Blade Element Theory in Forward Flight

- Although forward flight is very difficult to model using BET with certain simplifying assumptions we can obtain the leading terms of the rotor aerodynamic forces.

Blade Forces

- The incremental lift *dL* is:

$$dL = \tfrac{1}{2}\rho U^2 c C_l dy = \tfrac{1}{2}\rho U^2 c C_{l_\alpha}(\theta - \phi)dy$$

$$= \tfrac{1}{2}\rho U_T^2 c C_{l_\alpha}\left(\theta - \frac{U_P}{U_T}\right)dy$$

$$= \tfrac{1}{2}\rho c C_{l_\alpha}\left(\theta U_T^2 - U_P U_T\right)dy$$

- The incremental drag *dD* is:

$$dD = \tfrac{1}{2}\rho U^2 c C_d dy = \tfrac{1}{2}\rho U_T^2 c C_d dy$$

Blade Forces

- We had already seen that the incremental force perpendicular to the rotor plane:

$$dF_z = dL\cos\phi - dD\sin\phi \approx dL$$
$$= \tfrac{1}{2}\rho c C_{l_\alpha}\left(\theta U_T^2 - U_P U_T\right)dy$$

- The incremental force in the rotor plane

$$dF_x = dL\sin\phi + dD\cos\phi \approx dL\phi + dD$$
$$= \tfrac{1}{2}\rho c C_{l_\alpha}\left(\theta U_p U_T - U_p^2 + \frac{C_d}{C_{l_\alpha}} U_T^2\right)dy$$

Velocities

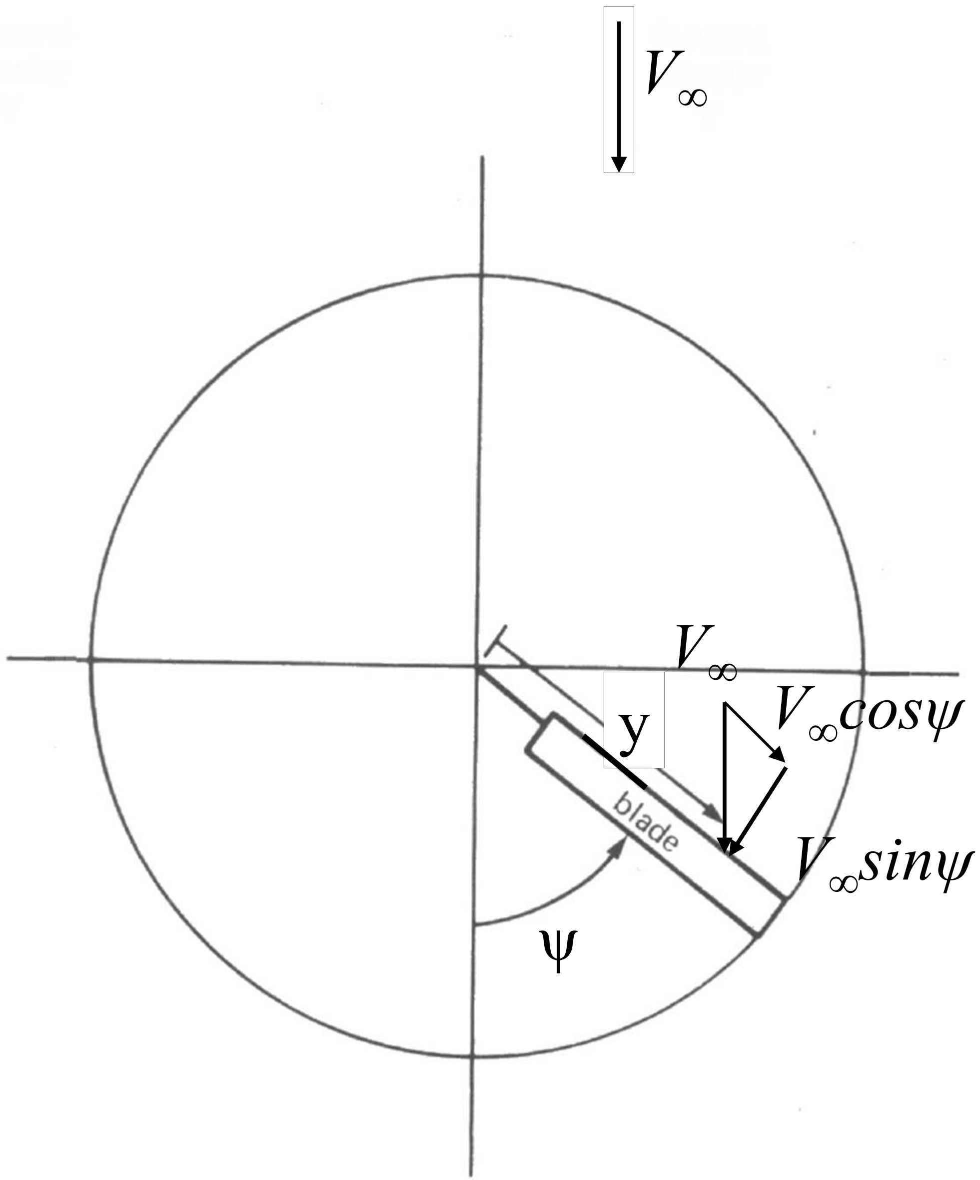
- In forward flight the velocities :
  - are periodic,
  - depend on the blade azimuthal position
  - the in plane velocity will have two components:
    - Due to the rotation
    - Due to the forward velocity
  - The out-of-plane velocity will have three components
    - Induced velocity
    - Due to flapping
    - Due to the coning angle

