numeric simulation

```
Quit[]
```

```
dispSimp = {a_[t] → a, Cos[a_] → c[a], Sin[a_] → s[a], 𝕚i_,zz -> Ii};
```

original equations:

```
(quadEqNominal = EulerEquations[L,
      {(*x1[t],y1[t],θ1[t],x2[t],y2[t],θ2[t],*)xp[t], yp[t], θp[t]}, t]
      (*[[All,1]]*)(*==Q*) // Simplify) // MatrixForm // TraditionalForm
```

$$\left(\begin{array}{l} \dfrac{k_1\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(t)+2\,x_1}{2\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p}} \\ \dfrac{k_1\left(-\frac{1}{2}\,h_p \cos(\theta_p(t))+\frac{1}{2}\,l_p \sin(\theta_p(t))-y_p(t)+y_1(t)\right)}{\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_i}} \\ \imath_{p,zz}\,\theta_p''(t)+\dfrac{k_1\,(h_p\,(x_1(t)\cos(\theta_p(t))-x_p(t)\cos(\theta_p(t))+(y_1(t)-y_p(t))\sin(\theta_p(t)))+l_p\,(x_1(t)\,(-\sin(\theta_p(t)))+x_p(t)\sin(\theta_p(t))+(y_1(}{2\sqrt{\frac{1}{4}\,(h_p \sin(\theta_p(t))+l_p \cos(\theta_p(t))-2\,x_p(}} \end{array}\right.$$

---

non - conserving forces :

aerodynamic $= f\left(\dot{x}_p, \dot{y}_p, \theta_p, w_x, w_y\right)$,

w for wind components. $= f\,(relV_x, relV_y)$, **relV is relative to air**

dumping (structural) $= f\,(\dot{l}_i) = f\left(\dot{x}_i, \dot{y}_i, \dot{x}_p, \dot{y}_p\right)$

---

non dim the full equations

```
(smallEqs =
   quadEqNominal /. terms2 /.
    terms3) // MatrixForm
```

$$\left(\begin{array}{l} \frac{\text{r1x}\, k_1\, (a-\text{L0}_1)}{\sqrt{a^2}}+\frac{\text{r2x}\, k_2\, (b-\text{L0}_2)}{\sqrt{b^2}} == m_p\, \text{x} \\ \frac{\text{r1y}\, k_1\, (a-\text{L0}_1)}{\sqrt{a^2}}+\frac{\text{r2y}\, k_2\, (b-\text{L0}_2)}{\sqrt{b^2}} == m_p\, (g\, + \\ \frac{\text{c1}\, k_1\, (a-\text{L0}_1)}{\sqrt{a^2}}+\frac{\text{c2}\, k_2\, (b-\text{L0}_2)}{\sqrt{b^2}} + i_{p,zz}\, \theta_p{}' \end{array}\right.$$

NonDimEq manually settings the terms:

$\tilde{y}_p[t] = y_p[t] \,/\, \text{L0}_1$ or any other of the lengths varialbes $(x_p, \text{r1x}, \text{r1y}, \text{r2x}, \text{r2y}, h_p, l_p)$

$t = \tau \,/\, \omega_s$

$\omega_s{}^2 = \frac{k_1}{m_p}\left[\frac{g}{l} = \frac{1}{s^2}\right]$

A is non - dimentional form of ' a '

B is non - dimentional form of ' b '

$$\left(\begin{array}{c} \frac{\left(1-\frac{1}{A}\right) k_1\, \text{L0}_1\, \text{r1x}}{m_p}+\frac{k_2\, \text{L0}_1\, \text{r2x} \left(1-\frac{\text{L0}_2}{B\, \text{L0}_1}\right)}{m_p} = \text{L0}_1\, \omega_s^2\, x_p''(t) \\ \frac{\left(1-\frac{1}{A}\right) k_1\, \text{L0}_1\, \text{r1y}}{m_p}+\frac{k_2\, \text{L0}_1\, \text{r2y} \left(1-\frac{\text{L0}_2}{B\, \text{L0}_1}\right)}{m_p} - g = \text{L0}_1\, \omega_s^2\, y_p''(t) \\ -\frac{\left(1-\frac{1}{A}\right) c_1\, k_1\, \text{L0}_1^2}{i_{p,\text{zz}}}-\frac{c_2\, k_2\, \text{L0}_1^2 \left(1-\frac{\text{L0}_2}{B\, \text{L0}_1}\right)}{i_{p,\text{zz}}} = \omega_s^2\, \theta_p''(t) \end{array}\right)$$

```
(*Delta_Equilibrium = m_p g / k_i *)
```

