 - given as system input. x,y coor. θ is not influential

quad 2 - given as system input. x,y coor. θ is not influential

payload (contrained to quads locations)

```
Quit[]
```

In[1]:=
```
Needs["VariationalMethods`"]
```

---

kinematics :

In[2]:=
$$\text{prop2D} = \left\{\left(X_i = \begin{pmatrix} x_i[t] \\ y_i[t] \\ 0 \end{pmatrix}\right) \text{ // MatrixForm // TraditionalForm}, \left(\text{Imat}_i = \begin{pmatrix} I_{i,xx} & 0 & 0 \\ 0 & I_{i,yy} & 0 \\ 0 & 0 & I_{i,zz} \end{pmatrix}\right) \text{ // MatrixForm // TraditionalForm}, \omega_i = D[\{0, 0, \theta_i[t]\}, t]\right\}$$

```
prop2D /. i → 1
prop2D /. i → 2
prop2D /. i → p
```

Out[2]= $\left\{\begin{pmatrix} x_i(t) \\ y_i(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbb{i}_{i,xx} & 0 & 0 \\ 0 & \mathbb{i}_{i,yy} & 0 \\ 0 & 0 & \mathbb{i}_{i,zz} \end{pmatrix}, \{0, 0, \theta_i'[t]\}\right\}$

Out[3]= $\left\{\begin{pmatrix} x_1(t) \\ y_1(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbb{i}_{1,xx} & 0 & 0 \\ 0 & \mathbb{i}_{1,yy} & 0 \\ 0 & 0 & \mathbb{i}_{1,zz} \end{pmatrix}, \{0, 0, \theta_1'[t]\}\right\}$

Out[4]= $\left\{\begin{pmatrix} x_2(t) \\ y_2(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbb{i}_{2,xx} & 0 & 0 \\ 0 & \mathbb{i}_{2,yy} & 0 \\ 0 & 0 & \mathbb{i}_{2,zz} \end{pmatrix}, \{0, 0, \theta_2'[t]\}\right\}$

Out[5]= $\left\{\begin{pmatrix} x_p(t) \\ y_p(t) \\ 0 \end{pmatrix}, \begin{pmatrix} \mathbb{i}_{p,xx} & 0 & 0 \\ 0 & \mathbb{i}_{p,yy} & 0 \\ 0 & 0 & \mathbb{i}_{p,zz} \end{pmatrix}, \{0, 0, \theta_p'[t]\}\right\}$

In[6]:=
$$(v_i = D[X_i, t]) \text{ // MatrixForm // TraditionalForm}$$

v_i /. i → 1

v_i /. i → 2

v_i /. i → p

Out[6]//TraditionalForm=
$$\begin{pmatrix} x_i'(t) \\ y_i'(t) \\ 0 \end{pmatrix}$$

Out[7]= $\{\{x_1'[t]\}, \{y_1'[t]\}, \{0\}\}$

Out[8]= $\{\{x_2'[t]\}, \{y_2'[t]\}, \{0\}\}$

Out[9]= $\{\{x_p'[t]\}, \{y_p'[t]\}, \{0\}\}$

rotations :

```
(*(Rp2I=(RotationMatrix[θp]))//MatrixForm;
HangPoint1=PayloadCenterPos-Rp2I.{lp/2,-hp/2}
    HangPoint2=PayloadCenterPos+Rp2I.{lp/2,hp/2}

     Quad1CenterPos = {xi,zi} /. i→1
       Quad2CenterPos = {xi,zi} /. i→2
        PayloadCenterPos = {xi,zi} /. i→p
         Δ1=(x1, y1)-(xp, yp)-Rp2I.{lp/2,-hp/2}*)
```

$\{3.98866, 4.52335\}$

$\{5.22548, 6.09506\}$

$\{0, 10\}$

$\{10, 10\}$

$\{5, 5\}$

$\{\{-6.01134\}, \{-0.476653 + y_1 - y_p\}\}$

enrgies :

```
Imat
Imat_i
Imat_i /. i → 1
```

Imat

$\{\{\mathbb{i}_{i,xx}, 0, 0\}, \{0, \mathbb{i}_{i,yy}, 0\}, \{0, 0, \mathbb{i}_{i,zz}\}\}$

$\{\{\mathbb{i}_{1,xx}, 0, 0\}, \{0, \mathbb{i}_{1,yy}, 0\}, \{0, 0, \mathbb{i}_{1,zz}\}\}$

```
a[e]
a[e] /. a[a_] → Cos[a]
a[e] /. a → Cos
```

a[e]

Cos[e]

Cos[e]

```
In[23]:= dispSimp = {a_[t] → a, Cos[a_] → c[a], Sin[a_] → s[a], i_{i_,zz} -> I_i};
```

In[10]:=
```
{(Rp2I = RotationMatrix[θp[t]]) // MatrixForm,
  x1dotSqr = (Transpose[vi].vi)[[1, 1]] /. i → 1,
 x2dotSqr = (Transpose[vi].vi)[[1, 1]] /. i → 2,
  IωSqr1 = ωi.Imati .ωi /. i → 1,
  IωSqr2 = ωi.Imati .ωi /. i → 2,
  xpdotSqr = (Transpose[vi].vi)[[1, 1]] /. i → p,
  IωSqrp = ωi.Imati .ωi /. i → p,
  r1[t] = {{x1[t]}, {y1[t]}} - ({{xp[t]}, {yp[t]}} + Rp2I.{-lp/2, hp/2}),
  r2[t] = {{x2[t]}, {y2[t]}} - ({{xp[t]}, {yp[t]}} + Rp2I.{lp/2, hp/2}),
  Δ1 = Sqrt[(r1[t][[1]])^2 + (r1[t][[2]])^2] - L01,

  Δ2 = Sqrt[(r2[t][[1]])^2 + (r2[t][[2]])^2] - L02};
(T = 1/2 m1 x1dotSqr + 1/2 IωSqr1 + 1/2 m2 x2dotSqr + 1/2 IωSqr2 + 1/2 mp xpdotSqr + 1/2 IωSqrp);
(*ri=li+Δl*)
V = m1 g (Xi[[2]] /. i → 1) +
   m2 g (Xi[[2]] /. i → 2) + mp g (Xi[[2]] /. i → p) + 1/2 k1 Δ1^2 + 1/2 k2 Δ2^2;
L = (T - V); (*Tquad#1+Tquad#2+Tpayload - (Vquad#1+Vquad#2+Vpayload+Vspring#1+Vspring#2)*)
L = (T - V)[[1]]
```

Out[13]=
$$-g\, m_1\, y_1[t] - g\, m_2\, y_2[t] - \frac{1}{2} k_1 \left(-L0_1 + \sqrt{\left(\left(\frac{1}{2} \mathrm{Sin}[\theta_p[t]]\, h_p + \frac{1}{2} \mathrm{Cos}[\theta_p[t]]\, l_p + x_1[t] - x_p[t]\right)^2 + \left(-\frac{1}{2} \mathrm{Cos}[\theta_p[t]]\, h_p + \frac{1}{2} \mathrm{Sin}[\theta_p[t]]\, l_p + y_1[t] - y_p[t]\right)^2\right)}\right)^2 -$$
$$\frac{1}{2} k_2 \left(-L0_2 + \sqrt{\left(\left(\frac{1}{2} \mathrm{Sin}[\theta_p[t]]\, h_p - \frac{1}{2} \mathrm{Cos}[\theta_p[t]]\, l_p + x_2[t] - x_p[t]\right)^2 + \left(-\frac{1}{2} \mathrm{Cos}[\theta_p[t]]\, h_p - \frac{1}{2} \mathrm{Sin}[\theta_p[t]]\, l_p + y_2[t] - y_p[t]\right)^2\right)}\right)^2 -$$
$$g\, m_p\, y_p[t] + \frac{1}{2} m_1 \left(x_1'[t]^2 + y_1'[t]^2\right) + \frac{1}{2} m_2 \left(x_2'[t]^2 + y_2'[t]^2\right) +$$
$$\frac{1}{2}$$
$$m_p \left(x_p'[t]^2 + y_p'[t]^2\right) +$$
$$\frac{1}{2} \mathbb{i}_{1,zz}\, \theta_1'[t]^2 + \frac{1}{2} \mathbb{i}_{2,zz}\, \theta_2'[t]^2 +$$
$$\frac{1}{2} \mathbb{i}_{p,zz}\, \theta_p'[t]^2$$

```
L //. dispSimp // TraditionalForm
```

$$-\frac{1}{2} k_1 \left(\sqrt{\left(\left(\frac{1}{2} l_p\, c(\theta_p) + \frac{1}{2} h_p\, s(\theta_p) - x_p + x_1\right)^2 + \left(-\frac{1}{2} h_p\, c(\theta_p) + \frac{1}{2} l_p\, s(\theta_p) - y_p + y_1\right)^2\right)} - \mathrm{L0}_1\right)^2 - \frac{1}{2} k_2 \left(\sqrt{\left(\left(-\frac{1}{2} l_p\, c(\theta_p) + \frac{1}{2} h_p\, s(\theta_p) - x_p + x_2\right)^2 + \left(-\frac{1}{2} h_p\, c(\theta_p) - \frac{1}{2} l_p\, s(\theta_p) - y_p + y_2\right)^2\right)} - \mathrm{L0}_2\right)^2 - g\, m_p\, y_p - g\, m_1\, y_1 - g\, m_2\, y_2 + \frac{1}{2} i_1 (\theta_1')^2 + \frac{1}{2} i_2 (\theta_2')^2 + \frac{1}{2} m_p \left((x_p')^2 + (y_p')^2\right) + \frac{1}{2} m_1 \left((x_1')^2 + (y_1')^2\right) + \frac{1}{2} m_2 \left((x_2')^2 + (y_2')^2\right) + \frac{1}{2} i_p (\theta_p')^2$$

