quad test driven development and testing

required : system of 2 quads and 1 payload

constraigns : not given expicitly. can make ones ..

test1

motion equations by Newton method == motion equations by Lagrangian method

quastions TODO :

how to paint vector for direction of forces ,and coordinate systems.

how to do it in *Mathematica*, in Python (blender,matplotlib)

how to paint moment arrow. same applications in question.

system elements :

quad 1 (6dof)

quad 2 (6dof)

payload (contrained to quads locations)

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kinematics :

In[1]:=

```
(X1 = (x1[t]
       y1[t]
       z1[t])) // MatrixForm // TraditionalForm
(Imat = (I11  0    0
         0    I12  0
         0    0    I13)) // MatrixForm // TraditionalForm
ω1 = {p1, q1, r1}
(X2 = (x2[t]
       y2[t]
       z2[t])) // MatrixForm // TraditionalForm
(Imat = (I11  0    0
         0    I12  0
         0    0    I13)) // MatrixForm // TraditionalForm
ω2 = {p2, q2, r2}
```

Out[1]//TraditionalForm=

$$\begin{pmatrix} x_1(t) \\ y_1(t) \\ z_1(t) \end{pmatrix}$$

Out[2]//TraditionalForm=

$$\begin{pmatrix} i_{11} & 0 & 0 \\ 0 & i_{12} & 0 \\ 0 & 0 & i_{13} \end{pmatrix}$$

Out[3]= $\{p_1, q_1, r_1\}$

Out[4]//TraditionalForm=

$$\begin{pmatrix} x_2(t) \\ y_2(t) \\ z_2(t) \end{pmatrix}$$

Out[5]//TraditionalForm=

$$\begin{pmatrix} i_{11} & 0 & 0 \\ 0 & i_{12} & 0 \\ 0 & 0 & i_{13} \end{pmatrix}$$

Out[6]= $\{p_2, q_2, r_2\}$

rotations :

In[7]:=
```
X_I = {1, 0, 0}
Y_I = {0, 1, 0}
Z_I = {0, 0, 1}
(Rx = RotationMatrix[ϕ[t], {1, 0, 0}]) // MatrixForm
(Ry = RotationMatrix[θ[t], {0, 1, 0}]) // MatrixForm
(Rz = RotationMatrix[ψ[t], {0, 0, 1}]) // MatrixForm
(R^I_B = Rz.Ry.Rx) /. Cos → C /. Sin → S // MatrixForm
(vec3 = (Rx. Ry).{0, 0, 1}) /. Cos → C /. Sin → S
(vec2 = (Rx).{0, 1, 0}) /. Cos → C /. Sin → S
(vec1 = {1, 0, 0})
(R^PQR_Euler = Transpose[Join[{vec1}, {vec2}, {vec3}]]) // MatrixForm
(R^euler_pqr = Inverse[R^PQR_Euler] // FullSimplify) // MatrixForm
(pqrvec = R^PQR_Euler.D[{ϕ[t], θ[t], ψ[t]}, t] // FullSimplify) // MatrixForm
(ω1 = R^PQR_Euler.D[{ϕ[t], θ[t], ψ[t]}, t] /. ϕ → ϕ1 /. θ → θ1 /. ψ → ψ1 // FullSimplify) //
 MatrixForm
(ω2 = R^PQR_Euler.D[{ϕ[t], θ[t], ψ[t]}, t] /. ϕ → ϕ2 /. θ → θ2 /. ψ → ψ2 // FullSimplify) //
 MatrixForm
(X1dot = D[X1, t]) // MatrixForm // TraditionalForm
(X2dot = D[X2, t]) // MatrixForm // TraditionalForm
```

Out[7]= {1, 0, 0}

Out[8]= {0, 1, 0}

Out[9]= {0, 0, 1}

Out[10]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \text{Cos}[\phi[t]] & -\text{Sin}[\phi[t]] \\ 0 & \text{Sin}[\phi[t]] & \text{Cos}[\phi[t]] \end{pmatrix}$$

Out[11]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\theta[t]] & 0 & \text{Sin}[\theta[t]] \\ 0 & 1 & 0 \\ -\text{Sin}[\theta[t]] & 0 & \text{Cos}[\theta[t]] \end{pmatrix}$$

Out[12]//MatrixForm=

$$\begin{pmatrix} \text{Cos}[\psi[t]] & -\text{Sin}[\psi[t]] & 0 \\ \text{Sin}[\psi[t]] & \text{Cos}[\psi[t]] & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Out[13]//MatrixForm=

$$\begin{pmatrix} C[\theta[t]]\,C[\psi[t]] & C[\psi[t]]\,S[\theta[t]]\,S[\phi[t]] - C[\phi[t]]\,S[\psi[t]] & C[\phi[t]]\,C[\psi[t]]\,S[\theta[t]] + S[\phi \\ C[\theta[t]]\,S[\psi[t]] & C[\phi[t]]\,C[\psi[t]] + S[\theta[t]]\,S[\phi[t]]\,S[\psi[t]] & -C[\psi[t]]\,S[\phi[t]] + C[\phi[t]]\,S[\\ -S[\theta[t]] & C[\theta[t]]\,S[\phi[t]] & C[\theta[t]]\,C[\phi[t]] \end{pmatrix}$$

Out[14]= {S[θ[t]], -C[θ[t]] S[ϕ[t]], C[θ[t]] C[ϕ[t]]}

Out[15]= {0, C[ϕ[t]], S[ϕ[t]]}

Out[16]= {1, 0, 0}

Out[17]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & \text{Sin}[\theta[t]] \\ 0 & \text{Cos}[\phi[t]] & -\text{Cos}[\theta[t]]\,\text{Sin}[\phi[t]] \\ 0 & \text{Sin}[\phi[t]] & \text{Cos}[\theta[t]]\,\text{Cos}[\phi[t]] \end{pmatrix}$$

