quad test driven development and testing

required : system of 2 quads and 1 payload

constraigns : not given expicitly. can make ones ..

test1

motion equations by Newton method == motion equations by Lagrangian method

quastions TODO :

how to paint vector for direction of forces ,and coordinate systems.

how to do it in *Mathematica*, in Python (blender,matplotlib)

how to paint moment arrow. same applications in question.

In[1]:=
```
(Rx = RotationMatrix[ϕ, {1, 0, 0}]) // MatrixForm
(Ry = RotationMatrix[θ, {0, 1, 0}]) // MatrixForm
(Rz = RotationMatrix[ψ, {0, 0, 1}]) // MatrixForm
```

Out[1]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \mathrm{Cos}[\phi] & -\mathrm{Sin}[\phi] \\ 0 & \mathrm{Sin}[\phi] & \mathrm{Cos}[\phi] \end{pmatrix}$$

Out[2]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cos}[\theta] & 0 & \mathrm{Sin}[\theta] \\ 0 & 1 & 0 \\ -\mathrm{Sin}[\theta] & 0 & \mathrm{Cos}[\theta] \end{pmatrix}$$

Out[3]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cos}[\psi] & -\mathrm{Sin}[\psi] & 0 \\ \mathrm{Sin}[\psi] & \mathrm{Cos}[\psi] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In[4]:=
```
Rx. Ry // MatrixForm
```

Out[4]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cos}[\theta] & 0 & \mathrm{Sin}[\theta] \\ \mathrm{Sin}[\theta]\,\mathrm{Sin}[\phi] & \mathrm{Cos}[\phi] & -\mathrm{Cos}[\theta]\,\mathrm{Sin}[\phi] \\ -\mathrm{Cos}[\phi]\,\mathrm{Sin}[\theta] & \mathrm{Sin}[\phi] & \mathrm{Cos}[\theta]\,\mathrm{Cos}[\phi] \end{pmatrix}$$

In[5]:=
$\left({}^{I}R_{B} = \mathrm{Rz}.\mathrm{Ry}\,.\,\mathrm{Rx}\right)$ // MatrixForm

Out[5]//MatrixForm=

$$\begin{pmatrix} \mathrm{Cos}[\theta]\,\mathrm{Cos}[\psi] & \mathrm{Cos}[\psi]\,\mathrm{Sin}[\theta]\,\mathrm{Sin}[\phi]-\mathrm{Cos}[\phi]\,\mathrm{Sin}[\psi] & \mathrm{Cos}[\phi]\,\mathrm{Cos}[\psi]\,\mathrm{Sin}[\theta]+\mathrm{Sin}[\phi]\,\mathrm{Sin}[\psi] \\ \mathrm{Cos}[\theta]\,\mathrm{Sin}[\psi] & \mathrm{Cos}[\phi]\,\mathrm{Cos}[\psi]+\mathrm{Sin}[\theta]\,\mathrm{Sin}[\phi]\,\mathrm{Sin}[\psi] & -\mathrm{Cos}[\psi]\,\mathrm{Sin}[\phi]+\mathrm{Cos}[\phi]\,\mathrm{Sin}[\theta]\,\mathrm{Sin}[\psi] \\ -\mathrm{Sin}[\theta] & \mathrm{Cos}[\theta]\,\mathrm{Sin}[\phi] & \mathrm{Cos}[\theta]\,\mathrm{Cos}[\phi] \end{pmatrix}$$

```
a[e]
a[e] /. a[a_] → Cos[a]
a[e] /. a → Cos
```

```
a[e]

Cos[e]

Cos[e]
```