Velocities

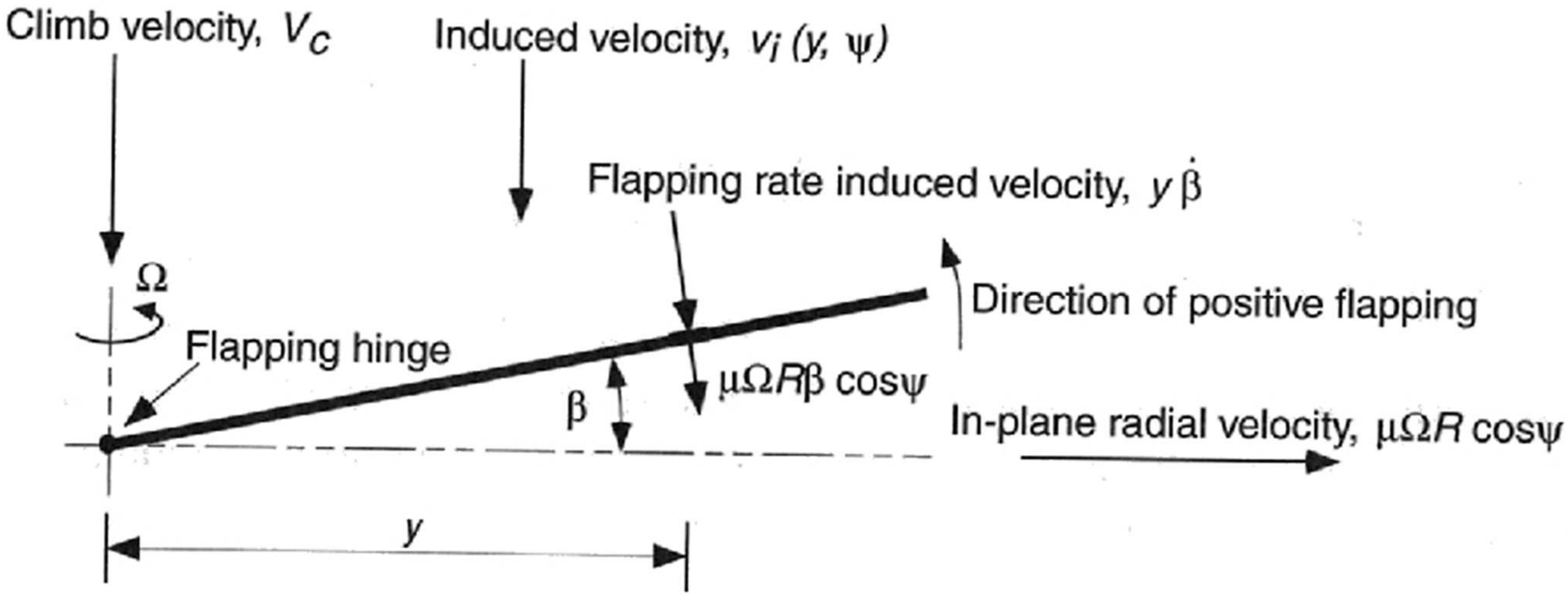
- The in plane velocity is:

$$U_T(y,\psi) = \Omega y + V_\infty \sin\Psi$$

$$= \Omega y + \mu\Omega R \sin\Psi$$



Velocities



- Out of plane velocity

$$U_P(y,\psi) = (\lambda_C + \lambda_i)\Omega R + y\dot{\beta}(\psi) + \mu\Omega R\beta(\psi)\cos\psi$$

