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Bending analysis of functionally graded plates under mechanical and thermal environment using non-polynomial shear deformation theory

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Abstract In this paper, a penalty-based C^0 finite element (FE) model for C^1 higher-order shear deformation theory (HSDT) five variables is proposed for linear bending analysis of functionally graded material (FGM) plates under mechanical and thermal loads. The rule of mixture is considered to model the material characteristics of the FGM. The principle of virtual work is utilized to obtain the weak form of governing equations for finite element formulation. A C^0 finite element mode is formulated using seven degrees-of-freedom per node to discretized the finite element mesh. The performance of proposed model is assessed by validation and comparison with available solutions in the literature. A parametric study is carried out to study the effect of design and material parameters such as material variation, side-to-thickness ratio, and aspect ratio of FGM plates. Further, a comparative study has been made between third-order shear deformation theory (TSDT) and a non-polynomial shear deformation theory (NPSDT). The present study provides a clear understanding of linear bending analysis of FGM plates.

Keywords Functionally graded plates (FGM) · Linear bending · Finite element method · Mechanical and thermal bending

1 Introduction

In recent years, the requirement from the structural components to perform in different challenging conditions has led to the development of advanced structural materials such as composites. Composite materials are vastly used in industrial and engineering sectors due to their specific-strength, specific-stiffness, durability, and better fatigue performance. However, these composite materials exhibit poor performance at high temperatures, as delamination and debonding can occur. The primary reason to these failures can be attributed through thickness mismatch of the material properties. To overcome all these limitations and to match the demands of efficient structural material, functionally graded material (FGM) has been developed [1]. The FGMs are in-homogeneous materials with varying compositions in a particular dimension to meet the structural requirement. They are typically made from a combination of metals and ceramics. Good thermal and corrosion resistance is provided by the ceramic, while the metallic component gives superior fracture toughness and weldability [2]. Apart from space application, FGMs are also used in different sectors such as the nuclear sector, biomedical application, energy sector, communication field, and manufacturing sectors. Thermal barriers in aircraft engines or re-entry capsules, rocket nozzle casings, turbine and compressor blades, and combustion chambers are some of the primary uses of FGMs in the aerospace industry [3].

Mostly, FGMs in the form of plate- and shell-like structures are utilized for various applications. In comparison

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with three-dimensional models, two-dimensional models are quite popular due to their reliable accuracy and lesser computation time. Various equivalent single-layer theories (ESLT) have been developed which include classical plate theory (CPT), first-order shear deformation theory (FSDT), and higher-order shear deformation theories (HSDT). The HSDTs can be further classified as polynomial shear deformation theories (PSDT) and non-polynomial shear deformation theories (NPSDT) based on the characteristics of shear strain function [4]. The accuracy of the PSDTs depends upon the degree of the polynomial used in it, which greatly affects the computation. On the other hand, the same level of accuracy can be achieved by using different non-polynomial shear strain functions which attract the attention of several researchers. Apart from HSDTs, quasi-3D theories are also seeking much attention. However, the present is limited to HSDTs which give reasonable accuracy for thin to moderately thick structures.

In the field of FG plates, numerous research studies have been reported to investigate the bending response of FG plates under mechanical and thermal loads using linear analysis. To explain them, Gupta and Talha [5] reviewed the recent development in the modeling and analysis of different FG materials and structures under mechanical and thermal loads. Thermo-mechanical analysis of FGM plates based on FSDT was done by Reddy [6]. Zenkour [7] conducted an investigation into the static response of plates made of functionally graded materials subjected to uniform force. The study utilized the generalized shear deformation theory and conducted comparisons with homogeneous isotropic plates. Later, Zenkour [8] conducted an investigation into the hygrothermal bending analysis of FGM plates resting on elastic foundations. Singha et al. [9] studied the bending response of FGM plates under the transverse mechanical load. They used a high precision plate bending finite element using C^1 continuous four-node quadrilateral plate. They also discussed the conditions of movable simply supported and movable clamped plates. Reddy and Berry [10] used CPT and FSDT for the axisymmetric bending analysis of plates with temperature-dependent material properties, as well as modified couple stress theory based on Hamilton's principle. Thai and Choi [11] investigated the linear bending of thick FGM plates using simplified FSDT. Sinusoidal plate theory was used by Hamidi et al. [12] in predicting the thermo-mechanical bending of FGM sandwich plates. The advantage of this technique is that, in addition to accounting for the thickness stretching effect, it has only five unknowns, as compared to seven in FSDT. Adhikari and Singh [13] used a quasi-3D theory to investigate the dynamic response of FGM plate which was resting on a two-parameter-based elastic foundation model. Mahi et al. [14] employed a novel hyperbolic shear deformation theory to analyze the bending and vibration characteristics of FGM plates. The authors provided a broad discussion on the significance of various higher-order theories. Now a days many researchers are actively working on utilizing higher-order non-polynomial shear deformation theories (NPSDT). Still, there need to be more exploration of the non-polynomial shear deformation theory for bending analysis in FGM plates. To effectively harness the advantages of these materials, it is important to have a comprehensive understanding of thermo-mechanical characteristic to explore the advantage of FGM plates. Consequently, extensive research on thermo-mechanical characteristics of FGM plates have been carried out using FSDT [16, 17], simplified HSDT [18–20], simplified quasi-3D theory [21], and higher-order shear and normal deformation theory (HSNDT) with 13 DOFs [22]. However, there have been limited studies on thermal bending analysis of FGM plates utilizing the finite element method in conjunction non-polynomial shear deformation theories.