In[6]:=
```
((Rz.Ry.Rx) /. Cos → C /. Sin → S) // MatrixForm
```

Out[6]//MatrixForm=

$$\begin{pmatrix} C[\theta]\,C[\psi] & C[\psi]\,S[\theta]\,S[\phi]-C[\phi]\,S[\psi] & C[\phi]\,C[\psi]\,S[\theta]+S[\phi]\,S[\psi] \\ C[\theta]\,S[\psi] & C[\phi]\,C[\psi]+S[\theta]\,S[\phi]\,S[\psi] & -C[\psi]\,S[\phi]+C[\phi]\,S[\theta]\,S[\psi] \\ -S[\theta] & C[\theta]\,S[\phi] & C[\theta]\,C[\phi] \end{pmatrix}$$

In[7]:=
```
vec3 = ((Rx. Ry).{0, 0, 1} /. Cos → C /. Sin → S)
vec2 = ((Rx).{0, 1, 0} /. Cos → C /. Sin → S)
vec1 = {1, 0, 0}
```

Out[7]= $\{S[\theta], -C[\theta]\,S[\phi], C[\theta]\,C[\phi]\}$

Out[8]= $\{0, C[\phi], S[\phi]\}$

Out[9]= $\{1, 0, 0\}$

In[10]:=
```
(R_Euler^PQR = Join[{vec1}, {vec2}, {vec3}]) // MatrixForm
```

Out[10]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & C[\phi] & S[\phi] \\ S[\theta] & -C[\theta]\,S[\phi] & C[\theta]\,C[\phi] \end{pmatrix}$$

In[12]:=
```
(R_pqr^euler = Transpose[Inverse[R_Euler^PQR]] // FullSimplify) // MatrixForm
```

Out[12]//MatrixForm=

$$\begin{pmatrix} 1 & \frac{S[\theta]\,S[\phi]}{C[\theta]\,(C[\phi]^2+S[\phi]^2)} & -\frac{C[\phi]\,S[\theta]}{C[\theta]\,(C[\phi]^2+S[\phi]^2)} \\ 0 & \frac{C[\phi]}{C[\phi]^2+S[\phi]^2} & \frac{S[\phi]}{C[\phi]^2+S[\phi]^2} \\ 0 & -\frac{S[\phi]}{C[\theta]\,(C[\phi]^2+S[\phi]^2)} & \frac{C[\phi]}{C[\theta]\,(C[\phi]^2+S[\phi]^2)} \end{pmatrix}$$

In[13]:=
```
R_pqr^euler.{p, q, r} // FullSimplify // MatrixForm
```

Out[13]//MatrixForm=

$$\begin{pmatrix} p+\frac{S[\theta]\,(-r\,C[\phi]+q\,S[\phi])}{C[\theta]\,(C[\phi]^2+S[\phi]^2)} \\ \frac{q\,C[\phi]+r\,S[\phi]}{C[\phi]^2+S[\phi]^2} \\ \frac{r\,C[\phi]-q\,S[\phi]}{C[\theta]\,(C[\phi]^2+S[\phi]^2)} \end{pmatrix}$$

In[14]:=
```
(pqrvec = Transpose[R_Euler^PQR].{ϕ, θ, ψ} // FullSimplify) // MatrixForm
```

Out[14]//MatrixForm=

$$\begin{pmatrix} \phi+\psi\,S[\theta] \\ \theta\,C[\phi]-\psi\,C[\theta]\,S[\phi] \\ \psi\,C[\theta]\,C[\phi]+\theta\,S[\phi] \end{pmatrix}$$

In[15]:= $\left(\left(\overset{P}{R}_{B} = ((Ry.Rz)\ /.\ \theta \to -\beta\ /.\ \psi \to -\gamma\ //\ Simplify)\right)\ /.\ Cos[a\_] \to C[a]\ /.\ Sin[a\_] \to S[a]\right)$ //
MatrixForm

Out[15]//MatrixForm=

$$\begin{pmatrix} C[\beta]\,C[\gamma] & C[\beta]\,S[\gamma] & -S[\beta] \\ -S[\gamma] & C[\gamma] & 0 \\ C[\gamma]\,S[\beta] & S[\beta]\,S[\gamma] & C[\beta] \end{pmatrix}$$

