



Contents lists available at ScienceDirect

### International Journal of Non-Linear Mechanics

journal homepage: www.elsevier.com/locate/nlm



## Geometrically nonlinear dynamic analysis of laminated composite plate using a nonpolynomial shear deformation theory



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#### ARTICLE INFO

# Keywords: Nonpolynomial shear deformation theory (NPSDT) Nonlinear dynamic analysis Green-Lagrange nonlinearity Steady state analysis Finite element analysis (FEA) Laminated composite plate

### ABSTRACT

A computationally efficient  $C^0$  finite element model in conjunction with the nonpolynomial shear deformation theory (NPSDT) is extended to examine the free and forced vibration behavior of laminated composite plates. The employed NPSDT assumes the nonlinear distribution of in-plane displacements which qualify the requirement of traction free boundary conditions at the top and bottom surfaces. The present formulation utilizes both von Kármán and Green-Lagrange type of strain-displacement relations to model the geometric nonlinearity. Using Hamilton's principle, the nonlinear governing equation of motion is derived and then discretized based on the nine-noded Lagrange element. The obtained equations are solved by utilizing unconditionally stable Newmark's scheme in conjunction with Newton-Raphson method. A damping effect in the transient analysis has been introduced in the framework of the Rayleigh damping model. The steady state forced vibration analysis has also been carried out by employing harmonic force with excitation frequency around the natural frequency. The arc-length continuation method is applied to obtained the frequency response. The present model has been validated for a wide range of problems and a detailed numerical study has been carried out for several types of boundary conditions under various types of loading with different magnitude of the load.

### 1. Introduction

Laminated composite materials are becoming one of the most effective and utilized materials for the aerospace, civil, and marine industry, etc. The preference of anisotropic composite material over the conventional isotropic material in a structure can be attributed to the various characteristic of composite materials like high specific modulus, high specific strength, and the tailoring capability for a specific application, etc. In general, composites are extensively utilized like a plate or shell form, which can be designed via three or two dimensional structural analysis. However, two-dimensional structural model which utilizes various plate theories is more preferred due to the high computation requirement of three-dimensional model. The increasing use of laminated composite structure in the industries has necessitated the need for an extensive study to understand the behavior of these structures, which leads to the development of many theories. Moreover, due to the low transverse shear moduli of the laminated composite plate, it possesses a greater transverse shear effect than a homogeneous isotropic plate. Hence, it is essential to incorporate the effect of transverse shear deformation during design, analysis, and

optimization of the laminated composite structures. Addressing this regard, many shear deformation theories have been developed.

The first effort to model the laminated composite is made through the classical laminated plate theory (CLPT), which is an extension of the classical plate theory of Kirchhoff [1]. The problem with the CLPT is that it ignores the transverse shear effect, which is only suitable for thin plates. However, for moderately thick and thick plates, the effect of transverse shear is prominent. Consequently, the prediction made through this method is not free from flaws. For example, CLPT under predicts the deflection and over predicts the frequency and buckling loads for moderately thick plates. Hence, the first-order shear deformation theory (FSDT) is proposed to overcome this problem. The FSDT is based on the Reissner [2] and the Mindlin [3] plate theory, which accommodates the transverse shear effects by taking independent field variable for rotation of transverse normal. Later, a shear correction factor is introduced to equivalently satisfy the traction-free boundary condition at the top and bottom surfaces of plates. However, the value of shear correction factor mainly depends on the material coefficients, geometry, stacking scheme, boundary conditions, and loading conditions, which is difficult to ascertain for practical problems [4]. Later

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