This paper proposes a penalty-based C^0 finite element model for C^1 higher-order shear deformation theory for linear thermo-mechanical bending analysis of functionally graded plates. The focus is on the non-polynomial shear deformation theory (NPSDT). This work also compares NPSDT with TSDT. The accuracy and reliability of the present approach are verified by comparing with available numerical results in the literature. The present results can be used as benchmark results for further studies on this topic. The paper is organized as follows. In Sect. 2, a penalty-based C^0 FE formulation is summarized. In Sect. 3, the accuracy of FE formulation is assessed by validation and comparison study. The paper is concluded in Sect. 4.

2 Mathematical formulation

Figure 1 depicts a schematic diagram of a functionally graded plate with dimensions $(a \times b \times h)$ with xy as the mid-plane where z points toward ceramic constituent.

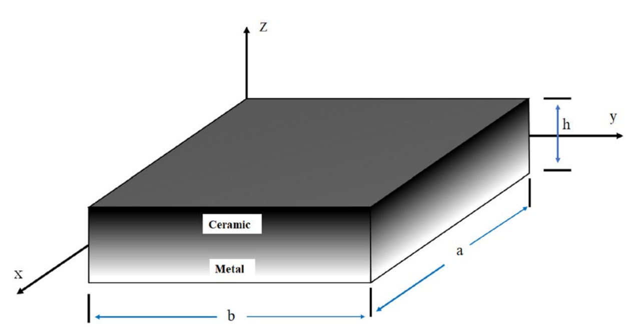

Fig. 1 Geometry and coordinates of functionally graded plate

2.1 Displacement field model

The HSDT model in a generalized form is defined as

$$\begin{aligned} u(x,y,z) &= u_0(x,y) - z\frac{\partial w_0}{\partial x} + f(z)\theta_x(x,y) \\ v(x,y,z) &= v_0(x,y) - z\frac{\partial w_0}{\partial y} + f(z)\theta_y(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \tag{1}$$

where displacement of the plate in the x, y, and z directions is denoted as u, v, and w, respectively; mid-plane displacements are denoted by u_0, v_0, and w_0; and θ_y and θ_x are the transverse shear deformations of cross section with thickness function $f(z)$. The function $f(z)$ depicts the nonlinear sense in transverse strain and plays an important role in the classification of HSDTs. If $f(z)$ is a non-polynomial function of z, then the theory is referred as a non-polynomial shear deformation theory (NPSDT) of a particular type. For this study, both third-order shear deformation theory and inverse hyperbolic shear deformation theory are considered. The expression of transverse shear function for TSDT [23] and NPSDT [24] are $f(z) = z - 4z^3/3h^2$ and $f(z) = sinh^{-1}(3z/h) - 6z/h\sqrt{13}$ with (r=3), respectively. The present formulation for the bending analysis of the FGM plate has been conducted utilizing these two transverse shear functions.

As most of the finite element commercial packages support only C^0 of field variables, the present study also restricts to this limitation. In order to reduce the C^1 continuity of w, a penalty-based approach is utilized which artificially assumes the field variables $\left(\phi_x = -\frac{\partial w_0}{\partial x}, \phi_y = -\frac{\partial w_0}{\partial y}\right)$. With the above assumption, the modified field variables now consist of seven field variables $\left(u_0, v_0, w_0, \phi_x, \phi_y, \theta_x, \theta_y\right)$.