In[16]:= $\left(\overset{B}{R}_{P} = Transpose\left[\overset{P}{R}_{B}\right]\ /.\ \theta \to -\beta\ /.\ \psi \to -\gamma\right)\ /.\ Cos[a\_] \to C[a]\ /.\ Sin[a\_] \to S[a]$ //
MatrixForm

Out[16]//MatrixForm=

$$\begin{pmatrix} C[\beta]\,C[\gamma] & -S[\gamma] & C[\gamma]\,S[\beta] \\ C[\beta]\,S[\gamma] & C[\gamma] & S[\beta]\,S[\gamma] \\ -S[\beta] & 0 & C[\beta] \end{pmatrix}$$

In[17]:= Inverse$\left[\overset{B}{R}_{P}\right]$ // Simplify // MatrixForm

Out[17]//MatrixForm=

$$\begin{pmatrix} Cos[\beta]\,Cos[\gamma] & Cos[\beta]\,Sin[\gamma] & -Sin[\beta] \\ -Sin[\gamma] & Cos[\gamma] & 0 \\ Cos[\gamma]\,Sin[\beta] & Sin[\beta]\,Sin[\gamma] & Cos[\beta] \end{pmatrix}$$

In[18]:= $\left(\overset{I}{R}_{P} = \overset{I}{R}_{B}.\overset{B}{R}_{P}\right)\ /.\ Cos[a\_] \to c[a]\ /.\ Sin[a\_] \to s[a]$ // TraditionalForm // MatrixForm

Out[18]//MatrixForm=

$$\begin{pmatrix} c(\beta)\,c(\gamma)\,c(\Theta)\,c(\psi) + c(\beta)\,s(\gamma)\,(c(\psi)\,s(\Theta)\,s(\phi) - c(\phi)\,s(\psi)) - s(\beta)\,(c(\phi)\,c(\psi)\,s(\Theta) + s(\phi)\,s(\psi \\ c(\beta)\,c(\gamma)\,c(\Theta)\,s(\psi) - s(\beta)\,(c(\phi)\,s(\Theta)\,s(\psi) - c(\psi)\,s(\phi)) + c(\beta)\,s(\gamma)\,(c(\phi)\,c(\psi) + s(\Theta)\,s(\phi)\,s(\psi \\ -c(\Theta)\,c(\phi)\,s(\beta) - c(\beta)\,c(\gamma)\,s(\Theta) + c(\beta)\,c(\Theta)\,s(\gamma)\,s(\phi) \end{pmatrix}$$

In[19]:= $\left(\left(\left(rIrel = \overset{I}{R}_{P}.\{r, 0, 0\}\right)\ /.\ Cos[a\_] \to c[a]\ /.\ Sin[a\_] \to s[a]\right)\right)$ // MatrixForm

Out[19]//MatrixForm=

$$\begin{pmatrix} r\,(c[\beta]\,c[\gamma]\,c[\Theta]\,c[\psi] + c[\beta]\,s[\gamma]\,(c[\psi]\,s[\Theta]\,s[\phi] - c[\phi]\,s[\psi]) - s[\beta]\,(c[\phi]\,c[\psi]\,s[\Theta] + s[\phi] \\ r\,(c[\beta]\,c[\gamma]\,c[\Theta]\,s[\psi] - s[\beta]\,(-c[\psi]\,s[\phi] + c[\phi]\,s[\Theta]\,s[\psi]) + c[\beta]\,s[\gamma]\,(c[\phi]\,c[\psi] + s[\Theta]\,s[\phi \\ r\,(-c[\Theta]\,c[\phi]\,s[\beta] - c[\beta]\,c[\gamma]\,s[\Theta] + c[\beta]\,c[\Theta]\,s[\gamma]\,s[\phi]) \end{pmatrix}$$

In[20]:= $\left(\left(\left(\overset{B}{R}_{P}.\{r, 0, 0\}\right)\ /.\ Cos[a\_] \to c[a]\ /.\ Sin[a\_] \to s[a]\right)\right)$ // MatrixForm

