20 Effect of hygrothermal environment on dynamic behaviour of folded laminated composite plate

Babu Ranjan Thakur[a], *Surendra Verma*[b], *D. K. Maiti*[c] *and B. N. Singh*[d]

Department of Aerospace Engineering, IIT Kharagpur, Kharagpur, West Bengal

Abstract

In this paper, the effects of hygrothermal environment on the free vibration and transient response of one-fold and two-fold folded laminated composite plates are analysed. A nine-noded C^0 continuity finite element approach utilizing a non-polynomial shear deformation theory is employed for the analysis. To model the induced stress due to thermal loading, the Green-Lagrange non-linearity has been employed. Newmark's method is employed to integrate the spatial-temporal partial differential governing equations. The effect of the thermal environment on the natural frequency and transient response of folded composite plates has been illustrated through various examples. The obtained solution has been validated with the available solution in the literature. The effect of fibre orientation, crank angle, and boundary condition is assessed extensively and some insightful solutions and interpretations have been presented.

Keywords: Dynamic analysis, folded plate, finite element method, hygrothermal environment, nonpolynomial shear deformation theory

Introduction

Structural engineering requires various kinds of structures to be analysed for the better design and implementation, one such type is the folded plate structure. This type of structure is widely employed in the aircraft fuselages, winglets, vehicle chassis, ship hulls, buildings and bridges, etc. (Thakur et al, 2020). Further, the advent of fibre-reinforced laminated composites brought a revolution in the field of structural design and manufacturing due to their high stiffness, high strength, and low weight properties. Due to this advancement in the material properties, the utilization of laminated composite plate has increased many folds. Further, due to various operating condition and the manufacturing processes, these structures often undergo change in their hygrothermal environmental condition. And these changes have tremendous effects on the behaviour of such structure. Moreover, the hygrothermal environment profoundly affects the behaviour of composite material due to the greater susceptibility of the matrix than the fibre under the elevated hygrothermal environment. Also, it has been observed that the elastic modulus and the strength of composite laminates degrade at elevated temperatures. And hence, it is imperative to make a proper assessment of the thermal and moisture effect on the structure to have proper design and analysis to safeguard the structural life. Further, the transverse shear deformation is an inherent characteristic found in the multilayered composite structure and therefore needs to be taken care of during the analysis (Gupta and Ghosh, 2019; Thakur et al, 2020).

---

[a]brtjsk@iitkgp.ac.in; brt.iitkgp@gmail.com; [b]surendraverm2501@iitkgp.ac.in; bnsingh@aero.iitkgp.ac.in; [d]dkmaiti@aero.iitkgp.ac.in

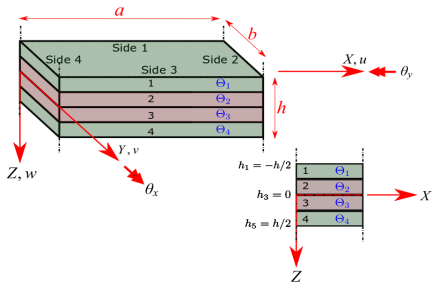


Figure 20.1 Schematic diagram of laminated composite plate

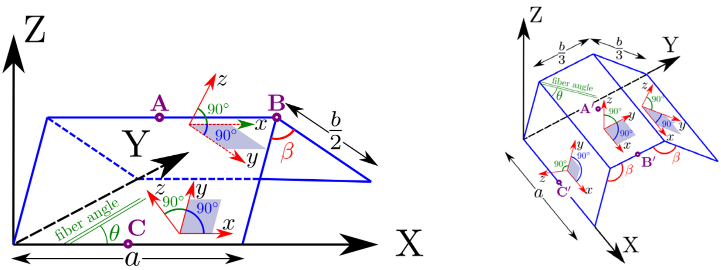


Figure 20.2 Schematic diagram of one-fold and two-fold folded plate

Recently, Thakur et al. (2021) have studied the dynamic analysis of folded laminated composite plate structure utilizing a nonpolynomial shear deformation theory. Further, it is observed in the literature that hygrothermal load has not been considered in most of the works pertaining to the folded plates. However, Das and Niyogi (2020) have considered the hygrothermal load, only for the free vibration analysis of folded plate.