2.2 Strain–displacement relation

The strain vector $\{\epsilon\}$ corresponding to modified field variables can be written as

$$\{\epsilon\}, = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 & z\frac{\partial}{\partial x} & 0 & f(z)\frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} & 0 & 0 & z\frac{\partial}{\partial y} & 0 & f(z)\frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 & z\frac{\partial}{\partial y} & z\frac{\partial}{\partial x} & f(z)\frac{\partial}{\partial y} & f(z)\frac{\partial}{\partial x} \\ 0 & 0 & \frac{\partial}{\partial y} & 0 & 1 & 0 & f'(z) \\ 0 & 0 & \frac{\partial}{\partial x} & 1 & 0 & f'(z) & 0 \end{bmatrix} \begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \phi_x \\ \phi_y \\ \theta_x \\ \theta_y \end{Bmatrix} \tag{2}$$

2.3 Constitutive relation

The generalized thermo-elastic stress–strain relation under plain stress condition is expressed as

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \\ \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \left(\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} - \Delta T \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \\ 0 \\ 0 \end{Bmatrix} \right) \tag{3}$$

The same can be rewritten in compact form as

$$\{\sigma\}=[Q]\left(\{\epsilon\} - \{\epsilon_{\text{th}}\}\right) = [Q](\{\epsilon\} - \{\alpha\}\Delta T)$$

where $\{\sigma\}$, $\{\epsilon\}$, and $\left[\overline{Q}\right]$ are the stress vector, the strain vector, and the materials matrix, respectively. The thermal expansion coefficient vector is represented by $\{\alpha\}$ with ΔT being the change in the temperature. As a result of the symmetry in the FG materials $Q_{21} = Q_{12}$ and $Q_{66} = Q_{44} = Q_{55}$ are considered. Each material matrix coefficient can be written as

$$Q_{11} = \frac{E}{1-\nu^2}; Q_{12} = \frac{\nu E}{1-\nu^2}; Q_{66} = \frac{E}{2(1+\nu)}$$

where $E(z)$ and $\nu(z)$ represent the Young's modulus and Poisson's ratio which are modeled using the simple rule of mixture as $E(z) = \left(E_c - E_m\right)\left(\frac{2z}{h} - 1\right)^n + E_m$ and $\nu(z) = \left(\nu_c - \nu_m\right)\left(\frac{2z}{h} - 1\right)^n + \nu_m$. Here, E_m and E_c represent the metallic and ceramic constituents, respectively. The same stands for Poisson's ratios ν_m and ν_c.

For static analysis of plates, the governing equations of equilibrium are derived from the principle of virtual work and the same is written as follows

$$\delta U + \delta U_\gamma - \delta W = 0 \tag{4}$$

where δU, δU_γ, and δW represent the virtual strain energy, strain energy caused by the imposition of an artificial constraint, and work done by the external force, respectively.

2.4 Finite element formulation

In present study, a nine-nodded isoparametric Lagrange element is utilized for finite element discretization as shown in Fig. 2. Following the definition of isoparametric mapping, the same basis functions are utilized for interpolation of plate displacement $\left\{u_0, v_0, w_0, \phi_x, \phi_y, \theta_x, \theta_y\right\}$ and geometry $\{x, y\}$, and the same is expressed as

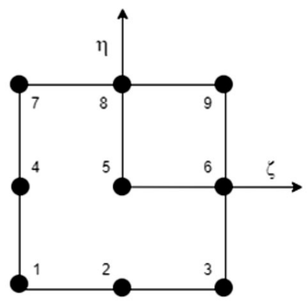

Fig. 2 Nine-nodded isoparametric finite element

$$\{q\} = \sum_{i=1}^{9} N_i\{q_i\}; \quad x = \sum_{i=1}^{9} N_i x_i; \quad y = \sum_{i=1}^{9} N_i y_i \tag{5}$$

where $\{q_i\} = \{u_{0i}, v_{0i}, w_{0i}, \phi_{xi}, \phi_{yi}, \theta_{xi}, \theta_{yi}\}$ and N_i are the displacement vector and shape function corresponding to i^{th} node.

Using Eqs. 1, 2,3, 5, the governing equations of static equilibrium are obtained by eliminating the virtual displacement $\{\delta q\}$ from Eqs. 4. The final set of equations is obtained as

$$\left([K] + \left[K_\gamma\right]\right)\{d\} - \{F_{th}\} = \{F_m\} \tag{6}$$

where $[K]$ and $\left[K_\gamma\right]$ are the structural stiffness matrix and stiffness due to artificial constraint, respectively; $\{F_m\}, \{F_{th}\}$, and $\{d\}$ are the applied mechanical load, thermal load, and elemental displacement vector, respectively.