Velocities

- There is still a radial velocity:

$$U_R = \mu \Omega R \cos\psi$$

- The Non-dimensional velocities are:

$$\frac{U_T}{\Omega R} = \frac{y}{R} + \mu \sin\Psi = r + \mu \sin\Psi$$

$$\frac{U_P}{\Omega R} = \left(\lambda_C + \lambda_i\right) + \frac{y\dot{\beta}}{\Omega R} + \mu\beta\cos\psi = \left(\lambda_C + \lambda_i\right) + r\frac{\dot{\beta}}{\Omega} + \mu\beta\cos\psi$$

$$\frac{U_R}{\Omega R} = \mu\cos\psi$$

Blade Element Theory in Forward Flight

- In the BET in forward flight we have taken into account:
  - Blade pitch
  - Flapping motion
- We need to know the induced velocity field which depends on the rotor wake which depends on the rotor thrust, flapping, trim state and airloads distribution.
- Let’s analysed the rotor performance using simple inflow models

Linear inflow model

- In-flight measurements on the time-average induced velocity showed that the longitudinal inflow variation to be approximately linear.
  - During the transition from hover to forward flight the is a region ($0.0 \leq \mu \leq 0.1$) where the induced velocity is non-uniform
  - In higher speed forward flight ($\mu \geq 0.1$) the time-average longitudinal inflow becomes more linear and can be represented by:

$$\lambda_i = \lambda_0 \left(1 + k_x \frac{x}{R} \right) = \lambda_0 \left(1 + k_x r \cos \psi \right)$$

Linear inflow model

- The value λ_0 is mean induced velocity at the center of the rotor, given by the momentum theory