$$\text{greekTerms} = \left\{ \frac{k_2}{k_1} \to \kappa,\ \frac{L0_2}{L0_1} \to \mathcal{L},\ \frac{k_1}{m_p} \text{ -> } \omega_s{}^2,\ \frac{m_p\, L0_1{}^2}{I_p}\left(= \frac{L0_1{}^2\, k_1}{I_p\, \omega_s{}^2}\right) \to \alpha,\ \frac{g}{L0_1\, \omega_s^2}\left(= \frac{g\, m_p}{L0_1\, k_1}\right) \to \gamma \right\}$$

$$\Bigg(\text{NonDimEq} = \Big\{ \omega_s^2\, L0_1 \left(1 - \frac{1}{A}\right) r1x + \kappa\, \omega_s^2\, L0_1 \left(1 - \frac{1}{B}\mathcal{L}\right) r2x == L0_1\, \omega_s^2\, x_p''[t],$$
$$\omega_s^2\, L0_1 \left(1 - \frac{1}{A}\right) r1y + \kappa\, \omega_s^2\, L0_1 \left(1 - \frac{1}{B}\mathcal{L}\right) r2y - \frac{g}{L0_1\, \omega_s^2} L0_1\, \omega_s^2 == L0_1\, \omega_s^2\, y_p''[t],$$
$$\frac{k_1}{-\mathbb{i}_{p,zz}} \frac{L0_1{}^2}{\omega_s^2} \omega_s^2 \left(1 - \frac{1}{A}\right) c_1 + \kappa \frac{k_1}{-\mathbb{i}_{p,zz}} \frac{L0_1{}^2}{\omega_s^2} \omega_s^2 \left(1 - \frac{1}{B}\mathcal{L}\right) c_2 == \omega_s^2\, \theta_p''[t] \Big\}\Bigg) \text{ // Flatten // MatrixForm // TraditionalForm}$$

$$\begin{pmatrix} \left(1-\frac{1}{A}\right) L0_1\, r1x\, \omega_s^2 + \kappa\, L0_1\, r2x \left(1 - \frac{\mathcal{L}}{B}\right) \omega_s^2 = L0_1\, \omega_s^2\, x_p''(t) \\ \left(1-\frac{1}{A}\right) L0_1\, r1y\, \omega_s^2 + \kappa\, L0_1\, r2y \left(1 - \frac{\mathcal{L}}{B}\right) \omega_s^2 - g = L0_1\, \omega_s^2\, y_p''(t) \\ -\frac{\left(1-\frac{1}{A}\right) c_1\, k_1\, L0_1^2}{i_{p,zz}} - \frac{c_2\, \kappa\, k_1\, L0_1^2 \left(1-\frac{\mathcal{L}}{B}\right)}{i_{p,zz}} = \omega_s^2\, \theta_p''(t) \end{pmatrix}$$

$$\ddot{\mathcal{X}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\alpha \end{pmatrix} \left(1 - \frac{1}{A}\right) \mathcal{V}_1 + \kappa \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -\alpha \end{pmatrix} \left(1 - \frac{1}{B}\mathcal{L}\right) \mathcal{V}_2 - \begin{pmatrix} 0 \\ \gamma \\ 0 \end{pmatrix}$$

$$\mathcal{X} = \begin{pmatrix} x_p[t] \\ y_p[t] \\ \theta_p[t] \end{pmatrix} (*\text{//Flatten}*)$$

$$\text{greekTermsSymetricCase} = \left\{ (*\tfrac{k_2}{k_1}\to*)\,\kappa \to 1,\ (*\tfrac{L0_2}{L0_1}\to*)\,\mathcal{L} \to 1 \right\}$$