Out[18]//MatrixForm=

$$\begin{pmatrix} 1 & \text{Sin}[\phi[t]]\,\text{Tan}[\theta[t]] & -\text{Cos}[\phi[t]]\,\text{Tan}[\theta[t]] \\ 0 & \text{Cos}[\phi[t]] & \text{Sin}[\phi[t]] \\ 0 & -\text{Sec}[\theta[t]]\,\text{Sin}[\phi[t]] & \text{Cos}[\phi[t]]\,\text{Sec}[\theta[t]] \end{pmatrix}$$

Out[19]//MatrixForm=

$$\begin{pmatrix} \phi'[t] + \text{Sin}[\theta[t]]\,\psi'[t] \\ \text{Cos}[\phi[t]]\,\theta'[t] - \text{Cos}[\theta[t]]\,\text{Sin}[\phi[t]]\,\psi'[t] \\ \text{Sin}[\phi[t]]\,\theta'[t] + \text{Cos}[\theta[t]]\,\text{Cos}[\phi[t]]\,\psi'[t] \end{pmatrix}$$

Out[20]//MatrixForm=

$$\begin{pmatrix} \phi_1{}'[t] + \text{Sin}[\theta_1[t]]\,\psi_1{}'[t] \\ \text{Cos}[\phi_1[t]]\,\theta_1{}'[t] - \text{Cos}[\theta_1[t]]\,\text{Sin}[\phi_1[t]]\,\psi_1{}'[t] \\ \text{Sin}[\phi_1[t]]\,\theta_1{}'[t] + \text{Cos}[\theta_1[t]]\,\text{Cos}[\phi_1[t]]\,\psi_1{}'[t] \end{pmatrix}$$

Out[21]//MatrixForm=

$$\begin{pmatrix} \phi_2{}'[t] + \text{Sin}[\theta_2[t]]\,\psi_2{}'[t] \\ \text{Cos}[\phi_2[t]]\,\theta_2{}'[t] - \text{Cos}[\theta_2[t]]\,\text{Sin}[\phi_2[t]]\,\psi_2{}'[t] \\ \text{Sin}[\phi_2[t]]\,\theta_2{}'[t] + \text{Cos}[\theta_2[t]]\,\text{Cos}[\phi_2[t]]\,\psi_2{}'[t] \end{pmatrix}$$

Out[22]//TraditionalForm=

$$\begin{pmatrix} x_1{}'(t) \\ y_1{}'(t) \\ z_1{}'(t) \end{pmatrix}$$

Out[23]//TraditionalForm=

$$\begin{pmatrix} x_2{}'(t) \\ y_2{}'(t) \\ z_2{}'(t) \end{pmatrix}$$

$$\mathbf{X_p} = \begin{pmatrix} \mathbf{x_1[t]} \\ \mathbf{y_1[t]} \\ \mathbf{z_1[t]} \end{pmatrix} + \mathbf{l_1} \begin{pmatrix} \textbf{Sin}[\beta[t]]\,\textbf{Cos}[\gamma[t]] \\ \textbf{Sin}[\beta[t]]\,\textbf{Sin}[\gamma[t]] \\ -\textbf{Cos}[\beta[t]] \end{pmatrix}$$

(when $\beta = 0$,

and $\gamma = 0$ then $X_p = \begin{pmatrix} 0 \\ 0 \\ -l_1 \end{pmatrix}$. it is the equlibrium state of the pendulum payload)

$0\,° < \beta < 180\,°$

$-180\,° < \gamma < 180\,°$

In[24]:=

$$\left(\mathbf{X_p} = \begin{pmatrix} \mathbf{x_1[t]} \\ \mathbf{y_1[t]} \\ \mathbf{z_1[t]} \end{pmatrix} + \mathbf{l_1} \begin{pmatrix} \textbf{Sin}[\beta[t]]\,\textbf{Cos}[\gamma[t]] \\ \textbf{Sin}[\beta[t]]\,\textbf{Sin}[\gamma[t]] \\ -\textbf{Cos}[\beta[t]] \end{pmatrix}\right) \;\textbf{// MatrixForm // TraditionalForm}$$

Out[24]//TraditionalForm=

$$\begin{pmatrix} l_1 \sin(\beta(t)) \cos(\gamma(t)) + x_1(t) \\ l_1 \sin(\beta(t)) \sin(\gamma(t)) + y_1(t) \\ z_1(t) - l_1 \cos(\beta(t)) \end{pmatrix}$$

(V_{tr} == D[X_p, t] // Simplify) // MatrixForm // TraditionalForm

$$V_{\text{tr}} = \begin{pmatrix} l_1 (\cos(\beta(t)) \cos(\gamma(t))\, \beta'(t) - \sin(\beta(t)) \sin(\gamma(t))\, \gamma'(t)) + x_1{}'(t) \\ l_1 (\cos(\beta(t)) \sin(\gamma(t))\, \beta'(t) + \cos(\gamma(t)) \sin(\beta(t))\, \gamma'(t)) + y_1{}'(t) \\ \sin(\beta(t))\, l_1\, \beta'(t) + z_1{}'(t) \end{pmatrix}$$

```
D[Xp // Flatten, t].D[Xp // Flatten, t] // MatrixForm // TraditionalForm
```

$$(l_1\,\beta'(t)\cos(\beta(t))\cos(\gamma(t)) - l_1\sin(\beta(t))\,\gamma'(t)\sin(\gamma(t)) + x_1{}'(t))^2 + (l_1\,\beta'(t)\cos(\beta(t))\sin(\gamma(t)) + l_1\sin(\beta(t))\,\gamma'(t)\cos(\gamma(t)) + y_1{}'(t))^2 + (l_1\,\beta'(t)\sin(\beta(t)) + z_1{}'(t))^2$$

In[25]:=
```
(VpVp = D[Xp // Flatten, t].D[Xp // Flatten, t] // Expand // Simplify) // MatrixForm //
 TraditionalForm
```