Out[20]//MatrixForm=

$$\begin{pmatrix} r\,c[\beta]\,c[\gamma] \\ r\,c[\beta]\,s[\gamma] \\ -r\,s[\beta] \end{pmatrix}$$

In[21]:= $(D[rIrel, \{\beta\}])\ /.\ Cos[a\_] \to c[a]\ /.\ Sin[a\_] \to s[a]$ // MatrixForm

Out[21]//MatrixForm=

$$\begin{pmatrix} r\,(-c[\gamma]\,c[\Theta]\,c[\psi]\,s[\beta] - s[\beta]\,s[\gamma]\,(c[\psi]\,s[\Theta]\,s[\phi] - c[\phi]\,s[\psi]) - c[\beta]\,(c[\phi]\,c[\psi]\,s[\Theta] + s[\phi \\ r\,(-c[\gamma]\,c[\Theta]\,s[\beta]\,s[\psi] - c[\beta]\,(-c[\psi]\,s[\phi] + c[\phi]\,s[\Theta]\,s[\psi]) - s[\beta]\,s[\gamma]\,(c[\phi]\,c[\psi] + s[\Theta]\,s[\\ r\,(-c[\beta]\,c[\Theta]\,c[\phi] + c[\gamma]\,s[\beta]\,s[\Theta] - c[\Theta]\,s[\beta]\,s[\gamma]\,s[\phi]) \end{pmatrix}$$

```
D[Cos[θ], θ]
D[Sin[θ], θ]
```

$-Sin[\theta]$

$Cos[\theta]$

In[22]:=
```
(a = pqrvec pqrvec // Expand) // MatrixForm
Sum[a, {i, 3}] // FullSimplify
```

Out[22]//MatrixForm=

$$\begin{pmatrix} \phi^2 + 2\,\phi\,\psi\,S[\theta] + \psi^2\,S[\theta]^2 \\ \theta^2\,C[\phi]^2 - 2\,\theta\,\psi\,C[\theta]\,C[\phi]\,S[\phi] + \psi^2\,C[\theta]^2\,S[\phi]^2 \\ \psi^2\,C[\theta]^2\,C[\phi]^2 + 2\,\theta\,\psi\,C[\theta]\,C[\phi]\,S[\phi] + \theta^2\,S[\phi]^2 \end{pmatrix}$$

Out[23]= $\{3\,(\phi + \psi\,S[\theta])^2,\ 3\,(\theta\,C[\phi] - \psi\,C[\theta]\,S[\phi])^2,\ 3\,(\psi\,C[\theta]\,C[\phi] + \theta\,S[\phi])^2\}$

In[24]:=
```
ω2 = (a[[1]] + a[[2]] + a[[3]]) /. C → Cos /. S → Sin // FullSimplify
```

Out[24]= $\theta^2 + \phi^2 + \psi^2 + 2\,\phi\,\psi\,\mathrm{Sin}[\theta]$

In[25]:= $T = \frac{1}{2}\,m\left[\dot{x}_1^{\,2} + \dot{y}_1^{\,2} + \dot{z}_1^{\,2}\right] + \frac{1}{2}\left(p^2\,I_{xx} + q^2\,I_{yy} + r^2\,I_{zz}\right)$

$V = m\,g\,z_1$

Out[25]= $\frac{1}{2}\,m\left[\dot{x}_1^{\,2} + \dot{y}_1^{\,2} + \dot{z}_1^{\,2}\right] + \frac{1}{2}\left(p^2\,\mathbb{i}_{xx} + q^2\,\mathbb{i}_{yy} + r^2\,\mathbb{i}_{zz}\right)$