$$\text{greekTermsGeneral} = \Big\{ (*\tfrac{k_2}{k_1}\to*)\,\kappa \to 1,\ (*\tfrac{L0_2}{L0_1}\to*)\,\mathcal{L} \to 1,\ (*\tfrac{k_1}{m_p}\text{->}*)\,\omega_s{}^2 \to 1,$$

```
(* mpL01^2/Ip (= L01^2 k1/(Ip ωs^2)) →*) α → 1,
(* q/(L01 ωs^2) (= q mp/(L01 k1)) → *) γ → 1 (* make sure it is not over-determined constant *)

}
(* already here : replacing all former hp,lp with new 2hp,2lp*)
A(*→Sqrt[r1x^2+r1y^2] *) = √((Sin[θp[t]] hp + Cos[θp[t]] lp + (x1[t] - xp[t]))^2 +
    (- Cos[θp[t]] hp + Sin[θp[t]] lp + (y1[t] - yp[t]))^2)

B(*→Sqrt[r2x^2+r2y^2] *) = √((Sin[θp[t]] hp - Cos[θp[t]] lp + (x2[t] - xp[t]))^2 +
    (- Cos[θp[t]] hp - Sin[θp[t]] lp + (y2[t] - yp[t]))^2)
(*c1(*→dr1+dr2*)=lp (-Sin[θp[t]] (x1[t]- xp[t])+Cos[θp[t]] (y1[t]-yp[t]))+
   hp (Cos[θp[t]]( x1[t]- xp[t])+Sin[θp[t]] (y1[t]-yp[t]))
   c2(*→dr3+dr4*)=lp (Sin[θp[t]] (x2[t]- xp[t])+Cos[θp[t]] (-y2[t]+yp[t]))+
   hp (Cos[θp[t]]( x2[t]- xp[t])+Sin[θp[t]] (y2[t]-yp[t]))*)
𝒱1(*={{r1x},{r1y},{c1}}*) =
 ( (Sin[θp[t]] hp + Cos[θp[t]] lp + (x1[t] - xp[t]))
   (- Cos[θp[t]] hp + Sin[θp[t]] lp + (y1[t] - yp[t]))
   lp (-Sin[θp[t]] (x1[t] - xp[t]) + Cos[θp[t]] (y1[t] - yp[t])) + hp (Cos[θp[t]] ( x1[t] - xp[
 )

𝒱2(*={{r2x},{r2y},{c2}}*) =
 ( (Sin[θp[t]] hp - Cos[θp[t]] lp + (x2[t] - xp[t]))
   (- Cos[θp[t]] hp - Sin[θp[t]] lp + (y2[t] - yp[t]))
   lp (Sin[θp[t]] (x2[t] - xp[t]) + Cos[θp[t]] (-y2[t] + yp[t])) + hp (Cos[θp[t]] ( x2[t] - xp[
 )
"equations with no general forces :"
EOM =
  D[𝒳, {t, 2}] == (({{1, 0, 0}, {0, 1, 0}, {0, 0, -α}} (1 - 1/A)).𝒱1 + (κ {{1, 0, 0}, {0, 1, 0}, {0, 0, -α}} (1 - 1/B ℒ)).𝒱2 - {{0}, {γ}, {0}} // Flatten;
{{xp[t]}, {yp[t]}, {θp[t]}}
{κ → 1, ℒ → 1}
{κ → 1, ℒ → 1, ωs^2 → 1, α → 1, γ → 1}
√((Sin[θp[t]] hp + Cos[θp[t]] lp + x1[t] - xp[t])^2 +
   (-Cos[θp[t]] hp + Sin[θp[t]] lp + y1[t] - yp[t])^2)
√((Sin[θp[t]] hp - Cos[θp[t]] lp + x2[t] - xp[t])^2 +
   (-Cos[θp[t]] hp - Sin[θp[t]] lp + y2[t] - yp[t])^2)
```

```
{{Sin[θp[t]] hp + Cos[θp[t]] lp + x1[t] - xp[t]},
 {-Cos[θp[t]] hp + Sin[θp[t]] lp + y1[t] - yp[t]},
 {lp (-Sin[θp[t]] (x1[t] - xp[t]) + Cos[θp[t]] (y1[t] - yp[t])) +
   hp (Cos[θp[t]] (x1[t] - xp[t]) + Sin[θp[t]] (y1[t] - yp[t]))}}

{{Sin[θp[t]] hp - Cos[θp[t]] lp + x2[t] - xp[t]},
 {-Cos[θp[t]] hp - Sin[θp[t]] lp + y2[t] - yp[t]},
 {hp (Cos[θp[t]] (x2[t] - xp[t]) + Sin[θp[t]] (y2[t] - yp[t])) +
   lp (Sin[θp[t]] (x2[t] - xp[t]) + Cos[θp[t]] (-y2[t] + yp[t]))}}
```

```
equations with no general forces :
```

```
nameChange = {lp → wp, a_[t] → a};
EOM /. nameChange /. greekTermsSymetricCase // Flatten
(*//MatrixForm*)(*//TraditionalForm*)
EOM /. nameChange /. greekTermsSymetricCase // Flatten
 (*//MatrixForm*) // TraditionalForm
```

```
{{xp''}, {yp''}, {θp''}} == {{(Sin[θp] hp + Cos[θp] wp + x1 - xp)
      (1 - 1 / (√((Sin[θp] hp + Cos[θp] wp + x1 - xp)^2 + (-Cos[θp] hp + Sin[θp] wp + y1 - yp)^2))) +
    (Sin[θp] hp - Cos[θp] wp + x2 - xp)
      (1 - 1 / (√((Sin[θp] hp - Cos[θp] wp + x2 - xp)^2 + (-Cos[θp] hp - Sin[θp] wp + y2 - yp)^2)))},
  {-γ + (1 - 1 / (√((Sin[θp] hp + Cos[θp] wp + x1 - xp)^2 + (-Cos[θp] hp + Sin[θp] wp + y1 - yp)^2)))
      (-Cos[θp] hp + Sin[θp] wp + y1 - yp) +
    (1 - 1 / (√((Sin[θp] hp - Cos[θp] wp + x2 - xp)^2 + (-Cos[θp] hp - Sin[θp] wp + y2 - yp)^2)))
      (-Cos[θp] hp - Sin[θp] wp + y2 - yp)},
  {-α (wp (-Sin[θp] (x1 - xp) + Cos[θp] (y1 - yp)) + hp (Cos[θp] (x1 - xp) + Sin[θp] (y1 - yp)))
      (1 - 1 / (√((Sin[θp] hp + Cos[θp] wp + x1 - xp)^2 + (-Cos[θp] hp + Sin[θp] wp + y1 - yp)^2))) -
    α (1 - 1 / (√((Sin[θp] hp - Cos[θp] wp + x2 - xp)^2 + (-Cos[θp] hp - Sin[θp] wp + y2 - yp)^2)))
      (hp (Cos[θp] (x2 - xp) + Sin[θp] (y2 - yp)) + wp (Sin[θp] (x2 - xp) + Cos[θp] (-y2 + yp)))}}
```