Out[25]//TraditionalForm=

$$l_1^2\left(\beta'(t)^2 + \sin^2(\beta(t))\,\gamma'(t)^2\right) + 2\,l_1\,(\sin(\beta(t))\,\gamma'(t)\,(\cos(\gamma(t))\,y_1{}'(t) - \sin(\gamma(t))\,x_1{}'(t)) + \beta'(t)\,(\cos(\beta(t))\cos(\gamma(t))\,x_1{}'(t) + \cos(\beta(t))\sin(\gamma(t))\,y_1{}'(t) + \sin(\beta(t))\,z_1{}'(t))) + x_1{}'(t)^2 + y_1{}'(t)^2 + z_1{}'(t)^2$$

```
(Vp^2 == VpVp) // MatrixForm // TraditionalForm
```

$$(g\,m_1\,z_1(t)_p)^2 = l_1^2\left(\beta'(t)^2 + \sin^2(\beta(t))\,\gamma'(t)^2\right) + 2\,l_1\,(\sin(\beta(t))\,\gamma'(t)\,(\cos(\gamma(t))\,y_1{}'(t) - \sin(\gamma(t))\,x_1{}'(t)) + \beta'(t)\,(\cos(\beta(t))\cos(\gamma(t))\,x_1{}'(t) + \cos(\beta(t))\sin(\gamma(t))\,y_1{}'(t) + \sin(\beta(t))\,z_1{}'(t))) + x_1{}'(t)^2 + y_1{}'(t)^2 + z_1{}'(t)^2$$

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enrgies :

In[26]:=

```
x1dotSqr = (Transpose[X1dot].X1dot)[[1, 1]]
x2dotSqr = (Transpose[X2dot].X2dot)[[1, 1]]
(T = 1/2 m1 x1dotSqr + 1/2 ω1.Imat.ω1 + 1/2 m2 x2dotSqr + 1/2 ω2.Imat.ω2 + 1/2 mp VpVp)
(*/.Cos→C/.Sin→S*)
V = m1 g (z1[t]) + m2 g (z2[t]) + mp g (z1[t] - l1 Cos[β[t]])
L = T - V
```

Out[26]= $x_1'[t]^2 + y_1'[t]^2 + z_1'[t]^2$

Out[27]= $x_2'[t]^2 + y_2'[t]^2 + z_2'[t]^2$

Out[28]=

$$\frac{1}{2} m_1 \left(x_1'[t]^2 + y_1'[t]^2 + z_1'[t]^2\right) + \frac{1}{2} m_p \Big(l_1^2 \left(\beta'[t]^2 + \mathrm{Sin}[\beta[t]]^2\, \gamma'[t]^2\right) + x_1'[t]^2 + y_1'[t]^2 + z_1'[t]^2 + 2\, l_1\, (\mathrm{Sin}[\beta[t]]\, \gamma'[t]\, (-\mathrm{Sin}[\gamma[t]]\, x_1'[t] + \mathrm{Cos}[\gamma[t]]\, y_1'[t]) + \beta'[t]\, (\mathrm{Cos}[\beta[t]]\, \mathrm{Cos}[\gamma[t]]\, x_1'[t] + \mathrm{Cos}[\beta[t]]\, \mathrm{Sin}[\gamma[t]]\, y_1'[t] + \mathrm{Sin}[\beta[t]]\, z_1'[t]))\Big) + \frac{1}{2} m_2 \left(x_2'[t]^2 + y_2'[t]^2 + z_2'[t]^2\right) + \frac{1}{2} \Big(\mathbb{i}_{13}\, (\mathrm{Sin}[\phi_1[t]]\, \theta_1'[t] + \mathrm{Cos}[\theta_1[t]]\, \mathrm{Cos}[\phi_1[t]]\, \psi_1'[t])^2 + \mathbb{i}_{11}\, (\phi_1'[t] + \mathrm{Sin}[\theta_1[t]]\, \psi_1'[t])^2 + \mathbb{i}_{12}\, (\mathrm{Cos}[\phi_1[t]]\, \theta_1'[t] - \mathrm{Cos}[\theta_1[t]]\, \mathrm{Sin}[\phi_1[t]]\, \psi_1'[t])^2\Big) + \frac{1}{2} \Big(\mathbb{i}_{13}\, (\mathrm{Sin}[\phi_2[t]]\, \theta_2'[t] + \mathrm{Cos}[\theta_2[t]]\, \mathrm{Cos}[\phi_2[t]]\, \psi_2'[t])^2 + \mathbb{i}_{11}\, (\phi_2'[t] + \mathrm{Sin}[\theta_2[t]]\, \psi_2'[t])^2 + \mathbb{i}_{12}\, (\mathrm{Cos}[\phi_2[t]]\, \theta_2'[t] - \mathrm{Cos}[\theta_2[t]]\, \mathrm{Sin}[\phi_2[t]]\, \psi_2'[t])^2\Big)$$

Out[29]= $g\, m_1\, z_1[t] + g\, m_p\, (-\mathrm{Cos}[\beta[t]]\, l_1 + z_1[t]) + g\, m_2\, z_2[t]$