Out[26]= $g\,m\,z_1$

$D\left[T,\ \dot{x}_1\right]$

$\dot{x}_1\,m'\left[\dot{x}_1^{\,2} + \dot{y}_1^{\,2} + \dot{z}_1^{\,2}\right]$

Pendulum element

In[27]:= $X_p = \begin{pmatrix} x_B \\ y_B \\ z_B \end{pmatrix} + \overset{I}{R}_B\,l_1 \begin{pmatrix} \mathrm{Sin}[\beta]\,\mathrm{Cos}[\gamma] \\ \mathrm{Sin}[\beta]\,\mathrm{Sin}[\gamma] \\ -\mathrm{Cos}[\beta] \end{pmatrix}$

when $\beta = 0$, and $\gamma = 0$ then $X_p = \begin{pmatrix} 0 \\ 0 \\ -l_1 \end{pmatrix}$

q

In[28]:= $X_p = \begin{pmatrix} 0 \\ 0 \\ z_1 \end{pmatrix} + l_1 \begin{pmatrix} \mathrm{Sin}[\beta[t]]\,\mathrm{Cos}[\gamma[t]] \\ \mathrm{Sin}[\beta[t]]\,\mathrm{Sin}[\gamma[t]] \\ -\mathrm{Cos}[\beta[t]] \end{pmatrix}$

Out[28]= $\{\{\mathrm{Cos}[\gamma[t]]\,\mathrm{Sin}[\beta[t]]\,l_1\},\ \{\mathrm{Sin}[\beta[t]]\,\mathrm{Sin}[\gamma[t]]\,l_1\},\ \{-\mathrm{Cos}[\beta[t]]\,l_1 + z_1\}\}$

$$\mathbf{L == T - V}$$

$$\mathbf{T == \frac{1}{2} m_p \dot{Xp}^2}$$

$$\mathbf{V == m_p\, g\, (z_1 - l_1\, Cos[\beta])}$$

$$\boldsymbol{\omega == \dot{\gamma}\, z + \dot{\beta}\, (-y)}$$

$$L == \left\{-g\, m_p\, (-Cos[\beta]\, l_1 + z_1) + \frac{1}{2} m_p \left(l_1^2 \left(\beta'[t]^2 + Sin[\beta[t]]^2\, \gamma'[t]^2\right) + 2\, Sin[\beta[t]]\, l_1\, \beta'[t]\, z_1'[t] + z_1'[t]^2\right)\right\}$$

$$\left\{\frac{1}{2} m_p \left(l_1^2 \left(\beta'[t]^2 + Sin[\beta[t]]^2\, \gamma'[t]^2\right) + 2\, Sin[\beta[t]]\, l_1\, \beta'[t]\, z_1'[t] + z_1'[t]^2\right)\right\} == \frac{1}{2} \dot{Xp}^2\, m_p$$

True

True

(deriv = D[X_p, t] // Simplify) /. Cos → c /. Sin → s // TraditionalForm // MatrixForm

$$\begin{pmatrix} l_1\, (c(\beta(t))\, c(\gamma(t))\, \beta'(t) - s(\beta(t))\, s(\gamma(t))\, \gamma'(t)) \\ l_1\, (c(\beta(t))\, s(\gamma(t))\, \beta'(t) + c(\gamma(t))\, s(\beta(t))\, \gamma'(t)) \\ s(\beta(t))\, l_1\, \beta'(t) \end{pmatrix}$$

(h = deriv deriv // Expand) // MatrixForm

(dSum = h[[1]] + h[[2]] + h[[3]] // FullSimplify) // MatrixForm // TraditionalForm

$$\begin{pmatrix} Cos[\beta[t]]^2\, Cos[\gamma[t]]^2\, l_1^2\, \beta'[t]^2 - 2\, Cos[\beta[t]]\, Cos[\gamma[t]]\, Sin[\beta[t]]\, Sin[\gamma[t]]\, l_1^2\, \beta'[t]\, \gamma'[t] + \\ Cos[\beta[t]]^2\, Sin[\gamma[t]]^2\, l_1^2\, \beta'[t]^2 + 2\, Cos[\beta[t]]\, Cos[\gamma[t]]\, Sin[\beta[t]]\, Sin[\gamma[t]]\, l_1^2\, \beta'[t]\, \gamma'[t] + \\ Sin[\beta[t]]^2\, l_1^2\, \beta'[t]^2 \end{pmatrix}$$