(*q= $\begin{pmatrix} x_1 \\ y_1 \\ \theta_1 \\ x_2 \\ y_2 \\ \theta_2 \\ x_p \\ y_p \\ \theta_p \end{pmatrix}$ [t]

$\begin{pmatrix} x_p \\ y_p \\ \theta_p \end{pmatrix}$ *)

$\{\{x_1\}, \{y_1\}, \{\theta_1\}, \{x_2\}, \{y_2\}, \{\theta_2\}, \{x_p\}, \{y_p\}, \{\theta_p\}\}[t]$

after setting L calculate the lagrangian derivatives and equations:

```
(quadEqNominal =
    EulerEquations[L, {x1[t], y1[t], θ1[t], x2[t], y2[t], θ2[t], xp[t], yp[t], θp[t]},
      t](*[[All,1]]*)(*==Q*) // Simplify) // MatrixForm // TraditionalForm
```

In[14]:=
```
(quadEqNominal = EulerEquations[L,
    {(*x1[t],y1[t],θ1[t],x2[t],y2[t],θ2[t],*)xp[t], yp[t], θp[t]}, t]
    (*[[All,1]]*)(*==Q*) // Simplify) // MatrixForm // TraditionalForm
```

Out[14]//TraditionalForm=

$$\left(\begin{array}{c} \dfrac{k_1\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1}{2\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p}} \\ \dfrac{k_1\left(-\frac{1}{2}\,h_p \cos(\theta_p(t))+\frac{1}{2}\,l_p \sin(\theta_p(t))-y_p(t)+y_1(t)\right)}{\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_i}} \\ i_{p,zz}\,\theta_p''(t)+\dfrac{k_1\,(h_p\,(x_1(t)\cos(\theta_p(t))-x_p(t)\cos(\theta_p(t))+(y_1(t)-y_p(t))\sin(\theta_p(t)))+l_p\,(x_1(t)\,(-\sin(\theta_p(t)))+x_p(t)\sin(\theta_p(t))+(y_1(}{2\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(}} \end{array}\right.$$

```
quadEqNominal // MatrixForm
```

$$\left(\begin{array}{c} \dfrac{k_1\;(\mathrm{Sin}[\theta_p[t]]\;h_p+\mathrm{Cos}[\theta_p[t]]\;l_p+2}{2\sqrt{\frac{1}{4}\;(}} \\ \dfrac{k_1\;\left(-\frac{1}{2}\,\mathrm{Cos}[\theta_p[t]]\;h_p+\frac{1}{2}\,\mathrm{Sin}[\theta_p[t]]\;l_p+y_1\right.}{\sqrt{\frac{1}{4}\;(\mathrm{Sin}[\theta}} \\ \dfrac{k_1\;(l_p\;(-\mathrm{Sin}[\theta_p[t]]\;x_1[t]+\mathrm{Sin}[\theta_p[t]]\;x_p[t]+\mathrm{Cos}[\theta_p[t]]\;(y_1[t]-y_p[t]))+h_p\;(\mathrm{Cos}[\theta_p[t]]\;x_1[t]-\mathrm{Cos}[\theta_p[t]]\;x_p[t]+\mathrm{Sin}[\theta_p[t]]\;(y_1[t}{2\sqrt{\frac{1}{4}\;(\mathrm{Sin}[\theta_p[t]]\;h_p+\mathrm{Cos}[\theta_p[t]]\;l_p+2\;x_1[t]-2\;x_p[t])^2+(}} \end{array}\right.$$

```
terms = {
   (*√(1/4 (Sin[θp[t]] hp+Cos[θp[t]] lp+2 x1[t]-2 xp[t])^2+
       (1/2 Cos[θp[t]] hp-1/2 Sin[θp[t]] lp-y1[t]+yp[t])^2)→dom1,*)
   1/4 (Sin[θp[t]] hp + Cos[θp[t]] lp + 2 x1[t] - 2 xp[t])^2 +
     (1/2 Cos[θp[t]] hp - 1/2 Sin[θp[t]] lp - y1[t] + yp[t])^2 → dom11,
   (*√((1/2 Sin[θp[t]] hp-1/2 Cos[θp[t]] lp+x2[t]-xp[t])^2+
       1/4 (Cos[θp[t]] hp+Sin[θp[t]] lp-2 y2[t]+2 yp[t])^2)→dom2,*)
   (1/2 Sin[θp[t]] hp - 1/2 Cos[θp[t]] lp + x2[t] - xp[t])^2 +
     1/4 (Cos[θp[t]] hp + Sin[θp[t]] lp - 2 y2[t] + 2 yp[t])^2 → dom22
  };
```

In[15]:=
```
terms2 = {
    (Sin[θp[t]] hp + Cos[θp[t]] lp + 2 x1[t] - 2 xp[t])^1 → (2 r1x),
   (-1/2 Cos[θp[t]] hp + 1/2 Sin[θp[t]] lp + y1[t] - yp[t])^1 → r1y,
   1/2 Cos[θp[t]] hp - 1/2 Sin[θp[t]] lp - y1[t] + yp[t] → -r1y,
   (1/2 Sin[θp[t]] hp - 1/2 Cos[θp[t]] lp + x2[t] - xp[t])^1 → r2x,
   (-1/2 Cos[θp[t]] hp - 1/2 Sin[θp[t]] lp + y2[t] - yp[t])^1 → r2y,
   Cos[θp[t]] hp + Sin[θp[t]] lp - 2 y2[t] + 2 yp[t] → (-2 r2y),
   lp (-Sin[θp[t]] x1[t] + Sin[θp[t]] xp[t] + Cos[θp[t]] (y1[t] - yp[t])) → dr1,
   hp (Cos[θp[t]] x1[t] - Cos[θp[t]] xp[t] + Sin[θp[t]] (y1[t] - yp[t])) → dr2,
   hp (Cos[θp[t]] x2[t] - Cos[θp[t]] xp[t] + Sin[θp[t]] (y2[t] - yp[t])) → dr4,
   lp (Sin[θp[t]] x2[t] - Sin[θp[t]] xp[t] + Cos[θp[t]] (-y2[t] + yp[t])) → dr3
  };
```

In[16]:=
```
(simpStep1 =
    (quadEqNominal(*//Simplify*)) /. terms2)
  (*//.dispSimp*)(*//Simplify*) //
 MatrixForm(*//TraditionalForm*)
```

Out[16]//MatrixForm=

$$\left(\begin{array}{c}
\frac{r1x\, k_1 \left(\sqrt{r1x^2+r1y^2}-L0_1\right)}{\sqrt{r1x^2+r1y^2}}+\frac{r2x\, k_2 \left(\sqrt{r2x^2+r2y^2}-L0_2\right)}{\sqrt{r2x^2+r2y^2}} == \mathrm{I} \\
\frac{r1y\, k_1 \left(\sqrt{r1x^2+r1y^2}-L0_1\right)}{\sqrt{r1x^2+r1y^2}}+\frac{r2y\, k_2 \left(\sqrt{r2x^2+r2y^2}-L0_2\right)}{\sqrt{r2x^2+r2y^2}} == m_p \\
\frac{(dr1+dr2)\, k_1 \left(\sqrt{r1x^2+r1y^2}-L0_1\right)}{2\sqrt{r1x^2+r1y^2}}+\frac{(dr3+dr4)\, k_2 \left(\sqrt{r2x^2+r2y^2}-L0_2\right)}{2\sqrt{r2x^2+r2y^2}}\ \text{-}
\end{array}\right.$$

In[17]:= **terms3 = {**
$\sqrt{r1x^2 + r1y^2} \to a$,
$\sqrt{r2x^2 + r2y^2} \to b$,
$(dr1 + dr2) \to (2\ c1)$,
$(dr3 + dr4) \to (2\ c2)$,
$r1x^2 + r1y^2 \to a^2$, $r2x^2 + r2y^2 \to b^2$,
$\sqrt{a^2} \to a$, $\sqrt{b^2} \to b$}

Out[17]= $\{\sqrt{r1x^2 + r1y^2} \to a,\ \sqrt{r2x^2 + r2y^2} \to b,\ dr1 + dr2 \to 2\ c1,$
$dr3 + dr4 \to 2\ c2,\ r1x^2 + r1y^2 \to a^2,\ r2x^2 + r2y^2 \to b^2,\ \sqrt{a^2} \to a,\ \sqrt{b^2} \to b\}$

```
(*simpStep1//InputForm*)

(*simpStep1//TreeForm*)
```

In[18]:=
```
(simpStep2 =
   (simpStep1 //. terms3) // Simplify) //
  MatrixForm(*//TraditionalForm*)
```