Out[30]=

$$-g\, m_1\, z_1[t] - g\, m_p\, (-\mathrm{Cos}[\beta[t]]\, l_1 + z_1[t]) - g\, m_2\, z_2[t] + \frac{1}{2} m_1 \left(x_1'[t]^2 + y_1'[t]^2 + z_1'[t]^2\right) + \frac{1}{2} m_p \Big(l_1^2 \left(\beta'[t]^2 + \mathrm{Sin}[\beta[t]]^2\, \gamma'[t]^2\right) + x_1'[t]^2 + y_1'[t]^2 + z_1'[t]^2 + 2\, l_1\, (\mathrm{Sin}[\beta[t]]\, \gamma'[t]\, (-\mathrm{Sin}[\gamma[t]]\, x_1'[t] + \mathrm{Cos}[\gamma[t]]\, y_1'[t]) + \beta'[t]\, (\mathrm{Cos}[\beta[t]]\, \mathrm{Cos}[\gamma[t]]\, x_1'[t] + \mathrm{Cos}[\beta[t]]\, \mathrm{Sin}[\gamma[t]]\, y_1'[t] + \mathrm{Sin}[\beta[t]]\, z_1'[t]))\Big) + \frac{1}{2} m_2 \left(x_2'[t]^2 + y_2'[t]^2 + z_2'[t]^2\right) + \frac{1}{2} \Big(\mathbb{i}_{13}\, (\mathrm{Sin}[\phi_1[t]]\, \theta_1'[t] + \mathrm{Cos}[\theta_1[t]]\, \mathrm{Cos}[\phi_1[t]]\, \psi_1'[t])^2 + \mathbb{i}_{11}\, (\phi_1'[t] + \mathrm{Sin}[\theta_1[t]]\, \psi_1'[t])^2 + \mathbb{i}_{12}\, (\mathrm{Cos}[\phi_1[t]]\, \theta_1'[t] - \mathrm{Cos}[\theta_1[t]]\, \mathrm{Sin}[\phi_1[t]]\, \psi_1'[t])^2\Big) + \frac{1}{2} \Big(\mathbb{i}_{13}\, (\mathrm{Sin}[\phi_2[t]]\, \theta_2'[t] + \mathrm{Cos}[\theta_2[t]]\, \mathrm{Cos}[\phi_2[t]]\, \psi_2'[t])^2 + \mathbb{i}_{11}\, (\phi_2'[t] + \mathrm{Sin}[\theta_2[t]]\, \psi_2'[t])^2 + \mathbb{i}_{12}\, (\mathrm{Cos}[\phi_2[t]]\, \theta_2'[t] - \mathrm{Cos}[\theta_2[t]]\, \mathrm{Sin}[\phi_2[t]]\, \psi_2'[t])^2\Big)$$

$$\left(\mathbf{T} = \frac{1}{2}\,\mathbf{m_1}\,\mathbf{x1dotSqr} + \frac{1}{2}\,\omega_1 . \mathbf{Imat} . \omega_1\right)$$

$$\mathbf{V} = \mathbf{m_1}\,\mathbf{g}\,(\mathbf{z_1}[\mathbf{t}])$$

$$\mathbf{L} = \mathbf{T} - \mathbf{V}$$

$$\frac{1}{2} m_1 \left(x_1'[t]^2 + y_1'[t]^2 + z_1'[t]^2\right) + \frac{1}{2}\left(\mathbb{i}_{13}\,(\text{Sin}[\phi_1[t]]\,\theta_1'[t] + \text{Cos}[\theta_1[t]]\,\text{Cos}[\phi_1[t]]\,\psi_1'[t])^2 + \mathbb{i}_{11}\,(\phi_1'[t] + \text{Sin}[\theta_1[t]]\,\psi_1'[t])^2 + \mathbb{i}_{12}\,(\text{Cos}[\phi_1[t]]\,\theta_1'[t] - \text{Cos}[\theta_1[t]]\,\text{Sin}[\phi_1[t]]\,\psi_1'[t])^2\right)$$

$$g\,m_1\,z_1[t]$$

$$-g\,m_1\,z_1[t] + \frac{1}{2} m_1 \left(x_1'[t]^2 + y_1'[t]^2 + z_1'[t]^2\right) + \frac{1}{2}\left(\mathbb{i}_{13}\,(\text{Sin}[\phi_1[t]]\,\theta_1'[t] + \text{Cos}[\theta_1[t]]\,\text{Cos}[\phi_1[t]]\,\psi_1'[t])^2 + \mathbb{i}_{11}\,(\phi_1'[t] + \text{Sin}[\theta_1[t]]\,\psi_1'[t])^2 + \mathbb{i}_{12}\,(\text{Cos}[\phi_1[t]]\,\theta_1'[t] - \text{Cos}[\theta_1[t]]\,\text{Sin}[\phi_1[t]]\,\psi_1'[t])^2\right)$$

$$\mathbf{q} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ \phi_1 \\ \theta_1 \\ \psi_1 \\ x_2 \\ y_2 \\ z_2 \\ \phi_2 \\ \theta_2 \\ \psi_2 \\ \beta \\ \gamma \end{pmatrix}$$

$$\{\{x_1\}, \{y_1\}, \{z_1\}, \{\phi_1\}, \{\theta_1\}, \{\psi_1\}, \{x_2\}, \{y_2\}, \{z_2\}, \{\{\{\phi_2\}, \{\theta_2\}, \{\psi_2\}\}\}, \{\beta\}, \{\gamma\}\}$$

In[31]:= **L**

Out[31]=

$$-g\,m_1\,z_1[t] - g\,m_p\,(-\text{Cos}[\beta[t]]\,l_1 + z_1[t]) - g\,m_2\,z_2[t] + \frac{1}{2} m_1 \left(x_1'[t]^2 + y_1'[t]^2 + z_1'[t]^2\right) + \frac{1}{2} m_p \left(l_1^2 \left(\beta'[t]^2 + \text{Sin}[\beta[t]]^2\,\gamma'[t]^2\right) + x_1'[t]^2 + y_1'[t]^2 + z_1'[t]^2 + 2\,l_1\,(\text{Sin}[\beta[t]]\,\gamma'[t]\,(-\text{Sin}[\gamma[t]]\,x_1'[t] + \text{Cos}[\gamma[t]]\,y_1'[t]) + \beta'[t]\,(\text{Cos}[\beta[t]]\,\text{Cos}[\gamma[t]]\,x_1'[t] + \text{Cos}[\beta[t]]\,\text{Sin}[\gamma[t]]\,y_1'[t] + \text{Sin}[\beta[t]]\,z_1'[t]))\right) + \frac{1}{2} m_2 \left(x_2'[t]^2 + y_2'[t]^2 + z_2'[t]^2\right) + \frac{1}{2}\left(\mathbb{i}_{13}\,(\text{Sin}[\phi_1[t]]\,\theta_1'[t] + \text{Cos}[\theta_1[t]]\,\text{Cos}[\phi_1[t]]\,\psi_1'[t])^2 + \mathbb{i}_{11}\,(\phi_1'[t] + \text{Sin}[\theta_1[t]]\,\psi_1'[t])^2 + \mathbb{i}_{12}\,(\text{Cos}[\phi_1[t]]\,\theta_1'[t] - \text{Cos}[\theta_1[t]]\,\text{Sin}[\phi_1[t]]\,\psi_1'[t])^2\right) + \frac{1}{2}\left(\mathbb{i}_{13}\,(\text{Sin}[\phi_2[t]]\,\theta_2'[t] + \text{Cos}[\theta_2[t]]\,\text{Cos}[\phi_2[t]]\,\psi_2'[t])^2 + \mathbb{i}_{11}\,(\phi_2'[t] + \text{Sin}[\theta_2[t]]\,\psi_2'[t])^2 + \mathbb{i}_{12}\,(\text{Cos}[\phi_2[t]]\,\theta_2'[t] - \text{Cos}[\theta_2[t]]\,\text{Sin}[\phi_2[t]]\,\psi_2'[t])^2\right)$$