$$\lambda_0 = \frac{C_T}{2\sqrt{\mu^2 + \lambda_i^2}}$$

- And the value of k_x=1.2

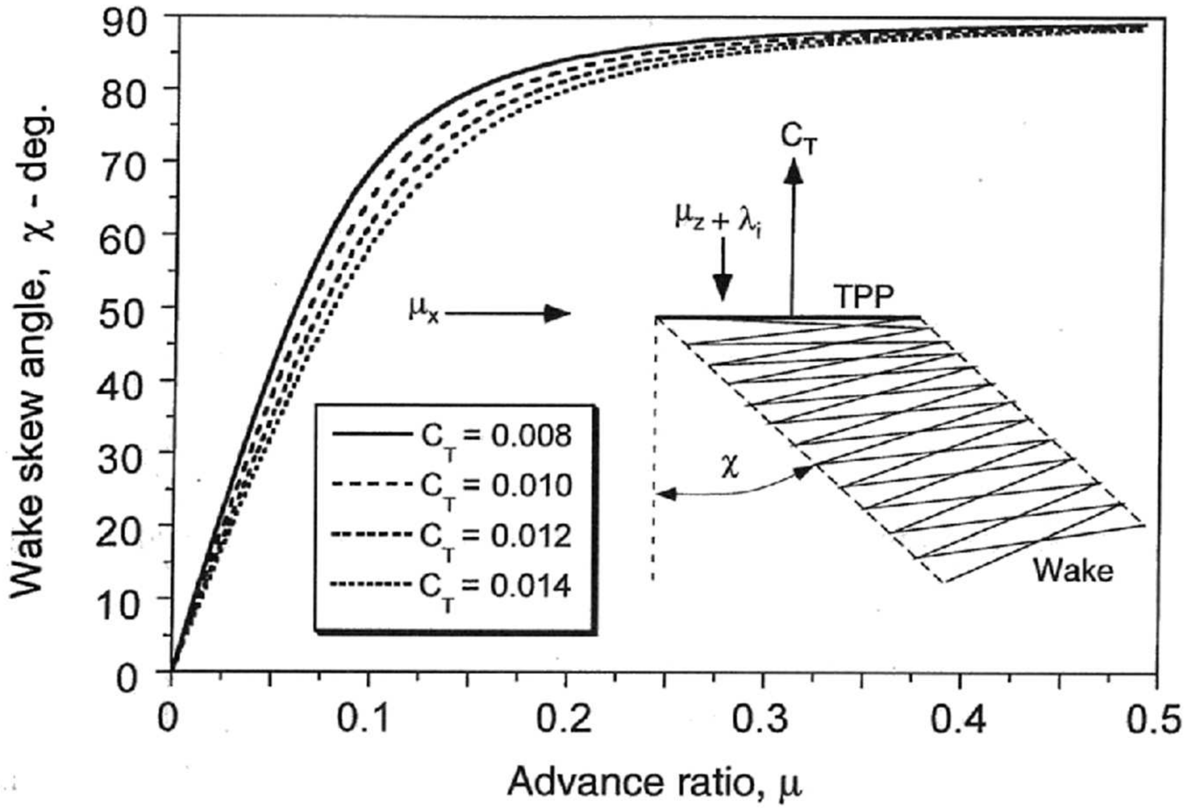
Linear inflow model

- A variation of these result can be expressed if we consider both longitudinal and lateral variation:

$$\lambda_i = \lambda_0\left(1 + k_x \frac{x}{R} + k_y \frac{y}{R}\right) = \lambda_0\left(1 + k_x r\cos\psi + k_y r\sin\psi\right)$$

Author(s)	k_x	k_y
Coleman et. Al.	$\tan(\chi/2)$	*0*
Drees	$\frac{4}{3}\left(1-\cos\chi-1.8\mu^2\right)/(\sin\chi)$	*-2μ*
Payne	$\frac{4}{3}\left((\mu/\lambda)/(1.2+\mu/\lambda)\right)$	*0*
White & Blake	$\sqrt{2}\sin\chi$	*0*
Pitt & Peters	$(15\pi/23)\tan(\chi/2)$	*0*
Howlett	$\sin^2\chi$	*0*

Wake Skew Angle



- In the previous expressions χ is the wake skew angle and is given by:

$$\chi = \tan^{-1}\left(\frac{\mu_x}{\mu_z + \lambda_i}\right)$$

Modelo de Mangler & Squire

- Mangler & Squire developed a model using incompressible linearized Euler equations to relate the pressure field across the disk to a inflow
- The disk loading is a Linear combination of two fundamental shapes:
  - Type 1 (m=1) Elliptical loading
  - Type 3 (m=3) that tends to zero at the blade tip and root
- The pressure loading is given by:

$$\Delta p_m \propto r^{m-1} \sqrt{1-r^2}$$

Inflow model of Mangler & Squire

- The inflow ratio is given by a Fourier series:

$$\lambda_i = \left(\frac{2C_T}{\mu}\right)\left[\frac{c_0}{2} + \sum_{n=1}^{\infty}(-1)^n c_n(r,\alpha)\cos n\psi\right]$$

- Where α is the disk angle of attack and c_n are constant that depends of the rotor loading

Inflow model of Mangler & Squire

- For the Type 1 loading the coefficients are:

$$c_0 = \frac{3}{4}\upsilon \qquad c_1 = -\frac{3\pi}{16}\sqrt{1-\upsilon^2}\left(\frac{1-\sin\alpha}{1+\sin\alpha}\right)^{1/2}$$

- For $n \geq 2$ even

$$c_n = (-1)^{\frac{n-2}{2}}\left(\frac{3}{4}\right)\left(\frac{\upsilon+n}{n^2-1}\right)\left(\frac{1-\upsilon}{1+\upsilon}\right)^{n/2}\left(\frac{1-\sin\alpha}{1+\sin\alpha}\right)^{n/2}$$

- For $n \geq 2$ odd $c_n = 0$

Inflow model of Mangler & Squire

- For Type 3 loading the coefficients are:

$$c_0 = \frac{15}{8}\upsilon\left(1-\upsilon^2\right)$$

$$c_1 = -\frac{15\pi}{256}\left(5-9\upsilon^2\right)\sqrt{1-\upsilon^2}\left(\frac{1-\sin\alpha}{1+\sin\alpha}\right)^{1/2}$$

$$c_3 = \frac{45\pi}{256}\left(1-\upsilon^2\right)^{3/2}\left(\frac{1-\sin\alpha}{1+\sin\alpha}\right)^{1/2}$$