Out[18]//MatrixForm=

$$\begin{pmatrix} \frac{\text{r1x}\,k_1\,(a-L0_1)}{\sqrt{a^2}}+\frac{\text{r2x}\,k_2\,(b-L0_2)}{\sqrt{b^2}} == m_p\,x_p''[t] \\ \frac{\text{r1y}\,k_1\,(a-L0_1)}{\sqrt{a^2}}+\frac{\text{r2y}\,k_2\,(b-L0_2)}{\sqrt{b^2}} == m_p\,(g+y_p''[t]) \\ \frac{\text{c1}\,k_1\,(a-L0_1)}{\sqrt{a^2}}+\frac{\text{c2}\,k_2\,(b-L0_2)}{\sqrt{b^2}}+\mathbb{i}_{p,zz}\,\theta_p''[t] == 0 \end{pmatrix}$$

In[26]:=
```
(simpStep3 =
   Map[Map[Times[#, a b ] &, #] &, simpStep2] // Expand //
    Simplify) // MatrixForm
```

Out[26]//MatrixForm=

$$\begin{pmatrix} \frac{\sqrt{a^2}\,b^2\,\text{r1x}\,k_1\,(a-L0_1)+a^2\sqrt{b^2}\,\text{r2x}\,k_2\,(b-L0_2)}{a\,b} == a\,b\,m_p\,x_p''[t] \\ \frac{\sqrt{a^2}\,b^2\,\text{r1y}\,k_1\,(a-L0_1)+a^2\sqrt{b^2}\,\text{r2y}\,k_2\,(b-L0_2)}{a\,b} == a\,b\,m_p\,(g+y_p''[t]) \\ \frac{\sqrt{a^2}\,b^2\,\text{c1}\,k_1\,(a-L0_1)+a^2\left(\sqrt{b^2}\,\text{c2}\,k_2\,(b-L0_2)+b^2\,\mathbb{i}_{p,zz}\,\theta_p''[t]\right)}{a\,b} == 0 \end{pmatrix}$$

```
simpStep3 //. dispSimp //
   Expand // MatrixForm //
  TraditionalForm
```

$$\begin{pmatrix} -\frac{\sqrt{a^2}\,b\,k_1\,\text{L0}_1\,\text{r1x}}{a}+\sqrt{a^2}\,b\,k_1\,\text{r1x}-\frac{a\sqrt{b^2} \\ -\frac{\sqrt{a^2}\,b\,k_1\,\text{L0}_1\,\text{r1y}}{a}+\sqrt{a^2}\,b\,k_1\,\text{r1y}-\frac{a\sqrt{b^2}\,k_2\,\text{L0}_2\,\text{r}}{b} \\ -\frac{\sqrt{a^2}\,b\,\text{c1}\,k_1\,\text{L0}_1}{a}+\sqrt{a^2}\,b\,\text{c1}\,k_1-\frac{a\sqrt{b^2}\,\text{c}}{b} \end{pmatrix}$$

$$\left(\texttt{simpStep4} = k_1\, b\,(a - L0_1)\begin{pmatrix} r1x \\ r1y \\ c1 \end{pmatrix} + k_2\, a\,(b - L0_1)\begin{pmatrix} r2x \\ r2y \\ c2 \end{pmatrix} + \begin{pmatrix} 0 \\ -m_p\, g\, a\, b \\ 0 \end{pmatrix} - a\, b\begin{pmatrix} m_p & 0 & 0 \\ 0 & m_p & 0 \\ 0 & 0 & -I_p \end{pmatrix}.\begin{pmatrix} x_p''[t] \\ y_p''[t] \\ \theta_p''[t] \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}\right) \texttt{ // MatrixForm}$$

$$\begin{pmatrix} b\, r1x\, k_1\,(a - L0_1) + a\, r2x\, k_2\,(b - L0_1) - a\, b\, m_p\, x_p''[t] \\ b\, r1y\, k_1\,(a - L0_1) + a\, r2y\, k_2\,(b - L0_1) - a\, b\, g\, m_p - a\, b\, m_p\, y_p''[t] \\ b\, c1\, k_1\,(a - L0_1) + a\, c2\, k_2\,(b - L0_1) + a\, b\, i_p\, \theta_p''[t] \end{pmatrix}$$

In[24]:=
```
terms2 //. dispSimp // MatrixForm // TraditionalForm
terms3 //. dispSimp // MatrixForm // TraditionalForm
```

Out[24]//TraditionalForm=

$$\begin{pmatrix} l_p\, c(\theta_p) + h_p\, s(\theta_p) - 2\, x_p + 2\, x_1 \to 2\,\text{r1x} \\ -\frac{1}{2}\, h_p\, c(\theta_p) + \frac{1}{2}\, l_p\, s(\theta_p) - y_p + y_1 \to \text{r1y} \\ \frac{1}{2}\, h_p\, c(\theta_p) - \frac{1}{2}\, l_p\, s(\theta_p) + y_p - y_1 \to -\text{r1y} \\ -\frac{1}{2}\, l_p\, c(\theta_p) + \frac{1}{2}\, h_p\, s(\theta_p) - x_p + x_2 \to \text{r2x} \\ -\frac{1}{2}\, h_p\, c(\theta_p) - \frac{1}{2}\, l_p\, s(\theta_p) - y_p + y_2 \to \text{r2y} \\ h_p\, c(\theta_p) + l_p\, s(\theta_p) + 2\, y_p - 2\, y_2 \to -2\,\text{r2y} \\ l_p\,((y_1 - y_p)\, c(\theta_p) + x_1\,(-s(\theta_p)) + x_p\, s(\theta_p)) \to \text{dr1} \\ h_p\,(x_1\, c(\theta_p) - x_p\, c(\theta_p) + (y_1 - y_p)\, s(\theta_p)) \to \text{dr2} \\ h_p\,(x_2\, c(\theta_p) - x_p\, c(\theta_p) + (y_2 - y_p)\, s(\theta_p)) \to \text{dr4} \\ l_p\,((y_p - y_2)\, c(\theta_p) + x_2\, s(\theta_p) - x_p\, s(\theta_p)) \to \text{dr3} \end{pmatrix}$$

Out[25]//TraditionalForm=

$$\begin{pmatrix} \sqrt{\text{r1x}^2 + \text{r1y}^2} \to a \\ \sqrt{\text{r2x}^2 + \text{r2y}^2} \to b \\ \text{dr1} + \text{dr2} \to 2\,\text{c1} \\ \text{dr3} + \text{dr4} \to 2\,\text{c2} \\ \text{r1x}^2 + \text{r1y}^2 \to a^2 \\ \text{r2x}^2 + \text{r2y}^2 \to b^2 \\ \sqrt{a^2} \to a \\ \sqrt{b^2} \to b \end{pmatrix}$$

$x_p, y_p, \theta_p = f\,(x_1, y_1, x_2, y_2, k_1, k_2, l_p, h_p)$

---

non - conver forces :

aerodynamic $= f\,(\dot{x}_p, \dot{y}_p, \theta_p, w_x, w_y)$,

w for wind components. $= f\,(\text{relV}_x, \text{relV}_y)$, **relV is relative to air**

dumping $= f\,(\dot{l}_i) = f\,(\dot{x}_i, \dot{y}_i, \dot{x}_p, \dot{y}_p)$

---

what needs to be done in order to keep horizontal payload? :

simpStep1 /. θ_p[t] → 0 /. dispSimp // MatrixForm // TraditionalForm

---

non - dimensional settings

$\tilde{y_p}[t] = y_p[t] \,/\, L0_1$ **or any other of the lengths varialbes** $(x_p, r1x, r1y, r2x, r2y, h_p, l_p)$

$t = \tau / \omega_s$

$\omega_s{}^2 = \frac{k_1}{m_p}\left[\frac{g}{l} = \frac{1}{s^2}\right]$

$k_{ratio} = \frac{k_2}{k_1}$

```
(simpStepNonDim =
    Map[Map[Divide[#, mp ] &, #] &, simpStep2] //
      Expand // Simplify) // MatrixForm
```

$$\begin{pmatrix} \frac{\sqrt{b^2}\ r1x\ k_1\ (a-L0_1)+\sqrt{a^2}\ r2x\ k_2\ (b-L0_2)}{\sqrt{a^2}\ \sqrt{b^2}\ m_p} == x_p''[t] \\ \frac{\sqrt{b^2}\ r1y\ k_1\ (a-L0_1)+\sqrt{a^2}\ r2y\ k_2\ (b-L0_2)}{\sqrt{a^2}\ \sqrt{b^2}\ m_p} == g + y_p''[t] \\ \frac{\sqrt{b^2}\ c1\ k_1\ (a-L0_1)+\sqrt{a^2}\ \left(c2\ k_2\ (b-L0_2)+\sqrt{b^2}\ \mathbb{i}_{p,zz}\ \theta_p''[t]\right)}{\sqrt{a^2}\ \sqrt{b^2}\ m_p} == 0 \end{pmatrix}$$

```
simpStepNonDim // Expand // MatrixForm
```

now all vars are non - dim :