In[32]:=
```
Needs["VariationalMethods`"]
```

In[33]:=
```
(quadEqNominal =
      EulerEquations[L, {x₁[t], y₁[t], z₁[t], ϕ₁[t], θ₁[t], ψ₁[t], x₂[t], y₂[t], z₂[t],
          ϕ₂[t], θ₂[t], ψ₂[t], β[t], γ[t]}, t][[All, 1]](*==Q*) // Simplify) /.
    Cos → c /. Sin → s // MatrixForm // TraditionalForm
```

Out[33]//TraditionalForm=

$$l_1\, m_p \left(2\, c(\beta(t))\, s(\gamma(\right.$$

$$l_1\, m_p \left(-c(\beta(t))\, s(\gamma(\right.$$

$$\frac{1}{2}\left((i_{12} - i_{13})\, c(\theta_1(t))^2\, s(2\, \phi\right.$$

$$\frac{1}{2}\left(s(2\, \theta_1(t))\, \psi_1'(t)^2 \left(-i_{13}\, c(\phi_1(t))^2 - i_{12}\, s(\phi_1(t))^2 + i_{11}\right) +\right.$$

$$(i_{12} - i_{13})\, c(\theta_1(t))^2\, (-s(2\, \phi_1(t)))\, \psi_1'(t)\, \phi_1'(t) + c(\theta_1(t))\, \theta_1'(t) \left(2\, s(\theta_1(t))\, \psi_1'(t) \left(i_{13}\, c(\phi_1(t))^2 + i_{12}\, s(\phi_1(t))^2 - i_{11}\right) - ((i_{13} - i_{12})\, c(\right.$$

$$\frac{1}{2}\left((i_{12} - i_{13})\, c(\theta_2(t))^2\, s(2\, \phi\right.$$

$$\frac{1}{2}\left(s(2\, \theta_2(t))\, \psi_2'(t)^2 \left(-i_{13}\, c(\phi_2(t))^2 - i_{12}\, s(\phi_2(t))^2 + i_{11}\right) +\right.$$

$$(i_{12} - i_{13})\, c(\theta_2(t))^2\, (-s(2\, \phi_2(t)))\, \psi_2'(t)\, \phi_2'(t) + c(\theta_2(t))\, \theta_2'(t) \left(2\, s(\theta_2(t))\, \psi_2'(t) \left(i_{13}\, c(\phi_2(t))^2 + i_{12}\, s(\phi_2(t))^2 - i_{11}\right) - ((i_{13} - i_{12})\, c(\right.$$

$$l_1\, m$$

In[34]:=
$$\textbf{Fthrust} = \begin{pmatrix} 0 \\ 0 \\ \text{Thrust}_1 + \text{Thrust}_2 + \text{Thrust}_3 + \text{Thrust}_4 \end{pmatrix}$$

$$\left(Q_1 = \left(\overset{I}{R_B}.\text{Fthrust}\right)[[1, 1]] (*X_I*) \text{ // Flatten // MatrixForm}\right)$$

$$\left(Q_2 = \left(\overset{I}{R_B}.\text{Fthrust}\right)[[2, 1]] (*Y_I*) \text{ // Flatten // MatrixForm}\right)$$

$$\left(Q_3 = \left(\overset{I}{R_B}.\text{Fthrust}\right)[[3, 1]] (*Z_I*) \text{ // Flatten // MatrixForm}\right)$$

$$(Q_4 = l_1 (\text{Thrust}_4 - \text{Thrust}_2) \text{ // MatrixForm})$$

$$(Q_5 = l_1 (\text{Thrust}_3 - \text{Thrust}_1) \text{ // MatrixForm})$$

$$(Q_6 = (\text{MotorMoment}_1 - \text{MotorMoment}_2 + \text{MotorMoment}_3 - \text{MotorMoment}_4) \text{ // MatrixForm})$$

Out[34]= $\{\{0\}, \{0\}, \{\text{Thrust}_1 + \text{Thrust}_2 + \text{Thrust}_3 + \text{Thrust}_4\}\}$

Out[35]//MatrixForm=

$$(\text{Cos}[\phi[t]]\,\text{Cos}[\psi[t]]\,\text{Sin}[\theta[t]] + \text{Sin}[\phi[t]]\,\text{Sin}[\psi[t]])\,(\text{Thrust}_1 + \text{Thrust}_2 + \text{Thrust}_3 + \text{Thrust}_4)$$

Out[36]//MatrixForm=

$$(-\text{Cos}[\psi[t]]\,\text{Sin}[\phi[t]] + \text{Cos}[\phi[t]]\,\text{Sin}[\theta[t]]\,\text{Sin}[\psi[t]])\,(\text{Thrust}_1 + \text{Thrust}_2 + \text{Thrust}_3 + \text{Thrust}_4)$$

Out[37]//MatrixForm=

$$\text{Cos}[\theta[t]]\,\text{Cos}[\phi[t]]\,(\text{Thrust}_1 + \text{Thrust}_2 + \text{Thrust}_3 + \text{Thrust}_4)$$