$$\begin{pmatrix} x_p'' \\ y_p'' \\ \theta_p'' \end{pmatrix} = \left(\begin{array}{r} (\sin(\theta_p)\, h_p + \cos(\theta_p)\, w_p + x_1 - x_p) \left(1 - \frac{}{\sqrt{(\sin(\theta_p)\, h_p + \cos(\theta_p)\, w_p + x_1 - x_p}} \right. \\ -\gamma + \left(1 - \frac{1}{\sqrt{(\sin(\theta_p)\, h_p + \cos(\theta_p)\, w_p + x_1 - x_p)^2 + (-\cos(\theta_p)\, h_p + \sin(\theta_p)\, w_p + y_1 - y_p)^2}} \right) (-\mathrm{co} \\ -\alpha\, (w_p\, (\cos(\theta_p)\, (y_1 - y_p) - \sin(\theta_p)\, (x_1 - x_p)) + h_p\, (\cos(\theta_p)\, (x_1 - x_p) + \sin(\theta_p)\, (y_1 - y_p))) \left(1 - \frac{}{\sqrt{(\sin(\theta_p)\, h_p + \cos(\theta_p)\, w_p + x_1 - x}} \right. \end{array} \right.$$

```
numerics = {hp → 0.1, wp → 1, x1[t] → 0, y1[t] → 0, x2[t] → 2 wp, y2[t] → 0, α → 0.04, γ → 2}
EOM //. greekTermsSymetricCase //. numerics //. nameChange // Flatten //
  TraditionalForm
```