$$\begin{pmatrix} r1x\ L0_1\ \frac{k_1}{m_p} + r2x\ L0_1\ \frac{k_2}{m_p} - r1x\ L0_1\ \frac{k_1}{m_p}\ \frac{L0_1}{a} - r2x\ L0_1 \\ L0_1\ \frac{r1y\ k_1}{m_p} + L0_1\ \frac{r2y\ k_2}{m_p} - L0_1\ \frac{r1y\ k_1\ L0_1}{m_p\ a} - L0_1\ \frac{r2y\ k_2\ L0_2}{m_p\ b} \\ L0_1{}^2\ c1\ k_1 + L0_1{}^2\ c2\ k_2 - L0_1{}^2\ \frac{c1\ k_1\ L0_1}{a} - L0_1{}^2\ \frac{c2\ k_2\ L0_2}{b} \end{pmatrix}$$

$$\begin{pmatrix} -\frac{k_1\, L0_1^2\, r1x}{a\, m_p} - \frac{k_2\, L0_2\, L0_1\, r2x}{b\, m_p} + \frac{k_1\, L0_1\, r1x}{m_p} + \frac{k_2\, L0_1\, r2x}{m_p} = L0_1\, \omega_s^2\, x_p''(t) \\ -\frac{k_1\, L0_1^2\, r1y}{a\, m_p} - \frac{k_2\, L0_2\, L0_1\, r2y}{b\, m_p} - g + \frac{k_1\, L0_1\, r1y}{m_p} + \frac{k_2\, L0_1\, r2y}{m_p} = L0_1\, \omega_s^2\, y_p''(t) \\ -\frac{c1\, k_1\, L0_1^3}{a} - \frac{c2\, k_2\, L0_2\, L0_1^2}{b} + c1\, k_1\, L0_1^2 + c2\, k_2\, L0_1^2 = -\omega_s^2\, \mathbb{i}_{p,zz}\, \theta_p''(t) \end{pmatrix}$$

turns to :

$$\begin{pmatrix} \mathrm{r1x}\left(1-\frac{\mathrm{L0}_1}{a}\right)+\mathrm{r2x}\,\frac{k_2}{k_1}\left(1-\frac{\mathrm{L0}_2}{b}\right)==x_p''[t] \\ \mathrm{r1y}\left(1-\frac{\mathrm{L0}_1}{a}\right)+\mathrm{r2y}\,\frac{k_2}{k_1}\left(1-\frac{\mathrm{L0}_2}{b}\right)-\frac{g}{\mathrm{L0}_1}\,\frac{1}{\omega_s{}^2}==y_p''[t] \\ -\frac{\mathrm{L0}_1{}^2\,k_1}{I_{p,zz}\,\omega_s{}^2}\left(c1\left(1-\frac{\mathrm{L0}_1}{a}\right)+c2\,\frac{k_2}{k_1}\left(1-\frac{\mathrm{L0}_2}{b}\right)\right)==\theta_p''[t] \end{pmatrix}$$

TraditionalForm

$$\begin{pmatrix} \frac{k_2\,\mathrm{r2x}\,(b-\mathrm{L0}_2)}{b\,k_1}+\mathrm{r1x}=\frac{\mathrm{L0}_1\,\mathrm{r1x}}{a}+x_p''(t) \\ \frac{k_2\,\mathrm{r2y}\,(b-\mathrm{L0}_2)}{b\,k_1}+\mathrm{r1y}=\frac{\mathrm{L0}_1\,\mathrm{r1y}}{a}+\frac{g}{\mathrm{L0}_1\,\omega_s^2}+y_p''(t) \\ \frac{\mathrm{L0}_1^2\,(b\,c1\,k_1\,(\mathrm{L0}_1-a)-a\,c2\,k_2\,(b+\mathrm{L0}_2))}{a\,b\,\omega_s^2\,i_{p,zz}}=\theta_p''(t) \end{pmatrix}$$

In[27]:= $\mathrm{terms4}=\left\{1-\frac{\mathrm{L0}_1}{a}\to A,\; 1-\frac{\mathrm{L0}_2}{b}\to B,\; \frac{k_2}{k_1}\to k,\; \frac{g}{\mathrm{L0}_1}\,\frac{1}{\omega_s{}^2}\to D,\; \frac{\mathrm{L0}_1{}^2\,k_1}{I_{p,zz}\,\omega_s{}^2}\to E\right\}$

Out[27]= $\left\{1-\frac{\mathrm{L0}_1}{a}\to A,\ 1-\frac{\mathrm{L0}_2}{b}\to B,\ \frac{k_2}{k_1}\to k,\ \frac{g}{\mathrm{L0}_1\,\omega_s^2}\to D,\ \frac{k_1\,\mathrm{L0}_1^2}{\omega_s^2\,i_{p,zz}}\to e\right\}$

$$\frac{g}{\mathrm{L0}_1}\,\frac{1}{\omega_s{}^2}==g\,\frac{1}{\mathrm{L0}_1}\,\frac{1}{k_1}\,m_p\left[\frac{m}{s^2}\,\frac{1}{m}\,\frac{1}{\frac{kg}{s^2}}\,kg\right]$$

$$\frac{\mathrm{L0}_1{}^2\,k_1}{I_{p,zz}\,\omega_s{}^2}==\frac{\mathrm{L0}_1{}^2\,k_1}{I_{p,zz}\,k_1}\,m_p\left[\frac{m^2\,kg}{kg\,m^2}\right]$$

$y_1=y_2$, $k_1=k_2$, $\mathrm{L0}_1=\mathrm{L0}_2$

$\theta_p=0$, $x_p=\frac{x_1+x_2}{2}$, $\frac{y_p}{\mathrm{L0}_1}=-\left(1+\frac{1}{2}D\right)$

trajectory :

$\tau=0$: $\ddot{y}=1\,m/s^2$ until $y_1=y_2=10\,\mathrm{L0}_1$

$\ddot{y}=-1\,m/s^2$ until $\dot{y}_1=\dot{y}_2=0$

$\ddot{x}_1=\ddot{x}_2=1\,m/s^2$ until $\dot{x}_1=\dot{x}_2=2\,m/s$

disterbunce can be input by x_1 += $5\,\mathrm{L0}_1$ over $\frac{1}{100\sqrt{\omega_s}}$

```
(* planar mass with springs *)
(trimmedEq = quadEqNominal /.
    {x1'[t] → 0, x2'[t] → 0, x1''[t] → 0, x2''[t] → 0, θ1'[t] → 0, θ2'[t] → 0,
     θ1''[t] → 0, θ2''[t] → 0, θ1[t] → 0, θ2[t] → 0}) // MatrixForm // TraditionalForm
```

$$\left(\begin{array}{l}
\frac{k_1\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))\left(\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t)}\right.}{2\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))^2+\left(\frac{1}{2}\,h_p \cos(\theta_p(t))-\right.}} \\
\frac{k_1\left(-\frac{1}{2}\,h_p \cos(\theta_p(t))+\frac{1}{2}\,l_p \sin(\theta_p(t))-y_p(t)+y_1(t)\right)\left(\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))^2+}\right.}{\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))^2+\left(\frac{1}{2}\,h_p \cos(\theta_p(t))-\frac{1}{2}\,l_p\right.}} \\
i_{p,zz}\,\theta_p''(t)+\frac{k_1\,(h_p\,(x_1(t)\cos(\theta_p(t))-x_p(t)\cos(\theta_p(t))+(y_1(t)-y_p(t))\sin(\theta_p(t)))+l_p\,(x_1(t)\,(-\sin(\theta_p(t)))+x_p(t)\sin(\theta_p(t))+(y_1(t)-y_p(t))\cos(\theta_p(t))))\left(\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t)}\right.}{2\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1(t))^2+\left(\frac{1}{2}\,h_p \cos(\theta_p(t))-\frac{1}{2}\,l_p \sin(\theta_p(t))+y_p(t)-y\right.}}
\end{array}\right.$$

```
eq2D =
 {trimmedEq[[1]]} /. θp[t] → 0 /. y1[t] → 0 /. y2[t] → 0 /. yp[t] → 0 /. lp → 0 /. hp → 0 /.
    dispSimp // Expand // Simplify // TraditionalForm
```

$$\left\{\frac{k_1\left((x_1-x_p)^2-\mathrm{L0}_1\sqrt{(x_1-x_p)^2}\right)}{x_1-x_p}+\frac{k_2\left((x_2-x_p)^2-\mathrm{L0}_2\sqrt{(x_2-x_p)^2}\right)}{x_2-x_p}=m_p\,x_p''\right\}$$

---

case for elastic pendulum:

in 2 D case the 2 DOF are x, y_p, looking at lumped mass payload.
X_1, X_2 → 0, k_2 → 0, l, h_p → 0 as well

```
L2D =
 L /. {x1[t] → 0, x1'[t] → 0, x1''[t] → 0, x2[t] → 0, x2'[t] → 0, x2''[t] → 0, θ1'[t] → 0,
      θ1''[t] → 0, θ2'[t] → 0, θ2''[t] → 0, θ1[t] → 0, θ2[t] → 0} /.
    {y1[t] → 0, y1'[t] → 0, y1''[t] → 0, y2[t] → 0, y2'[t] → 0, y2''[t] → 0} /.
   lp → 0 /. hp → 0 /. k2 → 0(*/.θp[t]→0*)
```

$$-g\,m_p\,y_p[t]-\frac{1}{2}\,k_1\left(-\mathrm{L0}_1+\sqrt{x_p[t]^2+y_p[t]^2}\right)^2+\frac{1}{2}\,m_p\left(x_p'[t]^2+y_p'[t]^2\right)+\frac{1}{2}\,\mathbb{i}_{p,zz}\,\theta_p'[t]^2$$