$$\left(l_1^2 \left(\beta'(t)^2 + \sin^2(\beta(t))\, \gamma'(t)^2\right) \right)$$

dSum

$$\left\{l_1^2 \left(\beta'[t]^2 + Sin[\beta[t]]^2\, \gamma'[t]^2\right)\right\}$$

$$\mathbf{T = \frac{1}{2} m_p\, dSum}$$

$$\mathbf{V = m_p\, g\, (z_1 - l_1\, Cos[\beta[t]])}$$

$$\left\{\frac{1}{2} l_1^2\, m_p \left(\beta'[t]^2 + Sin[\beta[t]]^2\, \gamma'[t]^2\right)\right\}$$

$$g\, m_p\, (-Cos[\beta[t]]\, l_1 + z_1)$$

Dt[Tt[t], {Tt, 1}] // TraditionalForm

$$Tt'(t)\, \frac{dt}{dTt}$$

$\frac{\mathbf{T}}{\boldsymbol{\beta}}$

$\frac{\mathbf{T}}{\boldsymbol{\gamma}}$

$\frac{\mathbf{T}}{\dot{\boldsymbol{\beta}}}$

$\frac{\mathbf{T}}{\dot{\boldsymbol{\gamma}}}$

$$\left\{\frac{l_1^2\, m_p \left(\beta'[t]^2 + \text{Sin}[\beta[t]]^2\, \gamma'[t]^2\right)}{2\, \beta}\right\}$$

$$\left\{\frac{l_1^2\, m_p \left(\beta'[t]^2 + \text{Sin}[\beta[t]]^2\, \gamma'[t]^2\right)}{2\, \gamma}\right\}$$

```
(dTdbeta = D[T, β[t]]) // TraditionalForm
(dTdbetaDot = D[T, β'[t]]) // TraditionalForm
(dTdgamma = D[T, γ[t]]) // TraditionalForm
(dTdgammaDot = D[T, γ'[t]]) // TraditionalForm
(dtdTdbetaDot = D[D[T, β'[t]], t]) // TraditionalForm
(dtdTdgammaDot = D[D[T, γ'[t]], t]) // TraditionalForm
(dVdbeta = D[V, β[t]]) // TraditionalForm
(dVdgamma = D[V, γ[t]]) // TraditionalForm
dtdTdbetaDot + dVdbeta - dTdbeta == 0 // TraditionalForm
dtdTdgammaDot + dVdgamma - dTdgamma == 0 // TraditionalForm
```

$$\left\{l_1^2\, m_p \sin(\beta(t)) \cos(\beta(t))\, \gamma'(t)^2\right\}$$

$$\left\{l_1^2\, m_p\, \beta'(t)\right\}$$

$$\{0\}$$

$$\left\{l_1^2\, m_p \sin^2(\beta(t))\, \gamma'(t)\right\}$$

$$\left\{l_1^2\, m_p\, \beta''(t)\right\}$$

$$\left\{2\, l_1^2\, m_p\, \beta'(t) \sin(\beta(t)) \cos(\beta(t))\, \gamma'(t) + l_1^2\, m_p \sin^2(\beta(t))\, \gamma''(t)\right\}$$

$$g\, l_1\, m_p \sin(\beta(t))$$

$$0$$

$$\left\{g\, l_1\, m_p \sin(\beta(t)) + l_1^2\, m_p\, \beta''(t) + l_1^2\, m_p \sin(\beta(t))\, (-\cos(\beta(t)))\, \gamma'(t)^2\right\} = 0$$

$$\left\{2\, l_1^2\, m_p\, \beta'(t) \sin(\beta(t)) \cos(\beta(t))\, \gamma'(t) + l_1^2\, m_p \sin^2(\beta(t))\, \gamma''(t)\right\} = 0$$