$\{h_p \to 0.1, w_p \to 1, x_1[t] \to 0, y_1[t] \to 0, x_2[t] \to 2\, w_p, y_2[t] \to 0, \alpha \to 0.04, \gamma \to 2\}$

$$\begin{pmatrix} x_p'' \\ y_p'' \\ \theta_p'' \end{pmatrix} = \left(\begin{array}{r} (0.1\sin(\theta_p) - \cos(\theta_p)\, w_p - x_p + 2)\left(1 - \frac{1}{\sqrt{(0.1\sin(\theta_p) - \cos(\theta_p)\, w_p - x_p + 2)^2 + (-0.1\cos(\theta_p) - \mathrm{s}}} \right. \\ \left(1 - \frac{1}{\sqrt{(0.1\sin(\theta_p) - \cos(\theta_p)\, w_p - x_p + 2)^2 + (-0.1\cos(\theta_p) - \sin(\theta_p)\, w_p - y_p)^2}}\right)(-0.1\cos(\theta_p) - \sin(\theta_p) \\ -0.04\left(1 - \frac{1}{\sqrt{(0.1\sin(\theta_p) - \cos(\theta_p)\, w_p - x_p + 2)^2 + (-0.1\cos(\theta_p) - \sin(\theta_p)\, w_p - y_p)^2}}\right)(w_p(\sin(\theta_p)(2 - x_p) + \cos(\theta_p)\, y_p) + 0.1(\cos(\theta_p)(2 - \end{array} \right.$$

```
EquibInputConditions = {x1[t] → 0, y1[t] → 0, x2[t] → 2 wp, y2[t] → y1[t]}
```

$\{x_1[t] \to 0, y_1[t] \to 0, x_2[t] \to 2\, w_p, y_2[t] \to y_1[t]\}$

```
(SymetricEquib = EOM /. nameChange /. greekTermsSymetricCase /. equibTerms //.
    EquibInputConditions) // MatrixForm // TraditionalForm
```

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \left(\begin{array}{r} (\sin(\theta_p(t))\, h_p - \cos(\theta_p(t))\, w_p + 2\, w_p - x_p(t))\left(1 - \frac{}{\sqrt{(\sin(\theta_p(t))\, h_p - \cos(\theta_p(t))}} \right. \\ -\gamma + \left(1 - \frac{1}{\sqrt{(\sin(\theta_p(t))\, h_p - \cos(\theta_p(t))\, w_p + 2\, w_p - x_p(t))^2 + (-\cos(\theta_p(t))\, h_p - \sin(\theta_p(t))\, w_p - y_p(t))^2}} \right. \\ -\alpha\left(1 - \frac{1}{\sqrt{(\sin(\theta_p(t))\, h_p - \cos(\theta_p(t))\, w_p + 2\, w_p - x_p(t))^2 + (-\cos(\theta_p(t))\, h_p - \sin(\theta_p(t))\, w_p - y_p(t))^2}}\right)(w_p(\sin(\theta_p(t))(2\, w_p - x_p(t)) + \cos(\theta_p(t))\, y_p(t)) + \end{array} \right.$$

```
NumericParametersTest =
  {k₁ → 200, k₂ → k₁ + 0, LO₁ → 2, LO₂ → LO₁ + 0, mₚ → 2, hₚ → 0.1, wₚ → 1, g → 9.81}
greekTermsGeneralForTest = {
   κ → k₂/k₁,
   ℒ → LO₂/LO₁,
   ωₛ^2 → k₁/mₚ,
   α → (3 LO₁^2)/(wₚ^2 + hₚ^2),
   γ -> ((g mₚ)/(LO₁ k₁))
  }
g/LO₁ /. NumericParametersTest
NumericTestParams = greekTermsGeneralForTest //. NumericParametersTest
```