In[33]:=
```
L2D =
 L /. {x1[t] → 0, x1'[t] → 0, x1''[t] → 0, x2[t] → 0, x2'[t] → 0, x2''[t] → 0, θ1'[t] → 0,
      θ1''[t] → 0, θ2'[t] → 0, θ2''[t] → 0, θ1[t] → 0, θ2[t] → 0} /.
    {y1[t] → 0, y1'[t] → 0, y1''[t] → 0, y2[t] → 0, y2'[t] → 0, y2''[t] → 0} /.
   lp → 0 /. hp → 0 /. k2 → 0(*/.θp[t]→0*)
```

Out[33]= $$-g\,m_p\,y_p[t]-\frac{1}{2}\,k_1\left(-\mathrm{L0}_1+\sqrt{x_p[t]^2+y_p[t]^2}\right)^2+\frac{1}{2}\,m_p\left(x_p'[t]^2+y_p'[t]^2\right)+\frac{1}{2}\,\mathbb{i}_{p,zz}\,\theta_p'[t]^2$$

In[34]:=
```
(quadEqNominal2D =
    EulerEquations[L2D, {x_p[t], y_p[t], θ_p[t]}, t](*[[
      All,1]]*)(*==Q*) // Expand // Simplify) /.
  dispSimp // MatrixForm // TraditionalForm
```

Out[34]//TraditionalForm=

$$\begin{pmatrix} k_1\, x_p \left(\frac{L0_1}{\sqrt{x_p^2+y_p^2}} - 1\right) = m_p\, x_p'' \\ k_1\, y_p \left(\frac{L0_1}{\sqrt{x_p^2+y_p^2}} - 1\right) = m_p\,(g + y_p'') \\ i_p\, \theta_p'' = 0 \end{pmatrix}$$

```
quadEqNominal2DEquib = quadEqNominal2D /. {x_p''[t] → 0, y_p''[t] → 0}
```

$$\left\{k_1\, x_p[t] \left(-1 + \frac{L0_1}{\sqrt{x_p[t]^2 + y_p[t]^2}}\right) == 0,\; k_1\, y_p[t] \left(-1 + \frac{L0_1}{\sqrt{x_p[t]^2 + y_p[t]^2}}\right) == g\, m_p,\; i_{p,zz}\, \theta_p''[t] == 0\right\}$$

```
Solve[quadEqNominal2DEquib, {x_p[t], y_p[t](*,θ_p[t]*)}]
```

$$\left\{\left\{x_p[t] \to 0,\; y_p[t] \to \frac{-k_1\, L0_1 - g\, m_p}{k_1}\right\},\; \left\{x_p[t] \to 0,\; y_p[t] \to \frac{k_1\, L0_1 - g\, m_p}{k_1}\right\}\right\}$$

assumption is y_p>0 fits to $y_p[t] \to \frac{k_1 L0_1 - g\, m_p}{k_1} = L0_1 - \frac{g\, m_p}{k_1}$,
so $L0_1 > \frac{g\, m_p}{k_1}$ otherwise it means the spring k_1 is to small and weak.

assumption is y_p<0 fits to $y_p[t] \to -\frac{k_1 L0_1 - g\, m_p}{k_1} = -L0_1 - \frac{g\, m_p}{k_1} = -\left(L0_1 + \frac{g\, m_p}{k_1}\right)$

$$y_p[t] \to -L0_1 - g\,\frac{m_p}{k_1}$$

$$\tilde{y}_p[t] \to -1 - \frac{g}{L0_1}\,\frac{m_p}{k_1} = -1 - B$$

```
Solve[a x^2 + b x + c == 0, x]
```

$$\left\{\left\{x \to \frac{-b - \sqrt{b^2 - 4\,a\,c}}{2\,a}\right\},\; \left\{x \to \frac{-b + \sqrt{b^2 - 4\,a\,c}}{2\,a}\right\}\right\}$$

In[60]:= $\text{Gterm} = \left\{G \to \frac{g}{L0_1}\,\frac{m_p}{k_1}\right\}$

Out[60]= $\left\{G \to \frac{g\, m_p}{k_1\, L0_1}\right\}$

In[190]:= **quadEqNominal2D(*//MatrixForm*)**

Out[190]= $$\left\{k_1\, x_p[t]\left(-1+\frac{L0_1}{\sqrt{x_p[t]^2+y_p[t]^2}}\right)==m_p\, x_p''[t],\; k_1\, y_p[t]\left(-1+\frac{L0_1}{\sqrt{x_p[t]^2+y_p[t]^2}}\right)==m_p\,(g+y_p''[t]),\; \mathbb{i}_{p,zz}\,\theta_p''[t]==0\right\}$$

In[197]:= **(equibTerms = {D[x_p[t], {t, 1}] → 0, D[x_p[t], {t, 2}] → 0, D[y_p[t], {t, 1}] → 0, D[y_p[t], {t, 2}] → 0}) // MatrixForm**
(equibMatrix = {quadEqNominal2D[[1]], quadEqNominal2D[[2]]} /. equibTerms) // MatrixForm

Out[197]//MatrixForm=

$$\begin{pmatrix} x_p'[t]\to 0 \\ x_p''[t]\to 0 \\ y_p'[t]\to 0 \\ y_p''[t]\to 0 \end{pmatrix}$$

Out[198]//MatrixForm=

$$\begin{pmatrix} k_1\, x_p[t]\left(-1+\frac{L0_1}{\sqrt{x_p[t]^2+y_p[t]^2}}\right)==0 \\ k_1\, y_p[t]\left(-1+\frac{L0_1}{\sqrt{x_p[t]^2+y_p[t]^2}}\right)==g\, m_p \end{pmatrix}$$

In[199]:= **Solve[equibMatrix, {x_p[t], y_p[t]}]**

Out[199]= $$\left\{\left\{x_p[t]\to 0,\; y_p[t]\to\frac{-k_1\, L0_1-g\, m_p}{k_1}\right\},\left\{x_p[t]\to 0,\; y_p[t]\to\frac{k_1\, L0_1-g\, m_p}{k_1}\right\}\right\}$$

In[203]:= **nn = 1; Series[√f[x, y] , {x, 0, nn}, {y, 0, nn}]**

Out[203]= $$\left(\sqrt{f[0,0]}+\frac{f^{(0,1)}[0,0]\,y}{2\sqrt{f[0,0]}}+O[y]^2\right)+\left(\frac{f^{(1,0)}[0,0]}{2\sqrt{f[0,0]}}+\left(-\frac{f^{(0,1)}[0,0]\,f^{(1,0)}[0,0]}{4\,f[0,0]^{3/2}}+\frac{f^{(1,1)}[0,0]}{2\sqrt{f[0,0]}}\right)y+O[y]^2\right)x+O[x]^2$$

In[215]:= **nn = 1; Series[1/√(x_p[t]^2 + y_p[t]^2) , {x_p[t], x_0, nn}, {y_p[t], y_0, nn}]**

Out[215]= $$\left(\frac{1}{\sqrt{x_0^2+y_0^2}}-\frac{y_0\,(y_p[t]-y_0)}{\left(x_0^2+y_0^2\right)^{3/2}}+O[y_p[t]-y_0]^2\right)+\left(-\frac{x_0}{\left(x_0^2+y_0^2\right)^{3/2}}+\frac{3\,x_0\,y_0\,(y_p[t]-y_0)}{\left(x_0^2+y_0^2\right)^{5/2}}+O[y_p[t]-y_0]^2\right)(x_p[t]-x_0)+O[x_p[t]-x_0]^2$$

In[216]:= $$\frac{1+y}{-y}\ /.\ y\to-(1+G)$$

Out[216]= $$\frac{G}{-1-G}$$

In[228]:= $\sqrt{\frac{G}{1+G}\frac{k_1}{m_p}}$ /. Gterm // Simplify

Out[228]= $\sqrt{\frac{g\,k_1}{k_1\,L0_1 + g\,m_p}}$

In[247]:= G /. Gterm /. $L0_1$ >> 1

In[248]:= G /. Gterm /. $k_1\,L0_1$ >> $g\,m_p$

ReplaceAll::reps : $\{k_1\,L0_1\}$ is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing. ≫

General::stream : $g\,m_p$ is not a string, InputStream[ ], or OutputStream[ ]. ≫

Out[248]= $\frac{g\,m_p}{k_1\,L0_1}$ /. $k_1\,L0_1$ >> $g\,m_p$

In[231]:= $\sqrt{G\frac{k_1}{m_p}}$ /. Gterm // Simplify

Out[231]= $\sqrt{\frac{g}{L0_1}}$

In[249]:= Series$\left[\frac{G}{1+G}, \{G, G\epsilon, 3\}\right]$

Series$\left[\frac{G}{1+G}, \{G, 0, 3\}\right]$

Series[a, {a, 0, 13}]

Out[249]= $\frac{G\epsilon}{1+G\epsilon} + \frac{G-G\epsilon}{(1+G\epsilon)^2} - \frac{(G-G\epsilon)^2}{(1+G\epsilon)^3} + \frac{(G-G\epsilon)^3}{(1+G\epsilon)^4} + O[G-G\epsilon]^4$