It is observed from literature that the dynamic behaviour of folded laminated composite is highly susceptible to the hygrothermal environment. As per authors' knowledge, however, work has not been adequately done in this field. Moreover, most of the works available in this regard are about polynomial shear deformation theory, and particularly with first-order shear deformation theory (FSDT). Hence, the present paper deals with the dynamic analysis of folded composite plate under the hygrothermal environment using a nonpolynomial shear deformation theory.

Mathematical Formulation

A rectangular laminated composite plate with dimensions $a \times b \times h$ and consisting of k orthotropic layers stacked in sequence ($\Theta_1/\Theta_2/\Theta_3/\Theta_4$...) as shown in Figure 20.1 is considered for the mathematical formulation. Further, a one-fold and two-fold folded plate schematic diagrams have also been shown in Figure 20.2.

The displacement field model of inverse hyperbolic shear deformation theory developed by Grover et al. (2013) is employed for the formulation.

$$
\begin{aligned}
u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + f(z)\theta_x(x,y,t) \\
v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + f(z)\theta_y(x,y,t) \\
w(x,y,z,t) &= w_0(x,y,t)
\end{aligned} \tag{1}
$$

Where $f(z) = g(z) + z\Omega;\ \ g(z) = sinh^{-1}(\frac{rz}{h})$ and$\Omega = \frac{-2r}{h\sqrt{r^2+4}},\ r = 3;$;

Where, u_0, v_0, and w_0 are the mid-plane displacements; θ_x and θ_y are the shear deformations at the mid-plane. The parameter, r, is the transverse shear strain parameter, and h is the plate thickness. To employ the C_0 continuity, the artificial constraints, $-\partial w_0/\partial x = \varphi_x$, and $-\partial w_0/\partial y = \varphi_y$ are considered.

Further, the constitutive equation for the kth layer in the hygrothermal environment can be written as follows.

$$\{\sigma_m\}^{(k)} = [\bar{Q}]^{(k)}\left\{\epsilon^{(k)} - \epsilon_{th}^{(k)}\right\} = \sigma^{(k)} - \sigma_{th}^{(k)} \tag{2}$$

where, $\{\sigma\}$, $\{\sigma_m\}$, $\{\sigma_{th}\}$, $\{\epsilon\}$, $\{\epsilon_{th}\}$, and $[\bar{Q}]$ are total stress vector, mechanical stress, hygrothermal stress, total strain vector, hygrothermal strain vector, and the material constants in the global coordinate, respectively.

Equation of motion

For arbitrary space variable and admissible virtual displacement, $\delta\{u, v, w\}$, Hamilton's principle of the given system using total Lagrangian approach is written as.

$$\delta \int_{t_i}^{t_f} L dt = \int_{t_i}^{t_f} (\delta K - \delta U + \delta W_{ext}) dt = 0 \tag{3}$$

Where äK, äU, and äW_{ext} are the virtual kinetic energy, virtual strain energy and virtual work done by external forces, respectively.

Folded plate formulation

The transformation matrix, [T] as mentioned in by Thakur et al, (2020) is operated on the elemental stiffness matrix, mass matrix, and force vector to get the respective matrices and vector in the global coordinate system of the folded plate. Following the transformation, the governing equations for the free and forced vibration problem can be stated as follows.

$$[\bar{M}]\ddot{q}' + [\bar{K}]q' = 0 \tag{4}$$

$$[\bar{M}]\ddot{q}' + [\bar{K}]q' = \overline{F_m}(t) \tag{5}$$

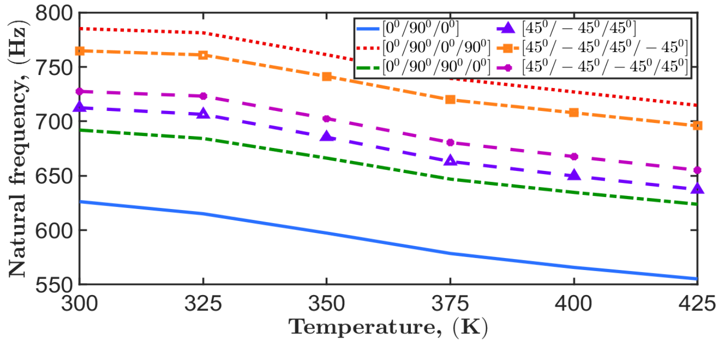
Where $\ddot{q}'$ and q' are the acceleration and displacement vector of the folded plate. $\bar{F}_m(t)$ is the transverse load applied at the surface of the plate. $[\bar{M}]$ and $[\bar{K}]$ are the global mass and stiffness matrices of the folded plate.