$\{k_1 \to 200,\ k_2 \to k_1,\ LO_1 \to 2,\ LO_2 \to LO_1,\ m_p \to 2,\ h_p \to 0.1,\ w_p \to 1,\ g \to 9.81\}$

$\left\{\kappa \to \frac{k_2}{k_1},\ \mathcal{L} \to \frac{LO_2}{LO_1},\ \omega_s^2 \to \frac{k_1}{m_p},\ \alpha \to \frac{3\,LO_1^2}{h_p^2 + w_p^2},\ \gamma \to \frac{g\,m_p}{k_1\,LO_1}\right\}$

4.905

$\left\{\kappa \to 1,\ \mathcal{L} \to 1,\ \omega_s^2 \to 100,\ \alpha \to 11.8812,\ \gamma \to 0.04905\right\}$

simple case testings:

```
(*trajectory:
   τ=0:  ÿ=1m/s²  until y₁=y₂=10LO₁
            ÿ=-1m/s² until ẏ₁=ẏ₂=0
           ẍ₁=ẍ₂=1m/s²  until ẋ₁=ẋ₂=2m/s
               disterbunce can be input by x₁+=5LO₁ over 1/(100 √ωₛ)*)
```

```
EOM // MatrixForm // TraditionalForm
nameChange
greekTermsSymetricCase
equibTerms
EquibInputConditions
```

$$\begin{pmatrix} x_p''(t) \\ y_p''(t) \\ \theta_p''(t) \end{pmatrix} = \begin{pmatrix} (\sin(\theta_p(t))\, h_p + \cos(\theta_p(t))\, l_p + x_1(t) - x_p(t)) \\ -\gamma + \left(1 - \frac{1}{\sqrt{(\sin(\theta_p(t))\, h_p + \cos(\theta_p(t))\, l_p + x_1(t) - x_p(t))^2 + (-\cos}}\right. \\ -\alpha\,(l_p\,(\cos(\theta_p(t))\,(y_1(t) - y_p(t)) - \sin(\theta_p(t))\,(x_1(t) - x_p(t))) + h_p\,(\cos(\theta_p(t))\,(x_1(t) - x_p(t)) + \sin(\theta_p(t))\,(y_1(t) - y_p(t))) \end{pmatrix}$$

```
{lp → wp}
{κ → 1, ℒ → 1}
{xp'[t] → 0, yp'[t] → 0, θp'[t] → 0, xp''[t] → 0, yp''[t] → 0, θp''[t] → 0}
{x1[t] → 0, y1[t] → 0, x2[t] → wp, y2[t] → y1[t]}
```

```
(* greekTermsSymetricCase must be used again
 because this is where the equilibrium point is refering to .
  EquibInputConditions might be turned of later for better investigation *)
(SymetricEOMtoInvestigate = EOM /. nameChange /. greekTermsSymetricCase //.
    EquibInputConditions) // MatrixForm // TraditionalForm
```

$$\begin{pmatrix} x_p''(t) \\ y_p''(t) \\ \theta_p''(t) \end{pmatrix} = \begin{pmatrix} (\sin(\theta_p(t))\, h_p - \cos(\theta_p(t))\, w_p + w_p - x_p(t)) \left(1 - \frac{}{\sqrt{(\sin(\theta_p(t))\, h_p - \cos(\theta_p(t}}\right. \\ -\gamma + \left(1 - \frac{1}{\sqrt{(\sin(\theta_p(t))\, h_p - \cos(\theta_p(t))\, w_p + w_p - x_p(t))^2 + (-\cos(\theta_p(t))\, h_p - \sin(\theta_p(t))\, w_p - y_p(t)}}\right. \\ -\alpha \left(1 - \frac{1}{\sqrt{(\sin(\theta_p(t))\, h_p - \cos(\theta_p(t))\, w_p + w_p - x_p(t))^2 + (-\cos(\theta_p(t))\, h_p - \sin(\theta_p(t))\, w_p - y_p(t))^2}}\right) (w_p\,(\sin(\theta_p(t))\,(w_p - x_p(t)) + \cos(\theta_p(t))\, y_p(t)) \end{pmatrix}$$