- For $n \geq 5$ odd $c_n = 0$

Inflow model of Mangler & Squire

- For $n \geq 2$ even

$$c_n = (-1)^{\frac{n-2}{2}} \left(\frac{15}{8}\right) \left[\left(\frac{\upsilon + n}{n^2 - 1}\right) \left(\frac{9\upsilon^2 + n^2 - 6}{n^2 - 9}\right) + \left(\frac{3\upsilon}{n^2 - 9}\right) \right] \left(\frac{1 - \upsilon}{1 + \upsilon}\right)^{n/2} \left(\frac{1 - \sin\alpha}{1 + \sin\alpha}\right)^{n/2}$$

- With

$$\upsilon = 1 - r^2$$

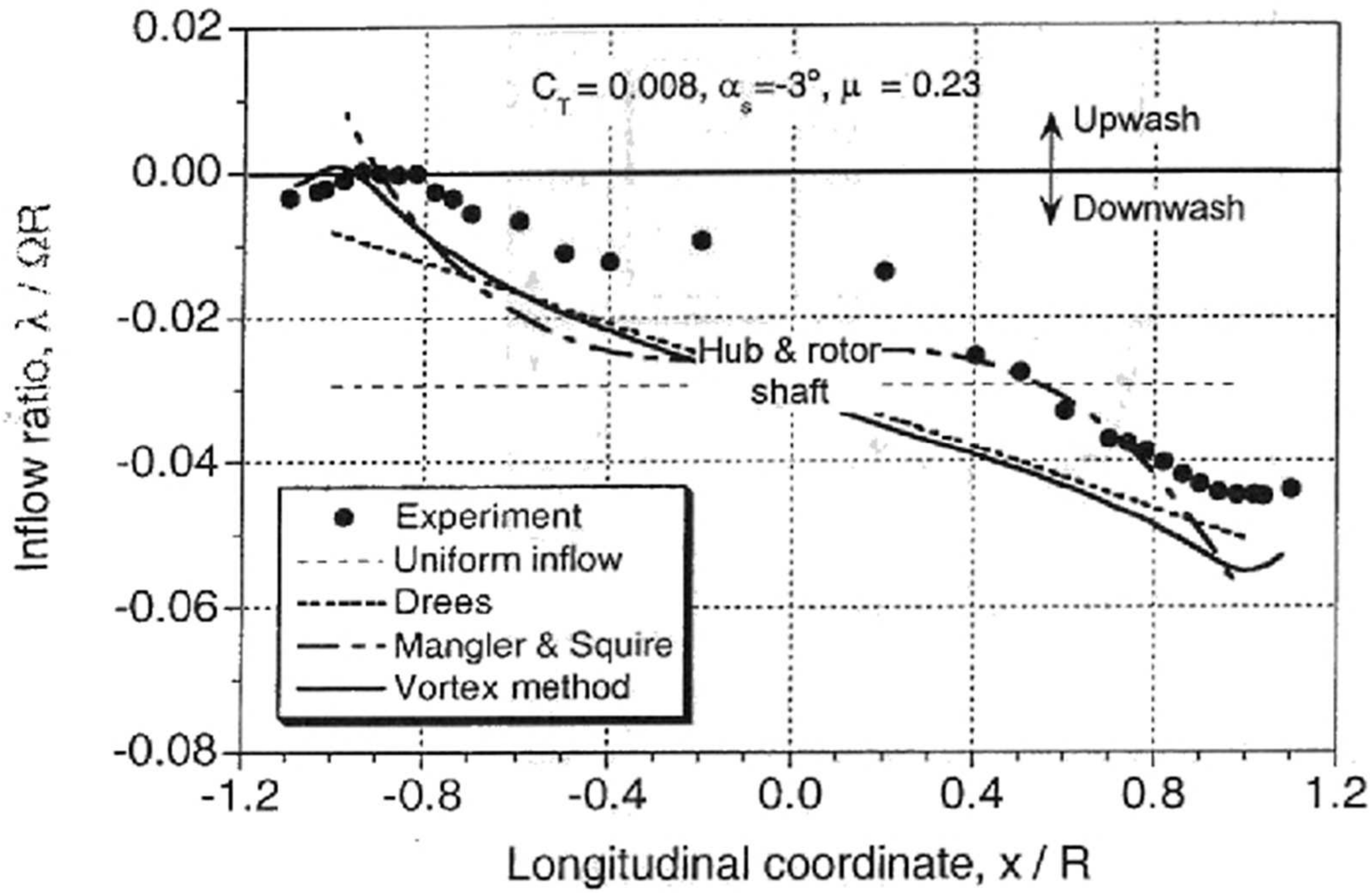
- And the disk loading is a linear combination of these two loadings:

$$\Delta p = w_1 \Delta p_1 + w_3 \Delta p_3 \qquad w_1 + w_3 = 1$$

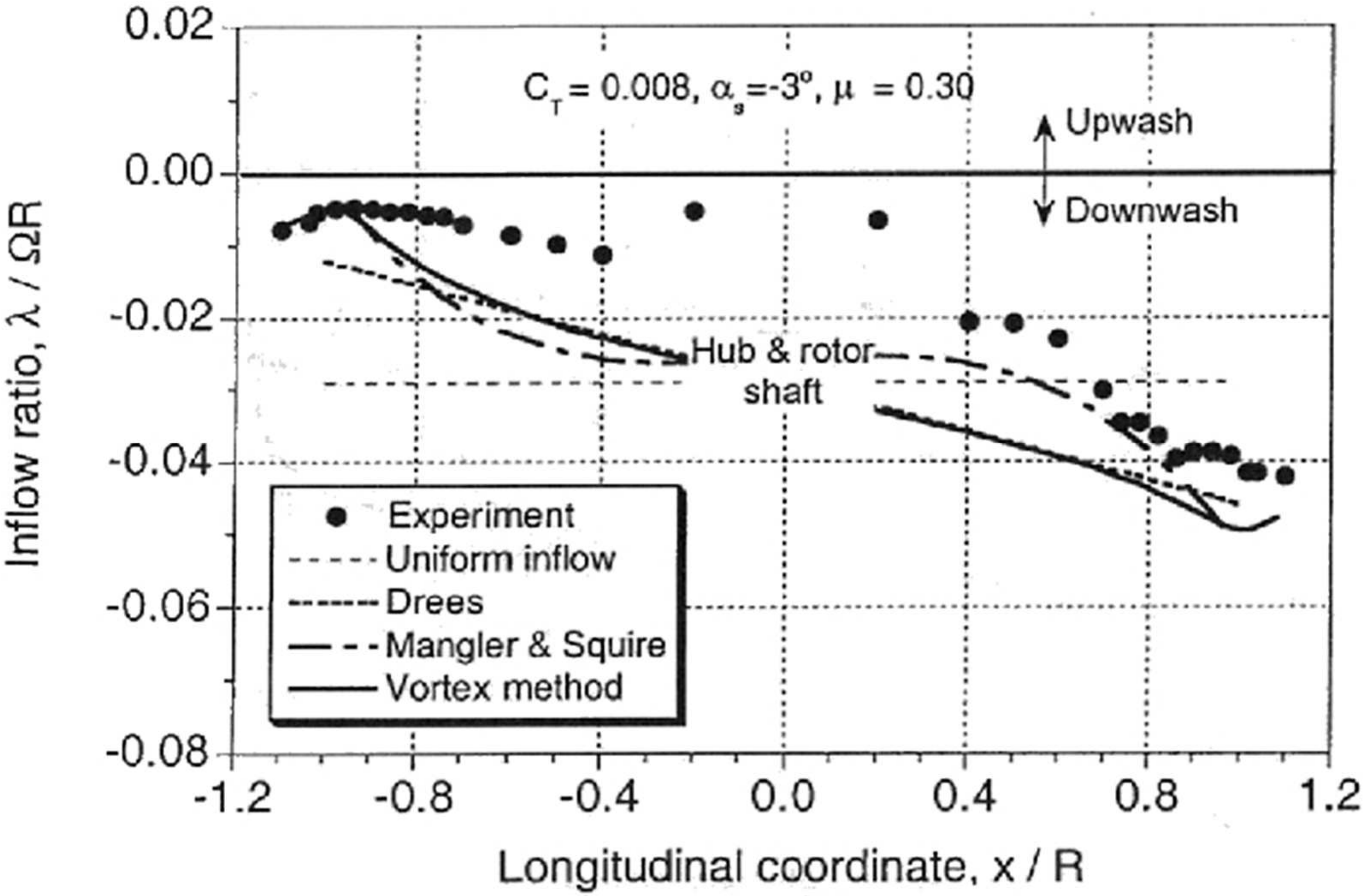
Inflow model of Mangler & Squire

- The main disadvantage of the theory is that it requires the aerodynamic loading on the rotor to be known or assumed a priori