Rescults and discussions

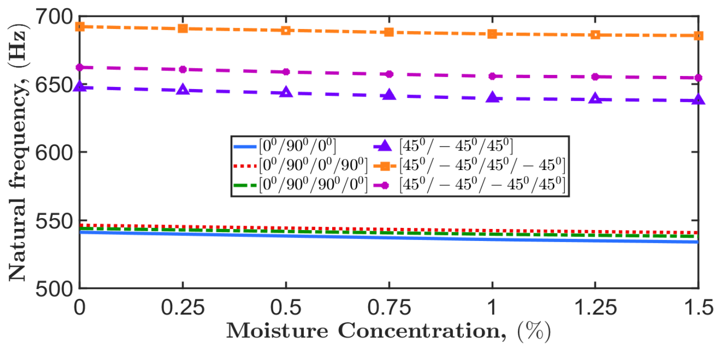
In this section, the result pertaining to free and forced vibration laminated composite folded plate are presented under the hygrothermal environment. The Newmark's scheme with $\alpha = 0.5$ and $\beta = 0.25$ is used for the transient analysis. The penalty parameter is kept as $\gamma = 10^8$. The various boundary conditions employed in this paper is mention by Thakur et al. (2021) and Niyogi et al. (1999) and has not been presented here for the sake of brevity.

Materal properties

The following material properties are used for the analysis.



(a) one-fold, crank angle 90°



(b) two-fold, crank angle 90°

Figure 20.3 Frequency variation of one-fold and two-fold folded laminated composite plate under uniform distribution of thermal for CCCC boundary condition

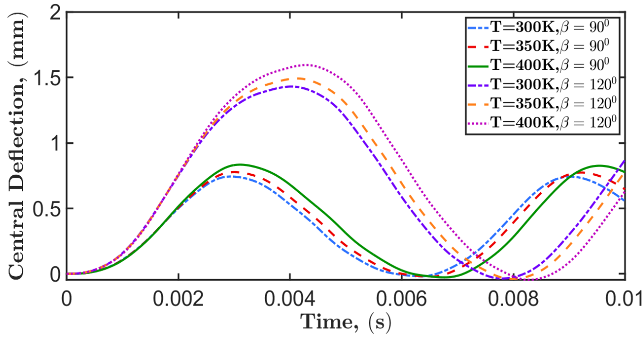
- *MM*1 Das and Niyogi (2020): *G*23 = *G*12, *G*23 = 0.5*G*12, ρ = 1600 *kg/m*3, ν12 = 0.3; other properties with respect to change in hygrothermal environment are mentioned by Das and Niyogi (2020).].

Natural frequency of one-fold and two-fold folded laminated composite plate under hygrothermal environment

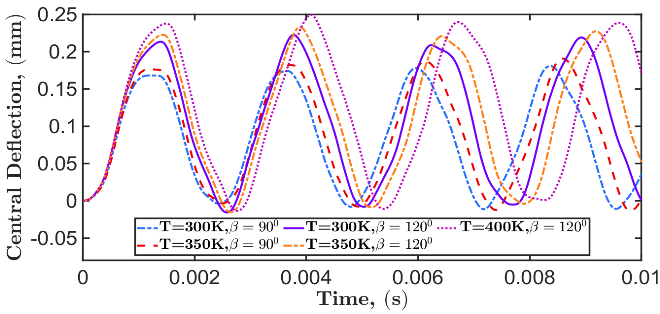
In this section, one-fold and two-fold folded laminated composite plates (as shown in Figure 20.2) having sides-length as a = b = 1.5m for one-fold, and a = 1.5m, b = 2.25m for two-fold, are considered. The plate is made up of hygrothermal dependent material properties MM1. The pattern of variation of natural frequency with respect to the temperature and moisture for cross-ply and angle-ply laminate is evaluated and shown in Figure 20.3. The side-to thickness ratio, a/h = 10 has been considered. Figure 20.3 reveals that the effect of folding on the natural frequencies for the angle-ply is less than the cross-ply.