3 Numerical results and discussion

In this section, several examples regarding linear thermo-mechanical bending analysis are solved to assess the accuracy and effectiveness of proposed FE formulation. The numerical study starts with preliminary validation of the proposed formulation against the available results in the literature. After the validation step, the parametric study is conducted to study the influence of geometric and material parameters. For numerical calculation, both third-order shear deformation theory (TSDT) and inverse hyperbolic shear deformation theory (NPSDT) are considered. The expression of transverse shear function for TSDT [23] and NPSDT [24] is $f(z) = z - 4z^3/3h^2$ and $f(z) = sinh^{-1}(3z/h) - 6z/h\sqrt{13}$ with (r=3), respectively. For numerical integration of the matrices and vector, selective Gauss–Legendre quadrature rule is employed with 2×2 for transverse shear energy and penalty stiffness matrices and rest remaining components are evaluated using 3×3 sampling points.

Table 1 Normalized deflection $\bar{w} = \frac{10E_c h^3}{P_0 a^4} w(\frac{a}{2}, \frac{b}{2})$ of simply supported square FGM plates subjected to sinusoidal transverse mechanical load

n	a/h	HSNDT [13]	FEM-TSDT*	FEM-NPSDT*
1	4	0.7284	0.7284	0.7267
	10	0.5889	0.5885	0.5887
	100	0.5625	0.5625	0.5625
4	4	1.1611	1.161	1.1617
	10	0.8817	0.8817	0.8820
	100	0.8287	0.8287	0.8287
10	4	1.3916	1.3916	1.3862
	10	1.0088	1.0089	1.0082
	100	0.9361	0.9362	0.9362

* represents present results

3.1 Validation problem under mechanical load

In order to establish the validation of proposed FE formulation, a result from Adhikari et al. [13] is replicated. This problem tries to study the influence of side-to-thickness ratio a/h and power index n on deflection response of simply supported (SSSS) square FG plates under sinusoidal transverse mechanical load $P = P_0 \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$. Material constituents for this problem are aluminum (E_m=70 GPa and n= 0.3) and alumina (E_c= 380 GPa and n= 0.3). Table 1 shows the comparison of normalized central deflection $\bar{w} = \frac{10E_c h^3}{P_0 a^4} w(\frac{a}{2}, \frac{b}{2})$ obtained by the present FE models with higher-order shear and normal deformation theory (HSNDT) solutions by Adhikari et al. [13]. From the above comparison, a good correlation between the present solution and published literature is observed. It is observed that the increase in the power index n increases the deflection, while increase in the side-to-thickness ratio a/h decreases the normalized deflection.

3.2 Validation problem under thermal load

In this section, a simply supported square FGM (Al/Al_2O_3) plate subjected to sinusoidal thermal load $T(x,y,z) = T_1\left(\frac{2z}{h}\right)\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$ is analyzed to illustrate the accuracy of the present FE model for linear thermal bending analysis. The material properties are identical to the one defined in previous problem along with $\alpha_m = 23 \times 10^{-6}$ //K and $\alpha_c = 7.4 \times 10^{-6}$ //K) as thermal expansion coefficient of metal and ceramic constituents, respectively. Normalized form of deflection is calculated by $\hat{w} = w/\left(h\alpha_c T_1\right)$. Table 2 presents the value of normalized central deflection obtained using present FE models for different side-to-thickness ratio a/h and power index n along with results using sinusoidal shear deformation theory (SSDT) by Xiaohui and Zhen [25].

Table 2 Normalized central deflection $\hat{w} = w_0\left(\frac{a}{2}, \frac{b}{2}\right)/\left(h\alpha_c T_1\right)$ of simply supported square FGM (Al/Al_2O_3) plates subjected to under thermal load $T(x, y, z) = T_1\left(\frac{2z}{h}\right)\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$

n	a/h	SSDT [25]	FEM-TSDT*	FEM-NPSDT*
0	5	3.227	3.2929	3.29299
	10	13.11	13.172	13.172
0.5	5	5.286	5.394	5.39384
	10	21.48	21.586	21.5858
1	5	5.842	5.9913	5.98961
	10	23.86	24.0107	24.0089

*Represents present results

It can be observed that the normalized deflection increases with increases in *a*/*h* and *n*. For validation study, the present obtained FE results for linear bending analysis of the FG plates are compared with the data obtained with solutions of higher-order shear and normal deformation theory (HSNDT) [13] and HSDT [26].