Out[250]= $G - G^2 + G^3 + O[G]^4$

Out[251]= $a + O[a]^{14}$

In[202]:= equibMatrix // MatrixForm

Out[202]//MatrixForm=

$$\begin{pmatrix} k_1\,x_p[t]\left(-1 + \frac{L0_1}{\sqrt{x_p[t]^2+y_p[t]^2}}\right) == 0 \\ k_1\,y_p[t]\left(-1 + \frac{L0_1}{\sqrt{x_p[t]^2+y_p[t]^2}}\right) == g\,m_p \end{pmatrix}$$

$$eq1 = \left(\delta x_p\left(-\frac{1}{1+G} - 1\right) == \delta\dot{x}_p\right)$$

$$eq2 = \left(-\delta y_p\left(1 - \frac{G}{1+G}\right) + \delta y_p\left(-\frac{1}{1+G} - 1\right) + 2 == \delta\dot{y}_p\right)$$

In[144]:=

```
(eqPerturb = ( (-1/(1+G) - 1)    0
               0                 (-1/(1+G) - 1) - (1 - G/(1+G)) ) . (δxp, δyp) + (0, 2) == D2 (δxp, δyp)) // MatrixForm
```

Out[144]//MatrixForm=

$$\left\{\left\{\left(-1-\frac{1}{1+G}\right)\delta x_p\right\},\left\{2+\left(-2-\frac{1}{1+G}+\frac{G}{1+G}\right)\delta y_p\right\}\right\} == \{\{D2\,\delta x_p\},\{D2\,\delta y_p\}\}$$

In[145]:=

```
K = -( (-1/(1+G) - 1)    0
       0                 (-1/(1+G) - 1) - (1 - G/(1+G)) )
M = IdentityMatrix[2]
F = (0, 2)
(*M ẍ+C ẋ+K x ==F*)
```

Out[145]= $\left\{\left\{1+\frac{1}{1+G},0\right\},\left\{0,2+\frac{1}{1+G}-\frac{G}{1+G}\right\}\right\}$

Out[146]= {{1, 0}, {0, 1}}

Out[147]= {{0}, {2}}

In[156]:=

```
K // Expand // Simplify // MatrixForm
```

Out[156]//MatrixForm=

$$\begin{pmatrix} 1+\frac{1}{1+G} & 0 \\ 0 & \frac{3+G}{1+G} \end{pmatrix}$$

In[185]:=

```
eq1 = Det[K - ω^2 M] == 0
eq2 = Det[K - ω^2 M] == 0 // Simplify
eq3 = Det[K - ω^2 M] == 0 // FullSimplify
(solution = Solve[eq1, ω])
  (*//Simplify//MatrixForm*) // TraditionalForm
```

Out[185]= $2+\frac{1}{(1+G)^2}-\frac{G}{(1+G)^2}+\frac{3}{1+G}-\frac{G}{1+G}-3\,\omega^2-\frac{2\,\omega^2}{1+G}+\frac{G\,\omega^2}{1+G}+\omega^4 == 0$

Out[186]= $\frac{1}{1+G}\left(6-5\,\omega^2+\omega^4+G^2\,(-1+\omega^2)^2+G\,(5-7\,\omega^2+2\,\omega^4)\right) == 0$

Out[187]= $4+\frac{2}{1+G}+\omega^4+G\,(-1+\omega^2)^2 == 5\,\omega^2$

Out[188]//TraditionalForm=

$$\left\{\left\{\omega\to-\frac{\sqrt{G+2}}{\sqrt{G+1}}\right\},\left\{\omega\to\frac{\sqrt{G+2}}{\sqrt{G+1}}\right\},\left\{\omega\to-\frac{\sqrt{G+3}}{\sqrt{G+1}}\right\},\left\{\omega\to\frac{\sqrt{G+3}}{\sqrt{G+1}}\right\}\right\}$$

$$\frac{\sqrt{G+2}}{\sqrt{G+1}} \quad \frac{G+2}{G+1}$$

In[163]:=
```
solution /. Gterm // Simplify
```

Out[163]=
$$\left\{\left\{\omega \to -\frac{1}{\sqrt{\frac{k_1\ LO_1+g\ m_p}{2\ k_1\ LO_1+g\ m_p}}}\right\},\ \left\{\omega \to \frac{1}{\sqrt{\frac{k_1\ LO_1+g\ m_p}{2\ k_1\ LO_1+g\ m_p}}}\right\},\ \left\{\omega \to -\frac{1}{\sqrt{\frac{k_1\ LO_1+g\ m_p}{3\ k_1\ LO_1+g\ m_p}}}\right\},\ \left\{\omega \to \frac{1}{\sqrt{\frac{k_1\ LO_1+g\ m_p}{3\ k_1\ LO_1+g\ m_p}}}\right\}\right\}$$

In[126]:=
$$\sqrt{\frac{2\ k_1\ LO_1 + g\ m_p}{k_1\ LO_1 + g\ m_p}}\ /.\ k_1 \to 0$$

$$\sqrt{\frac{2\ k_1\ LO_1 + g\ m_p}{k_1\ LO_1 + g\ m_p}}\ /.\ g \to 0$$

Out[126]= 1

Out[127]= $\sqrt{2}$

In[125]:=
```
Infinity
```

Out[125]= ∞

In[165]:=
$$\left\{\text{Limit}\left[\frac{\sqrt{2+G}}{\sqrt{1+G}},\ G\ \text{->}\ \text{Limit}\left[\frac{g\ m_p}{k_1\ LO_1},\ k_1 \to \text{Infinity}\right]\right]\right\}$$

$$\left\{\text{Limit}\left[\frac{\sqrt{3+G}}{\sqrt{1+G}},\ G\ \text{->}\ \text{Limit}\left[\frac{g\ m_p}{k_1\ LO_1},\ k_1 \to \text{Infinity}\right]\right]\right\}$$

Out[165]= $\{\sqrt{2}\}$

Out[166]= $\{\sqrt{3}\}$

$$\omega_{real} = \frac{g}{LO_1}$$

```
equibState = {xp[t] → 0 + epsX, yp[t] → -L01 - g mp/k1 + epsY}
quadEqNominal2D /. equibState
```

$$\left\{x_p[t] \to \text{epsX},\ y_p[t] \to \text{epsY} - \text{L0}_1 - \frac{g\, m_p}{k_1}\right\}$$

$$\left\{\text{epsX}\, k_1 \left(-1 + \frac{\text{L0}_1}{\sqrt{\text{epsX}^2 + \left(\text{epsY} - \text{L0}_1 - \frac{g\, m_p}{k_1}\right)^2}}\right) == m_p\, x_p''[t],\right.$$

$$k_1 \left(\text{epsY} - \text{L0}_1 - \frac{g\, m_p}{k_1}\right) \left(-1 + \frac{\text{L0}_1}{\sqrt{\text{epsX}^2 + \left(\text{epsY} - \text{L0}_1 - \frac{g\, m_p}{k_1}\right)^2}}\right) == m_p\, (g + y_p''[t]),$$

$$\left. \mathbb{i}_{p,zz}\, \theta_p''[t] == 0\right\}$$

```
Series[Sin[x + y], {x, 0, 3}, {y, 0, 3}]
```

$$\left(y - \frac{y^3}{6} + O[y]^4\right) + \left(1 - \frac{y^2}{2} + O[y]^4\right) x + \left(-\frac{y}{2} + \frac{y^3}{12} + O[y]^4\right) x^2 + \left(-\frac{1}{6} + \frac{y^2}{12} + O[y]^4\right) x^3 + O[x]^4$$

```
Series[f[y, x], {x, 0, 5}, {y, 0, 5}]
```

$$\left(f[0,0]+f^{(1,0)}[0,0]\,y+\frac{1}{2}f^{(2,0)}[0,0]\,y^2+\frac{1}{6}f^{(3,0)}[0,0]\,y^3+\frac{1}{24}f^{(4,0)}[0,0]\,y^4+\frac{1}{120}f^{(5,0)}[0,0]\,y^5+O[y]^6\right)+$$
$$\left(f^{(0,1)}[0,0]+f^{(1,1)}[0,0]\,y+\frac{1}{2}f^{(2,1)}[0,0]\,y^2+\frac{1}{6}f^{(3,1)}[0,0]\,y^3+\frac{1}{24}f^{(4,1)}[0,0]\,y^4+\frac{1}{120}f^{(5,1)}[0,0]\,y^5+O[y]^6\right)x+$$
$$\left(\frac{1}{2}f^{(0,2)}[0,0]+\frac{1}{2}f^{(1,2)}[0,0]\,y+\frac{1}{4}f^{(2,2)}[0,0]\,y^2+\frac{1}{12}f^{(3,2)}[0,0]\,y^3+\frac{1}{48}f^{(4,2)}[0,0]\,y^4+\frac{1}{240}f^{(5,2)}[0,0]\,y^5+O[y]^6\right)x^2+$$
$$\left(\frac{1}{6}f^{(0,3)}[0,0]+\frac{1}{6}f^{(1,3)}[0,0]\,y+\frac{1}{12}f^{(2,3)}[0,0]\,y^2+\frac{1}{36}f^{(3,3)}[0,0]\,y^3+\frac{1}{144}f^{(4,3)}[0,0]\,y^4+\frac{1}{720}f^{(5,3)}[0,0]\,y^5+O[y]^6\right)x^3+$$
$$\left(\frac{1}{24}f^{(0,4)}[0,0]+\frac{1}{24}f^{(1,4)}[0,0]\,y+\frac{1}{48}f^{(2,4)}[0,0]\,y^2+\frac{1}{144}f^{(3,4)}[0,0]\,y^3+\frac{1}{576}f^{(4,4)}[0,0]\,y^4+\frac{f^{(5,4)}[0,0]\,y^5}{2880}+O[y]^6\right)x^4+$$
$$\left(\frac{1}{120}f^{(0,5)}[0,0]+\frac{1}{120}f^{(1,5)}[0,0]\,y+\frac{1}{240}f^{(2,5)}[0,0]\,y^2+\frac{1}{720}f^{(3,5)}[0,0]\,y^3+\frac{f^{(4,5)}[0,0]\,y^4}{2880}+\frac{f^{(5,5)}[0,0]\,y^5}{14\,400}+O[y]^6\right)x^5+O[x]^6$$

```
nn = 1; Series[f[y, x], {x, 0, nn}, {y, 0, nn}]
```

$$\left(f[0,0]+f^{(1,0)}[0,0]\,y+O[y]^2\right)+\left(f^{(0,1)}[0,0]+f^{(1,1)}[0,0]\,y+O[y]^2\right)x+O[x]^2$$