A. Transient response of one-fold and two-fold folded laminated composite plate under hygrothermal environment

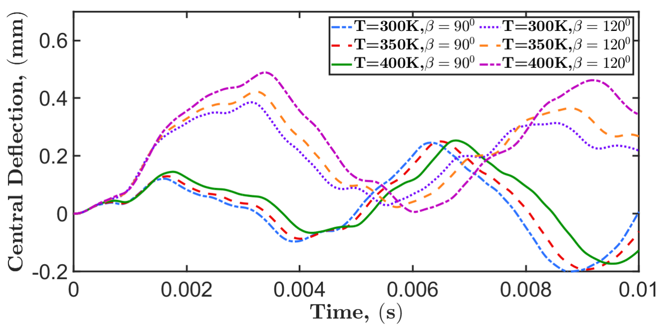
A one-fold and two-fold folded (as shown in Figure 20.2) twenty-layered cross-ply $[0/90]_{10}$ laminated composite plate with crank angle 90° and 120° is considered for the transient analysis under hygrothermal environment. The plates with dimensions, a = b



(a) Tip deflection, one-fold, FFFC



(b) Tip deflection, one-fold, FFCC



(c) Central tip (point B') deflection, two-fold, FFFC

Figure 20.4 Tip and central deflection of one-fold and two-fold folded twenty-layered $[0/90]_{10}$ laminated composite plate under FFCC boundary conditions and thermal load for applied step load of magnitude 10^5 N/m^2

= 1.5m for one-fold, and a =1.5m, b = 2.25m for two-fold, and side-to-thickness ratio, a/h = 10 are considered. The plate is made up of material MM1. The step load of magnitude P = 10^5 N/m^2 is applied for t_1 = 0.01s at temperature, T = 300K, 350K and 400K to obtain the transient response. The responses of the tip (Point B and B') deflection and central deflection (point A') for FFCC (both folded side clamped) and FFFC (one folded sided clamped) boundary conditions are shown in Figure 20.4 for one-fold and two-fold folded plate, respectively. It can be observed from Figure 20.4 that the effect of crank angle is quite significant along with the effect of boundary conditions. Apart from that the effect of fold angle and the thermal load in two-fold folded laminated composite plate is more prominent than the one-fold folded composite plate.

Conclusion

In this paper, the free and forced vibration analyses of folded laminated composite plates under the hygrothermal environment are investigated using a nonpolynomial shear deformation theory. The effect of hygrothermal load on the angle-ply laminate seems to be more susceptible than the cross-ply laminate. Further, the effect of various parameters such as thermal and moisture loads is analysed and observed that the natural frequency of three-layered and four-layered symmetric cross-ply laminate is found to be almost the same due to the absence of the extensional-bending coupling. In contrast to the cross-ply, the angle-ply laminate shows a different pattern and is highly influenced by the lamination scheme. The present work illustrates a holistic understanding of the dynamic behaviour of laminated composite folded plates under the hygrothermal environment, which can be relied upon for further benchmark analysis pertaining to higher-order shear deformation theory.

References

Das, S. and Niyogi, A. G. (2020). Free-vibration analysis of epoxy-based cross-ply laminated composite folded plates subjected to hygro-thermal loading. J. Inst. Eng. (India): C. 101(3):541–557. doi:10.1007/s40032-020-00573-8.

Gupta, A. and Ghosh, A. (2019). Isogeometric static and dynamic analysis of laminated and sandwich composite plates using nonpolynomial shear deformation theory. Compos. B: Eng. 176:107295. doi:10.1016/j.compositesb.2019.107295.

Grover, N., Singh, B., and Maiti, D. (2013). New nonpolynomial shear-deformation theories for structural behavior of laminated-composite and sandwich plates, AIAA J. 51(8):1861–1871. doi:10.2514/1.j052399.

Thakur, B. R., Verma, S., Singh, B., and Maiti, D. (2020). Maiti, Geometrically nonlinear dynamic analysis of laminated composite plate using a nonpolynomial shear deformation theory. Int. J. Non-Linear Mech. 106, 106083-106103 doi:10.1016/j.ijnonlinmec.2020.103635.

Thakur, B. R., Verma, S., Singh, B., and Maiti, D. (2020). Dynamic analysis of Folded Laminated Composite Plate Using Nonpolynomial Shear Deformation Theory. Aerospace Sci. Technol. 106083. doi: 10.1016/j.ast.2020.106083.

Thakur, B. R., Verma, S., Singh, B., and Maiti, D. (2021). Dynamic analysis of flat and folded laminated composite plates under hygrothermal environment using a nonpolynomial shear deformation theory. Compos. Struc. 274:114327. doi:10.1016/j.compstruct.2021.114327.