Out[38]//MatrixForm=

$$l_1\,(-\text{Thrust}_2 + \text{Thrust}_4)$$

Out[39]//MatrixForm=

$$l_1\,(-\text{Thrust}_1 + \text{Thrust}_3)$$

Out[40]//MatrixForm=

$$\text{MotorMoment}_1 - \text{MotorMoment}_2 + \text{MotorMoment}_3 - \text{MotorMoment}_4$$

In[41]:= $Q_{13} = Q_{14} = 0$

Out[41]= 0

limited cases tests :

$$\textbf{quadEqNominal /. } \psi_1 \to 0 \text{ /. } \theta_1 \to 0 \text{ // MatrixForm // TraditionalForm}$$

In[42]:= **quadEqNominal**

Out[42]=
$$\Big\{l_1\,m_p\,\big(\text{Cos}[\gamma[t]]\,\text{Sin}[\beta[t]]\,\beta'[t]^2 + 2\,\text{Cos}[\beta[t]]\,\text{Sin}[\gamma[t]]\,\beta'[t]\,\gamma'[t] + \text{Cos}[\gamma[t]]\,\text{Sin}[\beta[t]]\,\gamma'[t]^2 - \text{Cos}[\beta[t]]\,\text{Cos}[\gamma[t]]\,\beta''[t] + \text{Sin}[\beta[t]]\,\text{Sin}[\gamma[t]]\,\gamma''[t]\big) - (m_1 + m_p)\,x_1''[t],\ l_1\,m_p\,\big(\text{Sin}[\beta[t]]\,\text{Sin}[\gamma[t]]\,\beta'[t]^2 - 2\,\text{Cos}[\beta[t]]\,\text{Cos}[\gamma[t]]\,\beta'[t]\,\gamma'[t] + \text{Sin}[\beta[t]]\,\text{Sin}[\gamma[t]]\,\gamma'[t]^2 - \text{Cos}[\beta[t]]\,\text{Sin}[\gamma[t]]\,\beta''[t] - \text{Cos}[\gamma[t]]\,\text{Sin}[\beta[t]]\,\gamma''[t]\big) - (m_1 + m_p)\,y_1''[t],$$
$$-m_1\,(g + z_1''[t]) - m_p\,\big(g + l_1\,\big(\text{Cos}[\beta[t]]\,\beta'[t]^2 + \text{Sin}[\beta[t]]\,\beta''[t]\big) + z_1''[t]\big),$$
$$\frac{1}{2}\Big(-\text{Sin}[2\,\phi_1[t]]\,(\mathbb{i}_{12} - \mathbb{i}_{13})\,\theta_1'[t]^2 - 2\,\text{Cos}[\theta_1[t]]\,\big(\mathbb{i}_{11} + \text{Cos}[2\,\phi_1[t]]\,(\mathbb{i}_{12} - \mathbb{i}_{13})\big)\,\theta_1'[t]\,\psi_1'[t] + \text{Cos}[\theta_1[t]]^2\,\text{Sin}[2\,\phi_1[t]]\,(\mathbb{i}_{12} - \mathbb{i}_{13})\,\psi_1'[t]^2 - 2\,\mathbb{i}_{11}\,(\phi_1''[t] + \text{Sin}[\theta_1[t]]\,\psi_1''[t])\Big),$$