In[130]:=
```
nn = 1; Series[Sqrt[f[x, y]], {x, 0, nn}, {y, 0, nn}]
```

Out[130]=
$$\left(\sqrt{f[0,0]}+\frac{f^{(0,1)}[0,0]\,y}{2\sqrt{f[0,0]}}+O[y]^2\right)+\left(\frac{f^{(1,0)}[0,0]}{2\sqrt{f[0,0]}}+\left(-\frac{f^{(0,1)}[0,0]\,f^{(1,0)}[0,0]}{4\,f[0,0]^{3/2}}+\frac{f^{(1,1)}[0,0]}{2\sqrt{f[0,0]}}\right)y+O[y]^2\right)x+O[x]^2$$

$$\left(\sqrt{f[0,0]}+\frac{f^{(1,0)}[0,0]\,y}{2\sqrt{f[0,0]}}+O[y]^2\right)+\left(\frac{f^{(0,1)}[0,0]}{2\sqrt{f[0,0]}}+\left(-\frac{f^{(0,1)}[0,0]\,f^{(1,0)}[0,0]}{4\,f[0,0]^{3/2}}+\frac{f^{(1,1)}[0,0]}{2\sqrt{f[0,0]}}\right)y+O[y]^2\right)x+O[x]^2$$

$$\sqrt{x_p^2+y_p^2} = \sqrt{x_p^2+y_p^2} + \frac{2\,y_p}{2\sqrt{x_p^2+y_p^2}}\,y + \frac{2\,x_p}{2\sqrt{x_p^2+y_p^2}}\,x + O(\epsilon)$$

```
Series[f[x], {x, 0, 5}]
```

$$f[0] + f'[0]\,x + \frac{1}{2}\,f''[0]\,x^2 + \frac{1}{6}\,f^{(3)}[0]\,x^3 + \frac{1}{24}\,f^{(4)}[0]\,x^4 + \frac{1}{120}\,f^{(5)}[0]\,x^5 + O[x]^6$$

```
Series[√f[x], {x, 0, 5}]
```

$$\sqrt{f[0]} + \frac{f'[0]\,x}{2\sqrt{f[0]}} + \frac{\left(-f'[0]^2 + 2\,f[0]\,f''[0]\right)x^2}{8\,f[0]^{3/2}} + \frac{\left(3\,f'[0]^3 - 6\,f[0]\,f'[0]\,f''[0] + 4\,f[0]^2\,f^{(3)}[0]\right)x^3}{48\,f[0]^{5/2}} + \frac{1}{384\,f[0]^{7/2}} \left(-15\,f'[0]^4 + 36\,f[0]\,f'[0]^2\,f''[0] - 12\,f[0]^2\,f''[0]^2 - 16\,f[0]^2\,f'[0]\,f^{(3)}[0] + 8\,f[0]^3\,f^{(4)}[0]\right)x^4 + \frac{1}{3840\,f[0]^{9/2}} \left(105\,f'[0]^5 - 300\,f[0]\,f'[0]^3\,f''[0] + 180\,f[0]^2\,f'[0]\,f''[0]^2 + 120\,f[0]^2\,f'[0]^2\,f^{(3)}[0] - 80\,f[0]^3\,f''[0]\,f^{(3)}[0] - 40\,f[0]^3\,f'[0]\,f^{(4)}[0] + 16\,f[0]^4\,f^{(5)}[0]\right)x^5 + O[x]^6$$

```
Series[quadEqNominal2D[[1]], {xp[t], 0, 3}, {yp[t], 0, 3}] // Simplify
```

$$\left(\frac{k_1\,L0_1}{y_p[t]} - k_1 + O[y_p[t]]^4\right)x_p[t] + \left(-\frac{k_1\,L0_1}{2\,y_p[t]^3} + O[y_p[t]]^4\right)x_p[t]^3 + O[x_p[t]]^4 == m_p\,x_p''[t]$$

```
Series[quadEqNominal2D[[1]], {yp[t], 0, 3}, {xp[t], 0, 3}] // Simplify
```

$$\left(k_1\,L0_1 - k_1\,x_p[t] + O[x_p[t]]^4\right) + \left(-\frac{k_1\,L0_1}{2\,x_p[t]^2} + O[x_p[t]]^4\right)y_p[t]^2 + O[y_p[t]]^4 == m_p\,x_p''[t]$$

```
Series[L01/√(xp[t]^2 + yp[t]^2), {xp[t], 0, 3}, {yp[t], 0, 3}] // Expand // Simplify
```

$$\left(\frac{L0_1}{y_p[t]} + O[y_p[t]]^4\right) + \left(-\frac{L0_1}{2\,y_p[t]^3} + O[y_p[t]]^4\right)x_p[t]^2 + O[x_p[t]]^4$$

trials :

```
Series[f[x], {x, a, 3}]
```

$$f[a] + f'[a]\ (x - a) + \frac{1}{2} f''[a]\ (x - a)^2 + \frac{1}{6} f^{(3)}[a]\ (x - a)^3 + O[x - a]^4$$

$$\text{Delta}_{\text{Equilibrium}} = \frac{m_p\, g}{k_i}$$

In[28]:=
```
equalibriumTerms = {
  x1'[t] → 0, x1''[t] → 0,
  x2'[t] → 0, x2''[t] → 0,
  θ1'[t] → 0, θ1''[t] → 0,
  θ2'[t] → 0, θ2''[t] → 0,
  y1'[t] → 0, y1''[t] → 0,
  y2'[t] → 0, y2''[t] → 0,
  xp'[t] → 0, xp''[t] → 0,
  yp'[t] → 0, yp''[t] → 0,
  θp'[t] → 0, θp''[t] → 0
 }
```

Out[28]= $\{x_1'[t] \to 0,\ x_1''[t] \to 0,\ x_2'[t] \to 0,\ x_2''[t] \to 0,\ \theta_1'[t] \to 0,\ \theta_1''[t] \to 0,\ \theta_2'[t] \to 0,\ \theta_2''[t] \to 0,\ y_1'[t] \to 0,\ y_1''[t] \to 0,\ y_2'[t] \to 0,\ y_2''[t] \to 0,\ x_p'[t] \to 0,\ x_p''[t] \to 0,\ y_p'[t] \to 0,\ y_p''[t] \to 0,\ \theta_p'[t] \to 0,\ \theta_p''[t] \to 0\}$

In[29]:=
```
simpStep1 /. equalibriumTerms //. dispSimp // MatrixForm //
 TraditionalForm
```

Out[29]//TraditionalForm=

$$\begin{pmatrix} \dfrac{k_1\,\text{r1x}\left(\sqrt{\text{r1x}^2+\text{r1y}^2}-\text{L0}_1\right)}{\sqrt{\text{r1x}^2+\text{r1y}^2}}+\dfrac{k_2\,\text{r2x}\left(\sqrt{\text{r2x}^2+\text{r2y}^2}-\text{L0}_2\right)}{\sqrt{\text{r2x}^2+\text{r2y}^2}}=0 \\ \dfrac{k_1\,\text{r1y}\left(\sqrt{\text{r1x}^2+\text{r1y}^2}-\text{L0}_1\right)}{\sqrt{\text{r1x}^2+\text{r1y}^2}}+\dfrac{k_2\,\text{r2y}\left(\sqrt{\text{r2x}^2+\text{r2y}^2}-\text{L0}_2\right)}{\sqrt{\text{r2x}^2+\text{r2y}^2}}=g\,m_p \\ \dfrac{k_1\,(\text{dr1}+\text{dr2})\left(\sqrt{\text{r1x}^2+\text{r1y}^2}-\text{L0}_1\right)}{2\sqrt{\text{r1x}^2+\text{r1y}^2}}+\dfrac{k_2\,(\text{dr3}+\text{dr4})\left(\sqrt{\text{r2x}^2+\text{r2y}^2}-\text{L0}_2\right)}{2\sqrt{\text{r2x}^2+\text{r2y}^2}}=0 \end{pmatrix}$$