- The theory is valid for hover and high forward velocities. It is not valid for low advance velocities since the assumed velocity relation is not valid.

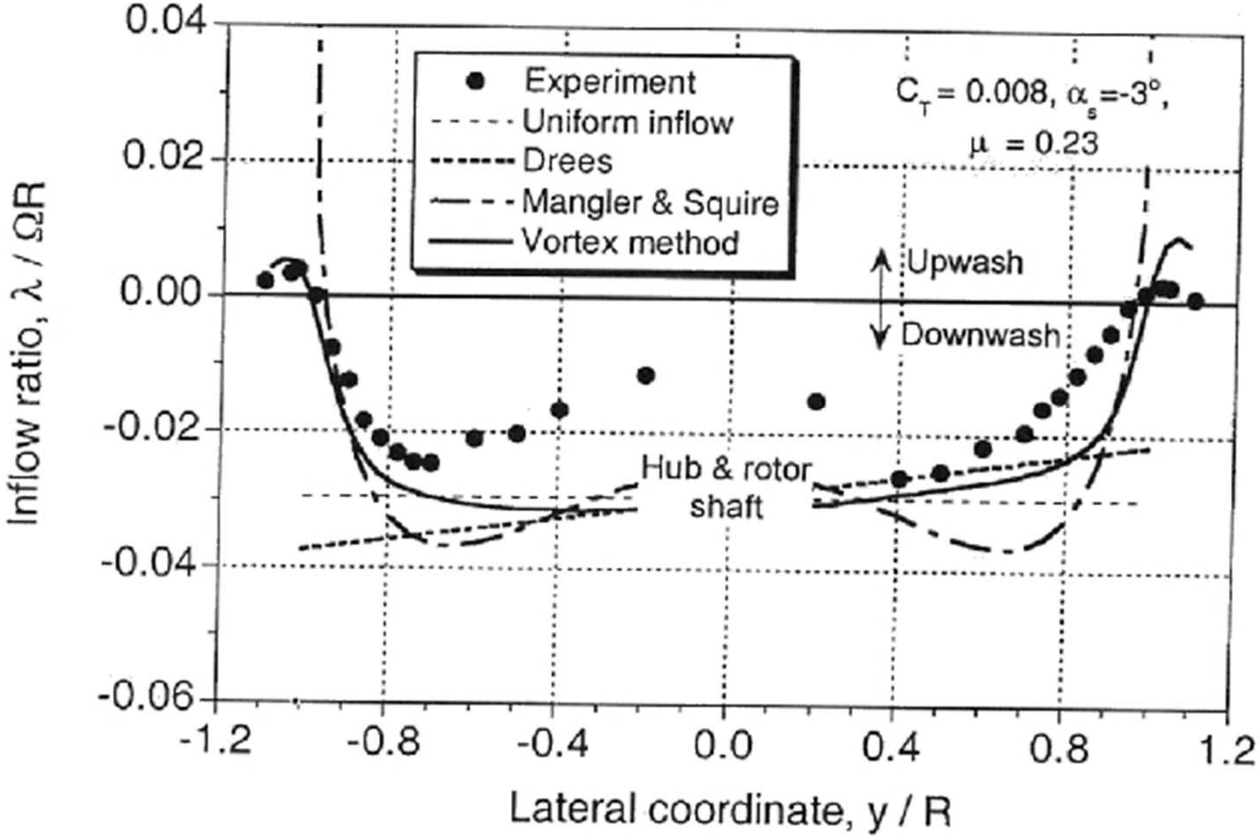
Model comparison



Model comparison



Model comparison



Model comparison