$$\mathrm{Sin}[2\,\phi_1[t]]\,(\mathbb{i}_{12}-\mathbb{i}_{13})\,\theta_1'[t]\,\phi_1'[t]+\mathrm{Cos}[\theta_1[t]]\,(\mathbb{i}_{11}+\mathrm{Cos}[2\,\phi_1[t]]\,(\mathbb{i}_{12}-\mathbb{i}_{13}))$$
$$\phi_1'[t]\,\psi_1'[t]+\frac{1}{2}\,(\mathrm{Sin}[2\,\theta_1[t]]\,(\mathbb{i}_{11}-\mathrm{Sin}[\phi_1[t]]^2\,\mathbb{i}_{12}-\mathrm{Cos}[\phi_1[t]]^2\,\mathbb{i}_{13})\,\psi_1'[t]^2-$$
$$2\,(\mathrm{Cos}[\phi_1[t]]^2\,\mathbb{i}_{12}+\mathrm{Sin}[\phi_1[t]]^2\,\mathbb{i}_{13})\,\theta_1''[t]+$$
$$\mathrm{Cos}[\theta_1[t]]\,\mathrm{Sin}[2\,\phi_1[t]]\,(\mathbb{i}_{12}-\mathbb{i}_{13})\,\psi_1''[t]),$$
$$-\frac{1}{2}\,\mathrm{Sin}[\theta_1[t]]\,\mathrm{Sin}[2\,\phi_1[t]]\,(\mathbb{i}_{12}-\mathbb{i}_{13})\,\theta_1'[t]^2-$$
$$\mathrm{Cos}[\theta_1[t]]^2\,\mathrm{Sin}[2\,\phi_1[t]]\,(\mathbb{i}_{12}-\mathbb{i}_{13})\,\phi_1'[t]\,\psi_1'[t]+$$
$$\mathrm{Cos}[\theta_1[t]]\,\theta_1'[t]\,(-(\mathbb{i}_{11}+\mathrm{Cos}[2\,\phi_1[t]]\,(-\mathbb{i}_{12}+\mathbb{i}_{13}))\,\phi_1'[t]+$$
$$2\,\mathrm{Sin}[\theta_1[t]]\,(-\mathbb{i}_{11}+\mathrm{Sin}[\phi_1[t]]^2\,\mathbb{i}_{12}+\mathrm{Cos}[\phi_1[t]]^2\,\mathbb{i}_{13})\,\psi_1'[t])+$$
$$\mathrm{Cos}[\theta_1[t]]\,\mathrm{Cos}[\phi_1[t]]\,\mathrm{Sin}[\phi_1[t]]\,\mathbb{i}_{12}\,\theta_1''[t]-\frac{1}{2}\,\mathrm{Cos}[\theta_1[t]]\,\mathrm{Sin}[2\,\phi_1[t]]\,\mathbb{i}_{13}\,\theta_1''[t]-$$
$$\mathrm{Sin}[\theta_1[t]]\,\mathbb{i}_{11}\,\phi_1''[t]-\mathrm{Sin}[\theta_1[t]]^2\,\mathbb{i}_{11}\,\psi_1''[t]-$$
$$\mathrm{Cos}[\theta_1[t]]^2\,\mathrm{Sin}[\phi_1[t]]^2\,\mathbb{i}_{12}\,\psi_1''[t]-\mathrm{Cos}[\theta_1[t]]^2\,\mathrm{Cos}[\phi_1[t]]^2\,\mathbb{i}_{13}\,\psi_1''[t],$$
$$-m_2\,x_2''[t],\ -m_2\,y_2''[t],\ -m_2\,(g+z_2''[t]),$$
$$\frac{1}{2}\,(-\mathrm{Sin}[2\,\phi_2[t]]\,(\mathbb{i}_{12}-\mathbb{i}_{13})\,\theta_2'[t]^2-$$
$$2\,\mathrm{Cos}[\theta_2[t]]\,(\mathbb{i}_{11}+\mathrm{Cos}[2\,\phi_2[t]]\,(\mathbb{i}_{12}-\mathbb{i}_{13}))\,\theta_2'[t]\,\psi_2'[t]+$$
$$\mathrm{Cos}[\theta_2[t]]^2\,\mathrm{Sin}[2\,\phi_2[t]]\,(\mathbb{i}_{12}-\mathbb{i}_{13})\,\psi_2'[t]^2-2\,\mathbb{i}_{11}\,(\phi_2''[t]+\mathrm{Sin}[\theta_2[t]]\,\psi_2''[t])),$$
$$\mathrm{Sin}[2\,\phi_2[t]]\,(\mathbb{i}_{12}-\mathbb{i}_{13})\,\theta_2'[t]\,\phi_2'[t]+\mathrm{Cos}[\theta_2[t]]\,(\mathbb{i}_{11}+\mathrm{Cos}[2\,\phi_2[t]]\,(\mathbb{i}_{12}-\mathbb{i}_{13}))$$
$$\phi_2'[t]\,\psi_2'[t]+\frac{1}{2}\,(\mathrm{Sin}[2\,\theta_2[t]]\,(\mathbb{i}_{11}-\mathrm{Sin}[\phi_2[t]]^2\,\mathbb{i}_{12}-\mathrm{Cos}[\phi_2[t]]^2\,\mathbb{i}_{13})\,\psi_2'[t]^2-$$
$$2\,(\mathrm{Cos}[\phi_2[t]]^2\,\mathbb{i}_{12}+\mathrm{Sin}[\phi_2[t]]^2\,\mathbb{i}_{13})\,\theta_2''[t]+$$
$$\mathrm{Cos}[\theta_2[t]]\,\mathrm{Sin}[2\,\phi_2[t]]\,(\mathbb{i}_{12}-\mathbb{i}_{13})\,\psi_2''[t]),$$
$$-\frac{1}{2}\,\mathrm{Sin}[\theta_2[t]]\,\mathrm{Sin}[2\,\phi_2[t]]\,(\mathbb{i}_{12}-\mathbb{i}_{13})\,\theta_2'[t]^2-$$
$$\mathrm{Cos}[\theta_2[t]]^2\,\mathrm{Sin}[2\,\phi_2[t]]\,(\mathbb{i}_{12}-\mathbb{i}_{13})\,\phi_2'[t]\,\psi_2'[t]+$$
$$\mathrm{Cos}[\theta_2[t]]\,\theta_2'[t]\,(-(\mathbb{i}_{11}+\mathrm{Cos}[2\,\phi_2[t]]\,(-\mathbb{i}_{12}+\mathbb{i}_{13}))\,\phi_2'[t]+$$
$$2\,\mathrm{Sin}[\theta_2[t]]\,(-\mathbb{i}_{11}+\mathrm{Sin}[\phi_2[t]]^2\,\mathbb{i}_{12}+\mathrm{Cos}[\phi_2[t]]^2\,\mathbb{i}_{13})\,\psi_2'[t])+$$
$$\mathrm{Cos}[\theta_2[t]]\,\mathrm{Cos}[\phi_2[t]]\,\mathrm{Sin}[\phi_2[t]]\,\mathbb{i}_{12}\,\theta_2''[t]-\frac{1}{2}\,\mathrm{Cos}[\theta_2[t]]\,\mathrm{Sin}[2\,\phi_2[t]]\,\mathbb{i}_{13}\,\theta_2''[t]-$$
$$\mathrm{Sin}[\theta_2[t]]\,\mathbb{i}_{11}\,\phi_2''[t]-\mathrm{Sin}[\theta_2[t]]^2\,\mathbb{i}_{11}\,\psi_2''[t]-$$
$$\mathrm{Cos}[\theta_2[t]]^2\,\mathrm{Sin}[\phi_2[t]]^2\,\mathbb{i}_{12}\,\psi_2''[t]-\mathrm{Cos}[\theta_2[t]]^2\,\mathrm{Cos}[\phi_2[t]]^2\,\mathbb{i}_{13}\,\psi_2''[t],\ l_1\,m_p$$
$$(-g\,\mathrm{Sin}[\beta[t]]+l_1\,(\mathrm{Cos}[\beta[t]]\,\mathrm{Sin}[\beta[t]]\,\gamma'[t]^2-\beta''[t])-\mathrm{Cos}[\beta[t]]\,\mathrm{Cos}[\gamma[t]]\,x_1''[t]-$$
$$\mathrm{Cos}[\beta[t]]\,\mathrm{Sin}[\gamma[t]]\,y_1''[t]-\mathrm{Sin}[\beta[t]]\,z_1''[t]),\ -\mathrm{Sin}[\beta[t]]\,l_1\,m_p$$
$$(l_1\,(2\,\mathrm{Cos}[\beta[t]]\,\beta'[t]\,\gamma'[t]+\mathrm{Sin}[\beta[t]]\,\gamma''[t])-\mathrm{Sin}[\gamma[t]]\,x_1''[t]+\mathrm{Cos}[\gamma[t]]\,y_1''[t])\}$$

system assumptions for trimming to 2D :