3.3 Parametric study

After establishing the accuracy of present method, parametric study is carried out to study the influence of the side-to-thickness ratio *a*/*h* and aspect ratio *a*/*b* on the deflection $\hat{w} = \frac{w\left(\frac{a}{2}, \frac{b}{2}, 0\right)}{h\alpha_c T_1}$ of the FGM (Al/Al_2O_3) plates under thermal load $T(x, y, z) = T_1\left(\frac{2z}{h}\right)\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$. The material properties are taken similar to problem in previous section. Variation of central deflection for different power index $n = 0, 0.5, 1.5, 2, 2.5, 3$ is plotted in Fig. 3 with $a/b = 1$. It can be observed that the central deflection increases with increase in side-to-thickness *a*/*h* due to the increase in the magnitude of end-moment.

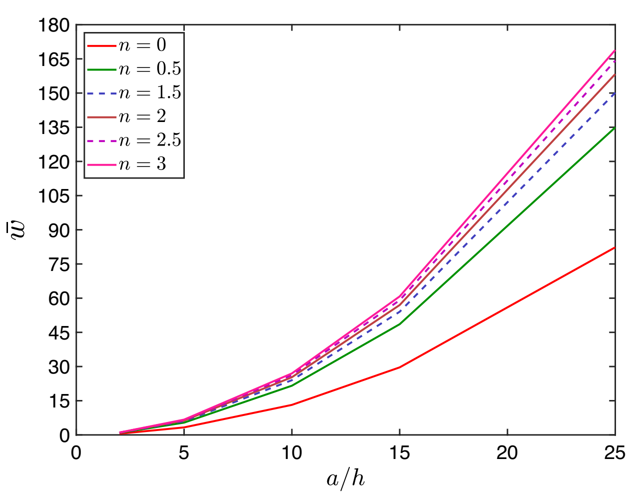

Fig. 3 Effect of thickness ratio (a/h) on normalized deflection of FGM plates under sinusoidal thermal load

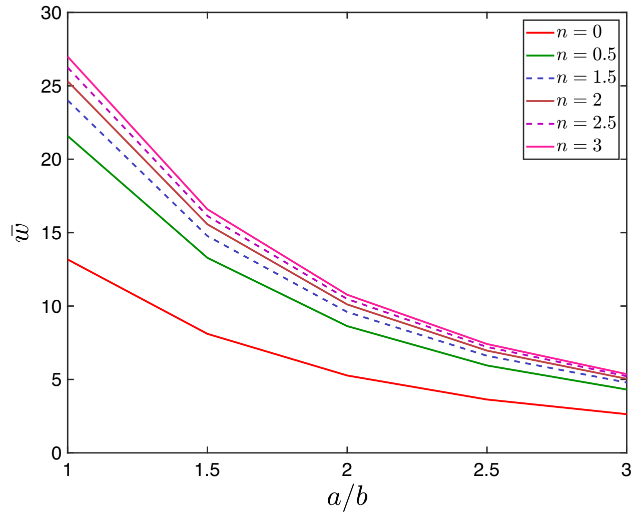

Fig. 4 Effect of aspect ratio (a/b) on central deflection of FGM plates under sinusoidal thermal load

In a similar manner, Fig. 4 illustrates the effect of aspect ratio (*a*/*b*) on the central deflection $w\left(\frac{a}{2}, \frac{b}{2}\right)$ of the FGM (Al/Al_2O_3) plates subjected to same thermal load ($T(x, y, z) = T_1\left(\frac{2z}{h}\right)\sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{b}\right)$. It is interesting to note that the central deflection of the plate decreases with increase in the aspect ratio *a*/*b* due to decrease in the effective end-moment for increase length.

4 Conclusion

This study proposes a penalty-based C^0 finite element formulation for C^1 higher-order shear deformation theory to analyze the linear thermo-mechanical response of functionally graded plates. A generalized HSDT temperate is considered, which can be reduced to particular shear deformation theory by substituting the transverse shear strain function $f(z)$. In this study, a nine-nodded isoparametric is utilized to carry out the validation and parametric study to study the influence of different geometric and material parameters on deflection of functionally graded plates under mechanical and thermal loads. The effectiveness of proposed finite element model has been assessed by validation and comparison studies with other available results in the literature. Due to the fact that alumina (ceramic) has a greater stiffness than aluminum (metal), the central displacement is lowest for a pure alumina plate, while it is highest for the pure aluminum plate. It is also noted that the center displacement grows with an increase in power index *n*. The present research work has the potential to give a stronger basis for comprehending the extensive breadth of the non-polynomial shear deformation theory (NPSDT) which can be useful for future investigations of linear structural analysis of FMG plates.

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