In[32]:= **terms2**

Out[32]=
$$\Big\{\mathrm{Sin}[\theta_p[t]]\,h_p + \mathrm{Cos}[\theta_p[t]]\,l_p + 2\,x_1[t] - 2\,x_p[t] \to 2\,r1x,$$
$$-\frac{1}{2}\mathrm{Cos}[\theta_p[t]]\,h_p + \frac{1}{2}\mathrm{Sin}[\theta_p[t]]\,l_p + y_1[t] - y_p[t] \to r1y,$$
$$\frac{1}{2}\mathrm{Cos}[\theta_p[t]]\,h_p - \frac{1}{2}\mathrm{Sin}[\theta_p[t]]\,l_p - y_1[t] + y_p[t] \to -r1y,$$
$$\frac{1}{2}\mathrm{Sin}[\theta_p[t]]\,h_p - \frac{1}{2}\mathrm{Cos}[\theta_p[t]]\,l_p + x_2[t] - x_p[t] \to r2x,$$
$$-\frac{1}{2}\mathrm{Cos}[\theta_p[t]]\,h_p - \frac{1}{2}\mathrm{Sin}[\theta_p[t]]\,l_p + y_2[t] - y_p[t] \to r2y,$$
$$\mathrm{Cos}[\theta_p[t]]\,h_p + \mathrm{Sin}[\theta_p[t]]\,l_p - 2\,y_2[t] + 2\,y_p[t] \to -2\,r2y,$$
$$l_p\,(-\mathrm{Sin}[\theta_p[t]]\,x_1[t] + \mathrm{Sin}[\theta_p[t]]\,x_p[t] + \mathrm{Cos}[\theta_p[t]]\,(y_1[t] - y_p[t])) \to dr1,$$
$$h_p\,(\mathrm{Cos}[\theta_p[t]]\,x_1[t] - \mathrm{Cos}[\theta_p[t]]\,x_p[t] + \mathrm{Sin}[\theta_p[t]]\,(y_1[t] - y_p[t])) \to dr2,$$
$$h_p\,(\mathrm{Cos}[\theta_p[t]]\,x_2[t] - \mathrm{Cos}[\theta_p[t]]\,x_p[t] + \mathrm{Sin}[\theta_p[t]]\,(y_2[t] - y_p[t])) \to dr4,$$
$$l_p\,(\mathrm{Sin}[\theta_p[t]]\,x_2[t] - \mathrm{Sin}[\theta_p[t]]\,x_p[t] + \mathrm{Cos}[\theta_p[t]]\,(-y_2[t] + y_p[t])) \to dr3\Big\}$$

In[31]:= **simpStep1 /. equalibriumTerms**

Out[31]=
$$\left\{\frac{r1x\,k_1\left(\sqrt{r1x^2+r1y^2}-L0_1\right)}{\sqrt{r1x^2+r1y^2}} + \frac{r2x\,k_2\left(\sqrt{r2x^2+r2y^2}-L0_2\right)}{\sqrt{r2x^2+r2y^2}} == 0,\right.$$
$$\frac{r1y\,k_1\left(\sqrt{r1x^2+r1y^2}-L0_1\right)}{\sqrt{r1x^2+r1y^2}} + \frac{r2y\,k_2\left(\sqrt{r2x^2+r2y^2}-L0_2\right)}{\sqrt{r2x^2+r2y^2}} == g\,m_p,$$
$$\left.\frac{(dr1+dr2)\,k_1\left(\sqrt{r1x^2+r1y^2}-L0_1\right)}{2\sqrt{r1x^2+r1y^2}} + \frac{(dr3+dr4)\,k_2\left(\sqrt{r2x^2+r2y^2}-L0_2\right)}{2\sqrt{r2x^2+r2y^2}} == 0\right\}$$

$$\ddot{\mathbf{X}} = \mathbf{f}\,(\mathbf{X})$$

$$\mathbf{x_p} = \mathbf{x_{p0}} + \delta\mathbf{x_p}$$

$$\mathbf{y_p} = \mathbf{y_{p0}} + \delta\mathbf{y_p}$$

Series$\left[\sqrt{1+x}\,,\ \{x,\,0,\,15\}\right]$

$$1+\frac{x}{2}-\frac{x^2}{8}+\frac{x^3}{16}-\frac{5\,x^4}{128}+\frac{7\,x^5}{256}-\frac{21\,x^6}{1024}+\frac{33\,x^7}{2048}-\frac{429\,x^8}{32\,768}+\frac{715\,x^9}{65\,536}-\frac{2431\,x^{10}}{262\,144}+$$
$$\frac{4199\,x^{11}}{524\,288}-\frac{29\,393\,x^{12}}{4\,194\,304}+\frac{52\,003\,x^{13}}{8\,388\,608}-\frac{185\,725\,x^{14}}{33\,554\,432}+\frac{334\,305\,x^{15}}{67\,108\,864}+O[x]^{16}$$

In[204]:= **Series**$\left[1\Big/\sqrt{y^2+x^2}\,,\ \{x,\,0,\,3\},\ \{y,\,0,\,3\}\right]$

In[208]:= **Series[1/ $\sqrt{y^2+x^2}$, {x, x0, 3}, {y, y0, 3}] // Simplify**

Out[208]=

$$\left(\frac{1}{\sqrt{x0^2+y0^2}}-\frac{y0\,(y-y0)}{\left(x0^2+y0^2\right)^{3/2}}+\frac{\left(-x0^2+2\,y0^2\right)\,(y-y0)^2}{2\,\left(x0^2+y0^2\right)^{5/2}}+\frac{\left(3\,x0^2\,y0-2\,y0^3\right)\,(y-y0)^3}{2\,\left(x0^2+y0^2\right)^{7/2}}+O[y-y0]^4\right)+$$
$$\left(-\frac{x0}{\left(x0^2+y0^2\right)^{3/2}}+\frac{3\,x0\,y0\,(y-y0)}{\left(x0^2+y0^2\right)^{5/2}}+\frac{3\,\left(x0^3-4\,x0\,y0^2\right)\,(y-y0)^2}{2\,\left(x0^2+y0^2\right)^{7/2}}-\frac{5\,\left(3\,x0^3\,y0-4\,x0\,y0^3\right)\,(y-y0)^3}{2\,\left(x0^2+y0^2\right)^{9/2}}+O[y-y0]^4\right)\,(x-x0)+$$
$$\left(\frac{2\,x0^2-y0^2}{2\,\left(x0^2+y0^2\right)^{5/2}}+\frac{3\,y0\,\left(-4\,x0^2+y0^2\right)\,(y-y0)}{2\,\left(x0^2+y0^2\right)^{7/2}}-\frac{3\,\left(4\,x0^4-27\,x0^2\,y0^2+4\,y0^4\right)\,(y-y0)^2}{4\,\left(x0^2+y0^2\right)^{9/2}}+\frac{5\,\left(18\,x0^4\,y0-41\,x0^2\,y0^3+4\,y0^5\right)\,(y-y0)^3}{4\,\left(x0^2+y0^2\right)^{11/2}}+O[y-y0]^4\right)\,(x-x0)^2+$$
$$\left(\frac{-2\,x0^3+3\,x0\,y0^2}{2\,\left(x0^2+y0^2\right)^{7/2}}+\frac{5\,\left(4\,x0^3\,y0-3\,x0\,y0^3\right)\,(y-y0)}{2\,\left(x0^2+y0^2\right)^{9/2}}+\frac{5\,\left(4\,x0^5-41\,x0^3\,y0^2+18\,x0\,y0^4\right)\,(y-y0)^2}{4\,\left(x0^2+y0^2\right)^{11/2}}-\frac{105\,\left(2\,x0^5\,y0-7\,x0^3\,y0^3+2\,x0\,y0^5\right)\,(y-y0)^3}{4\,\left(x0^2+y0^2\right)^{13/2}}+O[y-y0]^4\right)\,(x-x0)^3+O[x-x0]^4$$

In[133]:= **n**

Out[133]= n

In[137]:= **n = 13; Series[$\sqrt{y^2+x^2}$, {x, 0, n}, {y, 0, n}]**

Out[137]=

$$\left(y+O[y]^{14}\right)+\left(\frac{1}{2\,y}+O[y]^{14}\right)x^2+\left(-\frac{1}{8\,y^3}+O[y]^{14}\right)x^4+\left(\frac{1}{16\,y^5}+O[y]^{14}\right)x^6+\left(-\frac{5}{128\,y^7}+O[y]^{14}\right)x^8+\left(\frac{7}{256\,y^9}+O[y]^{14}\right)x^{10}+\left(-\frac{21}{1024\,y^{11}}+O[y]^{14}\right)x^{12}+O[x]^{14}$$

In[138]:= **n = 13; Series[$\sqrt{y^2+x^2}$, (*{x,0,n},*){y, 0, n}]**

Out[138]=

$$\sqrt{x^2}+\frac{y^2}{2\sqrt{x^2}}-\frac{\sqrt{x^2}\,y^4}{8\,x^4}+\frac{\sqrt{x^2}\,y^6}{16\,x^6}-\frac{5\sqrt{x^2}\,y^8}{128\,x^8}+\frac{7\sqrt{x^2}\,y^{10}}{256\,x^{10}}-\frac{21\sqrt{x^2}\,y^{12}}{1024\,x^{12}}+O[y]^{14}$$