ψ=0

ϕ=0 (or θ=0 ??)

y=0 (-> ~ ϕ=0)

γ=0

->

new trimmed rotation matrices

test cases are :

In[74]:=
```
trimSettingGeneral = {y → 0, ϕ → 0, ψ → 0, γ → 0}
trimSettingGeneralt = {y[t] → 0, ϕ[t] → 0, ψ[t] → 0, γ[t] → 0}
trimSetting = {
  y1[t] → 0, D[y1[t], t] → 0, D[y1[t], {t, 2}] → 0,
  ϕ1[t] → 0, D[ϕ1[t], t] → 0, D[ϕ1[t], {t, 2}] → 0,
  ψ1[t] → 0, D[ψ1[t], t] → 0, D[ψ1[t], {t, 2}] → 0,
  γ[t] → 0, D[γ[t], t] → 0, D[γ[t], {t, 2}] → 0,
  y2[t] → 0, D[y2[t], t] → 0, D[y2[t], {t, 2}] → 0,
  ϕ2[t] → 0, D[ϕ2[t], t] → 0, D[ϕ2[t], {t, 2}] → 0,
  ψ2[t] → 0, D[ψ2[t], t] → 0, D[ψ2[t], {t, 2}] → 0
 }
```

Out[74]= $\{y \to 0, \phi \to 0, \psi \to 0, \gamma \to 0\}$

Out[75]= $\{y[t] \to 0, \phi[t] \to 0, \psi[t] \to 0, \gamma[t] \to 0\}$

Out[76]= $\{y_1[t] \to 0, y_1'[t] \to 0, y_1''[t] \to 0, \phi_1[t] \to 0, \phi_1'[t] \to 0, \phi_1''[t] \to 0, \psi_1[t] \to 0, \psi_1'[t] \to 0, \psi_1''[t] \to 0, \gamma[t] \to 0, \gamma'[t] \to 0, \gamma''[t] \to 0, y_2[t] \to 0, y_2'[t] \to 0, y_2''[t] \to 0, \phi_2[t] \to 0, \phi_2'[t] \to 0, \phi_2''[t] \to 0, \psi_2[t] \to 0, \psi_2'[t] \to 0, \psi_2''[t] \to 0\}$

In[66]:=
```
quadEqNominal /. trimSetting
```

Out[66]= $\{l_1\, m_p\,(\mathrm{Sin}[\beta[t]]\,\beta'[t]^2 - \mathrm{Cos}[\beta[t]]\,\beta''[t]) - (m_1 + m_p)\, x_1''[t], 0, -m_1\,(g + z_1''[t]) - m_p\,(g + l_1\,(\mathrm{Cos}[\beta[t]]\,\beta'[t]^2 + \mathrm{Sin}[\beta[t]]\,\beta''[t]) + z_1''[t]), 0, -i_{12}\,\theta_1''[t], 0, -m_2\, x_2''[t], 0, -m_2\,(g + z_2''[t]), 0, -i_{12}\,\theta_2''[t], 0, l_1\, m_p\,(-g\,\mathrm{Sin}[\beta[t]] - l_1\,\beta''[t] - \mathrm{Cos}[\beta[t]]\, x_1''[t] - \mathrm{Sin}[\beta[t]]\, z_1''[t]), 0\}$

In[67]:=
```
quadEqNominal /. trimSetting // MatrixForm // TraditionalForm
```

Out[67]//TraditionalForm=

$$\begin{pmatrix} l_1\, m_p\left(\beta'(t)^2 \sin(\beta(t)) - \beta''(t)\cos(\beta(t))\right) - (m_p + m_1)\, x_1''(t) \\ 0 \\ -m_p\left(g + l_1\left(\beta''(t)\sin(\beta(t)) + \beta'(t)^2\cos(\beta(t))\right) + z_1''(t)\right) - m_1\,(g + z_1''(t)) \\ 0 \\ -i_{12}\,\theta_1''(t) \\ 0 \\ -m_2\, x_2''(t) \\ 0 \\ -m_2\,(g + z_2''(t)) \\ 0 \\ -i_{12}\,\theta_2''(t) \\ 0 \\ l_1\, m_p\,(-g\sin(\beta(t)) - l_1\,\beta''(t) - \cos(\beta(t))\, x_1''(t) - \sin(\beta(t))\, z_1''(t)) \\ 0 \end{pmatrix}$$

In[77]:=
```
R_B^I /. trimSettingGeneralt // MatrixForm // TraditionalForm
```

Out[77]//TraditionalForm=

$$\begin{pmatrix} \cos(\theta(t)) & 0 & \sin(\theta(t)) \\ 0 & 1 & 0 \\ -\sin(\theta(t)) & 0 & \cos(\theta(t)) \end{pmatrix}$$

In[70]:=
```
R_B^I // MatrixForm // TraditionalForm
```

Out[70]//TraditionalForm=

$$\begin{pmatrix} \cos(\theta(t))\cos(\psi(t)) & \sin(\theta(t))\cos(\psi(t))\sin(\phi(t)) - \sin(\psi(t))\cos(\phi(t)) & \sin(\theta(t))\cos(\psi(t))\cos(\phi(t)) + \sin(\psi(t))\sin(\phi(t)) \\ \cos(\theta(t))\sin(\psi(t)) & \sin(\theta(t))\sin(\psi(t))\sin(\phi(t)) + \cos(\psi(t))\cos(\phi(t)) & \sin(\theta(t))\sin(\psi(t))\cos(\phi(t)) - \cos(\psi(t))\sin(\phi(t)) \\ -\sin(\theta(t)) & \cos(\theta(t))\sin(\phi(t)) & \cos(\theta(t))\cos(\phi(t)) \end{pmatrix}$$